## Geometry Honors Chapter 1:

## Foundations for Geometry



Unit 1: Vocabulary

| 1) | point |  |
| :--- | :--- | :--- |
| 2) | line |  |
| 3$)$ | plane |  |
| 4) | segment |  |
| 5) | endpoint |  |
| 6) | ray |  |
| 7) | collinear |  |
| 8) | coplanar |  |
| 9$)$ | opposite ray |  |
| distance along a line |  |  |
| length |  |  |
| 14) | midpoint |  |
| 12) | congruent segments |  |


| 15) | (to) bisect |  |
| :---: | :---: | :---: |
| 16) | segment bisector |  |
| 17) | angle |  |
| 18) | vertex |  |
| 19) | acute angle |  |
| 20) | obtuse angle |  |
| 21) | straight angle |  |
| 22) | congruent angles |  |
| 23) | angle bisector |  |
| 24) | construct(ion) |  |
| 25) | adjacent angles |  |
| 26) | linear pair |  |
| 27) | complementary angles |  |
| 28) | supplementary angles |  |
| 29) | vertical angles |  |

## Day 1: Understanding Points, Lines, and Planes

G.CO. 1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Warm-Up Solve for t :
$5 t-2(t-5)=19$

The most basic figures in geometry are undefined terms, which cannot be defined by using other figures. The undefined terms point, line, and plane are the building blocks of geometry.

## Undefined Terms

| TERM | NAME | DIAGRAM |
| :--- | :--- | :---: |
| A point names a location <br> and has no size. It is <br> represented by a dot. | A capital letter <br> point $P$ |  |
| A line is a straight path <br> that has no thickness and <br> extends forever. | A lowercase letter or <br> two points on the line <br> line $\ell \overleftrightarrow{X Y}$ or $\overleftrightarrow{Y X}$ | $\leftarrow$ |
| A plane is a flat surface <br> that has no thickness and <br> extends forever. | A script capital letter <br> or three points not <br> on a line <br> plane $\mathcal{R}$ or plane $A B C$ |  |

Points that lie on the same line are collinear. $K, L$, and $M$ are collinear. $K, L$, and $N$ are noncollinear.


Points that lie on the same plane are coplanar. Otherwise they are noncoplanar.

## Sketches

| A line that is contained (lies in) in a plane | A line that intersects a plane in one point |
| :--- | :--- |
| Coplanar points |  |
|  |  |


| Segments and Rays |  |  |
| :---: | :---: | :---: |
| DEFINITION | NAME | DIAGRAM |
| A segment, or line segment, is the part of a line consisting of two points and all points between them. | The two endpoints $\overline{A B}$ or $\overline{B A}$ | $\rightarrow \rightarrow$ |
| An endpoint is a point at one end of a segment or the starting point of a ray. | A capital letter $C$ and $D$ | $\stackrel{\bullet}{C}$ |
| A ray is a part of a line that starts at an endpoint and extends forever in one direction. | Its endpoint and any other point on the ray $\overrightarrow{R S}$ |  |
| Opposite rays are two rays that have a common endpoint and form a line. | The common endpoint and any other point on each ray $\overrightarrow{E F}$ and $\overrightarrow{E G}$ | $\leftarrow F \quad G \quad G$ |

Model Problems Use the diagram at right.

1) Name a point.
2) Name the line that goes through point $E$ in two ways.
3) Name a segment.
4) Name three collinear points.
5) Name three non-collinear points.
6) Name the intersection of $\overleftrightarrow{E C}$ and the segment not on $\overleftrightarrow{E C}$.

7) Name the plane shown in the diagram.

## Exercise

1) Name a point.
2) Name the line that goes through point Z in three ways.
3) Name a segment.
4) Name three coplanar points.
5) Name three non-collinear points.
6) Name the intersection of line $m$ and $\overleftrightarrow{Y Z}$.
7) Name the plane shown in the diagram.
8) Name the points that determine this plane.
9) Name two lines that intersect line $m$.
10) Name a line that does not intersect line $m$.


## Postulates about Lines and Points

A postulate, or axiom, is a statement that is accepted as true without proof. Postulates about points, lines, and planes help describe geometric properties.

| Postulate | Sketch | Illustration |
| :--- | :--- | :--- |
| Two points determine a line. |  | Any two points are collinear. |
| Three points determine a plane. |  | Any three points are coplanar. <br> Think of a wobbly chair. It <br> will be stable if any three legs <br> are touching the ground. |


| If two points lie in a plane, then <br> the line containing those points <br> will lie in that plane too. |  | If you draw two points on a <br> piece of paper, the line that <br> connects them is on the paper <br> too. |
| :--- | :--- | :--- |
| The intersection of two lines is a <br> point. |  | - Street intersection <br> - Pivot of scissors <br> - The letter "X" |
| The intersection of two planes is <br> a line. |  | - The crease of a book <br> - The edge of a door <br> - Aver valley <br> The corner where two <br> walls meet |

## Check for Understanding

Two flat walls meet in the corner of a classroom. Which postulate best describes this situation?
(A) Through any three noncollinear points there is exactly one plane.
(B) If two points lie in a plane, then the line containing them lies in the plane.
(C) If two lines intersect, then they intersect in exactly one point.
(D) If two planes intersect, then they intersect in exactly one line.

Model Problems Draw and label each of the following.
A. Plane $\mathcal{H}$ that contains two lines that intersect at M
B. $\overleftrightarrow{S T}$ intersecting plane $\mathcal{M}$ at R

## Drawing Hints:

lines - have arrows on both sides.
rays - arrow on one side, first letter is endpoint
planes - are flat surfaces
points - are always

## Check for Understanding

Sketch a figure that shows two lines intersect in one point in a plane, but only one of the lines lies in the plane.

## Lesson Quiz

1. Two opposite rays
2. A point on $\overleftrightarrow{B C}$.
3. The intersection of plane $N$ and plane $T$
4. A plane containing $E, D$, and $B$.


Draw each of the following.
5. a line intersecting a plane at one point
6. a ray with endpoint $P$ that passes through $Q$

## Homework

## GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. Give an example from your classroom of three collinear points.
2. Make use of the fact that endpoint is a compound of end and point and name the endpoint of $\overrightarrow{S T}$.

1 Use the figure to name each of the following.
3. five points
4. two lines
5. two planes
6. point on $\overleftrightarrow{B D}$

2 Draw and label each of the following.

7. a segment with endpoints $M$ and $N$
8. a ray with endpoint $F$ that passes through $G$

SEE EXAMPLE 3 Use the figure to name each of the following.
p. 7
9. a line that contains $A$ and $C$
10. a plane that contains $A, D$, and $C$


SEE EXAMPLE 4 Sketch a figure that shows each of the following.
p. 8 11. three coplanar lines that intersect in a common point
12. two lines that do not intersect

## Day 2: Measuring and Constructing Segments

G.CO. 1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

## Warm-Up

a. Draw and label the following:
i. A line containing points X and Y
ii. A pair of opposite rays that both contain point $R$
b. Campers often use a cooking stove with three legs. Why might they prefer this design to a stove that has four legs?

The distance along a line between any two points is the absolute value of the difference of the coordinates. The coordinates can be measured in a variety of units, such as inches or centimeters.

If the coordinates of points $A$ and $B$ are $a$ and $b$, then the distance between $A$ and $B$ is $|a-b|$ or $|b-a|$. The distance between $A$ and $B$ is also called the length, or measure of $\overline{A B}$, or $A B$.

## Exercise

Find each length.

A. $B C$
B. $A C$
$\square$
Congruent Segments Congruent segments are segments that have the same length. In the diagram, $P Q=R S$, so you can write $\overline{P Q} \cong \overline{R S}$. This is read as "segment $P Q$ is congruent to segment $R S$."

Tick marks are used in a figure to show congruent segments.


## Constructing Congruent Segments

A construction is a special drawing that only uses a compass and a straightedge. Constructions can be justified by using geometric principles to create figures.

Model Problem Construct a segment congruent to $\overline{A B}$.


Exercise Construct a segment congruent to $\overline{A B}$. Then answer the questions below.


Think About It. Why does this construction result in a line segment with the same length as $\overline{A B}$ ?

## Betweenness

In order for you to say that a point $B$ is between two points $A$ and $C$, all three points must lie on the same line, and $A B+B C=A C$.

## Postulate 1-2-2 Segment Addition Postulate

If $B$ is between $A$ and $C$, then $A B+B C=A C$.


Model Problems
$A . \quad M$ is between $N$ and $O$. Find $N O$.

B. H is between I and J . If $\mathrm{HI}=3.9$ and $\mathrm{HJ}=6.2$, find IJ .

## Exercise

1. $E$ is between $D$ and $F$. Find $D F$.

2. $H$ is between $I$ and $J$. If $I J=25$ and $H I=13$, find $H J$.

## Midpoint

The midpoint $M$ of $\overline{A B}$ is the point that bisects, or divides the segment into two congruent segments. If $M$ is the midpoint of $\overline{A B}$, then $A M=M B$. So if $A B=6$, then $A M=3$ and $M B=3$.

Model Problems

A. $D$ is the midpoint of $\overline{E F}, E D=4 x+6$, and $D F=7 x-9$. Find $E D, D F$, and $E F$.
B. $B$ is the midpoint of $\overline{A C} . A B=8 v$, and $A C=2 v+42$. What is $B C$ ?


## Exercise

1. H is the midpoint of $\overline{I J}$.
i. If $\mathrm{IJ}=18$, find HI and HJ .
ii. If $\mathrm{IH}=10$, find HJ and IJ .

2. E is the midpoint of $\overline{\mathrm{DF}} . \mathrm{DE}=2 \mathrm{x}+4$ and $\mathrm{EF}=3 \mathrm{x}-1$. Find $\mathrm{DE}, \mathrm{EF}$, and DF .
$\qquad$
$\mathrm{EF}=$
DF =
3. X is the midpoint of $\overline{A T}$. If $\mathrm{AX}=4 \mathrm{x}$ and $\mathrm{AT}=3 \mathrm{x}+25$, find $\mathrm{AX}, \mathrm{XT}$, and AT .
4. S is between R and T . Does that mean that S must be a midpoint? Explain and sketch an appropriate diagram of $\overline{R T}$.

## Homework Day 2 Complete in notebook.

Holt - pg: 17-19/\#s 12, 14 - 15, 17-20, 22, 29, 32, 41
McDougal and Littell - pg: 33-35/ \#11, 21
G.CO. 1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

## Warm-Up

M is the midpoint of segment $\overline{A D}$. If $\mathrm{AM}=\mathrm{x}+3$ and $\mathrm{AD}=\mathrm{x}^{2}+6$, find: a) the value of $x$. b) $M D$

An angle is a figure formed by two rays, or sides, with a common endpoint called the vertex (plural: vertices).
You can name an angle several ways: by its vertex, by a point on each ray and the vertex, or by a number.
The set of all points between the sides of the angle is the interior of an angle. The exterior of an angle is the set of all points outside the angle.

## Model Problem

Name the angle at right in four ways.


Note: You cannot name an angle just by its vertex if the point is the vertex of more than one angle. In this case, you must use all three points to name the angle, and the middle point is always the vertex.

Wrong:
Right:


## Exercise

| 1) $\quad$ Draw and label $\angle D E F$ below. | 2)Draw and label $\angle Q R S$ with $\overrightarrow{R T}$ in the <br> interior of the angle. |
| :--- | :--- |

3) Write the different ways you can name each given angle in the diagram.
4) 


$\angle S V T$ : $\qquad$
$\angle S V R$ : $\qquad$
$\angle R V T$ : $\qquad$
2)

$\angle 3$ : $\qquad$
$\angle 4$ : $\qquad$
$\angle D V F$ : $\qquad$

## Measuring Angles

The measure of an angle is usually given in degrees. Since there are $360^{\circ}$ in a circle, one degree is $1 / 360$ of a circle.

When referring to the degree measure of angles, we write:

$$
m \angle A B C
$$

which is read, "the measure of angle ABC ."


## Types of Angles

## Types of Angles

$\xrightarrow{\text { Acute Angle }}$

Measures greater than $0^{\circ}$ and less than $90^{\circ}$

Measures $90^{\circ}$
measures


?
Measures greater than $90^{\circ}$ and less than $180^{\circ}$


Formed by two opposite rays and meaures $180^{\circ}$

## Congruent Angles

Congruent angles are angles that have the same degree measure.
In the diagram, $\mathrm{m} \angle A B C=\mathrm{m} \angle D E F$, so you can write $\angle A B C \cong \angle D E F$. This is read as "angle ABC is congruent to angle $D E F$."

Arc marks are used to show that the two angles are congruent.


## Check for Understanding

In the diagram, assume $\angle D E F$ is congruent to $\angle F E G$.

1) Explain what this means in your own words.
2) Mark the diagram at right to show this congruence.
3) Write " $\angle D E F$ is congruent to $\angle F E G$ " using symbols.

4) Write "the measure of $\angle D E F$ equals the measure of $\angle F E G$ " in symbols.

## Angle Addition Postulate

## Postulate 1-3-2 Angle Addition Postulate

If $S$ is in the interior of $\angle P Q R$, then $\mathrm{m} \angle P Q S+\mathrm{m} \angle S Q R=\mathrm{m} \angle P Q R$.
( $\angle$ Add. Post.)


Note that this is similar to segment addition:
"PART + PART = WHOLE"

Model Problem Mark up the diagram appropriately. Then answer the question below.
In the accompanying diagram, $\mathrm{m} \angle D E G=115^{\circ}, \mathrm{m} \angle D E F=2 \mathrm{x}-1^{\circ}$, $\mathrm{m} \angle G E F=3 \mathrm{x}+1^{\circ}$. Find $\mathrm{m} \angle D E F$ and $\mathrm{m} \angle G E F$.

## Exercise

Find $m \angle K L M$ if $m \angle K L B=26^{\circ}$
and $m \angle B L M=60^{\circ}$.


Find $m \angle W D C$ if $m \angle E D C=145^{\circ}$ and $m \angle E D W=61^{\circ}$.

$m \angle H G F=16 x+4, m \angle E G F=110^{\circ}$, and $m \angle H G E=3 x+11$. Find $x$.

$m \angle F C D=x+41, m \angle B C F=x+78$, and $m \angle B C D=95^{\circ}$. Find $x$.


## The Angle Bisector

An angle bisector is a ray that divides an angle into two congruent angles.

Given: $\overrightarrow{J K}$ bisects $\angle L J M$
Conclusion: $\angle L J K \cong \angle M J K$


Model Problems
A. $\overrightarrow{K M}$ bisects $\angle J K L, \mathrm{~m} \angle J K M=(4 x+6)^{\circ}$, and $\mathrm{m} \angle M K L=(7 x-12)^{\circ}$. Find $\mathrm{m} \angle J K M$.

B. Given: $\overrightarrow{Q S}$ bisects $\angle P Q R$.

1. Sketch and label $\angle P Q R$ first, then draw ray $\overrightarrow{Q S}$ from point Q .
2. $\mathrm{m} \angle P Q S=(5 y-1)^{\circ}$, and $\mathrm{m} \angle P Q R=(8 y+12)^{\circ}$. Find $\mathrm{m} \angle P Q S$.

## Exercise

$\overrightarrow{B D}$ bisects $\angle A B C$. Find $\mathrm{m} \angle A B D$ if $\mathrm{m} \angle A B D=(6 \mathrm{x}+4)^{\circ}$ and $\mathrm{m} \angle D B C=(8 \mathrm{x}-4)^{\circ}$.


Homework Day 3 Complete in notebook.
Holt: pages 24-27 \#'s 8, 10, 11, 18, 19-22 all, 29, 31, 32, 38, 44, 45, 47
Mc Dougal, Littell: pages 16-17 \#'s 15, 17, 21 and page 27 \# 14

## Day 4: Measuring and Constructing Angles

G.CO. 1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
G.CO. 12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.

## Warm-Up

$\overrightarrow{N T}$ bisects $\angle M N S$. Find $\mathrm{m} \angle T N S$ if $\mathrm{m} \angle M N T=(2 \mathrm{x}-30)^{\circ}$ and $\mathrm{m} \angle D B C=(2 \mathrm{x})^{\circ}$.


## Independent Practice Algebraic Review

$\overrightarrow{Q S}$ bisects $\angle P Q T$.

1. If $m \angle P Q T=60$ and $m \angle P Q S=4 x+14$, find the value of $x$.

2. If $m \angle P Q S=3 x+13$ and $m \angle S Q T=6 x-2$, find $m \angle P Q T$.


## Constructing Congruent Angles

Task: Construct an angle with vertex $X$ that has the same measure as $S$.

A In the space below, use a straightedge to draw a ray with endpoint $X$.

## $x \cdot$



B Place the point of your compass on $S$ and draw an arc that intersects both sides of the angle. Label the points of intersection $T$ and $U$.

C Without adjusting the compass, place the point of the compass on $X$ and draw an arc that intersects the ray. Label the intersection $Y$.


D Place the point of the compass on $U$ and open it to the distance $T U$.

E Without adjusting the compass, place the point of the compass on $Y$ and draw an arc. Label the intersection with the first arc $Z$.

F Use a straightedge to draw $\overrightarrow{X Z}$.

You Try:

## $X \cdot$



Why does this construction work?

## Practice

## Construct an angle with the same measure as the given angle.


2.
3. $\longrightarrow$

Think About It. If the angle you construct has longer sides than the original angle, can the two angles still have the same measure? Explain. $\qquad$
$\qquad$

## Constructing an Angle Bisector

## Task:

Construct the bisector of $\angle M$. Work directly on the angle at right.

A Place the point of your compass on point $M$. Draw an arc that intersects both sides of the angle. Label the points of intersection $P$ and $Q$.


B Place the point of the compass on $P$ and draw an arc in the interior of the angle.

C Without adjusting the compass, place the point of the compass on $Q$ and draw an arc that intersects the arc from Step B. Label the intersection of the arcs $R$.

D Use a straightedge to draw $\overrightarrow{M R}$.


Why does this construction work?

Practice Construct the bisector of each angle.
4.

5.

6.


## Practice (continued)

7) Explain how you can use a compass and straightedge to construct an angle that has twice measure of $\angle A$. Then do the construction in the space provided.

8) Explain how you can use a compass and straightedge to construct an angle that has $1 / 4$ the measure of $\angle B$. Then do the construction in the space provided.

9) Given the diagram at right. Write the number that names the same angle. If the angle does not exist in the diagram, write "does not exist." Some angles may be used more than once.
a) $\angle P M O$ $\qquad$
b) $\angle M N O$ $\qquad$
c) $\angle M P O$ $\qquad$
d) $\angle M O P$ $\qquad$
e) $\angle R P Q$ $\qquad$
f) $\angle Q R P$ $\qquad$
g) $\angle M P R$ $\qquad$
h) $\angle N M O$ $\qquad$
i) $\angle O N M$ $\qquad$
j) $\angle N O M$ $\qquad$


ALGEBRA In the figure $\overrightarrow{B A}$ and $\overrightarrow{B C}$ are opposite rays. $\overrightarrow{B F}$ bisects $\angle C B E$.
3. If $m \angle E B F=6 x+4$ and $m \angle C B F=7 x-2$, find $m \angle E B F$.

4. If $m \angle 3=4 x+10$ and $m \angle 4=5 x$, find $m \angle 4$.

5. If $m \angle 3=6 y+2$ and $m \angle 4=8 y-14$, find $m \angle C B E$.

6. Let $m \angle 1=m \angle 2$. If $m \angle A B E=100$ and $m \angle A B D=2(r+5)$, find $r$ and $m \angle D B E$.


Draw each of the following.
7. a line that contains $\overrightarrow{A B}$ and $\overrightarrow{C B}$
8. two different lines that intersect $\overline{M N}$
9. a plane and a ray that intersect only at $Q$
10. Critical Thinking Can an obtuse angle be congruent to an acute angle? Why or why not?
11. Short Response If an obtuse angle is bisected, are the resulting angles acute or obtuse? Explain.
G.CO. 1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

## Warm-Up

$\overrightarrow{R T}$ bisects $\angle Q R S$. If $\mathrm{m} \angle Q R T=x+15, m \angle T R S=2 y+10$, and $m \angle Q R S=2 x+2 y$, find the value of x and y .

Many pairs of angles have special relationships. Some relationships are because of the measurements of the angles in the pair. Other relationships are because of the positions of the angles in the pair.

## Pairs of Ancles

Adjacent angles are two angles in the same plane with a common vertex and a common side, but no common interior points. $\angle 1$ and $\angle 2$ are adjacent angles.

A linear pair of angles is a pair of adjacent angles whose noncommon sides are opposite rays. $\angle 3$ and $\angle 4$ form a linear pair.


Vertical angles are two nonadjacent angles formed by two intersecting lines.
$\angle 1$ and $\angle 3$ are vertical angles, as are $\angle 2$ and $\angle 4$.
Vertical angles are always congruent.



## Exercise

For \#1-2, tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

1) $\angle A E B$ and $\angle B E D$

2) $\angle A E B$ and $\angle D E C$
3) Using the diagram at right, name:
a) a pair of vertical angles
b) two angles that form a linear pair
c) two adjacent angles that do not form a linear pair


## Complementary and Supplementary Angles

## Complementary and Supplementary Ancles

Complementary angles are two angles whose measures have a sum of $90^{\circ}$. $\angle A$ and $\angle B$ are complementary.

Supplementary angles are two angles whose measures have a sum of $180^{\circ}$. $\angle A$ and $\angle C$ are supplementary.


We say that $\angle A$ is the complement of $\angle B$ and the supplement of $\angle C$.

If two angles form a $90^{\circ}$ angle (or a right angle), we often see this marked with a square corner:


If supplementary angles are also adjacent, they form a linear pair.


## Exercise

1) Explain the relationship between $\angle D Z Q$ and $\angle P Z Q$.
2) Find $m \angle D Z Q$ and $m \angle Q Z P$.

3) Explain the relationship between $\angle S T Q$ and $\angle P T Q$.

4) If $m \angle S T Q=2 x+30$ and $m \angle P T Q=8 x$, find $m \angle S T Q$.

## Algebraic Representations of Complements

| Given an angle of... | Calculate its complement: |
| :---: | :--- |
| $40^{\circ}$ |  |
| $60^{\circ}$ |  |
| $10^{\circ}$ |  |
| $\boldsymbol{x}^{\circ}$ |  |

Algebraic Representation of Supplements

| Given an angle of $\ldots$ | Calculate its supplement: |
| :---: | :--- |
| $40^{\circ}$ |  |
| $60^{\circ}$ |  |
| $10^{\circ}$ |  |
| $\boldsymbol{x}^{\circ}$ |  |

Model Problems Try these first on your own:

1) An angle is 10 more than 3 times the measure of its complement. Find the measure of the complement.
2) Five times the complement of an angle less twice the angle's supplement is 40 . Find the measure of the supplement.

## Ratio Problems

3) Two supplementary angles are in the ratio 11:7. Find the measure of each angle.

For Exercises 1-6, use the figure at the right. Name an angle or angle pair that satisfies each condition.

1. Name two acute vertical angles.
2. Name two obtuse vertical angles.
3. Name a linear pair.
4. Name two acute adjacent angles.
5. Name an angle complementary to $\angle E K H$.

6. Name an angle supplementary to $\angle F K G$.

Name an angle or angle pair that satisfies each condition.
7) Name two obtuse vertical angles.
8) Name a linear pair whose vertex is $B$.
9) Name an angle not adjacent to, but complementary to $\angle F G C$.
10) Name an angle adjacent and supplementary to $\angle D C B$.

11) If $m \angle P T Q=3 y-10$ and $m \angle Q T R=y$, find the value of $y$ so that $\angle P T R$ is a right angle.


Determine whether each statement can be assumed from the figure. Explain.
12) $\angle N Q O$ and $\angle O Q P$ are complementary.
13) $\angle S R Q$ and $\angle Q R P$ is a linear pair.
14) $\angle M Q N$ and $\angle M Q R$ are vertical angles.


Homework Day 5 Complete in notebook.
Holt: pages 31-33 \#9, 10, 11, 22, 27, 31, 32, 44, 45, 46
McDougal, Littell page 71 \#'s 18, 21, 25 page 103 \# 15
G.GPE. 4 Use coordinates to prove simple geometric theorems algebraically.

## Warm-Up

Which pair of angles are supplementary? F $\angle U S V, \angle V S W \quad$ G $\angle V S W, \angle W S R$ $\mathbf{H} \angle T S V, \angle V S W \quad \mathbf{J} \angle T S R, \angle U S W$


## The Distance Formula

$\overline{A B}$ has endpoints $A(2,5)$ and $B(-4,-3)$.
How can we find the length of this line segment?
Draw a right triangle and label the third point C .
Using the Pythagorean Theorem:
(1) $A B^{2}=A C^{2}+B C^{2}$
(2) $A B^{2}=ـ_{2}^{2}+{ }^{2}$
(3) $A B^{2}=$ $\qquad$
(4) $A B=$ $\qquad$


How did you find the lengths of $\overline{A C}$ and $\overline{B C}$ in step (2)? $\qquad$
How can you find these lengths using the coordinates? $\qquad$
How did you solve for AB in step (4)? $\qquad$

The length of a line segment with endpoints $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ is given by:

$$
\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}
$$

## Model Problem

A. Use the distance formula to find the length of $\overline{A B}$ :

1. Label the points: $(-2,-3)(4,5)$
2. Plug into the formula and simplify:

$$
\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}
$$



## Guided Practice

B. $\overline{C D}$ has coordinates $(-1,-2)$ and $(2,6)$.

Use the distance formula to find the length of $\overline{C D}$ to the nearest tenth.

1. Label the points: $(-1,-2)$
(2, 6 )
2. Plug into the formula and simplify:

$$
\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}
$$


C. $\overline{C D}$ has endpoints $\mathrm{C}(-2,4)$ and $\mathrm{D}(6,0)$.

Plot $\overline{C D}$ on the axes at right and find CD. Express your answer in simplest radical form.

D. What is the distance between points $(-1,-2)$ and $(5,0)$ ?

## Independent Practice



1) Find the distance between the points $(-1,-1)$ and $(2,-5)$.
$\square$
2) Find, in radical form, the distance between points $(-1,-2)$ and $(5,0)$.
3) Find the length of $\overline{P Q}$.

|  |  |  | $4{ }^{1}$ | 1 | T |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\cdots$ | $\bar{P}$ |  |
|  |  |  |  |  |  |  |
|  |  | - |  |  |  |  |
|  | $Q$ |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  | 0 |  |  |  | $\vec{x}$ |
|  |  |  | $\downarrow$ |  |  |  |

4) Find, in simplest radical form, the length of the line segment joining points $(1,5)$ and $(3,9)$.
5) Express, in radical form, the distance between the points $(2,4)$ and $(0,-5)$.

## Challenge!

The vertices of $\Delta \mathrm{ABC}$ are $\mathrm{A}(2,3), \mathrm{B}(5,7)$, and $\mathrm{C}(1,4)$.
Show that $\triangle \mathrm{ABC}$ is an isosceles triangle.

$\square$
Find the distance between each pair of points. Express in simplest radical form if necessary.

$$
(-2,3),(-7,-7)
$$

$$
(2,-9),(-1,4)
$$

$(5,9),(-7,-7)$
$(8,5),(-1,3)$
$(-10,-7),(-8,1)$
$(-6,-10),(-2,-10)$

Find the length of each line segment. Round to the nearest tenth if necessary.




G.GPE. 4 Use coordinates to prove simple geometric theorems algebraically.

## Warm-Up

Ben and Kate are making a map of their neighborhood. They decide to make one unit on the graph paper correspond to 100 yards. They put their homes on the map as shown below.

How many yards apart are Kate and Ben's homes?


## The Midpoint Formula

$\overline{A B}$ has coordinates $A(2,5)$ and $B(-4,-3)$. How can we find the midpoint of this segment?

1) Plot $\overline{A B}$ on the axes at right.
2) Find and plot the midpoint of $\overline{A B}$ by eye. Label this point M.
3) Explain how you know that M is the midpoint of $\overline{A B}$. Justify your answer mathematically.


## A. Finding the Midpoint Given the Endpoints

## Model Problem

Find the midpoint of the segment whose endpoints are $\mathrm{A}(-2,6)$ and B (6, -4).


## Exercise

1) Plot and find the coordinates of the midpoint of the segment whose endpoints are $(-5,1)$ and $(0,-5)$.

2) What are the coordinates of the midpoint of the segment joining (5. -3) and (6, 3)?

## B. Finding the Missing Endpoint Given the Midpoint

## Model Problem

The midpoint M of $\overline{A B}$ is $(-1,1)$. If the coordinates of A are $(2,-1)$, find the coordinates of endpoint B .

## Method \#1: Graphic Solution

Plot the known endpoint and the midpoint on the graph.
Extend the segment to the other endpoint.


Answer: $\qquad$

The midpoint M of $\overline{A B}$ is $(-1,1)$. If the coordinates of A are $(2,-1)$, find the coordinates of endpoint B .

## Method \#3: Algebraically

The midpoint M of $\overline{A B}$ is $(-1,1)$. If the coordinates of A are $(2,-1)$, find the coordinates of endpoint B .

Exercise Use any correct method. (The use of the graphs is optional.)
a) M is the midpoint of $\overline{C D}$. If the coordinates of C are $(6,4)$ and the coordinates of M are $(0,6)$, find the coordinates of point D.

b) The coordinates of the midpoint of a segment are $(-4,1)$ and the coordinates of one endpoint are $(-6,-5)$. Find the coordinates of the other endpoint.

c) The coordinates of the center of a circle are $(0,0)$. If one endpoint of the diameter is $(-3,4)$, find the coordinates of the other endpoint of the diameter. ${ }^{1}$ (Hint: SKETCH IT!!)

Independent Practice You may use graph paper if you wish.

1) What are the coordinates of the midpoint of the line segment that connects the points $(1,2)$ and $(6,7)$ ?
2) In a circle, the coordinates of the endpoints of the diameter are $(4,5)$ and $(10,1)$. What are the coordinates of the center of the circle?
3) What are the coordinates of the midpoint of the segment whose endpoints are $(-4,6)$ and $(-8,-2)$ ?

[^0]4) The coordinates of the midpoint of line segment AB are (1,2). If the coordinates of A are $(1,0)$, find the coordinates of point B.
5) The midpoint of $\overline{A B}$ is $M$. If the coordinates of $A$ are (2, -6 ), and the coordinates of $M$ are ( $5,-1$ ), find the coordinates of $B$.

Homework There are optional graphs at the end of this homework if you need them.

Find the midpoint of the line segment with the given endpoints.

$$
(-4,4),(5,-1)
$$

$$
(-1,-6),(-6,5)
$$

$(2,4),(1,-3)$

$$
(-4,4),(-2,2)
$$

Find the midpoint of each line segment.
1)

2)

3)

4)


Find the other endpoint of the line segment with the given endpoint and midpoint.

Endpoint: $(-1,9)$, midpoint: $(-9,-10)$

Endpoint: (5, 2), midpoint: $(-10,-2)$
Endpoint: $(2,5)$, midpoint: $(5,1)$

Endpoint: $(9,-10)$, midpoint: $(4,8)$

Endpoint: (-6, 4), midpoint: $(4,8)$
Endpoint: (-9, 7), midpoint: $(10,-3)$







## Warm-Up

The midpoint of line segment AB is $(-2,3)$. If point A has coordinates $(4,2)$, find the coordinates of point B.

## Understanding Ratios

Let's say that instead of dividing a line segment in half, we divide it into a ratio of 2:3. That means there will be five equal parts, because $2+3=5$ :


We can use this idea to partition line segments into any ratio we choose.

## Model Problem

The endpoints of $\overline{D E F}$ are $\mathrm{D}(1,4)$ and $\mathrm{F}(16,14)$. Determine and state the coordinates of point E , if $\mathrm{DE}: E F=2: 3$.

## Exercise

Directed line segment $\overline{P T}$ has endpoints whose coordinates are $\mathrm{P}(-2,1)$ and $\mathrm{T}(4,7)$. Determine the coordinates of point J that divides the segment in the ratio 2 to 1 .

## Independent Practice/Homework

1) The coordinates of the endpoints of $\overline{A B}$ are $\mathrm{A}(-6,-5)$ and $\mathrm{B}(4,0)$. Point P is on $\overline{A B}$. Determine and state the coordinates of point P , such that $\mathrm{AP}: \mathrm{PB}$ is 2:3.
2) What are the coordinates of the point on the directed line segment from $K(-5,-4)$ to $L(5,1)$ that partitions the segment into a ratio of 3 to 2 ?
3) Point $B$ is between $A(2,5)$ and $C(10,1)$. Find the coordinates of $B$, if $A B: B C=1: 3$.
4) A line segment has endpoints $(-6,7)$ and $(9,2)$. What are the coordinates of the point on this line segment that divides it into a ratio of $2: 3$ ?
5) Point X divides $\overline{M N}$ into a ratio of $3: 5$. If the coordinates of M are $(4,3)$ and the coordinates of N are $(20,11)$, find the coordinates of X .
6) Directed line segment $D E G$ has endpoints $D(0,8)$ and $G(-24,-16)$. Find the coordinates of point E such that $\mathrm{DE}: E G=5: 7$.
7) $\overline{S P}$ has endpoints $\mathrm{S}(-11,6)$ and $\mathrm{P}(10,-1)$. Point U is on $\overline{S P}$. Determine and state the coordinates of U such that $\mathrm{SU}: \mathrm{UP}=3: 4$.
8) Two points are located on a coordinate grid at $(8,1)$ and $(-2,16)$. Find the coordinates of the point that is $1 / 5$ of the directed distance from $(8,1)$ to $(-2,16)$.
9) Line segment $A B$ has endpoints $A(4,9)$ and $B(9,19)$. Is the point that divides $\overline{A B}$ into a ratio of 2:3 the same point that divides it into a ratio of 3:2? Explain.

## 10) Challenge

Point B on line segment AC divides $\overline{A C}$ into a ratio of $2: 3$. If the coordinates of A are $(4,-1)$ and the coordinates of $B$ are $(10,3)$, find the coordinates of point $C$.

As you read, underline all important terms from this unit. Remember to draw diagrams!

1. Draw and label plane $\mathcal{N}$ containing two lines that intersect at $B$.

Use the figure to name each of the following.
2. four noncoplanar points
3. line containing $B$ and $E$

4. The coordinate of $A$ is -3 , and the coordinate of $B$ is 0.5 . Find $A B$.
5. $E, F$, and $G$ represent mile markers along a straight highway. Find $E F$.

6. $J$ is the midpoint of $\overline{H K}$. Find $H J, J K$, and $H K$.

7. For (a)-(d), use the diagram at right.
a) Name the vertex to all the angles in the diagram.
b) Name the angle vertical to $\angle C B A$.
c) If $\angle F B E$ is a right angle, name two complementary angles.
d) Name the angle supplementary to $\angle D B C$. $\qquad$

9) $\overrightarrow{T V}$ bisects $\angle R T S$. If $m \angle R T V=(16 x-6)^{\circ}$ and $m \angle V T S=(13 x+9)^{\circ}$, what is $m \angle R T V$ ?
10) Find the distance between $A(-12,13)$ and $B(-2,-11)$.
11) Find the midpoint of the segment with endpoints $(-4,6)$ and $(3,2)$.
12) M is the midpoint of $\overline{L N}$. M has coordinates $(-5,1)$ and L has coordinates $(2,4)$. Find the coordinates of N .
13) Directed line segment $G H$ is divided by point $I$ into a ratio of $4: 5$. If point $G$ has coordinates $(-3,5)$ and point H has coordinates $(6,-13)$, determine and state the coordinates of point I .

## Constructions

14) Construct a segment congruent to $\overline{A B}$.

15) Copy angle $S$ at vertex $X$.

16) Construct the bisector of the angle below. Label it $\overrightarrow{Q T}$. Name two congruent angles.


[^0]:    ${ }^{1}$ BONUS \#1: Find the length of the radius of this circle.
    BONUS \#2: Find the area of this circle in terms of $\pi$.

