# Geometry Honors Chapter 1:

# Foundations for Geometry



# Unit 1: Vocabulary

1)	point	
2)	line	
3)	plane	
4)	segment	
5)	endpoint	
6)	ray	
7)	collinear	
8)	coplanar	
9)	opposite ray	
10)	distance along a line	
11)	length	
12)	congruent segments	
13)	between	
14)	midpoint	

15)	(to) bisect	
16)	segment bisector	
17)	angle	
18)	vertex	
19)	acute angle	
20)	obtuse angle	
21)	straight angle	
22)	congruent angles	
23)	angle bisector	
24)	construct(ion)	
25)	adjacent angles	
26)	linear pair	
27)	complementary angles	
28)	supplementary angles	
29)	vertical angles	

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

<u>Warm-Up</u> Solve for t:

5t - 2(t - 5) = 19

The most basic figures in geometry are **<u>undefined terms</u>**, which cannot be defined by using other figures. The undefined terms *point*, *line*, and *plane* are the building blocks of geometry.

Undefined Terms					
TERM	NAME	DIAGRAM			
A <b>point</b> names a location and has no size. It is represented by a dot.	A capital letter point <b>P</b>	Ρ.			
A <b>line</b> is a straight path that has no thickness and extends forever.	A lowercase letter or two points on the line line $\ell$ , $\overleftarrow{XY}$ or $\overleftarrow{YX}$	$\begin{array}{c} & & \\ & & \\ & X & & Y \end{array} \ell$			
A <b>plane</b> is a flat surface that has no thickness and extends forever.	A script capital letter or three points not on a line plane $\Re$ or plane <b>ABC</b>	А• С• <sub>Я</sub> В•			

Points that lie on the same line are <u>collinear</u>. *K*, *L*, and *M* are collinear. *K*, *L*, and *N* are *noncollinear*.



Points that lie on the same plane are **coplanar**. Otherwise they are *noncoplanar*.

#### **Sketches**

A line that is <i>contained</i> (lies in) in a plane	A line that intersects a plane in one point
r me mai is containea (nes m) m a plane	A fine that intersects a plane in one point
Coplanar points	Four non-conlanar points
Coplanar points	Four non-coplanar points

Seg	ments and Rays		
	DEFINITION	NAME	DIAGRAM
A is of	<b>segment</b> , or line segment, the part of a line consisting f two points and all points etween them.	The two endpoints	A B
A or st	n <mark>endpoint</mark> is a point at ne end of a segment or the arting point of a <i>ray.</i>	A capital letter C and D	c D
A th ar di	<b>ray</b> is a part of a line nat starts at an endpoint nd extends forever in one irection.	Its endpoint and any other point on the ray RS	R S S R
Opposite rays are two rays that have a common endpoint and form a line.		The common endpoint and any other point on each ray <b>ĒF</b> and <b>Ē</b> Ġ	$\overbrace{F \ E \ G}$

# Model Problems Use the diagram at right.

- 1) Name a point.
- 2) Name the line that goes through point E in two ways.
- 3) Name a segment.
- 4) Name three collinear points.
- 5) Name three non-collinear points.
- 6) Name the intersection of  $\overrightarrow{EC}$  and the segment not on  $\overrightarrow{EC}$ .
- 7) Name the plane shown in the diagram.



#### Exercise

- 1) Name a point.
- 2) Name the line that goes through point Z in *three* ways.
- 3) Name a segment.
- 4) Name three coplanar points.
- 5) Name three non-collinear points.
- 6) Name the intersection of line *m* and  $\overleftarrow{YZ}$ .
- 7) Name the plane shown in the diagram.
- 8) Name the points that determine this plane.
- 9) Name two lines that intersect line *m*.
- 10) Name a line that does not intersect line m.

#### **Postulates about Lines and Points**

A **postulate**, or *axiom*, is a statement that is accepted as true without proof. Postulates about points, lines, and planes help describe geometric properties.

Postulate	Sketch	Illustration		
Two points determine a line.		Any two points are collinear.		
Three points determine a plane.		Any three points are coplanar. Think of a wobbly chair. It will be stable if any three legs are touching the ground.		



If two points lie in a plane, then the line containing those points will lie in that plane too.	If you draw two points on a piece of paper, the line that connects them is on the paper too.
The intersection of two lines is a point.	<ul> <li>Street intersection</li> <li>Pivot of scissors</li> <li>The letter "X"</li> <li>A plus sign</li> </ul>
The intersection of two planes is a line.	<ul> <li>The crease of a book</li> <li>The edge of a door</li> <li>A river valley</li> <li>The corner where two walls meet</li> </ul>

#### Check for Understanding

Two flat walls meet in the corner of a classroom. Which postulate best describes this situation?

- A Through any three noncollinear points there is exactly one plane.
- (B) If two points lie in a plane, then the line containing them lies in the plane.
- C If two lines intersect, then they intersect in exactly one point.
- ① If two planes intersect, then they intersect in exactly one line.

Model Problems Draw and label each of the following.

- A. Plane  $\mathcal{H}$  that contains two lines that intersect at M
- B.  $\overrightarrow{ST}$  intersecting plane  $\mathcal{M}$  at R

#### Drawing Hints:

lines – have arrows on both sides.

rays – arrow on one side, first letter is endpoint

planes – are flat surfaces

points – are always 7 capital letters

#### Check for Understanding

Sketch a figure that shows two lines intersect in one point in a plane, but only one of the lines lies in the plane.

## **Lesson Quiz**

- **1.** Two opposite rays
- **2.** A point on  $\overrightarrow{BC}$ .
- **3.** The intersection of plane N and plane T
- **4.** A plane containing E, D, and B.



#### Draw each of the following.

**5.** a line intersecting a plane at one point

**6.** a ray with endpoint P that passes through Q

# Homework



G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

#### Warm-Up

- a. Draw and label the following:
  - i. A line containing points X and Y
  - ii. A pair of opposite rays that both contain point R
- b. Campers often use a cooking stove with three legs. Why might they prefer this design to a stove that has four legs?

The <u>distance along a line</u> between any two points is the absolute value of the difference of the coordinates. The coordinates can be measured in a variety of units, such as inches or centimeters.

If the coordinates of points *A* and *B* are *a* and *b*, then the distance between *A* and *B* is |a - b| or |b - a|. The distance between *A* and *B* is also called the **length**, or *measure* of  $\overline{AB}$ , or AB.

Exercise

#### Find each length.





**Congruent Segments** <u>Congruent segments</u> are segments that have the same length. In the

diagram, PQ = RS, so you can write  $\overline{PQ} \cong \overline{RS}$ . This is read as "segment PQ is congruent to segment RS."

*Tick marks* are used in a figure to show congruent segments.



# **Constructing Congruent Segments**

A **construction** is a special drawing that only uses a compass and a straightedge. Constructions can be justified by using geometric principles to create figures.

<u>Model Problem</u> Construct a segment congruent to  $\overline{AB}$ .



<u>Exercise</u> Construct a segment congruent to  $\overline{AB}$ . Then answer the questions below.



*Think About It.* Why does this construction result in a line segment with the same length as  $\overline{AB}$ ?

#### **Betweenness**

In order for you to say that a point *B* is <u>between</u> two points *A* and *C*, all three points must lie on the same line, and AB + BC = AC.



**B.** H is between I and J. If HI = 3.9 and HJ = 6.2, find IJ.

#### Exercise

1. *E* is between *D* and *F*. Find *DF*.



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2. H is between I and J. If IJ = 25 and HI = 13, find HJ.

# **Midpoint**

The <u>midpoint</u> M of  $\overline{AB}$  is the point that <u>bisects</u>, or divides the segment into two **congruent** segments. If M is the midpoint of  $\overline{AB}$ , then AM = MB. So if AB = 6, then AM = 3 and MB = 3.

#### Model Problems

A. *D* is the midpoint of  $\overline{EF}$ , ED = 4x + 6, and DF = 7x - 9. Find *ED*, *DF*, and *EF*.

B. B is the midpoint of  $\overline{AC}$ . AB = 8v, and AC = 2v + 42. What is BC?



#### **Exercise**

- 1. H is the midpoint of  $\overline{IJ}$ .
  - i. If IJ = 18, find HI and HJ.
  - ii. If IH = 10, find HJ and IJ.



2. E is the midpoint of  $\overline{DF}$ . DE = 2x + 4 and EF = 3x - 1. Find DE, EF, and DF.

DE =	_		
EF =		•	•
DF =			

3. X is the midpoint of  $\overline{AT}$ . If AX = 4x and AT = 3x + 25, find AX, XT, and AT.

4. S is between R and T. Does that mean that S must be a **midpoint**? Explain and sketch an appropriate diagram of  $\overline{RT}$ .

# Homework Day 2 Complete in notebook.

Holt - pg: 17 – 19/ #s 12, 14 – 15, 17- 20, 22, 29, 32, 41

McDougal and Littell - pg: 33 - 35/ #11, 21

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

#### Warm-Up

M is the midpoint of segment  $\overline{AD}$ . If AM = x + 3 and  $AD = x^2 + 6$ , find: a) the value of x. b) MD

An **angle** is a figure formed by two rays, or sides, with a common endpoint called the **vertex** (plural: vertices).

You can name an angle several ways: by its vertex, by a point on each ray and the vertex, or by a number.

The set of all points between the sides of the angle is the *interior of an angle*. The *exterior of an angle* is the set of all points outside the angle.

#### Model Problem

Name the angle at right in four ways.



**Note:** You cannot name an angle just by its vertex if the point is the vertex of more than one angle. In this case, you must use all three points to name the angle, and the middle point is always the vertex.

Wrong:

<u>Right:</u>



#### Exercise

<ol> <li>Draw and label ∠DEF below.</li> <li>Draw and label ∠QRS with RT in the <i>interior</i> of the angle.</li> </ol>	

3) Write the different ways you can name each given angle in the diagram.



# **Measuring Angles**

The measure of an angle is usually given in *degrees*. Since there are  $360^{\circ}$  in a circle, one degree is 1/360 of a circle.

When referring to the degree measure of angles, we write:

#### $m \angle ABC$

which is read, "the measure of angle ABC."



# **Types of Angles**



# **Congruent Angles**

Check for Understanding

<u>Congruent angles</u> are angles that have the same degree measure.

In the diagram,  $m \angle ABC = m \angle DEF$ , so you can write  $\angle ABC \cong \angle DEF$ . This is read as "angle ABC is congruent to angle *DEF*."

*Arc marks* are used to show that the two angles are congruent.



2) Mark the diagram at right to show this congruence.

In the diagram, assume  $\angle DEF$  is **congruent** to  $\angle FEG$ .

1) Explain what this means in your own words.

- 3) Write " $\angle DEF$  is congruent to  $\angle FEG$ " using symbols.
- 4) Write "the measure of  $\angle DEF$  equals the measure of  $\angle FEG$ " in symbols.

## **Angle Addition Postulate**



Note that this is similar to segment addition:

"PART + PART = WHOLE"

<u>Model Problem</u> Mark up the diagram appropriately. Then answer the question below.

In the accompanying diagram,  $m \angle DEG = 115^{\circ}$ ,  $m \angle DEF = 2x - 1^{\circ}$ ,  $m \angle GEF = 3x + 1^{\circ}$ . Find  $m \angle DEF$  and  $m \angle GEF$ .



Exercise

Find  $m \angle KLM$  if  $m \angle KLB = 26^{\circ}$ and  $m \angle BLM = 60^{\circ}$ .



Find  $m \angle WDC$  if  $m \angle EDC = 145^{\circ}$ and  $m \angle EDW = 61^{\circ}$ .



 $m \angle HGF = 16x + 4$ ,  $m \angle EGF = 110^{\circ}$ , and  $m \angle HGE = 3x + 11$ . Find *x*.

F E E H

 $m \angle FCD = x + 41, m \angle BCF = x + 78,$ and  $m \angle BCD = 95^{\circ}$ . Find *x*.



#### **The Angle Bisector**

An <u>angle bisector</u> is a ray that divides an angle into two **congruent** angles.

Given:  $\overrightarrow{JK}$  bisects  $\angle LJM$ 

Conclusion:  $\angle LJK \cong \angle MJK$ 



#### Model Problems

A.  $\overrightarrow{KM}$  bisects  $\angle JKL$ ,  $m \angle JKM = (4x + 6)^{\circ}$ , and  $m \angle MKL = (7x - 12)^{\circ}$ . Find  $m \angle JKM$ .



- **B.** Given:  $\overrightarrow{QS}$  bisects  $\angle PQR$ .
  - 1. Sketch and label  $\angle PQR$  first, then draw ray  $\overrightarrow{QS}$  from point Q.

2.  $m \angle PQS = (5y - 1)^\circ$ , and  $m \angle PQR = (8y + 12)^\circ$ . Find  $m \angle PQS$ .

Exercise

 $\overrightarrow{BD}$  bisects  $\angle ABC$ . Find m $\angle ABD$  if m $\angle ABD = (6x + 4)^{\circ}$  and m $\angle DBC = (8x - 4)^{\circ}$ .



<u>Homework Day 3</u> Complete in notebook.

Holt: pages 24- 27 #'s 8, 10, 11, 18, 19- 22 all, 29, 31, 32, 38, 44, 45, 47

Mc Dougal, Littell: pages 16-17 #'s 15, 17, 21 and page 27 # 14

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.

Warm-Up

 $\overrightarrow{NT}$  bisects  $\angle MNS$ . Find m $\angle TNS$  if m $\angle MNT = (2x - 30)^{\circ}$  and m $\angle DBC = (2x)^{\circ}$ .



Independent Practice Algebraic Review



**1.** If  $m \angle PQT = 60$  and  $m \angle PQS = 4x + 14$ , find the value of x.



**2.** If  $m \angle PQS = 3x + 13$  and  $m \angle SQT = 6x - 2$ , find  $m \angle PQT$ .



#### **Constructing Congruent Angles**

Task: Construct an angle with vertex X that has the same measure as S.

A In the space below, use a straightedge to draw a ray with endpoint *X*.

X۰

- **B** Place the point of your compass on *S* and draw an arc that intersects both sides of the angle. Label the points of intersection *T* and *U*.
- **C** Without adjusting the compass, place the point of the compass on *X* and draw an arc that intersects the ray. Label the intersection *Y*.
- **D** Place the point of the compass on *U* and open it to the distance *TU*.
- **E** Without adjusting the compass, place the point of the compass on *Y* and draw an arc. Label the intersection with the first arc *Z*.

**F** Use a straightedge to draw  $\overrightarrow{XZ}$ .

You Try:

X۰

Why does this construction work?







# Practice

Construct an angle with the same measure as the given angle.



Think About It. If the angle you construct has longer sides than the original angle, can the two angles still have the same measure? Explain.

#### **Constructing an Angle Bisector**

Task:

Construct the bisector of  $\angle M$ . Work directly on the angle at right.

- A Place the point of your compass on point *M*. Draw an arc that intersects both sides of the angle. Label the points of intersection *P* and *Q*.
- **B** Place the point of the compass on *P* and draw an arc in the interior of the angle.
- **C** Without adjusting the compass, place the point of the compass on *Q* and draw an arc that intersects the arc from Step B. Label the intersection of the arcs *R*.
- **D** Use a straightedge to draw  $\overrightarrow{MR}$ .

Why does this construction work?





<u>Practice</u> Construct the bisector of each angle.



#### Practice (continued)

7) Explain how you can use a compass and straightedge to construct an angle that has twice measure of  $\angle A$ . Then do the construction in the space provided.



8) Explain how you can use a compass and straightedge to construct an angle that has  $\frac{1}{4}$  the measure of  $\angle B$ . Then do the construction in the space provided.



#### Homework

1) Given the diagram at right. Write the number that names the same angle. If the angle does not exist in the diagram, write "does not exist." Some angles may be used more than once.





# ALGEBRA In the figure $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. $\overrightarrow{BF}$ bisects $\angle CBE$ .

**3.** If  $m \angle EBF = 6x + 4$  and  $m \angle CBF = 7x - 2$ , find  $m \angle EBF$ .



**4.** If  $m \angle 3 = 4x + 10$  and  $m \angle 4 = 5x$ , find  $m \angle 4$ .



5. If  $m \angle 3 = 6y + 2$  and  $m \angle 4 = 8y - 14$ , find  $m \angle CBE$ .

**6.** Let  $m \angle 1 = m \angle 2$ . If  $m \angle ABE = 100$  and  $m \angle ABD = 2(r + 5)$ , find r and  $m \angle DBE$ .



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Draw each of the following.

- 7. a line that contains  $\overrightarrow{AB}$  and  $\overrightarrow{CB}$
- 8. two different lines that intersect  $\overline{MN}$
- 9. a plane and a ray that intersect only at Q
- 10. **Critical Thinking** Can an obtuse angle be congruent to an acute angle? Why or why not?
- 11. **Short Response** If an obtuse angle is bisected, are the resulting angles acute or obtuse? Explain.

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

#### Warm-Up

 $\overrightarrow{RT}$  bisects  $\angle QRS$ . If  $m \angle QRT = x + 15$ ,  $m \angle TRS = 2y + 10$ , and  $m \angle QRS = 2x + 2y$ , find the value of x and y.

Many pairs of angles have special relationships. Some relationships are because of the *measurements* of the angles in the pair. Other relationships are because of the *positions* of the angles in the pair.



<u>Vertical angles</u> are two nonadjacent angles formed by two intersecting lines.

 $\angle 1$  and  $\angle 3$  are vertical angles, as are  $\angle 2$  and  $\angle 4$ .

Vertical angles are always congruent.







#### Exercise

For #1-2, tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

1)  $\angle AEB$  and  $\angle BED$ 



- 3) Using the diagram at right, name:
  - a) a pair of **vertical angles**
  - b) two angles that form a linear pair
  - c) two adjacent angles that do not form a **linear pair**





#### **Complementary and Supplementary Angles**



We say that  $\angle A$  is the <u>complement</u> of  $\angle B$  and the <u>supplement</u> of  $\angle C$ .

If two angles form a 90° angle (or a right angle), we often see this marked with a square corner:



If supplementary angles are also adjacent, they form a linear pair.



#### Exercise

1) Explain the relationship between  $\angle DZQ$  and  $\angle PZQ$ .



2) Find  $m \angle DZQ$  and  $m \angle QZP$ .

3) Explain the relationship between  $\angle STQ$  and  $\angle PTQ$ .



4) If  $m \angle STQ = 2x + 30$  and  $m \angle PTQ = 8x$ , find  $m \angle STQ$ .

#### Algebraic Representations of Complements

Given an angle of	Calculate its <b>complement</b> :
40°	
60°	
10°	
x°	

#### Algebraic Representation of Supplements

Given an angle of	Calculate its <b>supplement</b> :
40°	
60°	
10°	
x°	

<u>Model Problems</u> Try these first on your own:

1) An angle is 10 more than 3 times the measure of its complement. Find the measure of the complement.

2) Five times the complement of an angle less twice the angle's supplement is 40. Find the measure of the supplement.

Ratio Problems

3) Two supplementary angles are in the ratio 11:7. Find the measure of each angle.

# For Exercises 1–6, use the figure at the right. Name an angle or angle pair that satisfies each condition.

- 1. Name two acute vertical angles.
- 2. Name two obtuse vertical angles.
- 3. Name a linear pair.
- 4. Name two acute adjacent angles.
- **5.** Name an angle complementary to  $\angle EKH$ .
- **6.** Name an angle supplementary to  $\angle FKG$ .



# Name an angle or angle pair that satisfies each condition.

- 7) Name two obtuse vertical angles.
- 8) Name a linear pair whose vertex is B.
- 9) Name an angle not adjacent to, but complementary to  $\angle FGC$ .
- 10) Name an angle adjacent and supplementary to  $\angle DCB$ .
  - 11) If  $m \angle PTQ = 3y 10$  and  $m \angle QTR = y$ , find the value of y so that  $\angle PTR$  is a right angle.

# Determine whether each statement can be assumed from the figure. Explain.

- 12)  $\angle NQO$  and  $\angle OQP$  are complementary.
- 13)  $\angle SRQ$  and  $\angle QRP$  is a linear pair.
- 14)  $\angle MQN$  and  $\angle MQR$  are vertical angles.







<u>Homework Day 5</u> Complete in notebook.

Holt: pages 31-33 #9, 10, 11, 22, 27, 31, 32, 44, 45, 46

McDougal, Littell page 71 #'s 18, 21, 25 page 103 # 15

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

#### Warm-Up

Which pair of angles are supplementary? **F**  $\angle USV, \angle VSW$  **G**  $\angle VSW, \angle WSR$ **H**  $\angle TSV, \angle VSW$  **J**  $\angle TSR, \angle USW$ 



#### **The Distance Formula**

 $\overline{AB}$  has endpoints A(2,5) and B(-4,-3). How can we find the **length** of this line segment?

Draw a right triangle and label the third point C.

Using the Pythagorean Theorem:

- $(1) \quad AB^2 = AC^2 + BC^2$
- (2)  $AB^2 = \__2^2 + \__2^2$
- (3)  $AB^2 =$ \_\_\_\_\_
- (4)  $AB = \_$ \_\_\_\_

How did you find the **lengths** of  $\overline{AC}$  and  $\overline{BC}$  in step (2)? \_\_\_\_\_

How can you find these **lengths** using the coordinates?

How did you solve for AB in step (4)?

The length of a line segment with endpoints  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



#### Model Problem

- A. Use the **distance formula** to find the **length** of  $\overline{AB}$ :
- 1. Label the points: (-2, -3) (4, 5)
- 2. Plug into the formula and simplify:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



#### **Guided Practice**

B.  $\overline{CD}$  has coordinates (-1, -2) and (2, 6).

Use the **distance formula** to find the **length** of  $\overline{CD}$  to the nearest tenth.

- 1. Label the points: (-1, -2) (2, 6)
- 2. Plug into the formula and simplify:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



C.  $\overline{CD}$  has endpoints C (-2, 4) and D (6, 0).

Plot  $\overline{CD}$  on the axes at right and find CD. Express your answer in simplest radical form.



D. What is the **distance** between points (-1, -2) and (5, 0)?



#### **Independent Practice**

1) Find the **distance** between the points (-1, -1) and (2, -5).

2) Find, in radical form, the **distance** between points (-1, -2) and (5, 0).

3) Find the **length** of  $\overline{PQ}$ .



4) Find, in simplest radical form, the **length** of the line segment joining points (1, 5) and (3, 9).

5) Express, in radical form, the distance **between** the points (2, 4) and (0, -5).

# Challenge!

The vertices of  $\triangle$  ABC are A(2, 3), B(5, 7), and C(1, 4).

Show that  $\triangle ABC$  is an isosceles triangle.



Find the **distance** between each pair of points. Express in simplest radical form if necessary.

$$(-2, 3), (-7, -7)$$
  $(2, -9), (-1, 4)$ 

$$(5, 9), (-7, -7)$$
  $(8, 5), (-1, 3)$ 

$$(-10, -7), (-8, 1)$$
  $(-6, -10), (-2, -10)$ 

Find the length of each line segment. Round to the nearest tenth if necessary.







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G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

#### Warm-Up

Ben and Kate are making a map of their neighborhood. They decide to make one unit on the graph paper correspond to 100 yards. They put their homes on the map as shown below.

How many yards apart are Kate and Ben's homes?

				ŀу					
						E	Ben	's	
						House			
			0						x
K	ate	's ,							
٦H	lou	se							
			1	,					

#### The Midpoint Formula

 $\overline{AB}$  has coordinates A(2, 5) and B(-4, -3). How can we find the **midpoint** of this segment?

- 1) Plot  $\overline{AB}$  on the axes at right.
- 2) Find and plot the **midpoint** of  $\overline{AB}$  by eye. Label this point M.
- 3) Explain how you know that M is the **midpoint** of  $\overline{AB}$ . Justify your answer mathematically.



## A. Finding the Midpoint Given the Endpoints

#### Model Problem

Find the **midpoint** of the segment whose endpoints are A (-2, 6) and B (6, -4).





#### Exercise

1) Plot and find the coordinates of the **midpoint** of the segment whose endpoints are (-5, 1) and (0, -5).

2) What are the coordinates of the **midpoint** of the segment joining (5. -3) and (6, 3)?

#### **B.** Finding the Missing Endpoint Given the Midpoint

#### Model Problem

The **midpoint** M of  $\overline{AB}$  is (-1, 1). If the coordinates of A are (2, -1), find the coordinates of **endpoint** B.

#### Method #1: Graphic Solution

Plot the known **endpoint** and the **midpoint** on the graph.

Extend the segment to the other **endpoint**.



Answer: \_\_\_\_\_

The midpoint M of  $\overline{AB}$  is (-1, 1). If the coordinates of A are (2, -1), find the coordinates of endpoint B.

Method #3: Algebraically

The midpoint M of  $\overline{AB}$  is (-1, 1). If the coordinates of A are (2, -1), find the coordinates of endpoint B.

Exercise Use any correct method. (The use of the graphs is optional.)

a) M is the **midpoint** of  $\overline{CD}$ . If the coordinates of C are (6, 4) and the coordinates of M are (0, 6), find the coordinates of point D.



b) The coordinates of the **midpoint** of a segment are (-4, 1) and the coordinates of one **endpoint** are (-6, -5). Find the coordinates of the other **endpoint**.



c) The coordinates of the **center** of a circle are (0, 0). If one **endpoint** of the diameter is (-3, 4), find the coordinates of the other endpoint of the diameter.<sup>1</sup> (Hint: SKETCH IT!!)

Independent Practice You may use graph paper if you wish.

1) What are the coordinates of the **midpoint** of the line segment that connects the points (1,2) and (6, 7)?

2) In a circle, the coordinates of the **endpoints** of the diameter are (4,5) and (10,1). What are the coordinates of the **center** of the circle?

3) What are the coordinates of the **midpoint** of the segment whose **endpoints** are (-4, 6) and (-8, -2)?

<sup>&</sup>lt;sup>1</sup> BONUS #1: Find the length of the radius of this circle. BONUS #2: Find the area of this circle in terms of  $\pi$ .

4) The coordinates of the **midpoint** of line segment AB are (1,2). If the coordinates of A are (1,0), find the coordinates of point B.

5) The **midpoint** of  $\overline{AB}$  is M. If the coordinates of A are (2, -6), and the coordinates of M are (5, -1), find the coordinates of B.

**Homework** There are optional graphs at the end of this homework if you need them.

# Find the midpoint of the line segment with the given endpoints.

$$(-4, 4), (5, -1)$$
  $(-1, -6), (-6, 5)$ 

$$(2, 4), (1, -3) (-4, 4), (-2, 2)$$









#### Find the other endpoint of the line segment with the given endpoint and midpoint.

Endpoint: (-1, 9), midpoint: (-9, -10)

Endpoint: (2, 5), midpoint: (5, 1)

Endpoint: (5, 2), midpoint: (-10, -2)

Endpoint: (9, -10), midpoint: (4, 8)

Endpoint: (-9, 7), midpoint: (10, -3)

Endpoint: (-6, 4), midpoint: (4, 8)

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G.GPE.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

#### Warm-Up

The midpoint of line segment AB is (-2, 3). If point A has coordinates (4, 2), find the coordinates of point B.

#### **Understanding Ratios**

Let's say that instead of dividing a line segment in half, we divide it into a ratio of 2:3. That means there will be five equal parts, because 2 + 3 = 5:



We can use this idea to partition line segments into any ratio we choose.

#### Model Problem

The endpoints of  $\overline{DEF}$  are D(1,4) and F(16,14). Determine and state the coordinates of point E, if DE:EF = 2:3.

#### Exercise

Directed line segment  $\overline{PT}$  has endpoints whose coordinates are P(-2,1) and T(4,7). Determine the coordinates of point J that divides the segment in the ratio 2 to 1.

#### Independent Practice/Homework

1) The coordinates of the endpoints of  $\overline{AB}$  are A(-6,-5) and B(4,0). Point P is on  $\overline{AB}$ . Determine and state the coordinates of point P, such that AP:PB is 2:3.

2) What are the coordinates of the point on the directed line segment from K(-5,-4) to L(5,1) that partitions the segment into a ratio of 3 to 2?

3) Point B is between A(2, 5) and C(10,1). Find the coordinates of B, if AB:BC = 1:3.

4) A line segment has endpoints (-6, 7) and (9, 2). What are the coordinates of the point on this line segment that divides it into a ratio of 2:3?

5) Point X divides  $\overline{MN}$  into a ratio of 3:5. If the coordinates of M are (4, 3) and the coordinates of N are (20,11), find the coordinates of X.

6) Directed line segment DEG has endpoints D(0, 8) and G(-24, -16). Find the coordinates of point E such that DE:EG = 5:7.

7)  $\overline{SP}$  has endpoints S(-11, 6) and P(10, -1). Point U is on  $\overline{SP}$ . Determine and state the coordinates of U such that SU:UP = 3:4.

8) Two points are located on a coordinate grid at (8, 1) and (-2, 16). Find the coordinates of the point that is 1/5 of the directed distance from (8, 1) to (-2, 16).

9) Line segment AB has endpoints A(4, 9) and B(9, 19). Is the point that divides  $\overline{AB}$  into a ratio of 2:3 the same point that divides it into a ratio of 3:2? Explain.

#### 10) <u>Challenge</u>

Point B on line segment AC divides  $\overline{AC}$  into a ratio of 2:3. If the coordinates of A are (4, -1) and the coordinates of B are (10, 3), find the coordinates of point C.

As you read, underline all important terms from this unit. Remember to draw diagrams!

**1.** Draw and label plane  $\mathcal{N}$  containing two lines that intersect at *B*.

Use the figure to name each of the following.

- **2.** four noncoplanar points **3.** line containing *B* and *E*
- **4.** The coordinate of *A* is -3, and the coordinate of *B* is 0.5. Find *AB*.
- **5.** *E*, *F*, and *G* represent mile markers along a straight highway. Find *EF*.
- **6.** *J* is the midpoint of *HK*. Find *HJ*, *JK*, and *HK*.



9)  $\overrightarrow{TV}$  bisects  $\angle RTS$ . If  $m \angle RTV = (16x - 6)^\circ$  and  $m \angle VTS = (13x + 9)^\circ$ , what is  $m \angle RTV$ ?





10) Find the distance between A(-12, 13) and B(-2, -11).

11) Find the midpoint of the segment with endpoints (-4, 6) and (3, 2).

12) M is the midpoint of  $\overline{LN}$ . M has coordinates (-5, 1) and L has coordinates (2, 4). Find the coordinates of N.

13) Directed line segment GH is divided by point I into a ratio of 4:5. If point G has coordinates (-3, 5) and point H has coordinates (6, -13), determine and state the coordinates of point I.

#### **Constructions**

14) Construct a segment congruent to  $\overline{AB}$ .



15) Copy angle S at vertex X.



X•

16) Construct the bisector of the angle below. Label it  $\overrightarrow{QT}$ . Name two congruent angles.

