Geometry III: Constructions

Practice Question 12.1

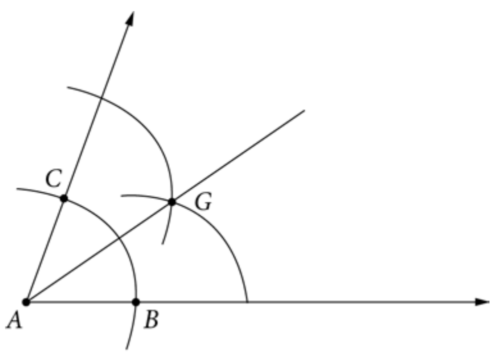
1. (i) Construct the following angles:

(ii) Construct the bisector of each of the angles in (i) above.

(a) 70°

<u>Steps:</u>

1. Draw a ray *AB*
2. Using a protractor, measure an angle of 70° at *A*
3. Placing the compass at *A*, draw the arc *BC*
4. With compass on *C* draw an arc
5. Without changing the width of the compass place, it on *B* and draw another arc
6. Mark the intersection point *G*
7. Join *B* to *G*

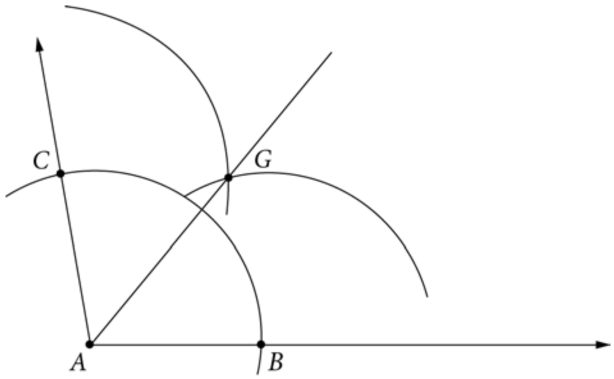


$\angle CAB = 70°$
$\angle CAG = \angle BAG$ 35°

(b) 100°

Steps:

1. Draw a ray AB
2. Using a protractor, measure an angle of 100° at A
3. Placing the compass at A, draw the arc BC
4. With compass on C draw an arc
5. Without changing the width of the compass, place it on B and draw another arc
6. Mark the intersection point G
7. Join B to G



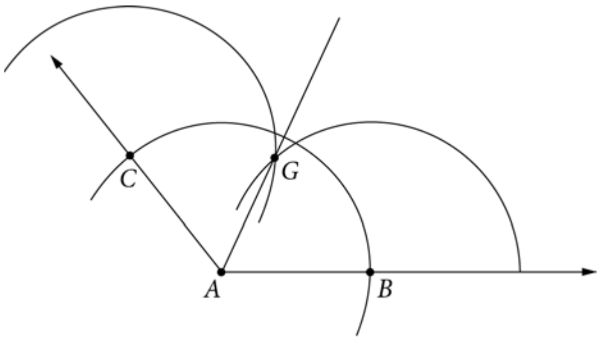
$\angle CAB = 100°$
$\angle CAG = \angle BAG = 50°$

(c) 130°

Steps:

1. Draw a ray AB
2. Using a protractor, measure an angle of 130° at A
3. Placing the compass at A, draw the arc BC
4. With compass on C draw an arc

5. Without changing the width of the compass, place it on *B* and draw another arc
6. Mark the intersection point *G*
7. Join *B* to *G*

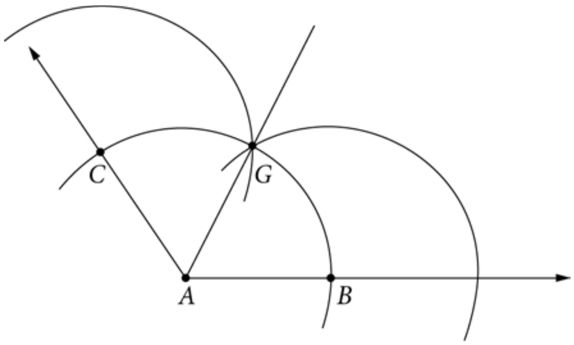


$\angle CAB = 130°$
$\angle CAG = \angle BAG = 65°$

(d) 124°

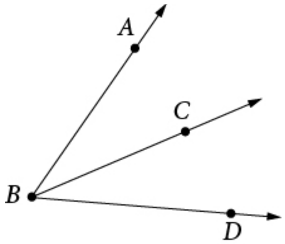
<u>Steps:</u>

1. Draw a ray *AB*
2. Using a protractor, measure an angle of 124° at *A*
3. Placing the compass at *A*, draw the arc *BC*
4. With compass on *C* draw an arc
5. Without changing the width of the compass, place it on *B* and draw another arc
6. Mark the intersection point *G*
7. Join *B* to *G*



$\angle CAB = 124°$
$\angle CAG = \angle BAG = 62°$

2. In the diagram, [*BC*] bisects the angle *ABD*.



(i) If $|\angle ABC| = 54°$, find $|\angle ABD|$.

$|\angle ABD| = |\angle ABC + |\angle DBC|$

$|\angle ABC| = |\angle DBC|$ [*BC*] bisects $\angle ABD$

$|\angle ABC| = 54°$ given

$|\angle ABD| = 54° + 54°$

$|\angle ABD| = 108°$

(ii) If $|\angle ABD| = 112°$, find $|\angle ABC|$.

$|\angle ABD| = |\angle ABC| + |\angle DBC|$

$|\angle ABC| = |\angle DBC|$ [*BC*] bisects $\angle ABD$

$|\angle ABD| = |\angle ABC| + |\angle ABC|$

$|\angle ABD| = 2|\angle ABC|$

$$\frac{|\angle ABD|}{2} = \frac{2|\angle ABC|}{2}$$ divide both sides by 2

$$\frac{|\angle ABD|}{2} = |\angle ABC|$$

$|\angle ABD| = 112°$ given

$$\frac{112°}{2} = |\angle ABC|$$

$56° = |\angle ABC|$

(iii) If $|\angle ABC| = (3x + 1)°$ and $|\angle ABD| = (5x + 19)°$, find the value of x.

$|\angle ABC| = 3x + 1$ given

$|\angle ABD| = 5x + 19$ given

$|\angle ABD| = 2\,|\angle ABC|$ from part (ii)

$5x + 19 = 2\,(3x + 1)$

$5x + 19 = 6x + 2$

$5x + 19 - 5x = 6x + 2 - 5x$ subtract $5x$ from both sides

$19 = x + 2$

$19 - 2 = x + 2 - 2$ subtract 2 from both sides

$17° = x$

(iv) If $|\angle ABC| = (5x - 3)°$ and $|\angle CBD| = (2x + 15)°$, find x.

$|\angle ABC| = 5x - 3$ given

$|\angle CBD| = 2x + 15$ given

$|\angle ABC| = |\angle CBD|$ $[BC]$ bisects $\angle ABD$

$5x - 3 = 2x + 15$

$5x - 3 - 2x = 2x + 15 - 2x$ subtract $2x$ from both sides

$3x - 3 = 15$

$3x - 3 + 3 = 15 + 3$ Add 3 to both sides

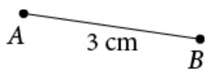
$3x = 18$

$\frac{3x}{3} = \frac{18}{3}$ Divided both sides by 3

$x = 6°$

3. Draw the following line segments and construct the perpendicular bisector of each one.

Measure each side to verify the line segment has been bisected.

(i) 

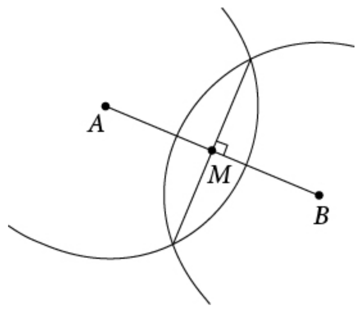
<u>Steps:</u>

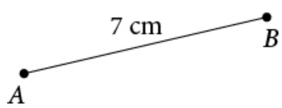
1. Draw [*AB*] 3 cm in length.
2. Plot the compass on *A* with width greater than half [*AB*] draw an arc.
3. Keeping compass width, the same and point on *B* draw another arc.
4. Draw a line through the points of intersection of these arcs.

$|AM| = 1{\cdot}5$ cm

$|BM| = 1{\cdot}5$ cm

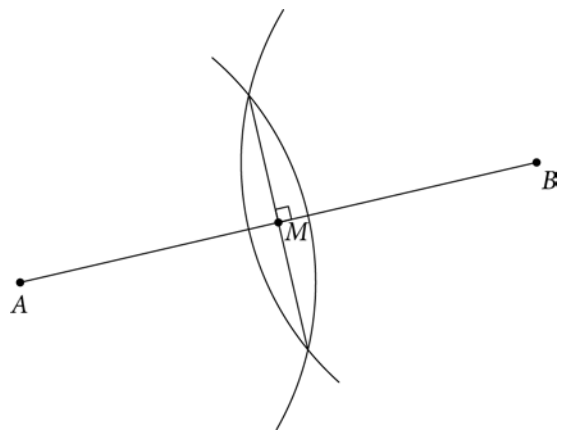
$|AM| = |BM|$



(ii) 

Steps:

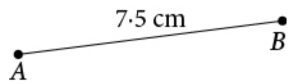
1. Draw [*AB*] 7 cm in length.
2. Plot the compass on *A* with width greater than half [*AB*] draw an arc.
3. Keeping compass width, the same and point on *B* draw another arc.
4. Draw a line through the points of intersection of these arcs.



$|AM| = 3{\cdot}5$ cm

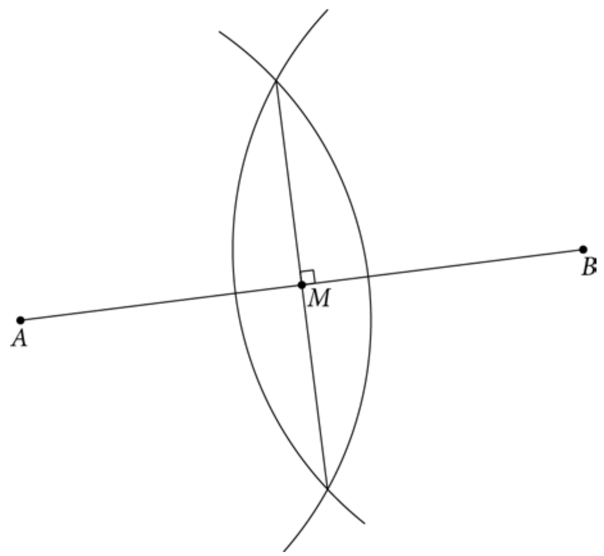
$|BM| = 3{\cdot}5$ cm

$|AM| = |BM|$

(iii) 

Steps:

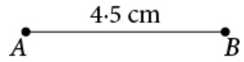
1. Draw [AB] 7·5 cm in length.
2. Plot the compass on A with width greater than half [AB] draw an arc.
3. Keeping compass width, the same and point on B draw another arc.
4. Draw a line through the points of intersection of these arcs.



$|AM| = 3·75$ cm

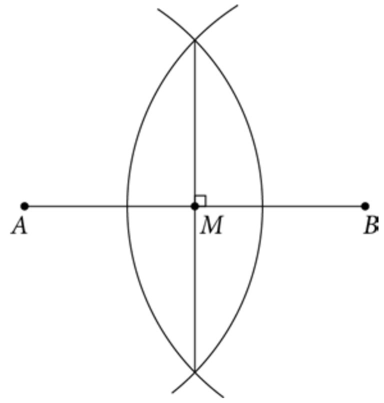
$|BM| = 3·75$ cm

$|AM| = |BM|$

(iv) 

Steps:

1. Draw [AB] 4·5 cm in length.
2. Plot the compass on A with width greater than half [AB] draw an arc.
3. Keeping compass width, the same and point on B draw another arc.
4. Draw a line through the points of intersection of these arcs.



$|AM| = 2·25$ cm

$|BM| = 2·25$ cm

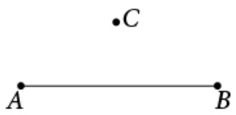
$|AM| = |BM|$

4. Copy the line segment [AB] and the point C shown in each case below. Construct the line segment through C that is:

(i) perpendicular to [AB]

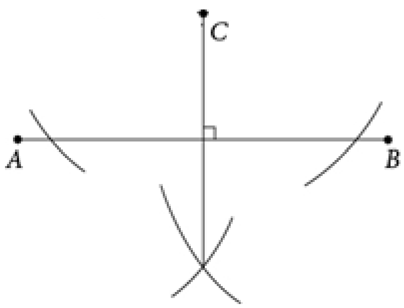
(ii) parallel to [AB].

(a)



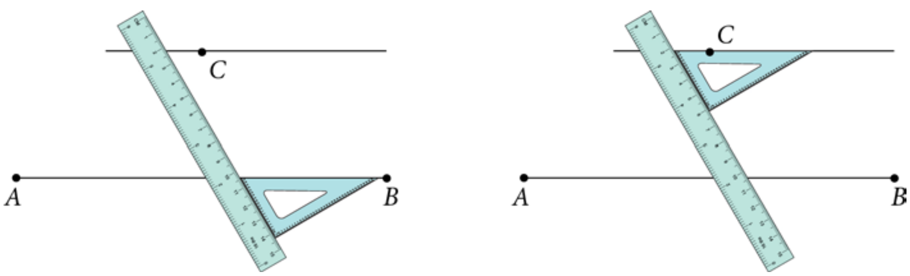
(i) Steps for perpendicular

1. Draw line [AB] and point C
2. With compass on C draw arcs on [AB]
3. With compass point on each arc on [AB] draw another 2 arcs
4. Join C to the point where the arcs intersect.

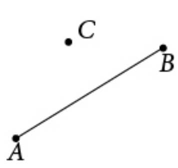


(ii) Steps for parallel line

1. Draw line segment [AB]
2. Put one edge of the set square on AB
3. Put a straight edge against one of the other edges
4. Slide the set square along the straight edge until it reaches the point C.
5. Draw a line through C.

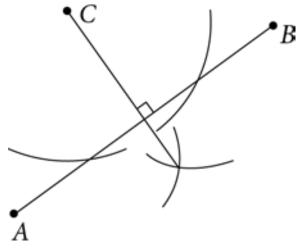


(b)



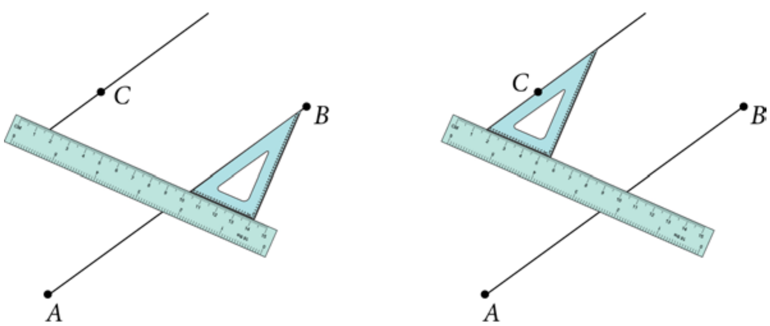
(i) Steps for perpendicular

1. Draw line [*AB*] and point *C*.
2. With compass on *C* draw arcs on [*AB*].
3. With compass point on each arc on [*AB*] draw another 2 arcs.
4. Join *C* to the point where the arcs intersect.

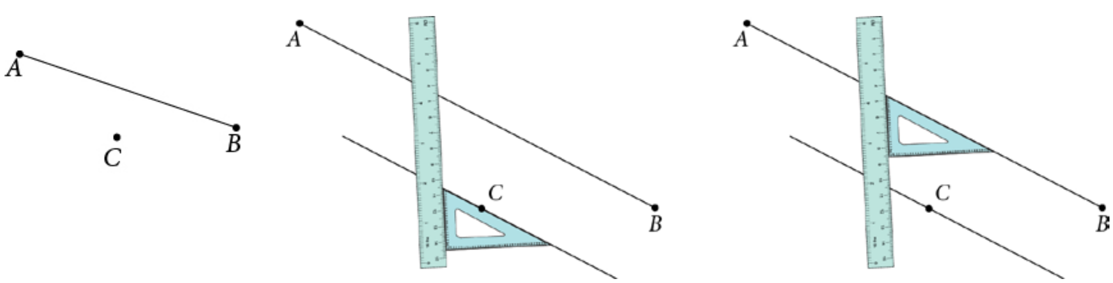


(ii) Steps for parallel line

1. Draw line segment [*AB*]
2. Put one edge of the set square on *AB*
3. Put a straight edge against one of the other edges
4. Slide the set square along the straight edge until it reaches the point *C*.
5. Draw a line through *C*.

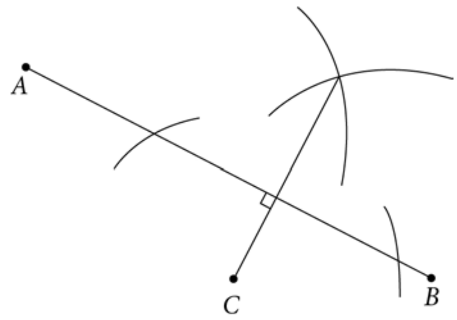


(c)



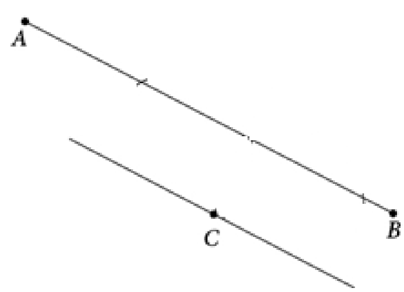
(i) Steps for perpendicular

1. Draw line [*AB*] and point *C*.
2. With compass on *C* draw arcs on [*AB*].
3. With compass point on each arc on [*AB*] draw another 2 arcs.
4. Join *C* to the point where the arcs intersect.

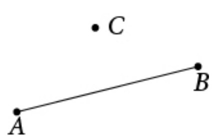


(ii) Steps for parallel line

1. Draw line segment [*AB*]
2. Put one edge of the set square on *AB*
3. Put a straight edge against one of the other edges
4. Slide the set square along the straight edge until it reaches the point *C*.
5. Draw a line through *C*.

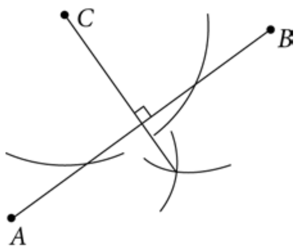


(d)



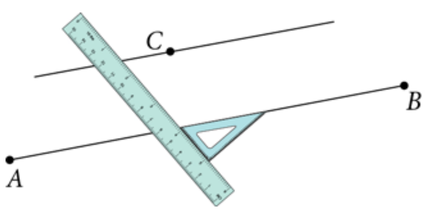
(i) Steps for perpendicular

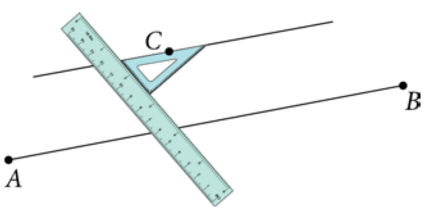
1. Draw line [*AB*] and point *C*.
2. With compass on *C* draw arcs on [*AB*].
3. With compass point on each arc on [*AB*] draw another 2 arcs.
4. Join *C* to the point where the arcs intersect.



(ii) Steps for parallel line

1. Draw line segment [*AB*]
2. Put one edge of the set square on *AB*
3. Put a straight edge against one of the other edges
4. Slide the set square along the straight edge until it reaches the point *C*.
5. Draw a line through *C*.



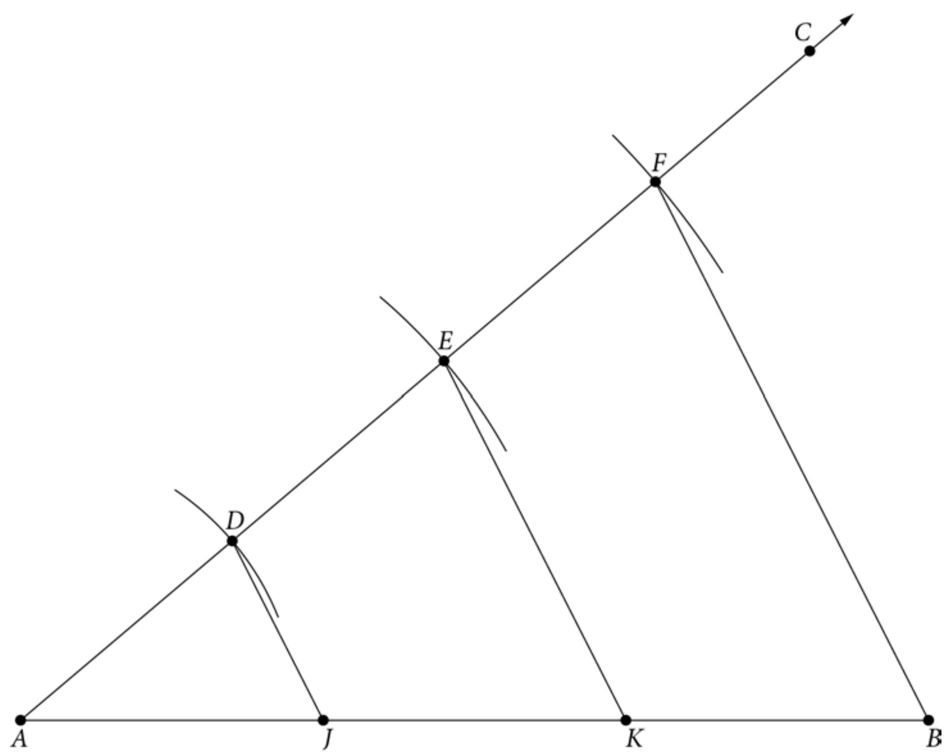


5. Draw the line segment [*AB*] of length given in each part below and then divide each line segment into three equal parts. Verify your result by measuring.

(i) 12 cm

<u>Steps:</u>

1. Draw [*AB*] = 12 cm
2. Pick a point *C*, not on [*AB*] and draw *AC*
3. With point of compass on *A* draw on arc on *AC*. Label as *D*.
4. With same compass width and point on *D* draw another arc. Label as *E*.
5. With same compass width and point on *E* draw a third arc. Label as *F*.
6. Join *B* to *F*
7. Join *E* to *K* and *D* to *J* making sure the lines arc parallel to *BF*.



$|AJ| = 4$ cm

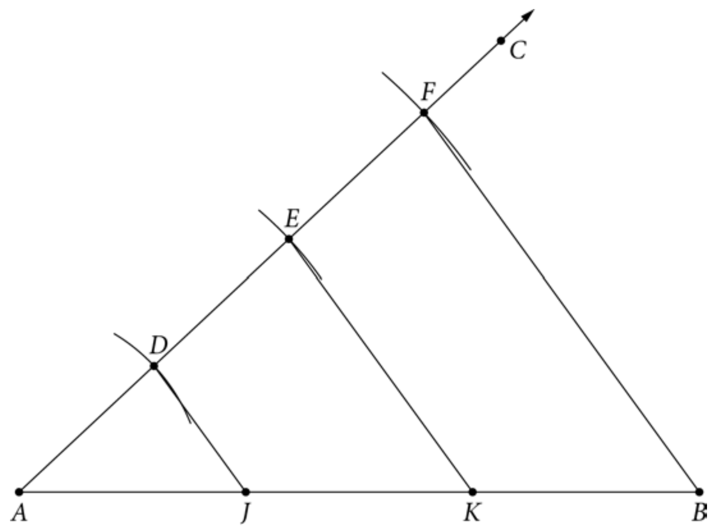
$|JK| = 4$ cm

$|KB| = 4$ cm

(ii) 9 cm

Steps:

1. Draw $[AB] = 9$ cm.
2. Pick a point C, not on $[AB]$ and draw AC
3. With point of compass on A draw on arc on AC. Label as D.
4. With same compass width and point on D draw another arc. Label as E.
5. With same compass width and point on E draw a third arc. Label as F.
6. Join B to F
7. Join E to K and D to J making sure the lines arc parallel to BF.



$|AJ| = 3$ cm

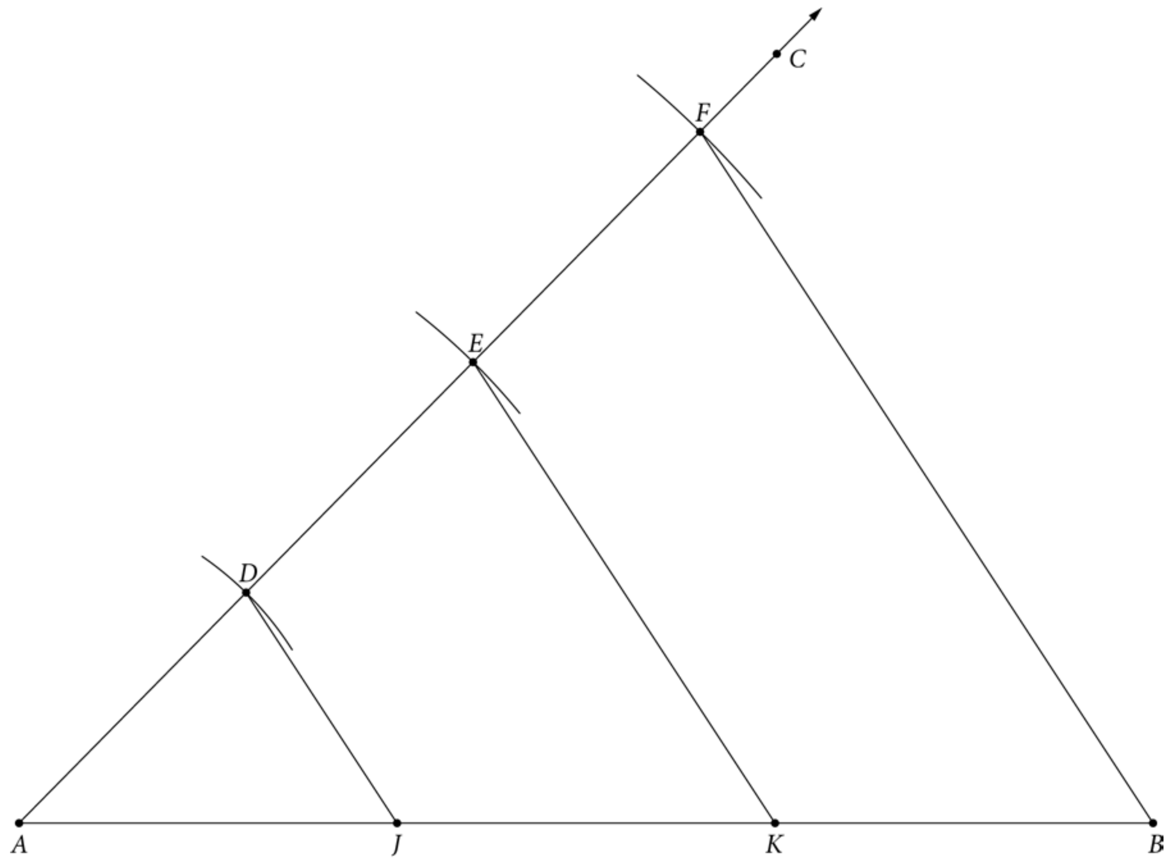
$|JK| = 3$ cm

$|KB| = 3$ cm

(iii) 15 cm

Steps:

1. Draw $[AB] = 15$ cm.
2. Pick a point C, not on $[AB]$ and draw AC
3. With point of compass on A draw on arc on AC. Label as D.
4. With same compass width and point on D draw another arc. Label as E.
5. With same compass width and point on E draw a third arc. Label as F.
6. Join B to F
7. Join E to K and D to J making sure the lines arc parallel to BF.



$|AJ| = 5$ cm

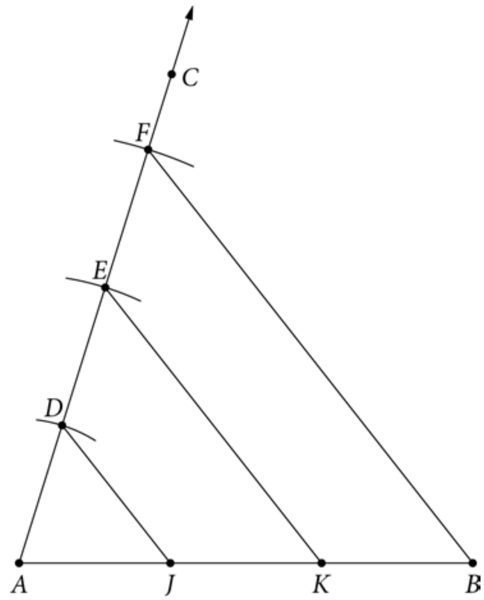
$|JK| = 5$ cm

$|KB| = 5$ cm

(iv) 6 cm

Steps:

1. Draw [*AB*] = 6 cm.
2. Pick a point *C*, not on [*AB*] and draw *AC*
3. With point of compass on *A*, draw on arc on *AC*. Label as *D*.
4. With same compass width and point on *D* draw another arc. Label as *E*.
5. With same compass width and point on *E* draw a third arc. Label as *F*.
6. Join *B* to *F*
7. Join *E* to *K* and *D* to *J* making sure the lines arc parallel to *BF*.



|*AJ*| = 2 cm

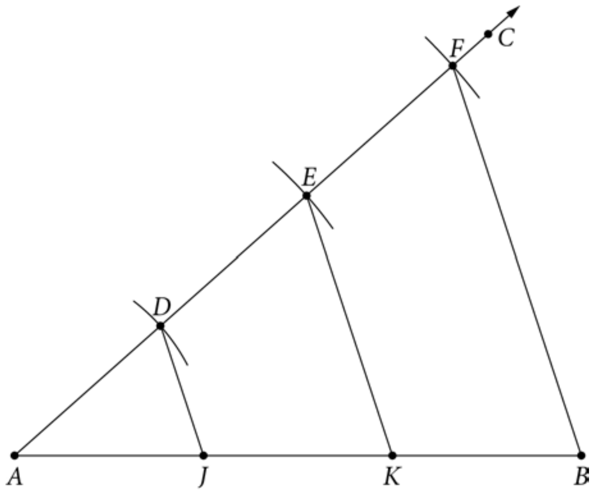
|*JK*| = 2 cm

|*KB*| = 2 cm

(v) 7·5 cm

Steps:

1. Draw [*AB*] = 7·5 cm.
2. Pick a point *C*, not on [*AB*] and draw *AC*
3. With point of compass on *A* draw on arc on *AC*. Label as *D*.
4. With same compass width and point on *D* draw another arc. Label as *E*.
5. With same compass width and point on *E* draw a third arc. Label as *F*.
6. Join *B* to *F*
7. Join *E* to *K* and *D* to *J* making sure the lines arc parallel to *BF*.



$|AJ| = 2{\cdot}5$ cm

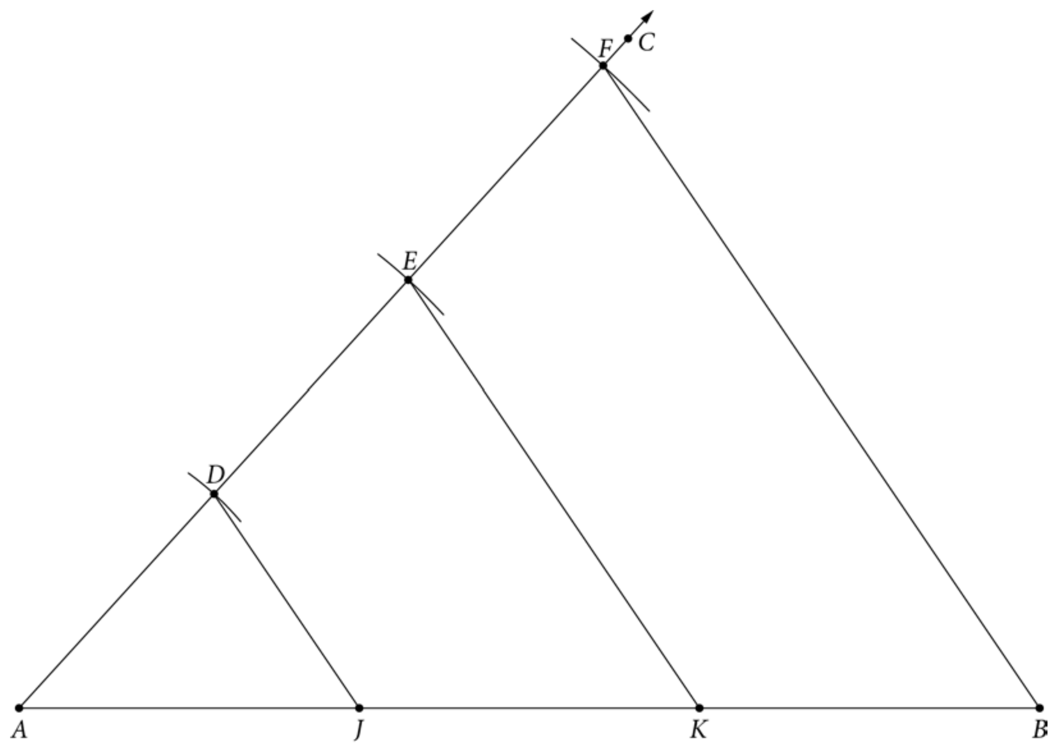
$|JK| = 2{\cdot}5$ cm

$|KB| = 2{\cdot}5$ cm

(vi) 13·5 cm

Steps:

1. Draw [*AB*] = 13·5 cm
2. Pick a point *C*, not on [*AB*] and draw *AC*
3. With point of compass on *A* draw on arc on *AC*. Label as *D*.
4. With same compass width and point on *D* draw another arc. Label as *E*.
5. With same compass width and point on *E* draw a third arc. Label as *F*.
6. Join *B* to *F*
7. Join *E* to *K* and *D* to *J* making sure the lines arc parallel to *BF*.



$|AJ| = 4{\cdot}5$ cm

$|JK| = 4{\cdot}5$ cm

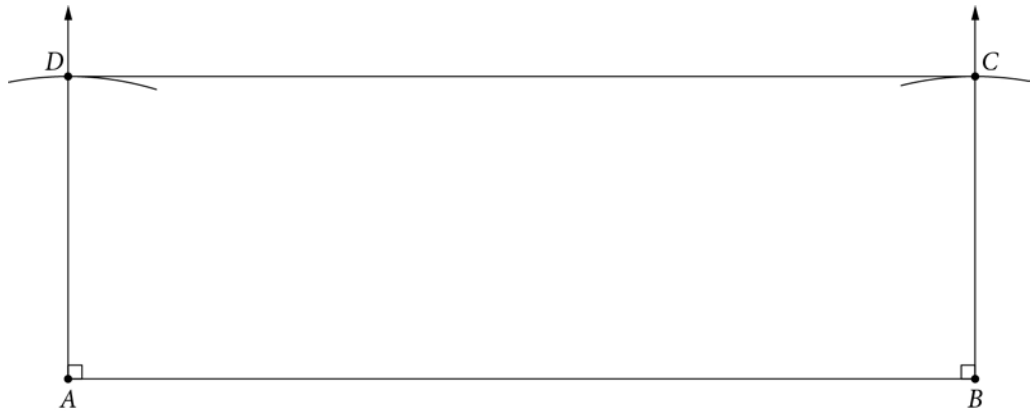
$|KB| = 4{\cdot}5$ cm

6. Construct a rectangle, *ABCD*, using the following measurements:

(i) $|AB| = 12$ cm, $|CB| = 4$ cm

<u>Steps:</u>

1. Draw $[AB] = 12$ cm
2. Draw a ray at *A* perpendicular to *AB*
3. With compass point at *A* and width = 4 cm draw an arc of the ray
4. Repeat step 2 and 3 at *B*
5. Join *D* to *C*.

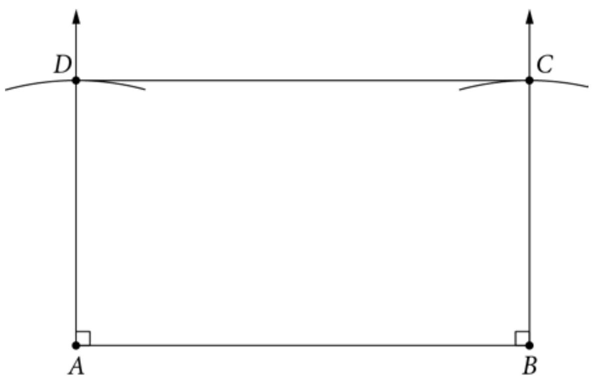


|AB| = 12 |AD| = 4 |DC| = 12 |CB|= 4

(ii) |AB| = 6 cm, |CB| = 3·5 cm

Steps:

1. Draw [AB] = 6 cm.
2. Draw a ray at A perpendicular to AB
3. With compass point at A and width = 3·5 cm draw an arc of the ray.
4. Repeat step 2 and 3 at B.
5. Join D to C

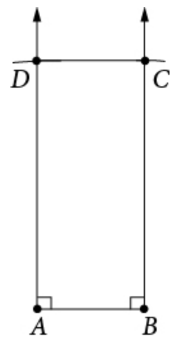


|AB| = 6 |AD| = 3·5 |DC| = 6 |BC| = 3·5

(iii) $|AB| = 2{\cdot}8$ cm, $|CB| = 6{\cdot}6$ cm

Steps:

1. Draw $[AB] = 2{\cdot}8$ cm.
2. Draw a ray at A perpendicular to AB
3. With compass point at A and width $= 6{\cdot}6$ cm draw an arc of the ray
4. Repeat step 2 and 3 at B
5. Join D to C.

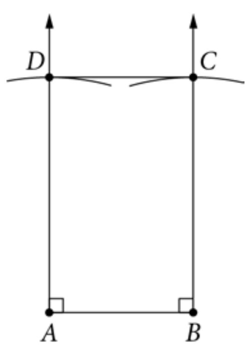


$|AB| = 2{\cdot}8$ cm $\quad |AD| = 6{\cdot}6$ cm $\quad |DC| = 2{\cdot}8$ cm $\quad |BC| = 6{\cdot}6$ cm

(iv) $|AB| = 1{\cdot}9$ cm, $|CB| = 3{\cdot}1$ cm

Steps:

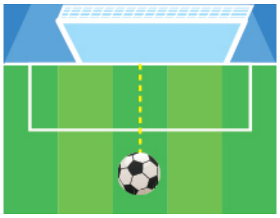
1. Draw $[AB] = 1{\cdot}9$ cm
2. Draw a ray at A perpendicular to AB
3. With compass point at A and width $= 3{\cdot}1$ cm draw an arc of the ray
4. Repeat step 2 and 3 at B
5. Join D to C.



$|AB| = 1{\cdot}9 \quad |AD| = 3{\cdot}1 \quad |DC| = 1{\cdot}9 \quad |BC| = 3{\cdot}1$

7. The penalty spot on a soccer pitch is located on the perpendicular bisector of the goal line. The width of the goal line is 7·3 m and the penalty spot is 11 m from the goal line.

Taking a scale of 1 m = 1 cm, show the location of the penalty spot on a scale diagram.



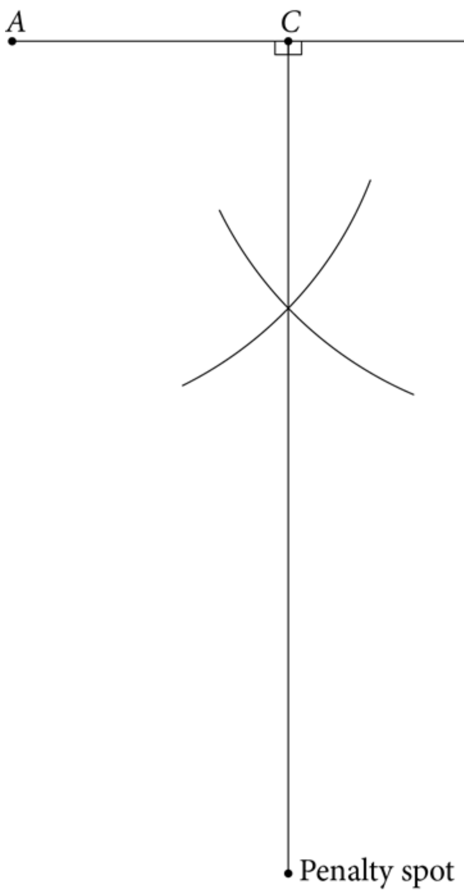
1 m = 1 cm Scale given

∴ 7·3 m = 7·3 cm

11 m = 11 cm

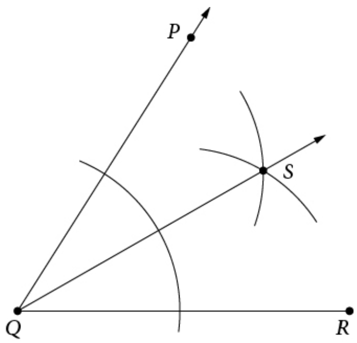
<u>Steps:</u>

1. Draw [*AB*] = 7·3 cm
2. With compass point on *A* and width greater than half [*AB*] draw an arc
3. Repeat step 2 at *B*, with compass width kept the same
4. Draw a line from *AB* through the intersection of the arcs. This line should be 11 cm long.



$|AC| = |BC|$

8. The diagram shows one of the constructions you have studied.



(i) Based on this construction, which of the following statements are true?

(a) $|\angle PQR| = |\angle RQS|$

False

(b) $|\angle RQS| = |\angle PQS|$

True

(c) $|\angle PQS| = \frac{1}{2}|\angle PQR|$

True

(d) $2|\angle PQR| = |\angle PQS|$

False

(ii) Change each of the untrue statements to make them true.

(a) $|\angle PQR| = 2\,|\angle RQS|$

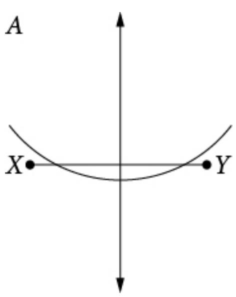
(d) $\frac{1}{2}|\angle PQR| = |\angle PQS|$

Other correct statements include:

$|\angle PQR| = |\angle PQS| + |\angle RQS|$ and $|\angle PQR| = 2\,|\angle PQS|$.

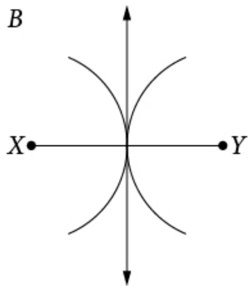
9. Which of the following diagrams represents the construction of the perpendicular bisector of a line?

A



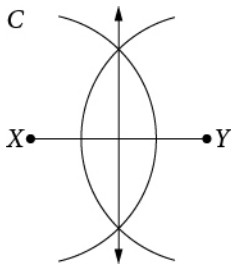
No – Arc drawn did not start at X or Y

B



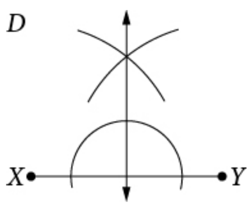
No – compass setting is not greater than half the length of [*XY*]

C



Yes

D



Top arcs would work. But semicircle at the line [*XY*] is not part of this construction.

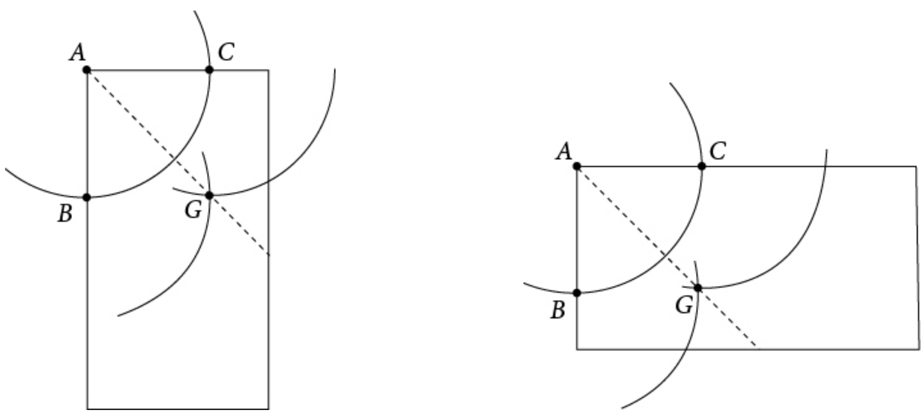
Diagram C represents the construction of the perpendicular bisector of a line.

10. As part of a construction studies project for his Leaving Certificate, Jack must build a frame using the four pieces of wood shown in the diagram. To join the corners neatly, he decides to cut each corner along the bisector of the angle as shown.

Using your compass and ruler, show how Jack bisected the corner angles of the four wooden pieces shown above to create the finished corner pieces.

<u>Steps to bisect the angle</u>

1. With compass point on A draw an arc. Mark points B and C.
2. Step compass width and with point on C draw another arc.
3. Keeping compass width, the same and point on B draw another arc. Label the intersection of these arcs G.
4. Join A to G.



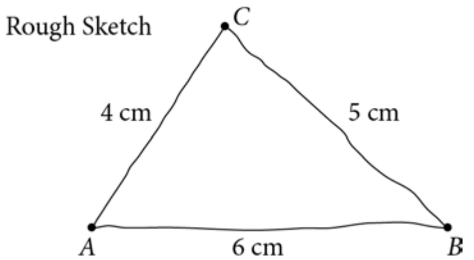
Practice Question 12.2

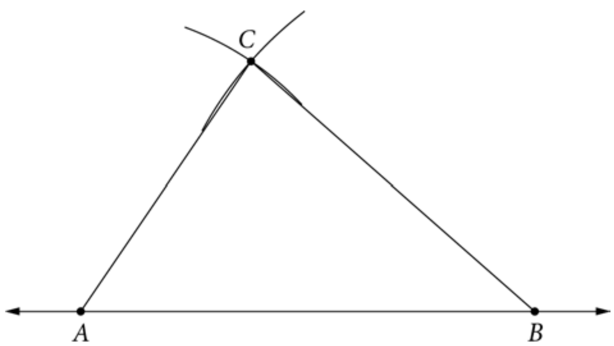
1. Construct the triangle *ABC* where:

(i) |*AB*| = 6 cm, |*BC*| = 5 cm and |*AC*| = 4 cm

<u>Steps:</u>

1. Draw a rough sketch
2. Draw [*AB*] = 6 cm
3. With compass point on *A* and compass width = 4 cm draw an arc
4. With compass point on *B* and compass width = 5 cm draw an arc
5. Mark the intersection of these arcs *C*
6. Join *A* to *C* and *B* to *C*.





|*AB*| = 6 cm |*BC*| = 5 cm |*AC*| = 4 cm

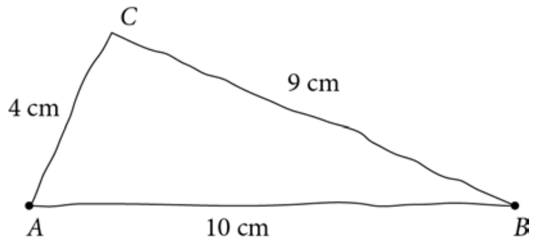
(ii) |*AB*| = 10 cm, |*AC*| = 4 cm and |*BC*| = 9 cm

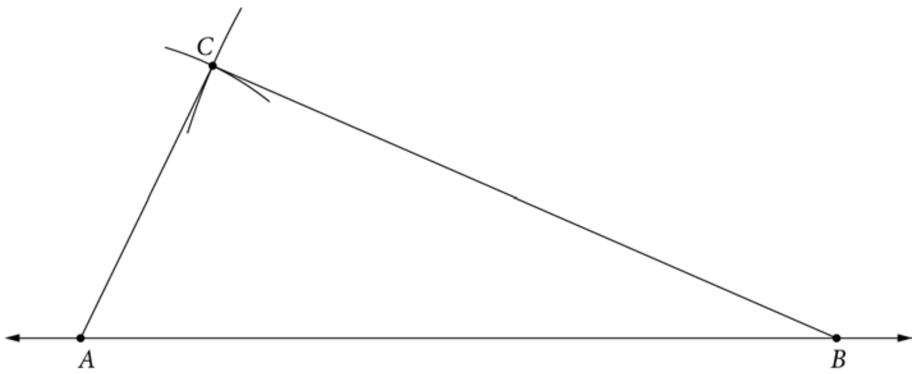
<u>Steps:</u>

1. Draw a rough sketch
2. Draw [*AB*] = 10 cm
3. With compass point on *A* and compass width = 4 cm draw an arc

4. With compass point on B and compass width = 9 cm draw an arc
5. Mark the intersection of these arcs C
6. Join A to C and B to C.

Rough sketch





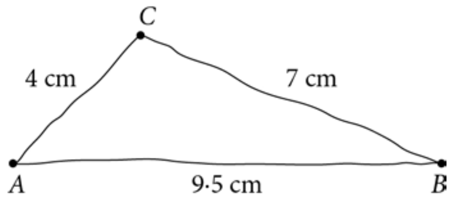
$|AB| = 10$ cm $|BC| = 9$ cm $|AC| = 4$ cm

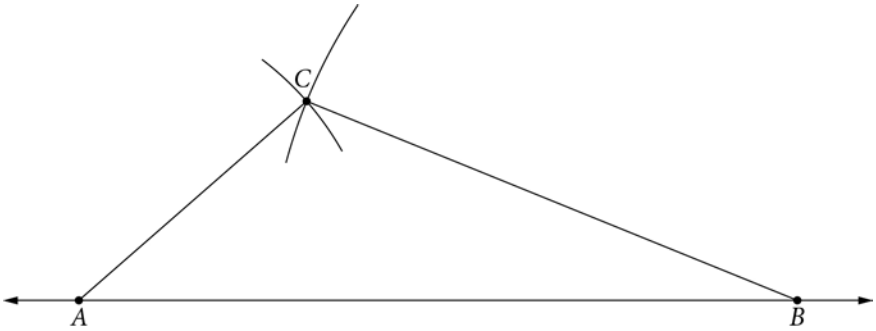
(iii) $|AB| = 9{\cdot}5$ cm, $|BC| = 7$ cm and $|AC| = 4$ cm

Steps:

1. Draw a rough sketch
2. Draw $[AB] = 9{\cdot}5$ cm
3. With compass point on A and compass width = 4 cm draw an arc
4. With compass point on B and compass width = 7 cm draw an arc
5. Mark the intersection of these arcs C
6. Join A to C and B to C.

Rough sketch





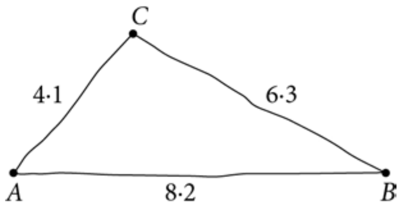
$|AB| = 9{\cdot}5$ cm $|BC| = 7$ cm $|AC| = 4$ cm

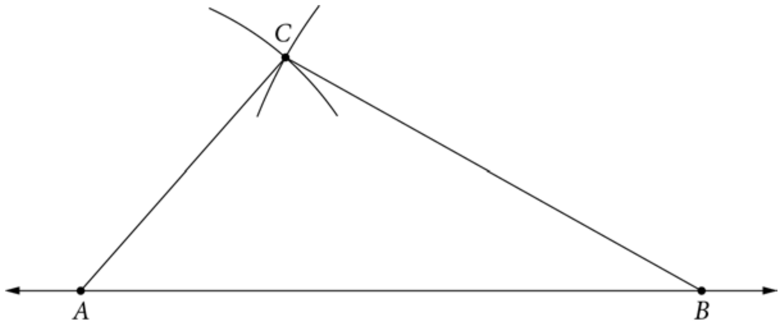
(iv) $|BC| = 6{\cdot}3$ cm, $|AB| = 8{\cdot}2$ cm and $|AC| = 4{\cdot}1$ cm

Steps:

1. Draw a rough sketch
2. Draw $[AB] = 8{\cdot}2$ cm
3. With compass point on A and compass width = 4·1 cm draw an arc
4. With compass point on B and compass width = 6·3 cm draw an arc
5. Mark the intersection of these arcs C
6. Join A to C and B to C.

Rough sketch





$|AB| = 8{\cdot}2$ cm $|BC| = 6{\cdot}3$ cm $|AC| = 4{\cdot}1$ cm

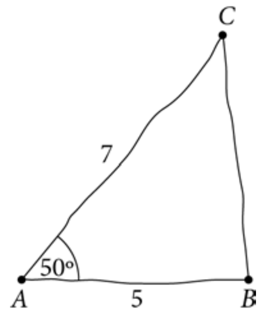
2. Construct the triangle ABC where:

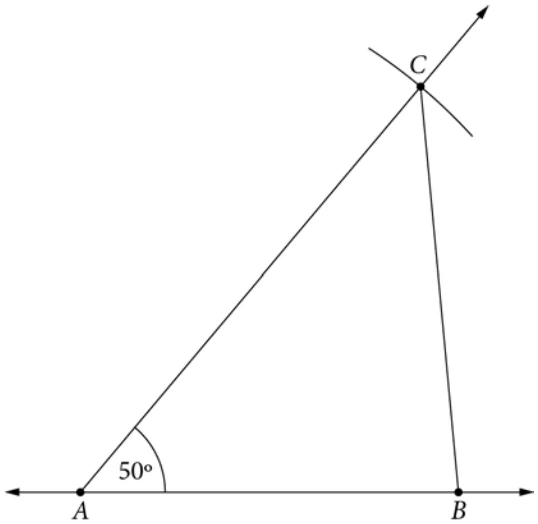
(i) $|AB| = 5$ cm, $|\angle BAC| = 50°$ and $|AC| = 7$ cm

<u>Steps:</u>

1. Draw a rough sketch
2. Draw $[AB] = 5$ cm
3. With a protractor on A draw angle = 50°
4. With compass point on A and compass width = 7 cm draw an arc. Mark as C.
5. Join A to C and B to C.

Rough sketch





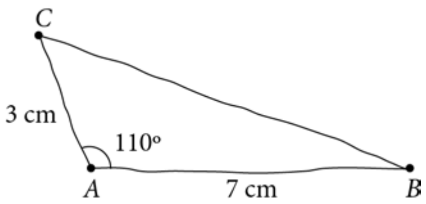
$|AB| = 5$ cm $|\angle BAC| = 50°$ $|AC| = 7$ cm

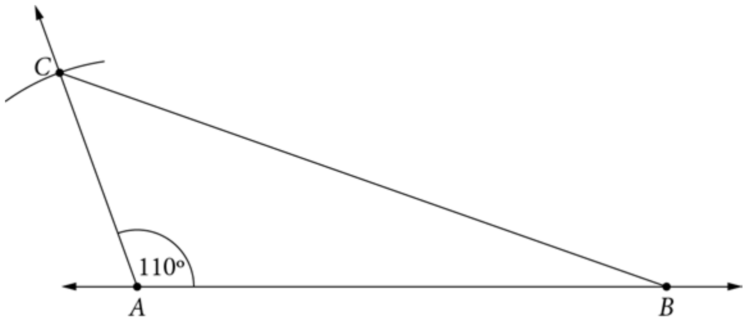
(ii) $|AB| = 7$ cm, $|\angle CAB| = 110°$ and $|AC| = 3$ cm

Steps:

1. Draw a rough sketch
2. Draw $[AB] = 7$ cm
3. With a protractor on A draw angle = 110°
4. With compass point on A and compass width = 3 cm draw an arc. Mark as C.
5. Join A to C and B to C.

Rough sketch





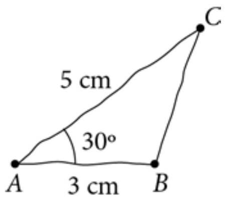
$|AB| = 7$ cm $|\angle BAC| = 110°$ $|AC| = 3$ cm

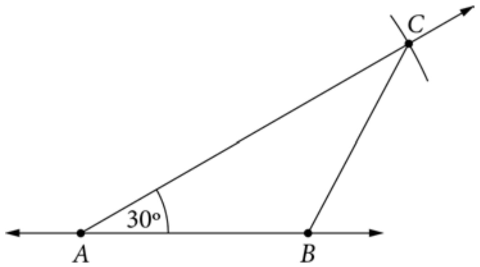
(iii) $|AC| = 5$ cm, $|AB| = 3$ cm and $|\angle BAC| = 30°$

Steps:

1. Draw a rough sketch
2. Draw $[AB] = 3$ cm
3. With a protractor on A draw angle = 30°
4. With compass point on A and compass width = 5 cm draw an arc. Mark as C.
5. Join A to C and B to C.

Rough sketch





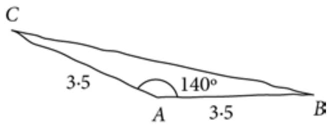
|*AB*| = 3 cm |∠*BAC*| = 30° |*AC*| = 5 cm

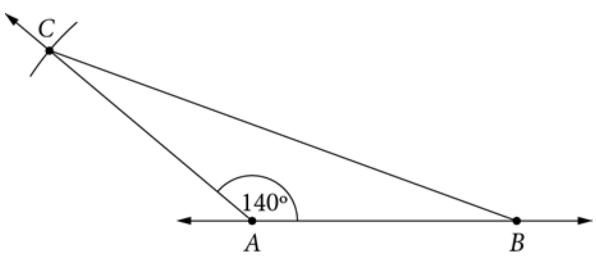
(iv) |*AB*| = 3·5 cm, |∠*BAC*| = 140° and |*AC*| = 3·5 cm

<u>Steps:</u>

1. Draw a rough sketch
2. Draw [*AB*] = 3·5 cm
3. With a protractor on *A* draw angle = 140°
4. With compass point on *A* and compass width = 3·5 cm draw an arc. Mark as *C*.
5. Join *A* to *C* and *B* to *C*.

Rough sketch





|*AB*| = 3·5 cm |∠*BAC*| = 140° |*AC*| = 3·5 cm

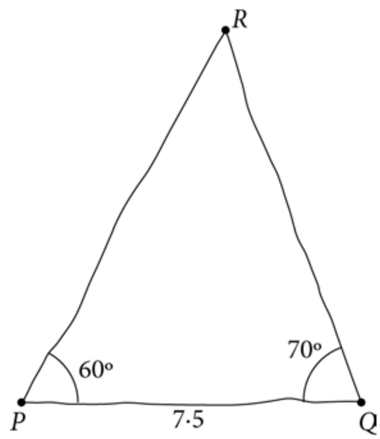
3. Construct the following triangles when given the information below:

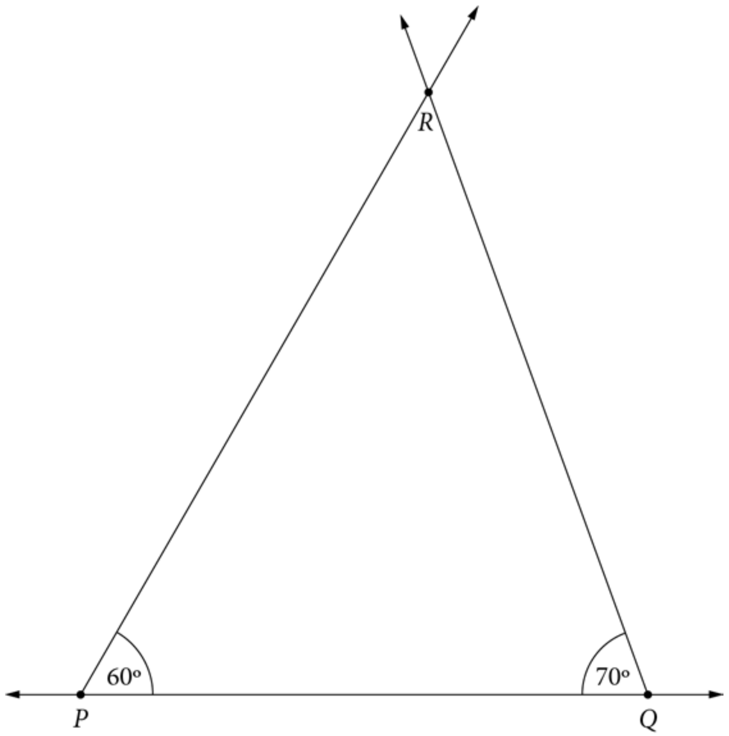
(i) $|\angle QPR| = 60°$, $|PQ| = 7{\cdot}5$ cm and $|\angle PQR| = 70°$

Steps:

1. Draw a rough sketch.
2. Draw $[PQ] = 7{\cdot}5$ cm
3. With protractor on P draw angle = 60°
4. With protractor on Q draw angle = 70°
5. Label the point of intersection of the arms of those angles R
6. Join P to R and Q to R.

Rough sketch





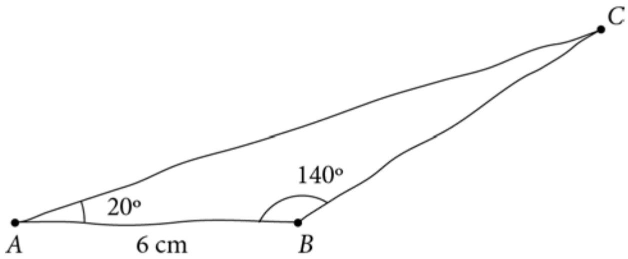
$|PQ| = 7{\cdot}5$ cm $|\angle QPR| = 60°$ $|\angle PQR| = 70°$

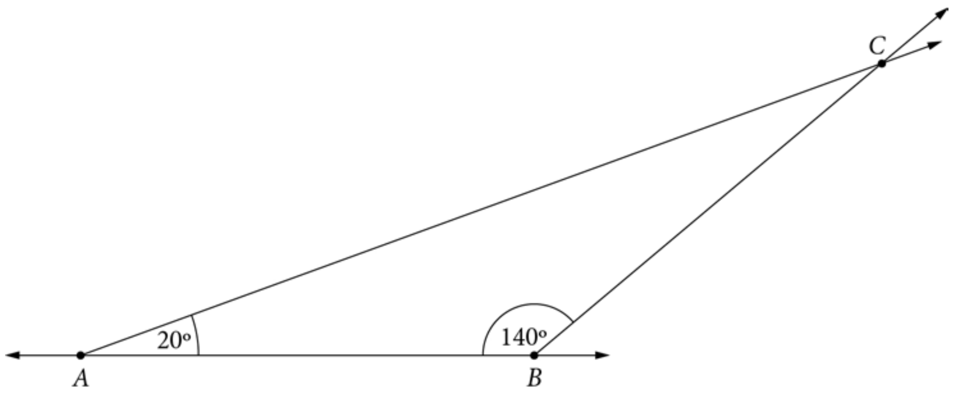
(ii) $|\angle BAC| = 20°$, $|AB| = 6$ cm and $|\angle ABC| = 140°$

Steps:

1. Draw a rough sketch
2. Draw $[AB] = 6$ cm
3. With protractor on A draw angle = 20°
4. With protractor on B draw angle = 140°
5. Label the point of intersection of the arms of those angles C
6. Join A to C and B to C.

Rough sketch





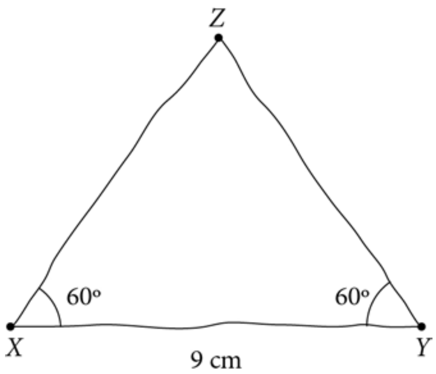
$|AB| = 6$ cm $|\angle BAC| = 20°$ $|\angle ABC| = 140°$

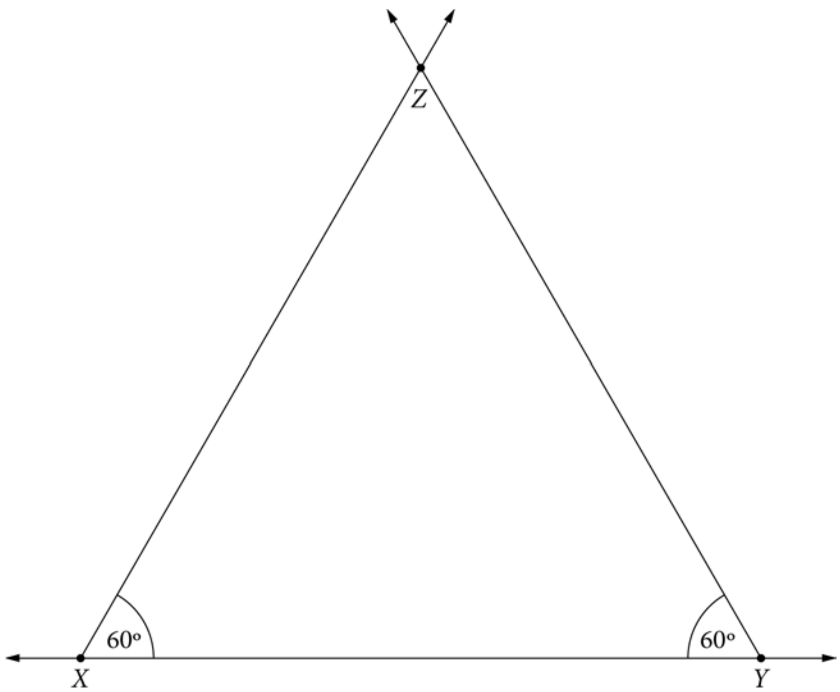
(iii) $|\angle YXZ| = 60°$, $|YX| = 9$ cm and $|\angle XYZ| = 60°$

Steps:

1. Draw a rough sketch
2. Draw $[XY] = 9$ cm
3. With protractor on X draw angle = 60°
4. With protractor on Y draw angle = 60°
5. Label the point of intersection of the arms of those angles Z
6. Join X to Z and Y to Z.

Rough sketch





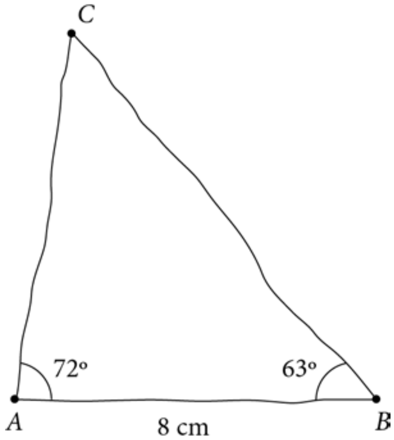
$|XY| = 9$ cm $|\angle YXZ| = 60°$ $|\angle XYZ| = 60°$

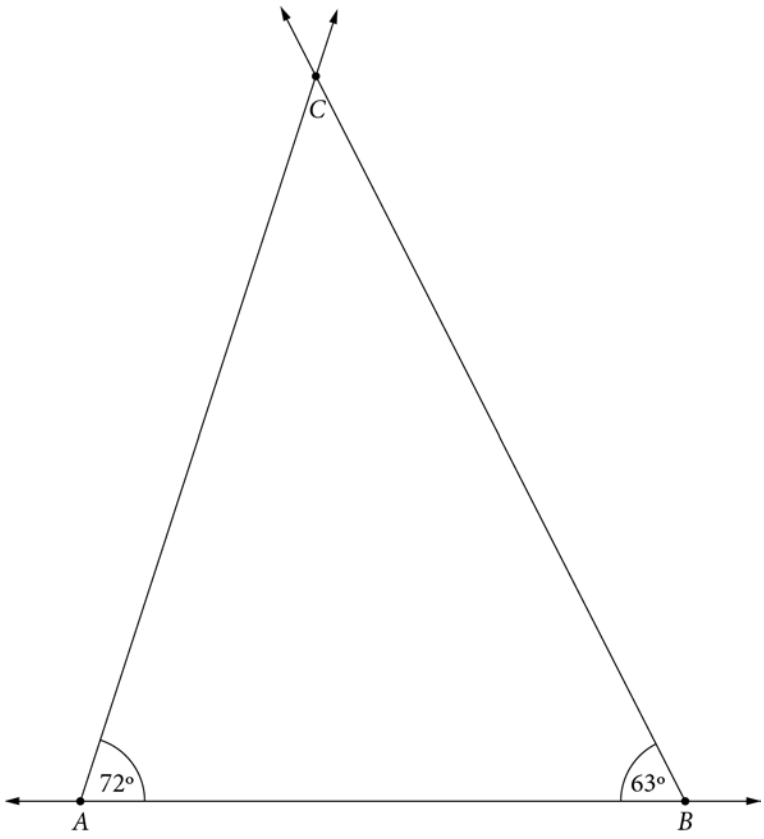
(iv) $|\angle BAC| = 72°$, $|AB| = 8$ cm and $|\angle ABC| = 63°$

<u>Steps:</u>

1. Draw a rough sketch
2. Draw $[AB] = 8$ cm
3. With protractor on A draw angle = 72°
4. With protractor on B draw angle = 63°
5. Label the point of intersection of the arms of those angles C
6. Join A to C and B to C.

Rough sketch





$|AB| = 8$ cm $\quad |\angle BAC| = 72°$ $\quad |\angle ABC| = 63°$

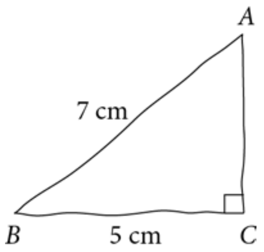
4. Construct the following right-angled triangles ABC when $|AB|$ is the hypotenuse.

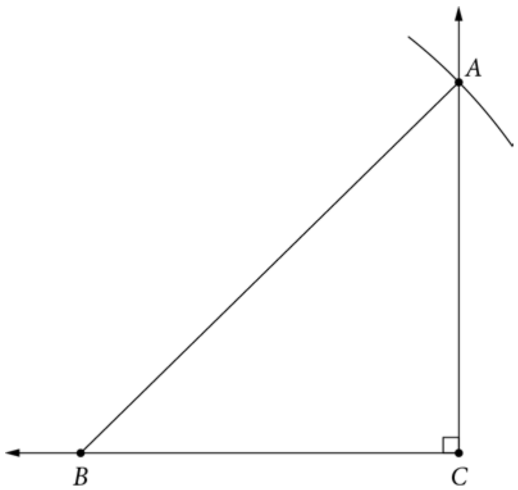
(i) $|AB| = 7$ cm, $|BC| = 5$ cm

Steps:

1. Draw a rough sketch
2. Draw $[BC] = 5$ cm
3. With protractor on C, draw $|\angle ACB| = 90°$
4. With compass point on B and width = 7 cm draw an arc
5. Mark the intersection of the arc and the arm of the angle A
6. Join C to A and B to A.

Rough sketch





$|BC| = 5$ cm

$|\angle BCA| = 90°$

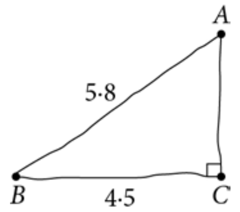
$|AB| = 7$ cm

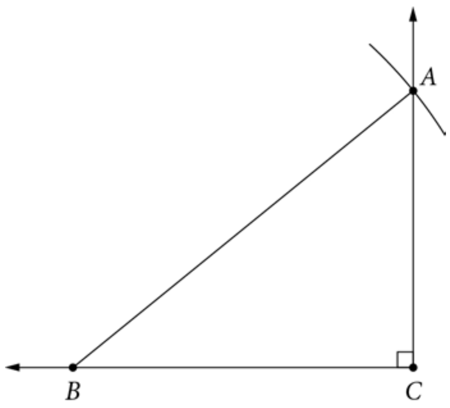
(ii) $|AB| = 5{\cdot}8$ cm, $|BC| = 4{\cdot}5$ cm

<u>Steps</u>:

1. Draw a rough sketch
2. Draw $[BC] = 4{\cdot}5$ cm
3. With protractor on C, draw $|\angle ACB| = 90°$
4. With compass point on B and width $= 5{\cdot}8$ cm draw an arc
5. Mark the intersection of the arc and the arm of the angle A.

Rough sketch





$|BC| = 4{\cdot}5$ cm $|\angle BCA| = 90°$ $|AB| = 5{\cdot}8$ cm

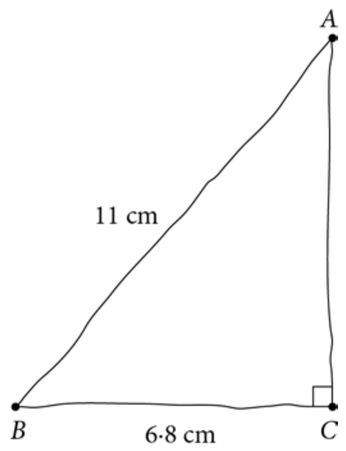
(iii) $|AB| = 11$ cm, $|BC| = 6{\cdot}8$ cm

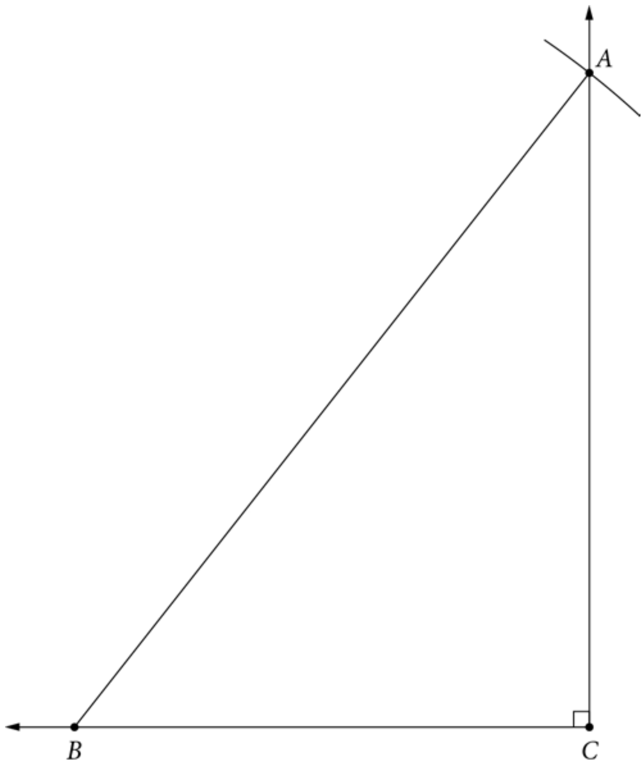
Rough sketch

<u>Steps:</u>

1. Draw a rough sketch
2. Draw $[BC] = 6{\cdot}8$ cm
3. With protractor on C, draw $|\angle ACB| = 90°$
4. With compass point on B and width $= 11$ cm draw an arc.

5. Mark the intersection of the arc and the arm of the angle *A*.





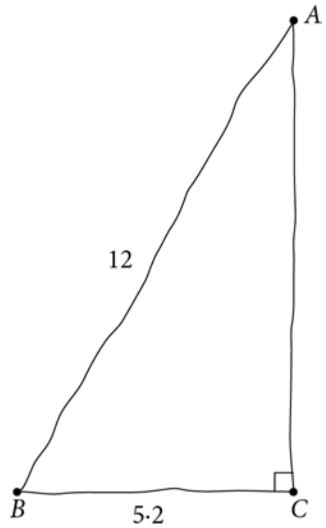
$|BC| = 6{\cdot}8$ cm $|\angle BCA| = 90°$ $|AB| = 11$ cm

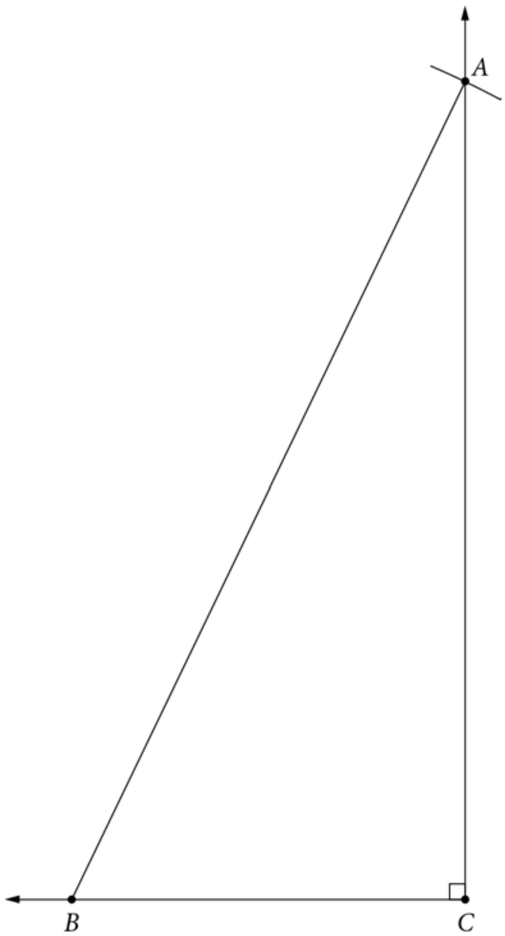
(iv) $|AB| = 12$ cm, $|BC| = 5{\cdot}2$ cm

Steps:

1. Draw a rough sketch
2. Draw $[BC] = 5{\cdot}2$ cm
3. With protractor on C, draw $|\angle ACB| = 90°$
4. With compass point on B and width = 12 cm, draw an arc
5. Mark the intersection of the arc and the arm of the angle A.

Rough sketch





$|BC| = 5{\cdot}2$ cm $\quad |\angle BCA| = 90°$ $\quad |AB| = 12$ cm

5. Construct the following right-angled triangles ABC when $|AB|$ is the hypotenuse.

(i) $|AB| = 8$ cm and $|\angle CAB| = 45°$

$|\angle BAC| + |\angle ABC| + |\angle ACB| = 180°$ Sum of angles in a triangle add to 180°

$|\angle BAC| = 45°$ given

$|\angle ACB| = 90°$ right angle

$45° + |\angle ABC| + 90° = 180°$

$|\angle ABC| + 135° = 180°$

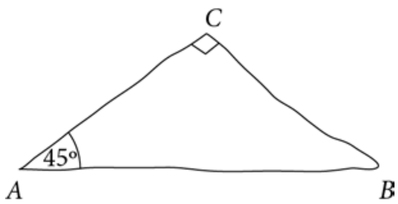
$|\angle ABC| + 135° - 135° = 180° - 135°$ Subtract 135° from both sides

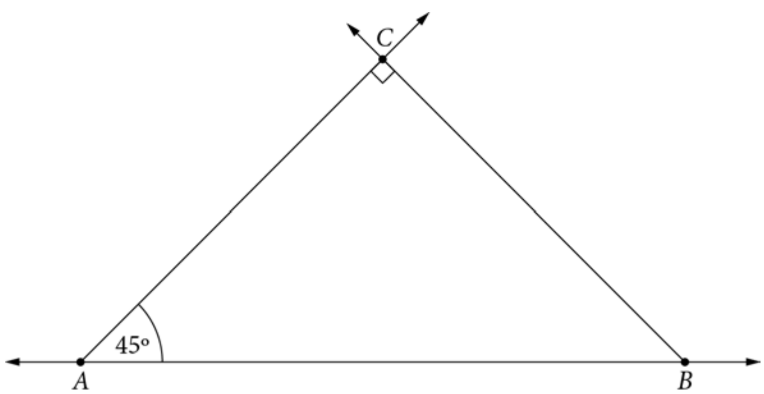
$|\angle ABC| = 45°$

<u>Steps:</u>

1. Draw a rough sketch
2. Draw $[AB] = 8$ cm
3. With protractor on A draw $|\angle BAC| = 45°$
4. Calculate $|\angle ABC| = 45°$ and draw angle at B
5. Mark the intersection C.

Rough sketch:





$|AB| = 8$ $\quad |\angle BAC| = 45°$ $\quad |\angle ABC| = 45°$ $\quad |\angle ACB| = 90°$

(ii) $|AB| = 7$ cm and $|\angle BAC| = 38°$

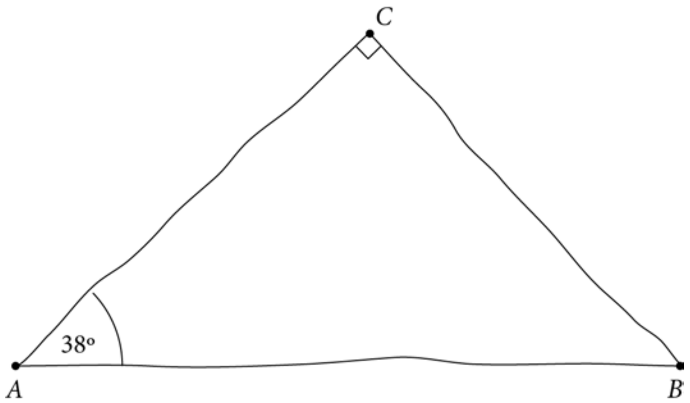
$|\angle BAC| + |\angle ABC| + |\angle ACB| = 180°$ Sum of angles in a triangle add to 180°

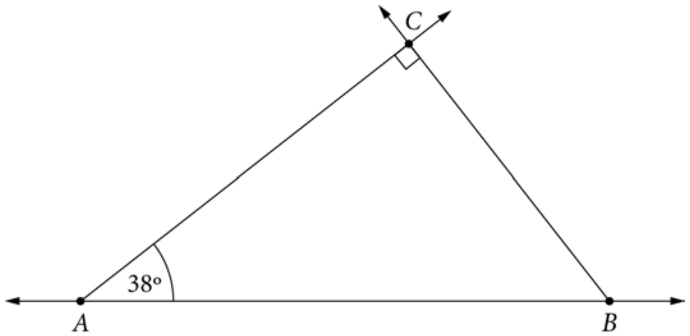
$$\begin{aligned}
|\angle BAC| &= 38° && \text{given} \\
|\angle ACB| &= 90° && \text{right angle} \\
38° + |\angle ABC| + 90° &= 180° \\
|\angle ABC| + 128° &= 180° \\
|\angle ABC| + 128° - 128° &= 180° - 128° && \text{Subtract 128° from both sides} \\
|\angle ABC| &= 52°
\end{aligned}$$

<u>Steps:</u>

1. Draw a rough sketch
2. Draw [AB] = 7 cm
3. With protractor on A draw $|\angle BAC| = 38°$
4. Calculate $|\angle ABC| = 52°$ and draw angle at B
5. Mark the intersection C.

Rough sketch





$|AB| = 7$ cm $|\angle BAC| = 38°$ $|\angle ABC| = 52°$ $|\angle ACB| = 90°$

(iii) $|AB| = 6$ cm and $|\angle CAB| = 56°$

$|\angle BAC| + |\angle ABC| + |\angle ACB| = 180°$ Sum of angles in a triangle add to 180°

$|\angle BAC| = 56°$ given

$|\angle ACB| = 90°$ right angle

$56° + |\angle ABC| + 90° = 180°$

$|\angle ABC| + 146° = 180°$

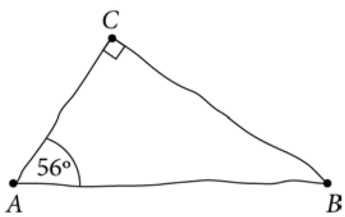
$|\angle ABC| + 146° – 146° = 180° – 146°$ Subtract 146° from both sides

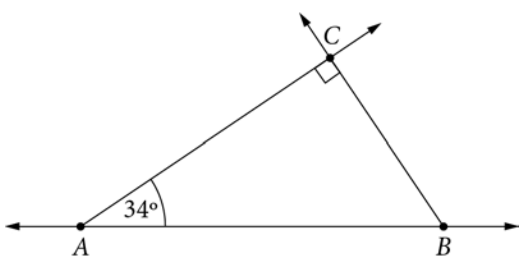
$|\angle ABC| = 34°$

<u>Steps:</u>

1. Draw a rough sketch
2. Draw $[AB] = 6$ cm
3. With protractor on A draw $|\angle BAC| = 56°$
4. Calculate $|\angle ABC| = 34°$ and draw angle at B
5. Mark the intersection C.

Rough sketch





$|AB| = 6$ cm $|\angle BAC| = 56°$ $|\angle ABC| = 34°$ $|\angle ACB| = 90°$

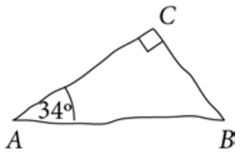
(iv) $|AB| = 4{\cdot}8$ cm and $|\angle BAC| = 34°$

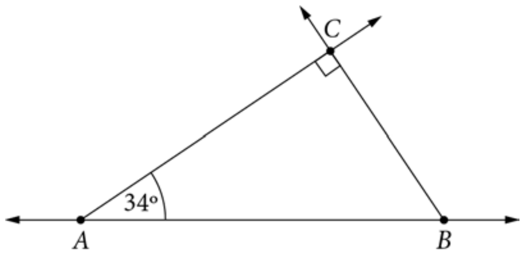
$\lvert\angle BAC\rvert + \lvert\angle ABC\rvert + \lvert\angle ACB\rvert$	$= 180°$	Sum of angles in a triangle add to 180°
$\lvert\angle BAC\rvert$	$= 34°$	given
$\lvert\angle ACB\rvert$	$= 90°$	right angle
$34° + \lvert\angle ABC\rvert + 90°$	$= 180°$	
$\lvert\angle ABC\rvert + 124°$	$= 180°$	
$\lvert\angle ABC\rvert + 124° - 124°$	$= 180° - 124°$	Subtract 124° from both sides
$\lvert\angle ABC\rvert$	$= 56°$	

<u>Steps:</u>

1. Draw a rough sketch
2. Draw $[AB] = 4{\cdot}8$ cm
3. With protractor on A draw $|\angle BAC| = 34°$
4. Calculate $|\angle ABC| = 56°$ and draw angle at B
5. Mark the intersection C.

Rough sketch



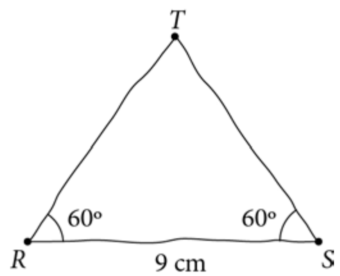


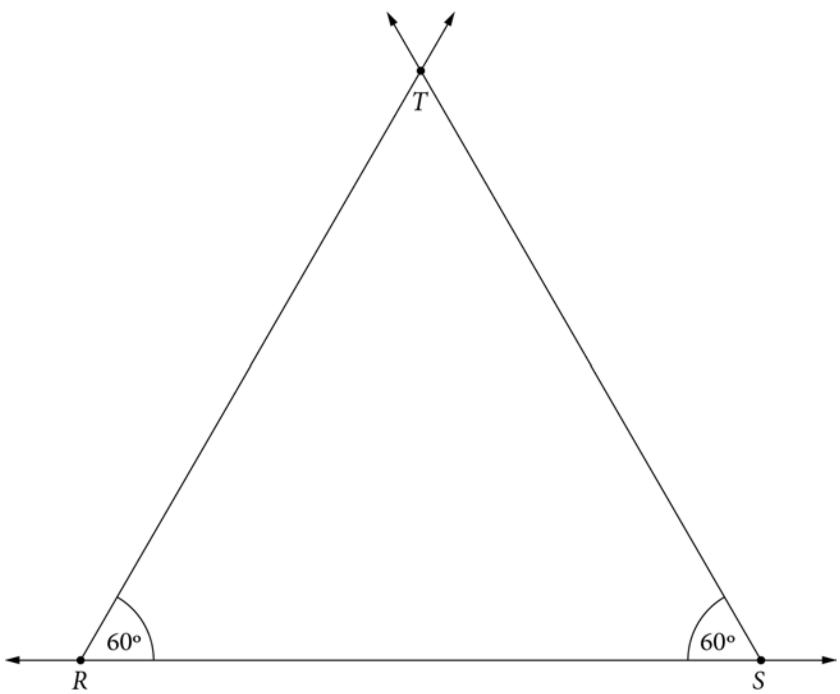
$|AB| = 4{\cdot}8$ $|\angle BAC| = 34°$ $|\angle ABC| = 56°$ $|\angle ACB| = 90°$

6. (i) Construct ΔRST where $|RS| = 9$ cm, $|\angle RST| = 60°$ and $|\angle SRT| = 60°$.

Steps:

1. Draw a rough sketch
2. Draw $[RS] = 9$ cm
3. With protractor on R draw $|\angle SRT| = 60°$
4. With protractor on S draw $|\angle RST| = 60°$
5. Label the intersection of the arms of the angle T.





(ii) What is the measure of angle $|\angle RTS|$?

By measurement using a protractor: $|\angle RTS| = 60°$

By calculation:

$|\angle RST| + |\angle SRT| + |\angle RTS| = 180°$	Sum of angles in a triangle add to 180°
$|\angle RST| = 60°$	given
$|\angle SRT| = 60°$	given
$60° + 60° + |\angle RTS| = 180°$	
$120° + |\angle RTS| = 180°$	
$120° + |\angle RTS| - 120° = 180° - 120°$	Subtract 120° from both sides
$|\angle RTS| = 60°$	

(iii) What type of triangle is ΔRTS?

ΔRST has three equal angles of 60° so it is an equilateral triangle

(iv) State how long the other sides of the triangle are without measuring them.

An equilateral triangle is a triangle in which all three sides are equal.

As $|RS| = 9$ cm, so also $|RT| = 9$ cm and $|ST| = 9$ cm

Practice Questions 12.3

1. Define fully each of the following and describe them as the point of intersection of something:

(i) Circumcentre

The circumcentre is the centre of the circumcircle; a circle that passes through the three vertices of a triangle. The circumcentre is the point at which the perpendicular bisectors of the sides of the triangle intersect.

(ii) Incentre

An incircle is a circle drawn inside a triangle so as to touch but not cross each side of the triangle. The incentre is the centre of the incircle. It is the point at which the bisectors of the angles of the triangle intersect.

(iii) Centroid

The centroid of a triangle is the centre of gravity of the triangle. It is the point of intersection of the medians of the triangle. The median of a triangle is a line that joins the vertex to the midpoint of the opposite side.

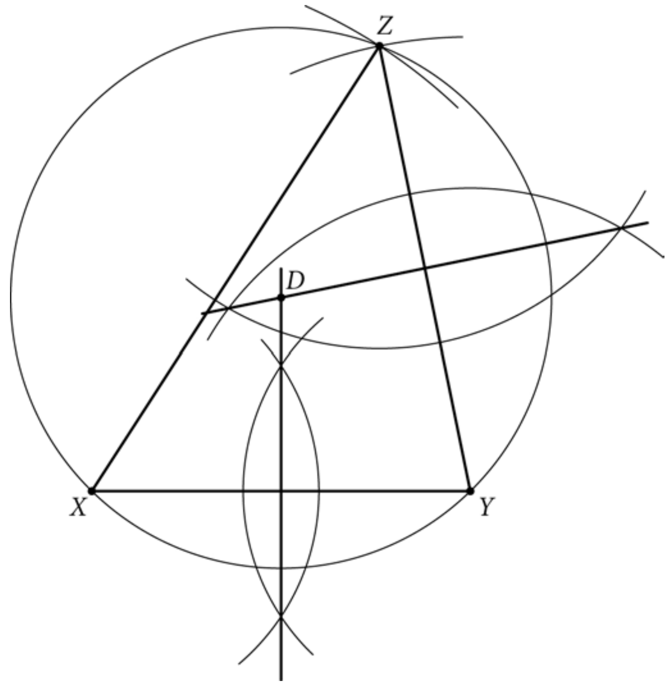
2. Construct the following triangles XYZ and hence construct the circumcircle of each.

(i) $|XY| = 5$ cm, $|YZ| = 6$ cm and $|XZ| = 7$ cm

<u>Steps:</u>

1. Draw $[XY] = 5$ cm
2. With compass point on X and width = 7 cm draw an arc
3. With compass point on Y and width = 6 cm draw another arc
4. Mark the intersection of the arcs Z and join Z to X and Y
5. With compass point on X and width greater than $\frac{1}{2}|XY|$ draw an arc through $[XY]$
6. With compass point on y and same width draw arc an $[XY]$
7. Join the intersection of these arcs to form the perpendicular bisector of $[XY]$
8. Repeat steps 5, 6 and 7 to form perpendicular bisector or $[YZ]$
9. Mark the intersection of the bisectors D

10. With compass point on D and width $|DY|$ draw the circumcircle

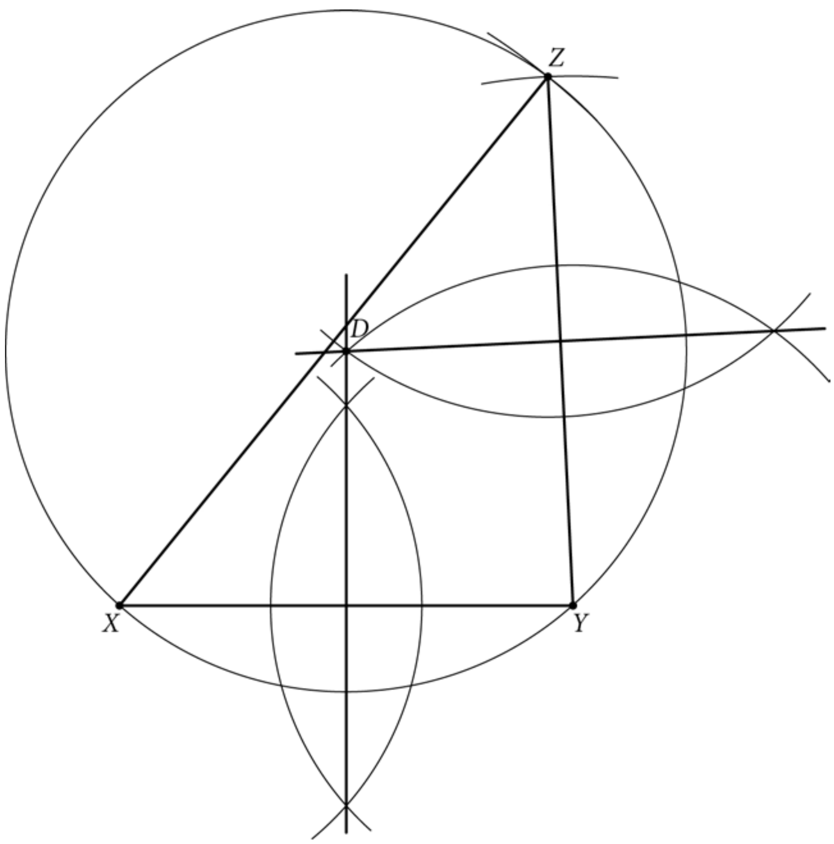


(ii) $|XY| = 6$ cm, $|YZ| = 7$ cm and $|XZ| = 9$ cm

<u>Steps:</u>

1. Draw $[XY] = 6$ cm
2. With compass point on X and width = 9 cm draw an arc
3. With compass point on Y and width = 7 cm draw another arc
4. Mark the intersection of the arcs Z and join Z to X and Y
5. With compass point on x and width greater than $\frac{1}{2}|XY|$ draw an arc through $[XY]$
6. With compass point on y and same width draw an arc through $[XY]$
7. Join the intersection of these arcs to form the perpendicular bisector of $[XY]$

8. Repeat steps 5, 6 and 7 to form perpendicular bisector or [*YZ*]
9. Mark the intersection of the bisectors *D*
10. With compass point on *D* and width |*DY*| draw the circumcircle

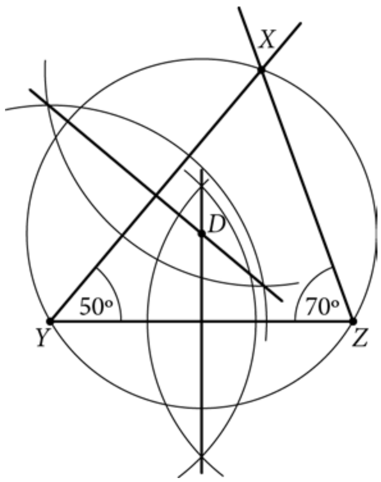


(iii) $|\angle XYZ| = 50°$, $|\angle YZX| = 70°$ and $|YZ| = 4$ cm

<u>Steps:</u>

1. Draw $[YZ] = 4$ cm
2. With protractor on *y* measure $|\angle XYZ| = 50°$
3. With protractor on *z* measure $|\angle YZX| = 70°$
4. Mark the point of intersection of the arms of the angles *X*
5. With compass on point on *y* and width greater than $\frac{1}{2}|XY|$, draw a large arc.
6. With compass point on *x* and same width as in step 5 draw another arc.

7. Join the points of intersection of these arcs to form the perpendicular of [XY].
8. Repeat steps 5, 6 and 7 to draw the perpendicular bisector or [YZ].
9. Mark the intersection of the bisectors D.
10. With compass point on D and width $|DY|$ draw the circumcircle.

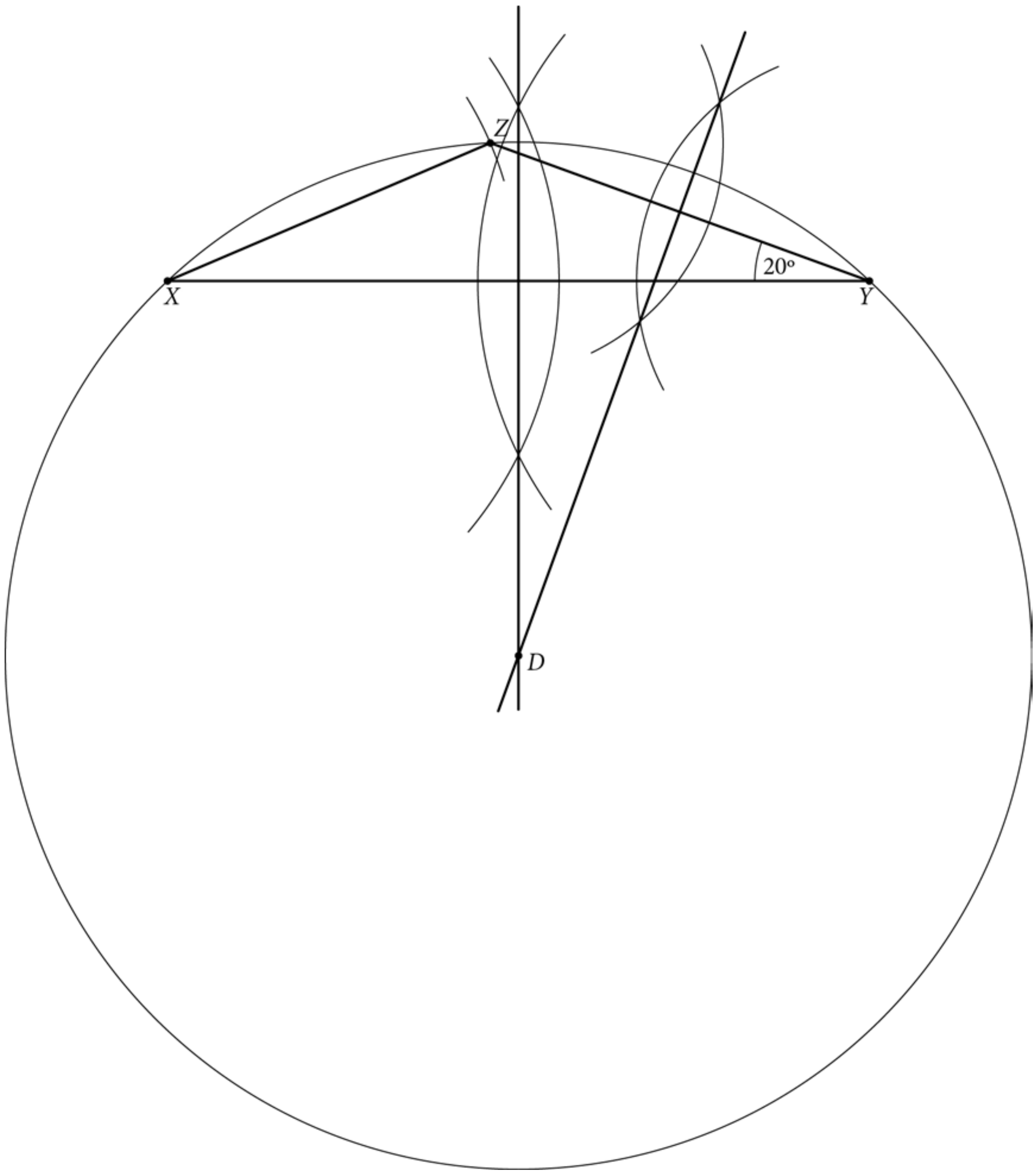


(iv) $|XY| = 10$ cm, $|YZ| = 5$ cm and $|\angle XYZ| = 20°$

Steps:

1 Draw [XY] = 10 cm
2. With protractor on Y draw $|\angle XYZ| = 20°$
3. With compass point on Y and width = 5 cm draw an arc on the arm of the angle mark as Z
4. Join Z to Y and X
5. With compass on point on Y and width greater than $\frac{1}{2}|XY|$ draw a large arc
6. With compass point on X and same width as in step 5 draw another arc
7. Join the points of intersection of these arcs to form the perpendicular of [XY]
8. Repeat steps 5, 6 and 7 to draw the perpendicular bisector or [YZ]

9. Mark the intersection of the bisectors D
10. With compass point on D and width $|DY|$ draw the circumcircle.

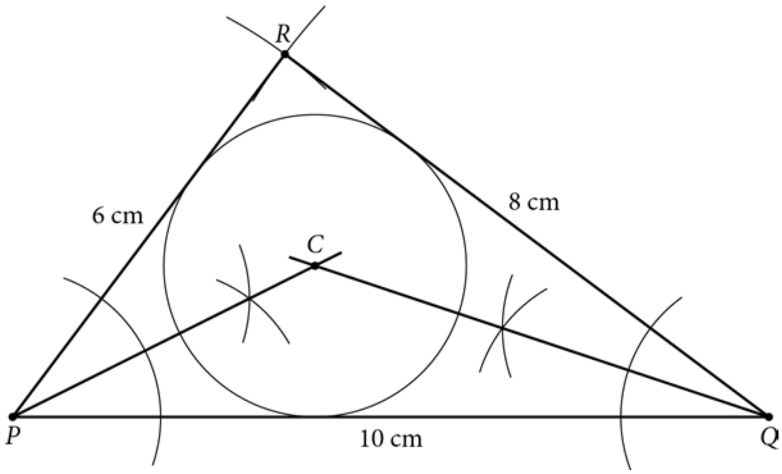


3. Construct the following triangles *PQR* and hence, construct the in circle of each.

(i) $|PQ| = 10$ cm, $|PR| = 6$ cm, $|QR| = 8$ cm

Steps:

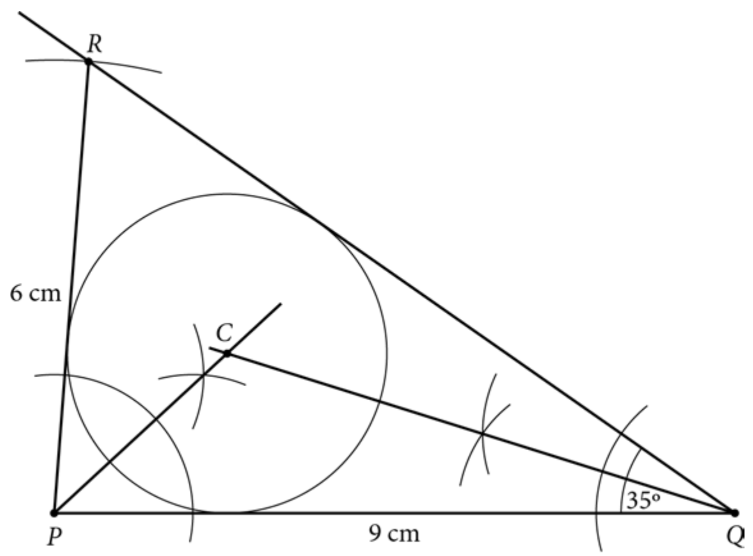
1. Draw $[PQ] = 10$ cm
2. With compass point on *P* and width = 6 cm, draw an arc
3. With compass point on *Q* and width = 8 cm, draw another arc
4. Mark the point of intersection of these arcs *R*
5. With compass point on *P* draw an arc that intersects $[PR]$ and $[PQ]$
6. Keeping compass width, the same and the compass point on the points of intersection on $[PR]$ and $[PQ]$ draw two more arcs
7. Join the point of intersection of arcs drawn in step 6 to *P* to form the bisector of the angle
8. Repeat steps 5, 6 and 7 at *Q*
9. Mark the point of intersection of the bisectors *C*
10. With compass point on *C* draw a circle that touches each side of the triangle.



(ii) $|PQ| = 9$ cm, $|PR| = 6$ cm, $|\angle PQR| = 35°$

Steps:

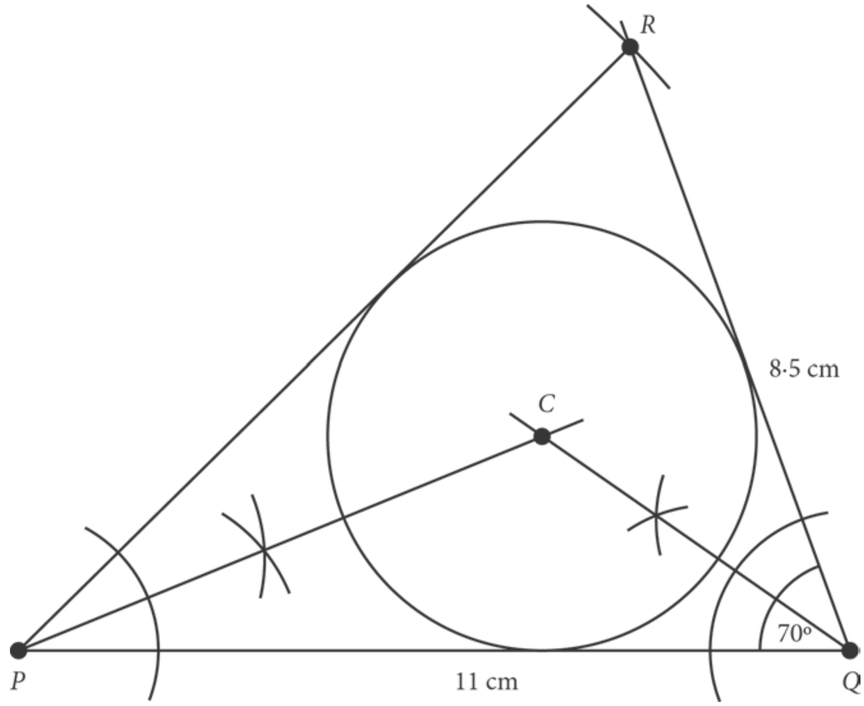
1. Draw $[PQ] = 9$ cm
2. With protractor on Q measure $|\angle PQR| = 35°$
3. With compass point on P and width = 7 cm draw an arc that intersects the arm of the angle. Mark the intersection point R.
4. With compass point on P draw an arc that intersects $[PR]$ and $[PQ]$
5. Keeping compass width, the same and the compass point on the points of intersection on $[PR]$ and $[PQ]$ draw two more arcs
6. Join the point of intersection of arcs drawn in step 6 to P to form the bisector of the angle
7. Repeat steps 5, 6 and 7 at Q
8. Mark the point of intersection of the bisectors C
9. With compass point on C draw a circle that touches each side of the triangle.



(iii) $|PQ| = 11$ cm, $|\angle PQR| = 70°$, $|PR| = 8{\cdot}5$ cm

Steps:

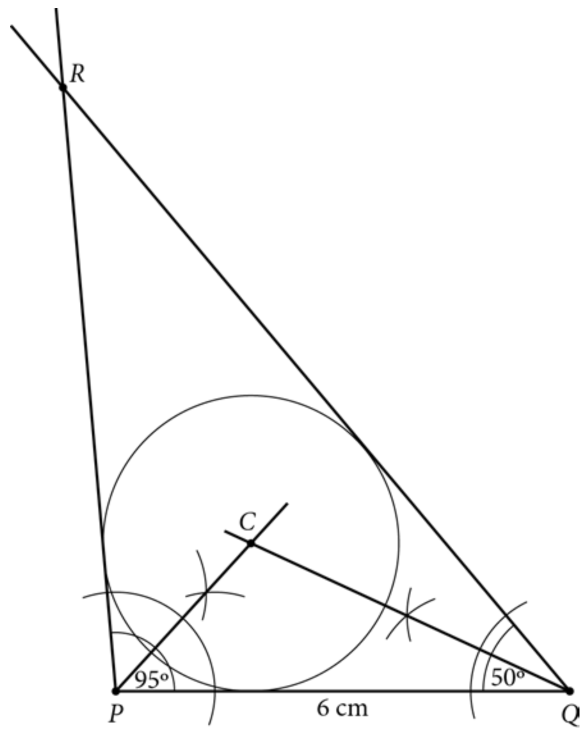
1. Draw $[PQ] = 11$ cm
2. With protractor on Q measure $|\angle PQR| = 70°$
3. With compass point on P and width 8·5 m draw an arc that intersects the arm of the angle. Mark the intersection point R.
4. With compass point on P draw an arc that intersects $[PR]$ and $[PQ]$
5. Keeping compass width the same, and the compass point on the points of intersection on $[PR]$ and $[PQ]$, draw two more arcs.
6. Join the point of intersection of arcs drawn in step 6 to P to form the bisector of the angle.
7. Repeat steps 5, 6 and 7 at Q.
8. Mark the point of intersection of the bisectors C
9. With compass point on C draw a circle that touches each side of the triangle.



(iv) $|\angle QPR| = 95°$,$|PQR| = 50°$ cm, and $|PQ| = 6$ cm

Steps:

1. Draw $[PQ] = 6$ cm
2. With protractor on P draw $|\angle QPR| = 95°$
3. With protractor on Q draw $|\angle PQR| = 50°$
4. Mark the intersection of the arms of the angles R
5. With compass point on P draw an arc that intersects $[PR]$ and $[PQ]$
6. Keeping compass width, the same and the compass point on the points of intersection on $[PR]$ and $[PQ]$ draw two more arcs
7. Join the point of intersection of arcs drawn in step 6 to P to form the bisector of the angle
8. Repeat steps 5, 6 and 7 at Q
9. Mark the point of intersection of the bisectors C
10. With compass point on C draw a circle that touches each side of the triangle.

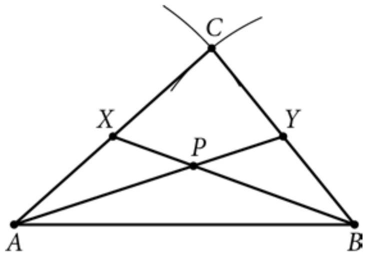


4. Construct the following triangles ABC and, hence, construct the centroid of each.

(i) $|AB| = 4{\cdot}5$ cm, $|AC| = 3{\cdot}5$ cm and $|BC| = 3$ cm

Steps:

1. Draw $[AB] = 4{\cdot}5$ cm
2. Compass on A and width $3{\cdot}5$ cm draw an arc
3. Compass on B and width 3 cm draw an arc
4. Make intersection of the arcs C and join to A and B
5. Find X the midpoint of $[AC]$ and Y the midpoint of $[BC]$
6. Join X to B and Y to A to from the medians
7. Label the point of intersection of the medians P. This is the centroid.

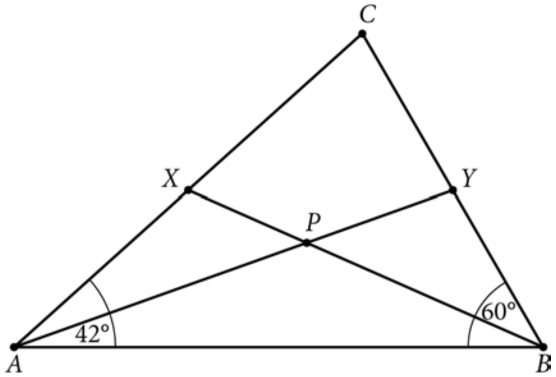


(ii) $|\angle ABC| = 60°$, $|\angle BAC| = 42°$ and $|AB| = 7$ cm

Steps:

1. Draw $[AB] = 7$ cm
2. With protractor on B draw $|\angle ABC| = 60°$
3. With protractor on A draw $|\angle BAC| = 42°$
4. Mark the intersection of the arms of the angle C and join to B and A
5. Find X the midpoint of $[AC]$ and Y the midpoint of $[BC]$
6. Join X to B and Y to A to from the medians.

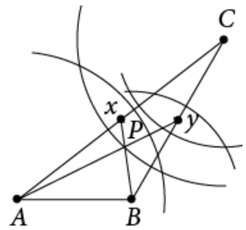
7. Label the point of intersection of the medians *P*. This is the centroid.



(iii) |*AB*| = 2·5 cm, |*BC*| = 4 cm and |∠*ABC*| = 121°

<u>Steps:</u>

1. Draw [*AB*] = 2·5 cm
2. With protractor on *B*, draw |∠*ABC*| = 121°
3. With compass on *B* and width = 4 cm, draw an arc on the angle arm.
4. Make *C* and join to *A* and *B*.
5. Find *X* the midpoint of [*AC*] and *Y* the midpoint of [*BC*].
6. Join *X* to *B* and *Y* to *A* to from the medians.
7. Label the point of intersection of the medians *P*. This is the centroid.

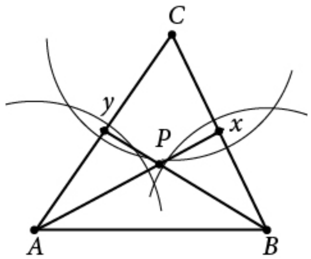


(iv) |∠*ABC*| = 65°, |∠*BAC*| = 55° and |*AB*| = 5 cm

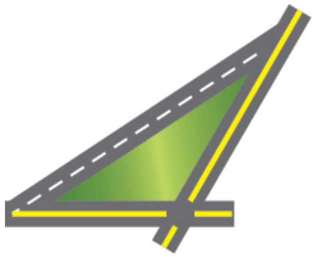
<u>Steps:</u>

1. Draw [*AB*] = 5 cm
2. With protractor on *B* draw |∠*ABC*| = 65°

3. With protractor on A draw $|\angle BAC| = 55°$
4. Mark the intersection of the arms of the angles C and join to A and B
5. Find X the midpoint of $[AC]$ and Y the midpoint of $[BC]$
6. Join X to B and Y to A to form the medians
7. Label the point of intersection of the medians P. This is the centroid.



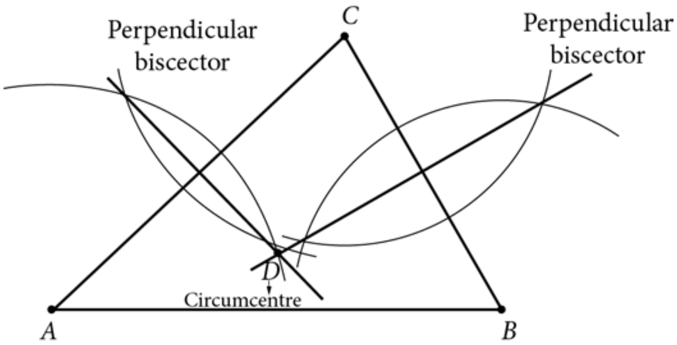
5. A local council wants to position a street light so that it is the same distance from each of the streets shown in the diagram.
Jane says that they should use the circumcentre of the triangle created by the three streets. Jack says this won't work.
Which person do you agree with? Justify your answer.



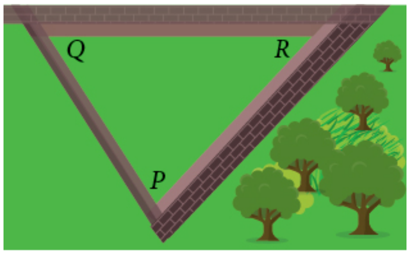
I agree with Jack.
Justification:
The circumcentre of a triangle is the point which is equidistant from the three vertices of the triangle.
Consider the ΔABC. D is the circumcentre of the triangle meaning that $[AD] = [BD] = [CD]$.

However, D is not equidistant from AB, BC and AC.

If the street light is positioned at the circumcentre, D, it is not the same distance from each of the streets.



6. Stephen wants to put the largest possible circular fish pond in his triangle shaped garden, shown below.



(i) Stephen measures the lengths of the garden and finds that $|QR| = 10{\cdot}5$ m, $|PR| = 9{\cdot}9$ m and $|PQ| = 8{\cdot}4$ m.

Using a scale of 1 m = 1 cm, show the construction required to produce the largest possible pond.

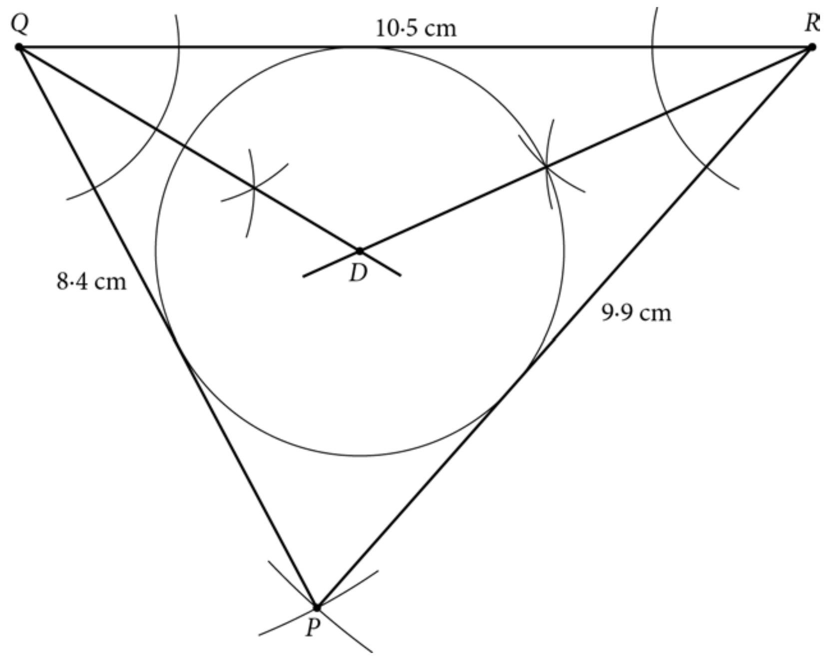
The largest pond possible is the incircle of the triangle given.

$|PQ| = 8{\cdot}4$ cm $|PR| = 9{\cdot}9$ cm $|QR| = 10{\cdot}5$ cm

Steps:

1. Draw $[QR] = 10{\cdot}5$ cm
2. With compass point on Q and width $= 8{\cdot}4$ cm draw an arc
3. With compass point on R and width $= 9{\cdot}9$ cm draw another arc
4. With compass point on Q draw an arc that intersects $[QP]$ and $[QR]$

5. With compass width the same and point on the intersection points of [QP] and [QR] respectively draw 2 more arcs
6. Join the intersection of these arcs to Q to from the bisector of $|\angle PQR|$
7. With compass point on R draw an arc that intersects [QR] and [RP]
8. With compass width, the same and point on the intersection points of [QR] and [RP] respectively draw 2 more arcs
9. Join the intersection of these arcs to R to from the bisector of $|\angle QRP|$
10. Mark the intersection of the bisectors of the angles D
11. With compass point on D draw a circle that touches each side of the triangle. This the largest pond possible in this garden.



(ii) Stephen decides to leave a path of width one metre around the pond.

Will the centre of the pond be in the same position as in part (i)? Justify your answer.

The incentre is the point at which the bisectors of the angles of a triangle intersect and is the centre of incircle.

In this scenario the largest possible fish pond will be the incircle of the ΔPQR. D is the incentre.

Placing a path around the pond will reduce the size of the pond.

But, the angles of the triangle do not change so the point at which the bisectors of those angle intersect will remain the same.

So the centre of the pond will be in the same position but the pond will be smaller.

7. Your friend says that the circumcentre of an equilateral triangle is also the incentre of the triangle. Is your friend correct? Explain your reasoning.

Yes.

An equilateral triangle is a symmetric figure. This means that the circumcentre and incentre are equidistant from the vertices.

Consider the equilateral triangle, ΔABC.

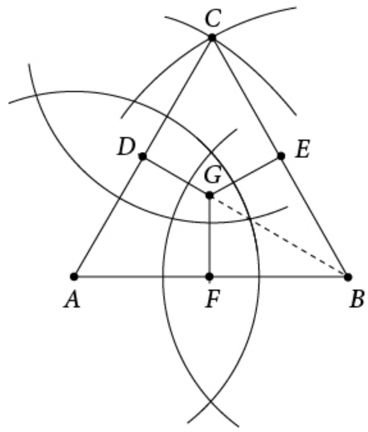
To find the circumcentre, G, of ΔABC we must find the perpendicular bisectors of the sides of the triangle.

A perpendicular bisector cuts a line segment in half so we have the midpoint of [*AC*], [*BC*] and [*AB*]. *D*, *E* and *F* respectively.

DG is the perpendicular bisectors of [*AC*]. If we continue the ray [*DG* we find it is also the angle bisector of $\angle ABC$

Similarly, *EG* is the perpendicular bisectors of [*BC*] and the ray [*EG* is the angle bisector of $\angle BAC$, and *FG* is the perpendicular bisectors of [*AB*] and the ray [*FG* is the angle bisector of $\angle ACB$

G is therefore also the point at which the bisectors of the triangles meet and is thus the incentre.



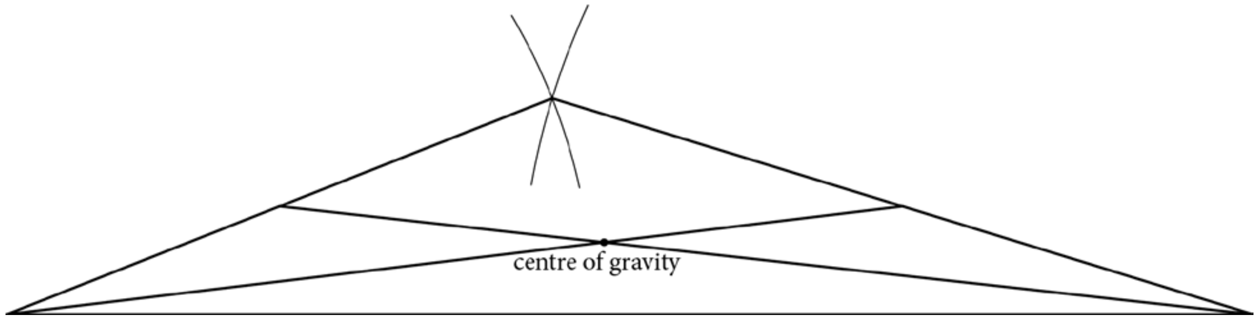
8. Construct the following triangles and hence construct the centre of gravity of each.

(i) Sides 8 cm, 10 cm and 17 cm

Steps:

1. Draw a line segment = 17 cm
2. With compass on one end and width 8 cm draw an arc
3. With compass on the other end and width 10 cm draw another arc
4. Join the intersection point of the arc to the end points
5. Find the midpoint of one side. Join to opposite vertex.
6. Find the midpoint of another side. Join to opposite vertex.

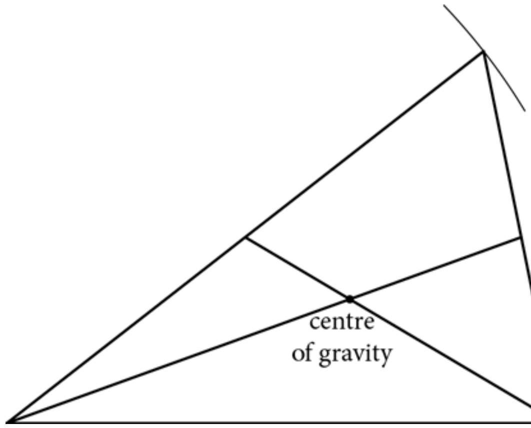
7. The intersection of these lines is the centre of gravity.



(ii) Sides 7·3 cm and 8 cm, angle 38°

Steps:

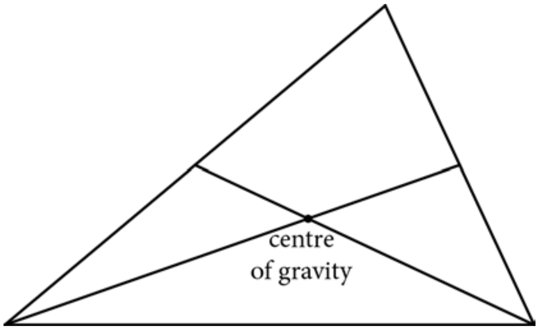
1. Draw a line segment = 7·3 cm
2. At one end of the line segment draw an angle = 38°
3. At the same end put the compass point with width = 8 cm and draw an arc on the angle arm.
4. Join this point to end of line segment.
5. Find the midpoint of one side. Join to opposite vertex.
6. Join the midpoint of another side. Join to opposite vertex.
7. The intersection of those two lines in the center of gravity.



(iii) Angles 40° and 65°, side 7 cm

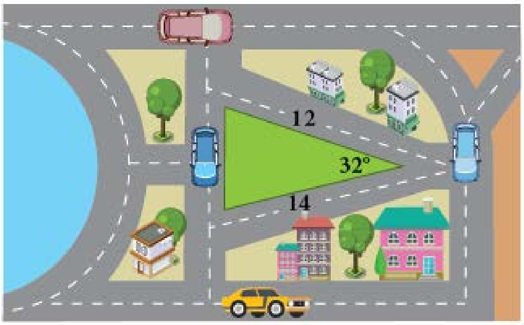
Steps:

1. Draw line segment = 7 cm
2. At one end draw angle = 40°
3. At the other end draw angle = 65°
4. Where angle arms intersection mark, a point and join to each end of the line segment
5. Find the midpoint of one side. Join to opposite vertex.
6. Join the midpoint of another side. Join to opposite vertex.
7. The intersection of those two lines in the center of gravity.



9. The residents of the area around the park shown in the scaled diagram want paths built through the park.

They suggest the paths should run from the midpoint of each side to its opposite corner.



(i) What is the mathematical term to describe each path?

The median of a triangle is a line that join the vertex to the midpoint of one opposite side.

(ii) The paths intersect at the point *P*. What is the mathematical term for the point *P*?

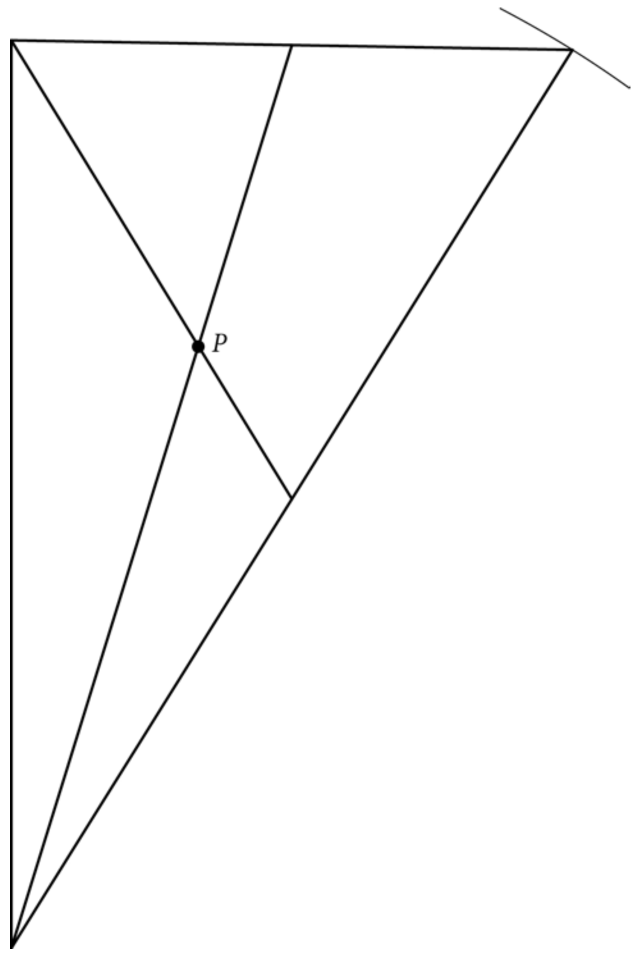
The centroid of a triangle is the point of intersection of the medians of the triangle.

(iii) Copy the diagram and construct the location of *P*.

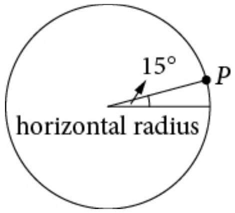
<u>Steps:</u>

1. Draw line segment = 12 cm
2. At one end of the line segment draw angle = 32°
3. Put the compass point at the same end and with a width = 14 cm draw an arc
4. Join the intersection point of the arc and the arm of the angle to the end of the line segment
5. Find the midpoints of one side. Join to opposite vertex.
6. Find the midpoints of the other side. Join to opposite vertex.

7. Mark the intersection of these lines P.



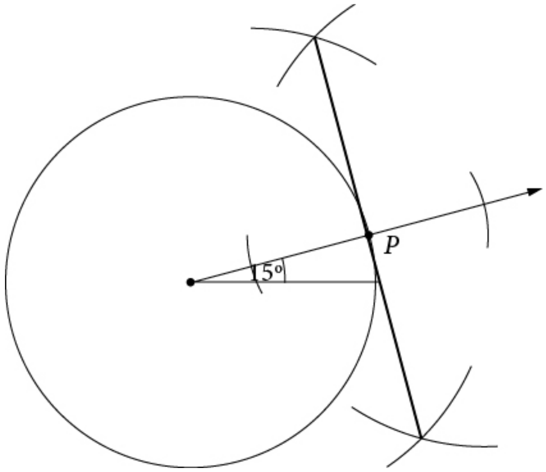
10. Construct the circles below and, using a protractor, mark the point P on the circle at the given angle from a horizontal radius (diagram for **(i)** is shown). Hence, construct the tangent to the circle at P.



(i) radius 4 cm, angle 15°

<u>Steps:</u>

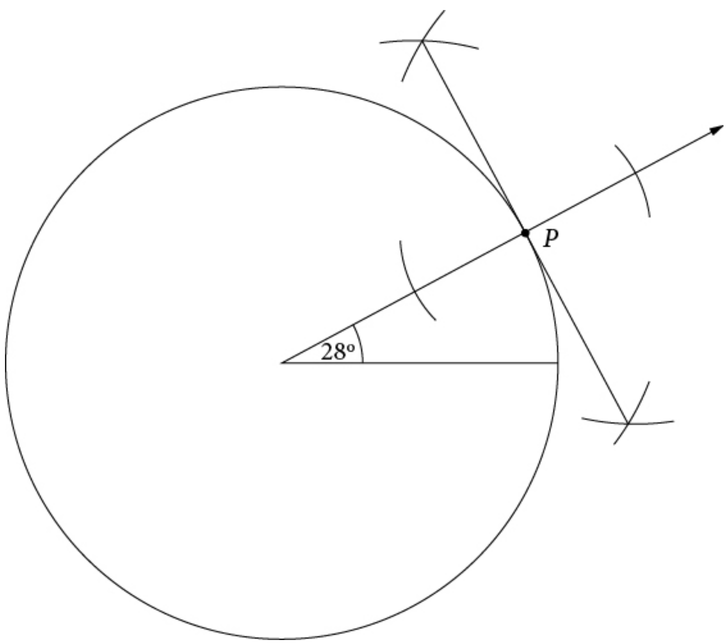
1. Draw a circle of radius = 4 cm.
2. Mark the point P = 15° above the radius.
3. Draw a ray from the centre through P.
4. With compass on P, draw 2 arcs either side of P on the ray.
5. Without changing compass width, put the point on each arc separately and draw another arc above and below P. (ii)
6. Use a straight edge to join P to the intersection of the arcs above and below it. This is the required tangent.



(ii) radius 6 cm, angle 28°

Steps:

1. Draw a circle of radius = 6 cm
2. Mark the point P = 28° above the radius
3. Draw a ray from the centre through P
4. With compass on P, draw 2 arcs either side of P on the ray
5. Without changing compass width, put the point on each arc separately and draw another arc above and below P (ii)
6. Use a straight edge to join P to the intersection of the arcs above and below it. This is the required tangent.

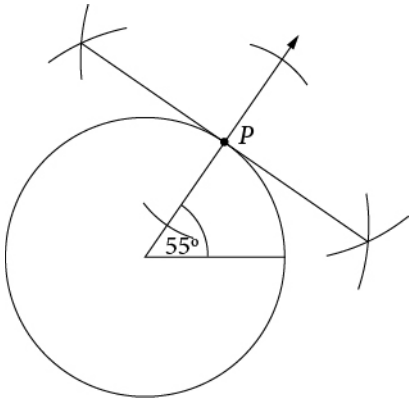


(iii) radius 3 cm, angle 55°

Steps:

1. Draw a circle of radius = 3 cm
2. Mark the point P = 55° above the radius
3. Draw a ray from the centre through P
4. With compass on P draw 2 arcs either side of P on the ray

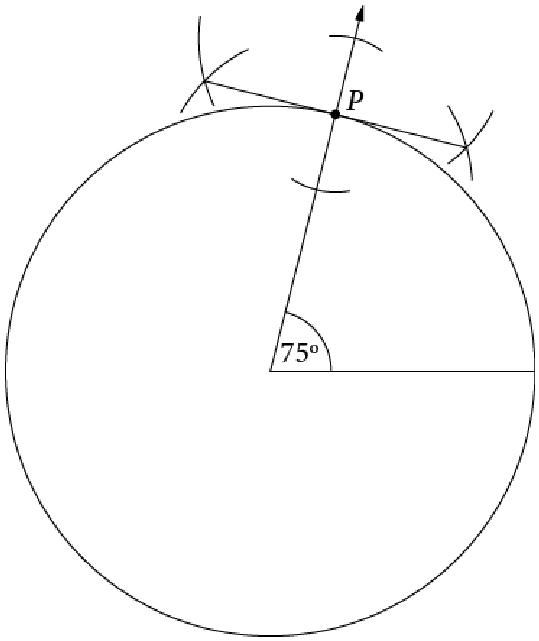
5. Without changing compass width put the point on each arc separately and draw another arc above and below *P* (ii)
6. Use a straight edge to join *P* to the intersection of the arcs above and below it. This is the required tangent.



(iv) radius 5 cm, angle 75°

<u>Steps:</u>

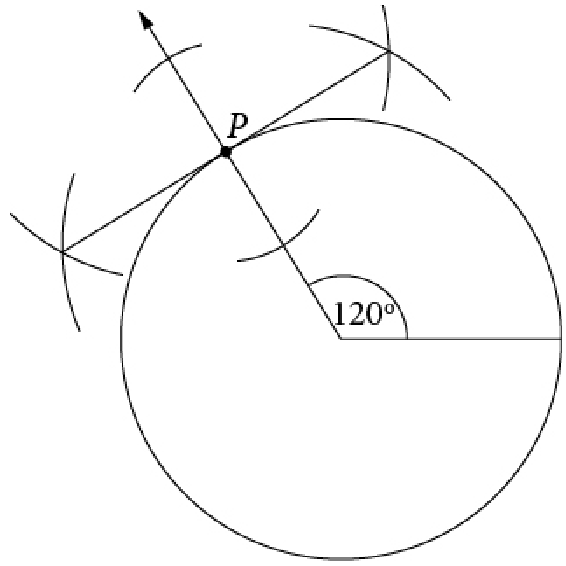
1. Draw a circle of radius = 5 cm
2. Mark the point *P* = 75° above the radius
3. Draw a ray from the centre through *P*
4. With compass on *P* draw 2 arcs either side of *P* on the ray
5. Without changing compass width put the point on each arc separately and draw another arc above and below *P* (ii)
6. Use a straight edge to join *P* to the intersection of the arcs above and below it. This is the required tangent.



(v) radius 3·5 cm, angle 120°

<u>Steps:</u>

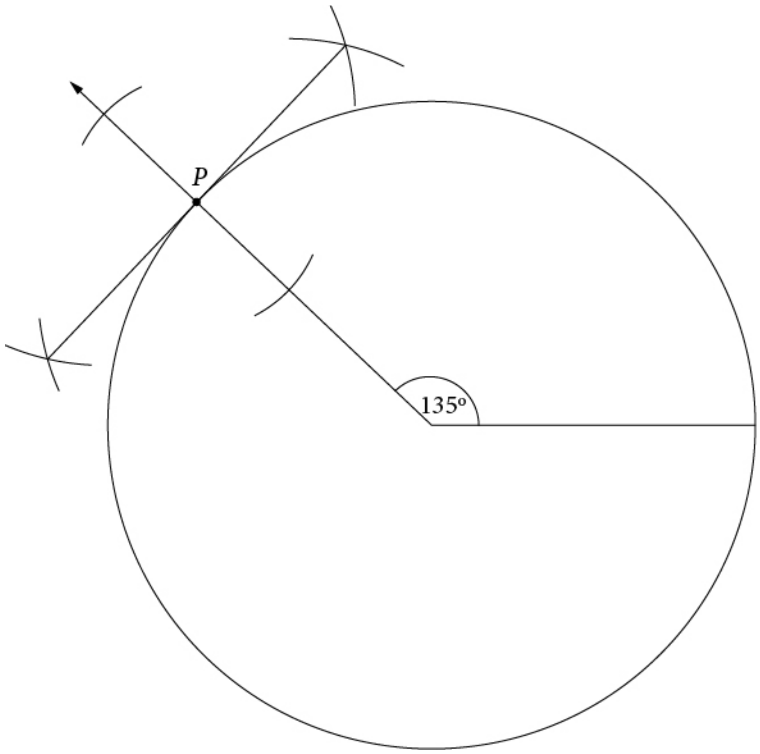
1. Draw a circle of radius = 3·5 cm
2. Mark the point P = 120° above the radius
3. Draw a ray from the centre through P
4. With compass on P, draw 2 arcs either side of P on the ray.
5. Without changing compass width, put the point on each arc separately and draw another arc above and below P. (ii)
6. Use a straight edge to join P to the intersection of the arcs above and below it. This is the required tangent.



(vi) radius 7 cm, angle 135°

<u>Steps:</u>

1. Draw a circle of radius = 7 cm
2. Mark the point *P* = 135° above the radius
3. Draw a ray from the centre through *P*
4. With compass on *P* draw 2 arcs either side of *P* on the ray
5. Without changing compass width put the point on each arc separately and draw another arc above and below *P* (ii)
6. Use a straight edge to join *P* to the intersection of the arcs above and below it. This is the required tangent.



Revision and Exam Style Questions – Sections A

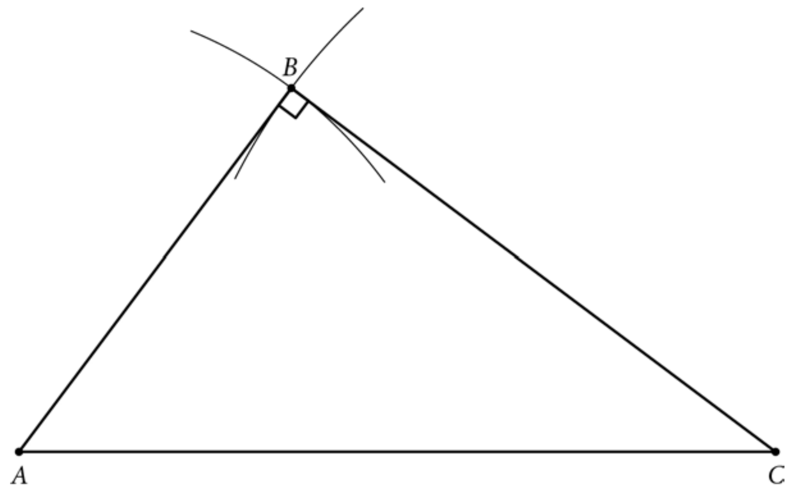
1. Construct a triangle *ABC* with |*AC*| = 10 cm, |*BC*| = 8 cm and |*AB*| = 6 cm.

What is the measure of the angle *ABC*?

<u>Steps:</u>

1. Draw a line segment [*AC*] = 10 cm
2. With compass point on *A* and compass width = 6 cm, draw an arc
3. With compass point on *C* and compass width = 8 cm, draw an arc
4. Mark the intersection of the arcs *B*

5. Join B to A and C to form the required triangle.



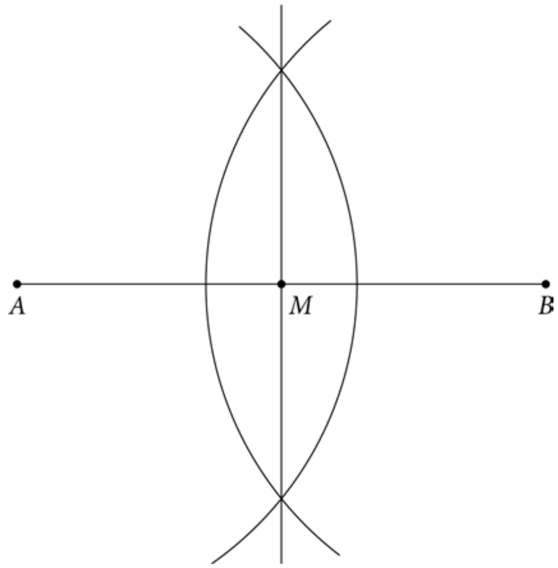
$|\angle ABC| = 90°$

2. Construct a line segment [RS] of 7 cm. Construct the perpendicular bisector of [RS].

Steps:

1. Draw [AB] = 7 cm
2. With compass point on A and compass width greater than $\frac{1}{2}$[AB], draw an arc
3. Keeping the compass width, the same and the point of the compass on B, draw an arc

4. Join the intersection points of the arcs.



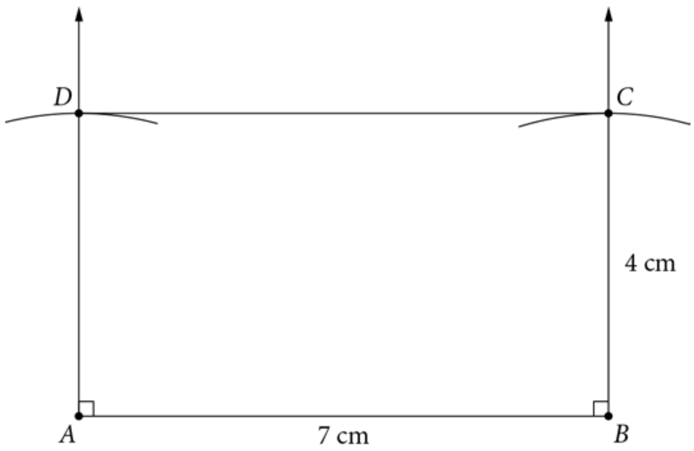
$|AM| = 3{\cdot}5$ cm $|BM| = 3{\cdot}5$ cm

$|AM| = |BM|$

3. Construct a rectangle, $ABCD$, where $|AB| = 7$ cm and $|BC| = 4$ cm.

Steps:

1. Draw $[AB] = 7$ cm
2. At B draw a ray perpendicular to $[AB]$
3. With compass point on B at compass width = 4 cm draw an arc on the ray. Mark the point of intersection C.
4. Repeat steps 2 and 3 at A and mark point D
5. Join D to C to form required rectangle.

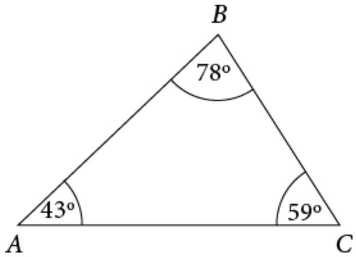


4. ABC is a triangle. $|\angle ABC| = 78°$, $|AC| = 7$ cm and $|\angle BAC| = 43°$. Construct this triangle.

$\lvert\angle ABC\rvert + \lvert\angle BAC\rvert + \lvert\angle ACB\rvert = 180°$	Sum of angles in a triangle add to 180°
$\lvert\angle ABC\rvert = 78°$	given
$\lvert\angle BAC\rvert = 43°$	given
$78° + 43° + \lvert\angle ACB\rvert = 180°$	
$121° + \lvert\angle ACB\rvert = 180°$	
$121° + \lvert\angle ACB\rvert - 121° = 180° - 121°$	Subtract 121° from both sides
$\lvert\angle ACB\rvert = 59°$	

Steps:

1. Draw $[AC] = 7$ cm
2. With protractor at A measure $|\angle BAC| = 43°$
3. Calculate $|\angle ACB| = 180° - (43° + 78°) = 59°$
4. With protractor at C measure $|\angle ACB| = 59°$
5. Mark the point of intersection of the arms of the angles as B to form required triangle.



5. Construct an equilateral triangle with sides of length 6 cm.

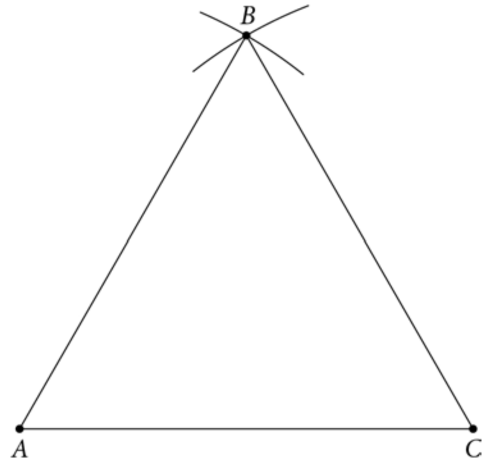
$|AB| = 6$ cm

$|BC| = 6$ cm

$|AC| = 6$ cm

Steps:

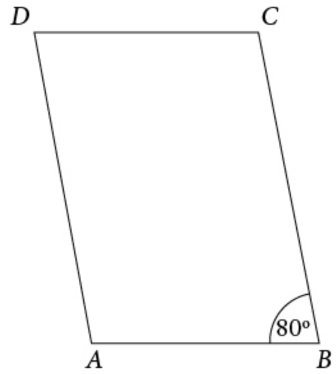
1. Draw $[AC] = 6$ cm
2. With Compass point on A and compass width = 6 cm, draw an arc
3. With compass point on C and compass width = 6 cm, draw an arc
4. Mark the intersection of the arcs B to form required triangle.



6. (i) Construct a parallelogram, $ABCD$, with sides $|AB| = 5$ cm and $|BC| = 7$ cm and $|\angle ABC| = 80°$.

<u>Steps:</u>

1. Draw $[AB] = 5$ cm.
2. With protractor at B measure an angle $= 80°$
3. With compass point at B and compass width $= 7$ cm draw an arc on the angle arm. Mark point C.
4. Place the side on the set square on $[BC]$ and slide along a straight edge to create a parallel line $[AD]$
5. Join D to C to form the parallelogram.



(ii) What is the measure of $\angle BAD$?

Opposite angles in parallelogram are equal.

$|\angle ABC| = |\angle ADC| = 80°$

Sum of all angles in a parallelogram = 360°

$|\angle ABC| + |\angle ADC| + |\angle BAD| + |\angle BCD| = 360°$

$80° + 80° + |\angle BAD| + |\angle BCD| = 360°$

$160° + |\angle BAD| + |\angle BCD| = 360°$

$|\angle BAD| + |\angle BCD| = 200°$ (Subtract 160° from both sides)

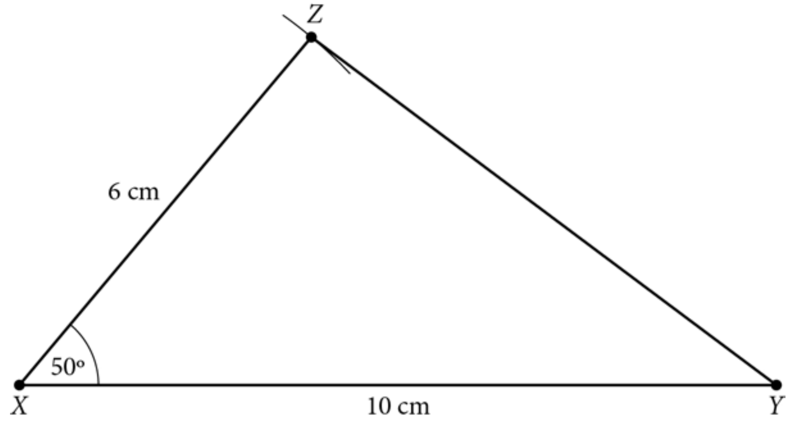
$|\angle BAD| = |\angle BCD|$ (Opposite angles)

$\Rightarrow$ $|\angle BAD| = 200° \div 2 = 100°$

7. (i) Construct a triangle XYZ in which $|XY| = 10$ cm, $|XZ| = 6$ cm and $|\angle ZXY| = 50°$. All construction lines must be clearly shown.

<u>Steps:</u>

1. Draw $[XY] = 10$ cm.
2. At X measure $|\angle ZXY| = 50°$
3. With compass point at X and compass width = 6 cm, draw an arc on the angle arm
4. Mark the intersection of the arc and the arm Z
5. Join Z to Y to form the required triangle.

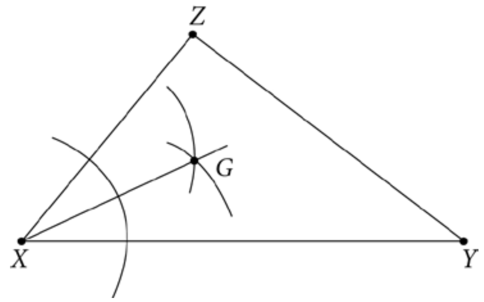


(ii) Show how to bisect $\angle XYZ$ without using a protractor.

<u>Steps:</u>

1. With compass point on X and any width draw an arc that intersects $[XZ]$ and $[XY]$
2. Keeping the compass width, the same put the point on the intersection of the arc and $[XZ]$ and draw an arc
3. Keeping the compass width, the same put the point on the intersection of the arc and $[XY]$ and draw an arc
4. Mark the intersection of these arcs G

5. Join X to G to form the angle bisector



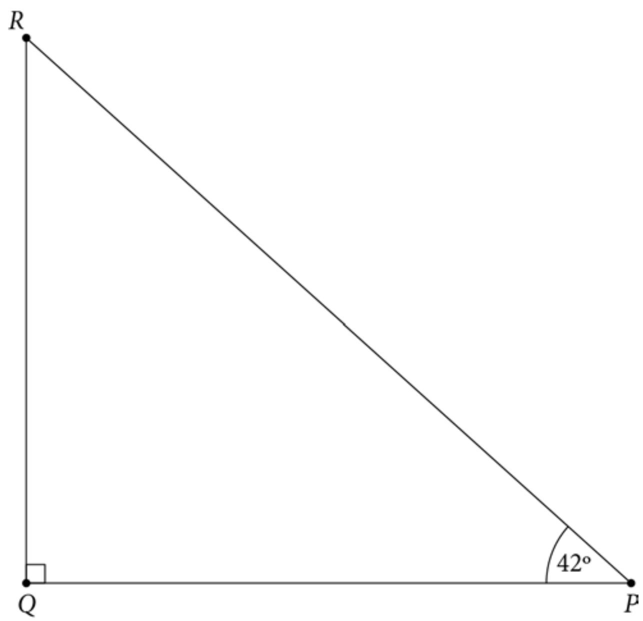
$|\angle ZXG| = 25°$

$|\angle YXG| = 25°$

8. Construct a right-angled triangle PQR, where $|\angle QPR| = 42°$, $|PQ| = 8$ cm and $|\angle RQP| = 90°$.

<u>Steps:</u>

1. Draw $[PQ] = 8$ cm
2. With protractor at Q draw $|\angle RQP| = 90°$
3. With protractor at P draw $|\angle QPR| = 42°$
4. Mark the intersection of the angle arms as R to form the required triangle.



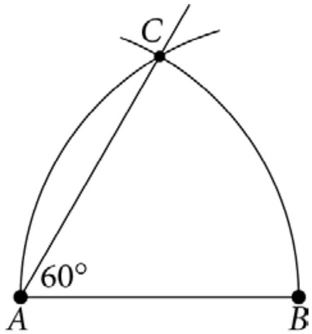
9. Construct, without using a protractor or set square, an angle of 60°.

Hence, construct, on the same diagram, and using a compass and straight edge only, an angle of 30°.

(i) Steps:

1. Draw a line segment [*AB*].
2. With compass point at *A*, draw an arc through *B*.
3. With compass point at *B*, draw an arc through *A*.
4. Mark the intersection of these arcs *C*.
5. Join *C* to *A*.

$|\angle CAB| = 60°$



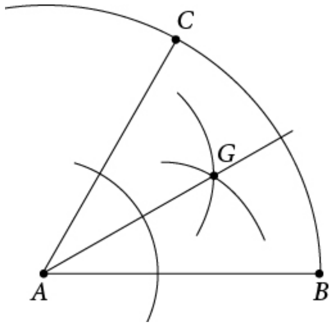
(ii) Construct 30^{o}

We can simply bisect the 60^{o} angle to construct a 30^{o} angle.

Steps:

1. With compass point on *A* and any width, draw an arc intersecting [*AC*] and [*AB*]
2. Keep compass width the same and point on the intersection of the arc and [*AC*] and draw an arc
3. Repeat step 2 on [*AB*]
4. Mark the intersection of these arcs *G*

5. Join A to G to form the required bisector.



$|\angle CAG| = 30°$

$|\angle BAG| = 30°$

10. (i) Construct ΔDEF such that, $|DE| = 6{\cdot}5$ cm, $|\angle DEF| = 50°$ and $|\angle DFE| = 30°$.

$|\angle DEF| + |\angle DFE| + |\angle EDF| = 180°$ Sum of angles in a triangle add to 180°

$|\angle DEF| = 50°$ given

$|\angle DFE| = 30°$ given

$50° + 30° + |\angle EDF| = 180°$

$80° + |\angle EDF| = 180°$

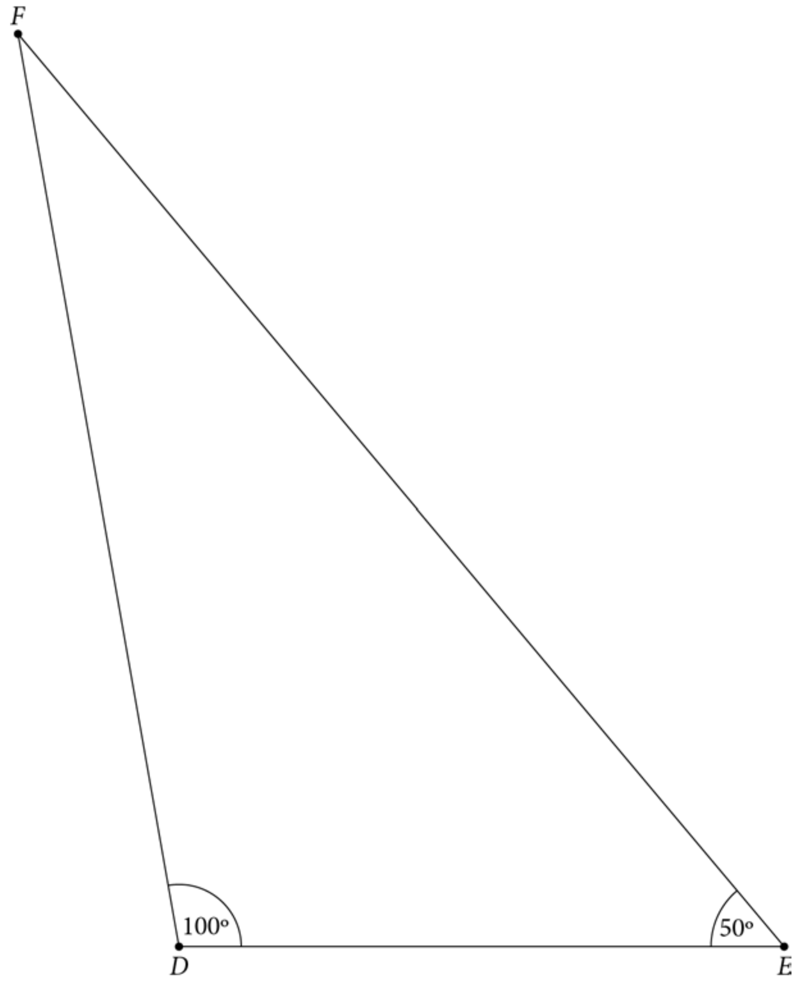
$80° + |\angle EDF| - 80° = 180° - 80°$ Subtract 80° from both sides

$|\angle EDF| = 100°$

(i) <u>Steps:</u>

1. Draw $[DE] = 8$ cm
2. At E measure $|\angle DEF| = 50°$
3. Calculate $|\angle EDF|$ as shown
4. At D measure $|\angle EDF| = 100°$

5. Mark the intersection of the angle arms F to the triangle.

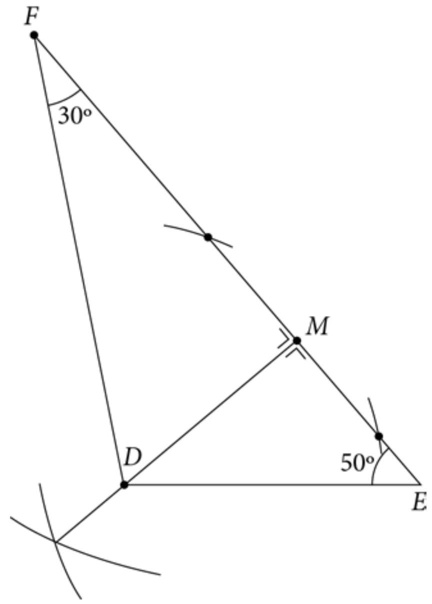


(ii) Construct the point M on $[EF]$ such that $[DM]$ is perpendicular to $[EF]$.

Steps:

1. With compass point of D, draw two arcs on $[EF]$
2. Increase the width of the compass, place point on each point of intersection of the arcs and $[EF]$ and draw 2 more arcs
3. Join the point of intersection of these arcs through D to $[EF]$.

4. Mark M.



(iii) Measure the length of [DM].

$|DM| = 5$ cm

Revision and Exam Style Questions – Section B

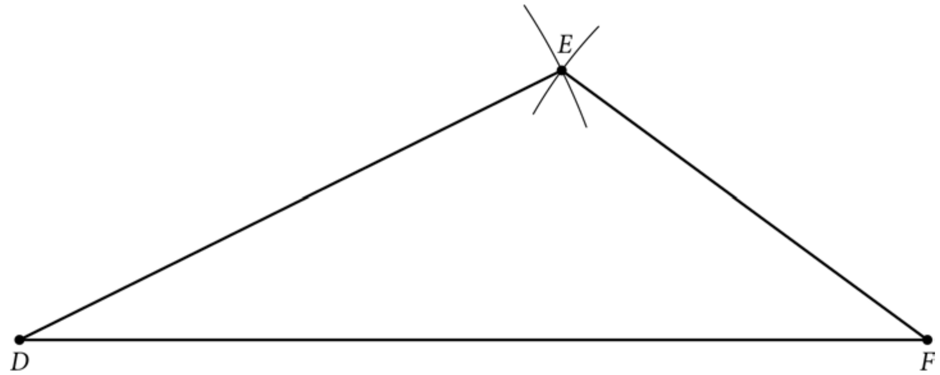
More challenging problems

1. (i) Construct the triangle DEF, such that $|DE| = 8$ cm, $|EF| = 6$ cm and $|DF| = 12$ cm.

Steps:

1. Draw [DF] = 12 cm
2. With compass point at D and width = 8 cm draw an arc.
3. With compass point at F and width = 6 cm draw an arc.
4. Mark the point E where the arcs intersect.

5. Join E to D and F to form triangle.

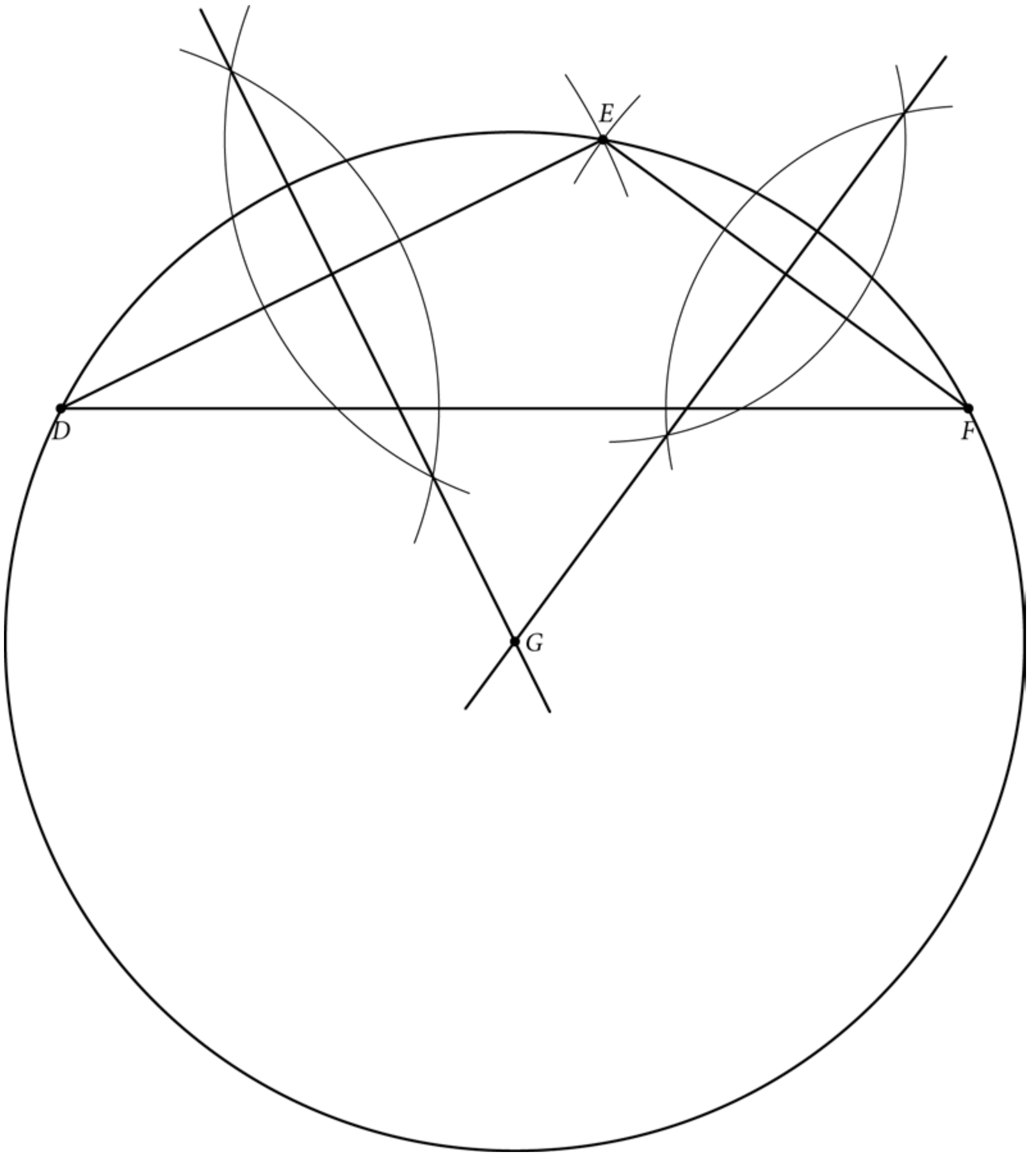


(ii) Construct the circumcircle of the triangle. Show all construction lines clearly.

Steps:

1. With compass point at D and width greater than $\frac{1}{2}[DE]$ draw a large arc
2. With compass width the same and point at E, draw another large arc.
3. Join the intersections of these arcs to form the bisector.
4. Repeat step 1, 2 and 3 on EF
5. Make the point of intersection of the perpendicular bisector G.

6. With point of the compass on G and compass width = [GE], draw a circle through D, E and F.



(iii) Under what condition(s) does the circumcentre of a triangle lie inside the triangle? Justify your answers.

The circumcentre lies inside a triangle if it is an acute triangle, i.e. all the angles of the triangle are smaller than a right angle (90°).

If one of the angles of a triangle is greater than 90° the triangle is obtuse and the circumcentre lies outside the triangle.

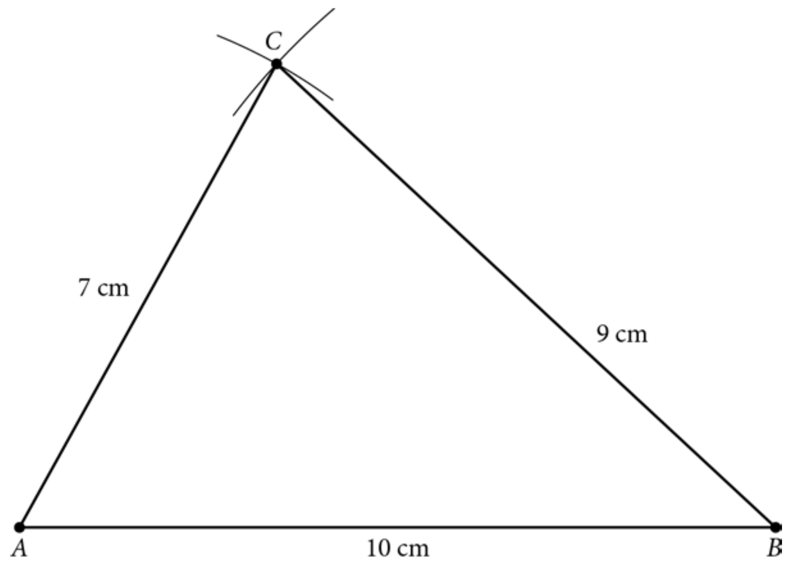
If the triangle is a right-angled triangle, the circumcentre lies on the hypotenuse.

In the triangle $\triangle DEF$ the angle $\angle DFF$ is greater than 90°, making it an obtuse triangle, with the circumcentre lying outside the triangle.

2. (i) Show how to construct the triangle ABC, with sides $|AB| = 10$ cm, $|BC| = 9$ cm and $|AC| = 7$ cm.

Steps:

1. Draw $[AB] = 10$ cm
2. With point of compass on A and width = 7 cm, draw an arc
3. With point of compass on B and width = 9 cm, draw an arc
4. Mark the intersection of these arcs C
5. Join C to A and B to form triangle:



(ii) Explain what is meant by the incentre.

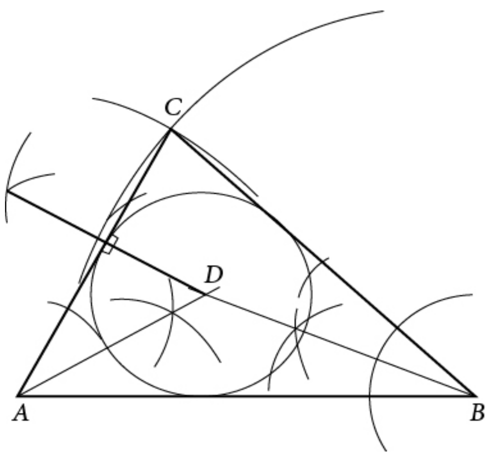
The incentre is the centre of the incircle; a circle drawn inside a triangle so as to touch but not cross each side of the triangle.

It is the point at which the bisectors of the angles of the triangle intersect.

(iii) Show how to construct the incircle of triangle *ABC*. All construction lines must be clearly shown.

Steps:

1. With compass point on *A* and any width, draw an arc that crosses [*AB*] and [*AC*]
2. Keep width the same, put point on the intersection of the arc and [*AB*] and draw another arc
3. Keep width the same, put point on the intersection of the arc and [*AC*] and draw another arc.
4. Mark the intersection of these arcs *D*. Join *D* to *A* to form the bisector.
5. Repeat steps 1–4 at *B*.
6. With *D* as the center draw a circle that touches each side of the triangle.



3. L, M and N are three villages in an area. The distance from L to M is 32·5 km, from M to N is 30 km and from L to N is 20 km.

(i) Using the scale 1 cm = 2·5 km, construct a scale diagram showing the locations of the three villages.

$L \rightarrow M = 32{\cdot}5$ km

$M \rightarrow N = 30$ km

$L \rightarrow N = 20$ km

2·5 km = 1 cm Scale

32·5 km = ?

$\frac{32{\cdot}5}{2{\cdot}5} = 13$ How many times does 2·5 go into 32·5?

$13 \times 1 = 13$ cm Increase 1 cm by same factor

30 km = ?

$\frac{30}{2{\cdot}5} = 12$ How many times does 2·5 go into 30?

$12 \times 1 = 12$ cm Increase 1 cm by same factor

20 km = ?

$\frac{20}{2{\cdot}5} = 8$ How many times does 2·5 go into 20?

$8 \times 1 = 8$ cm Increase 1 cm by same factor

$|LM| = 13$ cm

$|MN| = 12$ cm

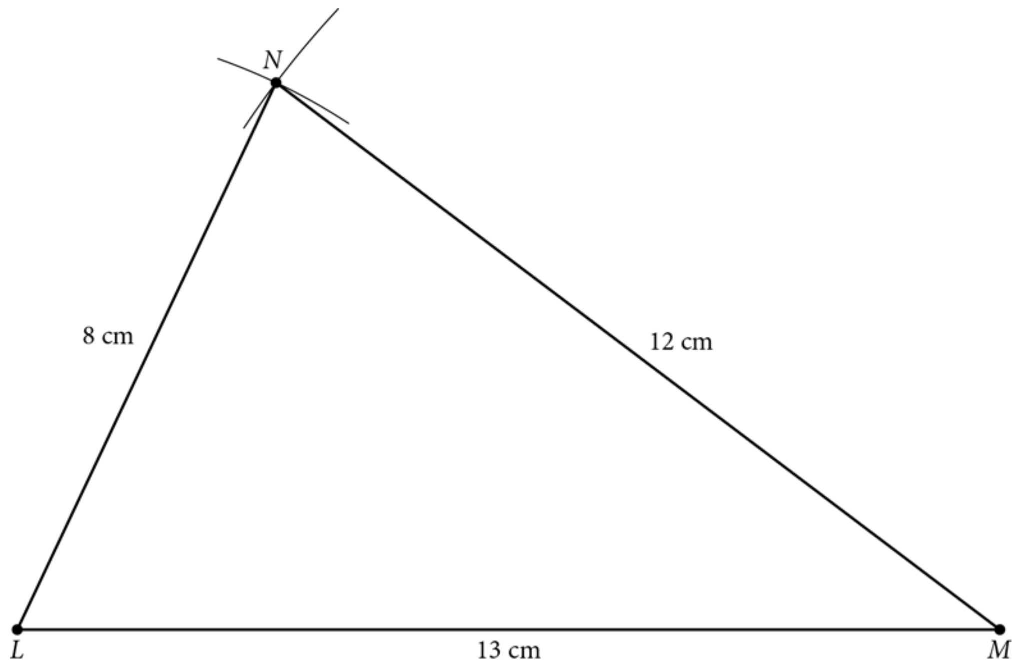
$|LN| = 8$ cm

Construct a scale diagram showing the locations of the three villages.

Steps:

1. Draw $[LM] = 13$ cm
2. At L with compass width = 8 cm draw an arc
3. At M with compass width = 12 cm draw an arc

4. Mark the intersection N
5. Join N to L and M.

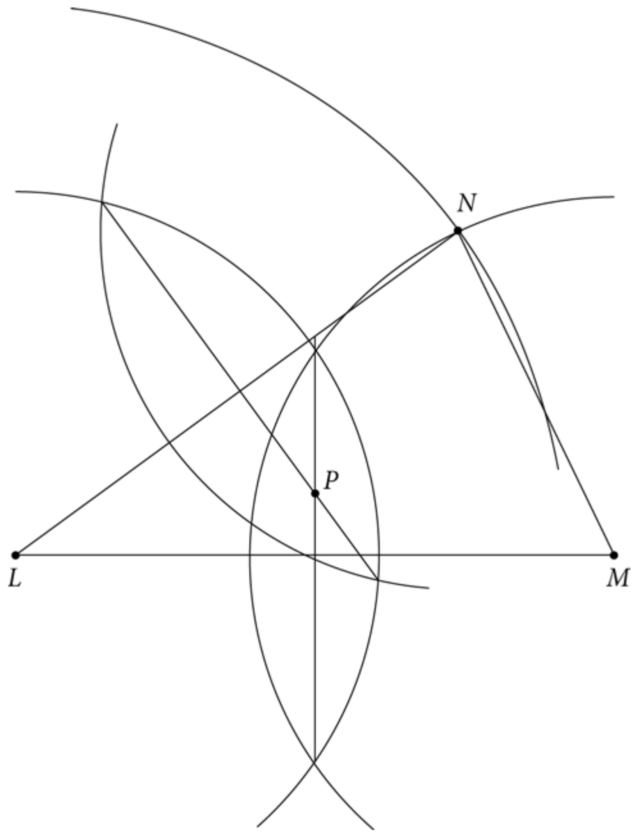


(ii) A new sports centre is to be built equidistant from each of the three villages. Indicate the location of the sports centre on your diagram. Show all construction lines clearly.

<u>Steps:</u>

1. With compass on L and width greater than $\frac{1}{2}[LN]$ draw a large arc
2. With compass width, the same and point at N draw another arc
3. Join the intersections of the arcs to form the perpendicular bisectors
4. Repeat steps 1–3 on $[LM]$

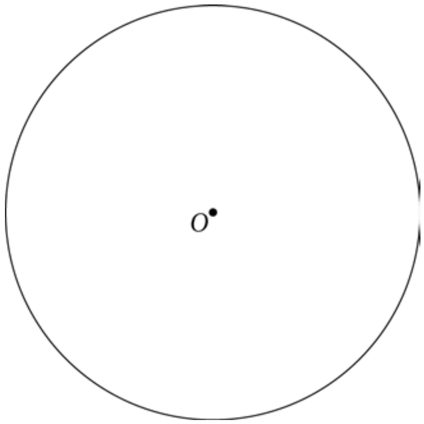
5. Mark *P* the point of intersection of the perpendicular bisectors.



(iii) What is the geometric name for this point?

Circumcentre.

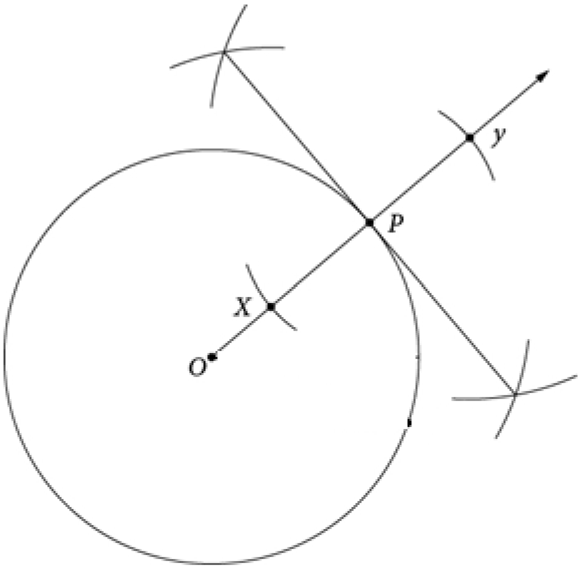
4. (i) Draw a circle of radius 4·5 cm.



(ii) Construct the tangent to the circle at a point P at any position on the circle.

<u>Steps:</u>

1. Mark a point P on the circle and draw a ray [OP
1. With compass on P draw 2 arcs on OP either side of P at x and y
2. With point on x draw 2 arcs above and below P
3. Keeping same width with point on y draw 2 arcs above and below P
4. Join P and and the intersections of the arcs above and below it to from the tangent.

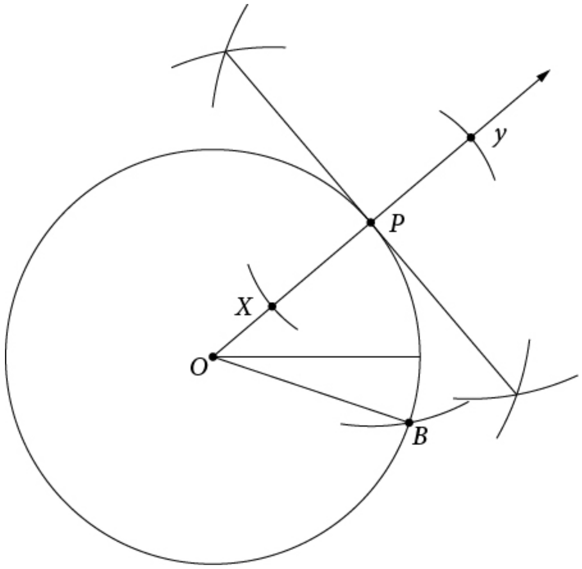


(iii) What theorem is used in the construction of the tangent?

Each tangent is perpendicular to the radius that to the point of contact

(iv) On the same diagram, construct $|\angle POB| = 60°$, where B is another point on the circle and O is the centre, without using a protractor or set square.

Use compass to measure OP, with compass point on P and width = $[OP]$ mark B on the circle. Check $[PB] = [OP]$



(v) What type of triangle is OPB? Give a reason for your answer.

Equilateral triangle

$\lvert OP\rvert = \lvert OB\rvert$	radii of circle
$\therefore$ base angles of Δ OPB are equal	Isosceles triangle
$\lvert\angle OPB\rvert = \lvert\angle PBO\rvert$	
$\lvert\angle POB\rvert + \lvert\angle OPB\rvert + \lvert\angle PBO\rvert = 180°$	Sum of angles is a triangle add to 180° given
$\lvert\angle POB\rvert = 60°$	
$60° + \lvert\angle OPB\rvert + \lvert\angle OPB\rvert = 180°$	
$60° + 2\lvert\angle OPB\rvert = 180°$	

$$60° + 2|\angle OPB| - 60° = 180° - 60° \quad \text{Subtract } 60° \text{ from both sides}$$

$$2|\angle OPB| = 120°$$

$$\frac{2\angle OPB}{2} = \frac{120^{\circ}}{2} \quad \text{divide both sides by 2}$$

$$|\angle OPB| = 60°$$

$|\angle OPB|$, $|\angle PBO|$ and $|\angle POB|$ are all 60°, $\therefore$ triangle ΔOPB is equilateral

5. (i) Complete each of the following statements.

(a) The circumcentre of a triangle is the point of intersection of …………

The circumcentre of a triangle is the point of intersection of the perpendicular bisectors of the sides of the triangle.

(b) The incentre of a triangle is the point of intersection of …………

The incentre of a triangle is the point of intersection of the bisectors of the angles of the triangle.

(c) The centroid of a triangle is the point of intersection of …………

The centroid of a triangle is the point of intersection of the medians of the triangle.

(ii) In an equilateral triangle, the circumcentre, the incentre and the centroid are all in the same place. Explain why this is the case.

To find the circumcentre, G, of the triangle ΔABC we must find the perpendicular bisectors of the sides of the triangle.

A perpendicular bisector cuts a line segment in half so we have the midpoint of $[AC]$, $[BC]$ and $[AB]$, D, E and F respectively.

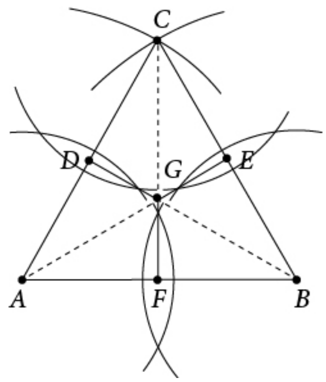
DG is the perpendicular it is bisector of $[AC]$. If we continue the ray $[DG$ we find it is also the angle bisector of $\angle ABC$.

DB is also a median of the triangle ΔABC as it joins the vertex to the midpoint of the opposite side.

Similarly, *EG* is the perpendicular bisector of [*BC*] is the ray [*EG* is the angle bisector of $\angle BAC$ and *EA* is a median of the triangle ΔABC.

FG is the perpendicular bisector of [*AB*], the ray [*FG* is the angle bisector of $\angle ACB$ and *FC* is a median of the triangle ΔABC.

G is therefore the centroid, incentre and circumcentre of the ΔABC.



6. Given a line segment [*AB*], 8 cm in length:

(i) construct the perpendicular bisector of [*AB*]

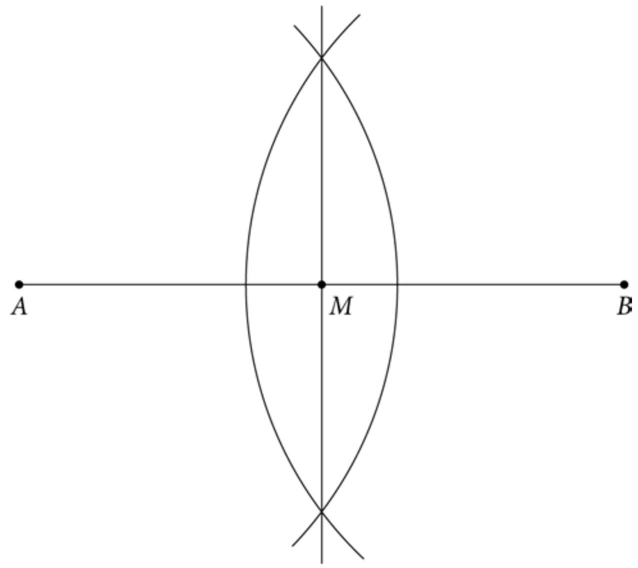
<u>Steps:</u>

1. Draw [*AB*] = 8 cm.

2. With compass point on *A* and width > $\frac{1}{2}$ |*AB*| draw a large arc.

3. With compass point on *B* and same width draw a large arc.

4. Join the intersections of these arcs to form the perpendicular bisector.

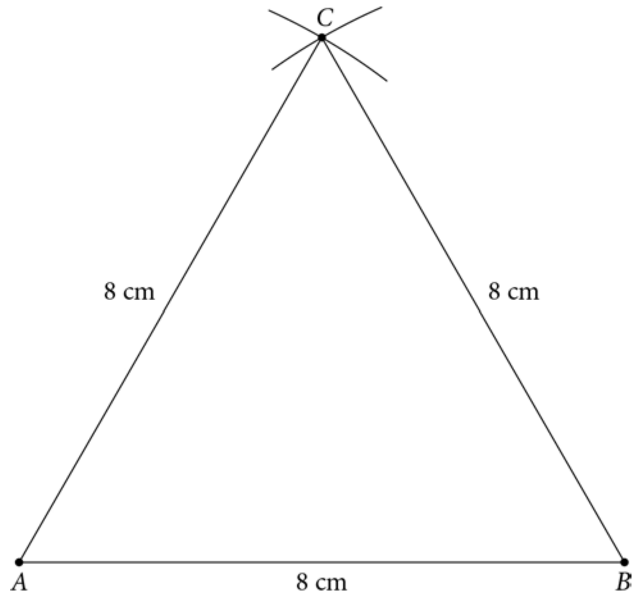


$|AM| = 4$ cm $\quad |BM| = 4$ cm

(ii) construct an equilateral triangle with side [*AB*].

<u>Steps:</u>

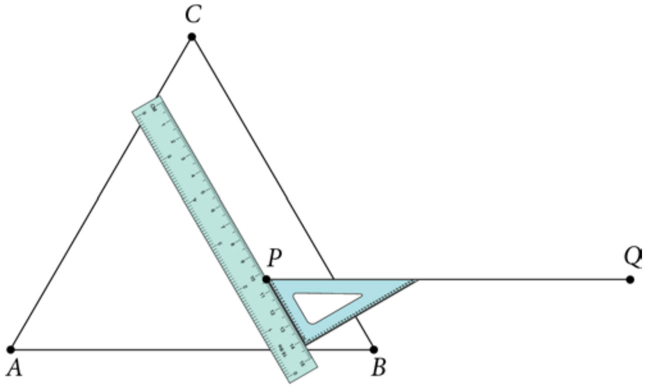
1. Draw [*AB*] = 8 cm
2. With point on *A* and width = 8 cm draw an arc
3. With point on *B* and width = 8 cm draw an arc
4. Make the point where the arcs insect *C*
5. Join *C* to *A* and *B* to form equilateral triangle.



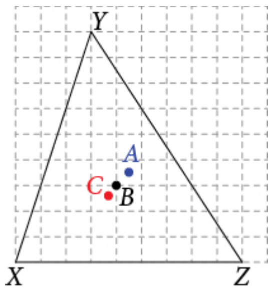
(iii) For a given point *P* not on *AB*, construct a line segment *PQ* parallel to *AB*, such that |*AB*| = |*PQ*|.

Steps:

1. Draw a point *P*, not on *AB*
2. Place the edge of the set square on [*AB*]
2. Place a straight edge on the other side of the set square
3. Slide the set square until the edge originally on [*AB*] passes through *P*
4. Draw [*PQ*] = 8 cm

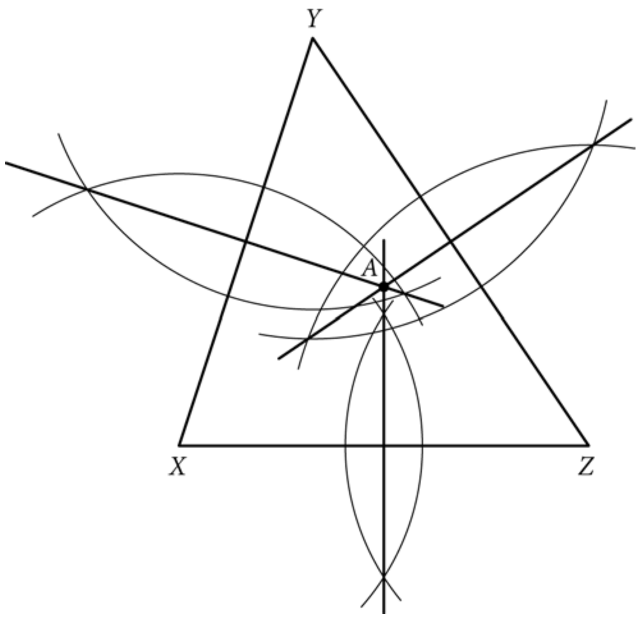


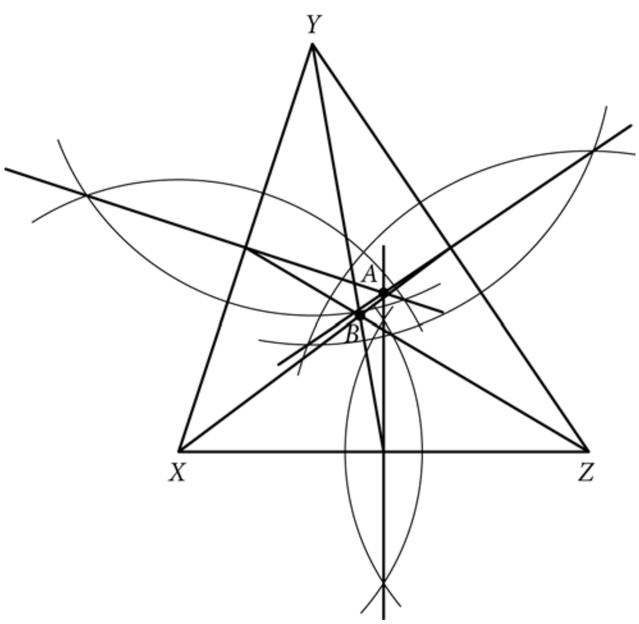
7. Copy the diagram on the right onto graph paper. The points *A*, *B*, and *C* represent the circumcentre, centroid and incentre.

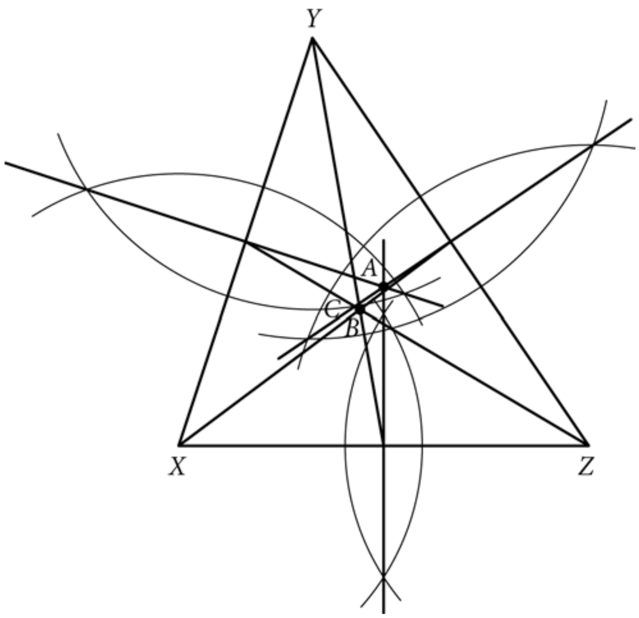


(i) Identify which point is the circumcentre, incentre and centroid.

Imagine the points X, Y and Z are three rural villages that share a fire station.







A = Circumcentre

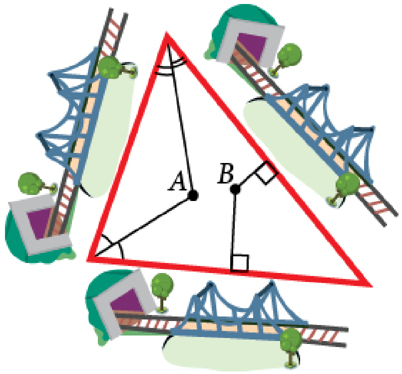
B = Centroid

C = Incentre

(ii) Where would be the fairest location for the fire station? Justify your answer.

The firestation should be located at ‘*A*’, the circumcentre of triangle Δ *XYZ*. The circumcentre is equidistant from each of the vertices of the triangle.

8. A platform at a busy train station is in the shape of a triangle, shown in red in the diagram. You are trying to talk to your friend on the phone while standing on this platform. The trains are very loud and you can barely hear. In order to hear better, you should stand at the point which is as far away as possible from **all** three tracks. Should you stand at *A* or *B*? Explain your choice.



A = Incentre ; point at which bisectors of the angles of the triangle intersect.

B = Circumcentre ; point at which the perpendicular bisectors of the sides of the triangle intersect.

Point *B* is equidistant from the three vertices of the triangle.

Point *A* is the centre of the triangle's incircle, the largest circle that fits inside the triangle and touches all sides of the triangle.

You should stand at point *A* to be as far away from all three tracks as possible.

9. A circus performer wants to balance a triangular piece of wood on the end of a stick as part of his act. He wants to mark the point where the piece will be balanced. The piece of wood has side lengths 0·75 m, 1 m and 1·25 m.

1 m = 6 cm

0·75 m = 4·5 cm

1·25 m = 7·5 cm

(i) Construct a scale diagram of the piece of wood, such that 1 m = 6 cm.

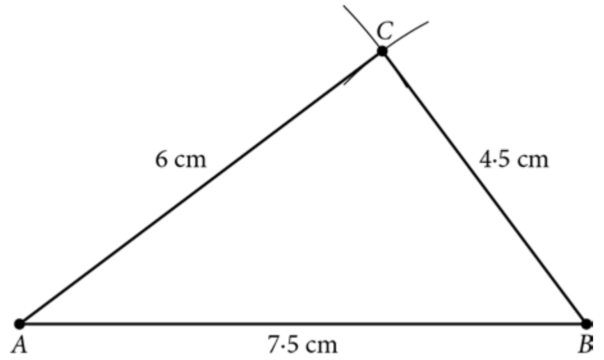
1 m = 6 cm

0·75 m = 4·5 cm

1·25 m = 7·5 cm

Steps:

1. Draw [*AB*] = 7·5 cm
2. With compass point on *A* and width = 6 cm draw an arc
3. With compass point on *B* and width = 4·5 cm draw another arc
4. Mark the intersection of the arcs *C* and form the required triangle.



(ii) On your diagram, mark the point the performer is looking for.

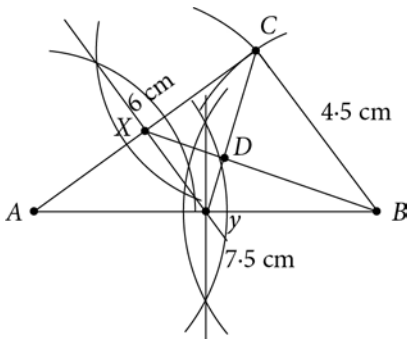
The point the performer is looking for is the centroid of the triangle.

Steps:

1. With compass point on *A* and width greater than $\frac{1}{2}$ [*AC*], draw a large arc
2. With compass point on *C* and same width draw another large arc
3. Draw a line through the intersects of these arcs to form the perpendicular bisector of [*AC*]
4. Mark the intersection the perpendicular and [*AC*], *x*
5. Join *x* to *B* to form the median of [*AC*]
6. Repeat step 1–5 on [*AB*] to form median of [*AB*]

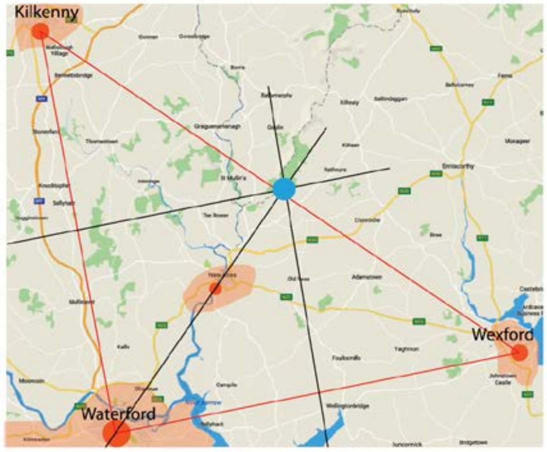
7. Mark the intersection of the medians D.

D = balance point.



10. As part of a marketing campaign for foreign tourists, Bórd Fáilte are promoting 'The Viking Triangle', formed by joining the cities of Kilkenny and Waterford and the town of Wexford, as shown on the map below.

Frank is asked to find a site for a visitor centre in this region. He proposes the site marked with a blue dot on the map.



(i) How did he construct the position of the site?

He drew the perpendicular bisector of each side of the triangle and found the intersection point.

(ii) What is the mathematical term for this proposed site?

The circumcenter.

(iii) Why, do you think, did Frank choose this site?

It is the same distance from Kilkenny, Waterford and Wexford.

(iv) In your opinion, is this a good location for the visitor centre?

Give a reason for your answer.

Yes.

As the site is the circumcentre of the triangle it is equidistant from the three vertices. This means that the visitor centre would be equidistant from the three locations that make up the Viking Triangle. However, it is not close to the main roads and therefore may not be easily accessible from all locations.