

Geometry/ Integrated Math II

2008

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by Indiana University-Purdue University Indianapolis

Indiana State Mathematics Contest

ICTM Website

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Next year's math contest date: April 25, 2009

1. In our number system the base is ten. If the base were changed to five you would count as follows: 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, ... The thirtieth number would be:
 - (A) 30
 - (B) 40
 - (C) 50
 - (D) 55
 - (E) 110

2. A chord which is the perpendicular bisector of a radius of length 12 in a circle, has length:
 - (A) $3\sqrt{3}$
 - (B) 27
 - (C) $6\sqrt{3}$
 - (D) $12\sqrt{3}$
 - (E) $4\sqrt{3}$

3. One thousand unit cubes are fastened together to form a large cube with edge length 10 units; this is painted and then separated into the original cubes. The number of these unit cubes which have at least one face painted is:
 - (A) 600
 - (B) 520
 - (C) 488
 - (D) 480
 - (E) 400

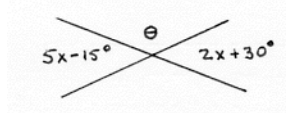
4. The number π is equal to:
 - (A) 3.14
 - (B) a rational fraction
 - (C) a finite decimal
 - (D) an infinite non-repeating decimal
 - (E) an infinite repeating decimal

5. In $\triangle ABC$ with right angle at C , altitude CH and median CM trisect the right angle. If the area of $\triangle CHM$ is K , then the area of $\triangle ABC$ is:
- (A) $6K$
 - (B) $4\sqrt{3}K$
 - (C) $3\sqrt{3}K$
 - (D) $3K$
 - (E) $4K$
6. The average (arithmetic mean) age of a group consisting of teachers and engineers is 40. If the teachers average 35 and the engineers 50 years old, then the ratio of the number of teachers to the number of engineers is:
- (A) 3 : 2
 - (B) 3 : 1
 - (C) 2 : 3
 - (D) 2 : 1
 - (E) 1 : 2
7. A sector with acute central angle θ is cut from a circle of radius 6. The radius of the circle circumscribed about the sector is:
- (A) $3\cos\theta$
 - (B) $3\sec\theta$
 - (C) $3\cos\frac{\theta}{2}$
 - (D) $3\sec\frac{\theta}{2}$
 - (E) 3
8. In the design of a swing set, the two metal posts that support the top bar each measure 8 feet. At the ground level, the posts are to be 6 feet apart. At what angle should the two metal posts be secured?
- (A) 44°
 - (B) 38°
 - (C) 32°
 - (D) 26°
 - (E) 20°

9. The negation of the statement, "No members of the team are male," is:

- (A) No members of the team are female.
- (B) Some members of the team are female.
- (C) Some members of the team are male.
- (D) All members of the team are male.
- (E) All members of the team are female.

10. What is $m\angle\theta$?



- (A) 15°
 - (B) 30°
 - (C) 60°
 - (D) 120°
 - (E) 240°
11. A circular grass plot 12 feet in diameter is cut by a straight gravel path 3 feet wide, one edge of which passes through the center of the plot. The number of square feet in the remaining grass area is:
- (A) $36\pi - 34$
 - (B) $30\pi - 15$
 - (C) $36\pi - 33$
 - (D) $35\pi - 9\sqrt{3}$
 - (E) $30\pi - 9\sqrt{3}$
12. If the sum of all the angles except one of a convex polygon is 2190 degrees, then the number of sides of the polygon must be:
- (A) 13
 - (B) 15
 - (C) 17
 - (D) 19
 - (E) 21

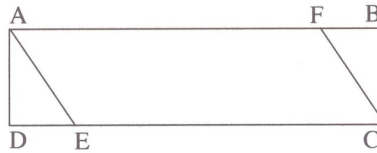
13. The volume of a pyramid whose base is an equilateral triangle of side length 6 and whose other edges are each of length $\sqrt{15}$ is:

- (A) 9
 (B) $9/2$
 (C) $27/2$
 (D) $\frac{9\sqrt{3}}{2}$
 (E) $\frac{9\sqrt{5}}{2}$

14. Which one of the following statements is incorrect?

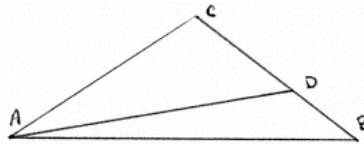
- (A) Doubling the base of a rectangle doubles the area.
 (B) Doubling the altitude of a triangle doubles the area.
 (C) Doubling the radius of a circle doubles the area.
 (D) Doubling the divisor of a fraction and dividing its numerator by 2 changes the quotient.
 (E) Doubling a quantity may make it less than it originally was.

15. On side \overline{AB} and \overline{DC} of rectangle $ABCD$, points E and F are chosen so that $AFCE$ is a rhombus. If $AB = 16$ and $BC = 12$, what is the length of AF ?



- (A) 15
 (B) 9
 (C) $\frac{7}{2}$
 (D) 12
 (E) $\frac{25}{2}$
16. Which one of the following statements is false? All equilateral triangles are:
- (A) equiangular
 (B) isosceles
 (C) regular polygons
 (D) congruent to each other
 (E) similar to each other

17. A 25 foot ladder is placed against a vertical wall of a building. The foot of the ladder is 7 feet from the base of the building. If the top of the ladder slips 4 feet, then the foot of the ladder will slide:
- (A) 9 ft.
 (B) 15 ft.
 (C) 5 ft.
 (D) 8 ft.
 (E) 4 ft.
18. The hypotenuse of a right triangle has length $2\sqrt{34}$ inches and the sum of the lengths of the legs is 16 inches. What is the area of this triangle?
- (A) 45 square inches
 (B) 40 square inches
 (C) 38 square inches
 (D) 34 square inches
 (E) 30 square inches
19. In triangle ABC , $\overline{AC} = \overline{CD}$ and $m\angle CAB - m\angle ABC = 30^\circ$. What is $m\angle BAD$?



- (A) 30°
 (B) 22°
 (C) 20°
 (D) 15°
 (E) 10°
20. The measure of $\angle C$ is twice the measure of its complement, $\angle D$. What is $3 \cdot m\angle C + 5 \cdot m\angle D$?
- (A) 90°
 (B) 300°
 (C) 330°
 (D) 480°
 (E) 520°

21. The area of the largest triangle that can be inscribed in a semi-circle whose radius is r is:
- (A) r^3
 - (B) $2r^3$
 - (C) r^2
 - (D) $2r^2$
 - (E) $\frac{r^2}{2}$
22. If four times the reciprocal of the circumference of a circle equals the diameter of the circle, then the area of the circle is:
- (A) $\frac{1}{\pi^2}$
 - (B) $\frac{1}{\pi}$
 - (C) 1
 - (D) π
 - (E) π^2
23. The distance around the earth's equator is 40,077 kilometers, and the population of the United States in 1999 was approximately 275,000,000 people. If this many people were spaced equally around the equator, what would be the length of the space each person would have, to the nearest centimeter?
- (A) 15 centimeters
 - (B) 69 centimeter
 - (C) 146 centimeters
 - (D) 686 centimeters
 - (E) 6,862 centimeters
24. The circumference of a toy balloon is increased from 20 inches to 25 inches. What is the increase in the radius of the balloon?
- (A) 5 inches
 - (B) $\frac{5}{\pi}$ inches
 - (C) 2.5 inches
 - (D) $\frac{5}{2\pi}$ inches
 - (E) $\frac{\pi}{5}$ inches

25. How many circular pipes with inside diameter of 1 inch will carry the same amount of water as a pipe with an inside diameter of 6 inches?
- (A) 3
(B) 6
(C) 12
(D) 36
(E) 48
26. A square is inscribed inside of a circle. The area of the square is what percent of the area of the circle (to the nearest tenth of a percent)?
- (A) 63.7%
(B) 31.8%
(C) 15.9%
(D) 40.6%
(E) 50%
27. In a triangle with sides of lengths a , b and c , $(a + b + c)(a + b - c) = 3ab$. The measure of the angle opposite the side of length c is:
- (A) 15°
(B) 30°
(C) 45°
(D) 60°
(E) 120°
28. $A(2a, 2b)$, $B(2c, 2d)$, and $C(0, 2e)$ are the vertices of a triangle. Find the slope of the altitude from B to \overline{AC} .
- (A) $\frac{e-b}{a}$
(B) $\frac{a}{e-b}$
(C) $\frac{b-e}{a}$
(D) $\frac{a}{b-e}$
(E) $\frac{2a}{e-b}$

29. How many faces, edges, and vertices does a pyramid with an n -gon base have?
- (A) $n, n+1, n+2$
(B) $n+1, 2n, n+1$
(C) $n, 2n, n+1$
(D) $n+1, n, 2n$
(E) $2n, n+1, n+2$
30. Each of two angles of a triangle is 60° and the included side is 4 cm. The area of the triangle, in square cm, is:
- (A) $8\sqrt{3}$
(B) 8
(C) $4\sqrt{3}$
(D) 4
(E) $2\sqrt{3}$
31. If the space diagonal of a cube has length 21, the edges of the cube all have length:
- (A) 7
(B) $\frac{21\sqrt{2}}{2}$
(C) $7\sqrt{2}$
(D) $\frac{7\sqrt{3}}{3}$
(E) $7\sqrt{3}$
32. A point P is outside a circle and is 13 inches from the center. A secant from P cuts the circle at Q and R so that the external segment of the secant PQ is 9 inches and \overline{QR} is 7 inches. The radius of the circle is:
- (A) 3 in
(B) 4 in
(C) 5 in
(D) 6 in
(E) 7 in