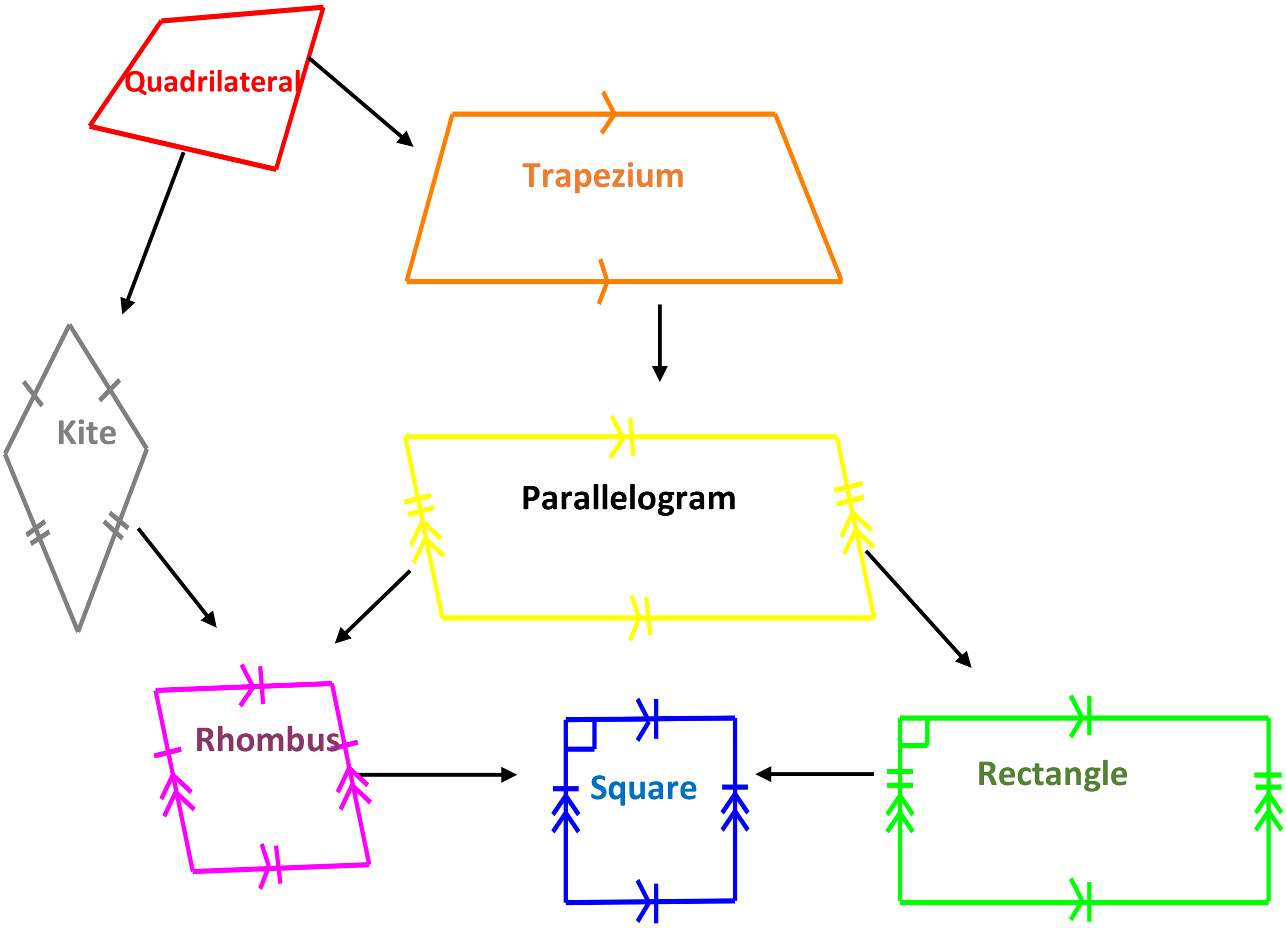
Geometry (Part 3)

Quadrilaterals

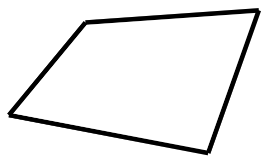
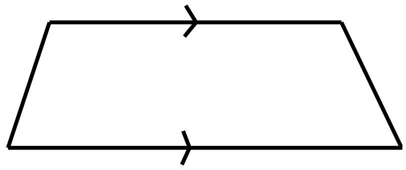
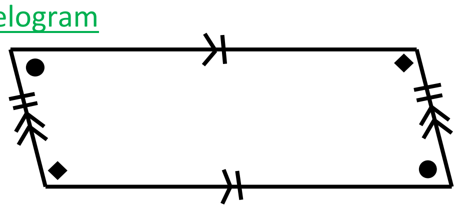
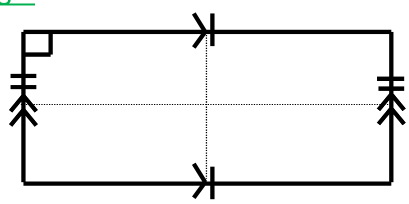
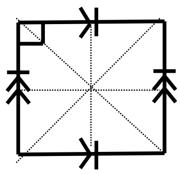
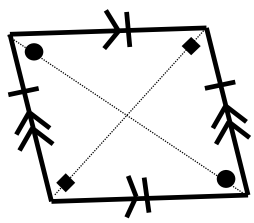
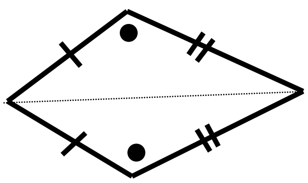
Quadrilateral family:

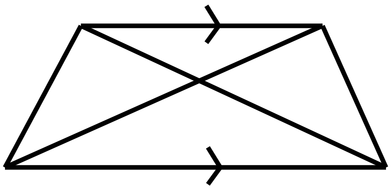
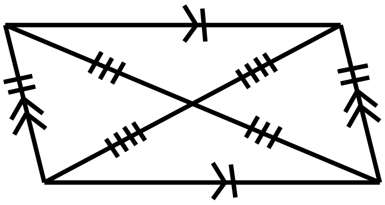
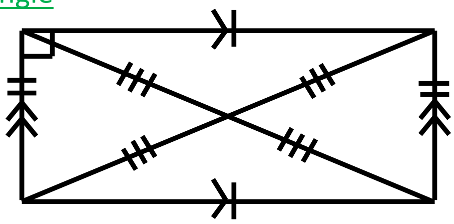
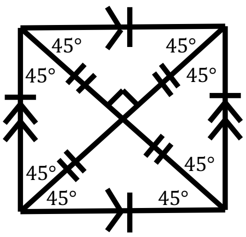
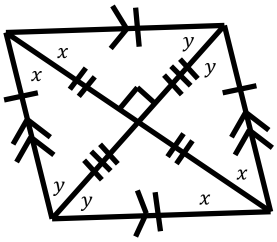
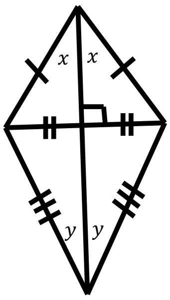


Using your knowledge of the properties of quadrilaterals, try to answer the following questions, with reasons:

1. Are all parallelograms trapeziums and vice versa (the other way around)?
2. Is a square a rectangle and vice versa (the other way around)?
3. Is a rectangle a parallelogram and vice versa (the other way around)?

Look at the back of the memo for the answers!

Properties of quadrilaterals	
Quadrilateral 	• Four closed sides • Interior angles add up to 360°
Trapezium 	• Only one pair of opposite sides parallel • No lines of symmetry
Parallelogram 	• Both pairs of opposite sides parallel • Both pairs of opposite sides equal in length • Both pairs of opposite interior angles equal in size • No lines of symmetry
Rectangle 	• Both pairs of opposite sides parallel • Both pairs of opposite sides equal in length • All interior angles equal to 90° • Two lines of symmetry
Sqaure 	• Both pairs of opposite sides parallel • All side equal to each other • All interior angles equal to 90° • Four lines of symmetry
Rhombus 	• Both pairs of opposite sides parallel • All sides equal in length • Both pairs of opposite interior angles equal in size • Two lines of symmetry
Kite 	• Two pairs of adjacent sides equal in length • One pair of opposite angles equal to each other where the short side meets the longer side • One line of symmetry

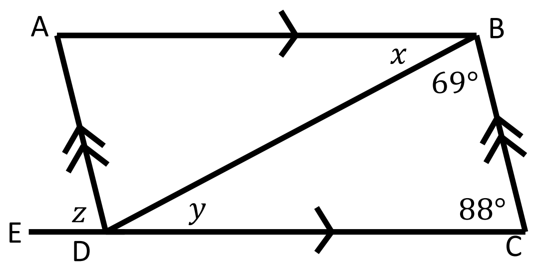
Properties of the diagonals of quadrilaterals	
Trapezium 	• No special properties Bisect means to divide into two equal sections
Parallelogram 	• The diagonals bisect each other • The diagonals are not equal in length
Rectangle 	• The diagonals bisect each other **and** is equal in length
Square 	• The diagonals bisect each other **perpendicularly and** is equal in length • The diagonals bisect the interior corner angles
Rhombus 	• The diagonals bisect each other **perpendicularly** • The diagonals bisect the interior opposite corner angles
Kite 	• The long diagonal bisect the short diagonal **perpendicularly** • The diagonals bisect the interior opposite corner angles **only** where the adjacent sides meet

Look out for the following when working with a…	
…trapezium, parallelogram, rectangle, square or rhombus…	They all have **<u>parallel sides</u>** which means you can use your **<u>FUN angles</u>** from Part 1.
…kite or square…	These shapes have a bunch of **<u>isosceles triangles</u>** in them. We learned in Part 2 that the **<u>base angles</u>** of an isosceles triangle are **<u>equal</u>** to each other.

Let's see in the example below how we will use the properties of quadrilaterals to help us solve geometrical problems. Remember to use everything that you've learn in Part 1 and Part 2 about lines, angles and triangles!

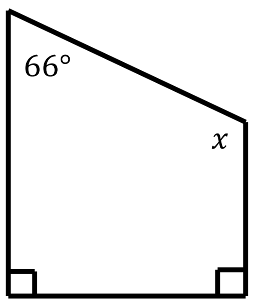
<u>Example 1:</u>

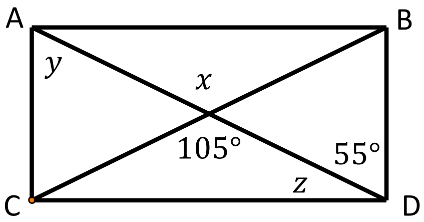
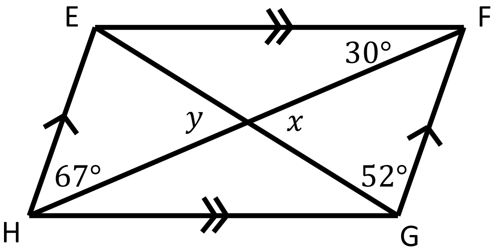
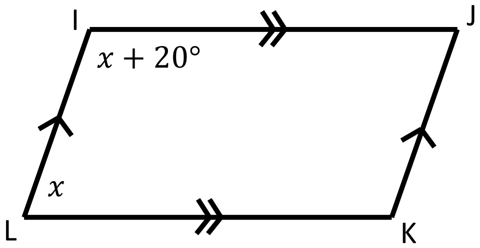
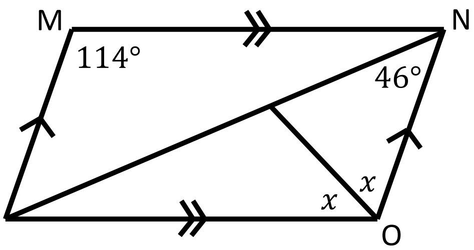
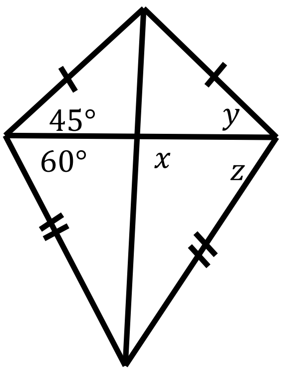
Determine, with reasons, the values of the unknown angles in the following:

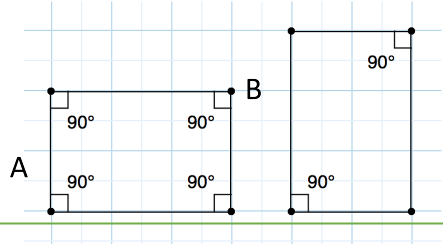
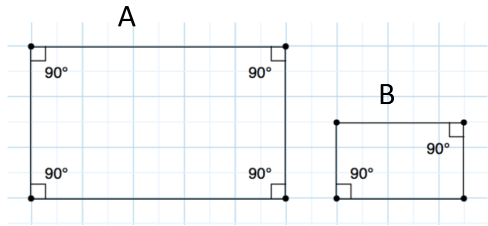
	Statement	**Reason**
	$x + 69° + 88° = 180°$ $x = 180° - 157°$ $x = 23°$ $y = 23°$ $z = 88°$	Co-interior ∠'s ; AB//EC Alternate ∠'s ; AB//EC Corresponding ∠'s ; AB//EC

<u>Exercise 1</u>: (*None of the diagrams are drawn to scale)*

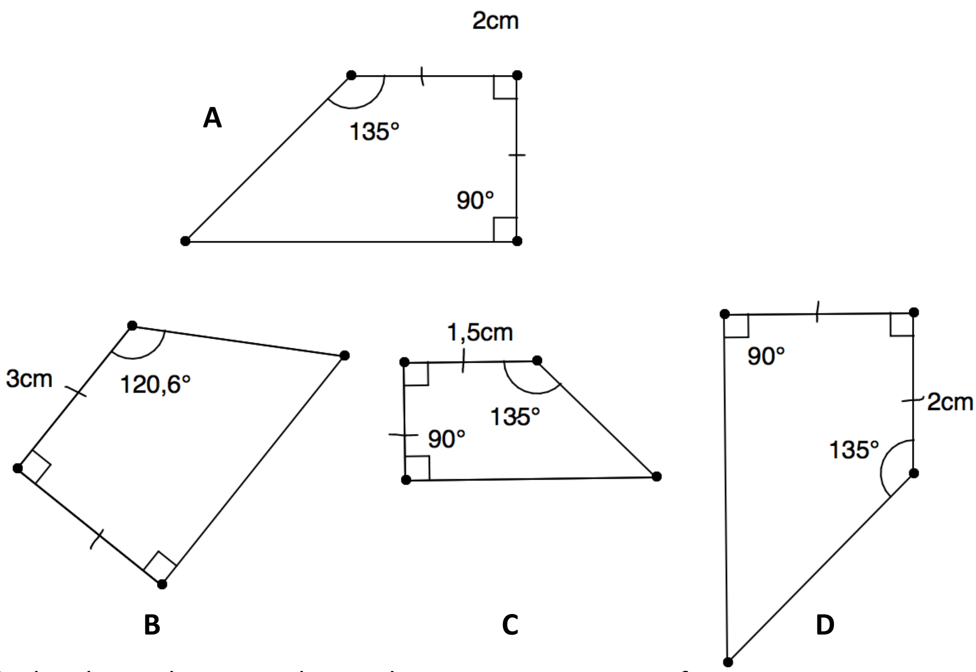
Determine, with reasons, the values of the unknown angles in the following:

	Statement	**Reason**
		

ABCD is a rectangle. 		
		
		
		
		

Congruency and Similarity of Quadrilaterals	
Two quadrilaterals are **congruent** when all corresponding sides and all corresponding angles of the two quadrilaterals are equal. *Rectangle A ≡ Rectangle B*	
Two quadrilaterals are **similar** when the corresponding angles of two quadrilaterals are equal, but the corresponding sides of the two quadrilaterals are not equal. The sides lengths of similar quadrilaterals will correspond in ratio. *Rectangle A |||Rectangle B*	

<u>Exercise 2:</u> Refer to the image below and answer the questions which follow:
Images are not drawn to scale.



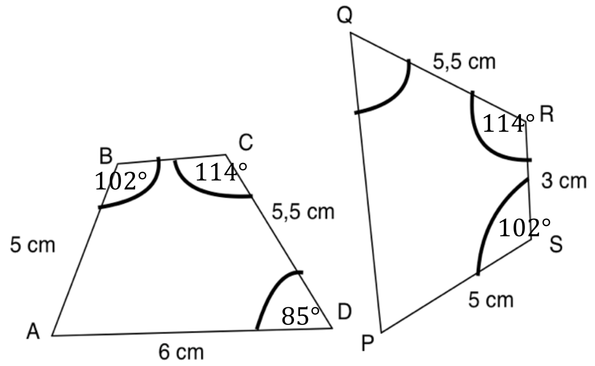
2.1 Identify the shape that is similar to Shape A. Give a reason for your answer.

2.2 Identify the shape that is congruent to Shape A. Give a reason for your answer.

Exercise 3: Answer the following questions on **congruence** and **similarity**:

3.1 $Quadrilateral\ ABCD \equiv Quadrilateral\ PQRS$

Calculate the following:

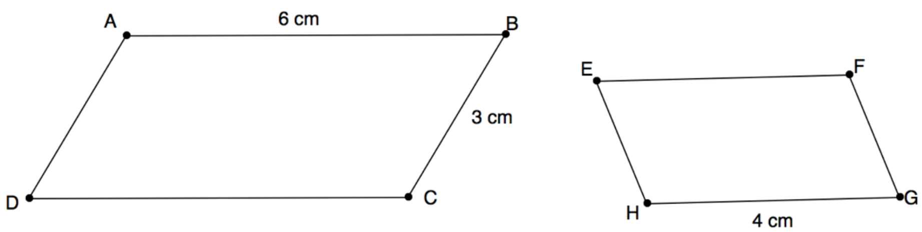


$\overleftrightarrow{QP}$

$\overleftrightarrow{BC}$

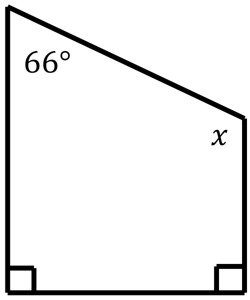
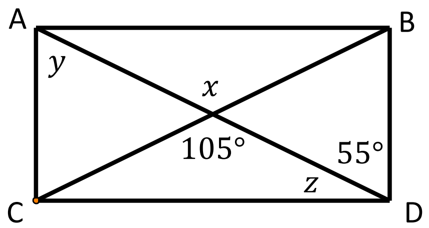
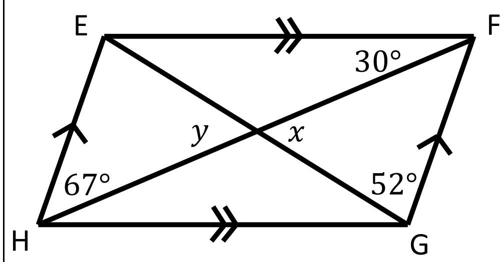
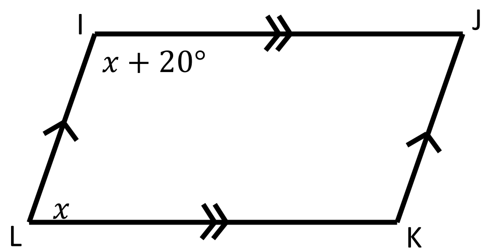
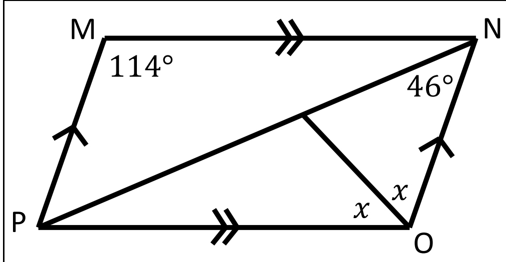
$\angle QPS$

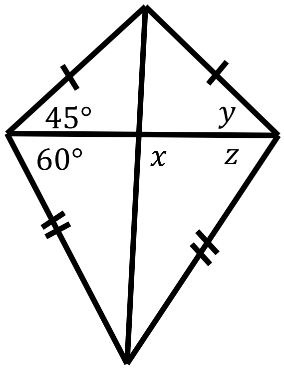
3.2 $Parallelogram\ ABCD\ |||\ Parallegrom\ EFGH$



Calculate the length of FG

MEMO

	Statement	Reason
	$x + 66^\circ + 90^\circ + 90^\circ = 360^\circ$ $x + 246^\circ = 360^\circ$ $x = 360^\circ - 246^\circ$ $x = 114^\circ$	Internal ∠'s of a quad
ABCD is a rectangle. 	$x = 105^\circ$ $y = 55^\circ$ $z = 90^\circ - 55^\circ$ $z = 35^\circ$	Vertically opposite ∠'s Alternate ∠'s ; AC // BD Internal ∠'s of a rectangle $= 90^\circ$
	$H\hat{F}G = 67^\circ$ $x + 52^\circ + 67^\circ = 180^\circ$ $x + 119^\circ = 180^\circ$ $x = 61^\circ$ $y = 61^\circ$	Alternate ∠'s ; EH // FG Internal ∠'s of a Δ Vertically opp ∠'s
	$x + 20^\circ + x = 180^\circ$ $2x + 20^\circ = 180^\circ$ $2x = 160^\circ$ $x = 80^\circ$	Co-interior ∠'s ; IJ // LK
	$x + x = 114^\circ$ $2x = 114^\circ$ $x = \frac{114^\circ}{2}$ $x = 57^\circ$	Opp ∠'s of parm =

	$x = 90°$ $y = 45°$ $z = 60°$	Diagonals of a kite bisect ⊥ Isosceles Δ Isosceles Δ

Reasons for angle calculations may vary as there may be other methods to calculate the angle sizes.

<u>Exercise 2:</u> Refer to the image below and answer the questions which follow:
Images are not drawn to scale.

2.1 Identify the shape that is similar to Shape A. Give a reason for your answer.

$Quadrilateral\ A\ |||\ Quadrilateral\ C$, because all the corresponding angles are equal in shape A and C and the corresponding sides are not equal, but the corresponding sides are in the same ratio.

2.2 Identify the shape that is congruent to Shape A. Give a reason for your answer.

$Quadrilateral\ A \equiv Quadrilateral\ D$, because all corresponding angles and sides in both shapes are equal.

<u>Exercise 3:</u> Answer the following questions on **congruence** and **similarity**:

3.1 $Quadrilateral\ ABCD \equiv Quadrilateral\ PQRS$

Calculate the following:

$\overleftrightarrow{QP} = 6cm$

$\overleftrightarrow{BC} = 3cm$

$\angle QPS = 360° - (102° + 114° + 85°)$

$\angle QPS = 360° - 301°$

$\angle QPS = 59°$

3.2 $Parallelogram\ ABCD\ |||\ Parallegrom\ EFGH$

Calculate the length of FG.

Ratio of AB : HG = 6 : 4 or 3 : 2

Therefore ratio of BC : FG will also be 3 : 2

If BC = 3cm then FG will be 2 cm in length

Using your knowledge of the properties of quadrilaterals, try to answer the following questions, with reasons:

1. A parallelogram is a trapezium, but a trapezium is not a parallelogram. A parallelogram has at least one pair of parallel sides (the properties of a trapezium).
2. A square is a rectangle, but a rectangle is not a square. A square has two pairs of equal, parallel sides and four right angles (the properties of a rectangle).
3. A rectangle is a parallelogram, but a parallelogram is not a rectangle. A rectangle has two pairs of equal, parallel sides and equal diagonally opposite angles (the properties of a parallelogram.)