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Geometry

Points, Lines, Planes & Angles

Part 2

2014-09-20

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Angles

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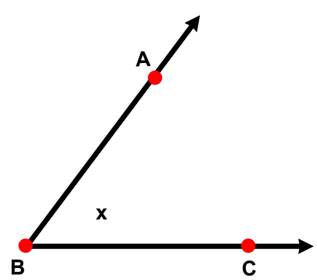
Angles

Definition 8: A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.

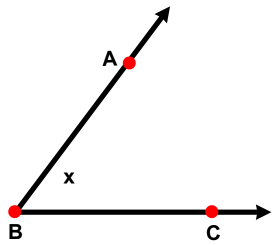
Whenever lines, rays or segments in a plane intersect, they do so at an angle.



Angles

The measure of angle is the amount that one line, one ray or segment would need to rotate in order to overlap the other.

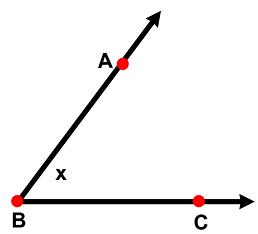
In this case, Ray BA would have to rotate through an angle of x in order to overlap Ray BC.



Angles

In this course, angles will be measured with degrees, which have the symbol ^{0}.

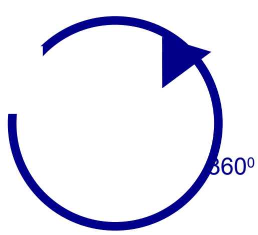
For a ray to rotate all the way around from BC, as shown, back to BC would represent a 360^0 angle.



Measuring angles in degrees

The use of 360 degrees to represent a full rotation back to the original position is arbitrary.

Any number could have been used, but 360 degrees for a full rotation has become a standard.



Measuring angles in degrees

The use of 360 for a full rotation is thought that it come from ancient Babylonia, which used a number system based on 60.

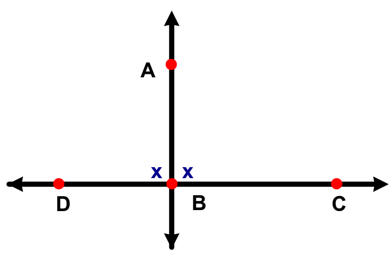
Their number system may also be linked to the fact that there are 365 days in a year, which is pretty close to 360.

360 is a much easier number to work with than 365 since it is divided evenly by many numbers.
These include 2, 3, 4, 5, 6, 8, 9, 10 and 12.

Right Angles

Definition 10: When a straight line standing on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands.

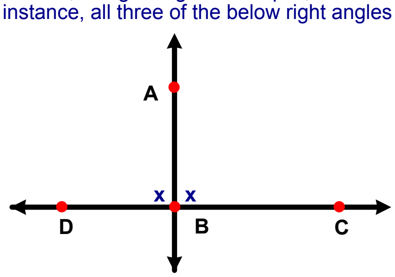
The only way that two lines can intersect as shown and form adjacent equal angles, such as shown here where Angle ABC = Angle ABD, is if there are right angles, 90^0.

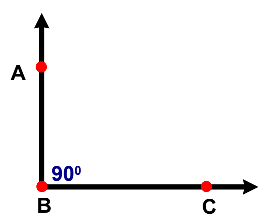


Right Angles

Fourth Postulate: That all right angles are equal to one another.

Not only are adjacent right angles equal to each other as shown below, all right angles are equal, even if they are not adjacent, for instance, all three of the below right angles are equal to one another.





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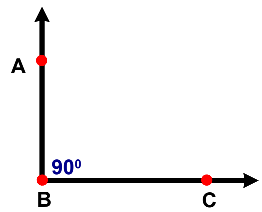
Right Angles

This definition is unchanged today and should be familiar to you.
Perpendicular lines, segments or rays form right angles.

If lines intersect to form adjacent equal angles, then they are perpendicular and the measure of those angles is 90^{0}.



When perpendicular lines meet, they form equal adjacent angles and their measure is 90^{0}.

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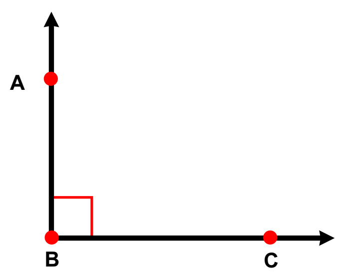
Right Angles

There is a special indicator of a right angle.

It is shown in red in this case to make it easy to recognize.



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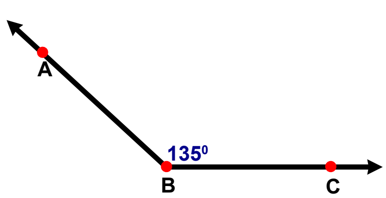
Obtuse Angles

Definition 11: An obtuse angle is an angle greater than a right angle.



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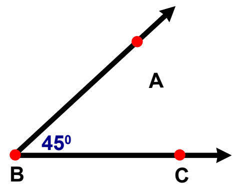
Acute Angles

Definition 12: An acute angle is an angle less than a right angle.



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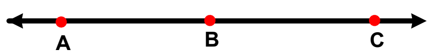
Straight Angle

A definition that we need that was not used in The Elements is that of a "straight angle." That is the angle of a straight line.



Answer

2 questions to discuss with a partner:

Is this an acute or obtuse angle?

What is the degree measurement of the angle?

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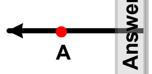
Straight Angle

A definition that we need th
of a "straight angle."



Answer

This is a type of obtuse angle

180^0

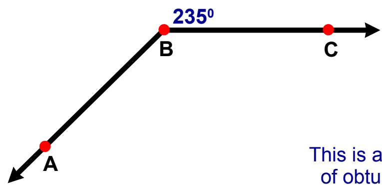
2 questions

Is this an acu

What is the degree measurement of the angle?

Reflex Angle

Another modern definition that was not used in *The Elements* is that of a "reflex angle." That is an angle that is greater than 180^0.



This is also a type of obtuse angle.

Angles

In the next few slides we'll use our responders to review the names of angles by showing angles from 0^0 to 360^0 in 45^0 increments.

Angles can be of any size, not just increments of 45^0, but this is just to give an idea for what a full rotation looks like.

1 This is an example of a (an) ________ angle. Choose all that apply.

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight

Answer

1 **This is an example of a (an) ________ angle. Choose all that apply.**

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight

Answer

A

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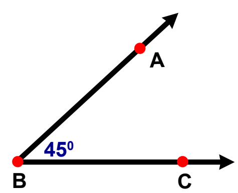
2 **This is an example of a (an) ________ angle. Choose all that apply.**

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight



Answer

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2 **This is an example of a (an) angle. Choose all that apply.**

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight

Answer

A

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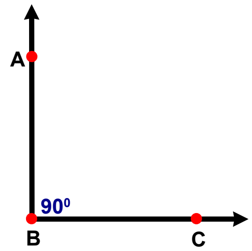
3 **This is an example of a (an) ________ angle. Choose all that apply.**

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight



Answer

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3 **This is an example of a (an) ________ angle. Choose all that apply.**

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight

Answer

C

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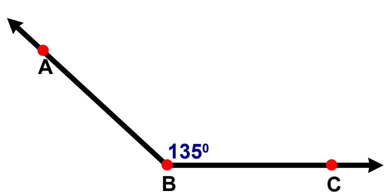
4 **This is an example of a (an) ________ angle. Choose all that apply.**

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight



Answer

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4 **This is an example of a (an) ________ angle.**
Choose all that apply.

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight

Answer

B

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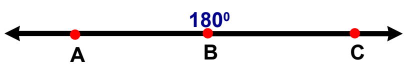
5 **This is an example of a (an) ________ angle.**
Choose all that apply.

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight



Answer

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5 **This is an example of a (an) ________ angle.**
Choose all that apply.

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight

Answer

B and E

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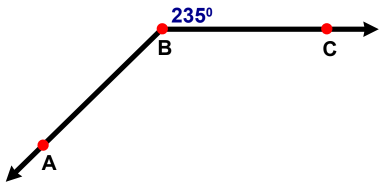
6 **This is an example of a (an) ________ angle. Choose all that apply.**

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight



Answer

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6 **This is an example of a (an) ________ angle. Choose all that apply.**

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight

Answer

B and D

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7 **This is an example of a (an) ________ angle. Choose all that apply.**

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight

270^0 B C A

Answer

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7 This is an example of a (an) ________ angle. Choose all that apply.

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight

Answer

B and D

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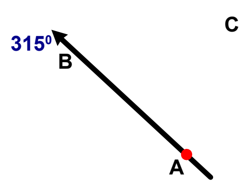
8 This is an example of a (an) ________ angle. Choose all that apply.

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight



Answer

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8 This is an example of a (an) ________ angle. Choose all that apply.

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight

Answer

B and D

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9 This is an example of a (an) ________ angle. Choose all that apply.

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight

Answer

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9 This is an example of a (an) ________ angle. Choose all that apply.

- ☐ acute
- ☐ obtuse
- ☐ right
- ☐ reflex
- ☐ straight

Answer

B and D

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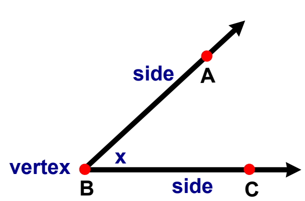
Naming Angles

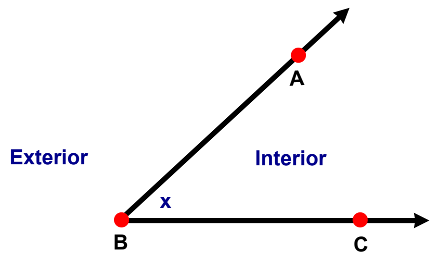
An angle has three parts, it has two sides and one vertex, where the sides meet.

In this example, the sides are the rays BA and BC and the vertex is B.



Interior of Angles

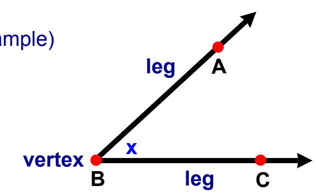
Any angle with a measure of less than 180⁰ has an interior and exterior, as shown below.



Naming Angles

An angle can be named in three different ways:

- By its vertex (B in the below example)
- By a point on one leg, its vertex and a point on the other leg (either ABC or CBA in the below example)



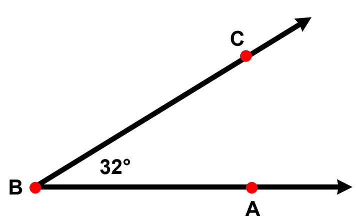
- Or by a letter or number placed inside the angle (x in the below)

Naming Angles

The angle shown can be called ∠ABC , ∠CBA, or ∠B.

When there is no chance of confusion, the angle may also be identified by its vertex B.

The sides of ∠ABC are rays BC and BA



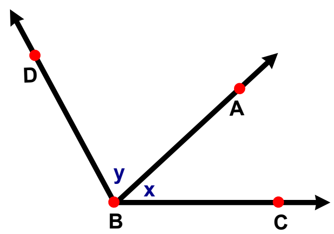
The measure of ∠ABC is 32 degrees, which can be rewritten as m∠ABC = 32°.

Naming Angles

Using the vertex to name an angle doesn't work in some cases. Why would it it would be unclear to use the vertex to name the angle in the image below?

Answer

How many angles do you count in the image?



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Naming Angles

Using the verte... ome cases. Wh... vertex to...

Answer

there is more than 1 angle with B as its vertex.

There are 3 angles

How many angles ... you count in the image?

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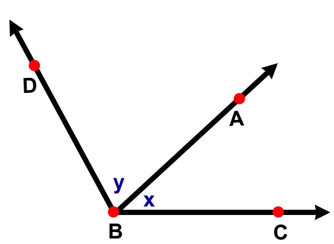
Naming Angles

What other ways could you name ∠ABC, ∠ABD and ∠DBC in the case below? (using the side - vertex - side method)

Answer



How could you name those 3 angles using the letters placed inside the angles?

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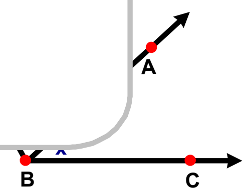
Naming Angles

Wh ∠ABD and ∠DBC in the - side method)

Answer

∠CAB, ∠DBA and ∠CBD

x, y, x+y

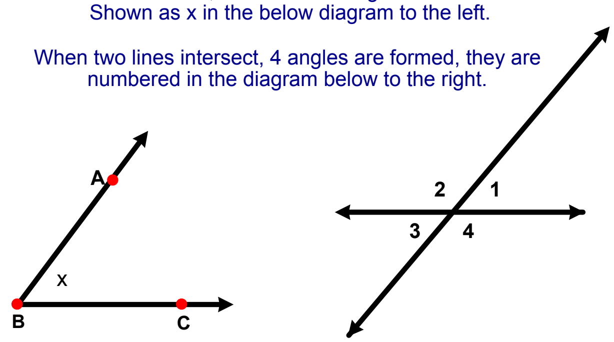


How could you name those 3 angles using the letters placed inside the angles?

Intersecting Lines Form Angles

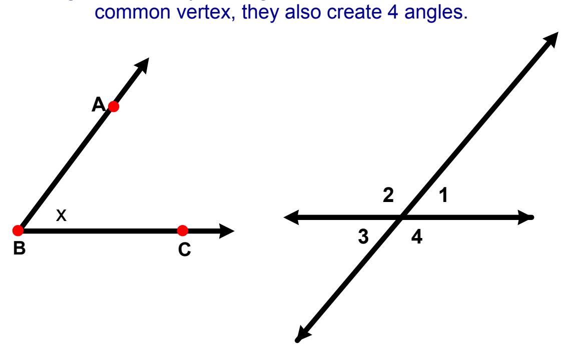
When an angle is formed by either two rays or segments with a shared vertex, one included angle is formed. Shown as x in the below diagram to the left.

When two lines intersect, 4 angles are formed, they are numbered in the diagram below to the right.



Intersecting Lines Form Angles

These numbers used have no special significance, but just show the 4 angles. When rays or segments intersect but do not have a common vertex, they also create 4 angles.



10 **Two lines ________________ meet at more than one point.**

- A Always
- B Sometimes
- C Never

Answer

10 **Two lines ________________ meet at more than one point.**

- A Always
- B Sometimes
- C Never

Answer

B

11 **An angle that measures 90 degrees is __________ a right angle.**

- A Always
- B Sometimes
- C Never

Answer

11 **An angle that measure**
a right angle.

- A Always
- B Sometimes
- C Never

Answer

A

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12 **An angle that is less than 90 degrees is ___________ obtuse.**

- A Always
- B Sometimes
- C Never

Answer

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12 **An angle that is less than 90 degrees is ___________ obtuse.**

- A Always
- B Sometimes
- C Never

Answer

C

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13 **An angle that is greater than 180 degrees is ________ referred to as a reflex angle.**

- **A** Always
- **B** Sometimes
- **C** Never

Answer

13 **An angle that is greater than 180 degrees is ________ referred to as**

- **A** Always
- **B** Sometimes
- **C** Never

Answer

A

Congruent Angles

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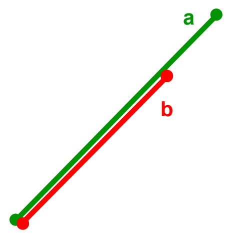
Congruence

We learned earlier that if two line segments have the same length, they are congruent.

Also, all line segments with the same length are congruent.

Are these two segments congruent?



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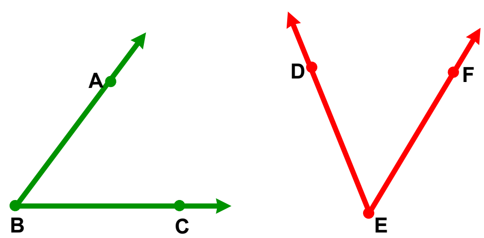
Congruence

How about two angles which are formed by two rays with common vertices. Are all of those congruent?

What would have to be the same for each of them to be congruent?



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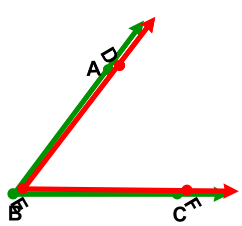
Congruence

If two angles have the same measure, they are congruent since they can be rotated and moved to overlap at every point.



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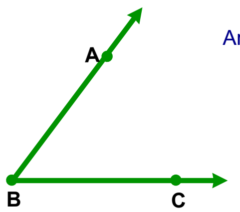
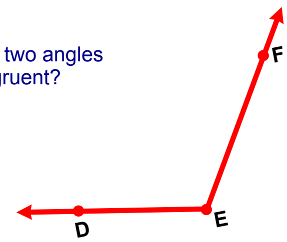
Congruence

However, if their included angles do not have equal measure, they cannot be made to overlap at every point.

For angles to be congruent, they need to have equal measures.

Are these two angles congruent?





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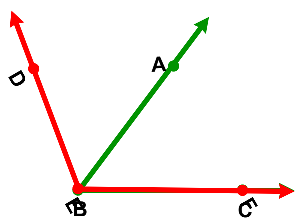
Congruence

However, if their included angles do not have the same measure, they cannot be made to overlap at every point.

For angles to be congruent, they need to have the same measure.



Here you can see clearly when we rotate the two angles from the previous slide, they do not have the same angle measure.

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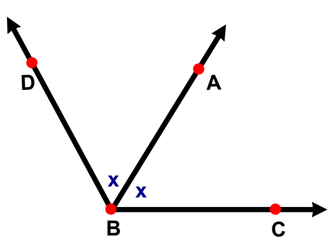
Congruent Angles

One way to indicate that two angles have the same measure is to label them with the same variable.

For instance, labeling both of these angles x indicates that they have the same measure.



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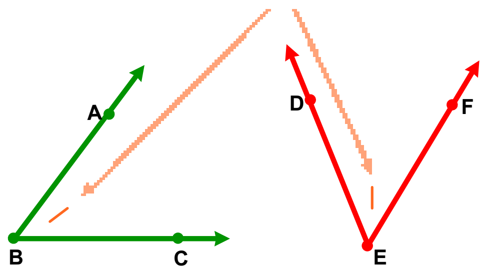
Congruent Angles

Another way to show angles are congruent is to mark the angle with a line. If there are 2 equal sets of angles, the second set could be marked with two lines.



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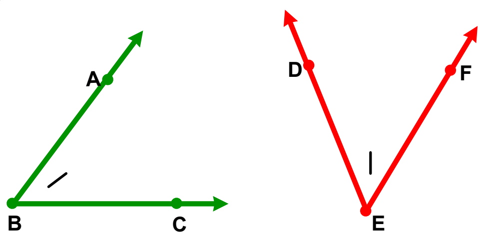
14 **Is ∠B congruent to ∠E ?**

❍ Yes

❍ No



Answer

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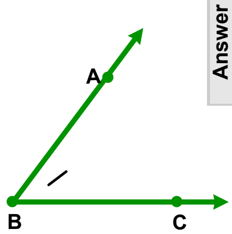
14 **Is ∠B congruent to ∠E ?**

❍ Yes

❍ No



Answer

YES

E

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15 **Congruent angles ___________ have the same measure.**

- A Always
- B Sometimes
- C Never

Answer

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15 **Congruent angles ___________ have the same measure.**

- A Always
- B Sometimes
- C Never

Answer

A

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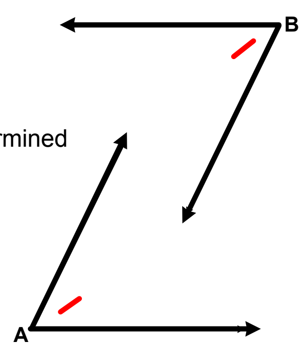
16 **∠A and ∠B are ______.**

- A Congruent
- B Not Congruent
- C Cannot be determined



Answer

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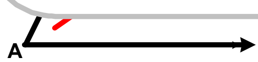
16 ∠A and ∠B are ______.

- ○ **A** Congruent
- ○ **B** Not Congruent
- ○ **C** Cannot be det

Answer

A



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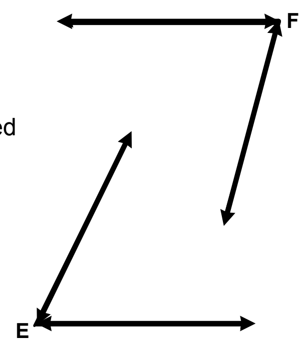
17 ∠E and ∠F are _______.

- ○ **A** Congruent
- ○ **B** Not Congruent
- ○ **C** Cannot be determined



Answer

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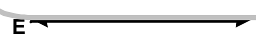
17 ∠E and ∠F are _______.

- ○ **A** Congruent
- ○ **B** Not Congruent
- ○ **C** Cannot be determ

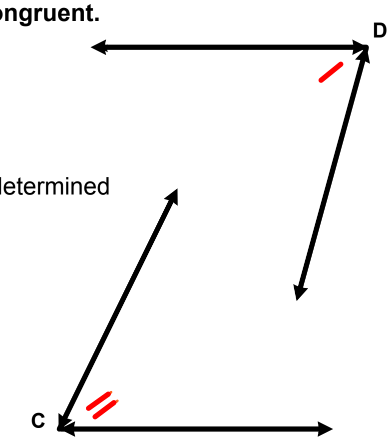
Answer

C



18 ∠C and ∠D are congruent.

- A True
- B False
- C Cannot be determined



Answer

18 ∠C and ∠D are congruent.

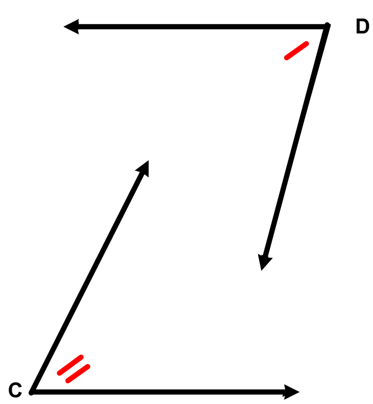
- A True
- B False
- C Cannot be deter

Answer

B

19

- True
- False



Answer

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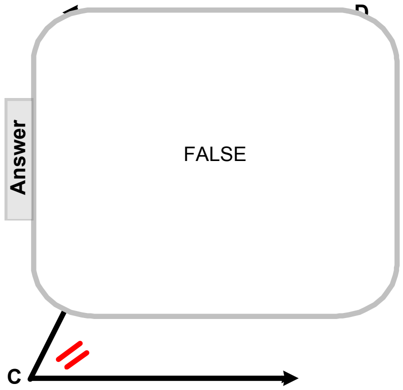
19

- True
- False



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Angles & Angle Addition Postulate

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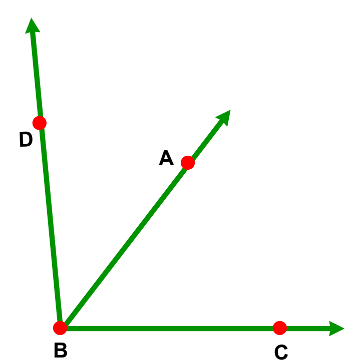
Adjacent Angles

Adjacent angles share a vertex and a side.

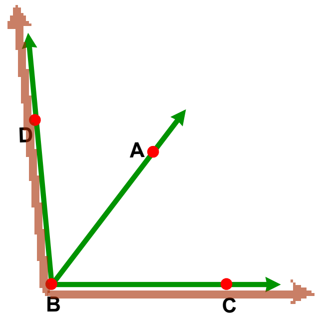
The two angles are side by side, or adjacent.

In this case, Angle DBA is adjacent to Angle ABC.



Angle Addition Postulate

The angle addition postulate says that the measures of two adjacent angles add together to form the measure of the angle formed by their exterior rays.

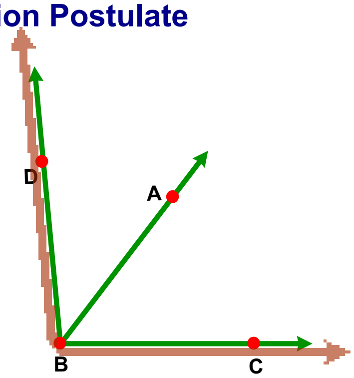


In this case, Angle DBC = Angle DBA + Angle ABC

Angle Addition Postulate

Further, it says that if any point lies in the interior of an angle, then the ray connecting that point to the vertex creates two adjacent angles that sum to the original angle.

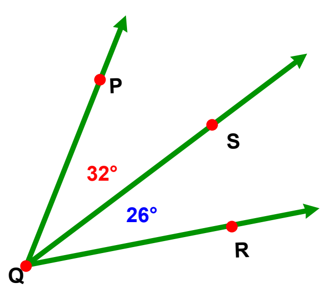
If A lies in the interior of Angle DBC then Angle DBA + Angle ABC= Angle DBC



Which yields the same result we had before.
Angle DBC = Angle DBA + Angle ABC

Angle Addition Postulate Example

m∠PQS = 32°
m∠SQR = 26°



Answer

What's the measure of ∠PQR?

Angle Addition Postulate Example

Answer

m∠PQS = 32°

m∠SQR = 26°

m∠PQR = 58^0

Q

What's the measure of ∠PQR?

Angle Addition Postulate Example

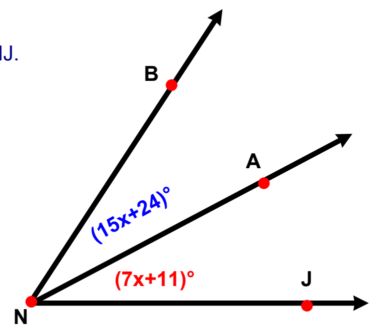
A is in the interior of BNJ.

If ∠ANJ = (7x +11)°,

∠ANB = (15x + 24)°,

and ∠BNJ = (9x+204)°.

Solve for x.



Answer

Angle Addition Postulate Example

A is in the interior c

If ∠ANJ = (7x +11)°

∠ANB = (15x + 2

and ∠BNJ = (9x+20

Solve for x.

Answer

∠ANJ + ∠ANB = ∠BNJ

$(7x + 11)° + (15x + 24)° = (9x + 204)°$

$22x + 35 = 9x + 204$

$13x + 35 = 204$

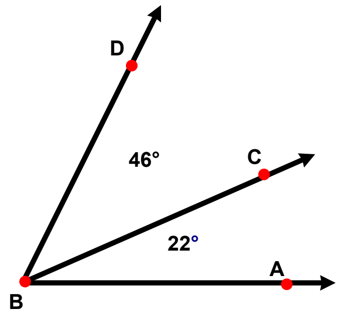
$13x = 169$

$x = 13$

N

20 Given m#ABC = 22° and m#DBC = 46°.

Find m#ABD.



Answer

20 Given m#ABC = 22° and m#DBC = 46°.

Find m#ABD.

Answer

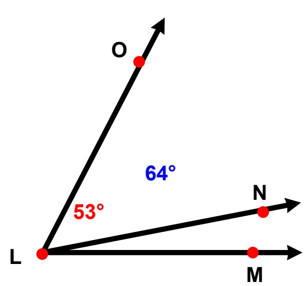
#ABD = #ABC + #DBC

#ABD = 22 + 46

#ABD = 68°

21 Given m#OLM = 64° and m#OLN = 53°.
Find m#NLM.

- A 28
- B 15
- C 11
- D 117



Answer

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21 **Given m#OLM = 64° and m#OLN = 53°.**
Find m#NLM.

- A 28
- B 15
- C 11
- D 117

Answer

#OLM = #OLN + #NLM

64 = 53 + #NLM

#NLM = 11°

C

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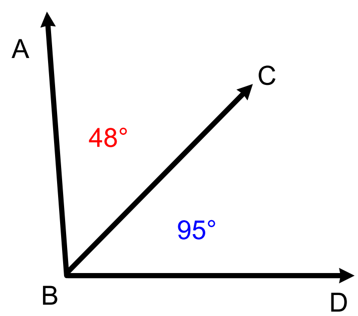
22 **Given m#ABD = 95° and m#CBA = 48°.**

Find m#DBC.



Answer

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22 **Given m#ABD = 95° and m#CBA = 48°.**

Find m#DBC.

A

Answer

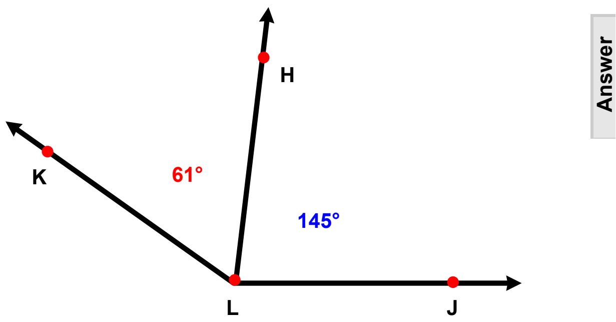
47°

B

D

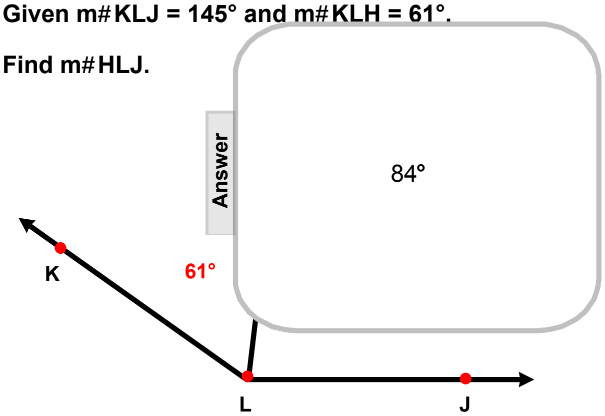
23 Given m#KLJ = 145° and m#KLH = 61°.

Find m#HLJ.



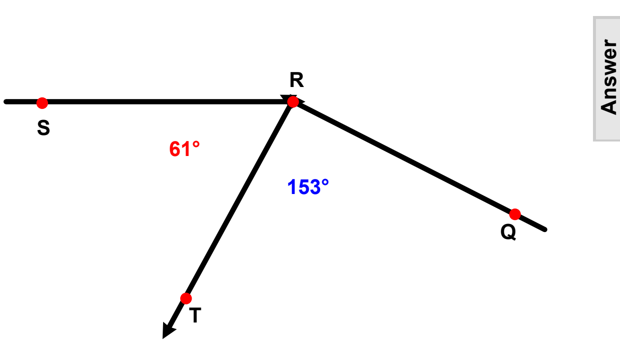
23 Given m#KLJ = 145° and m#KLH = 61°.

Find m#HLJ.



24 Given m#TRS = 61° and m#SRQ = 153°.

Find m#QRT.



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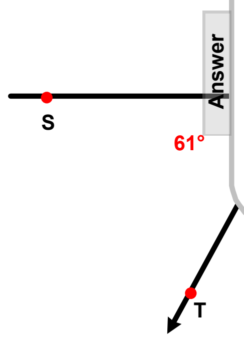
24 **Given m#TRS = 61° and m#SRQ = 153°.**

Find m#QRT.



92°

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25 **C is in the interior of #TUV.**

If m#TUV = (10x + 72)#,

m#TUC = (14x + 18)# and

m#CUV = (9x + 2)#

Solve for x.

Answer

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25 **C is in the interior of #TUV.**

If m#TUV = (10x + 72)#,

m#TUC = (14x + 18)# and

m#CUV = (9x + 2)#

Solve for x.

Answer

$10x + 72 = 14x + 18 + 9x + 2$

$10x + 72 = 23x + 20$

$13x = 52$

$x = 4$

26 D is in the interior of #ABC.

If m#CBA = (11x + 66)#,

m#DBA = (5x + 3)# and

m#CBD= (13x + 7)#

Solve for x.

Answer

26 D is in the interior of #ABC.

If m#CBA = (11x +

m#DBA = (5x + 3)#

m#CBD= (13x + 7

Solve for x.

Answer

$11x + 66 = 5x + 3 + 13x + 7$

$11x + 66 = 18x + 10$

$7x = 56$

$x = 8$

27 F is in the interior of #DQP.

m#DQP = (3x + 44)#

m#FQP = (8x + 3)#

m#DQF= (5x + 1)#

Solve for x.

Answer

27 F is in the interior of #DQP

m#DQP = (3x + 44)

m#FQP = (8x + 3)#

m#DQF= (5x + 1)#

Solve for x.

Answer

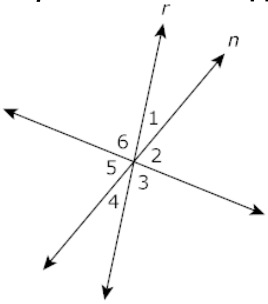
$3x + 44 = 8x + 3 + 5x + 1$

$3x + 44 = 13x + 4$

$10x = 40$

$x = 4$

28 The figure shows lines *r, n,* and *p* intersecting to form angles numbered 1, 2, 3, 4, 5, and 6. All three lines lie in the same plane. Based on the figure, which of the individual statements would provide enough information to conclude that line *r* is perpendicular to line *p*? Select *all* that apply.



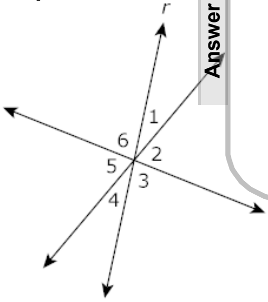
From PARCC sample test

not to scale

- A. $m\angle 2 = 90^\circ$
- B. $m\angle 6 = 90^\circ$
- C. $m\angle 3 = m\angle 6$
- D. $m\angle 1 + m\angle 6 = 90^\circ$
- E. $m\angle 3 + m\angle 4 = 90^\circ$
- F. $m\angle 4 + m\angle 5 = 90^\circ$

Answer

28 The figure shows lines *r, n,* and *p* intersecting to form angles numbered 1, 2, 3, 4, 5, and 6. All three lines lie in the same plane. Based on the figure, which of the individual statements would provide enough information to conclude that line *r* is perpendicular to line *p*? Select *all* that apply.



From PARCC sample test

not to scale

Answer

Select **all** that apply.

- ✗ A. $m\angle 2 = 90^\circ$
- ✓ B. $m\angle 6 = 90^\circ$
- ✗ C. $m\angle 3 = m\angle 6$
- ✗ D. $m\angle 1 + m\angle 6 = 90^\circ$
- ✗ E. $m\angle 3 + m\angle 4 = 90^\circ$
- ✓ F. $m\angle 4 + m\angle 5 = 90^\circ$

- E. $m\angle 3 + m\angle 4 = 90^\circ$
- F. $m\angle 4 + m\angle 5 = 90^\circ$

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Protractors

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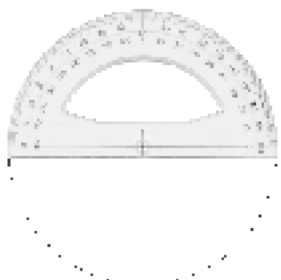
Protractors

Angles are measured in degrees, using a protractor.

Every angle has a measure from 0 to 180 degrees.

Angles can be drawn in any size.



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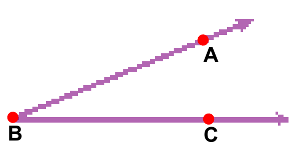
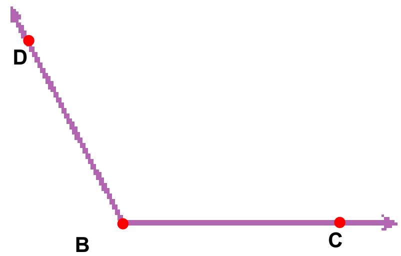
Protractors



#ABC is a 23° degree angle

The measure of #ABC is 23° degrees

Protractors



∠DBC is a 118° angle.
The measure of ∠DBC is 118°.

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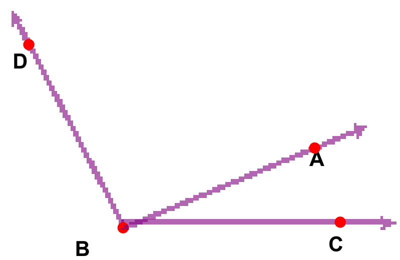
Protractors



From our prior results we know that Angle DBC = 118^0 and Angle ABC = 23^0.

So, the Angle Addition Postulatetells us that Angle DBA must be what?

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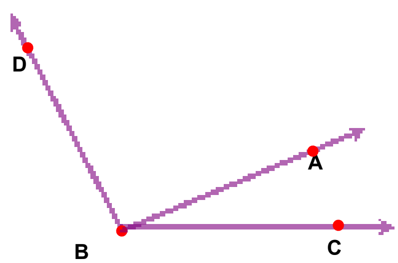
Protractors



Without those prior results, we could just read the values of 118^0 and 23^0 from the protractor to get the included angle to be 95^0.

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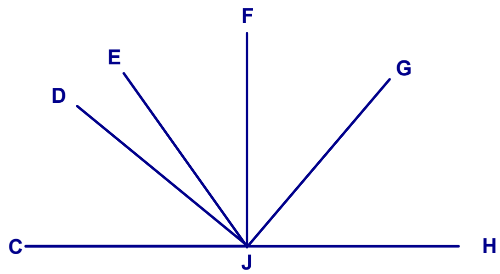
29 **What is the m∠CJD?**

- 39°
- 54°
- 130°
- 180°



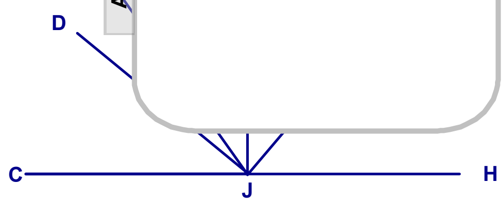
Answer

29 **What is the m∠CJD?**

- 39°
- 54°
- 130°
- 180°

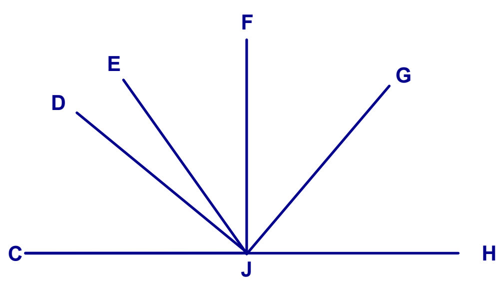
Answer

A



30 **What is the m∠CJG**

- 39°
- 54°
- 130°
- 180°



Answer

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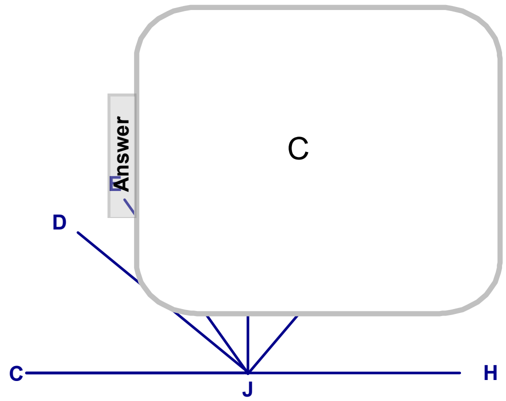
30 **What is the m∠CJG**

- 39°
- 54°
- 130°
- 180°



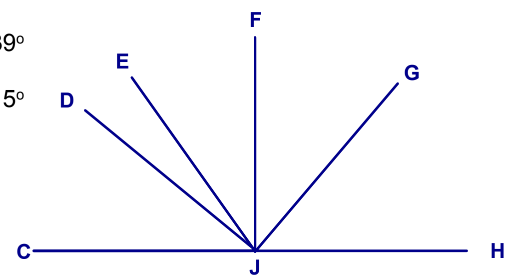
Answer

C

Slide 78 / 185

31 **What is the m∠DJE?**

- 141°
- 54°
- 39°
- 15°

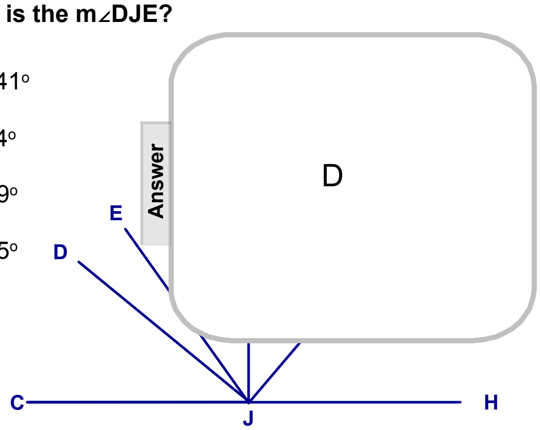


Answer

Slide 78 (Answer) / 185

31 **What is the m∠DJE?**

- 141°
- 54°
- 39°
- 15°



Answer

D

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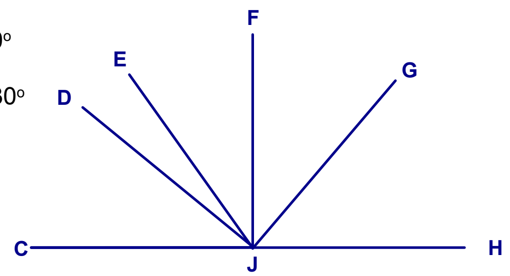
32 **What is the m∠EJG?**

- 54°
- 76°
- 90°
- 130°

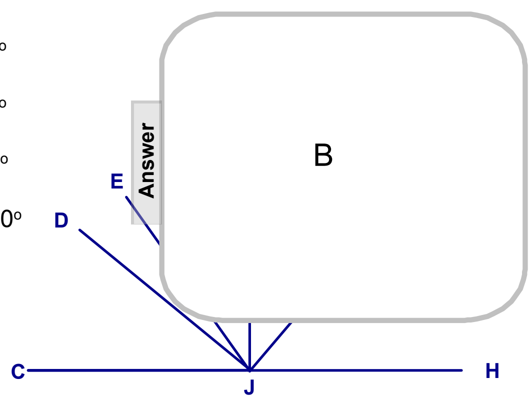


Answer

Slide 79 (Answer) / 185

32 **What is the m∠EJG?**

- 54°
- 76°
- 90°
- 130°



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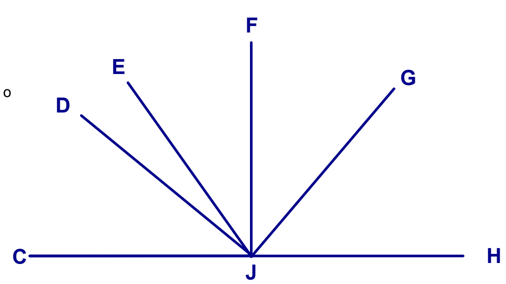
33 **What is the m∠DJF?**

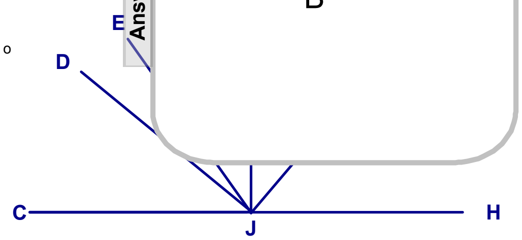
- 39°
- 51°
- 90°
- 141°



Answer

33 **What is the m∠DJF?**

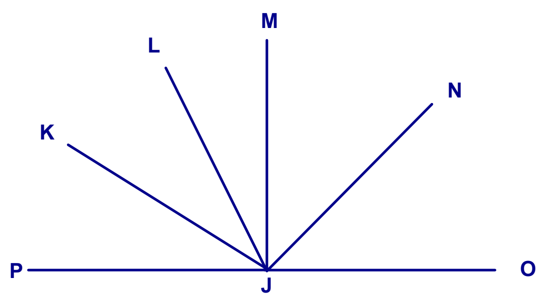
- 39°
- 51°
- 90°
- 141°



Answer

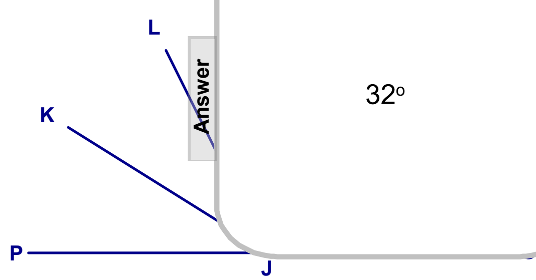
B

34 **#PJK =**



Answer

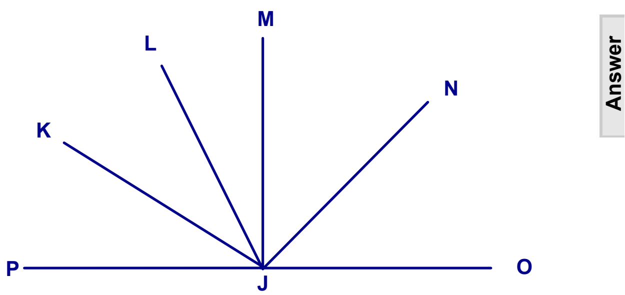
34 **#PJK =**



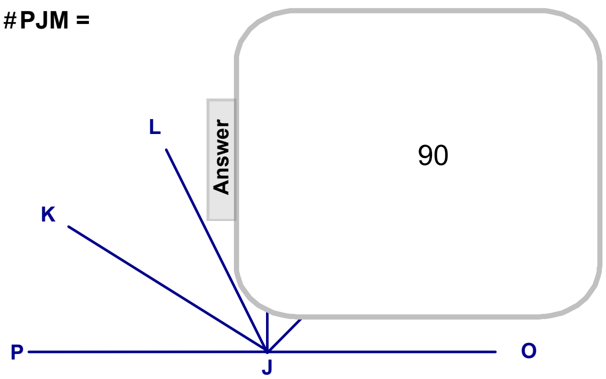
Answer

32°

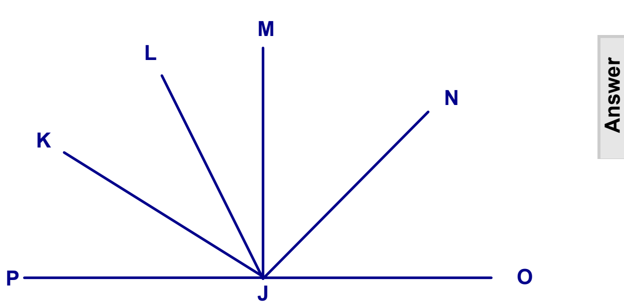
35 #PJM =



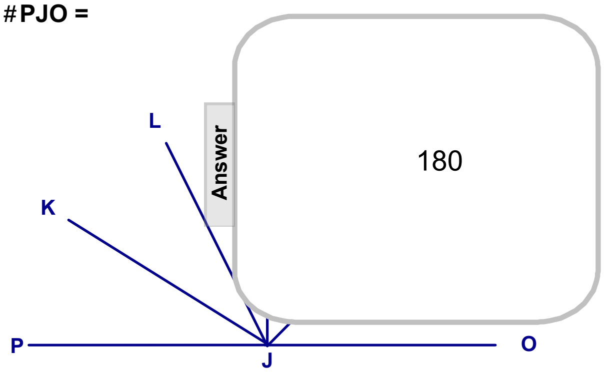
35 #PJM =



36 #PJO =



36 #PJO =



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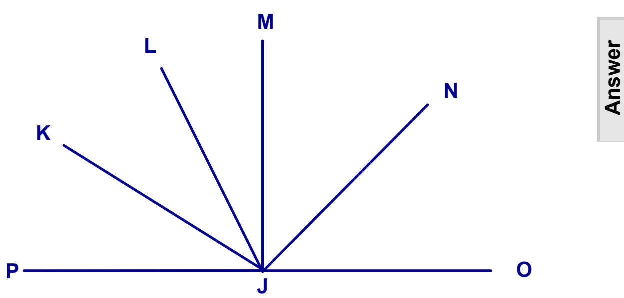
37 #PJL =



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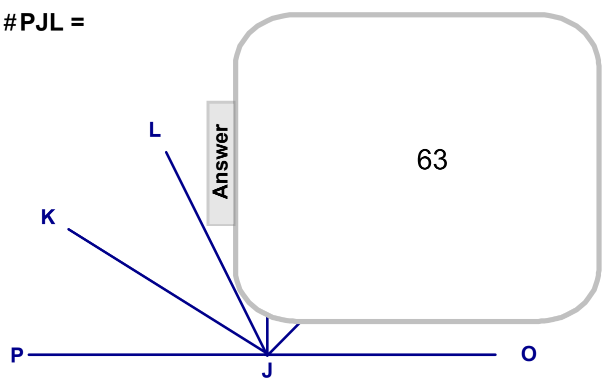
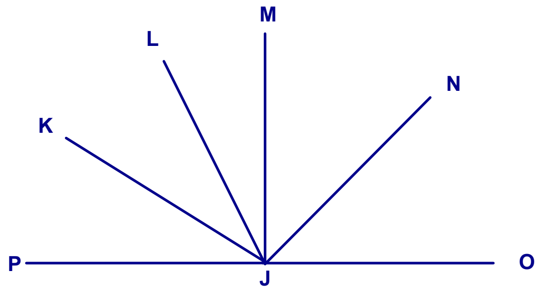
37 #PJL =



Slide 84 (Answer) / 185

Slide 85 / 185

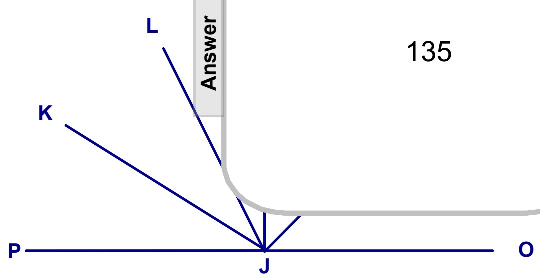
38 **#PJN =**



Answer

Slide 85 (Answer) / 185

38 **#PJN =**



Answer

135

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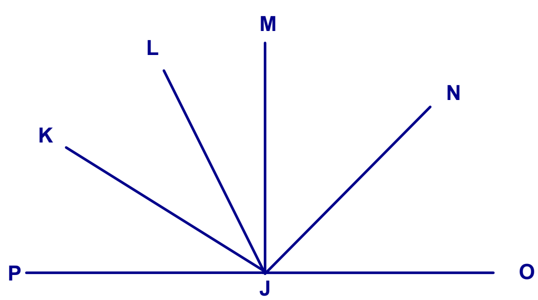
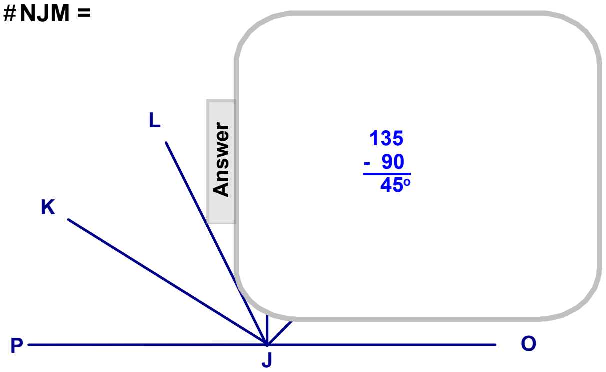
39 **#NJM =**



Answer

Slide 86 (Answer) / 185

39 #NJM =



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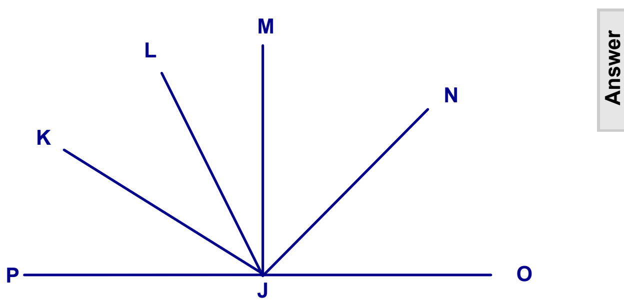
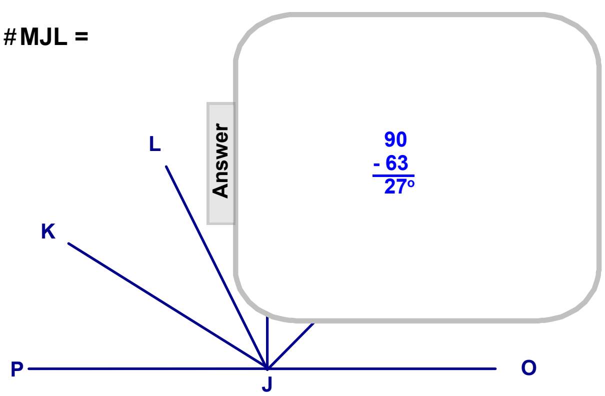
40 #MJL =

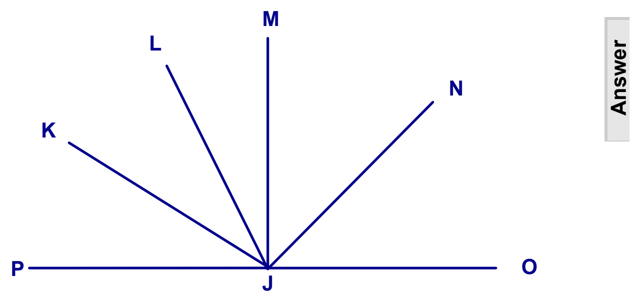


Slide 87 (Answer) / 185

40 #MJL =

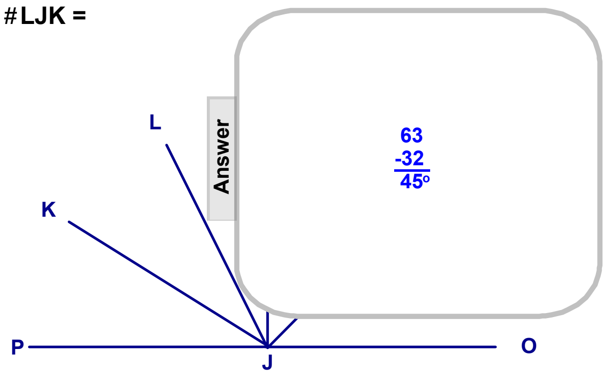


41 #LJK =



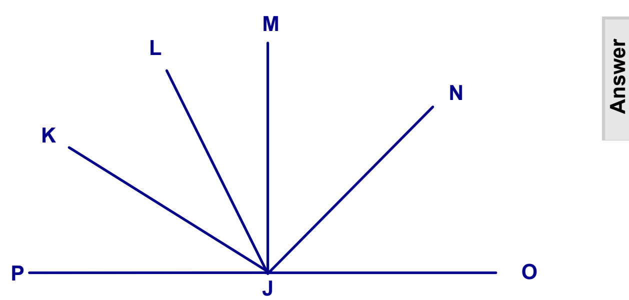
Slide 88 / 185

41 #LJK =



Slide 88 (Answer) / 185

42 #NJK =



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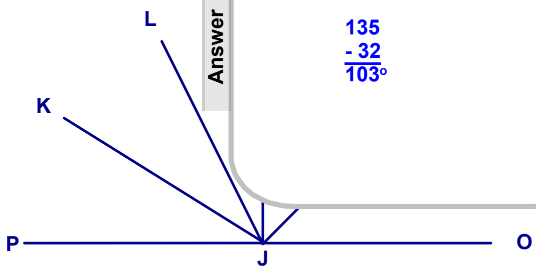
Slide 89 (Answer) / 185

42 ∠NJK =



Answer

135
- 32
103°

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Special Angle Pairs

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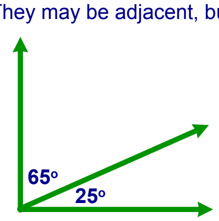
Complementary Angles

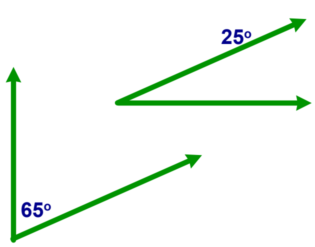
Complementary angles are angles whose sum measures 90^0.

One such angle is said to complement the other.

They may be adjacent, but don't need to be.



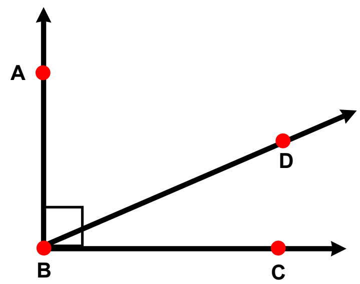
Complementary adjacent



Complementary nonadjacent

Complementary Angles

Adjacent complementary angles form a right angle.



Angle ABD and Angle DBC are complementary since they comprise Angle ABC, which is a right angle.

43 **What is the complement of an angle whose measure is 72⁰?**

Answer

43 **What is the complement of an angle whose measure is 7**

Answer

18⁰

44 What is the complement of an angle whose measure is 28^0?

Answer

44 What is the complement of an angle whose measure is 28

Answer

62^0

Example

Two angles are complementary.
The larger angle is twice the size of the smaller angle.
What is the measure of both angles?

Let x = the smaller angle and the larger angle = 2x.

Answer

Example

Two ang
The larger angle is
What is th

Answer

Let x = the smalle

Since the angles are complementary we know their sum must equal 90 degrees.

$90 = 2x + x$
$90 = 3x$
$30 = x$

Slide 95 (Answer) / 185

45 **An angle is 34° more than its complement.**

What is its measure?

Answer

Slide 96 / 185

45 **An angle is 34° more than its complement.**

What is its measure

Answer

angle = complement + 34
angle = (90 - x) + 34
x = 90 - x +34
2x = 124
x = 62

Slide 96 (Answer) / 185

46 **An angle is 14° less than its complement.**

What is the angle's measure?

Answer

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46 **An angle is 14° less than its complement.**

What is the angle's measure?

Answer

angle = complement - 14
angle = (90 - x) - 14
x = 90 - x - 14
2x = 90 - 14
2x = 76
x = 38

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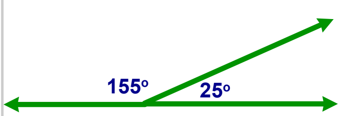
Supplementary Angles

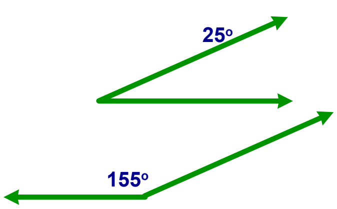
Supplementary angles are angles whose sum measures 180^0.

Supplementary angles may be adjacent, but don't need to be.

One angle is said to supplement the other.



Supplementary adjacent a.k.a. Linear Pair



Supplementary nonadjacent

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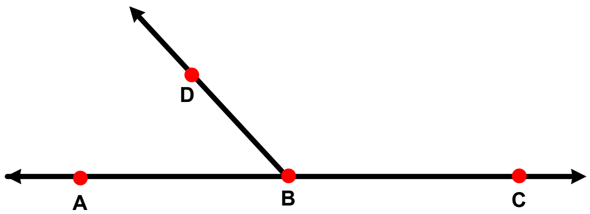
Supplementary Angles

Any two angles that add to a straight angle are supplementary.

Or, two adjacent angles whose exterior sides are opposite rays, are supplementary.



If Angle ABC is a straight angle, its measure is 180^0.

Then Angle ABD and Angle DBC are supplementary since their measures add to 180^0.

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47 What is the supplement of an angle whose measure is 72^0?

Answer

Slide 100 (Answer) / 185

47 What is the supplement of an angle whose measure is 72^0?

Answer

108^0

48 What is the supplement of an angle whose measure is 28^0?

Answer

48 What is the supplement of an angle whose measure is 28^0?

Answer

152^0

49 The measure of an angle is 98^0 more than its supplement.

What is the measure of the angle?

Answer

Slide 102 (Answer) / 185

49 The measure of an angle is 98⁰ more than its supplement.

What is the meas

Answer

angle = (180 - x) + 98
x = 180 - x + 98
2x = 278
x = 139

Slide 103 / 185

50 An measure of angle is 74° less than its supplement.

What is the angle?

Answer

Slide 103 (Answer) / 185

50 An measure of angle is 74° less than its supplement.

What is the angle

Answer

angle = supplement - 74
x = (180 - x) - 74
2x = 180 - 74
2x = 106
x = 53

51 **The measure of an angle is 26° more than its supplement.**

What is the angle?

Answer

51 **The measure of an angle is 26° more than its supplement.**

What is the angle?

Answer

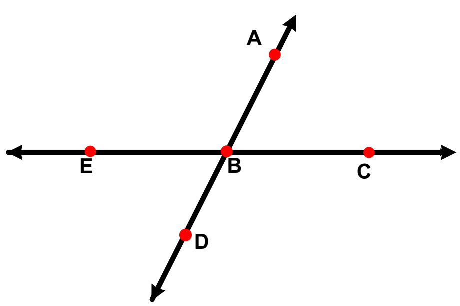
angle = supplement + 26
x = (180 - x) + 26
2x = 180 + 26
2x = 206
x = 103

Vertical Angles

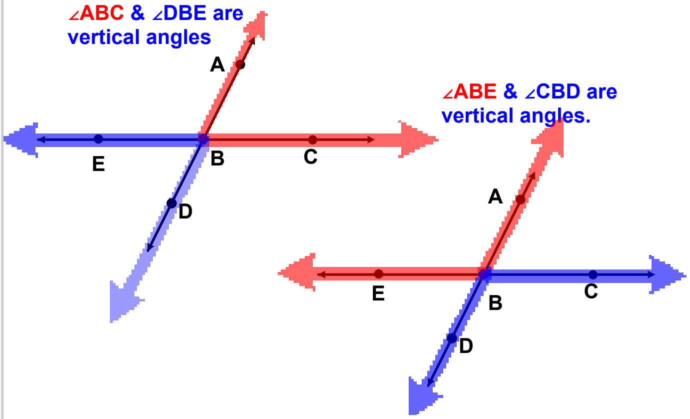
Vertical Angles are two angles whose sides form two pairs of opposite rays

Whenever two lines intersect, two pairs of vertical angles are formed.

∠ABC & ∠DBE are vertical angles, and ∠ABE & ∠CBD are vertical angles.



Vertical Angles



Vertical Angles

We can prove some important propeties about these three special cases: angles which are complementary, supplementary or vertical.

Two column proofs use one column to make a statement and the column next to it to provide the reason, as shown below.

We're going to use those a lot, so we're going to use this example to both prove three theorems.

Proofs

Special Angles

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Two Column Proofs

Proofs all start out with a goal: what it is we are trying to prove.

They are not open-ended explorations, but are directed towards a specific end.

We know the last statement of every proof when we start, it is what we are trying to prove.

We don't know the reason in advance.

Complementary Angles Theorem

Theorem: Angles which are complementary to the same angle are equal.

Given: Angles 1 and 2 are complementary
Angles 1 and 3 are complementary

Prove: m#2 = m#3

Complementary Angles Theorem

Theorem: Angles which are complementary to the same angle are equal.

Statement 1	Reason 1
Angles 1 and 2 are complementary Angles 1 and 3 are complementary	Given

What do we know about the sum of the measures of complementary angles?

Complementary Angles Theorem

Statement 2
m#1 + m#2 = 90
m#1 + m#3 = 90

Reason 2
Definition of complementary angles

Now, we can set the left sides equal by substituting for 90

Complementary Angles Theorem

Statement 3
m#1 + m#2 = m#1 + m#3

Reason 3
Substitution property of equality

And, now subtract m#1 from both sides.

Complementary Angles Theorem

Statement 4
m#2 = m#3

Reason 4
Subtraction property of equality

Which is what we set out to prove

Complementary Angles Theorem

Given: Angles 1 and 2 are complementary
Angles 1 and 3 are complementary

Prove: m 2 = m 3

Statement	Reason
Angles 1 and 2 are complementary Angles 1 and 3 are complementary	Given
m#1 + m#2 = 90 m#1 + m#3 = 90	Definition of complementary angles
m#1 + m#2 = m#1 + m#3	Substitution Property of Equality
m#2 = m#3	Subtraction Property of Equality

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Supplementary Angles Theorem

Theorem: Angles which are supplementary to the same angle are equal.

Given: Angles 1 and 2 are supplementary
Angles 1 and 3 are supplementary

Prove: m#2 = m#3

This is so much like the last proof, that we'll do this by just examining the total proof.

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Supplementary Angles Theorem

Given: Angles 1 and 2 are supplementary
Angles 1 and 3 are supplementary

Prove: m#2 = m#3

Statement	Reason
Angles 1 and 2 are supplementary Angles 1 and 3 are supplementary	Given
m#1 + m#2 = 180 m#1 + m#3 = 180	Definition of supplementary angles
m#1 + m#2 = m#1 + m#3	Substitution property of equality
m#2 = m#3	Subtraction property of equality

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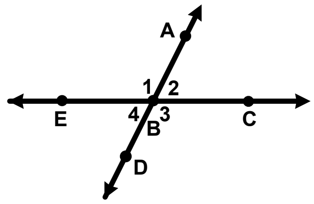
Vertical Angles Theorem

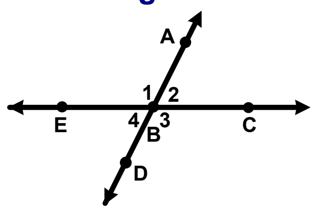
Vertical angles have equal measure



Given: line AD and line EC are straight lines that intersect at Point B and form angles 1, 2, 3 and 4

Prove: m#1 = m#3 and m#2 = m#4

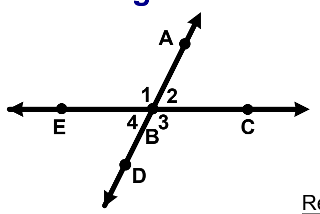
Vertical Angles Theorem



The first statement will focus on what we are given which makes this situation unique.

In this case, it's just the Givens.

Vertical Angles Theorem



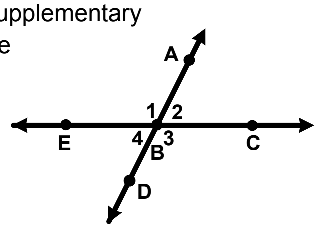
Statement 1	Reason 1
line AD and line EC are straight lines that intersect at Point B and form angles 1, 2, 3 and 4	Given

Then, we know we want to know something about the relationship between the pairs of vertical angles: #1 & #3 and #2 & #4.

What do you know about these four angles that the givens can help us with.

52 We know that angles _____________.

- ○ A #1 & #4 are supplementary
- ○ B #1 & #3 are supplementary
- ○ C #2 & #3 are supplementary
- ○ D #3 & #4 are supplementary
- ○ E All of the above



Answer

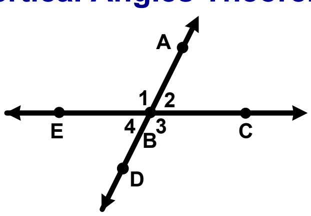
52 We know that angles _____________.

- ○ A #1 & #4 are suppl
- ○ B #1 & #3 are sup
- ○ C #2 & #3 are sup
- ○ D #3 & #4 are su
- ○ E All of the abov

Answer

E

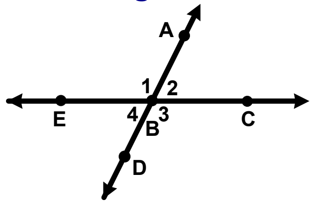
Vertical Angles Theorem



Statement 2	Reason 2
#1 & #2 are supplementary #1 & #4 are supplementary #2 & #3 are supplementary #3 & #4 are supplementary	Angles that form a linear pair are supplementary

What do you know about two angles which are supplementary to the same angle, like #2 & #4 which are both supplements of #1?

Vertical Angles Theorem



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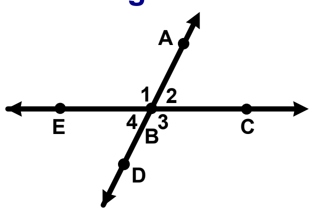
Statement 2	Reason 2
#1 & #2 are supplementary #1 & #4 are supplementary #2 & #3 are supplementary #3 & #4 are supplementary	Angles that form a linear pair are supplementary

Let's look at the fact that #2 & #4 are both supplementary to #1 and that 1 & 3 are both supplementary to #4, since that relates to the vertical angles we're interested in.

Vertical Angles Theorem



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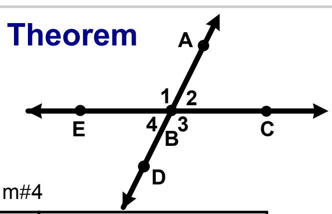
Statement 3	Reason 3
m#1 = m#3 m#2 = m#4	Two angles supplementary to the same angle are equal

But those are the pairs of vertical angles which we set out to prove are equal.

So, our proof is complete: vertical angles are equal

Vertical Angles Theorem

Given: AD and EC are straight lines that intersect at Point B and form angles 1, 2, 3 and 4



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Prove: m#1 = m#3 and m#2 = m#4

Statement	Reason
line AD and line EC are straight lines that intersect at Point B and form angles 1, 2, 3 and 4	Given
#1 & #2 are supplementary #1 & #4 are supplementary #2 & #3 are supplementary #3 & #4 are supplementary	Angles that form a linear pair are supplementary
m#1 = m#3 and m#2 = m#4	Two angles supplementary to the same angle are equal

Vertical Angles Theorem

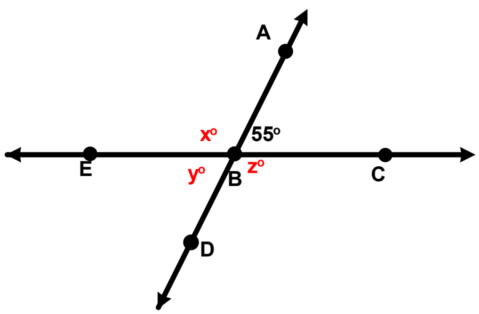
We have proven that vertical angles are congruent.

This becomes a theorem we can use in future proofs.

Also, we can solve problems with it.

Vertical Angles

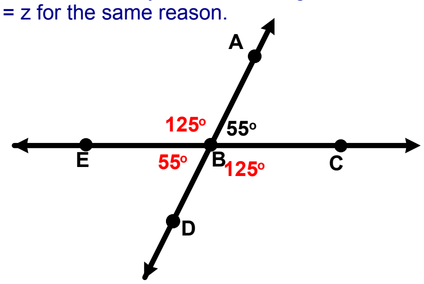
Given: m∠ABC = 55°, solve for x, y and z.



Vertical Angles

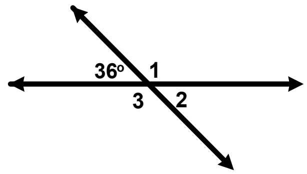
Given: m∠ABC = 55°

We know that x + 55 = 180°, since they are supplementary.
And that y = 55°, since they are vertical angles.
And that x = z for the same reason.



Example

Find m#1, m#2 & m#3. Explain your answer.



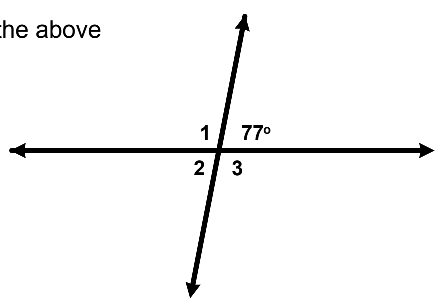
36 + m#1 = 180
m#1 = 144°
Linear pair angles are supplementary

m#2 = 36°; Vertical angles are congruent (original angle & m#2)
m#3 = 144°; Vertical angles are congruent (m#1 & m#3)

53 What is the measure of angle 1?

- ❑ A 77°
- ❑ B 103°
- ❑ C 113°
- ❑ D none of the above

Answer

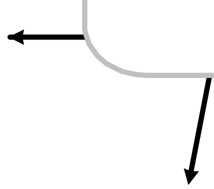


53 What is the measure of angle 1?

- ❑ A 77°
- ❑ B 103°
- ❑ C 113°
- ❑ D none of the ab

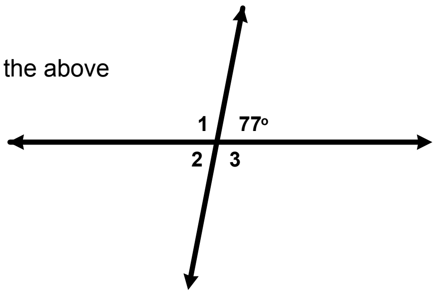
Answer

B



54 What is the measure of angle 2?

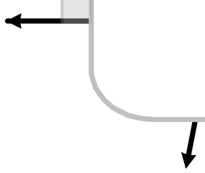
- ❑A 77°
- ❑B 103°
- ❑C 113°
- ❑D none of the above



Answer

54 What is the measure of angle 2?

- ❑A 77°
- ❑B 103°
- ❑C 113°
- ❑D none of the above

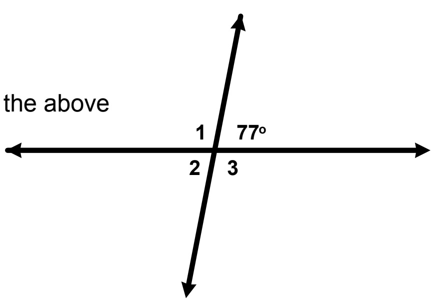


Answer

A

55 What is the measure of angle 3?

- ❑A 77°
- ❑B 103°
- ❑C 113°
- ❑D none of the above



Answer

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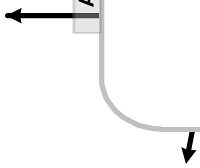
55 What is the measure of angle 3?

- ❑ A 77°
- ❑ B 103°
- ❑ C 113°
- ❑ D none of the abo[ve]

Answer

B



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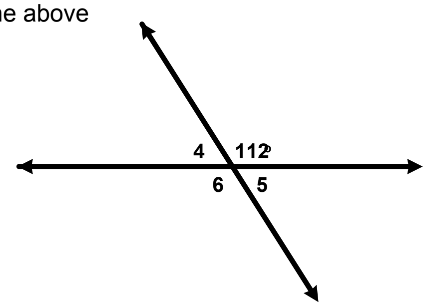
56 What is the measure of angle 4?

- ❑ A 112°
- ❑ B 78°
- ❑ C 102°
- ❑ D none of the above



Answer

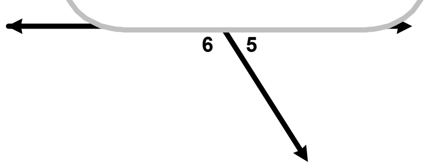
Slide 133 (Answer) / 185

56 What is the measure of angle 4?

- ❑ A 112°
- ❑ B 78°
- ❑ C 102°
- ❑ D none of the ab[ove]

Answer

D) m < 4 = 68°



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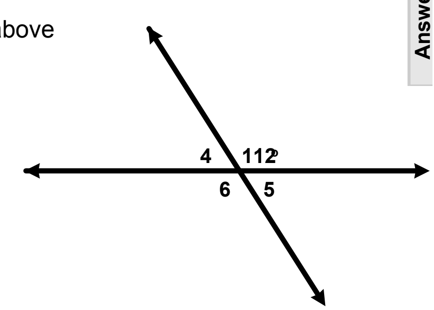
57 What is the measure of angle 5?

- ❑A 112°
- ❑B 68°
- ❑C 102°
- ❑D none of the above



Answer

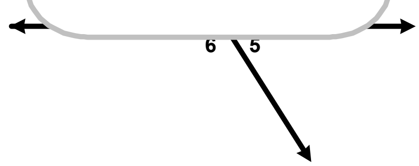
Slide 134 (Answer) / 185

57 What is the measure of angle 5?

- ❑A 112°
- ❑B 68°
- ❑C 102°
- ❑D none of the above

Answer

B



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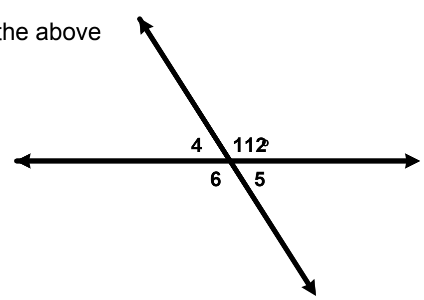
58 What is the m∠6?

- ❑A 102°
- ❑B 78°
- ❑C 112°
- ❑D none of the above



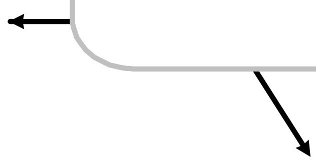
Answer

58 What is the m∠6?

- ☐ A 102°
- ☐ B 78°
- ☐ C 112°
- ☐ D none of the abo

Answer

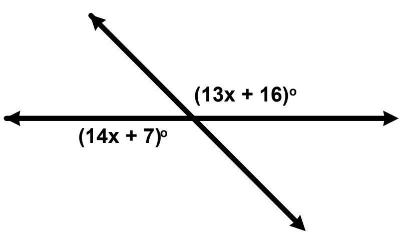
C



Example

Find the value of x.

The angles shown are vertical, so they are congruent.

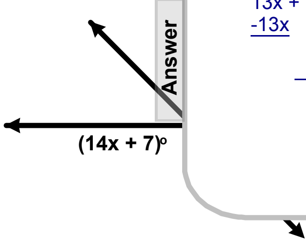


Answer

Example

Find the value of x.

The angles shown are vertical,



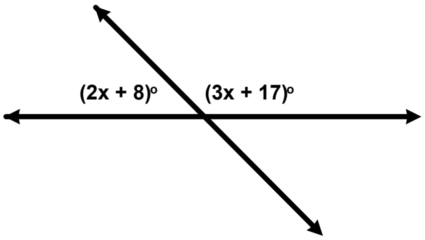
Answer

$$13x + 16 = 14x + 7$$
$$-13x \qquad -13x$$
$$16 = x + 7$$
$$-7 \qquad -7$$
$$9 = x$$

Example

Find the value of x.

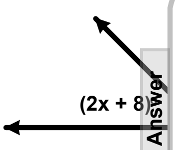
The angles shown are supplementary



Answer

Example

Find the value of x.



Answer

$2x + 8 + 3x + 17 = 180$

$5x + 25 = 180$

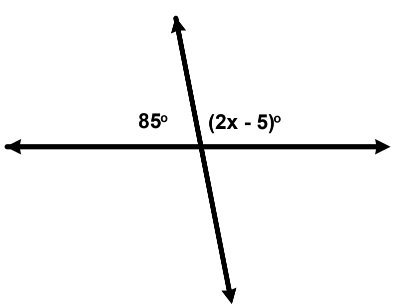
$-25 \quad -25$

$\frac{5x}{5} = \frac{155}{5}$

$x = 31$

59 Find the value of x.

- ❑A 95
- ❑B 50
- ❑C **45**
- ❑D 40



Answer

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59 **Find the value of x.**

- ☐ A 95
- ☐ B 50
- ☐ C 45
- ☐ D 40

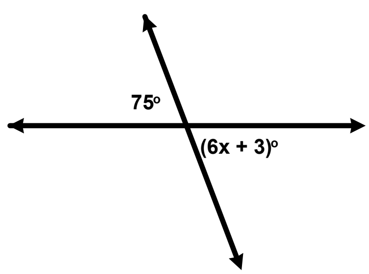
Answer

B

Slide 139 / 185

60 **Find the value of x.**

- ☐ A 75
- ☐ B 17
- ☐ C 13
- ☐ D 12



Answer

Slide 139 (Answer) / 185

60 **Find the value of x.**

- ☐ A 75
- ☐ B 17
- ☐ C 13
- ☐ D 12

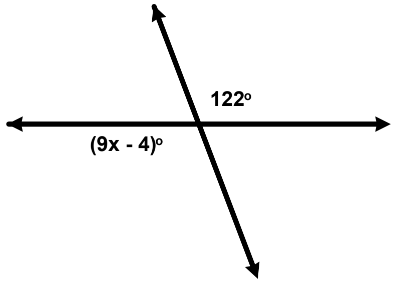
Answer

D

Slide 140 / 185

61 **Find the value of x.**

- ❑A 13.1
- ❑B 14
- ❑C 15
- ❑D 122



Answer

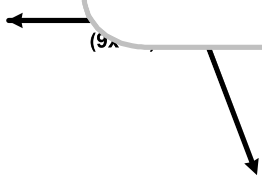
Slide 140 (Answer) / 185

61 **Find the value of x.**

- ❑A 13.1
- ❑B 14
- ❑C 15
- ❑D 122

Answer

B



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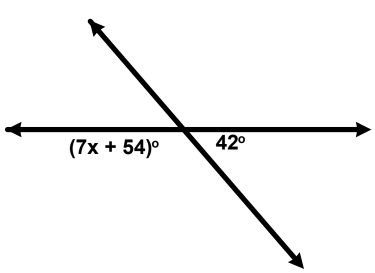
62 **Find the value of x.**

- ❑A 12
- ❑B 13
- ❑C 42
- ❑D 138

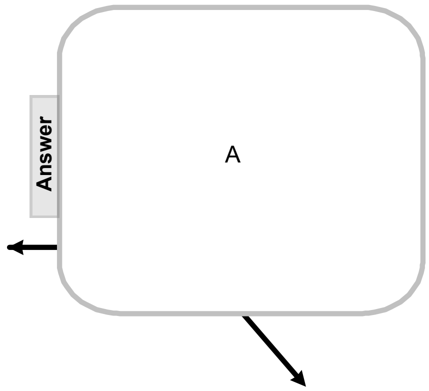


Answer

Slide 141 (Answer) / 185

62 Find the value of x.

- ❑A 12
- ❑B 13
- ❑C 42
- ❑D 138



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Angle Bisectors

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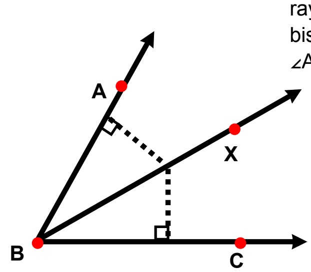
Angle Bisector

ray BX bisects ∠ABC

An angle bisector is a ray or line which starts at the vertex and cuts an angle into two equal halves

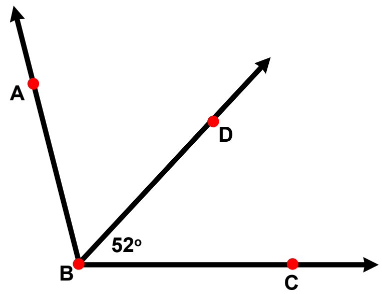


Bisect means to cut it into two equal parts. The 'bisector' is the thing doing the cutting.

The angle bisector is equidistant from the sides of the angle when measured along a segment perpendicular to the sides of the angle.

Finding the missing measurement.

Example: ∠ABC is bisected by ray BD. Find the measures of the missing angles.



Answer

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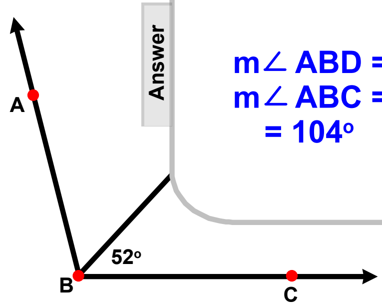
Finding the missing measurement.

Example: ∠ABC is bisected by ray BD. Find the measures of the missing angles.



Answer

m∠ ABD = 52°
m∠ ABC = 2(52)
= 104°

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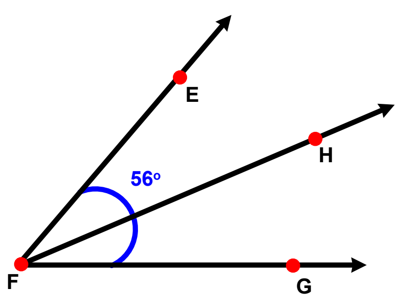
63 **#EFG is bisected by $\overrightarrow{FH}$. The m#EFG = 56^0. Find the measures of the missing angles.**



Answer

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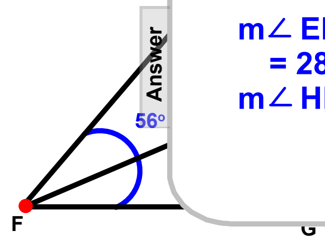
Slide 145 (Answer) / 185

63 #EFG is bisected by $\overrightarrow{FH}$. The m#EFG = 56°. Find the measures of the missing angles.



m∠ EFH = 56/2
= 28°
m∠ HFG = 28°

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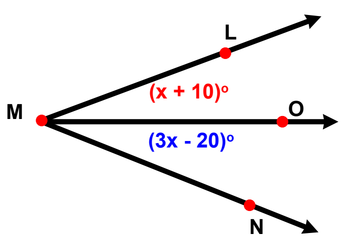
64 $\overrightarrow{MO}$ bisects #LMN. Find the value of x.



Answer

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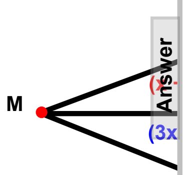
64 $\overrightarrow{MO}$ bisects #LMN. Find the value of x.



m∠LMO = m∠OMN

$x + 10 = 3x - 20$

$-x \quad -x$

$10 = 2x - 20$

$+20 \quad +20$

$\frac{30}{2} = \frac{2x}{2}$

$15 = x$

65 Ray NP bisects ∠MNO Given that ∠MNP = 57º, what is ∠MNO?

Answer

Hint:

click to reveal

65 Ray NP bisects ∠MNO [partially covered] what is ∠MNO?

Answer

$m\angle MNO = 2(57)$
$= 114°$

Hint:

click to reveal

66 Ray RT bisects ∠QRS Given that ∠QRT = 78º, what is ∠QRS?

Answer

Slide 148 (Answer) / 185

66 Ray RT bisects ∠QRS Given that ∠QRT = 78°, what is ∠QRS?

Answer

m∠QRS = 2(78)
= 156°

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67 Ray VY bisects ∠UVW. Given that ∠UVW = 165°, what is ∠UVY?

Answer

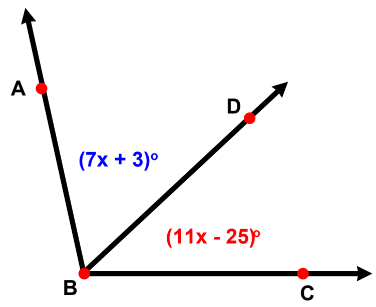
Slide 149 (Answer) / 185

67 Ray VY bisects ∠UVW. Given that ∠UVW = 165°, what is ∠UVY?

Answer

m∠UVY = 165/2
= 82.5°

68 **Ray BD bisects ∠ABC. Find the value of x.**



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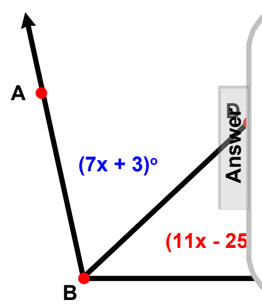
68 **Ray BD bisects ∠ABC. Find the value of x.**



$7x + 3 = 11x - 25$

$3 = 4x - 25$

$28 = 4x$

$7 = x$

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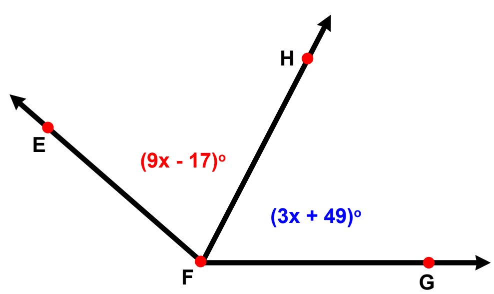
69 **Ray FH bisects ∠EFG. Find the value of x.**



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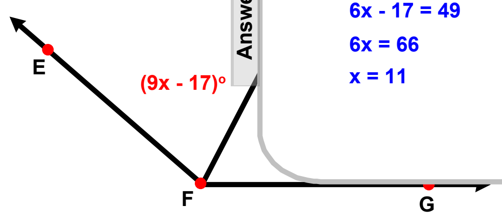
Slide 151 (Answer) / 185

69 Ray FH bisects ∠EFG. Find the value of x.



$9x - 17 = 3x + 49$

$6x - 17 = 49$

$6x = 66$

$x = 11$

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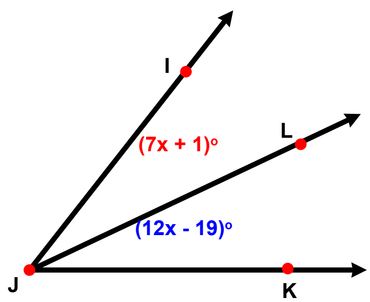
70 Ray JL bisects ∠IJK. Find the value of x.



Answer

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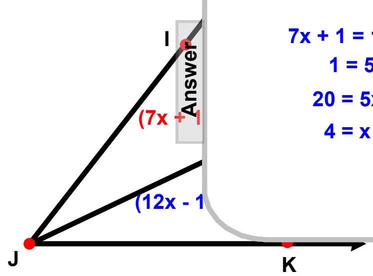
70 Ray JL bisects ∠IJK. Find the value of x.



$7x + 1 = 12x - 19$

$1 = 5x - 19$

$20 = 5x$

$4 = x$

Locus & Angle Constructions

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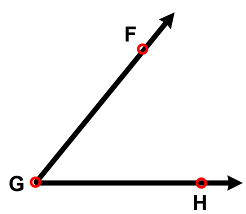
Constructing Congruent Angles

Given: ∠**FGH**
Construct: ∠ABC such that ∠AB₵ ∠**FGH**

Our approach will be based on the idea that the measure of an angle is how much we would have rotate one ray it overlap the other.

The larger the measure of the angle, the farther apart they are as you move away from the vertex.

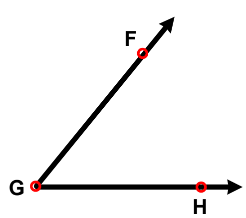


Constructing Congruent Angles

So, if we go out a fixed distance from the vertex on both rays and draw points there, the distance those points are apart from one another defines the measure of the angle.

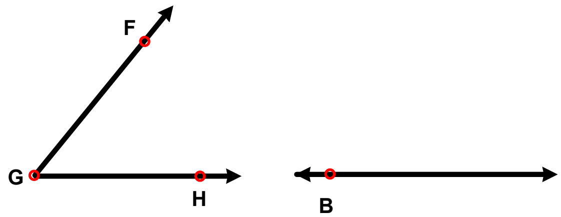
The bigger the distance, the bigger the measure of the angle.

If we construct an angle whose rays are the same distance apart at the same distance from the vertex, it will be congruent to the first angle.



Constructing Congruent Angles

1. Draw a reference line with your straight edge. Place a reference point (B) to indicate where your new ray will start on the line.

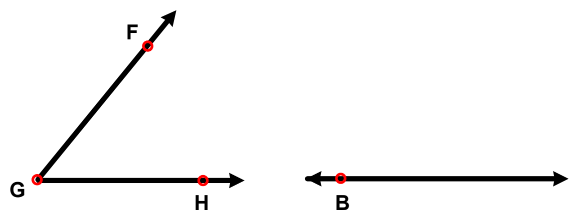


Constructing Congruent Angles

2. Place the compass point on the vertex G and stretch it to any length so long as your arc will intersect both rays .

3. Draw an arc that intersects both rays of ∠FGH.

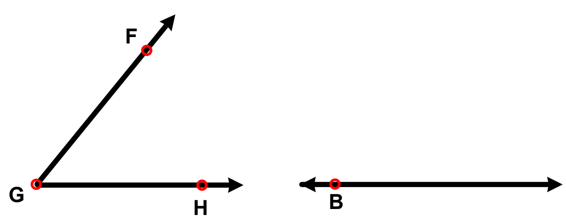
(This defines a common distance from the vertex on both rays since the arc is part of a circle and all its points are equidistant from the center of the circle.)



Constructing Congruent Angles

4. Without changing the span of the compass, place the compass tip on your reference point B and swing an arc that goes through the line and above it.

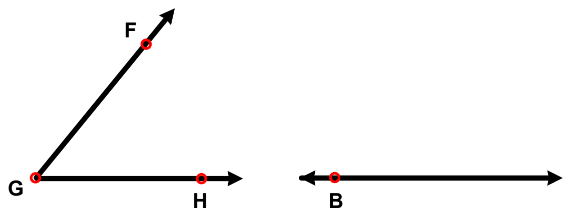
(This defines that same distance from the vertex on both our reference ray and the ray we will draw as we used for the original angle.)



Constructing Congruent Angles

5. Now place your compass where the arc intersects one ray of the original angle and set it so it can draw an arc where it crosses the other ray.

(This defines how far apart the rays are at that distance from the vertex.)



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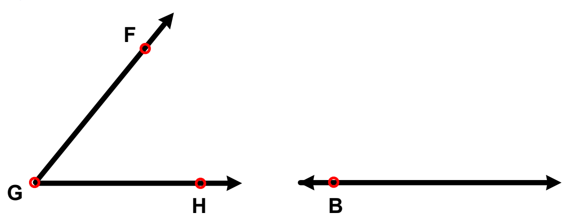
Constructing Congruent Angles

6. Without changing the span of the compass place the point of the compass where the first arc crosses the first ray and draw an arc that intersects the arc above the ray.

(This will make the separation between the rays the same at the same distance from the new vertex as was the case for the original angle.)



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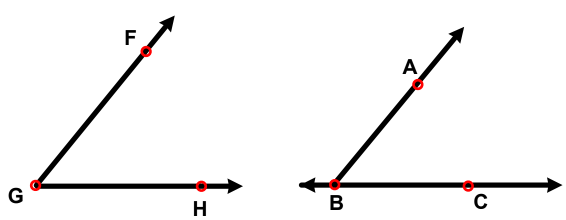
Constructing Congruent Angles

6. Now, use your straight edge to draw the second ray of the new angle which is congruent with the first angle.



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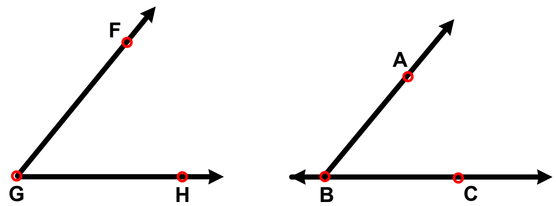
Constructing Congruent Angles

It should be clear that these two angles are congruent. Ray FG would have to be rotated the same amount to overlap Ray GH as would Ray AB to overlap Ray BC.

Notice that where we place the points is not relevant, just the shape of the angle indicates congruence.



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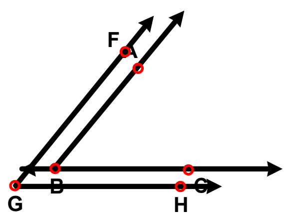
Constructing Congruent Angles

We can confirm that by putting one atop the other.



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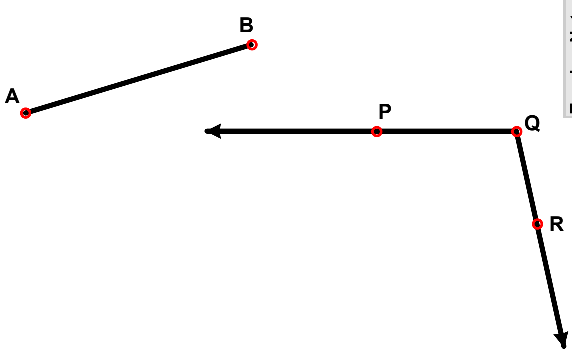
Try this!

Construct a congruent angle on the given line segment.

5)



Teacher Notes

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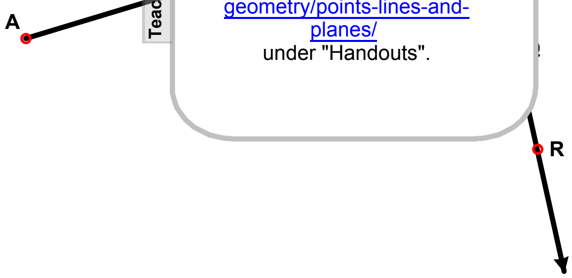
Construct a congruent ang

5)



Teacher Notes

The file for the "Try This!" problems is located on the NJCTL website:
https://njctl.org/courses/math/geometry/points-lines-and-planes/
under "Handouts".

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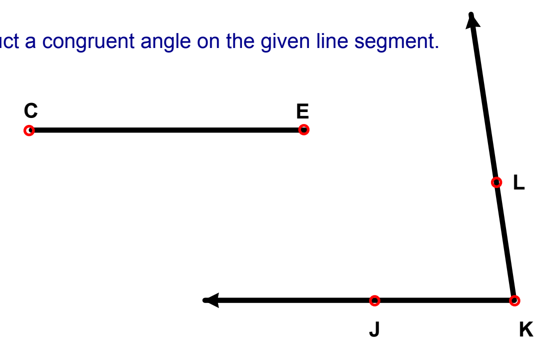
Try this!

Construct a congruent angle on the given line segment.

6)



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Video Demonstrating Constructing Congruent Angles using Dynamic Geometric Software

Click here to see video

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Angle Bisectors & Constructions

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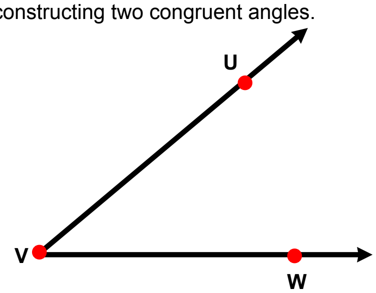
Constructing Angle Bisectors

As we learned earlier, an angle bisector divides an angle into two adjacent angles of equal measure.

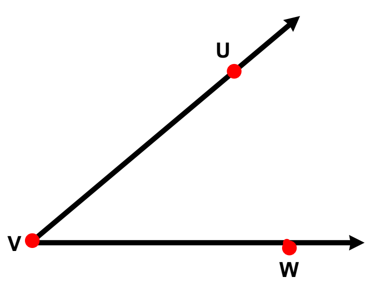
To create an angle bisector we will use an approach similar to that used to construct a congruent angle, since, in this case, we will be constructing two congruent angles.



Constructing Angle Bisectors

1. With the compass point on the vertex, draw an arc that intersects both rays.

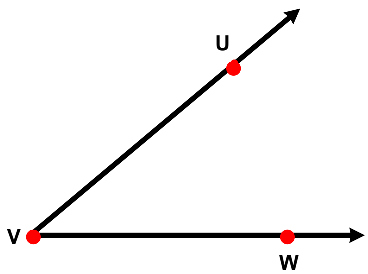
(This will establish a fixed distance from the vertex on both rays.



Constructing Angle Bisectors

2. Without changing the compass setting, place the compass point on the intersection of each arc and ray and draw a new arc such that the two new arcs intersect in the interior of the angle.

(This fixes the distance from each original ray to the new ray to be the same, so that the two new angles will be congruent.)



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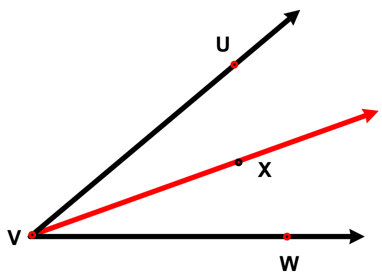
Constructing Angle Bisectors

3. With a straightedge, draw a ray from the vertex through the intersection of the arcs and label that point.

Because we know that the distance of each original ray to the new ray is the same, at the same distance from the vertex, we know the measures of the new angles is the same and that m∠UVX = m∠XVW



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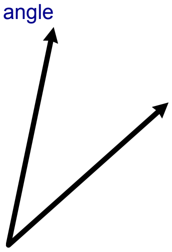
Try This!

Bisect the angle

7)



Teacher Notes

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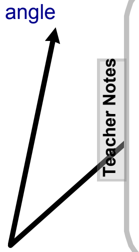
Try This!

Bisect the angle

7)



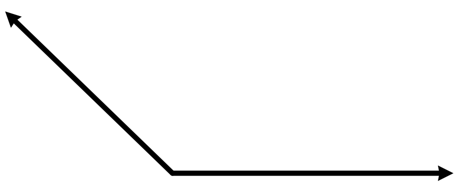
Teacher Notes

The file for the "Try This!" problems is located on the NJCTL website: https://njctl.org/courses/math/geometry/points-lines-and-planes/ under "Handouts".

Try This!

Bisect the angle

8)

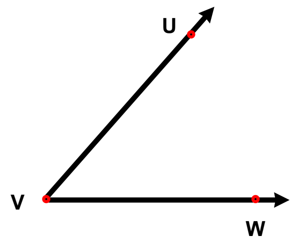


Constructing Angle Bisectors w/ string, rod, pencil & straightedge

Everything we do with a compass can also be done with a rod and string. In both cases, the idea is to mark a center (either the point of the compass or the rod) and then draw an part of a circle by keeping a fixed radius (with the span of the compass or the length of the string.

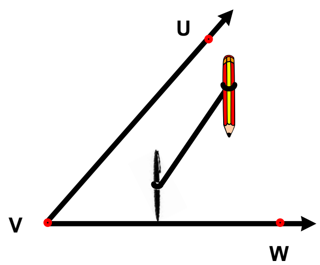
Constructing Angle Bisectors w/ string, rod, pencil & straightedge

1. With the rod on the vertex, draw an arc across each side.



Constructing Angle Bisectors w/ string, rod, pencil & straightedge

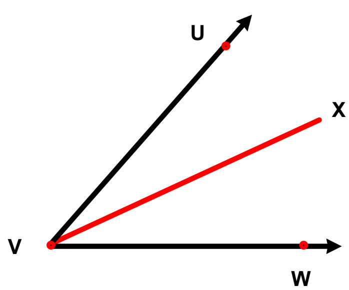
2. Place the rod on the arc intersections of the sides & draw 2 arcs, one from each side showing an intersection point.



Constructing Angle Bisectors w/ string, rod, pencil & straightedge

3. With a straightedge, connect the vertex to the arc intersections. Label your point.

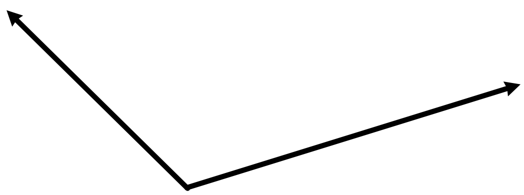
$m\angle UVX = m\angle XVW$



Try This!

Bisect the angle with string, rod, pencil & straightedge.

9)



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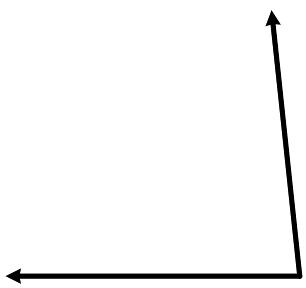
Try This!

Bisect the angle with string, rod, pencil & straightedge.

10)



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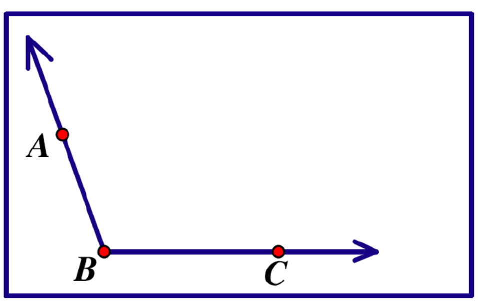
Constructing Angle Bisectors by Folding

1. On patty paper, create any angle of your choice. Make it appear large on your patty paper. Label the points A, B & C.



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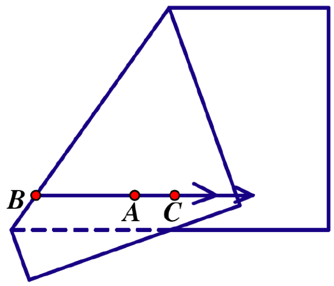
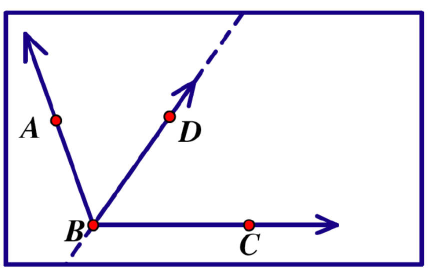
Constructing Angle Bisectors by Folding

2. Fold your patty paper so that ray BA lines up with ray BC.
Crease the fold.



Constructing Angle Bisectors by Folding

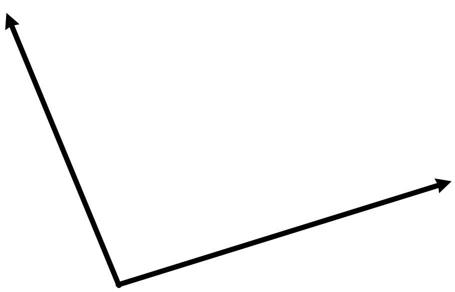
3. Unfold your patty paper. Draw a ray along the fold, starting at point B. Draw and label a point on your ray.



Try This!

Bisect the angle with folding.

11)



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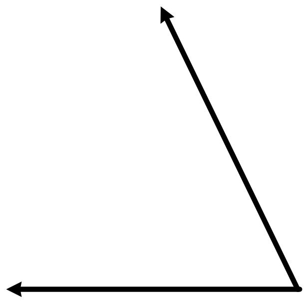
Try This!

Bisect the angle with folding.

12)



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Videos Demonstrating Constructing Angle Bisectors using Dynamic Geometric Software

Click here to see video using a *compass and segment tool*

Click here to see video using the *menu options*