GEOMETRY PROOFS

General Steps to Complete a Proof (www.wyzant.com)

1. Read the problem over carefully. Write down the information that is given. Also, make note of the conclusion to be proved because that is the final step of your proof.

2. Draw an illustration of the problem to help you visualize what is given and what you want to prove, if not already drawn. Include marks that will help you see congruent angles, congruent segments, parallel lines, or other important details if necessary.

3. Use the information given to help you deduce the preliminary steps of your proof. Every step must be shown,

4. Use the conclusion, or argument to be proven, to help guide the statements you make. Remember to support your statements with reasons, which can include definitions, postulates, or theorems.

5. Review your Proof.

Requirements in Performing Proofs (Prepared by: Earl L. Whitney, FSA, MAAA, Mathguy.us)

- Each proof starts with a set of "givens," statements that you are supplied and from which you must derive a "conclusion." Your mission is to start with the givens and to proceed logically to the conclusion, providing reasons for each step along the way.
- Each step in a proof builds on what has been developed before. Initially, you look at what you can conclude from the" givens." Then as you proceed through the steps in the proof, you are able to use additional things you have concluded based on earlier steps.
- Each step in a proof must have a valid reason associated with it. So, each statement in the proof must be furnished with an answer to the question: "Why is this step valid?"

Tips for Successful Proof Development (Prepared by: Earl L. Whitney, FSA, MAAA, Mathguy.us)

- At each step, think about what you know and what you can conclude from that information. Do this initially without regard to what you are being asked to prove. Then look at each thing you can conclude and see which ones move you closer to what you are trying to prove.
- Go as far as you can into the proof from the beginning. If you get stuck, …
- Work backwards from the end of the proof. Ask yourself what the last step in the proof is likely to be. For example, if you are asked to prove that two triangles are congruent, try to see which of the several theorems about this is most likely to be useful based on what you were given and what you have been able to prove so far.
- Continue working backwards until you see steps that can be added to the front end of the proof. You may find yourself alternating between the front end and the back end until you finally bridge the gap between the two sections of the proof.
- Don't skip any steps. Some things appear obvious, but have a mathematical reason for being true. For example, $a = a$, might seem obvious, but "obvious" is not a valid reason in a geometry proof. The reason for $a = a$ is a property of algebra called the "reflexive property of equality."

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Geometry Common Reasons List (www.studyit.org.nz) **– Quarter 1+2**

Reason	**Abbreviation**
Vertically opposite angles are equal	vert opp ∠s
Adjacent angles on a straight line add to 180º	Adj, ∠s on str. line
Angles at a point add to 360º	∠s at pt
Angles in a triangle add to 180º	∠ sum of Δ
The exterior angle of a triangle equals the sum of the two interior opposite angles	ext ∠ of Δ
The base angles of an isosceles triangle are equal	base ∠s isos. Δ
The angle sum of an isosceles triangle is 180º	∠ sum isos. Δ
Each angle in an equilateral triangle is 60º	∠ in equilat. Δ
Corresponding angles on parallel lines are equal	corresp ∠s, // lines
Alternate angles on parallel lines are equal	alt. ∠s, // lines
Co-interior angles on parallel lines are supplementary (add to 180º)	co-int ∠s, // lines
The interior angles of a polygon add to 180(n – 2)º, where n is the number of sides	int ∠ sum of polygon
The exterior angles of a polygon add to 360	ext ∠ sum of polygon

Geometry Common Reasons List (jaglerbpmath.wordpress.com) -- Organized by Types

Algebra Related	**Basic Angles**
• TRANSITIVE PROPERTY ♦ SUBSTITUTION PROPERTY ♦ GIVEN ♦ SIMPLIFY/COMBINE LIKE TERMS ♦ ADDITION POE ♦ SUBTRACTION POE ♦ MULTIPLICATION POE ♦ DIVISION POE ♦ SYMMETRIC PROPERTY ♦ DISTRIBUTIVE PROPERTY	• ANGLE ADDITION POSTULATE ♦ SEGMENT ADDITION POSTULATE ♦ ALL RIGHT ANGLES ARE CONGRUENT. ♦ TWO PERPENDICULAR LINES FORM FOUR RIGHT ANGLES. ♦ DEFINITION OF RIGHT ANGLES ♦ DEFINITION OF PERPENDICULAR ♦ VERTICAL ANGLES ARE CONGRUENT. ♦ SUPPLEMENTS OF THE SAME ANGLE ARE CONGRUENT. ♦ COMPLEMENTS OF THE SAME ANGLE ARE CONGRUENT. ♦ SUPPLEMENTS OF CONGRUENT ANGLES ARE CONGRUENT. ♦ COMPLEMENTS OF CONGRUENT ANGLES ARE CONGRUENT. ♦ 180-360 POSTULATE
Geometry Definitions	**Parallel Lines**
♦ DEFINITION OF COMPLEMENTARY ♦ DEFINITION OF SUPPLEMENTARY ♦ DEFINITION OF CONGRUENCY ♦ DEFINITION OF SEGMENT BISECTOR ♦ DEFINITION OF MIDPOINT ♦ DEFINITION OF ANGLE BISECTOR • REFLEXIVE PROPERTY	♦ IF TWO PARALLEL LINES ARE CUT BY A TRANSVERSAL, THEN: 1. ALTERNATE INTERIOR ANGLES ARE CONGRUENT. 2. ALTERNATE EXTERIOR ANGLES ARE CONGRUENT. 3. CORRESPONDING ANGLES ARE CONGRUENT. 4. SAME-SIDE INTERIOR ANGLES ARE CONGRUENT. ♦ TWO LINES CUT BY A TRANSVERSAL ARE PARALLEL IF: 1. ALTERNATE INTERIOR ANGLES ARE CONGRUENT. 2. ALTERNATE EXTERIOR ANGLES ARE CONGRUENT. 3. CORRESPONDING ANGLES ARE CONGRUENT. 4. SAME-SIDE INTERIOR ANGLES ARE SUPPLEMENTARY.
Other	
♦ IN A PLANE, IF TWO LINES ARE PERPENDICULAR TO THE SAME LINE, THEN THEY ARE PARALLEL. ♦ **THE TRIANGLE SUM THEOREM – THE SUM OF THE MEASURES OF THE ANGLES IN A TRIANGLE IS 180 DEGREES.** ♦ **THIRD ANGLES THEOREM – IF TWO ANGLES IN A TRIANGLE ARE CONGRUENT TO TWO ANGLES IN A 2ND TRIANGLE, THEN THE THIRD ANGLES ARE ALSO CONGRUENT.**	

Common Properties Explained (Prepared by: Earl L. Whitney, FSA, MAAA, Mathguy.us)

Properties of Equality and Congruence.

Property	Definition for Equality For any real numbers **a**, **b**, and **c**:	Definition for Congruence For any geometric elements **a**, **b** and **c**. (e.g., segment, angle, triangle)
Reflexive Property	$a = a$	$a \cong a$
Symmetric Property	$If\ a = b, then\ b = a$	$If\ a \cong b, then\ b \cong a$
Transitive Property	$If\ a = b\ and\ b = c, then\ a = c$	$If\ a \cong b\ and\ b \cong c, then\ a \cong c$
Substitution Property	If $a = b$, then either can be substituted for the other in any equation (or inequality).	If $a \cong b$, then either can be substituted for the other in any congruence expression.

More Properties of Equality. For any real numbers **a**, **b**, and **c**:

Property	Definition for Equality
Addition Property	$If\ a = b, then\ a + c = b + c$
Subtraction Property	$If\ a = b, then\ a - c = b - c$
Multiplication Property	$If\ a = b, then\ a \cdot c = b \cdot c$
Division Property	$If\ a = b\ and\ c \neq 0, then\ a \div c = b \div c$

Properties of Addition and Multiplication. For any real numbers **a**, **b**, and **c**:

Property	Definition for Addition	Definition for Multiplication
Commutative Property	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative Property	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributive Property	$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$	

Theorems by Chapter (Mrs. Daniel)

Chapter 2

C-1 Linear Pair Conjecture - If two angles form a linear pair, then the measures of the angles add up to 180°.

C-2 Vertical Angles Conjecture - If two angles are vertical angles, then they are congruent (have equal measures).

C-3a Corresponding Angles Conjecture (CA) - If two parallel lines are cut by a transversal, then corresponding angles are congruent.

C-3b Alternate Interior Angles Conjecture (AIA)- If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

C-3c Alternate Exterior Angles Conjecture (AEA) - If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.

C-3 Parallel Lines Conjecture - If two parallel lines are cut by a transversal, then corresponding angles are congruent, alternate interior angles are congruent, and alternate exterior angles are congruent.

C-4 Converse of the Parallel Lines Conjecture - If two lines are cut by a transversal to form pairs of congruent corresponding angles, congruent alternate interior angles, or congruent alternate exterior angles, then the lines are parallel.

Chapter 3

C-5 Perpendicular Bisector Conjecture - If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints.

C-6 Converse of the Perpendicular Bisector Conjecture - If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

C-7 Shortest Distance Conjecture - The shortest distance from a point to a line is measured along the perpendicular segment from the point to the line.

C-8 Angle Bisector Conjecture - If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

C-9 Angle Bisector Concurrency Conjecture - The three angle bisectors of a triangle are concurrent (meet at a point).

C-10 Perpendicular Bisector Concurrency Conjecture - The three perpendicular bisectors of a triangle are concurrent.

C-11 Altitude Concurrency Conjecture - The three altitudes (or the lines containing the altitudes) of a triangle are concurrent.

C-12 Circumcenter Conjecture - The circumcenter of a triangle is equidistant from the vertices.

C-13 Incenter Conjecture - The incenter of a triangle is equidistant from the sides.

C-14 Median Concurrency Conjecture - The three medians of a triangle are concurrent.

C-15 Centroid Conjecture - The centroid of a triangle divides each median into two parts so that the distance from the centroid to the vertex is twice the distance from the centroid to the midpoint of the opposite side.

C-16 Center of Gravity Conjecture - The centroid of a triangle is the center of gravity of the triangular region.

Chapter 4

C-17 Triangle Sum Conjecture - The sum of the measures of the angles in every triangle is 180°.

C-18 Third Angle Conjecture - If two angles of one triangle are equal in measure to two angles of another triangle, then the third angle in each triangle is equal in measure to the third angle in the other triangle.

C-19 Isosceles Triangle Conjecture - If a triangle is isosceles, then its base angles are congruent. C-20 Converse of the Isosceles Triangle Conjecture - If a triangle has two congruent angles, then it is an isosceles triangle.

C-21 Triangle Inequality Conjecture - The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

C-22 Side-Angle Inequality Conjecture - In a triangle, if one side is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

C-23 Triangle Exterior Angle Conjecture - The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.

C-24 SSS Congruence Conjecture - If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.
C-25 SAS Congruence Conjecture - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.
C-26 ASA Congruence Conjecture - If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.
C-27 SAA Congruence Conjecture - If two angles and a non-included side of one triangle are congruent to the corresponding angles and side of another triangle, then the triangles are congruent.

C-29 Equilateral/Equiangular Triangle Conjecture - Every equilateral triangle is equiangular. Conversely, every equiangular triangle is equilateral.

Chapter 5

C-30 Quadrilateral Sum Conjecture - The sum of the measures of the four angles of any quadrilateral is 360°.

C-31 Pentagon Sum Conjecture - The sum of the measures of the five angles of any pentagon is 540°.

C-32 Polygon Sum Conjecture - The sum of the measures of the n interior angles of an n-gon is

$$(n-2) \bullet 180.$$

C-33 Exterior Angle Sum Conjecture - For any polygon, the sum of the measures of a set of exterior angles is 360°.

C-34 Equiangular Polygon Conjecture - You can find the measure of each interior angle of an equiangular ngon by using either of these formulas:

$$\frac{(n-2) \bullet 180}{n} \ or \ \left(180 - \frac{360°}{n}\right)$$

C-39 Trapezoid Consecutive Angles Conjecture - The consecutive angles between the bases of a trapezoid are supplementary.

C-40 Isosceles Trapezoid Conjecture - The base angles of an isosceles trapezoid are congruent.

C-41 Isosceles Trapezoid Diagonals Conjecture - The diagonals of an isosceles trapezoid are congruent.

C-42 Three Midsegments Conjecture - The three midsegments of a triangle divide it into four congruent triangles.

C-43 Triangle Midsegment Conjecture - A midsegment of a triangle is parallel to the third side and half the length of the third side.

C-45 Parallelogram Opposite Angles Conjecture - The opposite angles of a parallelogram are congruent.

C-46 Parallelogram Consecutive Angles Conjecture - The consecutive angles of a parallelogram are supplementary.

C-47 Parallelogram Opposite Sides Conjecture - The opposite sides of a parallelogram are congruent.

C-48 Parallelogram Diagonals Conjecture - The diagonals of a parallelogram bisect each other.

C-50 Rhombus Diagonals Conjecture - The diagonals of a rhombus are perpendicular and they bisect each other.

C-51 Rhombus Angles Conjecture - The diagonals of a rhombus bisect the angles of the rhombus.

C-52 Rectangle Diagonals Conjecture - The diagonals of a rectangle are congruent and bisect each other.

C-53 Square Diagonals Conjecture - The diagonals of a square are congruent, perpendicular,and bisect each other.