Geometry
Tessellations Project

BACKGROUND: Maurits Cornelis Escher, born in Leeuwarden, Holland in 1898, created unique and fascinating works of art that explore and exhibit an array of mathematical ideas. Among his greatest admirers were mathematicians, who recognized in Escher's work an extraordinary visualization of mathematical principles. This was quite remarkable as Escher had no formal mathematics training beyond secondary school.

Tessellations, or regular divisions of the plane, are arrangements of closed shapes that completely cover the plane without overlapping and without leaving gaps. For shapes to fill the plane without overlaps or gaps, their angles, when arranged around a point, must have measures that add up to exactly 360°. Typically, the shapes making up a tessellation are polygons or similar regular shapes (like square tiles used on floors). Escher exploited these basic patterns in his tessellations, applying reflections, translations, and rotations to obtain a greater variety of patterns. He also "distorted" these shapes to form animals, birds, and other figures. These distortions had to obey the three, four, or six-fold symmetry of the underlying pattern in order to preserve the tessellation.

PROJECT OBJECTIVE: Students will create a tessellation that demonstrates their knowledge of the properties of geometric transformations, such as translations, rotations, and reflections.

PROCEDURE:

Tessellation

Use one or more transformations to create an Escher-esque Tessellation on a sheet of paper (12 x 18 inches). You will need to make a template to trace. The final product will be graded according to "complexity points", much like "degree of difficulty" in competition diving or gymnastics. You earn more points for attempting a more difficult tessellation. You may not simply take a polygon and slide, rotate, or reflect it over and over again to create your tessellation. You must create a template based on one of the methods in this packet. You can use a computer to create your template, but the **final tessellation must be drawn by hand**.

Written Response

Type a one-page response that specifically answers each of the questions below and uses formal writing language:

1. What is the idea/theme behind your tessellation? (Why did you choose the pattern? The color(s)? Did anything inspire you?)
2. What polygon(s) did you start with and how did you alter it?
3. What transformations (reflection(s), translation(s), rotation(s) did you use?
4. In your opinion, was M.C. Escher an artist or a mathematician? Justify your answer.
5. Provide some examples of where tessellations are found in the world around us. Hint: Look for examples in nature, art, architecture, business, or advertising. Include two photos or two illustrations to support your claim.
6. Please cite your source(s) in the format of your choice.

Use the Project Planning Worksheet to generate ideas and help focus your project on a specific theme. Remember, you are creating an ***original* piece of work.

ASSESSMENT:

Tessellation (20 points)

- Polygon(s) and transformations used
- Complexity and creativity
- Visual appeal (neatness and color)
- Follows directions (meets project requirements)

CATEGORY	**4**	**3**	**2**	**1**
Organization	Tessellations are presented in an organized manner. Their construction is complete and accurate.	Tessellations are presented in a somewhat organized manner. Their construction is fairly complete.	Tessellations are organized, but not well constructed.	There is no organization to the tessellation.
Complexity of Design	Tessellation was created with non-polygon shapes that connect to create an intricate and complex pattern.	Tessellation was created with complex polygon shapes that connect to create an intricate and complex pattern.	Tessellation was created with simple shapes that connect to create a pattern	Tessellation is simple and pattern is not complex or interesting.
Completeness of Tessellation	All areas of the tessellation are covered by the complex pattern to completely fit together.	All areas of the tessellation are covered by the simple pattern with few or small holes in the pattern.	Most areas of the tessellation are covered by the simple pattern with some holes in the pattern.	There are major holes in the tessellation and it does not fit together.
Creativity	Tessellation uses unique design and patterns of unusual shapes to form a complex and interesting design.	Tessellation uses a mixture of polygons and interesting shapes to create a nice design.	Tessellation uses some shapes that fit together to form a simple pattern	There is no creativity to the design.
Follows Directions	Tessellation covers a 12" x 18" sheet of paper completely with interesting display and presentation.	Tessellation covers a 12" x 18" sheet of paper and has a complete presentation.	Tessellation mostly covers a 12" x 18" sheet of paper and has no presentation	Tessellation does not cover the 12" x 18" paper completely.

Written Response 10 pts.

Participation 10 pts.

You will have one day in class to work on your project. You will receive a participation grade of up to 10 points for how efficiently you use your time in class. The entire project will be worth **40 pts.**

WRITING GUIDELINES

- Each answer in your written response should be numbered to correspond with the question.
- Questions need to be clearly answered, showing insight and understanding, and they must be supported by sufficient detail.
- Response must contain fluent use of mathematical terms, such as reflection, translation, rotation, etc.
- Response must be carefully worded with minimal distracting spelling or grammatical errors.
- Must be typed.

Tessellations Project → Planning Worksheet

Use one or more tessellations to create an Escher-esque Tessellation on 12" x 18" paper. You must create a template based on one of the methods below. You should use a 3" x 3" square to start. You can also use other polygons approximately this size.

**You can use the computer to create your template, but the tessellation must be drawn by hand.

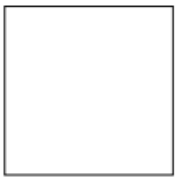
Consider the following when planning your tessellation:

- Do you want to use one polygon or more than one?
- How complex do you want to make your original figure?
- Do you want to use one transformation or a combination to tessellate your figure?
- How are you going to use color to alter your tessellation and create more patterns?
- What is the idea or theme that you want to express in your piece?
- What is the title of your piece?

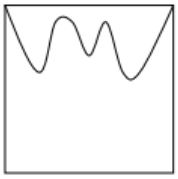
Translation Tessellation (Easy) Template

For simple translation tessellations, the starting polygon should have opposite sides that are parallel and congruent. Squares, hexagons, and parallelograms work best here.

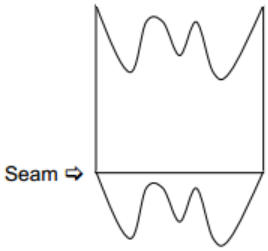
1. Start with a square.



2. Draw a design on one side of the square.



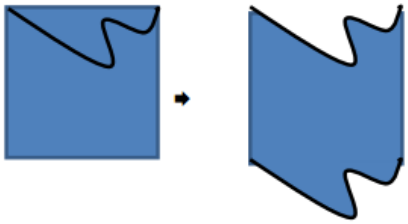
3. Cut the design piece out and translate (slide) it across to the opposite side of the square. Secure it there with tape—be careful not to overlap the piece or make a gap. It should fit perfectly at the seam.



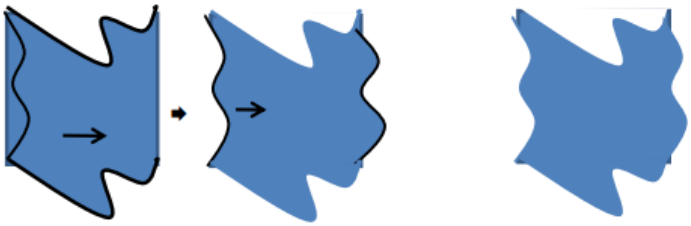
Translation Tessellation (Hard) Template

1. Start with a square.

2. Draw a design on one side of the square, translate (slide) it to the opposite side, and secure it with tape.



3. Draw another design on the adjacent side of the square and translate (slide) it to the opposite side. Secure it with tape.



Glide Reflection Tessellation Template

For glide reflection tessellations, the starting polygon should have opposite sides that are parallel and congruent. Squares, hexagons, and parallelograms work best here.

1. Start with a square.

2. Draw a design on one side of the square.

3. Cut the design piece out and translate (slide) it across to the opposite side of the square. Reflect (flip) the cut piece on its vertical axis.

4. Secure the cutout piece with tape to the opposite side of the square to complete the pattern.

*This pattern needs to be rotated or reflected as it is traced.

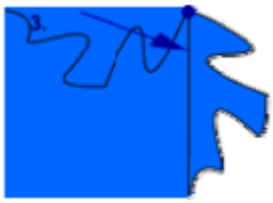
Rotation Tessellation Template

For rotation tessellations, the adjacent sides of the polygon must be congruent. Squares, equilateral triangles, regular hexagons, and rhombuses work best here.

1. Start with a square.

2. Draw a design on one side of the square.

3. Cut the design piece out and rotate (turn) it on an end point until it lies evenly with an adjacent side of the square.



4. Secure the cutout piece to the adjacent side of the square to complete the pattern. This pattern (template) needs to be rotated as it is traced.