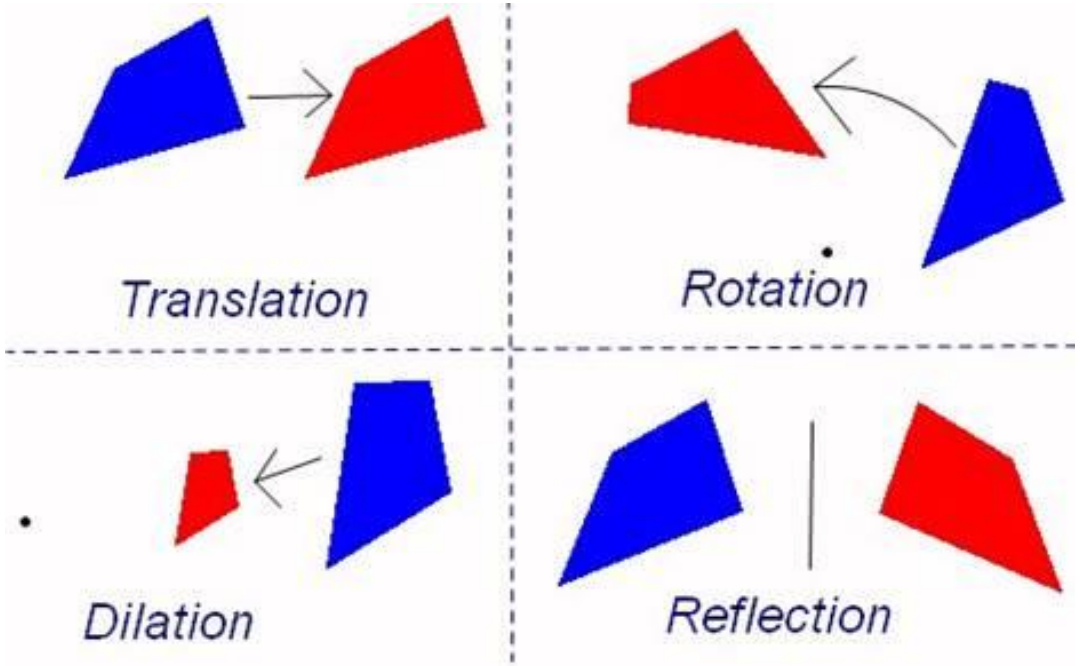
Geometry
Unit 1: Transformations in the Coordinate Plane

Guided Notes

Standard: MGSE9–12.G.CO.1 Know precise definitions

Essential Question: What are the undefined terms essential to any study of geometry?

Transformation: The mapping, or movement, of all points of a figure in a plane according to a common operation, such as translation, reflection or rotation.



Standard: MGSE9–12.G.CO.1 Know precise definitions

Essential Question: What are the undefined terms essential to any study of geometry?

Pre–image: A figure before a transformation has taken place.

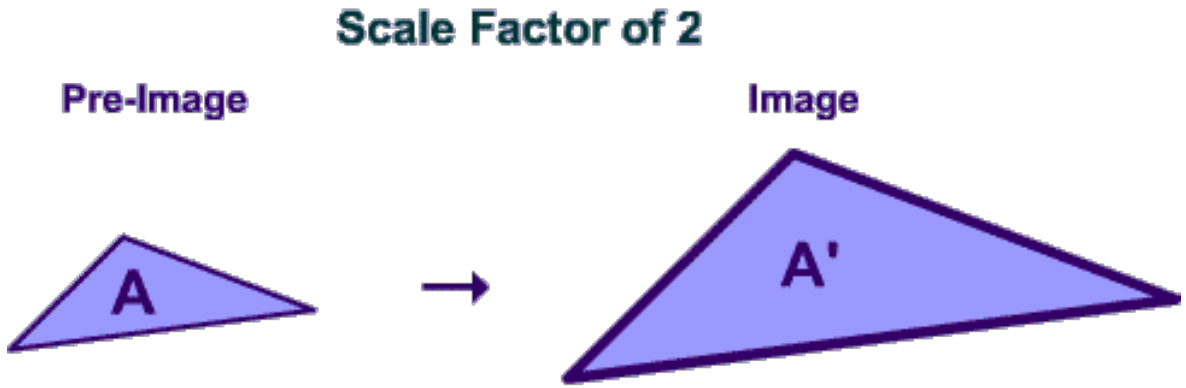


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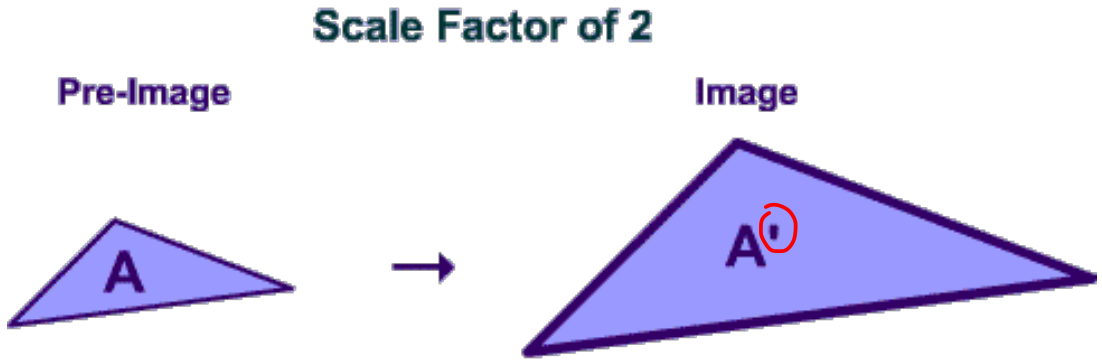
Image: The result of a transformation.



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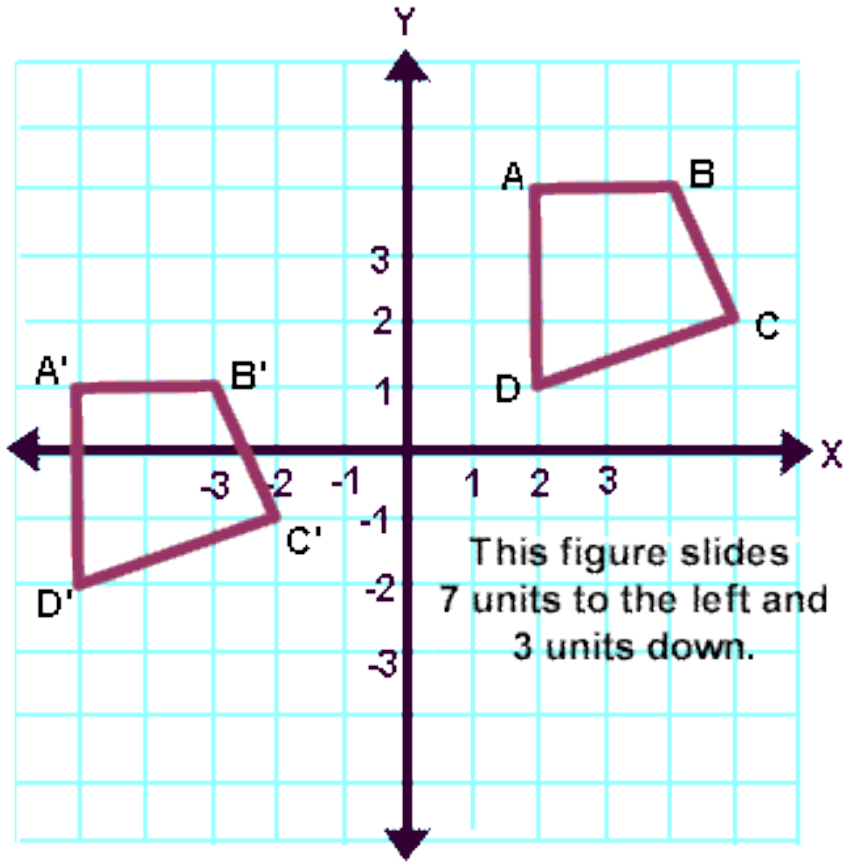
Essential Question: What are the undefined terms essential to any study of geometry?

Translation: A transformation that slides each point of a figure the same distance in the same direction.



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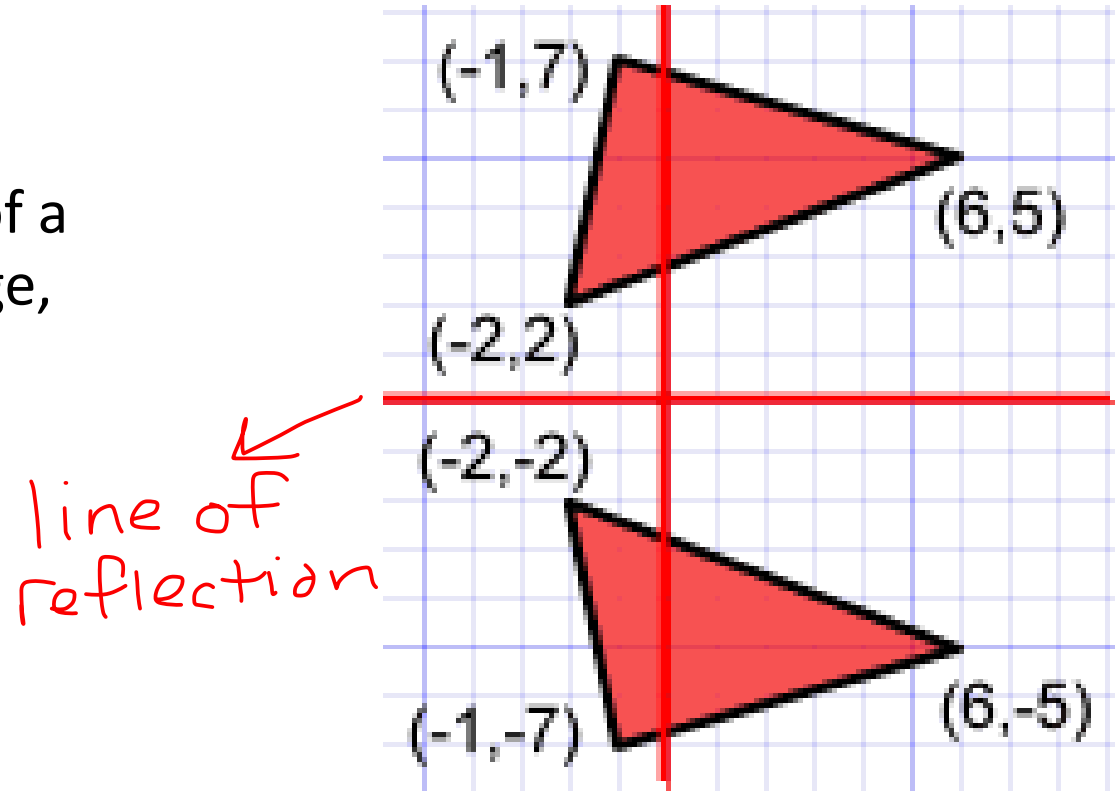
Essential Question: What are the undefined terms essential to any study of geometry?

Reflection: A transformation of a figure that creates a mirror image, "flips," over a line.



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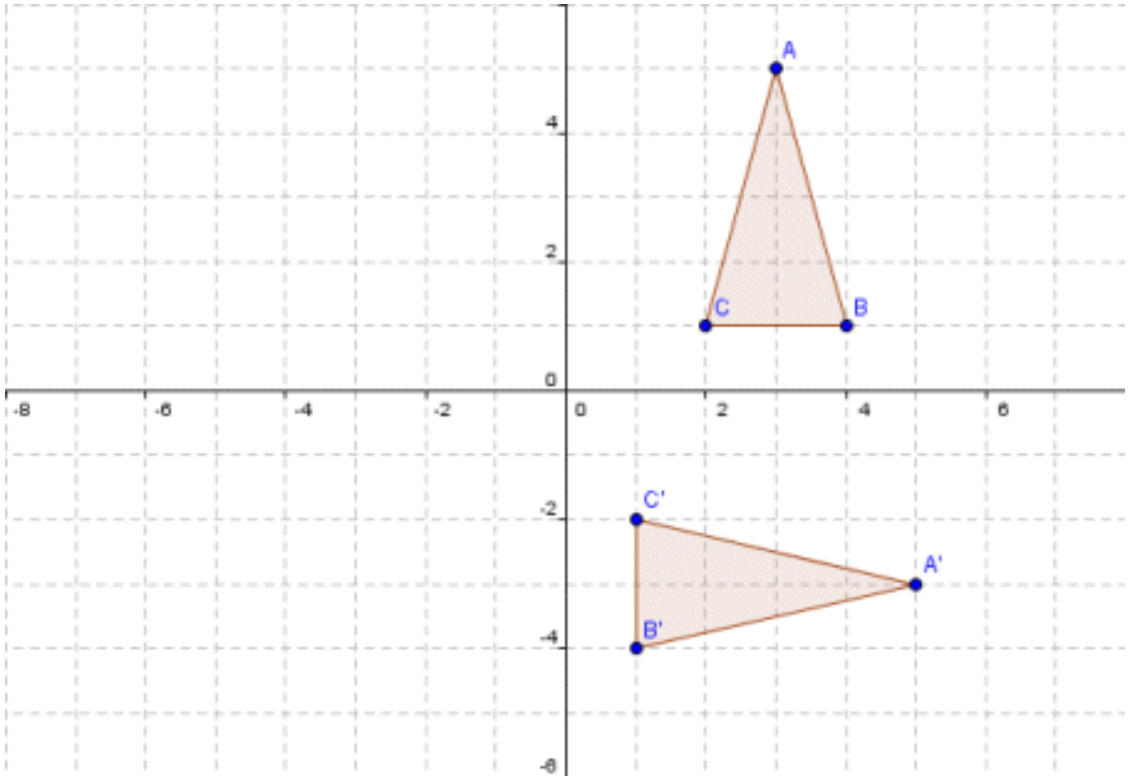
Essential Question: What are the undefined terms essential to any study of geometry?

Rotation: A transformation that turns a figure about a fixed point through a given angle and a given direction, such as 90° clockwise.



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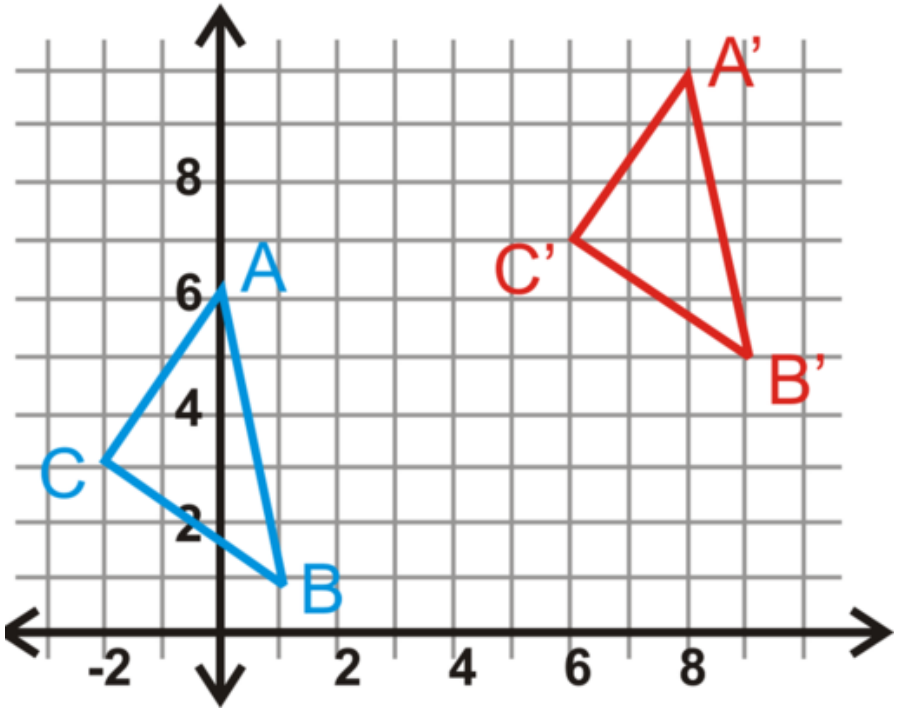
Isometry: a distance preserving map of a geometric figure to another location using a reflection, rotation or translation.



Standard: MGSE9–12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Essential Question: How do we translate geometric figures in the coordinate plane?

Translation: A transformation that slides each point of a figure the same distance in the same direction.

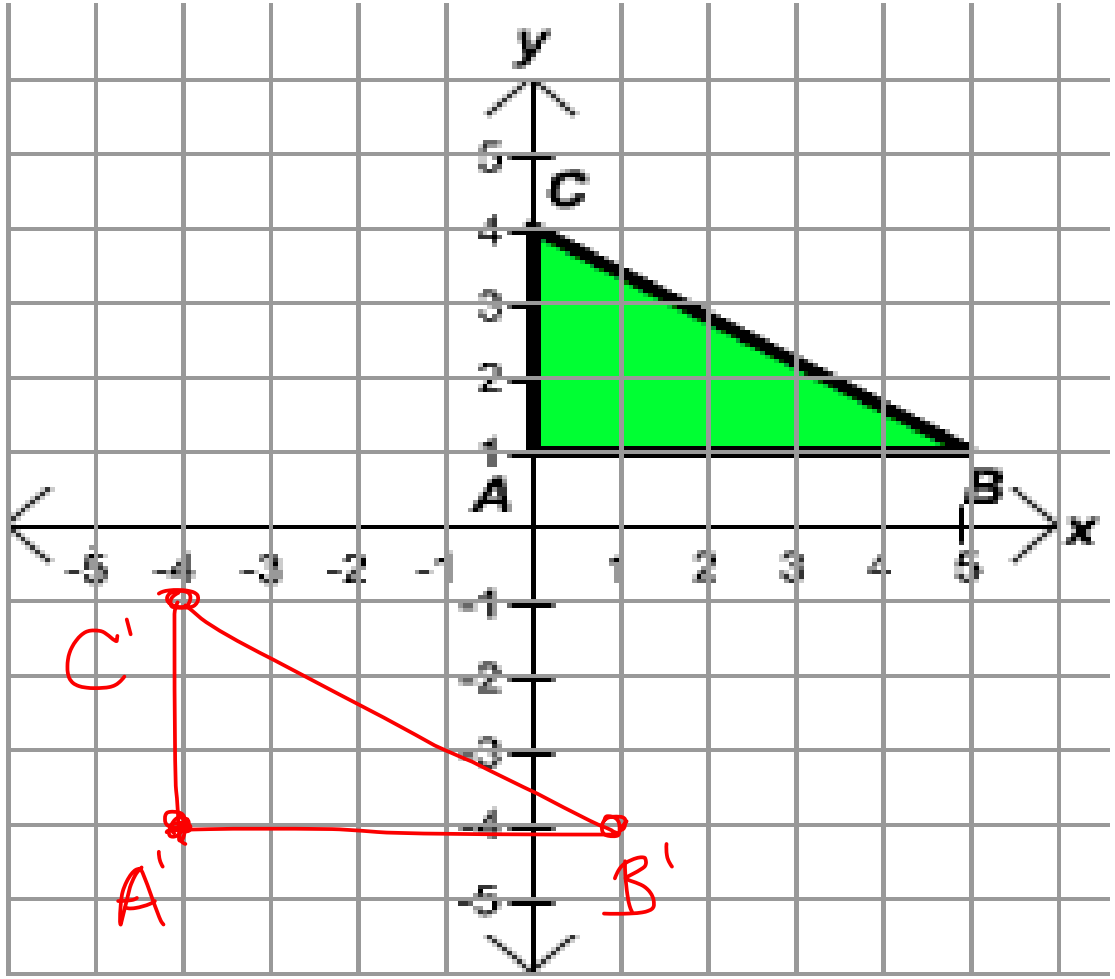
Translations do not change the map of the figure! They are isometries!

EX: Translate ΔABC 5 units down and 4 units left.

$(x, y) \rightarrow (x - 4, y - 5)$

A(0, 1)
B(5, 1)
C(0, 4)

A' (-4, -4)
B' (1, -4)
C' (-4, -1)



Standard: MGSE9–12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Essential Question: How do we reflect geometric figures in a coordinate plane?

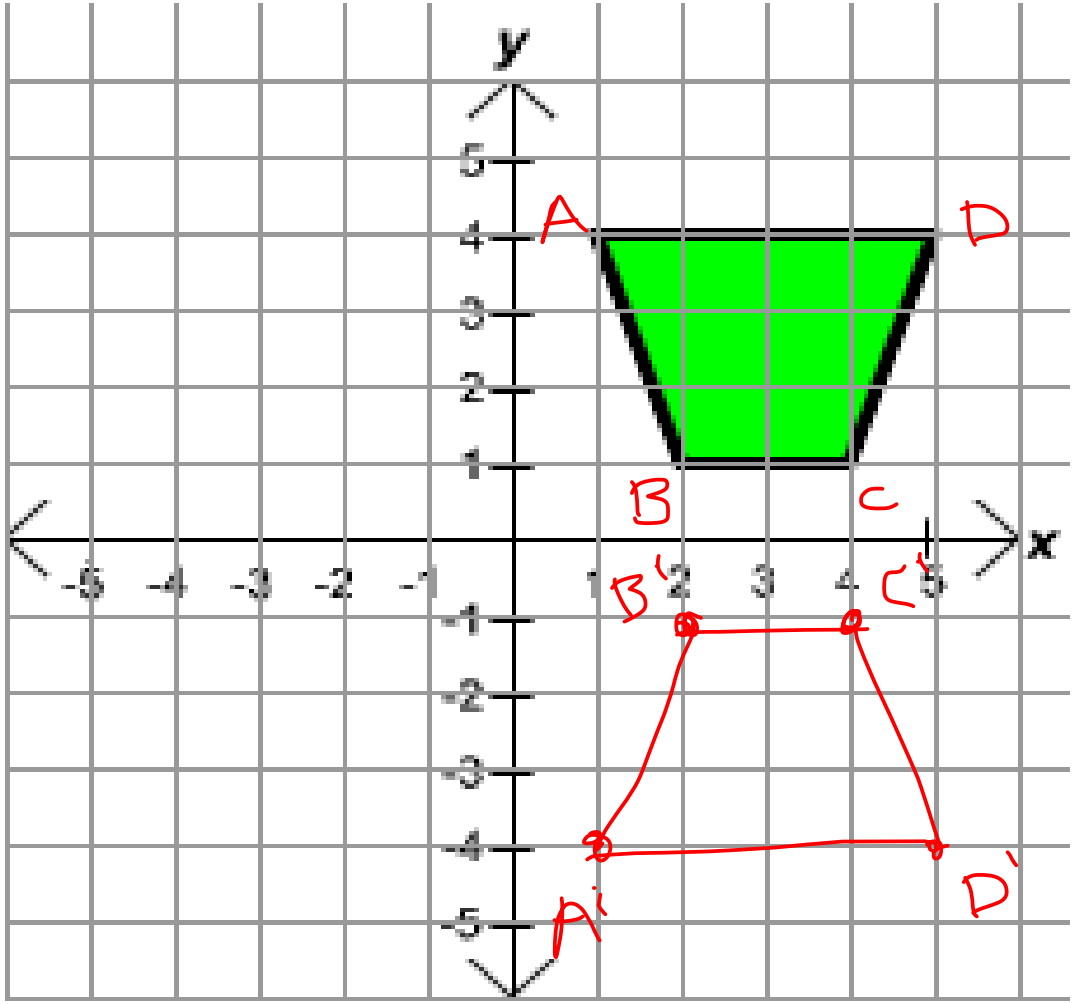
Reflection: A transformation of a figure that creates a mirror image, "flips," over a line.

Reflections do not change the map of the figure! They are isometries!

EX: Reflect the trapezoid about the x-axis. To reflect about the x-axis, (x, y) → (x, -y).

A (1, 4)
B (2, 1)
C (4, 1)
D (5, 4)

A' (1, -4)
B' (2, -1)
C' (4, -1)
D' (5, -4)



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Essential Question: How do we reflect geometric figures in a coordinate plane?

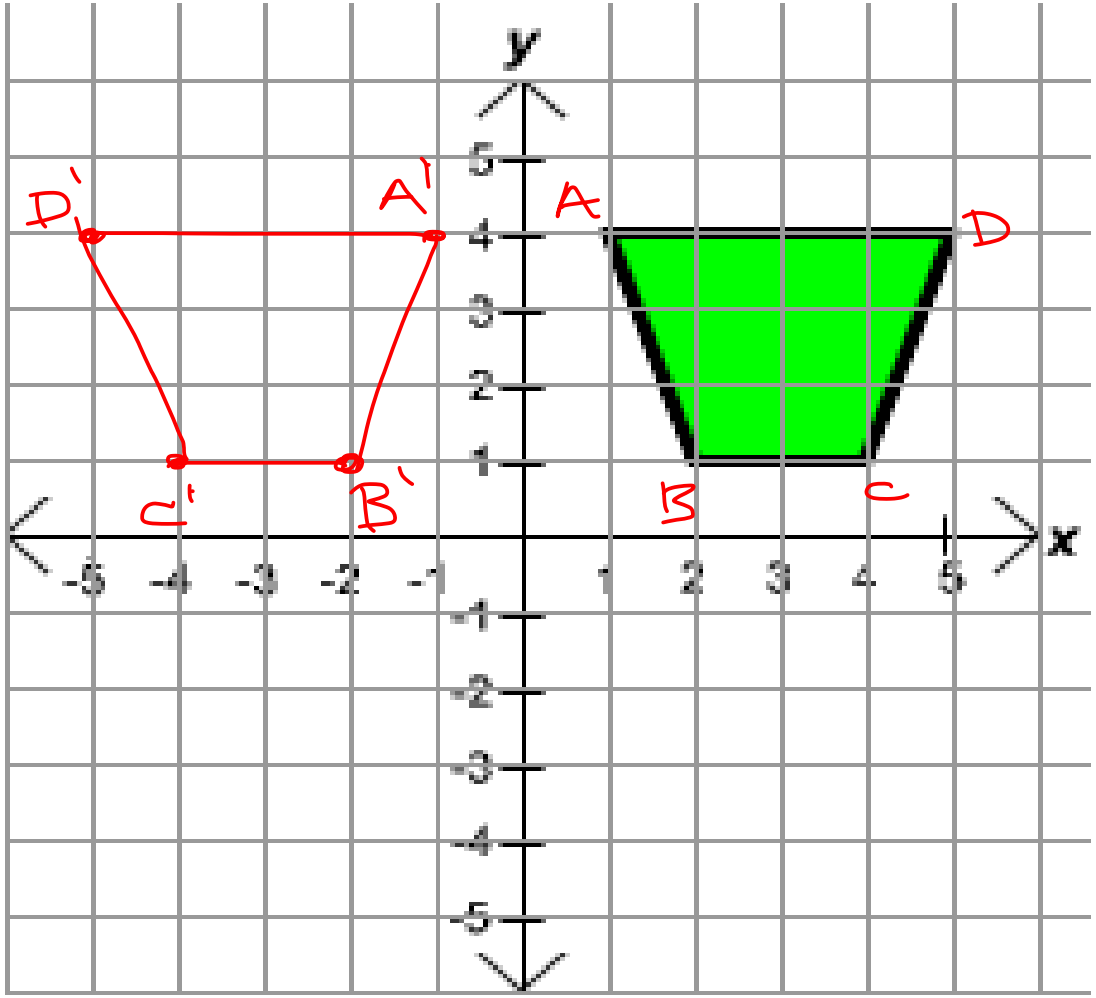
EX: Reflect the trapezoid about the y-axis. To reflect about the y-axis, (x, y) → (-x, y).

A(1, 4) A' (-1, 4)

B(2, 1) B' (-2, 1)

C(4, 1) C' (-4, 1)

D(5, 4) D' (-5, 4)



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Essential Question: How do we reflect geometric figures in a coordinate plane?

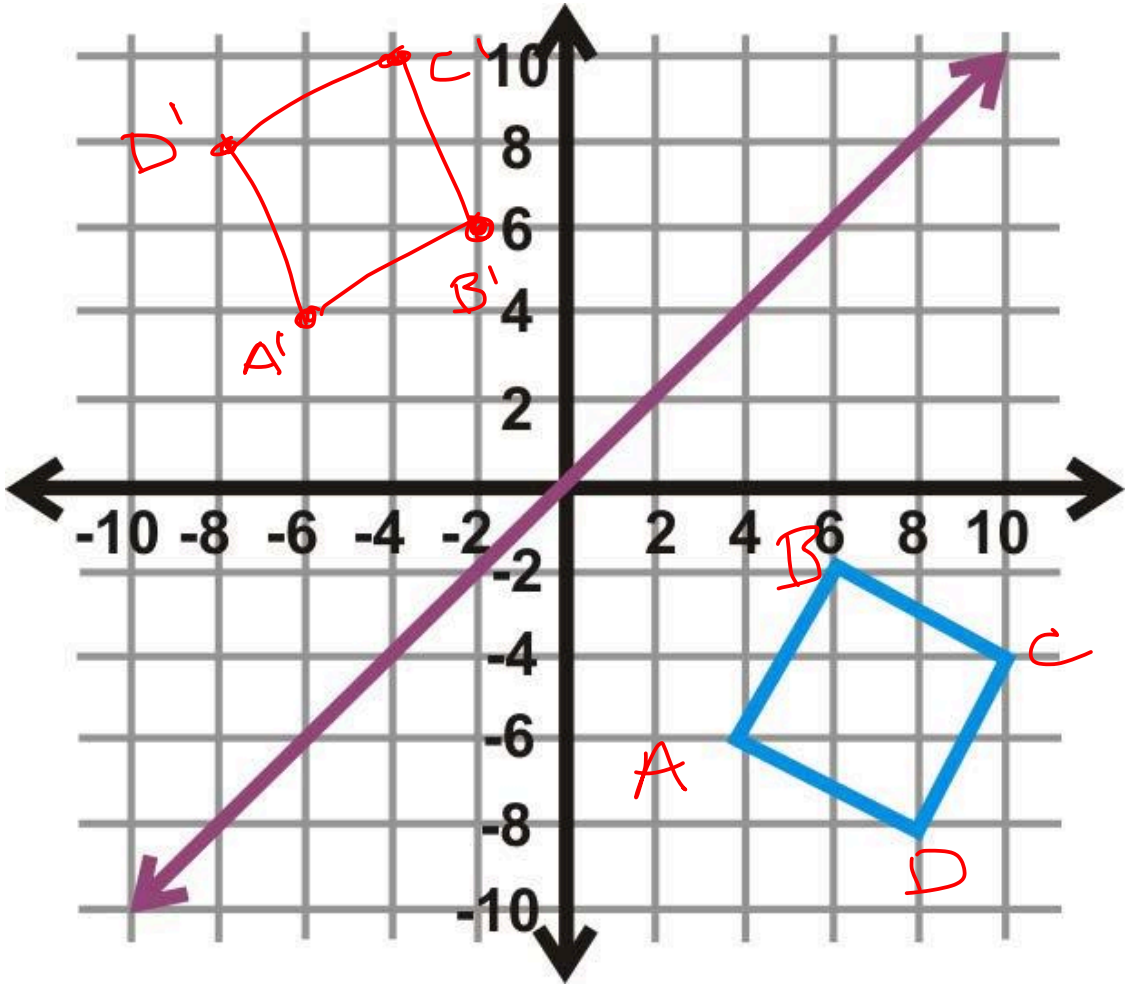
EX: Reflect the square about the line y = x. To reflect about y = x, (x, y) → (y, x).

A(4, −6) A' (−6, 4)

B(6, −2) B' (−2, 6)

C(10, −4) C' (−4, 10)

D(8, −8) D' (−8, 8)



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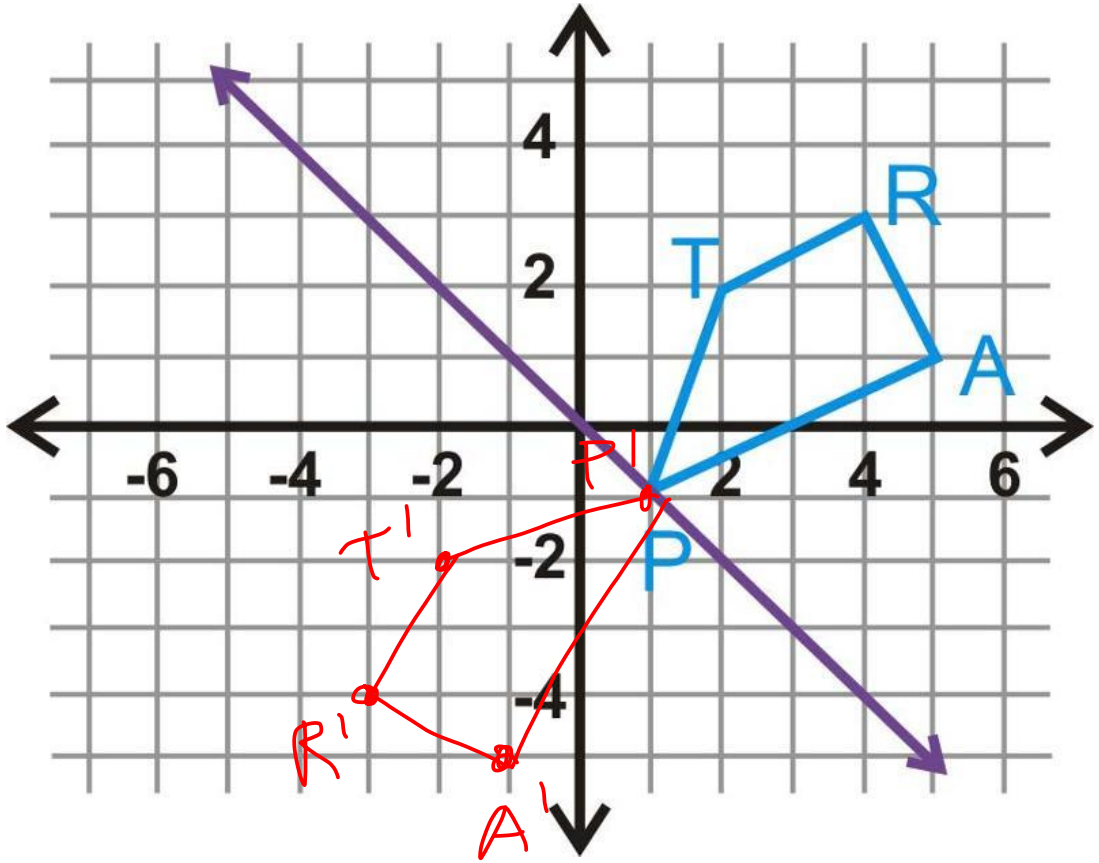
Essential Question: How do we reflect geometric figures in a coordinate plane?

EX: Reflect the trapezoid about the line y = -x.
To reflect about y= -x, (x, y) → (-y, -x).

P (1, -1) P'(1, -1)
A (5, 1) A' (-1, -5)
R (4, 3) R' (-3, -4)
T (2, 2) T' (-2, -2)



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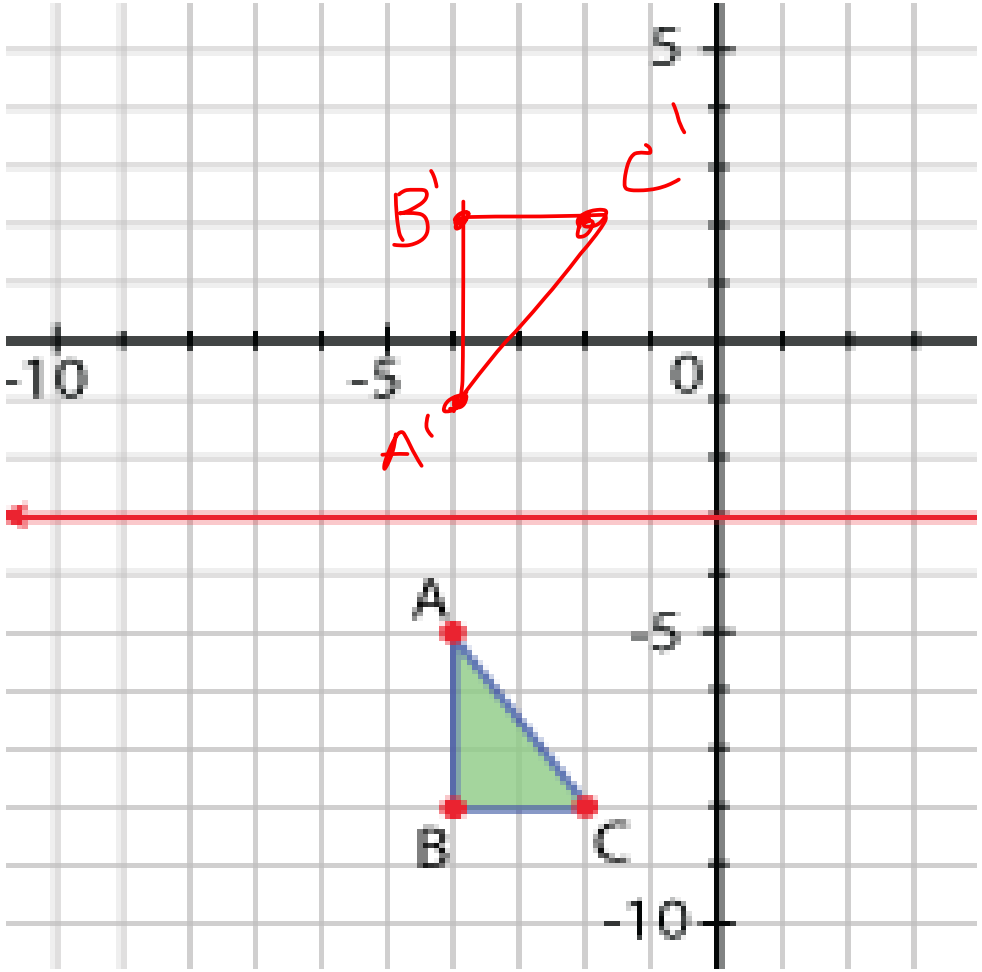
Essential Question: How do we reflect geometric figures in a coordinate plane?

EX: Reflect the triangle about the line y = -3. To reflect about y = -3, (x, y) → (x, 2(-3) – y).

A (-4, -5) A' (-4, -1)

B (-4, -8) B' (-4, 2)

C (-2, -8) C' (-2, 2)



Standard: MGSE9–12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Essential Question: How do we reflect geometric figures in a coordinate plane?

EX: Reflect the trapezoid about the line x = 6.

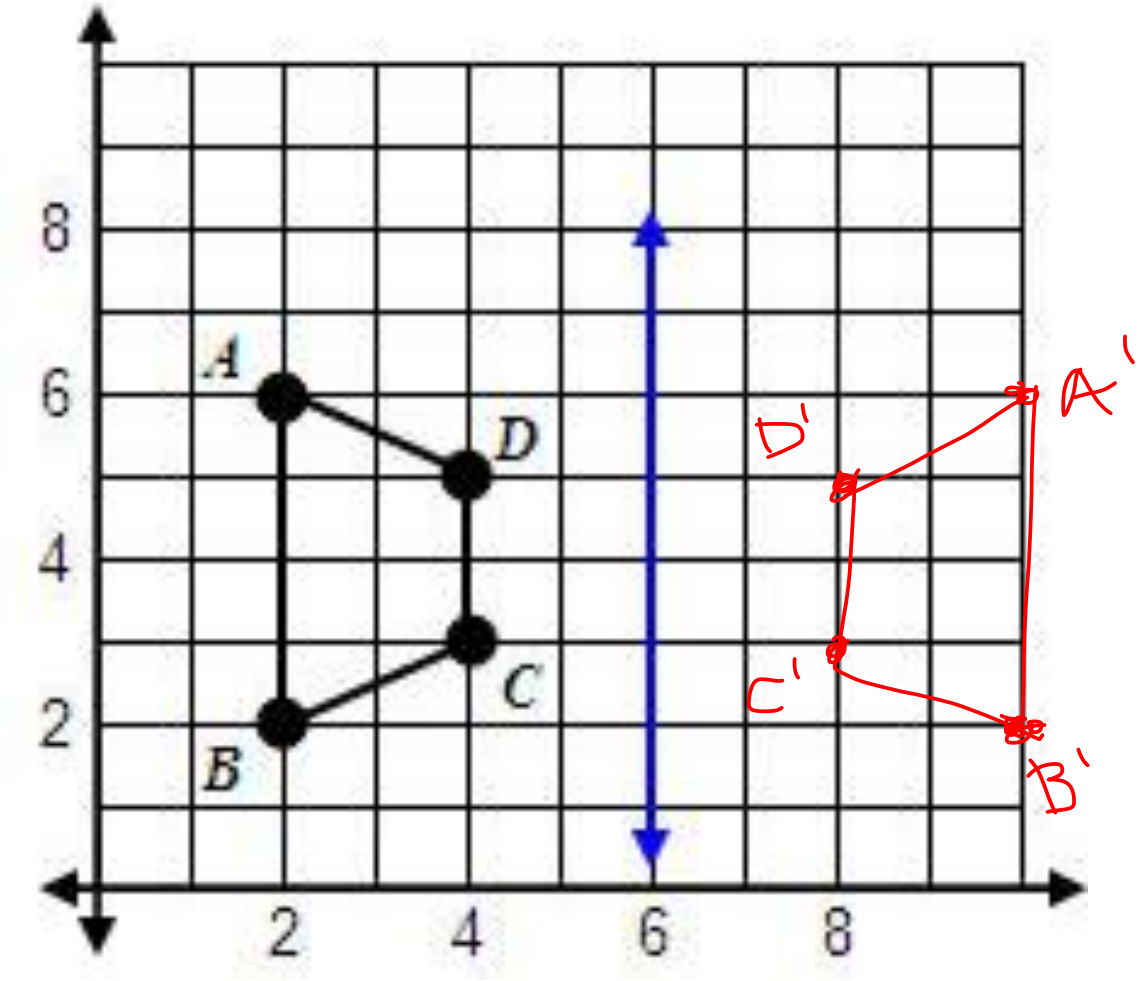
To reflect about x = 6, (x, y) → (2(6) – x, y).

A (2, 6) A' (10, 6)

B (2, 2) B' (10, 2)

C (4, 3) C' (8, 3)

D (4, 5) D' (8, 5)

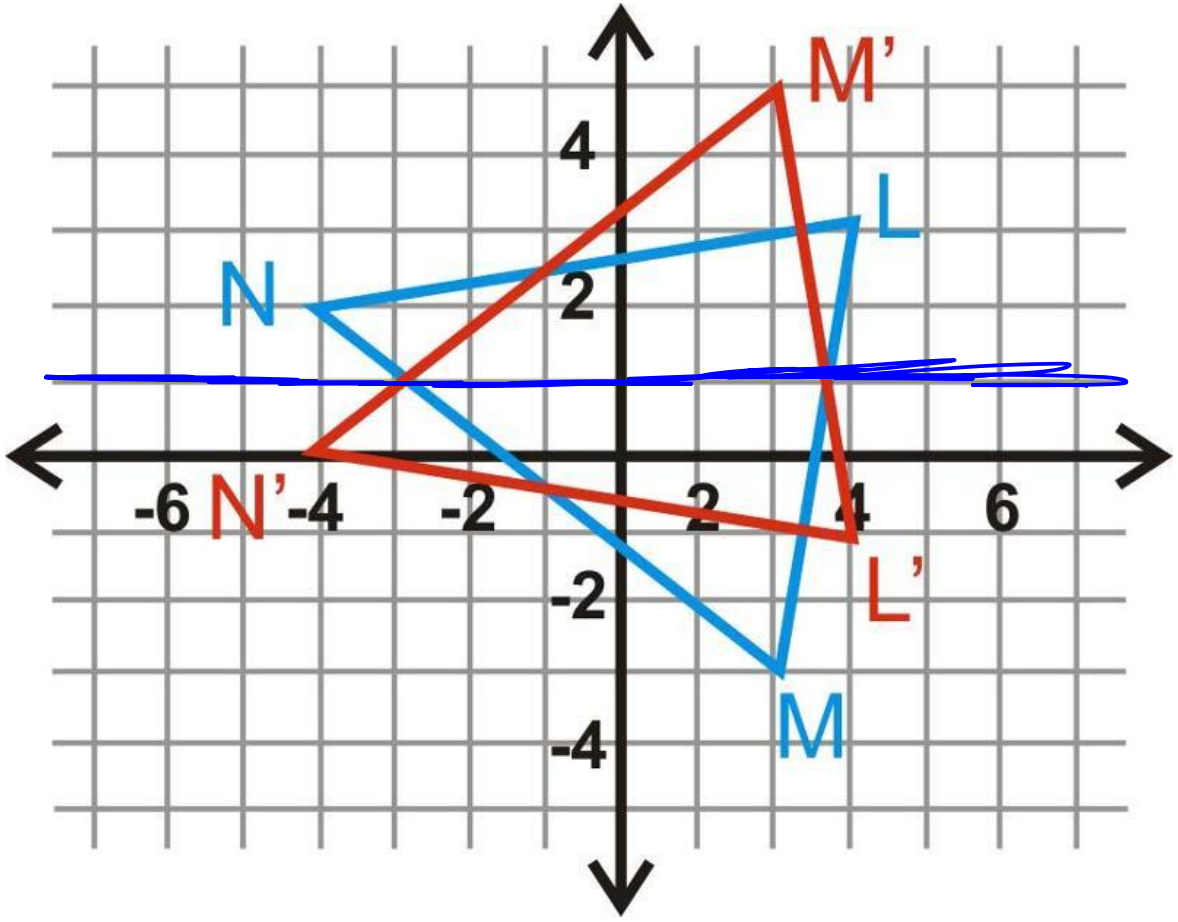


Standard: MGSE9–12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Essential Question: How do we identify the line of reflection in a coordinate plane?

EX: What is the line of reflection in the figure?

$y = 1$



Standard: MGSE9–12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Essential Question: How do we rotate geometric figures in a coordinate plane?

Rotation: A transformation that turns a figure about a fixed point through a given angle and a given direction

Rotations do not change the map of the figure! They are isometries!

EX: Rotate the trapezoid counterclockwise 90° about the origin. To rotate 90° counterclockwise about the origin,

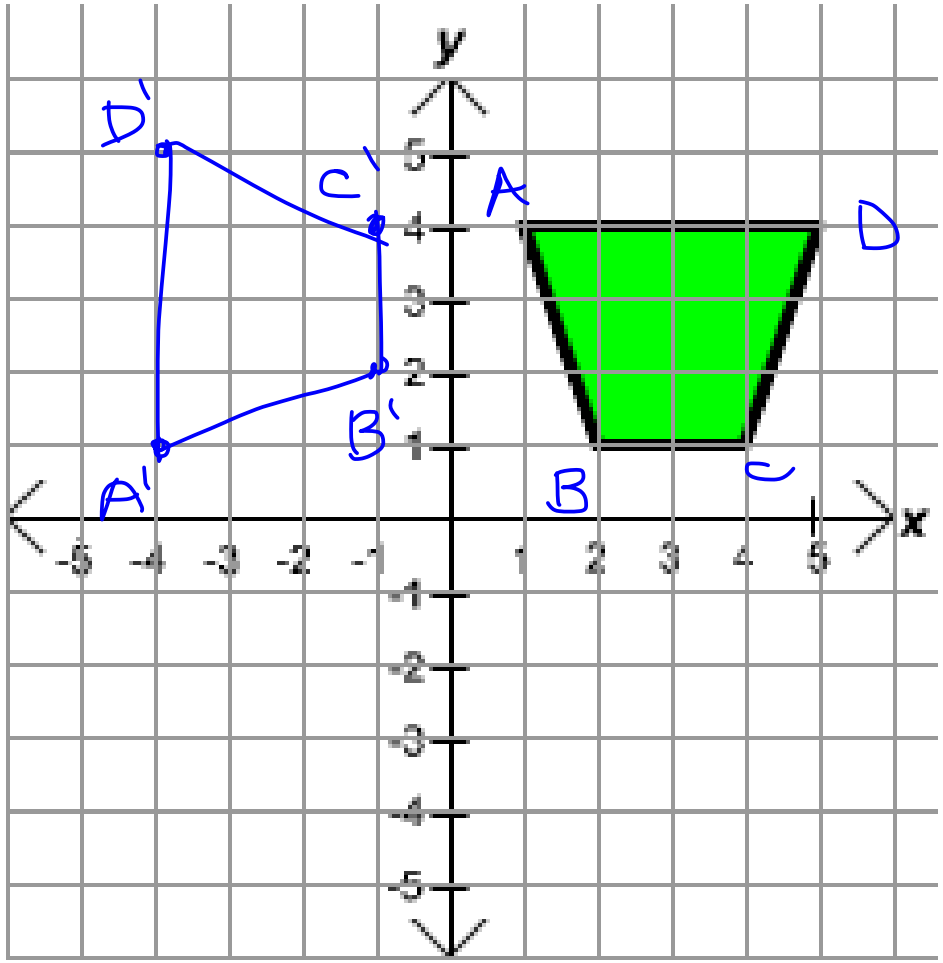
(x, y) → (-y, x).

A(1, 4) A' (-4, 1)

B(2, 1) B' (-1, 2)

C(4, 1) C' (-1, 4)

D(5, 4) D' (-4, 5)



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Essential Question: How do we rotate geometric figures in a coordinate plane?

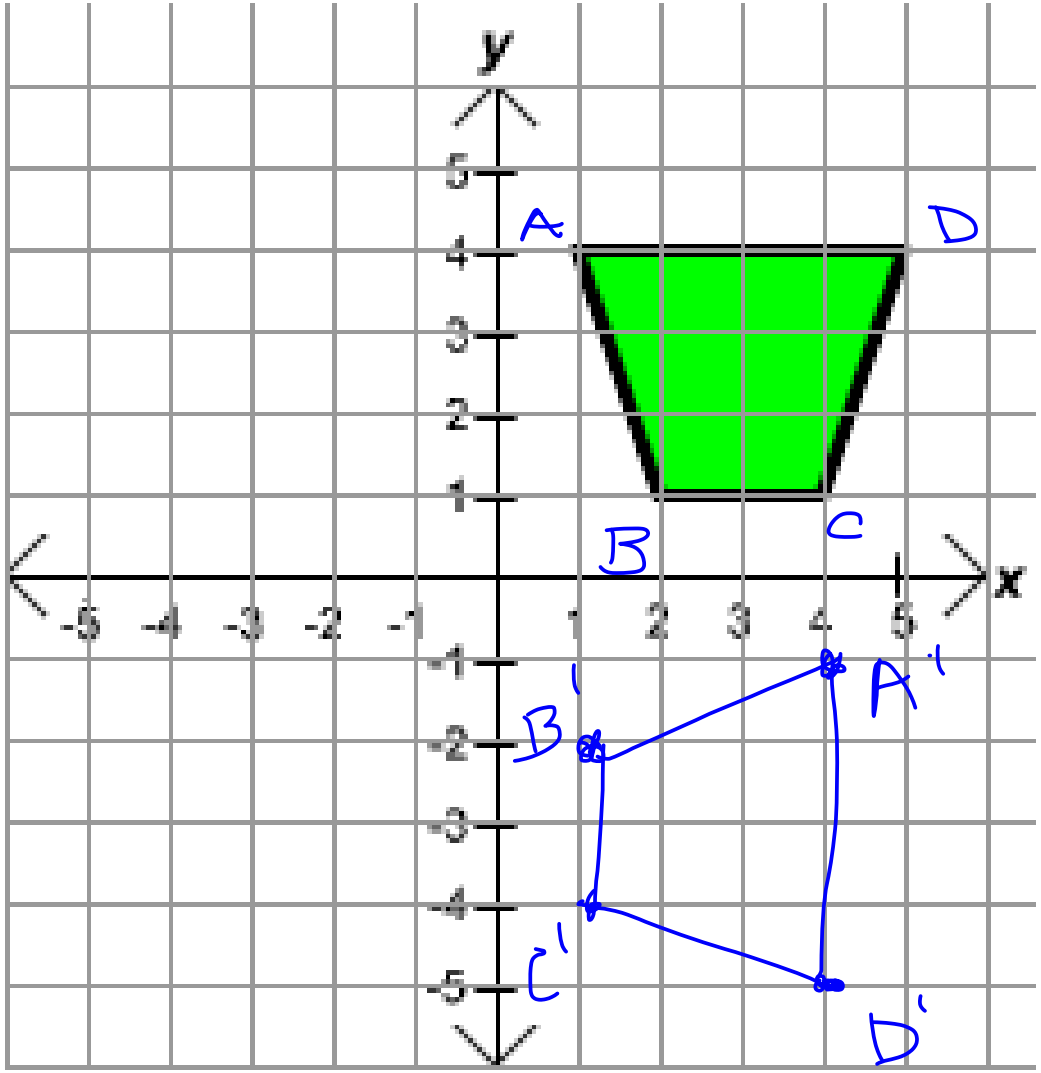
EX: Rotate the trapezoid clockwise 90° about the origin. To rotate 90° clockwise about the origin, (x, y) → (y, -x).

A (1, 4) A' (4, -1)

B (2, 1) B' (1, -2)

C (4, 1) C' (1, -4)

D (5, 4) D' (4, -5)



EOC Example

Rectangle *ABCD* has points *A*(2, 2), *B*(6, 2), *C*(6, 8), and *D*(2, 8). The rectangle maps to *A'B'C'D'* such that $(x, y) \longrightarrow (y, -x)$.

Which statement is true about the transformation of *ABCD* to *A'B'C'D'*?

A. *ABCD* maps to *A'B'C'D'* by a reflection over the *x*-axis and *B'* is located at (2, −6).

B. *ABCD* maps to *A'B'C'D'* by a reflection over the *x*-axis and *B'* is located at (6, −2).

C. *ABCD* maps to *A'B'C'D'* by a 90 degree clockwise rotation about the origin and *B'* is located at (2, −6).

D. *ABCD* maps to *A'B'C'D'* by a 90 degree clockwise rotation about the origin and *B'* is located at (6, −2).

Standard: MGSE9–12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Essential Question: How do we rotate geometric figures in a coordinate plane?

EX: Rotate the trapezoid 180° counterclockwise about the origin. To rotate 180° ***clockwise or counterclockwise*** about the origin**,**

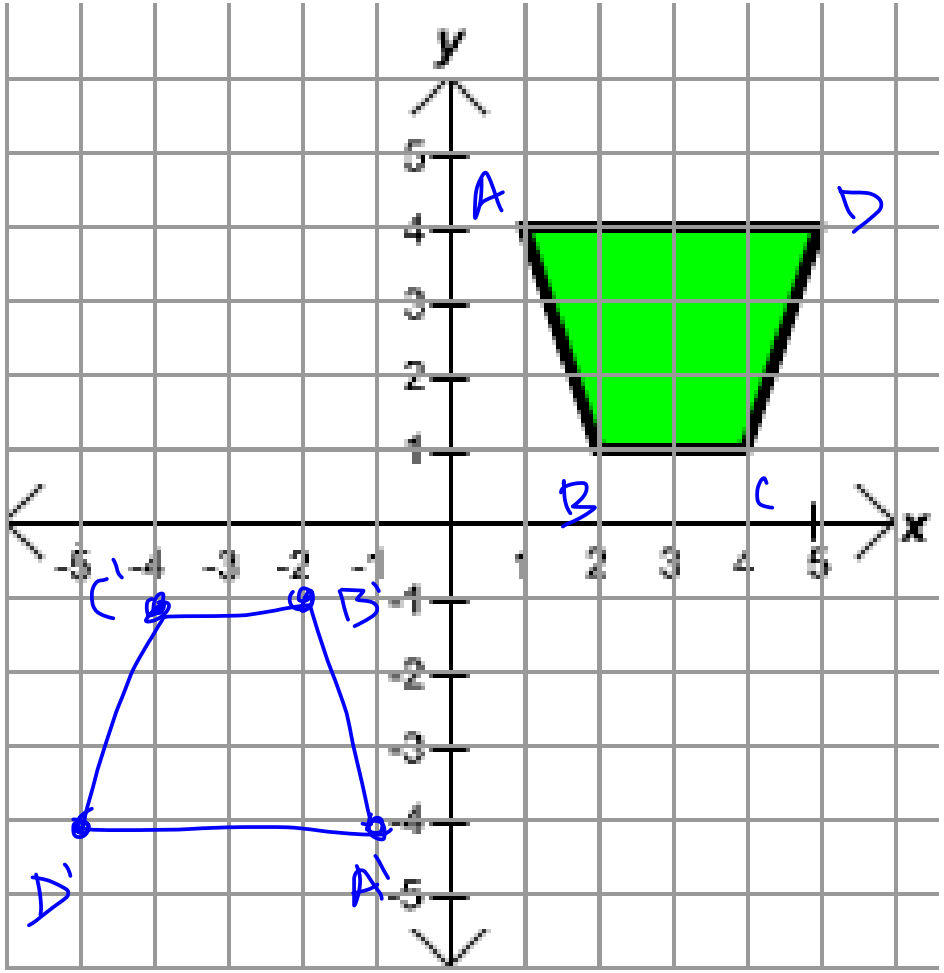
$(x, y) \rightarrow (-x, -y)$.

A (1, 4) A' (-1, -4)

B (2, 1) B' (-2, -1)

C (4, 1) C' (-4, -1)

D (5, 4) D' (-5, -4)



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Essential Question: How do we rotate geometric figures in a coordinate plane?

EX: Rotate the trapezoid 270° counterclockwise about the origin. To rotate 270° counterclockwise about the origin,

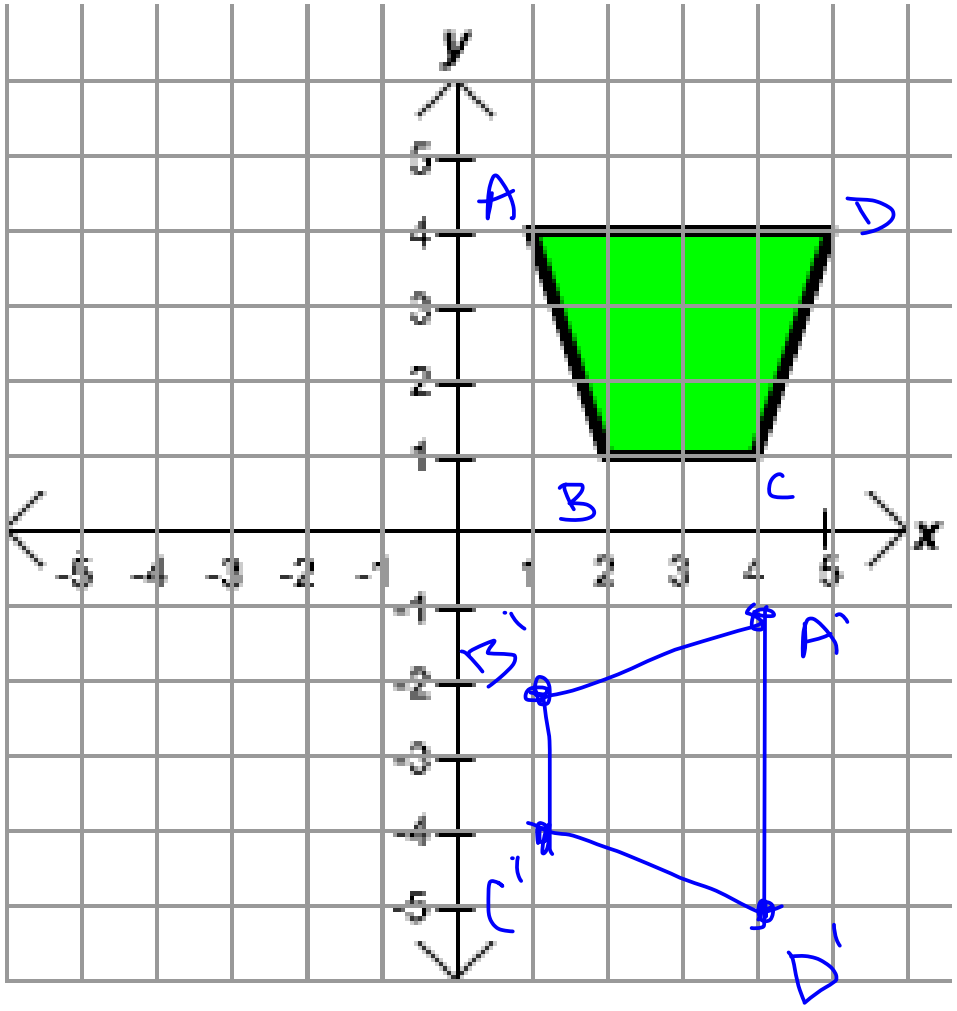
$(x, y) \rightarrow (y, -x)$.

A (1,4) A' (4,-1)

B (2,1) B' (1,-2)

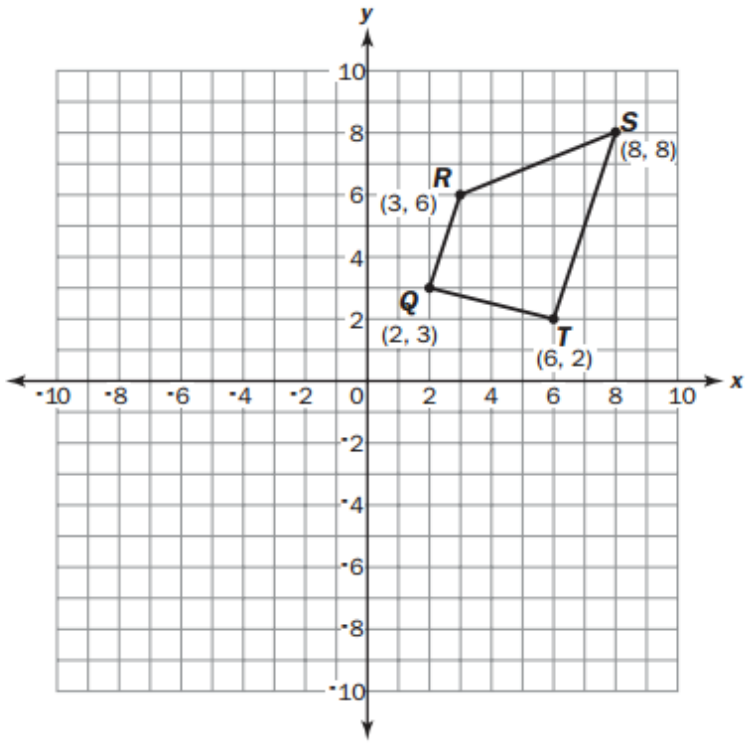
C (4,1) C' (1,-4)

D (5,4) D' (4,-5)



EOC Example

Look at quadrilateral *QRST*.



What is the image of point *R* after a counterclockwise rotation of 270 degrees about the origin?

A. (6, –3)
B. (–3, 6)
C. (–6, 3)
D. (3, –6)

Standard: MGSE9–12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Essential Question: How do we rotate geometric figures in a coordinate plane?

EX: Rotate the trapezoid 270° clockwise about the origin. To rotate 270° clockwise about the origin,

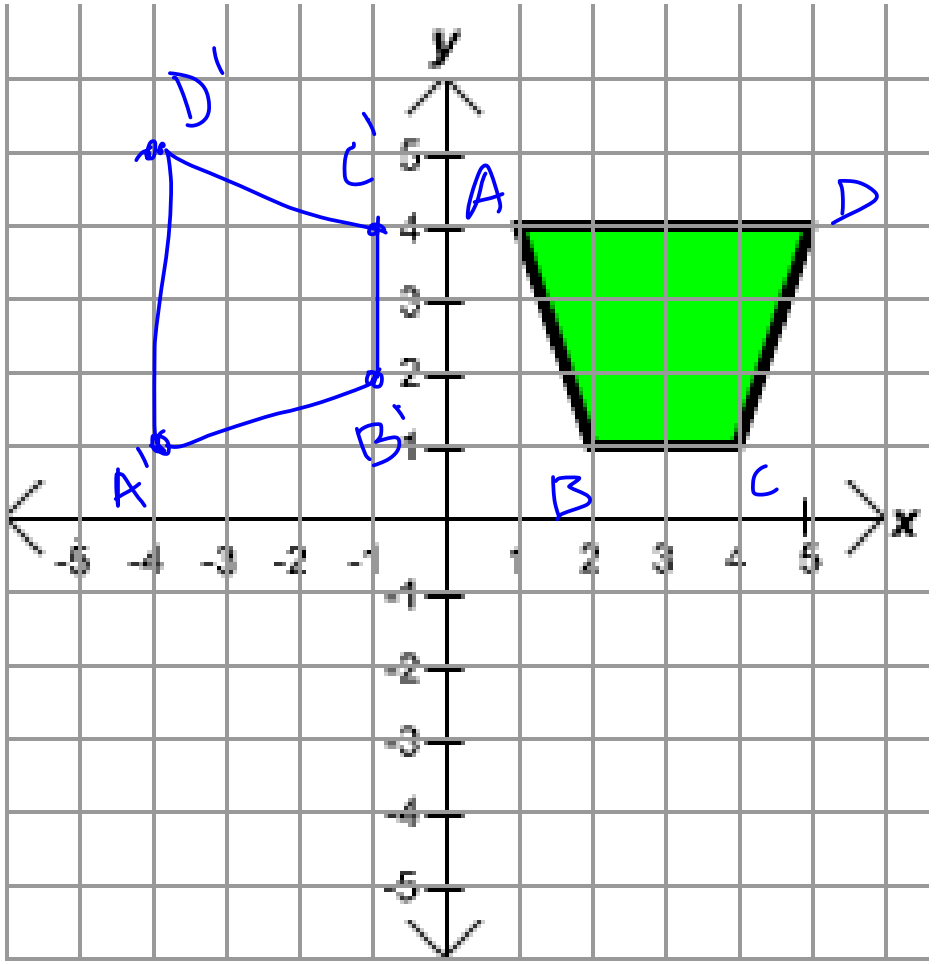
$(x, y) \rightarrow (-y, x)$.

A (1, 4) A' (−4, 1)

B (2, 1) B' (−1, 2)

C (4, 1) C' (−1, 4)

D (5, 4) D' (−4, 5)



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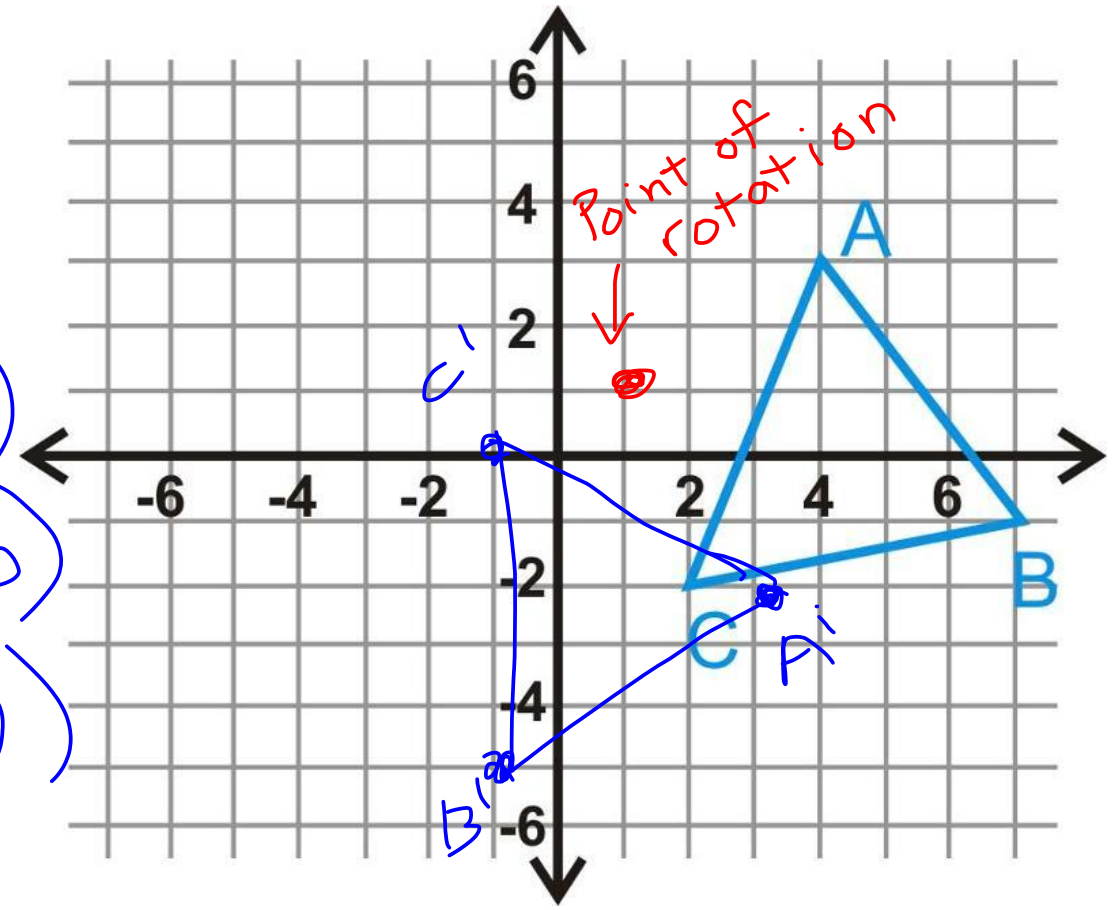
Essential Question: How do we rotate geometric figures in a coordinate plane about a given point?

EX: Rotate the triangle 90° clockwise about the point (1, 1).

A (4,3) A' (3,-2)

B (7,-1) B' (-1,-5)

C (2,-2) C' (-2,0)



Standard: MGSE9–12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

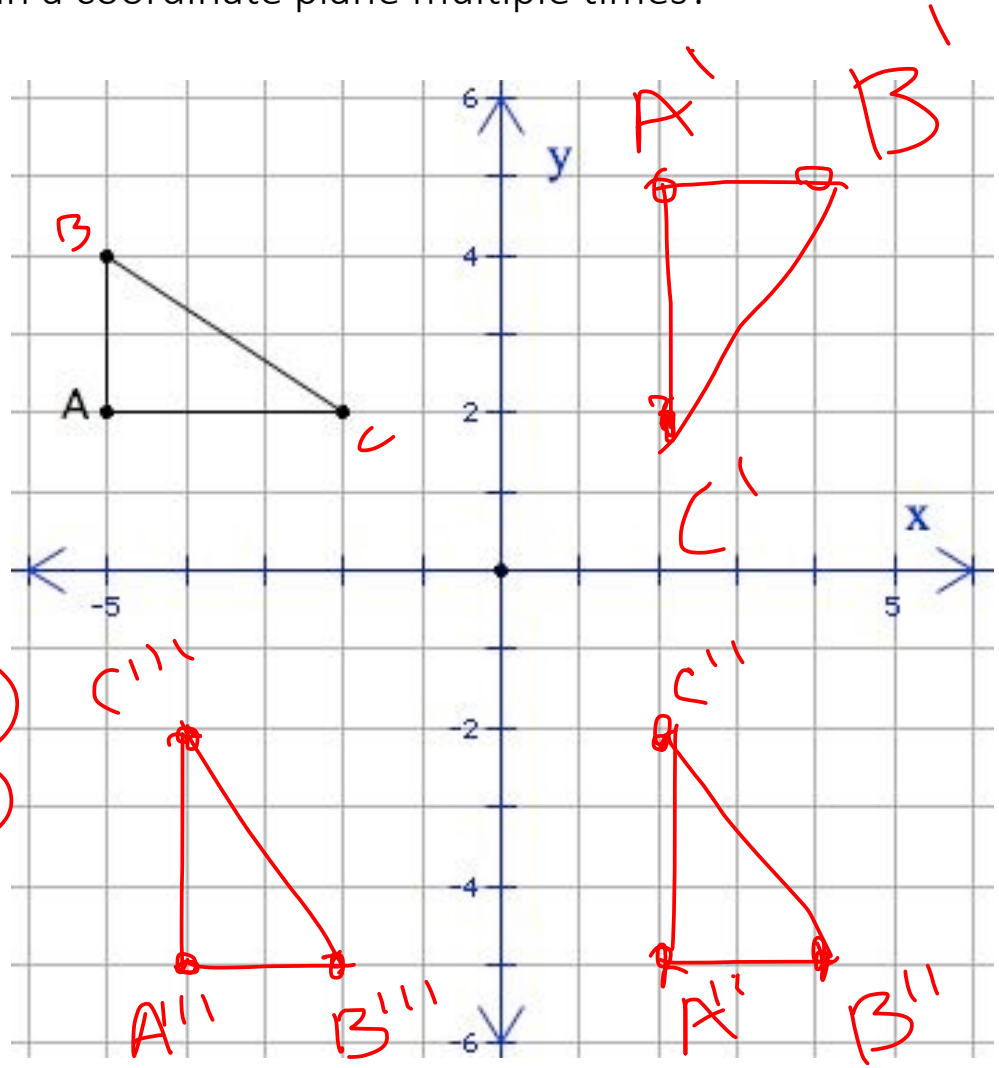
Essential Question: How do we transform geometric figures in a coordinate plane multiple times?

EX: Rotate the triangle 90° clockwise, reflect the figure about the x-axis, then translate the figure 6 units to the left.

A (-5,2) A' (2,5) A'' (2,-5) A''' (-4,-5)

B (-5,4) B' (4,5) B'' (4,-5) B''' (-2,-5)

C (-2,2) C' (2,2) C'' (2,-2) C''' (-4,-2)



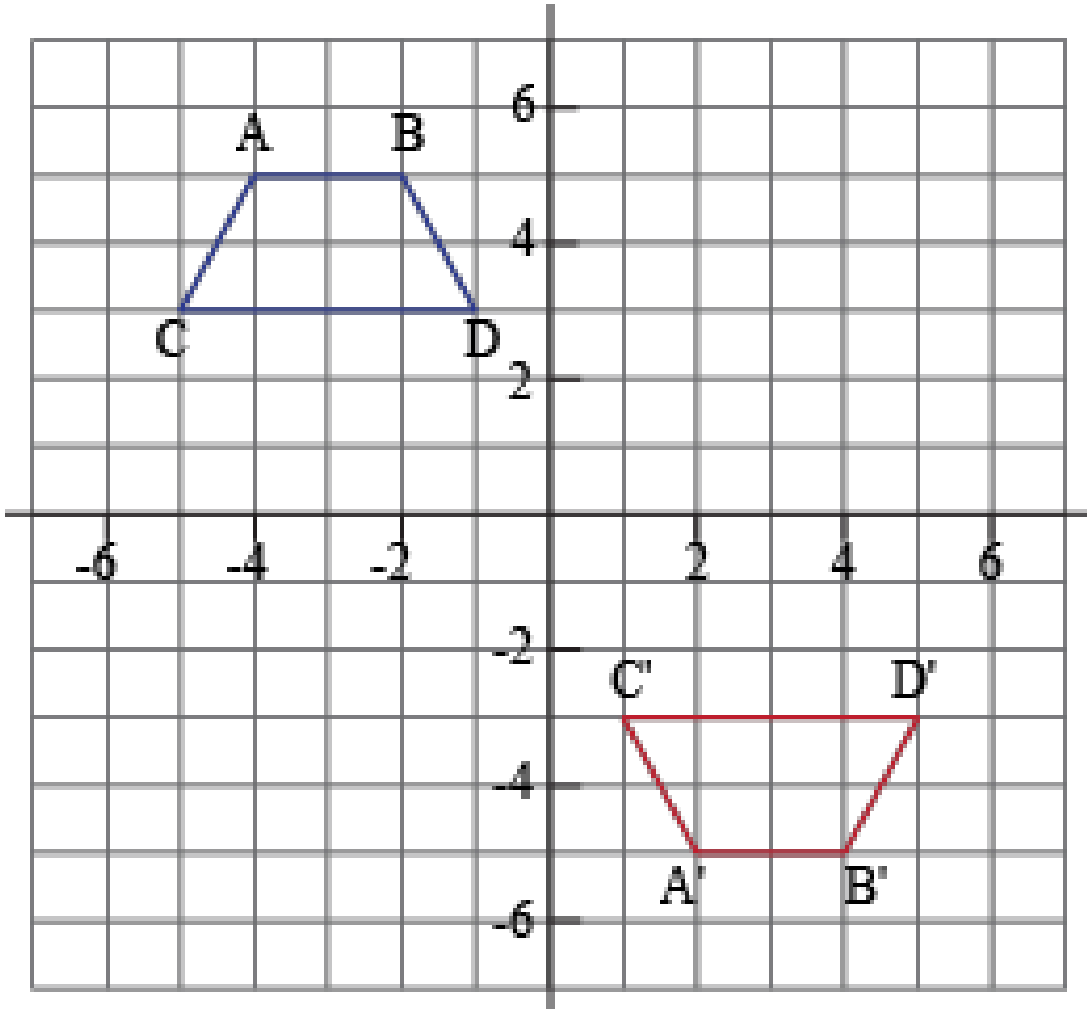
Standard: MGSE9-12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Essential Question: How do we transform geometric figures in a coordinate plane multiple times?

EX: Specify the sequence of transformations that will map ABCD to A'B'C'D'.

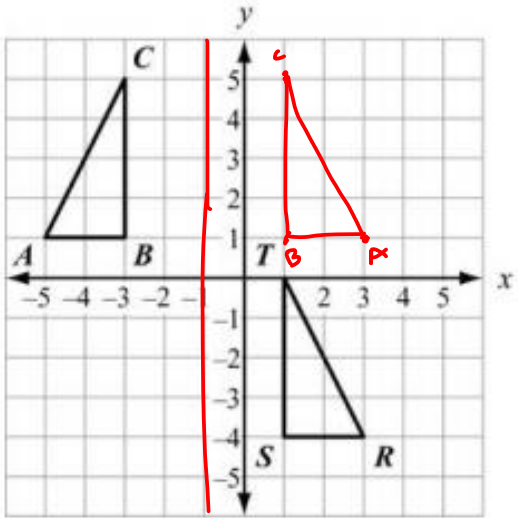
1. Reflection about x-axis

2. Translation 6 units to the right.



EOC Example

Which sequence of transformations maps △*ABC* to △*RST*?



A. Reflect △*ABC* across the line $x = -1$. Then translate the result 1 unit down.

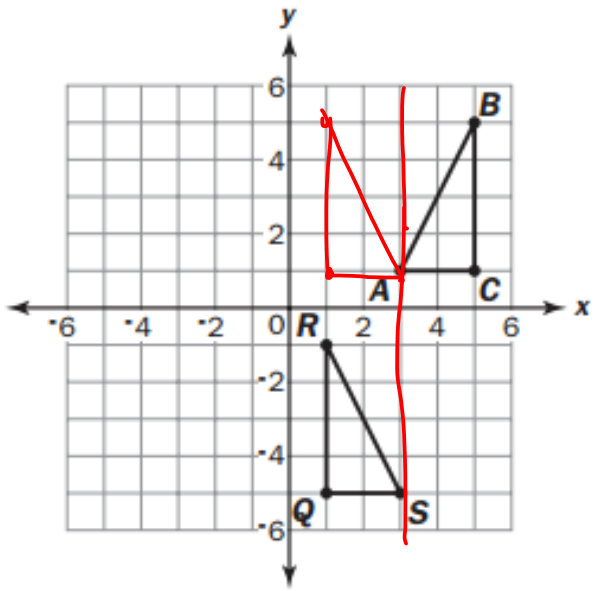
B. Reflect △*ABC* across the line $x = -1$. Then translate the result 5 units down.

C. Translate △*ABC* 6 units to the right. Then rotate the result 90° clockwise about the point (1, 1).

D. Translate △*ABC* 6 units to the right. Then rotate the result 90° counterclockwise about the point (1, 1).

EOC Example

What is the sequence of transformations that carry triangle *ABC* to triangle *QRS*?



A. Triangle *ABC* is reflected across the line $x = 3$. Then it is translated 2 units down.

B. Triangle *ABC* is reflected across the line $x = 3$. Then it is translated 6 units down.

C. Triangle *ABC* is translated 2 units to the left. Then it is rotated 90 degrees counterclockwise about the point (1, 1).

D. Triangle *ABC* is translated 2 units to the right. Then it is rotated 90 degrees counterclockwise about the point (1, 1).

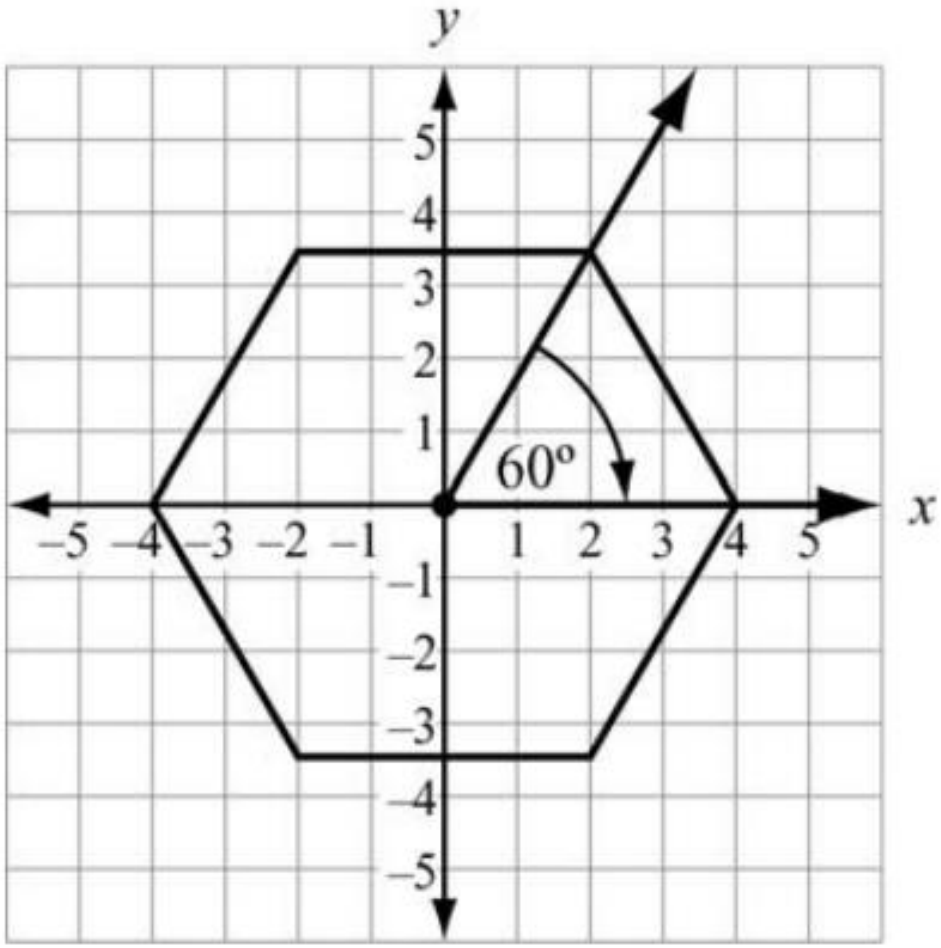
Standard: MGSE9–12.G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Essential Question: Which transformations carry figures onto themselves?

All angles are congruent. $\cong$

EX: What transformation maps the regular hexagon to itself?

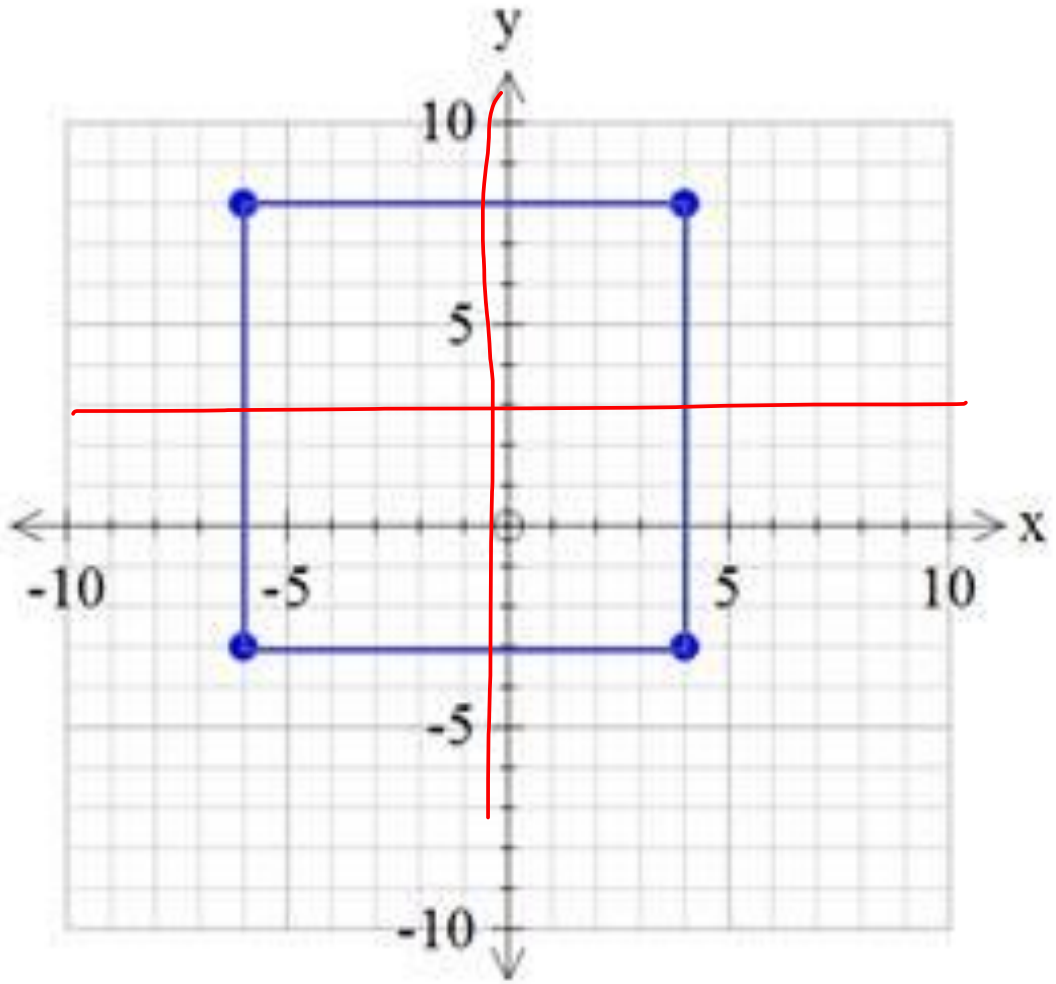
1) Reflection about x-axis.
2) Reflection about y-axis.
3) 60° rotation about origin.
4) 120° " " "
5) 180° " " "
6) 240° " " "
7) 300° " " "
8) 360° " " "



Standard: MGSE9–12.G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Essential Question: Which transformations carry figures onto themselves?

EX: What transformations map the ~~square~~ rectangle onto itself?

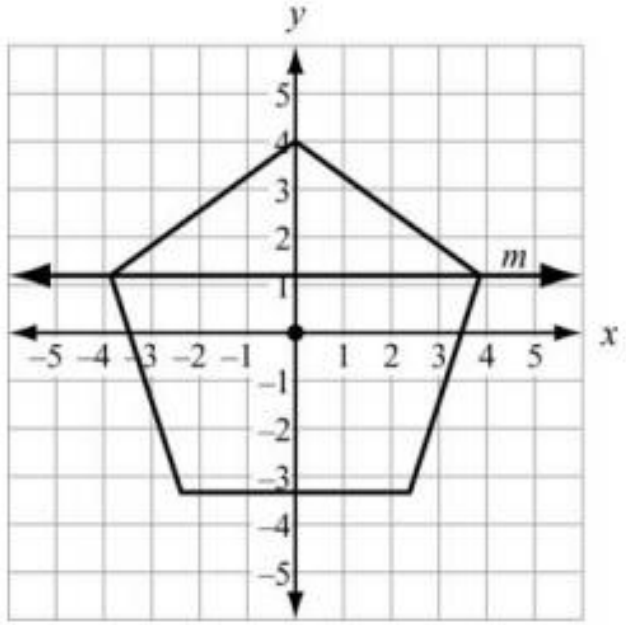


1) Reflection about $x = -.5$
2) Reflection about $y = 3$
3) 90° rotation about $(-.5, 3)$
4) 180°
5) 270°
6) 360°

EOC Example

A regular pentagon is centered about the origin and has a vertex at (0, 4).

All angles ≅ at 72°.



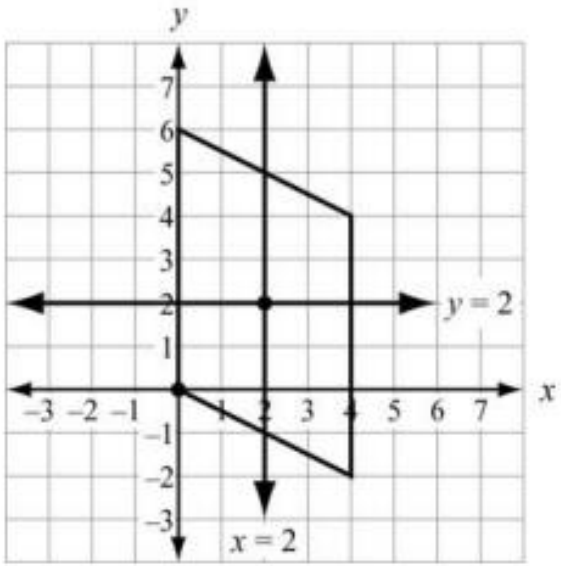
Which transformation maps the pentagon to itself?

A. a reflection across line *m*
B. a reflection across the *x*-axis
C. a clockwise rotation of 100° about the origin
D. a clockwise rotation of 144° about the origin

multiple of 72

EOC Example

A parallelogram has vertices at (0, 0), (0, 6), (4, 4), and (4, –2).



Which transformation maps the parallelogram to itself?

A. a reflection across the line $x = 2$
B. a reflection across the line $y = 2$
C. a rotation of 180° about the point (2, 2)
D. a rotation of 180° about the point (0, 0)

EOC Example

Which transformation on quadrilateral *ABCD* produces an image that does not preserve distance between points in quadrilateral *ABCD*?

A. reflection across $y = x$
B. translation 3 units down and 4 units to the right
C. dilation by a scale factor of 2
D. rotation of 270 degrees

Reflections, translations, and rotations preserve size and shape of a figure. Dilations do not.