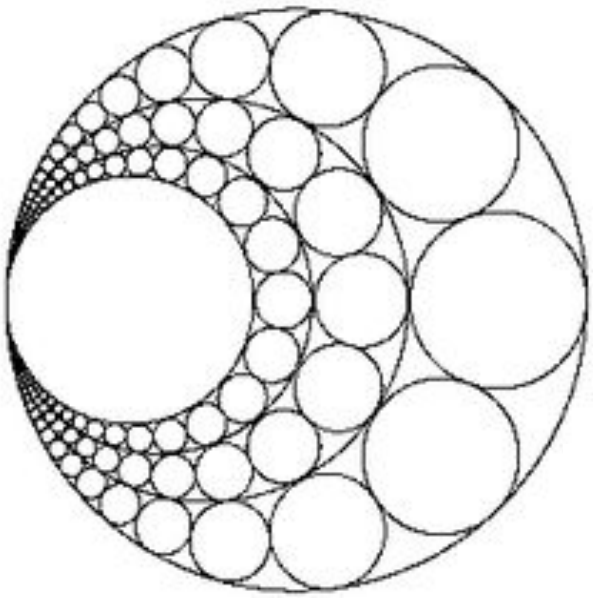
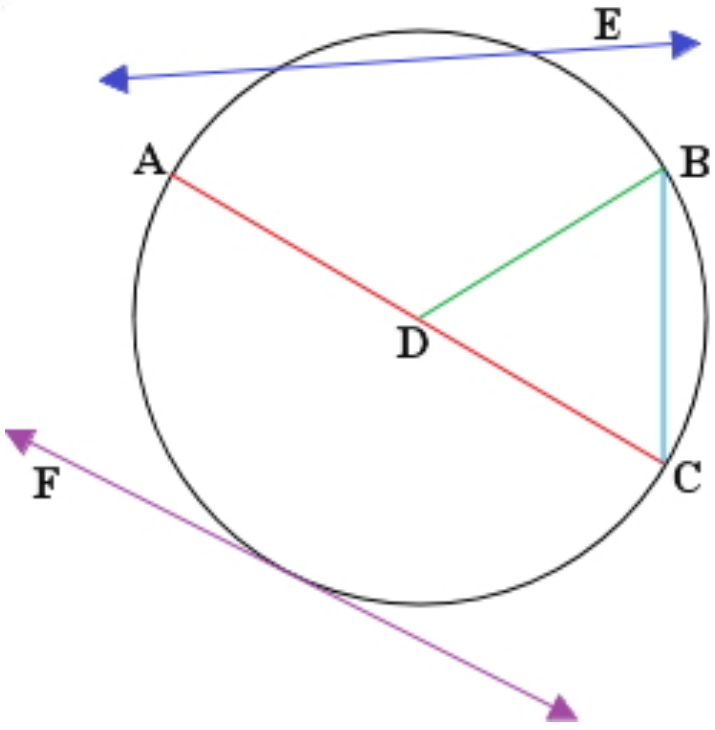
Geometry
Unit 10: Circles



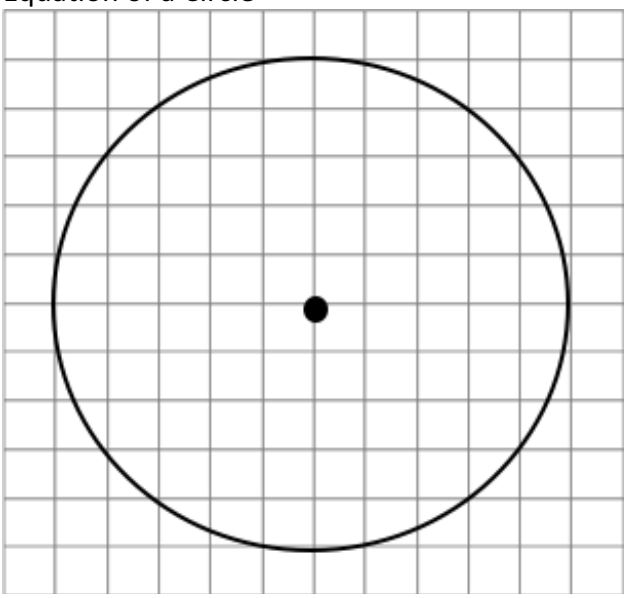


Name_____

Helpful Vocabulary

Word	Definition/Explanation	Examples/Helpful Tips

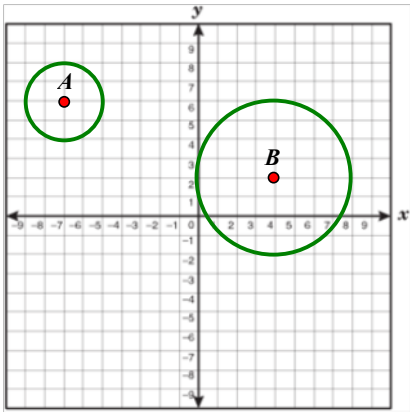
Equation of a Circle



Determine the center and radius of the given circles:

1.$(x-7)^2 + (y+10)^2 = 81$ center_____ radius___	2. $(x+5)^2 + (y+1)^2 = 25$ center_____ radius___	3. $(x-4)^2 + (y+5)^2 = 9$ center_____ radius___
4. $(x+9)^2 + (y-5)^2 = 12$ center_____ radius___	5. $(x-1)^2 + (y-5)^2 = 16$ center_____ radius___	6. $(x+5)^2 + (y+6)^2 = 9$ center_____ radius___

7.



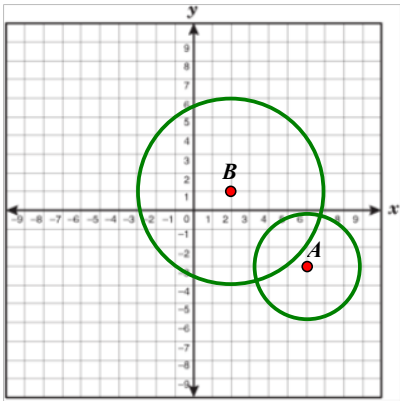
A) center_____radius_____

equation_____

B) center_____radius_____

equation_____

8.



A) center_____radius_____

equation_____

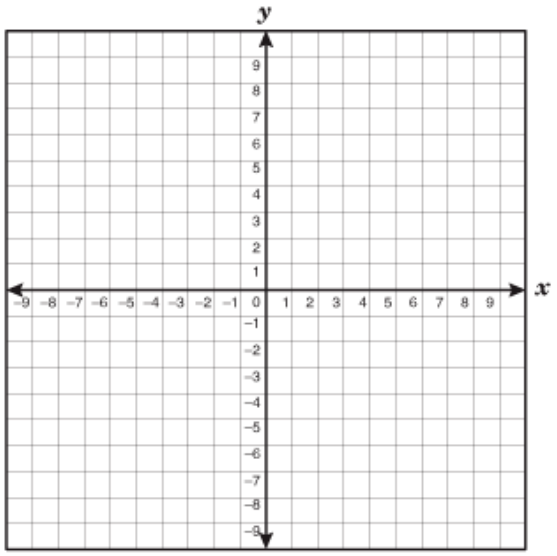
B) center_____radius_____

equation_____

Graph the following circles. State the center and radius.

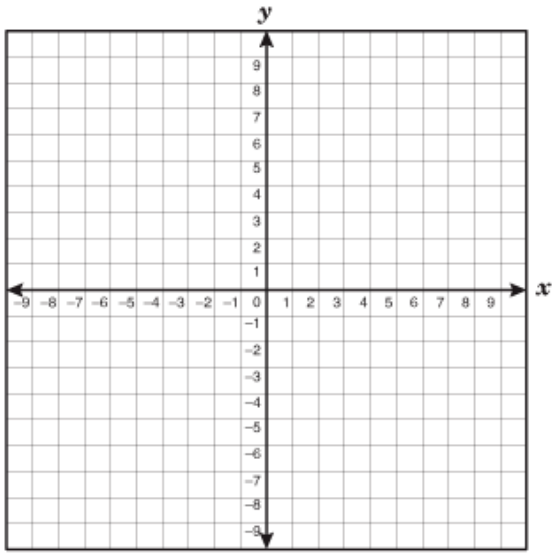
1. a) $(x - 3)^2 + (y + 2)^2 = 4$

b) $(x + 6)^2 + (y - 7)^2 = 9$



2. a) $x^2 + (y - 7)^2 = 25$

b) $(x - 4)^2 + (y + 5)^2 = 9$



Standard Form and Perfect Square Trinomials

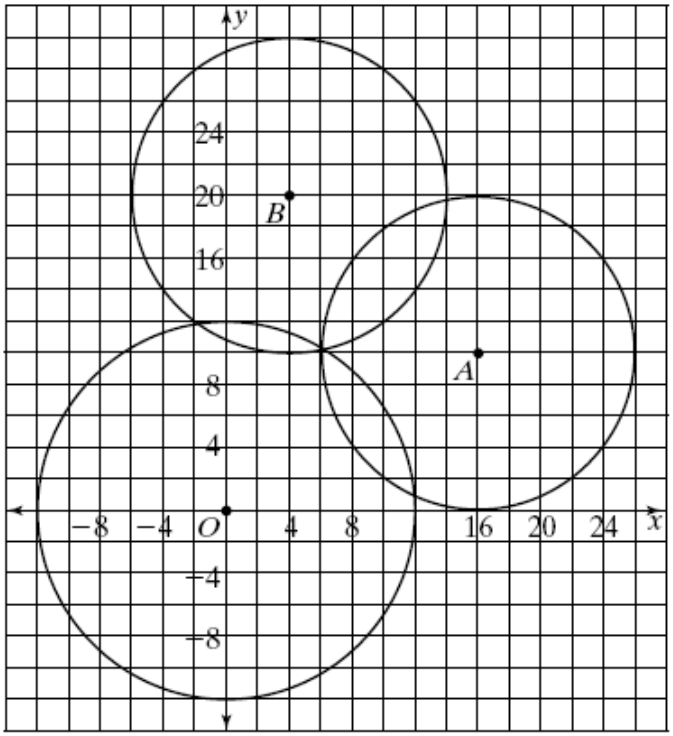
1. $(x - 2)^2$	2. $(x + 5)^2$	3. $(x - 9)^2$
a = _____ b= _____ c= _____	a = _____ b= _____ c= _____	a = _____ b= _____ c= _____

Completing the Square

Determine the value of the constant term, c, to create a perfect square trinomial then write the trinomial in factored form.

1. $x^2 + 4x + \_\_\_$	2. $x^2 + 10x + \_\_\_$	3. $x^2 + 14x + \_\_\_$
Factored Form _____	Factored Form _____	Factored Form _____
4. $x^2 - 12x + \_\_\_$	5. $x^2 - 8x + \_\_\_$	6. $x^2 - 2x + \_\_\_$
Factored Form _____	Factored Form _____	Factored Form _____

Do Now

Finding the Equation of a Circle	
1. Circle A center_____ radius_____ equation _____	
2. Circle B center_____ radius_____ equation _____	
3. Circle O center_____ radius_____ equation _____	

Using Completing the Square with Quadratic Equations to Rewrite from Standard Form to Vertex Form

1.	2.	3.
$x^2 + 6x + 3 = 0$	$x^2 + 10x + 20 = 0$	$x^2 - 8x - 3 = 0$

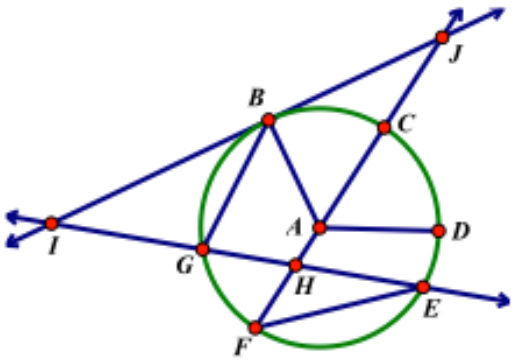
Using Completing the Square with Circle Equations to Rewrite from Standard Form to Center Radius Form

Determine the center and radius of the given circles	
1. $x^2 + y^2 + 4x - 16y + 52 = 0$	2. $x^2 + 10x + y^2 - 16 = 0$

Practice

Determine the center and radius of the given circles	
1. $x^2 + y^2 + 2x + 18y + 1 = 0$	2. $x^2 + y^2 + 18x + 17 = 0$
3. $x^2 - 14x + y^2 - 2y - 50 = 0$	4. $x^2 + y^2 - 10x + 10y = -48$

Circle Vocabulary & Activity



Do Now

Using correct circle vocabulary, name each line:

AC =

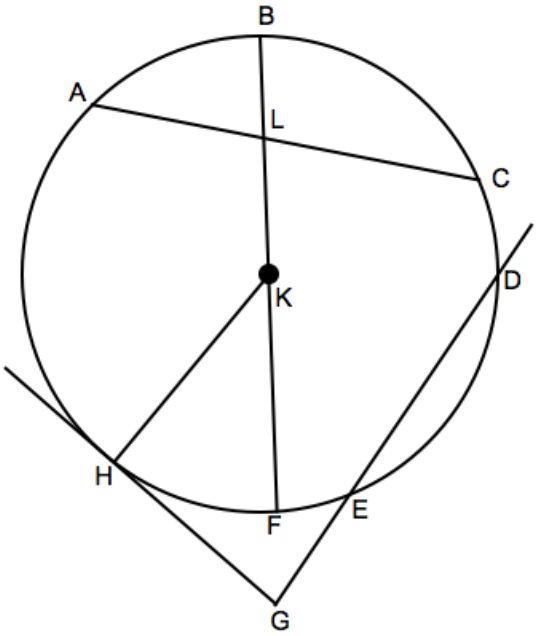
HK =

GD =

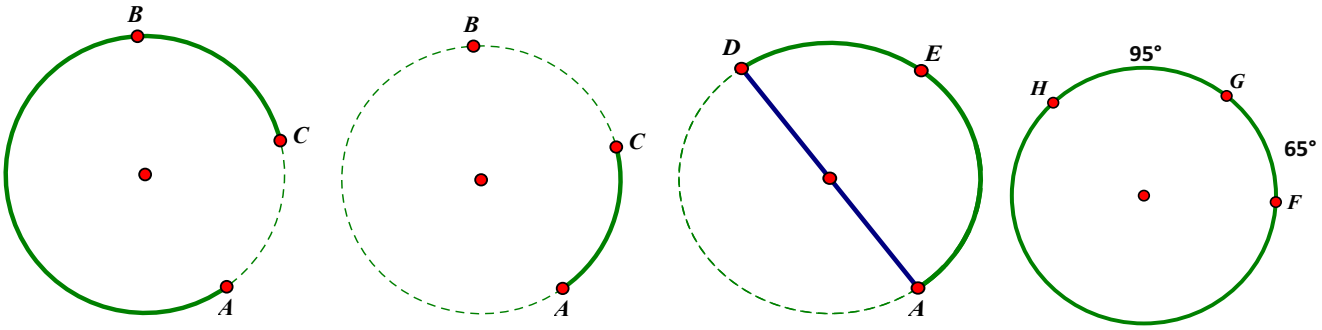
GH =

FB =

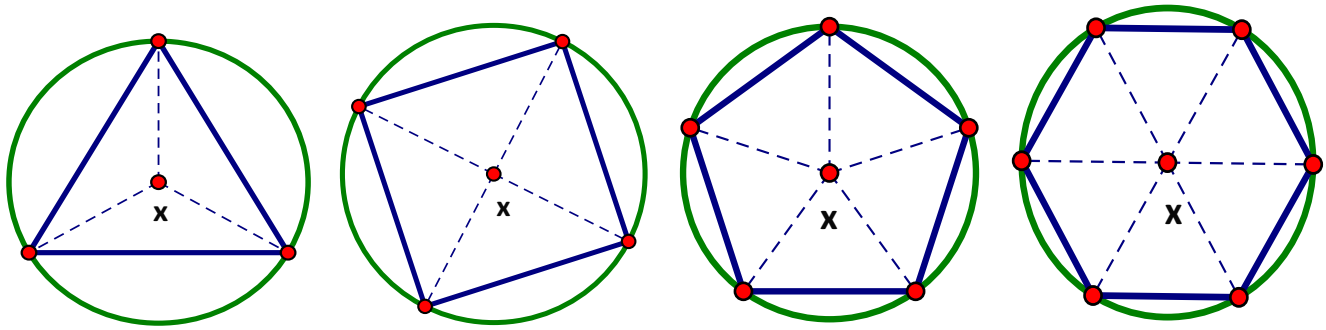
ED =

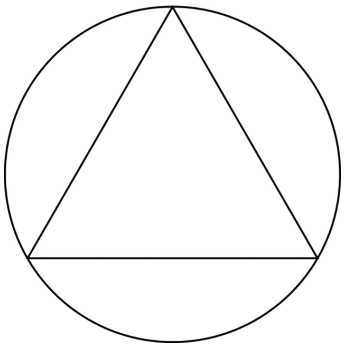


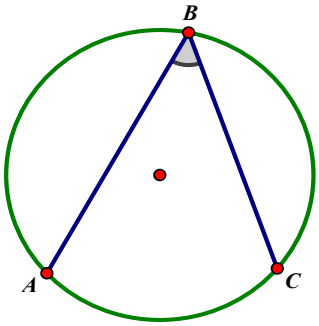
Arc Measure

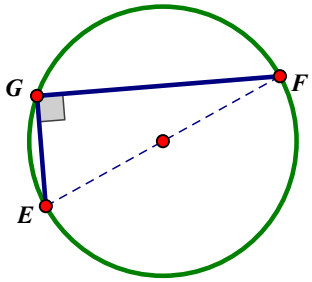


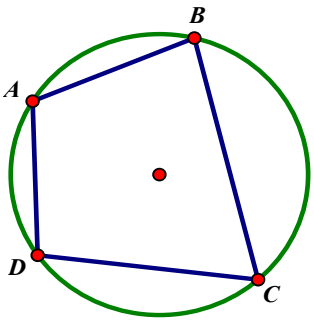
Central Angle









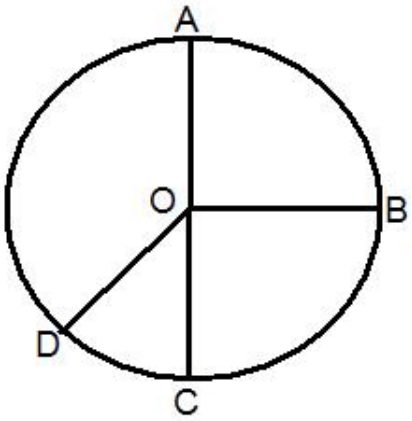


Practice

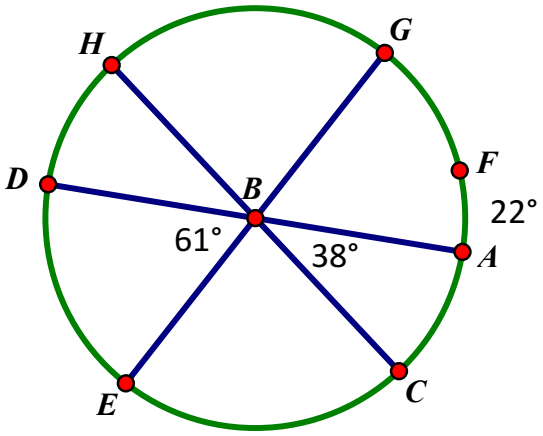
1. In a circle O, m∠AOB = 87, m∠BOC = 93, and m∠COD = 35. Find the measure of each of the following:

a) ∠DOA_____ b) $\overset{\frown}{AB}$ _____ c) $\overset{\frown}{BC}$ _____ d) $\overset{\frown}{ABC}$ _____

e) $\overset{\frown}{DC}$ _____ f) $\overset{\frown}{AD}$ _____ g) $\overset{\frown}{BCD}$ _____ h) $\overset{\frown}{CDB}$ _____ i) $\overset{\frown}{DBC}$ _____



2. Given Circle B with diameters $\overline{HC}$, $\overline{EG}$ and $\overline{DA}$.



a) m∠DBH = _____

b) $m\overset{\frown}{DCE}$ = _____

c) $m\overset{\frown}{HG}$ = _____

d) $m\overset{\frown}{HCF}$ = _____

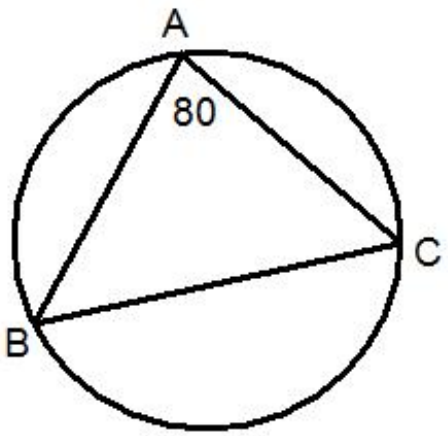
e) m∠HBA = _____

f) m∠DBA = _____

3. Triangle ABC is inscribed in a circle, mA = 80, m $\overset{\frown}{AC}$ = 88. Find:

a) m $\overset{\frown}{BC}$ _____ b) m∠B _____

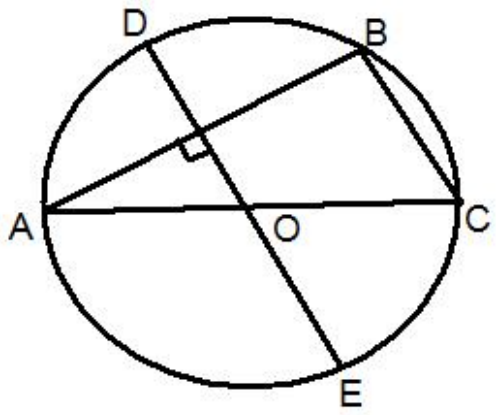
c) m∠C_____ d) m $\overset{\frown}{AB}$ _____



4. Diameter $\overline{DOE}$ ⊥ chords $\overline{AB}$ at F, $\overline{DOE}$ is a diameter and $\overline{BC}$ is a chord of circle O. If m $\overset{\frown}{BC}$ = 60, find:

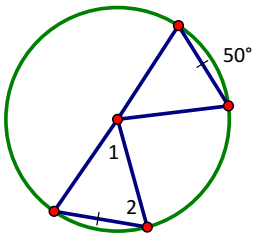
a) m $\overset{\frown}{AB}$ _____b) m∠A_____c) m∠C _____

d) m $\overset{\frown}{AD}$ _____e) m∠AOD____f) m $\overset{\frown}{CE}$ _____



Extra Practice

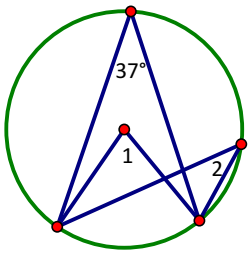
1.



m∠1 = _____

m∠2 = _____

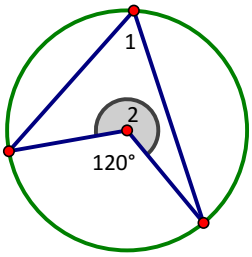
2.



m∠1 = _____

m∠2 = _____

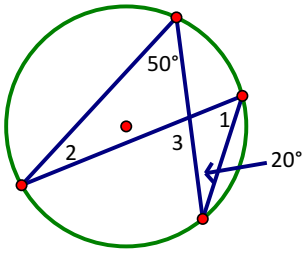
3.



m∠1 = _____

m∠2 = _____

4.

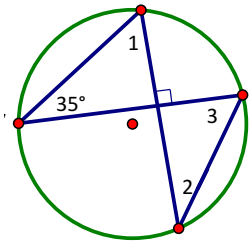


m∠1 = _____

m∠2 = _____

m∠3 = _____

5.

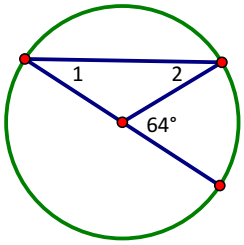


$m\angle 1 =$ _____

$m\angle 2 =$ _____

$m\angle 3 =$ _____

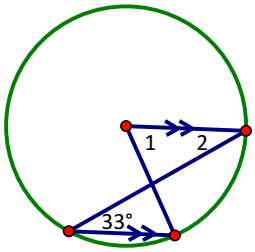
6.



$m\angle 1 =$ _____

$m\angle 2 =$ _____

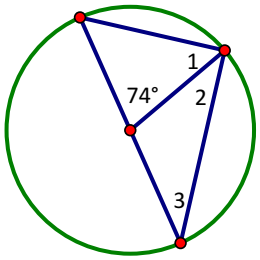
7.



$m\angle 1 =$ _____

$m\angle 2 =$ _____

8.

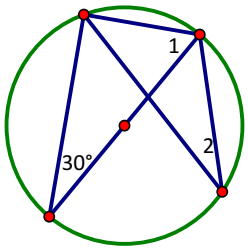


$m\angle 1 =$ _____

$m\angle 2 =$ _____

$m\angle 3 =$ _____

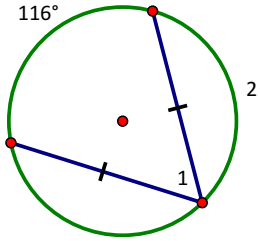
9.



$m\angle 1 =$ _____

$m\angle 2 =$ _____

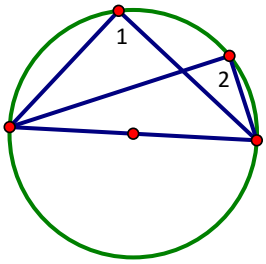
10.



$m\angle 1 =$ _____

$m2 =$ _____

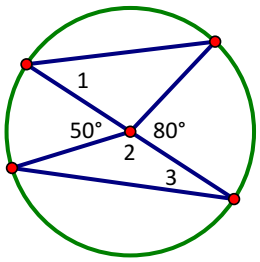
11.



$m\angle 1 =$ _____

$m\angle 2 =$ _____

12.

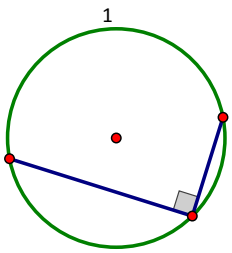


$m\angle 1 =$ _____

$m\angle 2 =$ _____

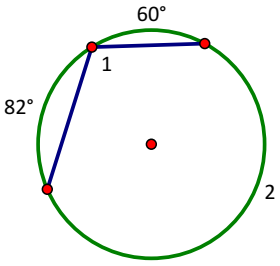
$m\angle 3 =$ _____

13.



$m1 =$ _____

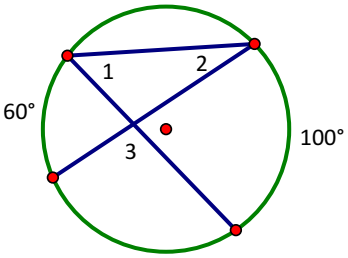
14.



$m\angle 1 =$ _____

$m2 =$ _____

15.

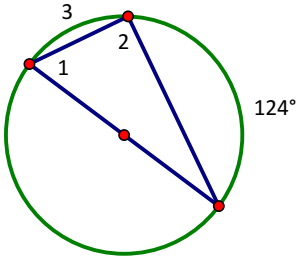


$m\angle 1 =$ _____

$m\angle 2 =$ _____

$m\angle 3 =$ _____

16.

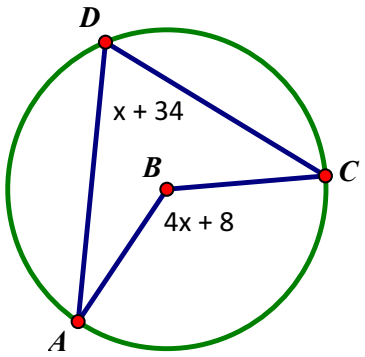


$m\angle 1 =$ _____

$m\angle 2 =$ _____

$m3 =$ _____

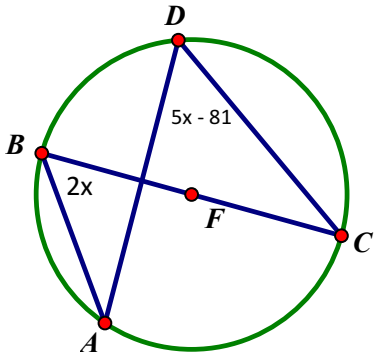
1.



x = _____

m∠ADC = _____

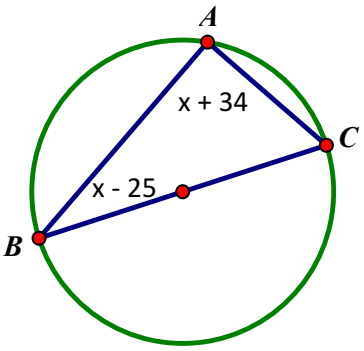
2.



x = _____

m∠ABC = _____

3.



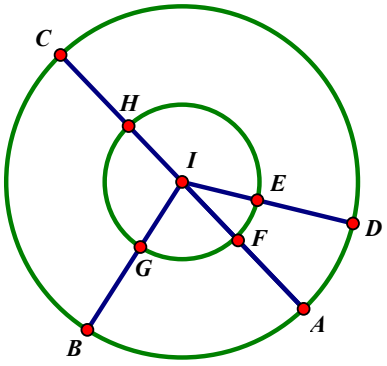
x = _____

m∠ACB = _____

4. Given concentric circles with mGF = 76°, m∠HIE = 147° and $\overline{CA}$ & $\overline{FH}$ are diameters.

$m\overset{\frown}{CB}$ = _____ $m\overset{\frown}{HE}$ = _____

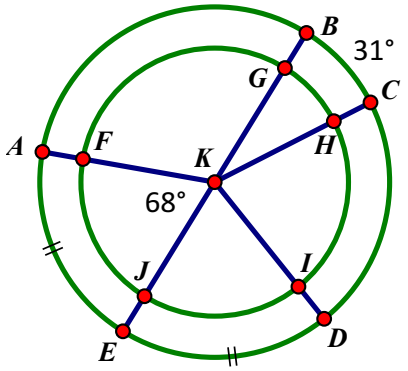
$m\overset{\frown}{BDC}$ = _____ m∠CIB = _____



5. Given concentric circles with mBC = 31°, m∠FKJ = 68° and $\overline{EB}$ is a diameter.

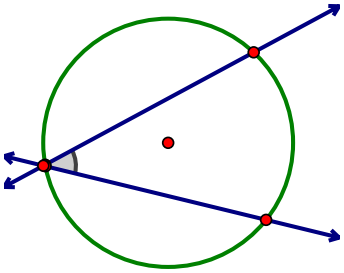
$m\overset{\frown}{ED}$ = _____ m∠GKH= _____

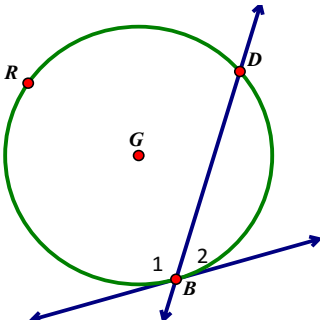
$m\overset{\frown}{ABD}$ = _____ m∠AKB = _____

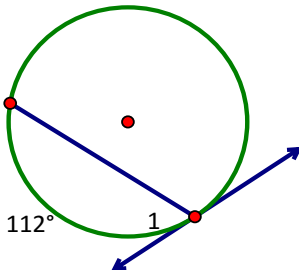


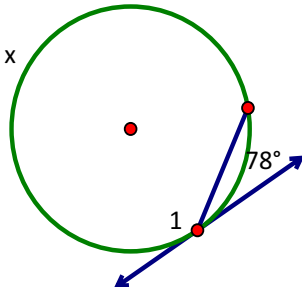
Angles Formed Inside and Outside a Circle

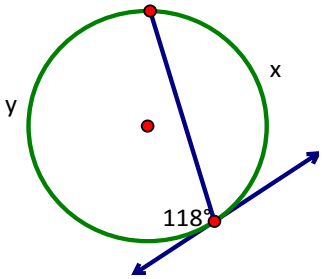
Angles Formed ON a Circle



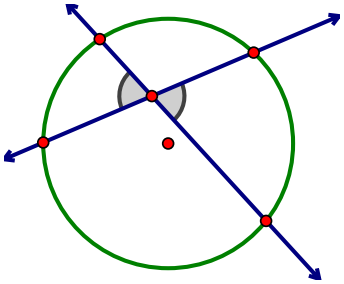


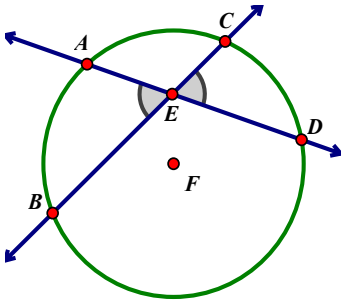


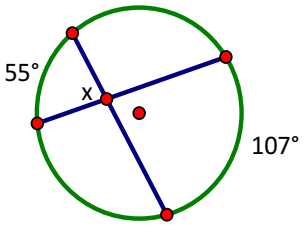


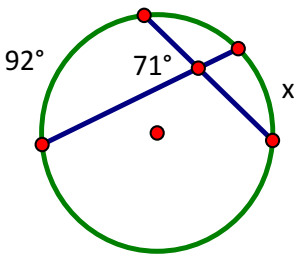


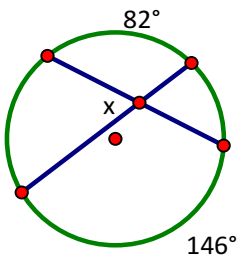
Angles Formed INSIDE a Circle



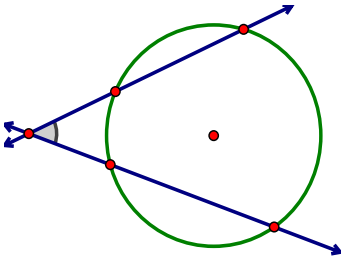


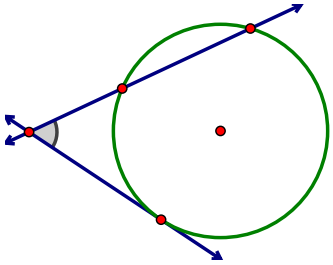


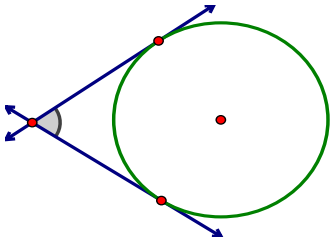


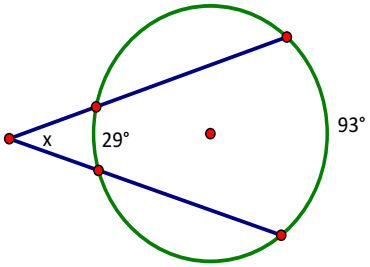


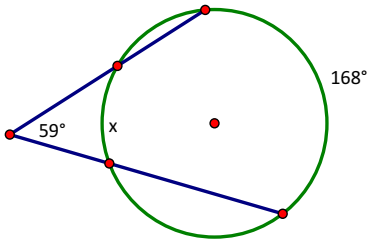
Angles Formed OUTSIDE a Circle

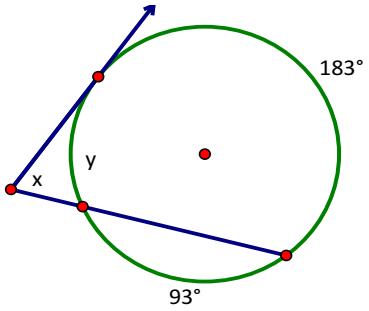




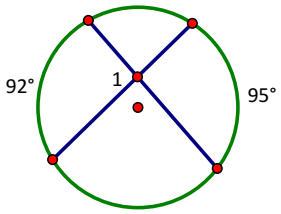
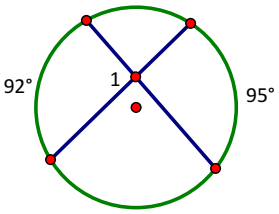
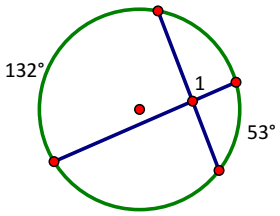
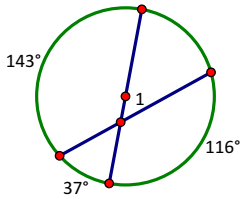
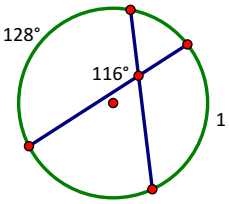
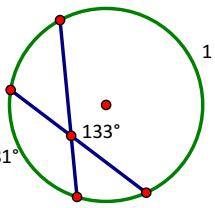
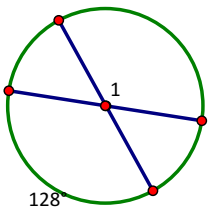
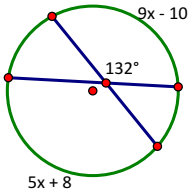




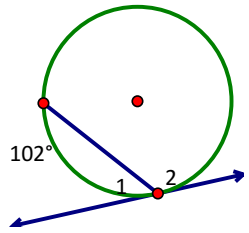




Practice

1.  m∠1 = _____	2.  m∠1 = _____	3.  m∠1 = _____	4.  m∠1 = _____
5.  *m*1 = _____	6.  *m*1 = _____	7.  m∠1 = _____	8.  x = _____

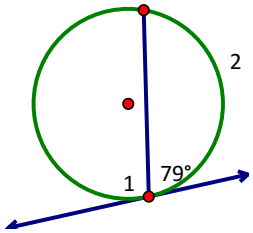
9.



m∠1 = _____

m∠2 = _____

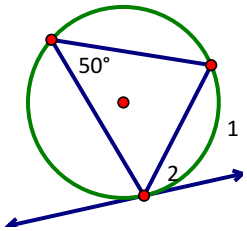
10.



m∠1 = _____

*m*2 = _____

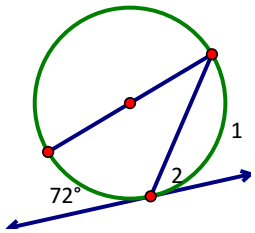
11.



*m*1 = _____

m∠2 = _____

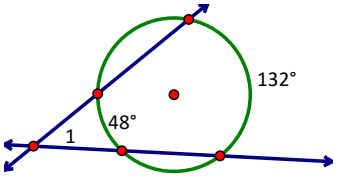
12.



*m*1 = _____

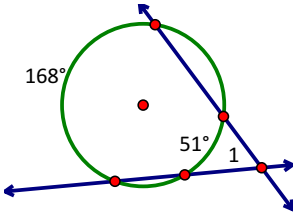
m∠2 = _____

13.



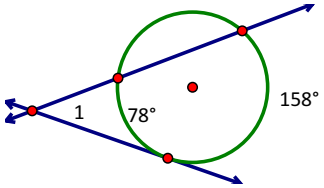
m∠1 = _____

14.



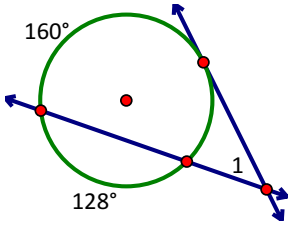
m∠1 = _____

15.



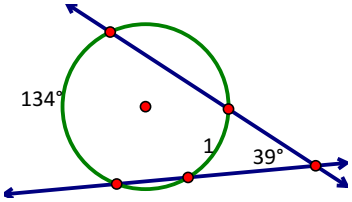
m∠1 = _____

16.



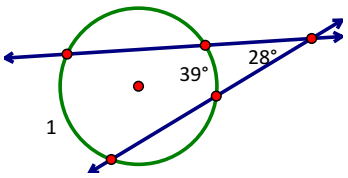
m∠1 = _____

17.



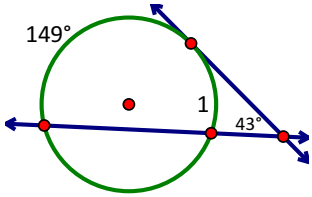
*m*1 = _____

18.



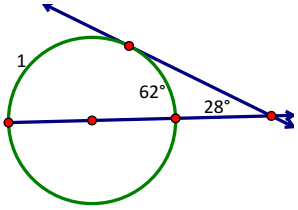
*m*1 = _____

19.



*m*1 = _____

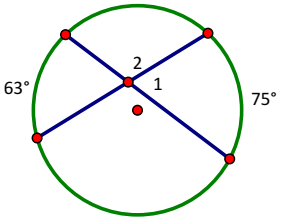
20.



*m*1 = _____

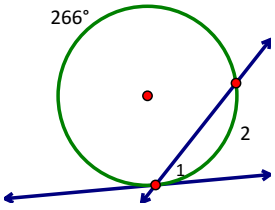
Extra Practice

1.



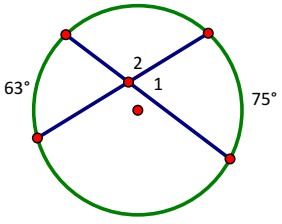
m∠1 = _____ m∠2 = _____

2.



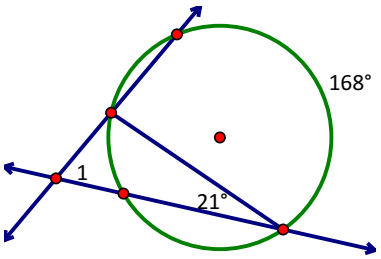
m∠1 = _____ *m*2 = _____

3.



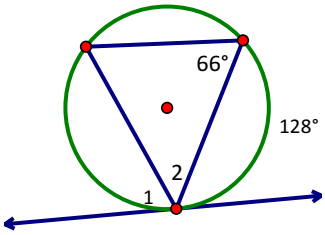
m∠1 = _____ m∠2 = _____

4.



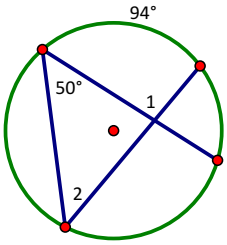
m∠1 = _____

5.



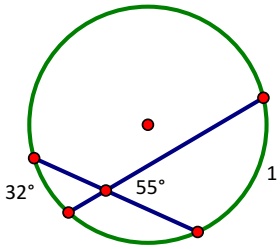
m∠1 = _____ m∠2 = _____

6.



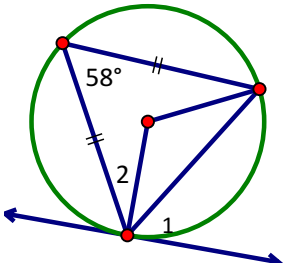
m∠1 = _____ m∠2 = _____

7.



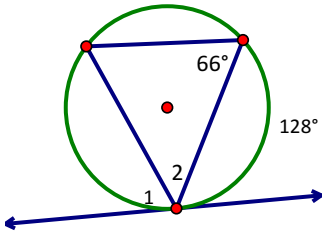
*m*1 = _____

8.



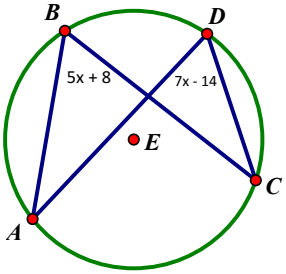
m∠1 = _____ m∠2 = _____

9.



m∠1 = _____ *m*2 = _____

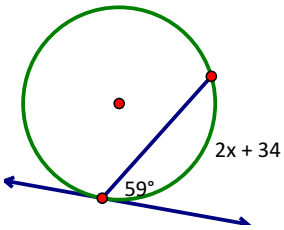
10.



x = _____

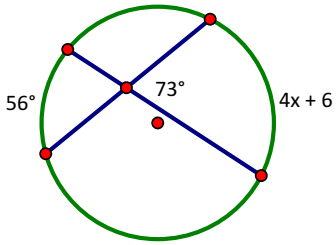
m∠ABC = _____

11.



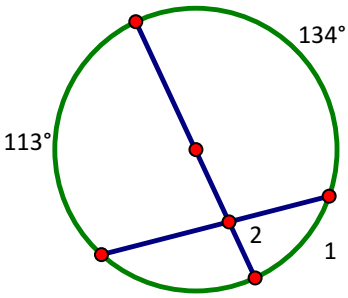
x = _____

12.



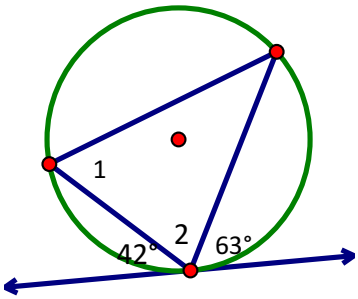
x = _____

13.



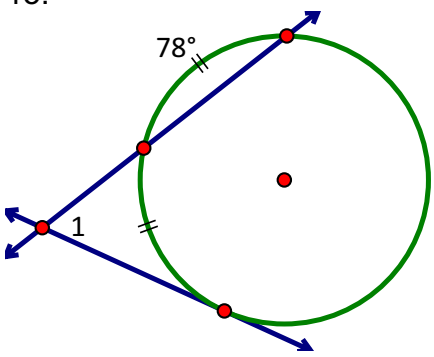
*m*1 = _____ m∠2 = _____

14.



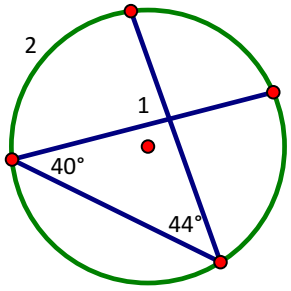
m∠1 = _____ m∠2 = _____

15.



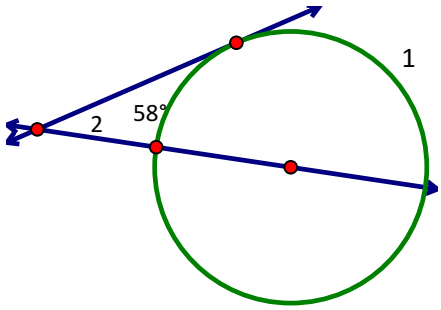
m∠1 = _____

16.



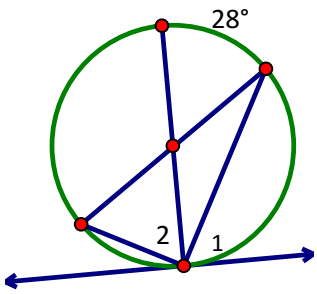
m∠1 = _____ $m2$ = _____

17.



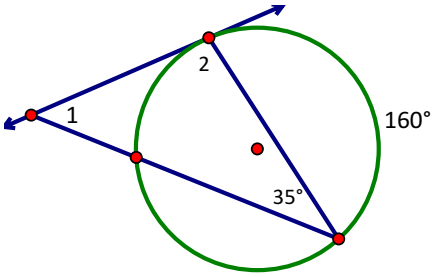
$m1$ = _____ m∠2 = _____

18.



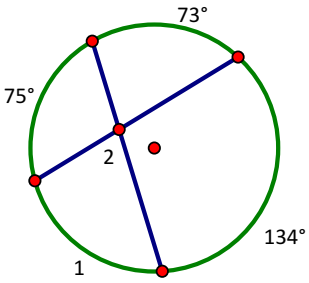
m∠1 = _____ m∠2 = _____

19.



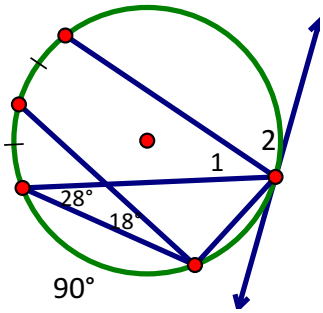
m∠1 = _____ m∠2 = _____

20.



$m1$ = _____ m∠2 = _____

21.



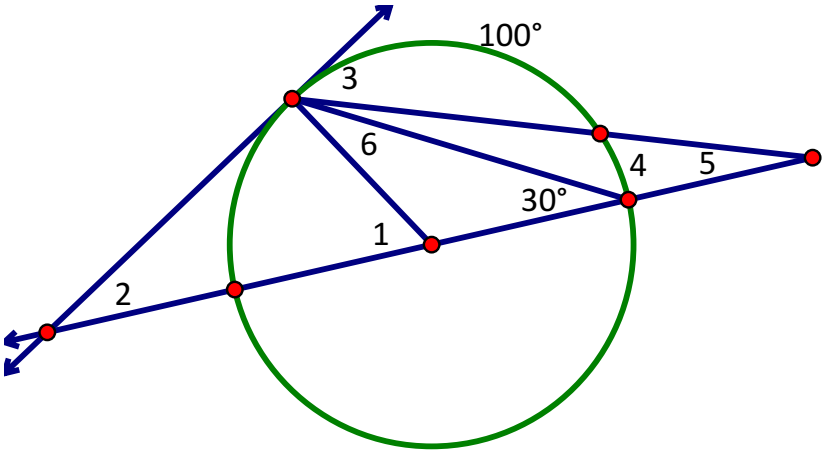
m∠1 = _____ m∠2 = _____

22. Solve for the missing values.

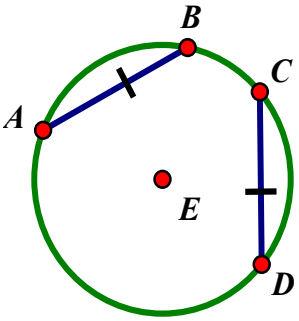
a) m∠1 = _____ b) m∠2 = _____

c) m∠3 = _____ d) $m4$ = _____

e) m∠5 = _____ f) m∠6 = _____

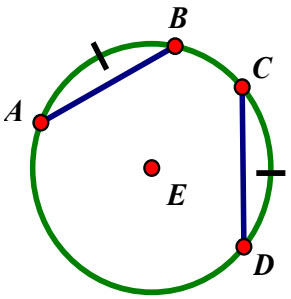


Congruent Chords/Arcs



If....
$\overline{AB} \cong \overline{CD}$ $AB = CD$

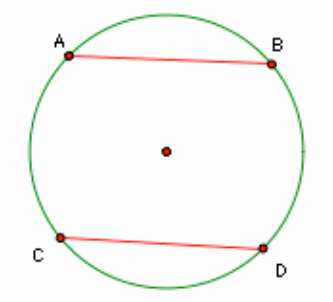
Then....
$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ $m\overset{\frown}{AB} = m\overset{\frown}{CD}$



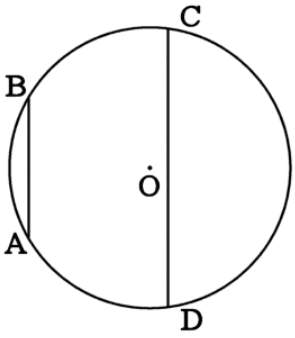
If.....
$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ $m\overset{\frown}{AB} = m\overset{\frown}{CD}$

Then....
$\overline{AB} \cong \overline{CD}$ $AB = CD$

Parallel Chords

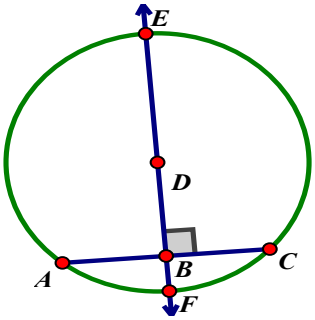


If
$\overline{AB} \parallel \overline{CD}$ $AB \parallel CD$
Then
$AC \cong BD$ $mAC \cong mBD$

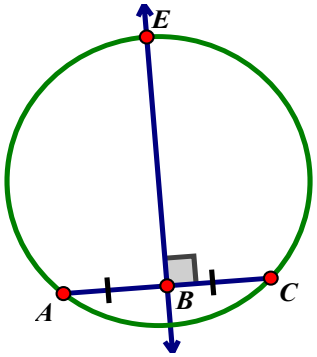


If
$AC \cong BD$ $mAC \cong mBD$
Then
$\overline{AB} \parallel \overline{CD}$ $AB \parallel CD$

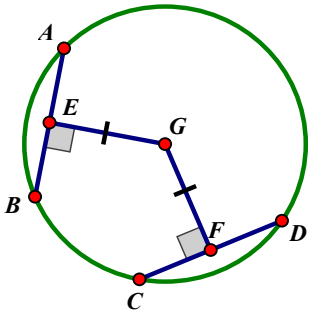
Chords and Diameters/Radii



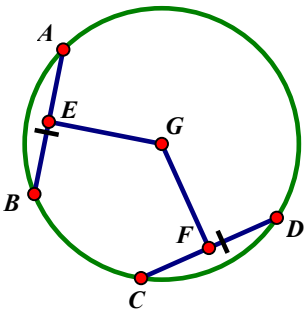
If diameter $\overline{BE}$ is $\perp$ to a chord,
then $\overline{AB} \cong \overline{BC}$ $(AB = BC)$
And $\overset{\frown}{AF} \cong \overset{\frown}{FC}$ $(m\overset{\frown}{AF} = m\overset{\frown}{FC})$



If $\overline{BE}$ is the perpendicular bisector of $\overline{AC}$, then $\overline{BE}$ is a diameter.



If $\overline{GE} \cong \overline{GF}$ and
$\overline{GE} \perp \overline{AB}$ and
$\overline{GF} \perp \overline{CD}$,
then
$\overline{AB} \cong \overline{CD}$

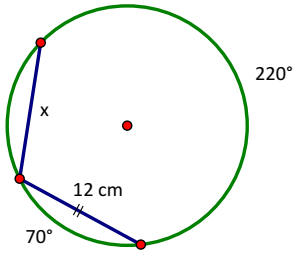


If $\overline{AB} \cong \overline{CD}$

Then

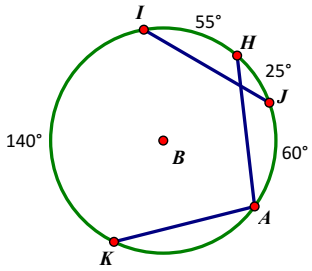
$\overline{GE} \cong \overline{GF}$

1.



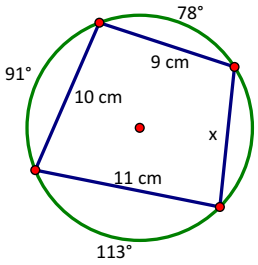
x = _____

2.



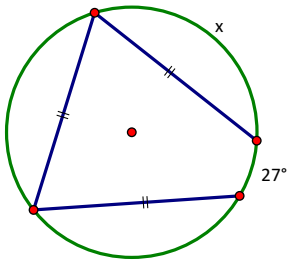
AK = _____

3.



x = _____

4.



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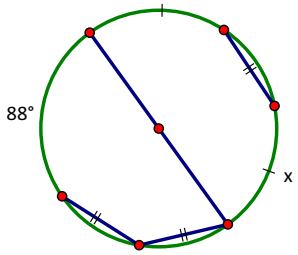
5.



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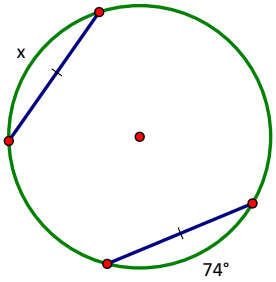
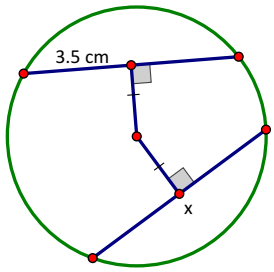


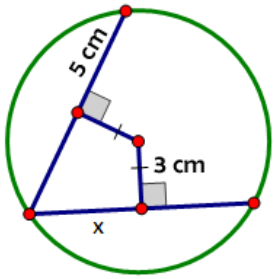
x = _____

7.



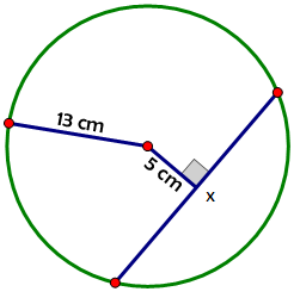
x = _____

8.



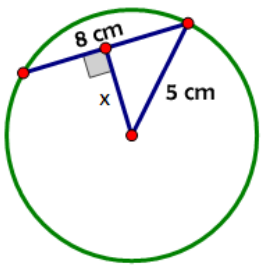
x = _____

9.



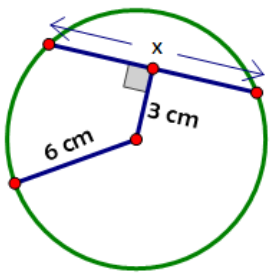
x = _____

10.



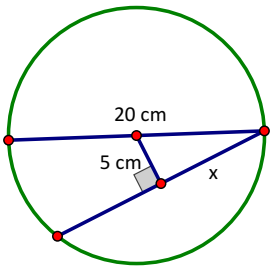
x = _____

11.



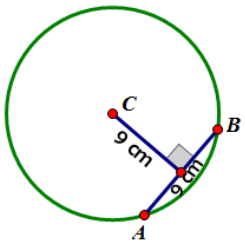
x = _____

12.



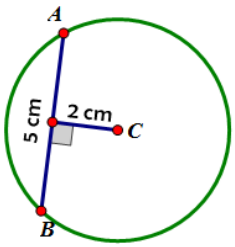
x = _____

13.



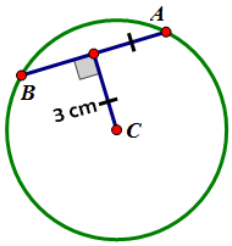
AC = _____(2 dec.)

14.



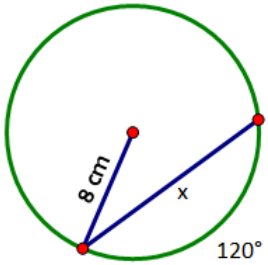
AC = _____(2 dec.)

15.



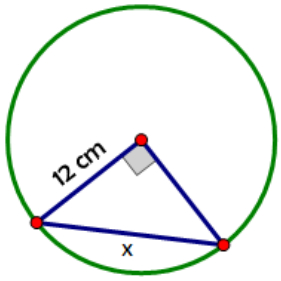
AC = _____(E)

16.



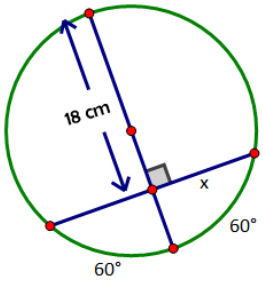
x = _____(E)

17.



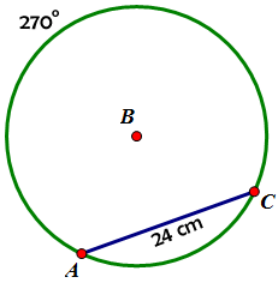
x = _____(E)

18.



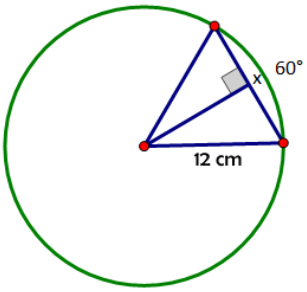
x = _____(E)

19.



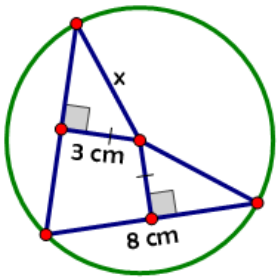
AB = _____(E)

20.

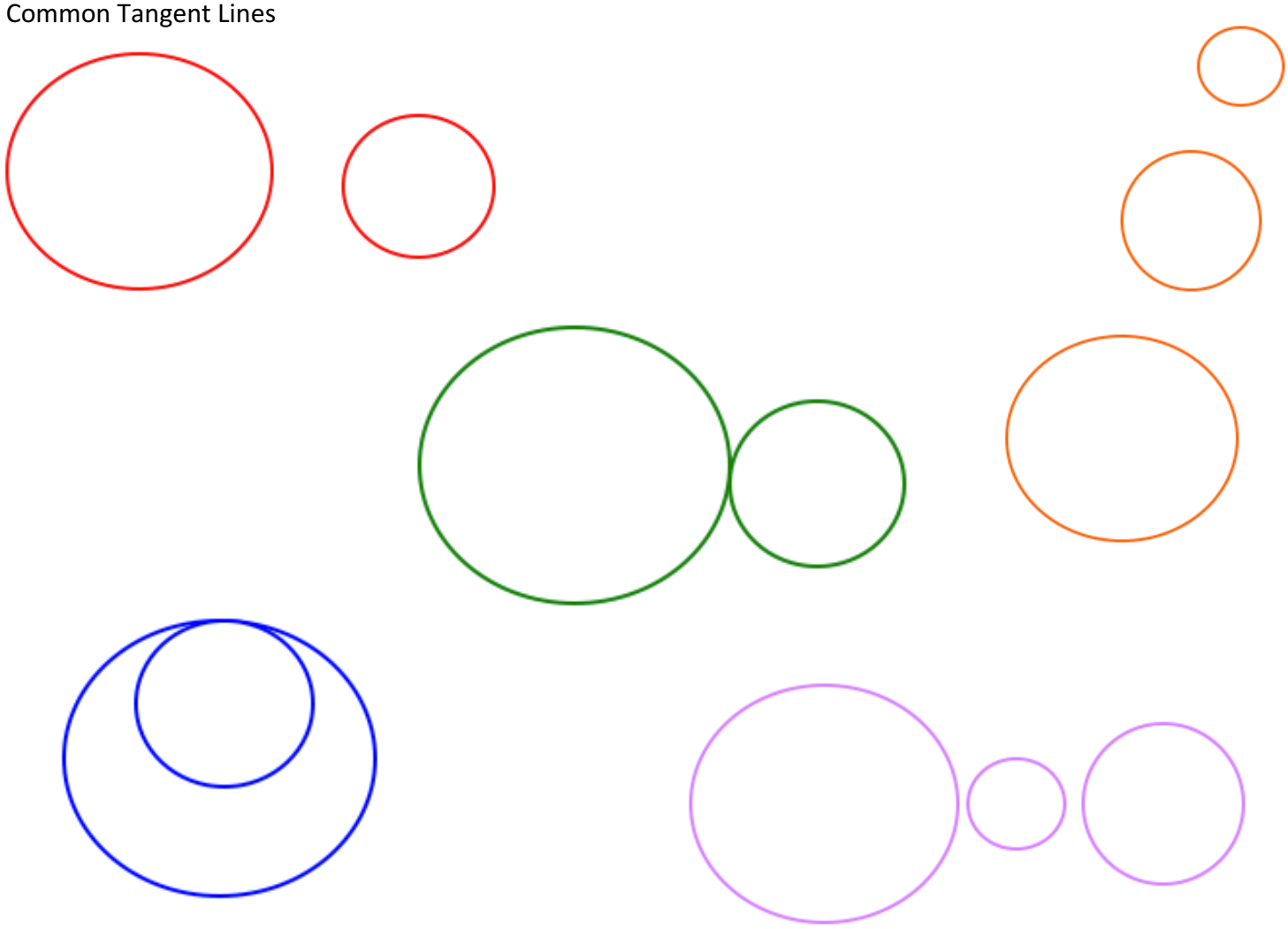


x = _____

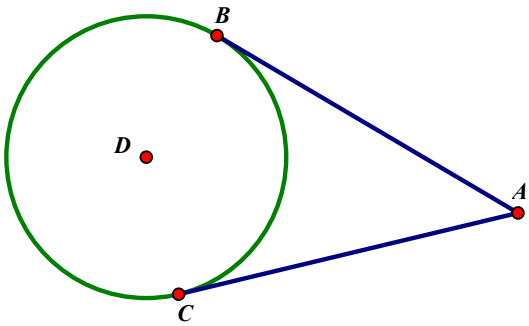
21.



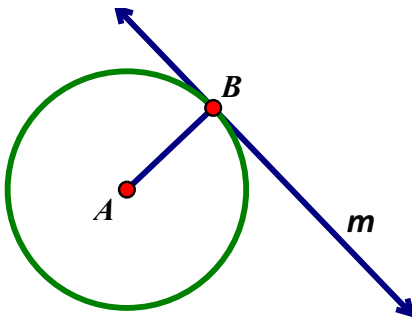
x = _____(E)



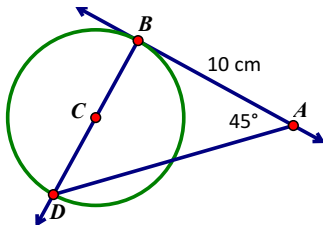
Two Tangents



Tangent to a Radius/Diameter

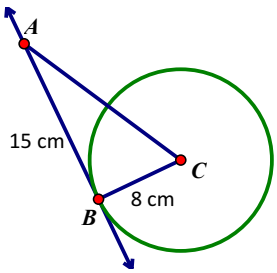


1.



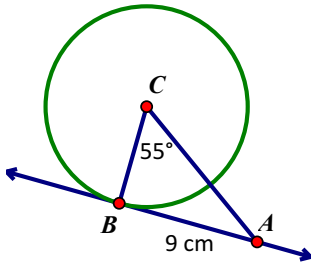
CB = _____

2.



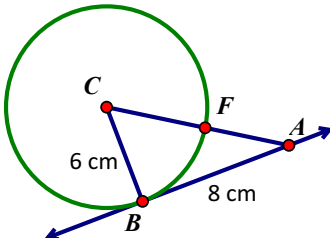
AC = _____

3.



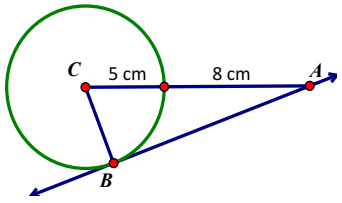
CB = _____

4.



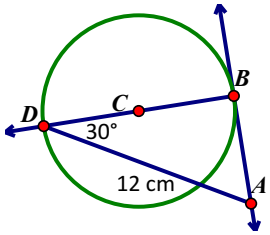
FA = _____

5.



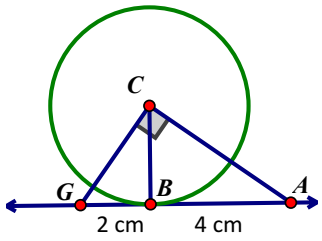
AB = _____

6.



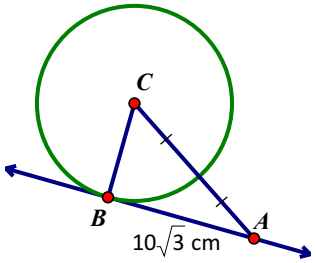
CB = _____ (E)

7.



CB = _____ (2 dec)

8.

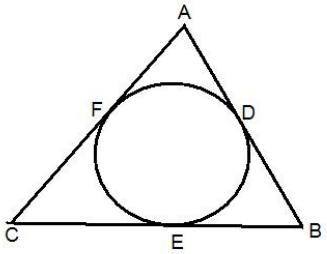


CB = _____

9. Triangle ABC is circumscribed about a circle, and D, E, and F are points of tangency. Let AD = 5, EB = 5, and CF = 10.

a) Find the lengths 0f AB, BC, and CA

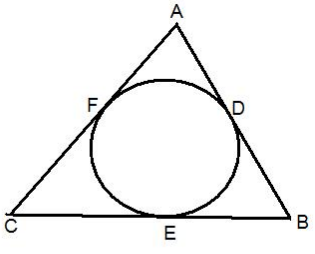
b) Show that ΔABC is isosceles.



10. Triangle ABC is circumscribed about a circle, and D, E, and F are points of tangency. Let AF = 10, CE = 20, and BD = 30.

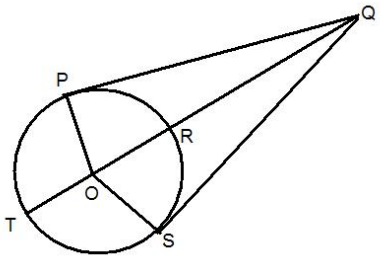
a) Find the lengths 0f AB, BC, and CA

b) Show that ΔABC is a right triangle.

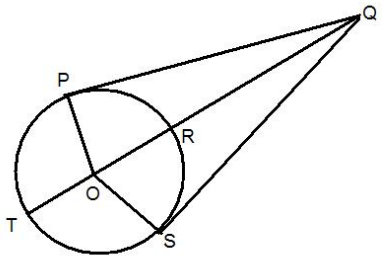


11. $\overline{PQ}$ is tangent to circle O at P, $\overline{ST}$ is tangent to circle O at S, and $\overline{OQ}$ intersect circle O at T and R. If OP = 15 and PQ = 20, find:

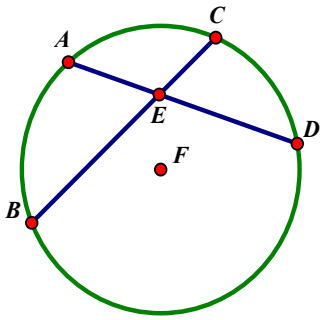
a) $\overline{OQ}$_____ b) $\overline{QS}$_____

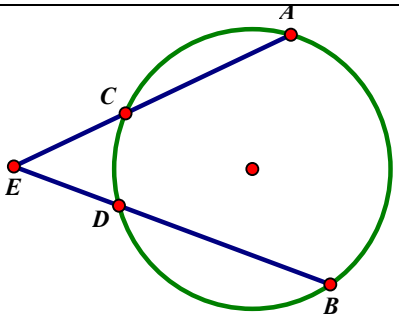


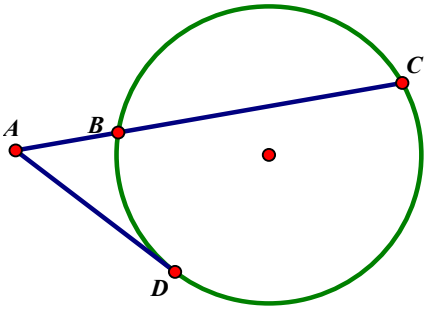
12. $\overline{PQ}$ is tangent to circle O at P, $\overline{ST}$ is tangent to circle O at S, and $\overline{OQ}$ intersect circle O at T and R. If OQ = 25 and PQ = 24, find PO.



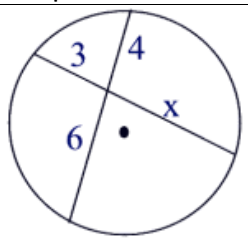
Segments Formed INDISE and OUTSIDE a Circle

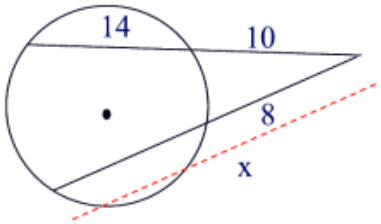


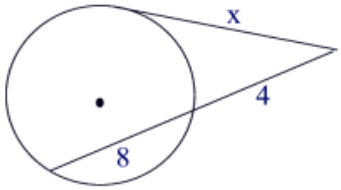




Examples

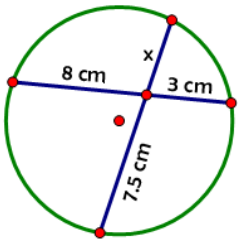






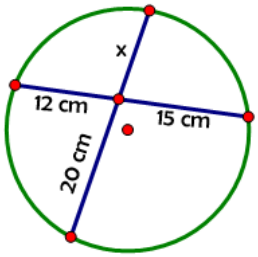
Practice

1.



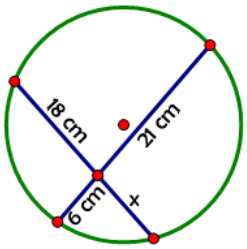
x = _____

2.



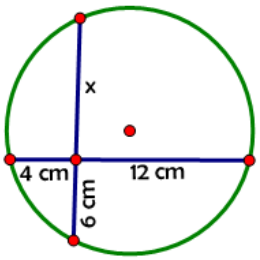
x = _____

3.



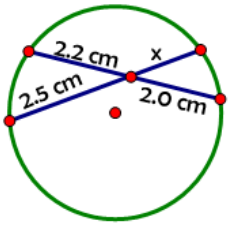
x = _____

4.



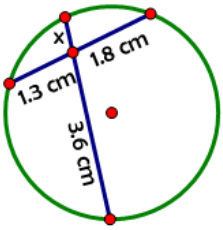
x = _____

5.



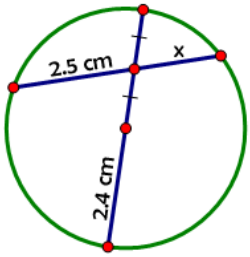
x = _____

6.



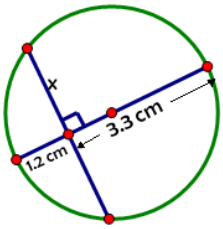
x = _____

7.



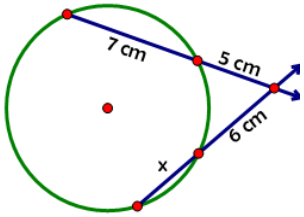
x = _____

8.



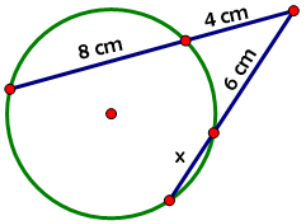
x = _____(2 dec.)

9.



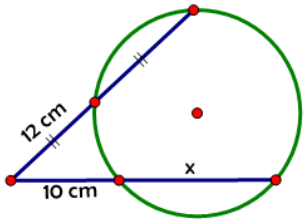
x = _____

10.

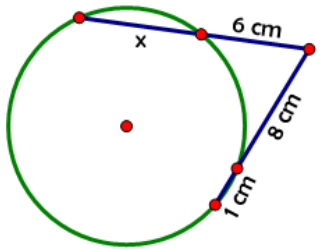
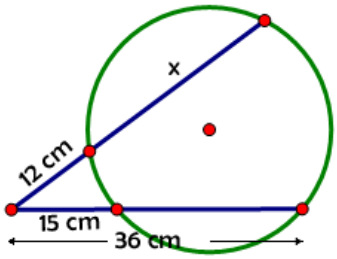
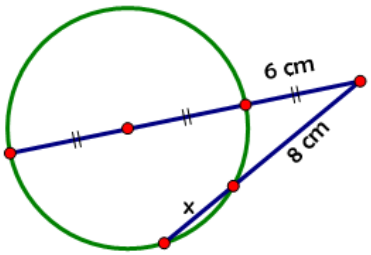
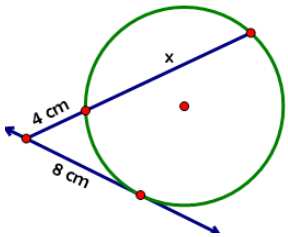
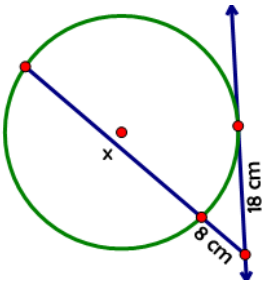
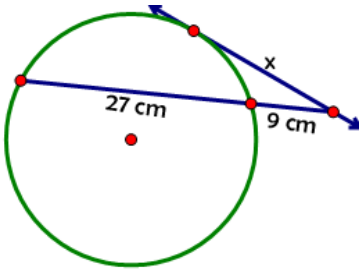


x = _____

11.



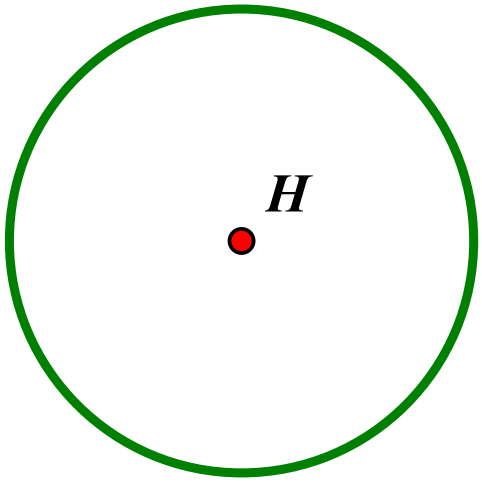
x = _____

12.  x = _____	13.  x = _____	14.  x = _____
15.  x = _____	16.  x = _____	17.  x = _____

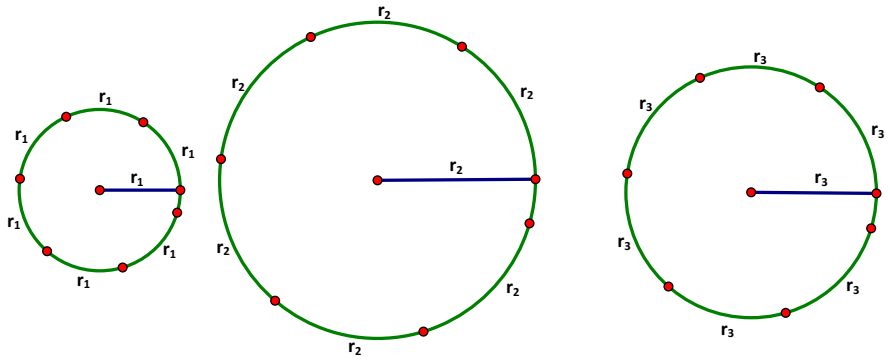
Radian Measure

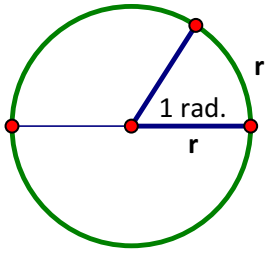
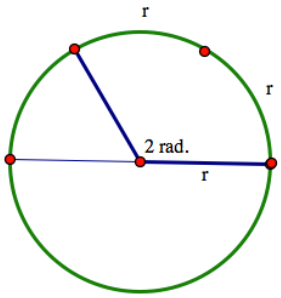
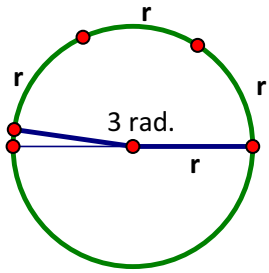
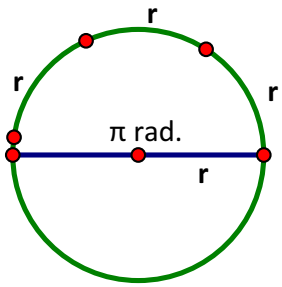
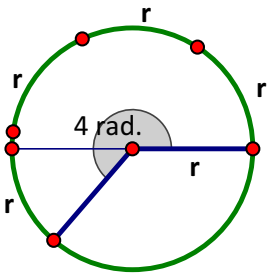
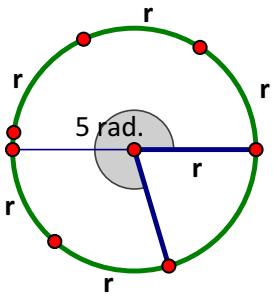
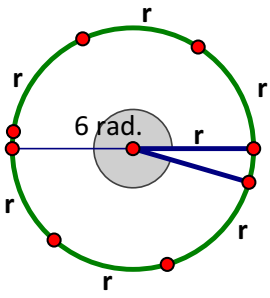
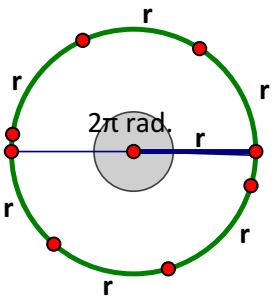
Do Now

About how many radii fit around a circle? (Hint: think back to an inscribed hexagon construction)



What does Radian mean?



1 Radian	2 Radians	3 Radians	π Radians
			
4 Radians	**5 Radians**	**6 Radians**	**2π Radians**
			

Converting Degree to Radian and Radian to Degree

Unit Conversions	Proportion

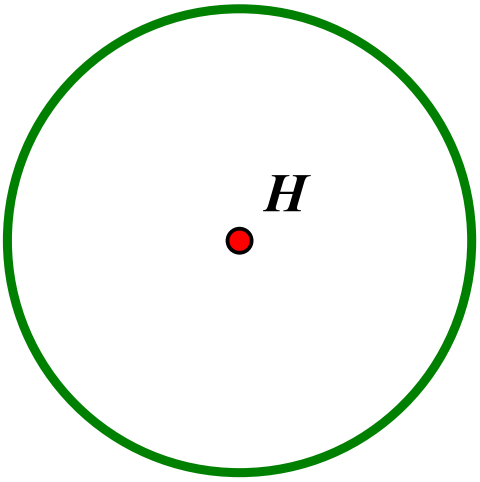
Example

1. Express in radian measure an angle of 75°.	2. Find the degree measure of an angle of $\frac{\pi}{4}$ rad.

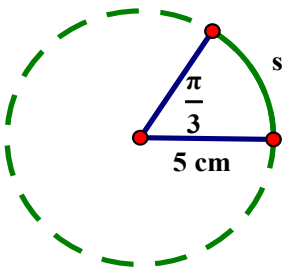
Practice

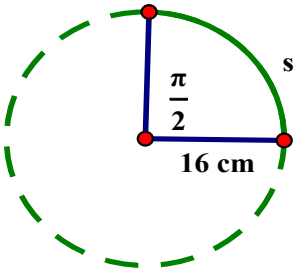
Find the radian measure or the degree measure of the following:				
1. 30^o	2. 90^o	3. 45^o	4. 120^o	5. 160^o
6. $\frac{\pi}{3}$	7. $\frac{\pi}{9}$	8. $\frac{\pi}{10}$	9. $\frac{2\pi}{5}$	10. $\frac{\pi}{2}$

Arc Length



Example





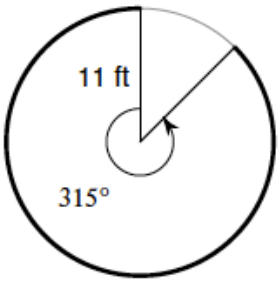
Practice

1. Central Angle of 30°, radius of 3 cm s = _____ (E)	2. Central Angle of 90°, radius of 8 cm s = _____ (E)	3. Central Angle of 72°, radius of 10 cm s = _____ (E)
4. Central Angle of $\frac{\pi}{4}$ *rad.*, radius of 12 cm s = _____ (E)	5. Central Angle of $\frac{2\pi}{3}$ *rad.*, radius of 15 cm s = _____ (E)	6. Central Angle of $\frac{4\pi}{5}$ *rad.*, radius of 10 cm s = _____ (E)

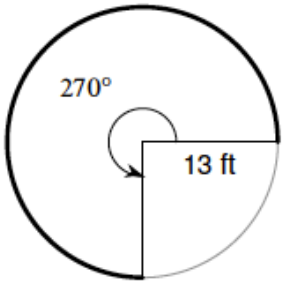
Extra Practice

Determine the arc length of the following circles.

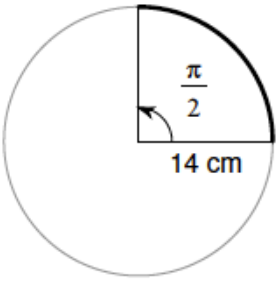
1.



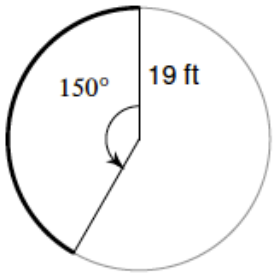
2.



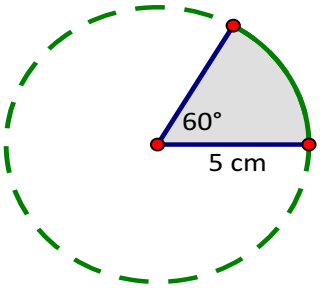
3.

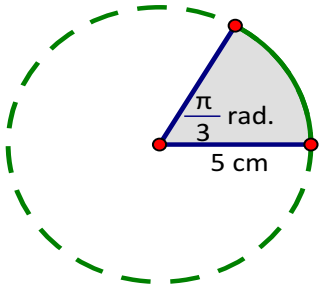


4.

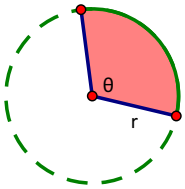
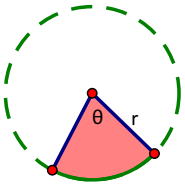
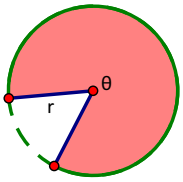


Area of a Sector



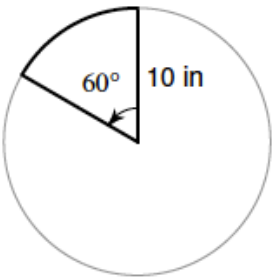


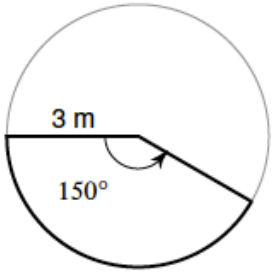
Practice

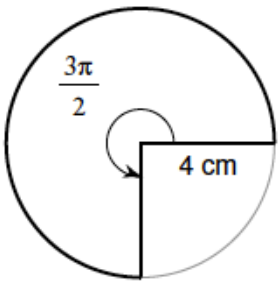
1. r = 10 cm, Θ = 2 rad.  Area = _____	2. $A = 6\pi$ cm^2, $\Theta = \frac{\pi}{3}$ rad.  r = _____	3. r = 3 cm, Θ = 300°  Area = _____

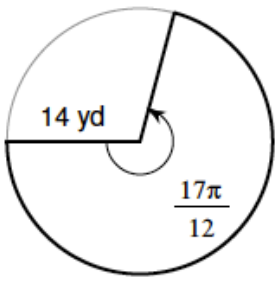
Extra Practice

Determine the area of the following sectors.

1. 

2. 

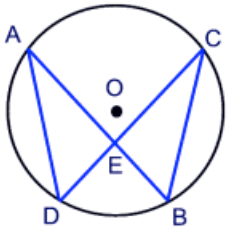
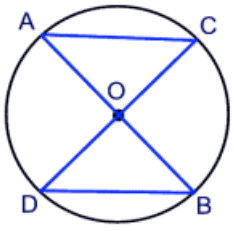
3. 

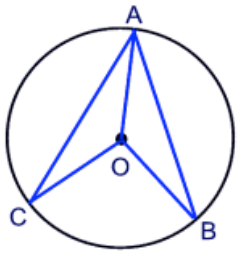
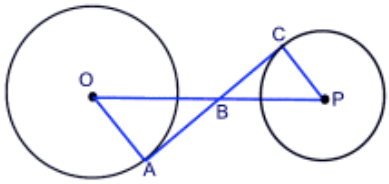
4. 

Proofs

Radius	1) In a circle, a radius perpendicular to a chord bisects the chord and the arc. 2) In a circle, a radius that bisects a chord is perpendicular to the chord. 3) In a circle, the perpendicular bisector of a chord passes through the center of the circle. 4) If a line is tangent to a circle, it is perpendicular to the radius draw to the point of tangency
Chords	5) In a circle, congruent chords are equidistant from the center. 6) In a circle, congruent chords have congruent arcs. 7) In a circle, congruent arcs have congruent chords. 8) In a circle, parallel chords intercept congruent arcs. 9) In a circle, congruent central angles have congruent chords.
Tangents	10) Tangents segments to a circle from the same external point are congruent.
Arcs	11) In a circle, congruent central angles have congruent arcs.
Angles	12) An inscribed in a semi-circle is a right angle. 13) The opposite angles of a quadrilateral are supplement.

Examples

1. 	Given: Chords $\overline{AB}$, $\overline{CD}$, $\overline{AD}$, and $\overline{CB}$ Prove: AE•EB = CE•ED
Statement	Reasons
2. 	Given: Diameters $\overline{AB}$ and $\overline{CD}$ Prove: $\widehat{AC} \cong \widehat{BD}$
Statement	Reasons

3. 	Given: $\overline{AB} \cong \overline{AC}$ Prove: $\Delta AOC \cong \Delta AOB$
Statement	Reasons
4. 	Given: Tangent $\overline{AC}$ Prove: $\angle O \cong \angle P$
Statement	Reasons