Geometry
Unit 6: Similarity and Trigonometry

Time Frame: Approximately four weeks

Unit Description

This unit addresses the measurement side of the similarity relationship which is extended to the Pythagorean theorem, its converse, and their applications. The three basic trigonometric relationships are defined and applied to right triangle situations.

Student Understandings

Students apply their knowledge of similar triangles to finding the missing measures of sides of similar triangles, and to using the Pythagorean theorem to find the length of missing sides in a right triangle. The converse of the Pythagorean theorem is used to determine whether a given triangle is a right, acute, or obtuse triangle. Students can use *sine*, *cosine*, and *tangent* to find lengths of sides or measures of angles in right triangles and the relationship to similarity.

Guiding Questions

1. Can students use proportions to find the lengths of missing sides of similar triangles?
2. Can students use similar triangles and other properties to prove and apply the Pythagorean theorem and its converse?
3. Can students relate trigonometric ratio use to knowledge of similar triangles?
4. Can students use *sine, cosine*, and *tangent* to find the measures of missing sides or angle measures in a right triangle?

Unit 6 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
Number and Number Relations	
3.	Define *sine*, *cosine*, and *tangent* in ratio form and calculate them using technology (N-6-H)
4.	Use ratios and proportional reasoning to solve a variety of real-life problems including similar figures and scale drawings (N-6-H) (M-4-H)
Measurement	
8.	Model and use trigonometric ratios to solve problems involving right triangles (M-4-H) (N-6-H)
Geometry	
9.	Construct 2- and 3-dimensional figures when given the name, description, or attributes, with and without technology (G-1-H)
12.	Apply the Pythagorean theorem in both abstract and real-life settings (G-2-H)
13.	Solve problems and determine measurements involving chords, radii, arcs, angles, secants, and tangents of a circle (G-2-H)
18.	Determine angle measures and side lengths of right and similar triangles using trigonometric ratios and properties of similarity, including congruence (G-5-H) (M-4-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)
CCSS for Mathematical Content	
CCSS #	**CCSS Text**
Similarity, Right Triangles, and Trigonometry	
G.SRT.7	Explain and use the relationship between the sine and cosine of complementary angles.
Expressing Geometric Properties with Equations	
G.GPE.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
G.GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
Modeling with Geometry	
G.MG.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
ELA CCSS	
CCSS #	**CCSS Text**
Reading Standards for Literacy in Science and Technical Subjects 6-12	
RST.9-10.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects 6-12	

WHST.9-10.2d	Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers.
WHST.9-10.10	Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

Sample Activities

Activity 1: Striking Similarity (GLE: 4; CCSS: WHST.9-10.10)

Materials List: pencil, paper, grid paper, ruler, protractor, Striking Similarity BLM

Have students work in pairs. Each pair should be given a copy of the Striking Similarity BLM. Students should also be given a piece of grid paper which has squares sized differently than the BLM (larger or smaller) but that has the same number of squares. For instance, on the BLM, the grid is 8 x 10 and each square is 1 centimeter. The students should be given a section of grid paper that also has 8 squares by 10 squares but the squares are a different size. Students will then reproduce the shapes on the blank grid by drawing the segments in the corresponding squares on the blank grid. Once students have enlarged or reduced the figures (The students should not reduce the figures too much), have students measure the segments and angles of both the original drawing on the BLM and the new drawing. Have students participate in a discussion that describes the relationship between pairs of corresponding angles and segments in the original and enlarged/reduced figures. Remind students about the information obtained in the previous unit on corresponding sides and angles of similar triangles and have the students develop a definition for similar figures (similar figures are figures in where all corresponding angles are congruent and corresponding sides are proportional).

Have students complete *RAFT writing* (view literacy strategy descriptions) to apply their knowledge of similar figures. This form of writing gives students the freedom to project themselves into unique roles and look at content from unique perspectives. From these roles and perspectives, *RAFT writing* is used to explain processes, describe a point of view, envision a potential job or assignment, or solve a problem. This kind of writing should be creative *and* informative.

Students should write the following *RAFT*:

R – Role—the role of the writer is a regular polygon like an equilateral triangle, a square, or some other regular polygon (the polygon can be assigned to each student or chosen by the student).

A – Audience—the regular polygon will be writing to other polygons in their family. For instance, equilateral triangles should write to scalene triangles or non-equilateral isosceles triangles; squares should write to non-square rectangles, non-square parallelograms, trapezoids, etc.; other polygons should write to non-regular polygons in their same family.
F – Form—the form of this writing is a letter
T – Topic—the focus of this writing is to explain why the regular polygon cannot be the non-regular polygon's partner because they are not similar.

In their *RAFTed* letters, students should include the definition of similar figures and an explanation of why the two figures are not similar. They can include drawings to help their explanation if they choose. These writings may be shared with the class or included in their portfolios. Students should listen for accuracy and logic in the *RAFT*s and be invited to ask questions. Clarify misconceptions and correct inaccurate content as appropriate.

Activity 2: Similarity and Ratios (GLE: 4; CCSS: RST.9-10.3, WHST.9-10.2d, WHST.9-10.10)

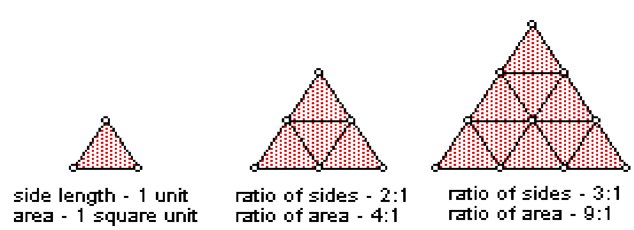
Materials List: pencil, paper, pattern blocks, Similarity and Ratios BLM, centimeter cubes or sugar cubes, *learning logs*

In groups of three or four, instruct students to use equilateral triangle pattern blocks and cubes to make generalizations about the ratios of sides, areas, and volumes in similar figures, using an activity like the one below. Give each student a copy of the Similarity and Ratios BLM so he/she can follow the directions and answer the questions that follow.

- Given an equilateral triangle, use pattern blocks to create a similar triangle so the ratio of side lengths is 2:1. If there are not enough pattern blocks for each group to create the correct triangle, have students trace the pattern blocks to create the similar triangles. Ask: "What is the ratio of areas of the two similar triangles?" Next, have students use pattern blocks to create a triangle similar to the original triangle so the ratio of side lengths is 3:1. Ask: "What is the ratio of the areas of these two similar triangles?"

  The sketches which follow are not included in the BLM but are provided here to illustrate what the students should be creating at their desks as they are working through the BLM.

Sample sketches:



- Have students use other pattern block shapes to investigate other similar polygons in the same manner as described above and record their findings in the tables provided on the Similarity and Ratios BLM. An example of the table is provided below for easy reference. The easiest pattern blocks to use would be parallelograms and rhombi. This can be accomplished using hexagons as well. Students can use triangles and rhombi to fill in "empty" space and to know how the area of the rectangles and rhombi relate to the area of the hexagon. Prior to completing this activity in class, "experiment" with other types of polygons to determine what obstacles students may encounter.

description of similar shapes	ratio of sides	ratio of areas
.	.	.
.	.	.
.	.	.

- Have students form generalizations based on their investigations in the two activities and have them answer the following question: If the ratio of sides of two similar polygons is *n*:1, what would the ratio of areas be?
- Given a cube, have students create a similar cube with a ratio of edges 2:1 using cm or sugar cubes. What is the ratio of volumes? Then have students create a similar cube with ratio of edges 3:1. What is the ratio of volumes? Have students record their observations in the tables on the Similarity and Ratios BLM and use their observations to answer the question, "If the edges of two cubes were in a ratio of *n*:1, what would the ratio of volumes be?" An example of the table is provided below.

description of similar 3-D shapes	ratio of edges	ratio of volumes
.	.	.
.	.	.
.	.	.
.	.	.

At the completion of this activity, have students answer the following prompt in their math *learning logs* (view literacy strategy descriptions)**:**

> Using what you have learned about the relationships between the ratio of the sides and the ratio of the areas of similar figures, determine the relationship between the ratio of the sides and the ratio of the perimeters of those same similar figures. Be sure to explain your reasoning and provide examples/calculations to aid your explanation.

A *learning log* is a notebook students keep in order to record ideas, questions, reactions, and new understandings. Students should use their math *learning logs* other times in class in addition to those listed throughout the curriculum. This will provide opportunities to demonstrate understanding. Have a few students share their responses with the class and lead a discussion about the validity of their responses.

Activity 3: Applying Similar Figures (GLEs: 4, <u>18</u>; CCSS G.MG.1)

Materials List: pencil, paper, calculator

Note: This activity was not changed as it already addressed the required modeling in the listed CCSS.

Give students various real-life situations in which similar triangles are used to find missing measures (i.e. shadow problems, distance across a river, width of a lake). The types of triangles should vary. Have students discuss why the triangles are similar before finding the requested missing measures. Provide students with practice in determining the missing sides of other pairs of similar figures in real-life settings.

Example:

> Alex is having a snapshot of his grandparents enlarged. The original snapshot is 4 inches by 6 inches. He needs the enlarged photo to be at least 13 inches on the shortest side. What must the minimum length be of the longer side?
> *Solution: 19.5 in.*

Activity 4: Proportional Parts of Triangles (GLEs: <u>4</u>, 19; CCSS: G.GPE.6, <u>G.GPE.7</u>, <u>G.MG.1</u>)

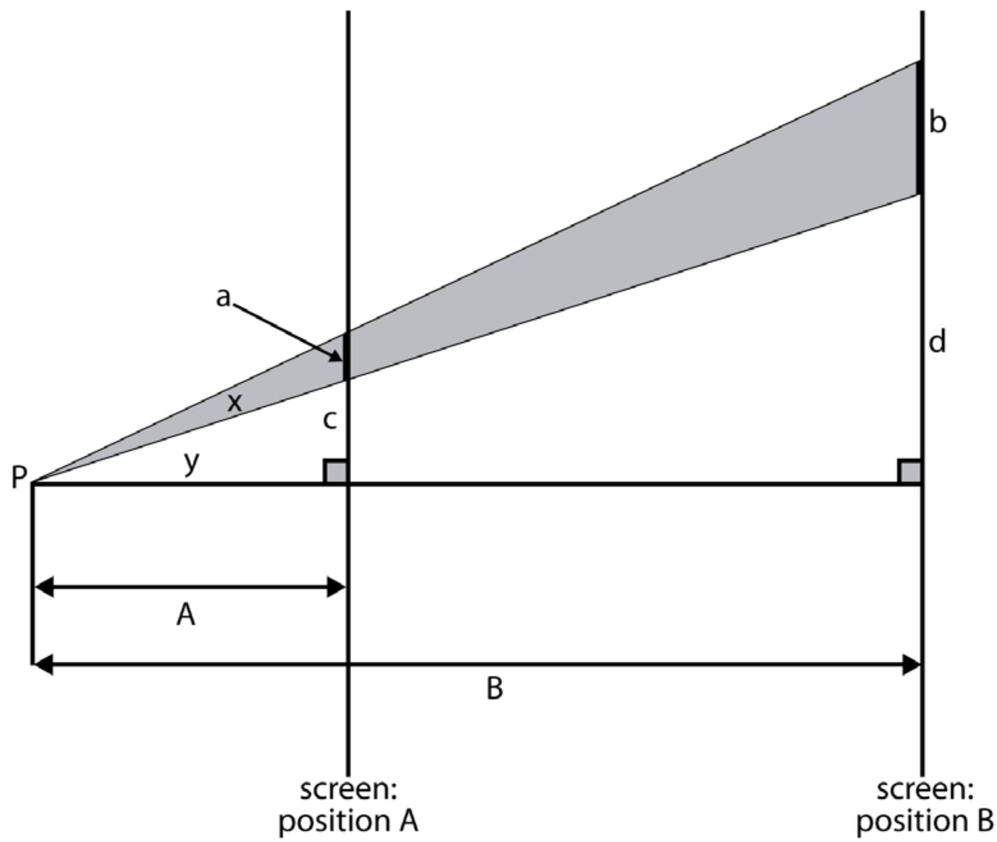
Materials List: pencil, paper, graph paper, Spotlight on Similarity BLM, overhead projector

Begin this lesson by having students draw $\triangle ABC$ with vertices $A(-5,-3), B(-1,5),$ and $C(11,-3)$. Then, have students find the location of point *D* on $\overline{AB}$ so that it is $\frac{3}{4}$ of the distance of *AB* from *B*; students also need to find point *E* on $\overline{CB}$ so that it is $\frac{3}{4}$ of the distance of *CB* from *B*. Have students connect points *D* and *E*. *Solution:* $D(-4,-1)$ and $E\ (8,-1)$. Ask students, "What do you know about the measures *BD, AD, BE,* and *BC*?" Students should see that the ratio $\frac{BD}{AD}$ is equal to the ratio $\frac{BE}{CE}$. Ask students to make some observations about $\overline{DE}$ in relation to $\overline{AC}$. Students should see that the segments are parallel (have students justify this using slope). Then, ask students to make a conjecture about the relationship between $\triangle ABC$ and $\triangle DBE$. Students should conjecture that the two triangles are similar, and they should be able to justify their conjecture using the methods from Unit 5. They may also find *DE* and *AC* to assist with their justifications that the triangles are similar. Discuss with students how they know that the corresponding angles are congruent using facts about parallel lines and the reflexive property of congruence. Next, have students find the perimeters of the two triangles and make observations about the relationship between the perimeters (ratio of the perimeter of $\triangle DBE$ to the perimeter of $\triangle ABC$ is the same as the ratio discussed earlier).

Next, give students copies of other diagrams that have a line segment drawn so that it is intersecting two sides and is parallel to the third side. Have students conjecture what is true about all pairs of triangles they have been given. After discussing with students that the diagrams all have pairs of similar triangles, have them state the Side-Splitter Theorem—if a segment intersects two sides of a triangle and is parallel to the third side, then the segment divides the sides proportionally. Students can also add why it makes the two triangles similar.

Then use the Spotlight on Similarity BLM to make a transparency for the overhead (or a copy can be made for each student) to help students investigate the following problem:

- A spotlight at point P throws out a beam of light.
- The light shines on a screen that can be moved closer to or farther from the light. The screen at position A is a horizontal distance A from the light and at position B is a horizontal distance B.
- The lengths *a* and *b* indicate the lengths of the light patch on the screen.



- Show that the ratio of the length *b* to the length *a* depends only on the distances A and B, and not on the measure of angle *y* of the beam to the perpendicular, nor on the measure of angle *x* of the beam itself.

Use similar triangle relationships, $\frac{a+c}{A}=\frac{b+d}{B}$. This means $\frac{a}{A}+\frac{c}{A}=\frac{b}{B}+\frac{d}{B}$. and $\frac{c}{A}=\frac{d}{B}$ because the triangles are similar. Hence, $\frac{a}{A}=\frac{b}{B}$, which is equivalent to $\frac{b}{a}=\frac{B}{A}$. The ratio $\frac{B}{A}$ is independent of the angles *x* and *y* and is the *scale factor* relating the distances of the two screens and the sizes of the images on the two screens.

Also, introduce students to the corollary of the Side-Splitter Theorem: When three parallel lines intersect two transversals, then the segments intercepted on the transversal are proportional. Have students develop a proof of the corollary based on the Side-Splitter Theorem and similar triangles.

Once students develop an understanding of the term scale factor, the Side-Splitter Theorem, and the corollary, give students diagrams to practice finding measures of the proportional parts of similar triangles and proportional parts of parallel lines.

Activity 5: Midsegment Theorem for Triangles (GLEs: 4, 18, 19)

Materials List: pencil, tracing or patty paper, scissors, ruler, protractors

Separate the class into groups of four. Give each group a sheet of tracing paper (or patty paper) and have each draw a triangle of any type and cut it out. Have students:

1. Find the midpoints of any two sides of the triangle by folding.
2. Fold (or draw) the segment that connects the two midpoints. Tell students that this is the midsegment and have them define the term based on what they have done so far.
3. Unfold the triangle and make any observations that seem to be true about the triangle and the midsegment. Students should use rulers and protractors to verify the observations they make concerning the midsegment of the triangle.
4. Fold and unfold the remaining two midsegments of the triangle.
5. Have students make observations regarding the three midsegments of the triangle. Ask them to look for a geometrical relationship between the midsegments and the sides, and to determine the numerical relationship between the lengths of the midsegments and the lengths of the sides. Each midsegment is parallel to one side of the triangle, and each midsegment has a length that is one-half the length of the parallel side.

If there is access to a computer drawing program such as *Geometer's Sketchpad*®, use it to construct the midsegments of several other triangles to determine if the observations made in the part above hold true.

Lead the class in a discussion which includes the use of similar triangles to prove the Midsegment theorem (the segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half its length). Ask students to discuss the relationship of the triangle formed by the three midsegments to the original triangle (i.e., the inner triangle is similar to the original and its perimeter is half the perimeter of the original triangle). Have students develop a proof of this relationship.

Activity 6: Parts of Similar Triangles (GLEs: 4)

Materials List: pencil, paper, automatic drawing program (optional), ruler, protractor, learning logs, expert attire (optional)

Students should investigate how the lengths of the special segments (altitude, median, angle bisector) in similar triangles relate to the measures of the sides of the similar triangles. Have them construct similar triangles, either with a drawing program or by hand, and draw the altitudes, angle bisectors, and medians. Instruct students to determine the scale factors of the sides and compare them to the ratios of the special segments. Have students refer to their math *learning log* (view literacy strategy descriptions) entries relating to the ratio of the perimeters of the similar triangles from Activity 2. Lead a class discussion to summarize that the ratios of the sides, altitudes, medians, angle bisectors, medians, and perimeters in similar figures are equal.

After the summary, use *professor know-it-all* (view literacy strategy descriptions) to review all of the concepts taught concerning similar figures. The *professor know-it-all* strategy allows students to question "experts" concerning a topic that has been studied through reading from a text, a lecture, a field trip, or any other information source. The only modification that may be needed is that students could be called "Math Masters" rather than *professors know-it-all* as high school students might be more receptive to the name. To implement the strategy, divide the class into groups of three or four. Give the students time to review the material covered in the last seven activities concerning similar figures. Tell the students, groups will be called on randomly to come to the front of the room and provide "expert" answers to questions from their peers about similar figures. Ask the groups to generate 3 – 5 questions about similar figures they think they might be asked or that they would like to ask other experts. Provide novelty items like ties, graduation caps, lab coats, clipboards, etc., to don when the students are the Math Masters.

After giving the students time to review material and create their questions, call a group to the front of the room and ask its members to face the class standing shoulder to shoulder. The Math Masters invite questions from the other groups. With the first question, model how the Math Masters should answer their peers' questions. Students should huddle together after each question to discuss and decide upon the answer; then have the spokesperson give the answer.

Direct the students to think carefully about the answer they receive and to challenge or correct the Math Masters if the answers are not correct or need additional information.

After 5 minutes or so, have a new group take its place as Math Masters and continue the process.

Some questions that might be asked are as follows:

- What is the definition of similar figures?
- What information must be provided to prove that two triangles are similar?
- Are all triangles similar?
- Are all quadrilaterals similar?
- Are squares similar to rectangles?
- How do the areas of similar figures relate to the scale factor of the figures?
- How do the perimeters of similar figures relate to the scale factor of the figures?
- How do the volumes of similar solids relate to the scale factor of the given solids?
- How does the measure of the midsegment of a triangle relate to the measure of the side?

The activity is complete when all groups have had a chance to be Math Masters or the peers have no new questions to ask the experts.

Activity 7: Pythagorean Theorem (GLEs: <u>12</u>, 19)

Materials List: pencil, paper

Provide students with a pair of similar right triangles whose leg measures are known. Ask students to determine if the triangles are similar and, if so, to provide a proof (i.e., the right angles are congruent and the legs in the two triangles are proportional) to support their ideas. Have students calculate the length of the hypotenuse of each triangle. Ask: "What is the scale factor between the two similar triangles? Is the hypotenuse of one triangle a multiple of the hypotenuse of the second triangle? What is the multiple?" Students should recognize and use common Pythagorean triples (e.g., 3-4-5, 5-12-13, 7-24-25, 8-15-17) and their multiples as shortcuts to solving problems. For example, if a right triangle has lengths of 15, ____, 39, the missing side is 36 since 15, 36, 39 is three times 5-12-13.

Activity 8: Application of the Converse of the Pythagorean Theorem (GLE: <u>12</u>)

Materials List: pencil, paper, automatic drawing program, protractors (if drawing program is unavailable)

In this activity, students should apply the converse of the Pythagorean theorem to determine if a triangle is right, acute, or obtuse. Begin by reviewing the converse of the Pythagorean theorem as a method of determining whether three segment measures could represent the measures of a right triangle. Then, give the students several different sets of measures that form a triangle (be sure that most of them are NOT right triangles). Have students apply the converse of the Pythagorean theorem to determine which of the trios forms a right triangle. Using a computer drawing program like *Geometer's Sketchpad*, have students construct triangles using side lengths that do not form right triangles to determine that some triangles are acute while others are obtuse. Have students make a conjecture about the sum of the squares of the smaller sides in relation to the square of the largest side, in both acute and obtuse triangles. Students should explain that if $a^2 + b^2 < c^2$, then the triangle is obtuse and if $a^2 + b^2 > c^2$, then the triangle is acute. Ask students to classify other triangles based only on the lengths of their sides.

If a drawing program is not available, provide students with diagrams in which the triangles have been drawn to scale and the lengths of the sides are labeled. Have students apply the Pythagorean theorem (or the rules concerning Pythagorean triples) to determine which triangles are right triangles. For the remaining triangles, have students use a protractor to measure angles, classify each triangle as acute or obtuse, and then determine the relationship between $a^2 + b^2$ and c^2 in the two types of triangles.

Activity 9: Discovering Trigonometry (Using Technology) (GLEs: 3, 8, 12)

Materials List: the Internet access for each student (or pairs of students), pencil, paper

Begin this activity by taking students through a *lesson impression* (view literacy strategy descriptions). *Lesson impressions* help create situational interest in the content by capitalizing on students' curiosity. In this activity, students will be given a list of words they will be exposed to throughout their work with trigonometry and asked to form a written impression of the content they will be learning prior to being exposed formally to the content. By asking students to form a written impression of the topic, they become eager to discover how closely their impression text matches the actual content.

Present students with the following list of words: trigonometry, relationships, angles, shadow, ladder, acute, ratios, hypotenuse, adjacent, angles of depression, and tangent.

Tell students they are to use the words to make a guess as to what will be covered in class. Students should be encouraged to write a paragraph (3-5 sentences) using the words. When students finish their *impression texts*, invite volunteers to present what they have written to the class. Allow several students to share their impressions in order to heighten the anticipation, leaving students to wonder whose impression is the closest to the actual content. Then present the content using the website described next. Direct students to keep track of the similarities and differences between the information presented and their *impression texts*. Students may wish to use a Venn diagram to track these similarities and differences with one circle's containing their ideas, the other circle's containing the actual information, and the overlap's containing the common ideas.

The website, http://catcode.com/trig/index.html, provides a series of activities that define and help explain the uses of trigonometry. The activities help students to expand their understandings of similar figures as they apply to the study of trigonometry. Only the first five activities and the activity titled "A Quick Review" should be viewed. Be sure that the computers students will be working on have Java enabled, so students can use the interactive activities. This cannot be printed because the interactivity with the figures will be lost. If necessary, students may be paired depending on class size and the number of available computers. If students do not have access to the Internet, present this information using presentation equipment. Create notes from the information presented on the website which students will be able to use for the remainder of the activity.

Employ a *directed learning-thinking activity (DL-TA)* (view literacy strategy descriptions) to aid students in reading and processing the information presented in the website. The *DL-TA* approach invites students to make predictions and check their predictions during and after the learning. It requires students to pause as they read/learn the information to ask/answer questions. Give students a copy of the *DL-TA* BLM and have them fill in the title "Discovering Trigonometry." Then take the students through the following steps:

- *Introduce background knowledge.* Begin the lesson with a discussion about trigonometry. Elicit information students may already know about trigonometry. Many students may have limited prior knowledge of trigonometry, and that is okay. Discuss the title of the activity. Record students' ideas on the board or chart paper.
- *Make predictions.* Ask questions that invite predictions, such as: "What do you expect to learn from this activity? Based on what we have learned already, what information do you think will be included?" Students should record the prediction questions in the proper place on the BLM. Have students write their predictions in the Before Reading box on the BLM. Students' predictions may include some ideas from the *lesson impression* they created at the beginning of the activity.
- *Read a section of text, stopping at predetermined places to check and revise predictions.* The first stopping point may be after students read the Frequently Asked Questions About Trigonometry. Students should reread their predictions and change them if they feel it is necessary. If they decide to change their predictions, they should cite the new evidence for doing so. Their new predictions should be recorded in the During Reading boxes on the BLM. Repeat this cycle as the students read through the information on trigonometry. Other recommended "stopping" points are after *Shadows and Triangle* (students should click on "See the difference here" to understand the effect of the moving sun on shadows), *Measuring the Sides, sine and cosine*, and *A Quick Review* (instruct the students to go back to the *Index* and click on *A Quick Review* to skip the other information at this time—the remaining information is too much to include at this point). Have students consider the following key questions: "What have you learned so far from the material?" (summarize) "Can you support your summary with evidence from the material?" "What do you expect to learn next?"
- *Once the exposure to the content is completed, use student predictions as a discussion tool.* Ask students to reflect on their original predictions and to track their changes in thinking and understanding trigonometry, as they confirmed or revised their predictions. Students should write their statements of overall understanding in the After Reading box on the BLM.

Following the discussion to summarize the learning, have students revisit the *lesson impression* from the beginning of the activity. Ask students if they have encountered every word given to them in order to form their *lesson impression*. They will not have encountered every term at this point. Have students discuss how close their *impressions* were based on what they have learned so far. Instruct students to keep their *impressions* available throughout the next 3 activities as they should see all of the terms throughout the remaining activities.

Activity 10: Special Right Triangles (GLEs: <u>3</u>, 12, <u>18</u>)

Materials List: pencil, paper, construction materials (compass and straightedge or patty paper), scientific calculator

Have students explore special right triangles by starting with an equilateral triangle with side lengths of 2 units. Have students construct an altitude to create two 30°-60°-90° triangles (discuss with students why constructing the altitude in an equilateral triangle produces two 30°-60°-90° triangles). Identify the parts of the 30°-60°-90° triangle as short leg, long leg, and hypotenuse. The resulting right triangles have a short leg of 1 unit (remind students that the altitude of an equilateral triangle is also a median which is why the right triangles have a short leg of 1 unit). A 30°-60°-90° triangle whose short leg is 1 is called the unit triangle. Have students use the Pythagorean theorem to calculate the length of the long leg (side opposite 60°) in simplified radical form. A review of simplifying radicals may be necessary.

Next, have students create a unit triangle for 45°-45°-90°, using 1 unit as the length of each of the two legs. Using the Pythagorean theorem, students will calculate the length of the hypotenuse in simplified radical form.

Repeat the activity several times, but use different measures for the sides of the equilateral triangle. Start with equilateral triangles whose sides are 4 units, and then 6 units and also use the isosceles right triangle. Do this several times until students see a pattern in the numbers. The goal is to have students write these as formulas: $\textit{short leg} = \frac{1}{2} \times \textit{hypotenuse}$ and $\textit{long leg} = \sqrt{3} \times \textit{short leg}$ in 30°-60°-90° triangles. For 45°-45°-90° triangles, the relationship is $\textit{hypotenuse} = \sqrt{2}$ x $\textit{leg}$. Additionally, show students how proportions are an alternative way of calculating the same values.

To help students become familiar with the definition of *sine* and *cosine*, have them calculate the ratios using the side lengths of special right triangles. Have students use the examples from the above activity and the definition of *sine* to determine that $\sin(30) = \frac{1}{2}$.

Allow them to use the calculator's sine function to verify this. This step may require instruction on the use of the calculator. Help students to understand that formulas, proportions, and trig functions are related to each other, and that each is a different way to write the ratios that exist. See that students become familiar with the idea that trigonometric functions represent ratios of sides in a right triangle.

Activity 11: Trigonometry (GLEs: 3, <u>8</u>, 12, <u>18</u>; CCSS: G.MG.1)

Materials List: pencil, paper, trig tables, scientific calculators (minimum)

Note: This activity was not changed as it already addressed the required modeling in the listed CCSS.

Extend Activity 10 to define the *cosine* and *tangent* ratios in right triangles. Be sure to include information which shows students how to find the measures of the acute angles in a right triangle if the side lengths are known. Assist students in learning how to use the calculator to find trig ratios and to use the ratios to solve problems. In order to facilitate understanding, have students read information from a standard trig table. For example, in order to solve $\tan x = \frac{12}{17}$, students need to understand that there is one angle which has the same decimal ratio as 12 divided by 17. Looking through the list of tangent ratios to find this number helps students understand that the calculator has these ratios stored in its memory. When the student requests $\tan^{-1}(\frac{12}{17})$, he/she is requesting the calculator to search for the angle whose ratio is the same as 12 divided by 17.

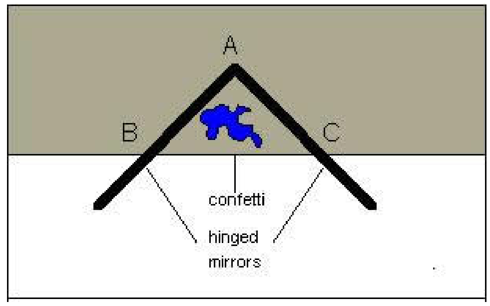
Have students practice finding the measures of missing sides and angles by applying the trigonometric ratios to right triangles. Once an understanding of the process is mastered, have students apply the trigonometric ratios to real-life problems. These problems can be finding the differences between the heights of two buildings, the distance two boats are apart from each other, the construction of airplanes, angles of elevation or depression, etc.

Activity 12: Area of Regular Polygons (GLEs: 9, <u>12</u>, 13, <u>18</u>)

Materials List: hinged mirrors, protractors, rulers, pencil, paper, compass

In this activity students use hinged mirrors, protractors, and rulers to draw regular polygons and to investigate the measures of their central angles. This will be pivotal to helping find the lengths of apothems that are not given when finding the area of regular polygons. A similar activity with activity sheets is available at http://illuminations.nctm.org/index_d.aspx?id=379.

1. Have students draw a line on a plain sheet of paper. Position the hinged mirror so that the sides of the mirror intersect the line at two points that are equal distances from the hinge of the mirror.



2. Remind students, to sketch what they see and then place the protractor on top of the mirror to determine the angle of the mirror. Record this measurement; this is the measure of a central angle of the polygon that they see in the mirror.
3. Have the students open the mirror wider; students must make sure that the mirror intersects the line at equal distances from the hinge. They should observe what happens to the figure, sketch the new figure, and record the new angle measurement.
4. Have students open or close the mirror until a regular hexagon is formed and record the angle of the mirror. Then students should answer the following questions about the six-sided figure:
    a. What is the sum the angle measures for those angles whose vertex is the mirror's hinge?
    b. How many angles are there in the image for each number of sides, and what is the measure of each of those angles?
    c. How does the measure of the hinged mirror compare to the measure you calculated in b above?
5. Have students use the hinged mirror to find the central angle for the following regular polygons: triangle, square, pentagon, octagon, decagon, and dodecagon.

Direct students to construct a circle using a compass and to inscribe a regular hexagon in the circle by making 6 congruent arcs along the circle. Have students divide the hexagon into six congruent triangles and write the formula for the area of the hexagon. Lead a class discussion to determine that students were able to generate the formula as $A = 6\left(\frac{1}{2}sa\right)$, where s is the measure of one side of the hexagon, and a is the height of the triangle, or the *apothem* of the hexagon. The apothem is the distance from the center of a regular polygon to the midpoint of a side. As part of the discussion, students need to see that the sides of the triangle that extend from the center of the circle are also radii so they should use the measure of the radii to find the measure of the height of the triangle. Once they know the height and the radii measure, they can use the Pythagorean theorem to find the length of the base. Once the formula has been stated in general terms, ask students to consider the meaning of $6s$ as this expression relates to the hexagon. Once it has been established that 6s is the perimeter of the hexagon, rewrite the formula for the area of a hexagon as $A = \frac{1}{2}Pa$, where *A* is the area of the polygon, *P* is the perimeter of the polygon, and *a* is the apothem of the polygon. This formula can be used for all regular polygons. Complete this process with other regular polygons to help students see that the formula $A = \frac{1}{2}Pa$ will work for all regular polygons.

Have students practice calculating the area of other regular polygons using the formula they have derived. Provide diagrams with labeled measurements or provide real objects in which students can measure the parts needed for the formula. Provide instances in which students must use special right triangles or trigonometric ratios to find the apothem or the side measure of the polygon. Students can find the length of the apothem of a regular polygon by using trigonometry. The formula for the length of the apothem of a regular polygon is $a = \frac{1}{2}s\tan\left(\frac{90^\circ(n-2)}{n}\right)$, where *s* is the length of the side of the polygon and *n* is the number of sides of the polygon.

<u>2013-2014</u>
Activity 13: Trigonometry of Complementary Angles (CCSS: <u>G.SRT.7</u>)

Materials List: pencil, paper

Review with students the definition of complementary angles and the fact that in right triangles, the two acute angles are complementary. Then have students find the sine and cosine of both acute angles in a set of given triangles. Next, have students make observations about the sine and cosine of complementary angles. Students should see that the sine and cosine of complementary angles are equal. Have students provide an explanation for why this is true. Formally stated, $\sin\theta = \cos(90-\theta)$ and $\cos\theta = \sin(90-\theta)$. Define these functions as cofunctions of each other (sine is the cofunction of cosine) hence the name **co**sine.

Sample Assessments

General Assessments

- The student will create a portfolio containing samples of his/her activities. For instance, the student could choose a particular drawing from class and enlarge it using a given scale. In this entry he/she would also explain the process and how to prove that the new drawing is similar to the given drawing.
- The student will complete math *learning log* entries for this unit. For example:
  - Discuss the proof for the special right triangles: 30°-60°-90° and 45°-45°-90°. In your discussion, explain why this information can be generalized to all triangles that have these an angle measures.

  - Explain how the Pythagorean theorem can be used to determine if a triangle is a right, obtuse, or acute triangle.
  - Verify that the formula for the area of a regular polygon will work for a square and an equilateral triangle. Provide an explanation of why it works.
- The student will find pictures of similar figures in magazines, newspapers, or other publications and will explain how he/she knows that the figures are similar. The teacher will challenge the student to find pairs of similar figures that are not congruent.

Activity-Specific Assessments

- Activity 1: Give each student a floor plan for a house. The floor plan should not have any measurements on it. The student will enlarge the floor plan to the size of a poster using a given scale. The student will find the actual dimensions of the rooms and the dimensions of the entire house from the scale used to create the floor plan.

- Activity 3: Provide instructions for making and using a hypsometer given on the Making a Hypsometer BLM. The student will write a rationale for the proportion that is given in the instructions. The student will determine the height of various objects throughout school showing all calculations necessary to indirectly find the height of the chosen object.

- Activity 9: The student will use a clinometer to determine the height of something on the grounds of the school (e.g., flag pole, light post, goal post) using the trigonometric functions. The student will produce a scaled diagram of the measurements made and show all calculations used to indirectly calculate the height of the chosen object using trig functions. Instructions for making a clinometer can be found in most geometry textbooks and on numerous websites, such as http://www.wikihow.com/Make-a-Clinometer.