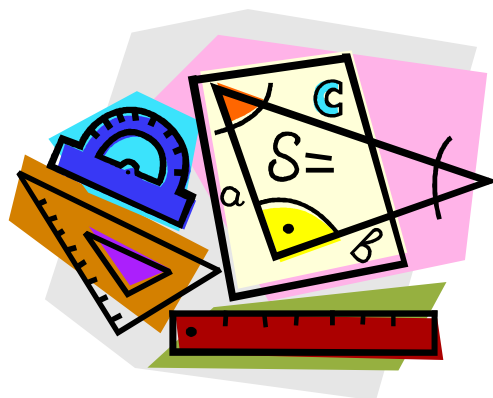


Geometry

Unit 4

Congruent Triangles



Name: _____

Section 4 – 1: Classifying Triangles

Notes

Parts of a Triangle:

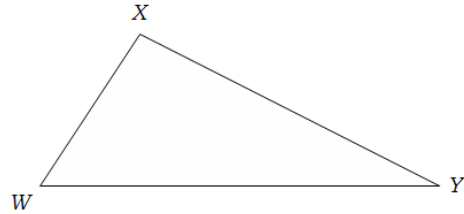
Triangle – a three-sided polygon

Name –

Sides –

Vertices –

Angles –



Classifying Triangles by Angles:

Acute Δ

Obtuse Δ

Right Δ

Equiangular Δ -

Classifying Triangles by Sides:

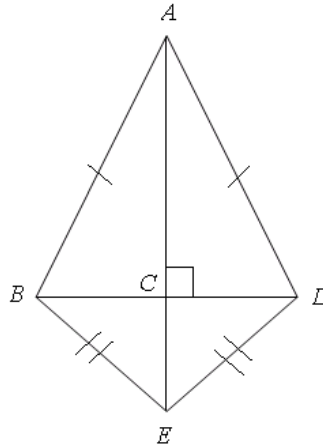
Scalene Δ

Isosceles Δ

Equilateral Δ

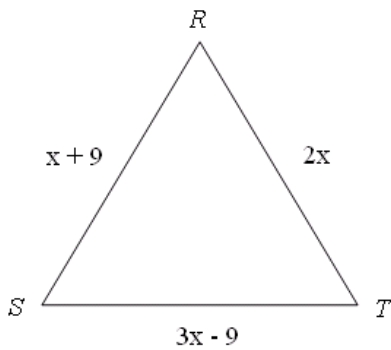
Example #1: Identify the indicated type of triangle in the figure.

a.) isosceles triangles

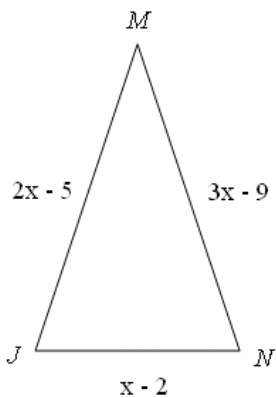


b.) scalene triangles

Example #2: Find x and the measure of each side of equilateral triangle RST .



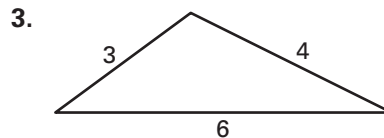
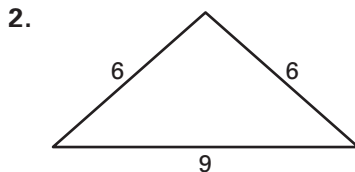
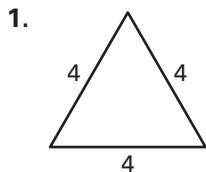
Example #3: Find x , JM , MN , and JN if $\triangle JMN$ is an isosceles triangle with $\overline{JM} \cong \overline{MN}$.



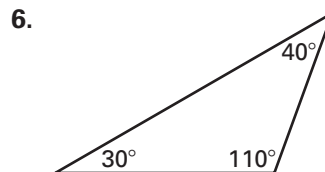
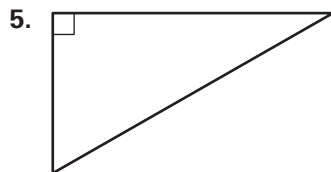
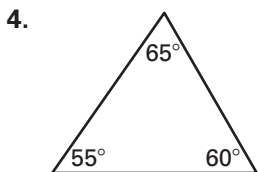
Practice B

For use with pages 173–178

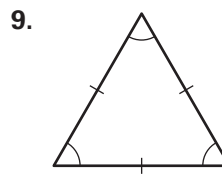
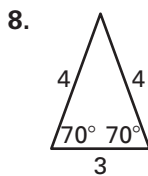
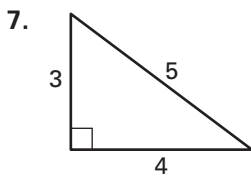
Classify the triangle by its sides.



Classify the triangle by its angles.



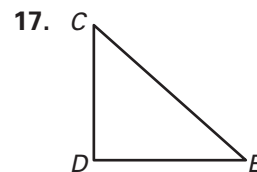
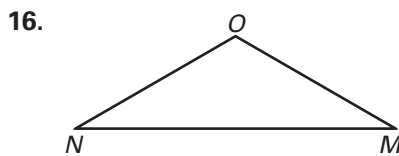
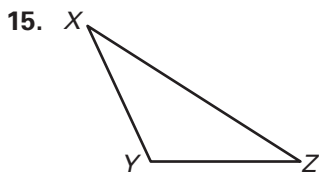
Classify the triangle by its angles and by its sides.



Match the triangle description with the most specific name.

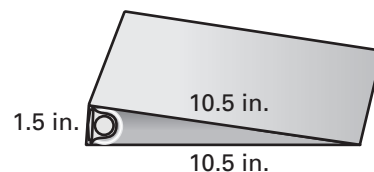
- | | |
|-------------------------------------|--------------|
| 10. Side lengths: 6 cm, 7 cm, 8 cm | A. Isosceles |
| 11. Side lengths: 9 cm, 10 cm, 9 cm | B. Obtuse |
| 12. Angle measures: 35°, 55°, 90° | C. Acute |
| 13. Angle measures: 13°, 27°, 140° | D. Right |
| 14. Angle measures: 59°, 60°, 61° | E. Scalene |

Identify which side is opposite each angle.



In Exercises 18 and 19, use the notebook binder at the right.

18. Use the lengths shown to classify the triangle formed by the bottom edges of the binder by sides.
19. If the triangle formed by the bottom edges of the binder has angles that measure 86°, 86°, and 8°, classify the triangle by angles.

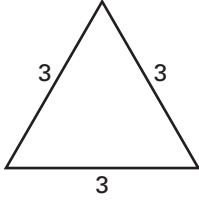


Practice A

For use with pages 173–178

Match the triangle with its classification by sides.

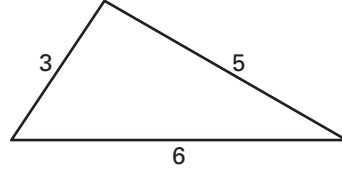
1.



2.



3.



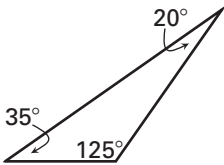
A. Isosceles Triangle

B. Scalene Triangle

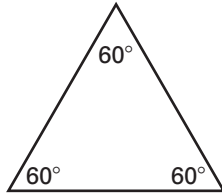
C. Equilateral Triangle

Match the triangle with its classification by angles.

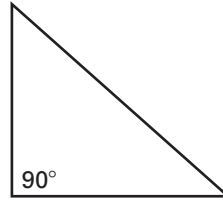
4.



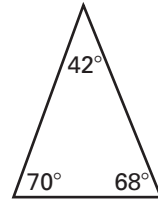
5.



6.



7.



A. Acute Triangle

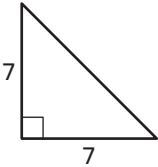
B. Equiangular Triangle

C. Obtuse Triangle

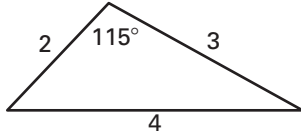
D. Right Triangle

Classify the triangle by its angles and by its sides.

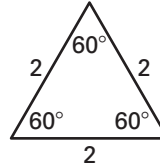
8.



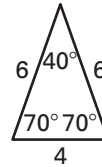
9.



10.

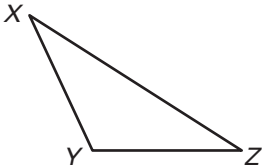


11.

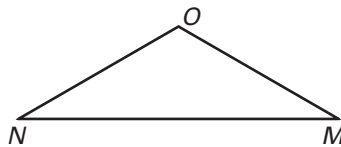


Name the side that is opposite the angle.

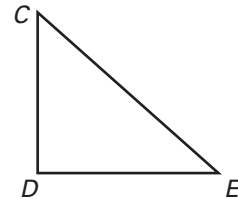
12. $\angle X$



13. $\angle M$



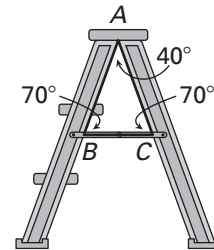
14. $\angle C$



The diagram at the right shows the side view of a stepladder. $\triangle ABC$ is formed by the legs and by the side brace of the stepladder.

15. Classify $\triangle ABC$ by its angles.

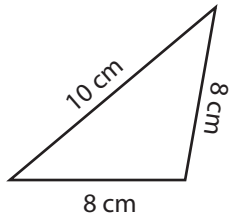
16. If $AB = 28$ in., $AC = 28$ in., and $BC = 16$ in., classify $\triangle ABC$ by its sides.



Identifying Triangles

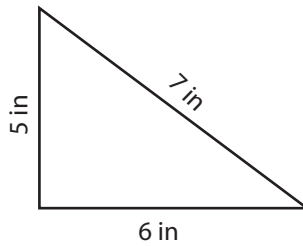
Identify each triangle based on both sides and angles.

1)

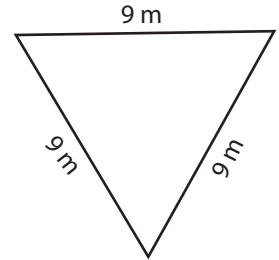


Isosceles obtuse triangle

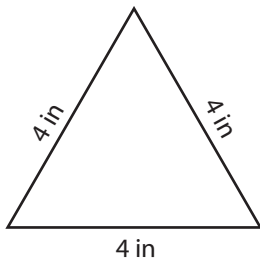
2)



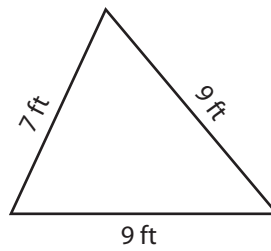
3)



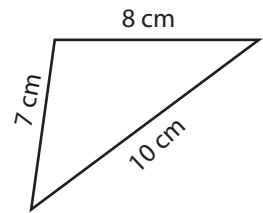
4)



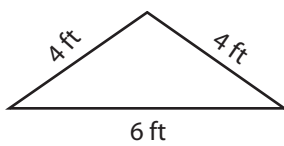
5)



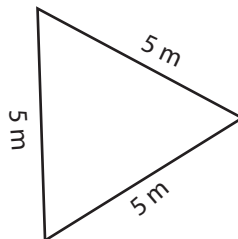
6)



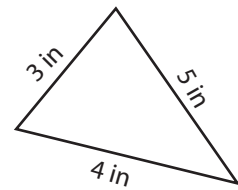
7)



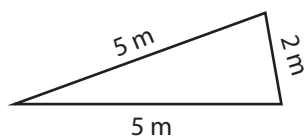
8)



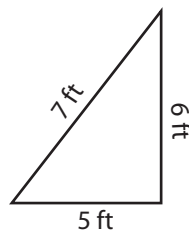
9)



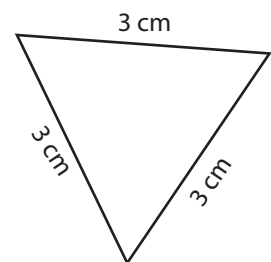
10)



11)



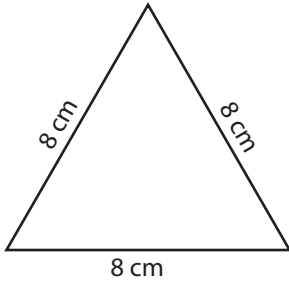
12)



Identifying Triangles

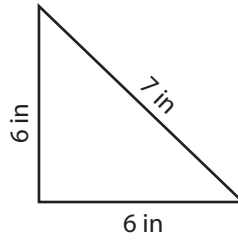
Identify each triangle based on both sides and angles.

1)

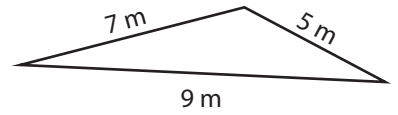


Equilateral acute triangle

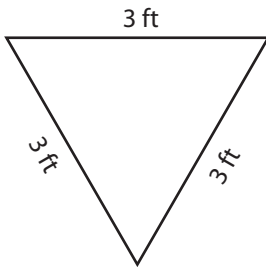
2)



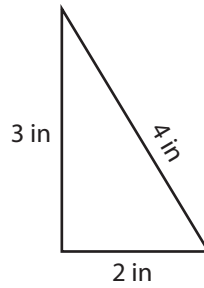
3)



4)



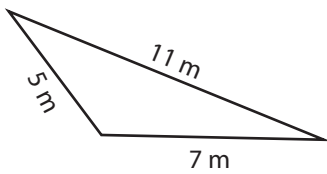
5)



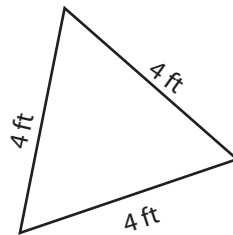
6)



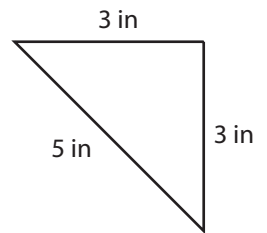
7)



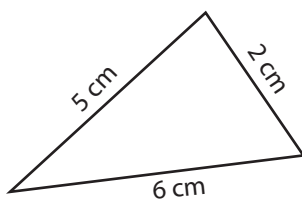
8)



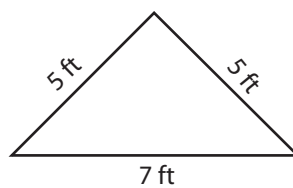
9)



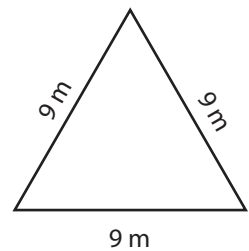
10)



11)



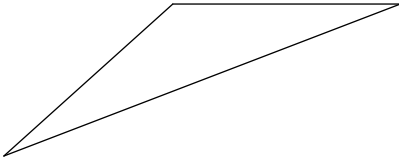
12)



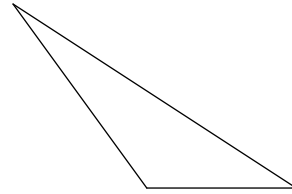
Classifying Triangles

Classify each triangle by each angles and sides. Base your decision on the actual lengths of the sides and the measures of the angles.

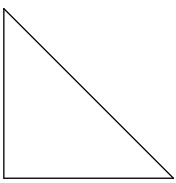
1)



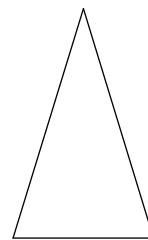
2)



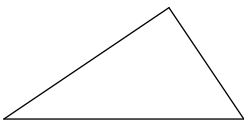
3)



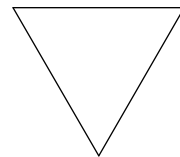
4)



5)

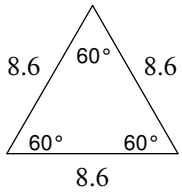


6)

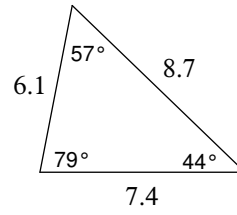


Classify each triangle by each angles and sides.

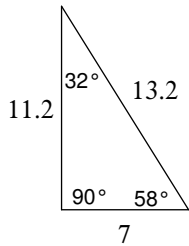
7)



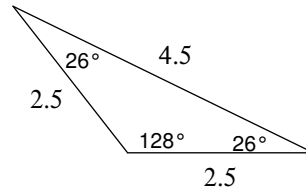
8)



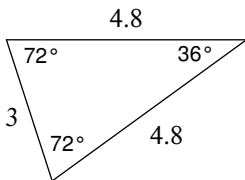
9)



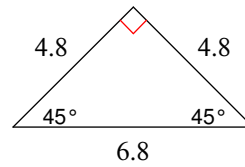
10)



11)

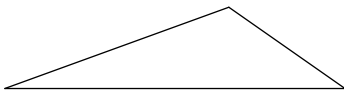


12)

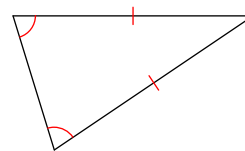


Classify each triangle by each angles and sides. Equal sides and equal angles, if any, are indicated in each diagram.

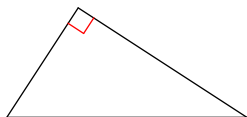
13)



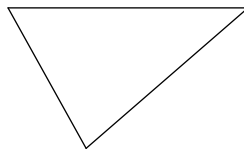
14)



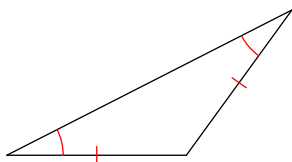
15)



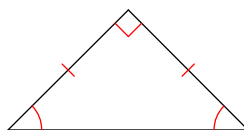
16)



17)



18)



Sketch an example of the type of triangle described. Mark the triangle to indicate what information is known. If no triangle can be drawn, write "not possible."

19) acute isosceles

20) right scalene

21) right isosceles

22) right equilateral

23) acute scalene

24) obtuse scalene

25) right obtuse

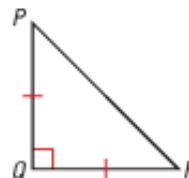
26) equilateral

Vocabulary Check

1. What is the difference between an *obtuse triangle* and an *acute triangle*?

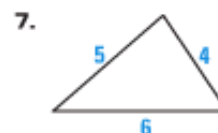
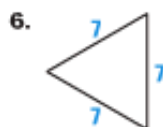
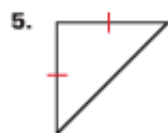
In Exercises 2–4, use the diagram.

2. Name the side *opposite* $\angle P$.
3. Name the side *opposite* $\angle Q$.
4. Classify the triangle by its sides.

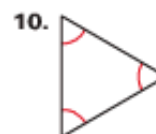
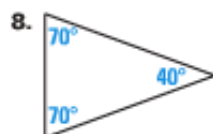


Skill Check

Classify the triangle by its sides.



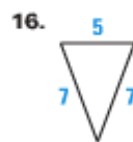
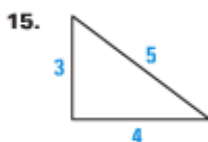
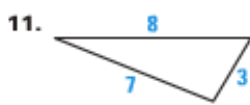
Classify the triangle by its angles.



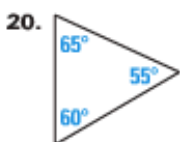
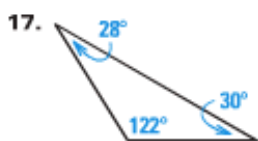
Extra Practice

See p. 681.

Classifying Triangles

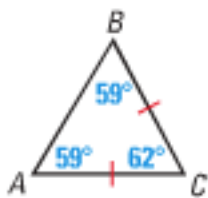
 Classify the triangle by its sides.

Classifying Triangles

 Classify the triangle by its angles.

Classifying Triangles Classify the triangle by its angles and by its sides.

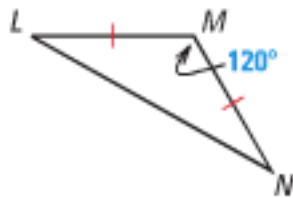
24.



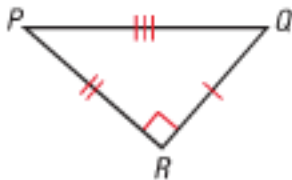
25.



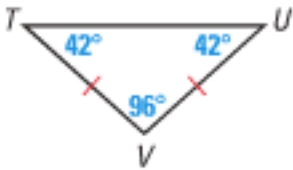
26.



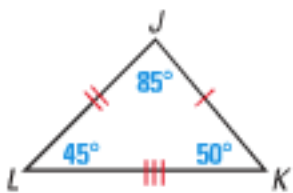
27.



28.




29.



Matching Triangles In Exercises 30–36, use the example above to match the triangle description with the most specific name.

- | | |
|---|----------------|
| 30. Side lengths: 2 cm, 3 cm, 4 cm | A. Equilateral |
| 31. Side lengths: 3 cm, 2 cm, 3 cm | B. Scalene |
| 32. Side lengths: 4 cm, 4 cm, 4 cm | C. Obtuse |
| 33. Angle measures: 60° , 60° , 60° | D. Equiangular |
| 34. Angle measures: 30° , 60° , 90° | E. Isosceles |
| 35. Angle measures: 20° , 145° , 15° | F. Acute |
| 36. Angle measures: 50° , 55° , 75° | G. Right |

Visualize It!  Draw an example of the triangle.

48. obtuse scalene

49. right isosceles

50. acute scalene

51. right scalene

52. acute isosceles

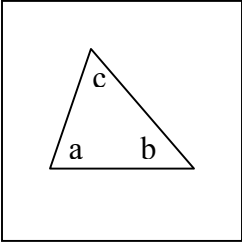
53. obtuse isosceles

Angles of Triangles Section 4-2

Angle Sum Activity

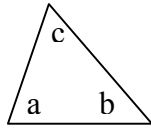
Draw a large triangle on your paper. (Use half the sheet of 8 ½ x 11 paper)

STEP 1



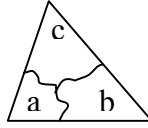
Write a, b and c in the interiors of the three angles of the triangle.

STEP 2



Carefully cut out the triangle.

STEP 3



Tear off the three angles.

STEP 4

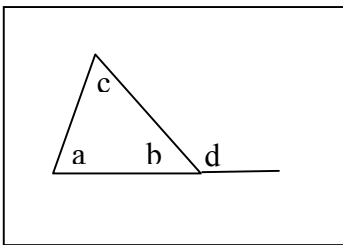
Arrange the three angles in such a way as to show their sum.

CONJECTURE: Sum of the angles of any triangle is _____

Exterior Angle Activity

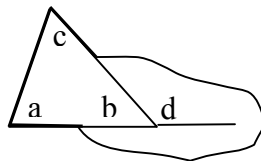
Draw a large triangle on your paper. (Use half the sheet of 8 ½ x 11 paper). Extend one side of the triangle to form an exterior angle. (See diagram in step 1)

STEP 1



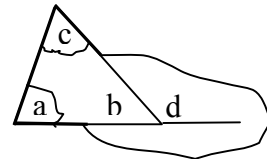
Write a, b, and c in the interiors of the three angles, and d in the exterior angle formed.

STEP 2



Carefully cut out the triangle and extended side as shown in the diagram.

STEP 3



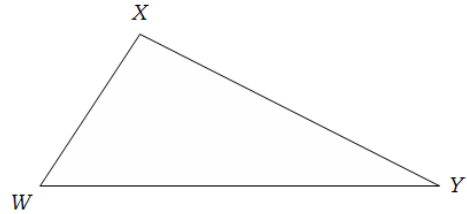
Tear off angles a and c only. Arrange angles a and c in such a way as to show their relationship to angle d.

CONJECTURE: The measure of the exterior angle of any triangle is

Section 4 – 2: Angles of Triangles
Notes

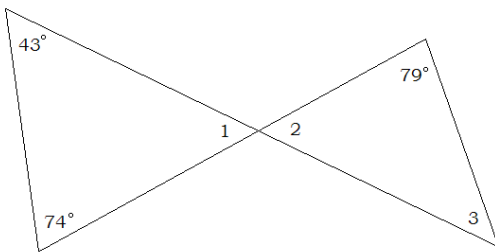
Angle Sum Theorem:

- The sum of the measures of the angles of a _____ is _____.

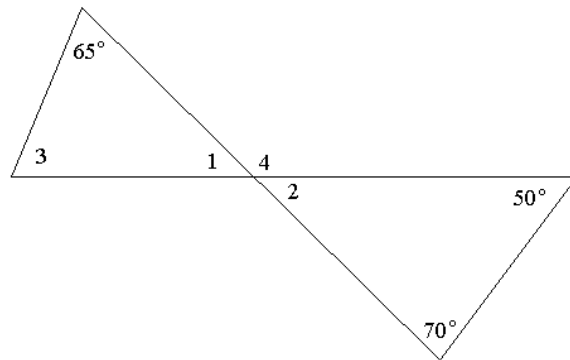


Example #1: Find the missing angle measures.

a.)

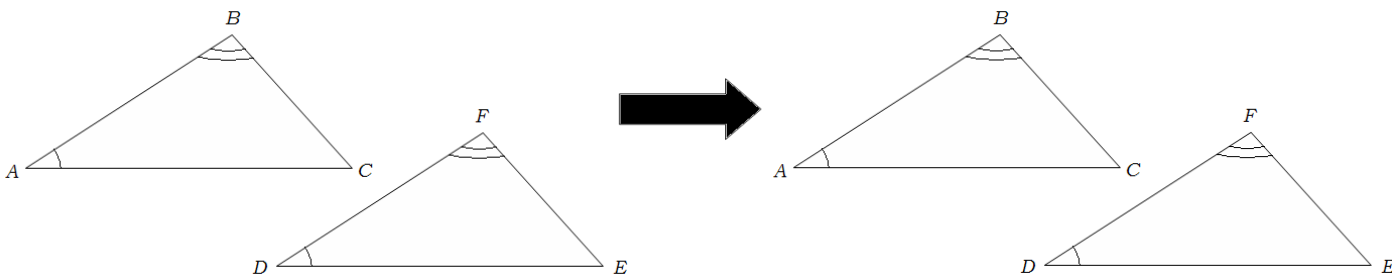


b.)



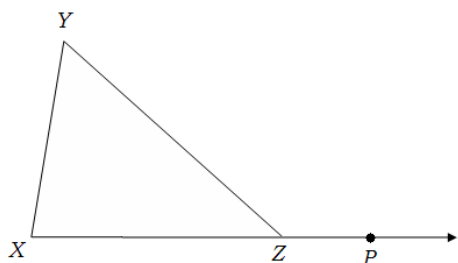
Third Angle Theorem:

- If two angles of one triangle are _____ to two angles of a second triangle, then the third angles of the triangles are _____.



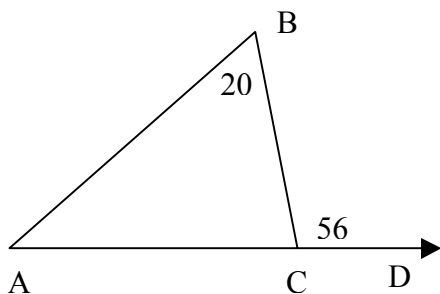
Exterior Angle Theorem:

- An **exterior angle** is formed by one side of a _____ and the extension of another _____.
- **Remote interior angles** are the angles of a triangle that are not _____ to a given _____ angle.
- The measure of an exterior angle of a triangle is _____ to the sum of the measures of the two _____ interior angles.



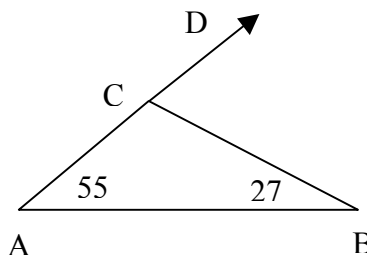
Example #2: Find the measure of each of the following angles.

a.)



$$m\angle A =$$

b.)



$$m\angle DCB =$$

Name : _____

Score : _____

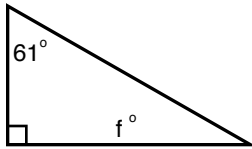
Teacher : _____

Date : _____

Triangle Angle Sum

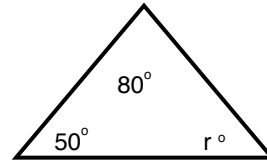
Solve for the given variable.

1)



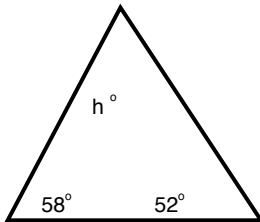
$$f = \underline{\hspace{2cm}}$$

5)



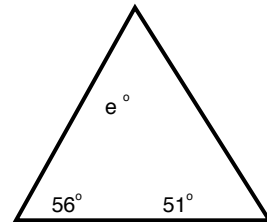
$$r = \underline{\hspace{2cm}}$$

2)



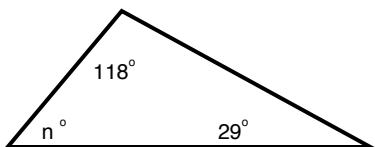
$$h = \underline{\hspace{2cm}}$$

6)



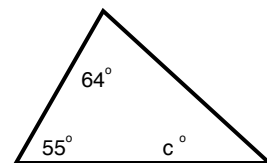
$$e = \underline{\hspace{2cm}}$$

3)



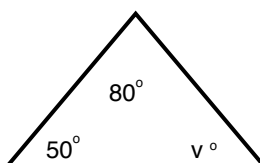
$$n = \underline{\hspace{2cm}}$$

7)



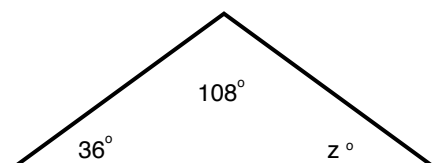
$$c = \underline{\hspace{2cm}}$$

4)



$$v = \underline{\hspace{2cm}}$$

8)



$$z = \underline{\hspace{2cm}}$$



Name : _____

Score : _____

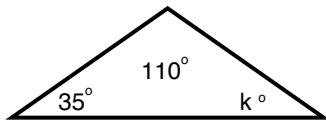
Teacher : _____

Date : _____

Triangle Angle Sum

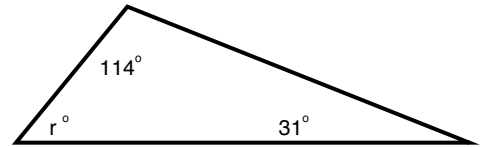
Solve for the given variable.

1)



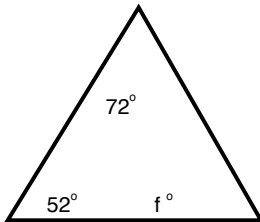
$$k = \underline{\hspace{2cm}}$$

5)



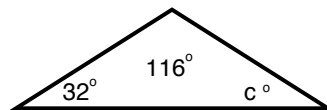
$$r = \underline{\hspace{2cm}}$$

2)



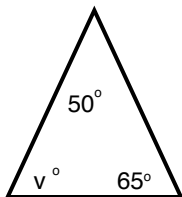
$$f = \underline{\hspace{2cm}}$$

6)



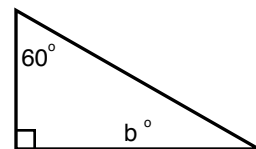
$$c = \underline{\hspace{2cm}}$$

3)



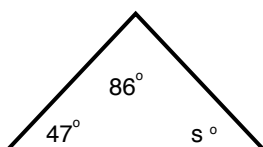
$$v = \underline{\hspace{2cm}}$$

7)



$$b = \underline{\hspace{2cm}}$$

4)



$$s = \underline{\hspace{2cm}}$$

8)



$$x = \underline{\hspace{2cm}}$$

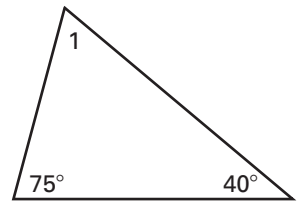


Practice A

For use with pages 179–184

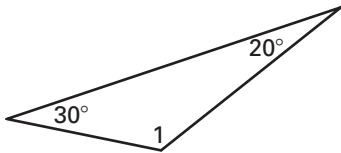
1. Which equation can be used to find $m\angle 1$ in the diagram?

- A. $75^\circ + 40^\circ = m\angle 1$ B. $m\angle 1 + 40^\circ + 75^\circ = 180^\circ$
 C. $75^\circ + m\angle 1 = 40^\circ$ D. $m\angle 1 + 40^\circ = 75^\circ$

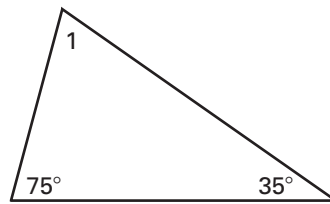


Find the measure of $\angle 1$.

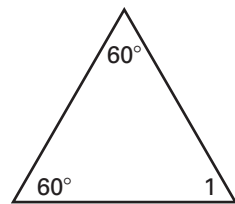
2.



3.

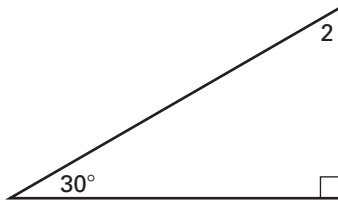


4.

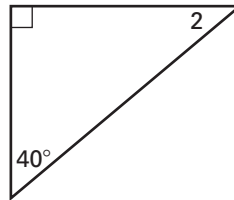


Find the measure of $\angle 2$.

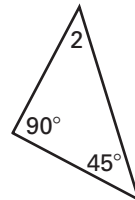
5.



6.

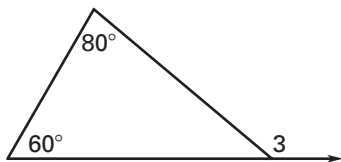


7.

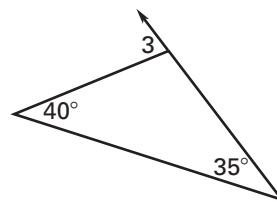


Find the measure of $\angle 3$.

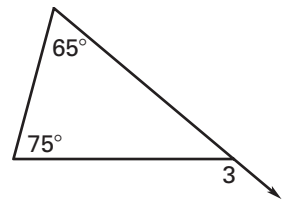
8.



9.

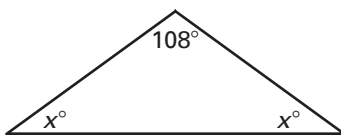


10.

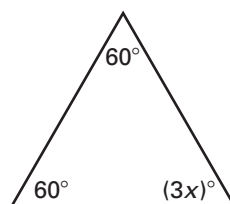


Find the value of x .

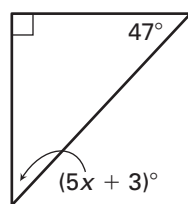
11.



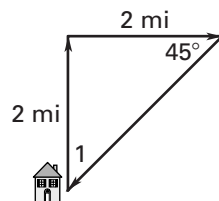
12.



13.



14. From your house, you walk north for two miles. Then you walk east for two miles. Next, you turn 45° to your right and walk back to your house. What is the measure of $\angle 1$, as shown in the diagram at the right?



Skill Check

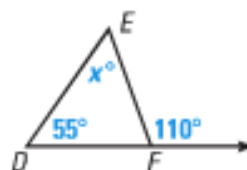
2. Use the diagram to determine which equation can be used to find $m\angle DEF$.

A. $55^\circ + x^\circ = 110^\circ$

B. $55^\circ + 110^\circ = x^\circ$

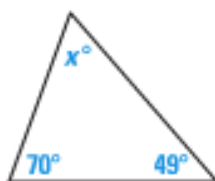
C. $55^\circ - x^\circ = 110^\circ$

D. $55^\circ - 110^\circ = x^\circ$

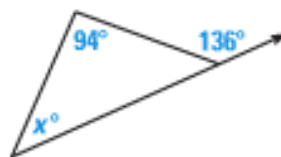


Find the value of x .

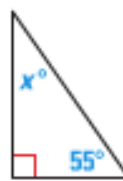
3.



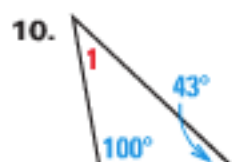
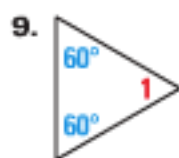
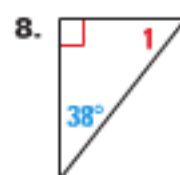
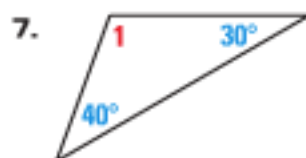
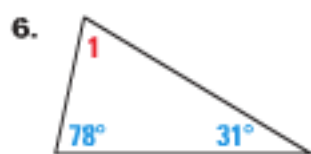
4.



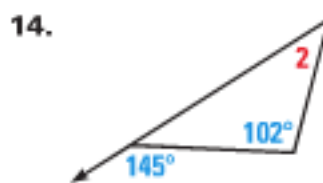
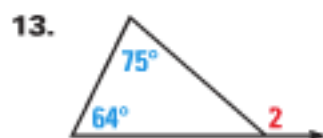
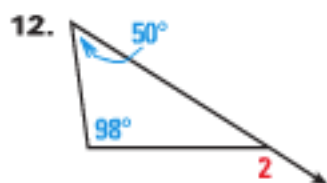
5.



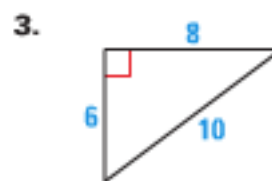
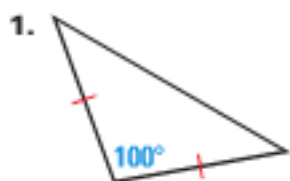
Finding Angle Measures Find the measure of $\angle 1$.



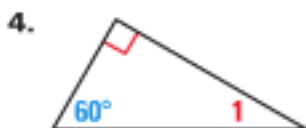
Exterior Angles Find the measure of $\angle 2$.



Classify the triangle by its angles and by its sides. (*Lesson 4.1*)

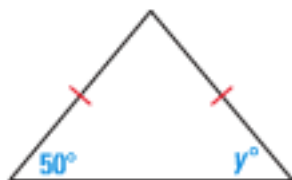


Find the measure of $\angle 1$. (*Lesson 4.2*)

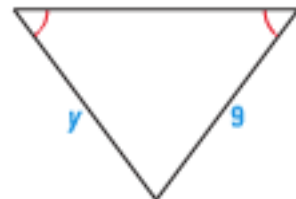


Find the value of y .

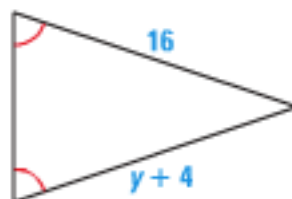
1.



2.

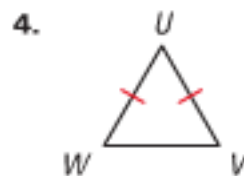
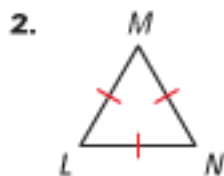


3.



1. What is the difference between *equilateral* and *equiangular*?

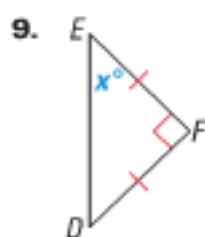
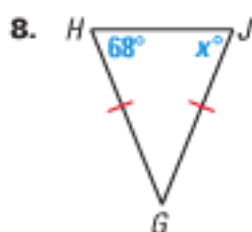
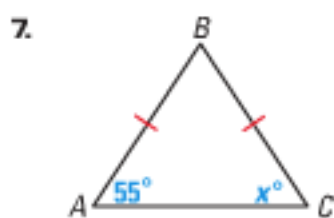
Tell which sides and angles of the triangle are congruent.



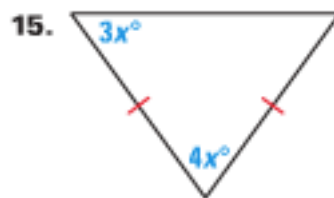
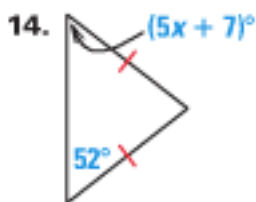
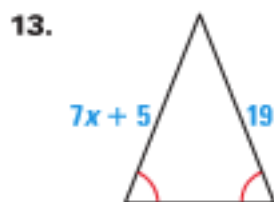
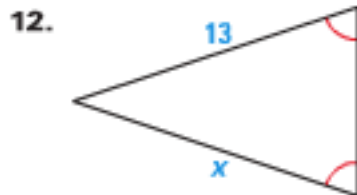
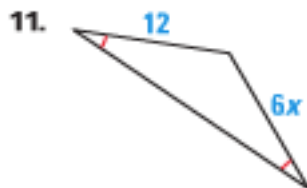
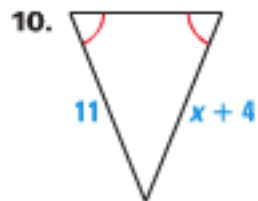
Find the value of x . Tell what theorem(s) you used.



Finding Measures Find the value of x . Tell what theorem(s) you used.

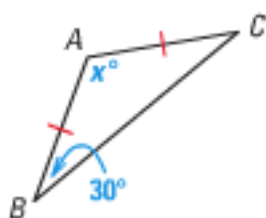


Using Algebra Find the value of x .

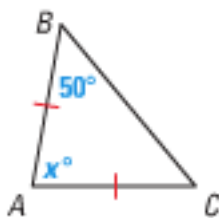


xy Using Algebra Find the measure of $\angle A$.

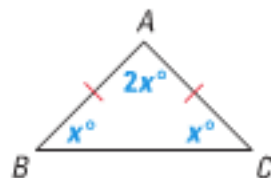
17.



18.



19.

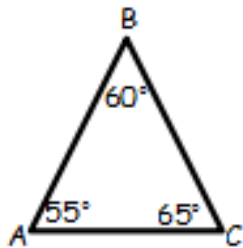


The **longest side** of a triangle is opposite its _____ angle.

The **shortest side** of a triangle is opposite its _____ angle.

The **largest angle** is opposite the _____ side.

The **smallest angle** is opposite the _____ side.



Longest side _____

Smallest Angle _____

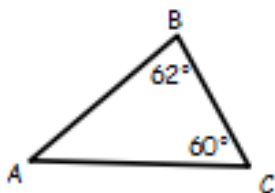
Shortest Side _____

Smallest Angle _____

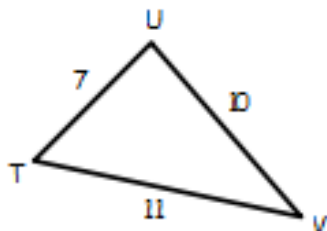
Theorem: If one side of a triangle is longer than another side, then the angle opposite the _____
_____ is larger than the angle opposite the _____.

Theorem: If one angle of a triangle is larger than another angle, then the side opposite the _____
_____ is longer than the side opposite the _____.

Example #1: Write the sides of the triangle in order from shortest to longest.



Example #2: Write the angles in order from smallest to largest.



Theorem: The sum of the lengths of any two sides of a triangle is greater than the third side. (Start with the two smaller sides, if they satisfy the theorem, then the remaining sums will also satisfy it.)

Example #3: Could the following be the side lengths of a triangle?

a. 27, 18, 12

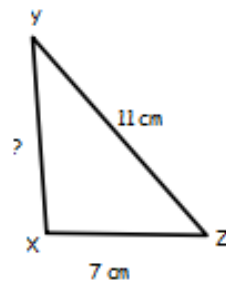
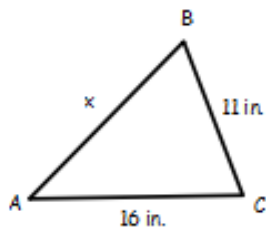
b. 5, 14, 7

c. 12, 14, 26

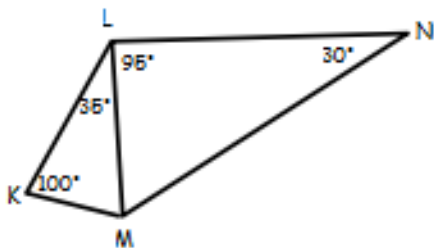
d. 2.001, 3, 5

e. $\sqrt{5}$, 2, 5

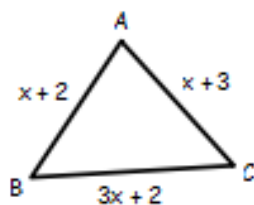
Example #4: Find the possible values for the 3rd side of the triangle.



Example #5: List the sides in order from shortest to longest.



Example #6: Find the range of values for x .



Name : _____

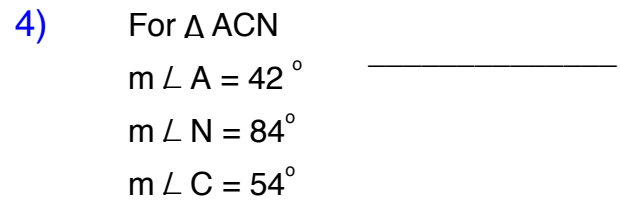
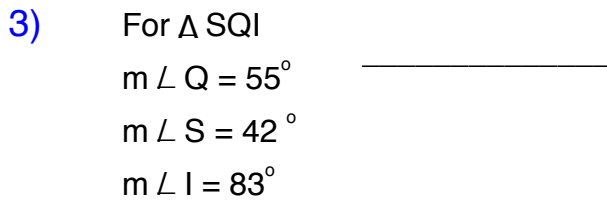
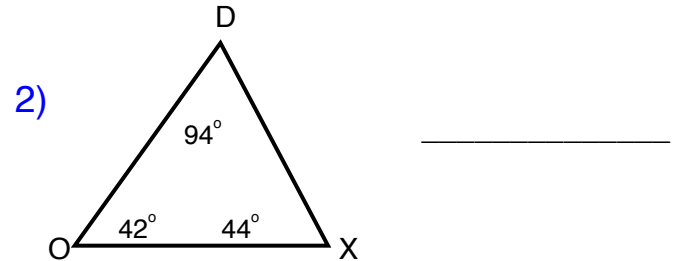
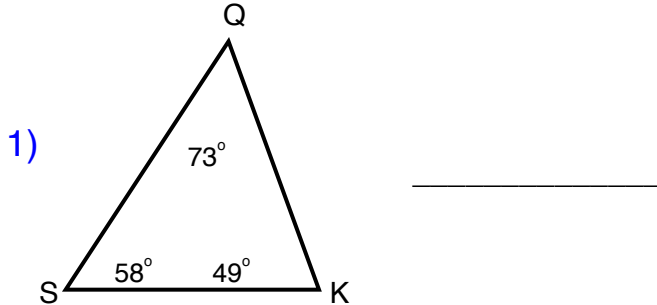
Score : _____

Teacher : _____

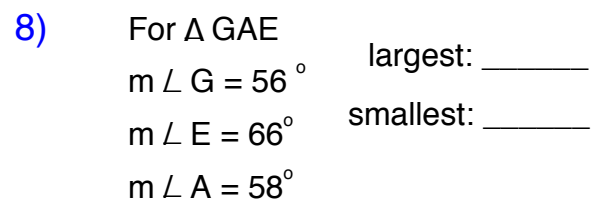
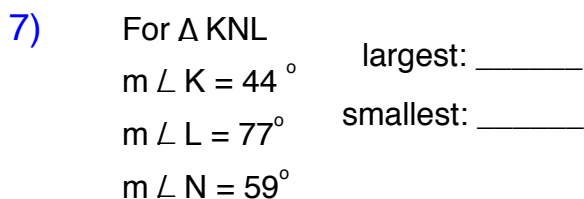
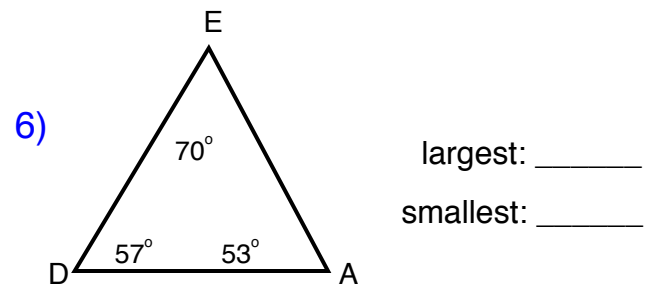
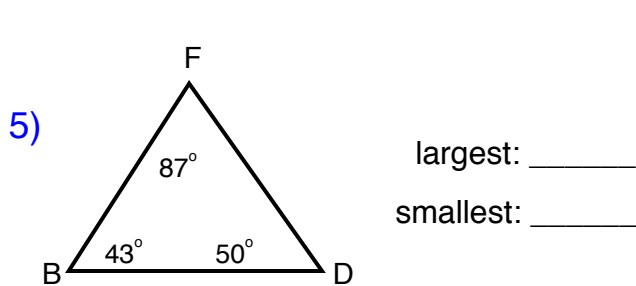
Date : _____

Triangle Inequality of Sides

Order each triangle's sides from largest to smallest.



Name the largest and smallest side for each triangle.



Name : _____

Score : _____

Teacher : _____

Date : _____

Triangle Inequality Theorem

State if each set of three numbers can be the lengths of the sides of a triangle.

1) 3, 12, 10

6) 5, 7, 10

2) 4, 5, 12

7) 3, 13, 20

3) 5, 8, 10

8) 7, 10, 16

4) 13, 3, 13

9) 7, 9, 21

5) 9, 5, 19

10) 13, 2, 16

Given are the lengths of two sides of a triangle. Find the range of lengths for the third side.

11) 3, 6

14) 4, 12

12) 2, 8

15) 13, 7

13) 12, 2

16) 10, 12



Name : _____

Score : _____

Teacher : _____

Date : _____

Triangle Inequality Theorem

State if each set of three numbers can be the lengths of the sides of a triangle.

1) 6, 11, 22

6) 5, 11, 19

2) 4, 7, 12

7) 2, 6, 10

3) 8, 13, 13

8) 10, 3, 18

4) 11, 5, 10

9) 7, 8, 3

5) 12, 9, 26

10) 2, 3, 10

Given are the lengths of two sides of a triangle. Find the range of lengths for the third side.

11) 9, 4

14) 13, 4

12) 11, 5

15) 3, 10

13) 2, 6

16) 8, 4



Name : _____

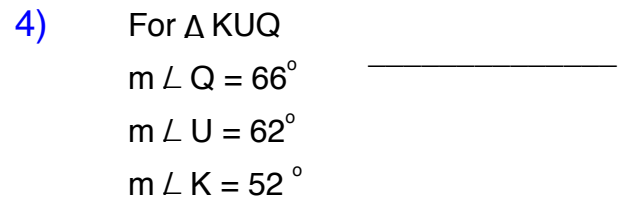
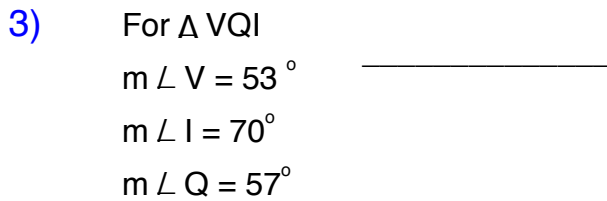
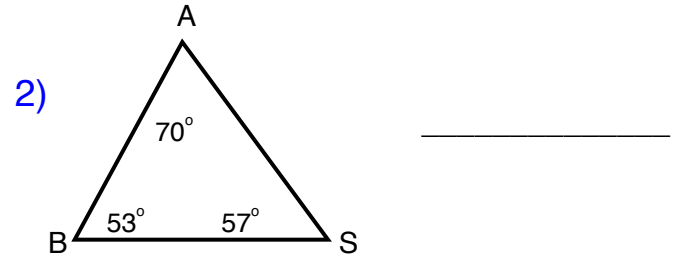
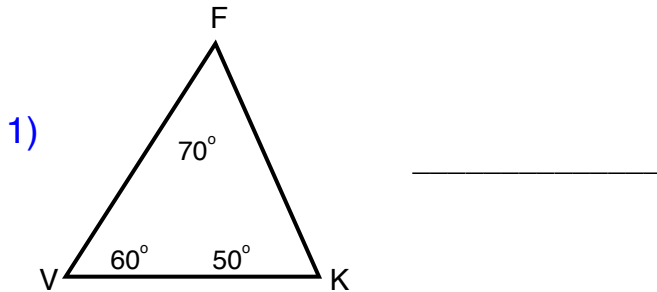
Score : _____

Teacher : _____

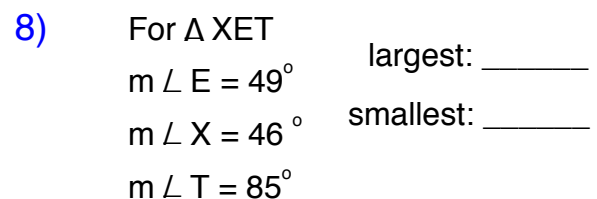
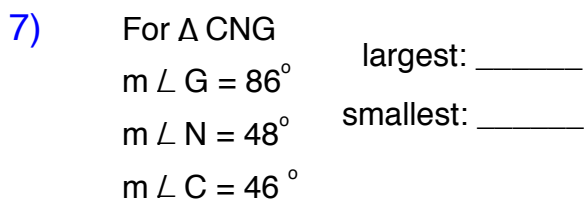
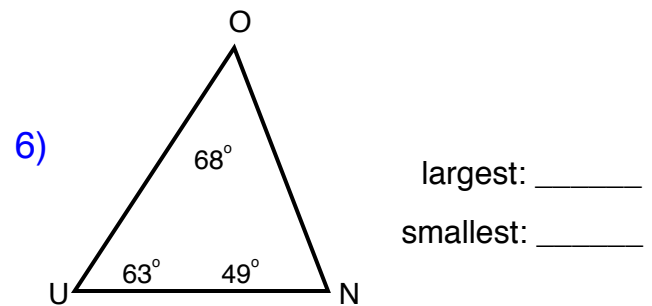
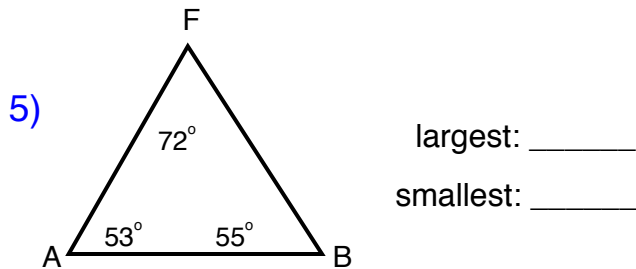
Date : _____

Triangle Inequality of Sides

Order each triangle's sides from largest to smallest.



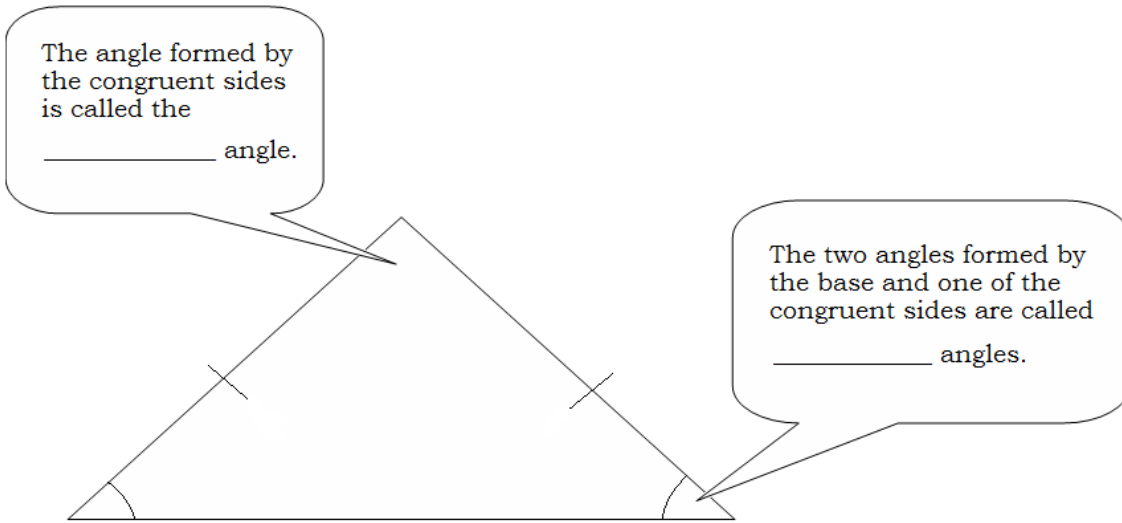
Name the largest and smallest side for each triangle.



Section 4 - 6: Isosceles Triangles

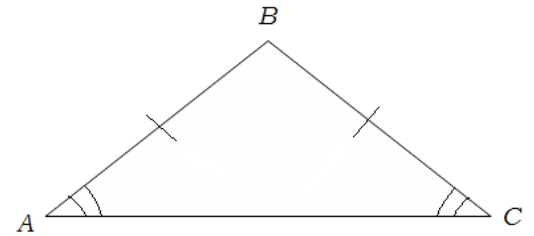
Notes

Isosceles Triangle: A triangle with at least _____ sides congruent.

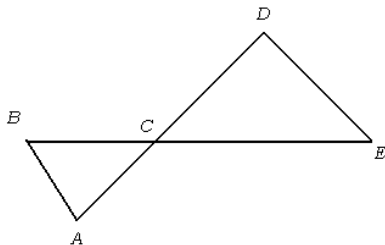


Isosceles Triangle Theorem: If two sides of a triangle are _____, then the angles opposite those sides are _____.

Ex:

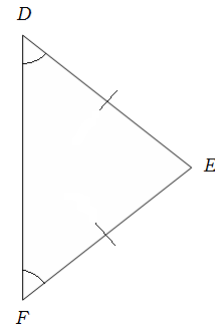


Example #1: If $\overline{DE} \cong \overline{CD}$, $\overline{BC} \cong \overline{AC}$, and $m\angle CDE = 120$, what is the measure of $\angle BAC$?

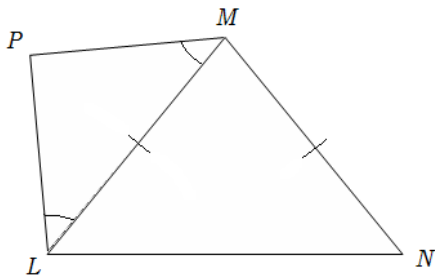


Theorem 4.10: If two angles of a _____ are congruent, then the sides opposite those angles are _____.

Ex:

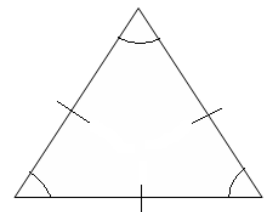


Example #2:

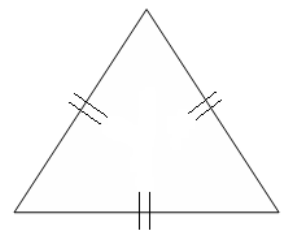


- Name all of the congruent angles.
- Name all of the congruent segments.

Corollary 4.3: A triangle is _____ if and only if it is _____.

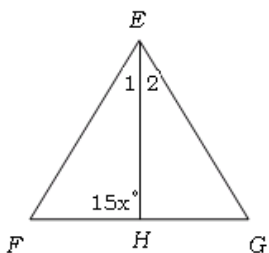


Corollary 4.4: Each angle of an equilateral triangle measures _____.



Example #3: $\triangle EFG$ is equilateral, and \overline{EH} bisects $\angle E$.

- Find $m\angle 1$ and $m\angle 2$.
- Find x .





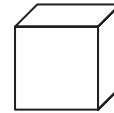
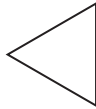
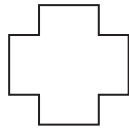
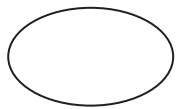
Vocabulary

Review

- Underline the correct word to complete the sentence.

A *polygon* is a two-dimensional figure with two / three or more segments that meet exactly at their endpoints.

- Cross out the figure(s) that are NOT *polygons*.



Vocabulary Builder

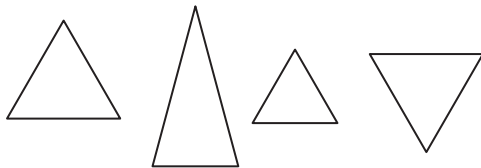
congruent (adjective) kahng GROO unt

Main Idea: **Congruent** figures have the same size and shape.

Related Word: congruence (noun)

Use Your Vocabulary

- Circle the triangles that appear to be *congruent*.



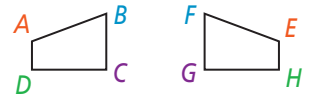
Write T for *true* or F for *false*.

- 4.** *Congruent* angles have different measures.
- 5.** A prism and its net are *congruent* figures.
- 6.** The corresponding sides of *congruent* figures have the same measure.

Take note

Key Concept Congruent Figures

Congruent polygons have congruent corresponding parts—their matching sides and angles. When you name congruent polygons, you must list corresponding vertices in the same order.



$$ABCD \cong EFGH$$

7. Use the figures at the right to complete each congruence statement.

$\overline{AB} \cong$ <input type="text"/>	$\overline{BC} \cong$ <input type="text"/>	$\overline{CD} \cong$ <input type="text"/>	$\overline{DA} \cong$ <input type="text"/>
$\angle A \cong$ <input type="text"/>	$\angle B \cong$ <input type="text"/>	$\angle C \cong$ <input type="text"/>	$\angle D \cong$ <input type="text"/>



Problem 1 Using Congruent Parts

Got It? If $\triangle WYS \cong \triangle MKV$, what are the congruent corresponding parts?

8. Use the diagram at the right.
Draw an arrow from each vertex of the first triangle to the corresponding vertex of the second triangle.

$$\triangle WYS \cong \triangle MKV$$

9. Use the diagram from Exercise 8 to complete each congruence statement.

Sides	$\overline{WY} \cong$ <input type="text"/>	$\overline{YS} \cong$ <input type="text"/>	$\overline{WS} \cong$ <input type="text"/>
Angles	$\angle W \cong$ <input type="text"/>	$\angle Y \cong$ <input type="text"/>	$\angle S \cong$ <input type="text"/>



Problem 2 Finding Congruent Parts

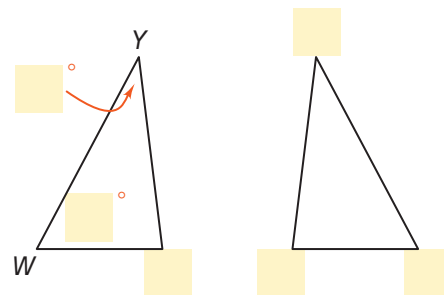
Got It? Suppose that $\triangle WYS \cong \triangle MKV$. If $m\angle W = 62$ and $m\angle Y = 35$, what is $m\angle V$? Explain.

Use the congruent triangles at the right.

10. Use the given information to label the triangles. Remember to write corresponding vertices in order.

11. Complete each congruence statement.

$\angle W \cong$ <input type="text"/>
$\angle Y \cong$ <input type="text"/>
$\angle S \cong$ <input type="text"/>



12. Use the Triangle Angle-Sum theorem.

$$m\angle S + m\text{ } + m\text{ } = 180, \text{ so } m\angle S = 180 - (\text{ } + \text{ }), \text{ or } \text{ }.$$

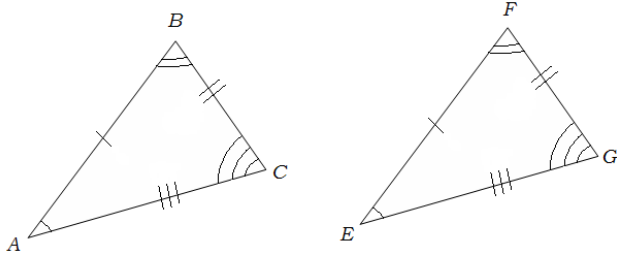
13. Complete.

Since $\angle S \cong$ and $m\angle S =$, $m\angle V =$.

Section 4 - 3: Congruent Triangles
Notes

Congruent Triangles: triangles that are the same _____ and _____

- Each triangle has three _____ and three _____.
- If all _____ of the corresponding parts of two triangles are _____, then the triangles are _____.



Congruent Triangles:

Corresponding Congruent Angles:

Corresponding Congruent Sides:

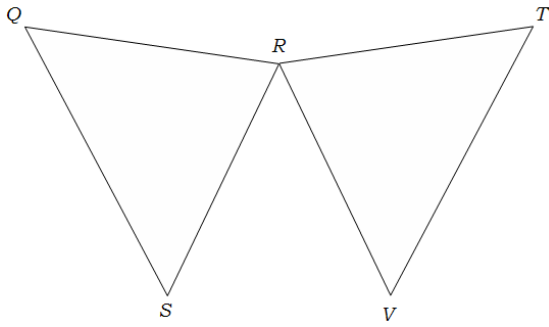
Definition of Congruent Triangles (CPCTC):

- Two triangles are congruent if and only if their corresponding parts are _____.
- *CPCTC* - Corresponding parts of congruent triangles are congruent

Example #1: In the following figure, $QR = 12$, $RS = 23$, $QS = 24$, $RT = 12$,

$TV = 24$, and $RV = 23$.

Name the corresponding congruent angles and sides.



Name the congruent triangles.

Properties of Triangle Congruence:

<u>Reflexive</u>	<u>Symmetric</u>	<u>Transitive</u>

Example #2: If $\triangle WXZ \cong \triangle STJ$, name the congruent angles and congruent sides.

Angles -

Sides -

4-1 Additional Problems

Congruent Figures

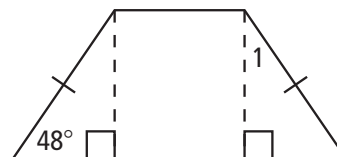
Problem 1

If $RSTU \cong WXYZ$, what are the congruent corresponding parts?

Problem 2

The sides of a roof suggest congruent triangles. What is $m\angle 1$?

- A. 90
- B. 48
- C. 42
- D. 32

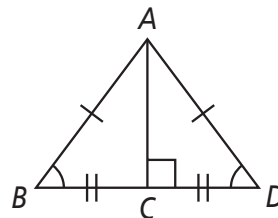


4-1 Additional Problems (continued)

Congruent Figures

Problem 3

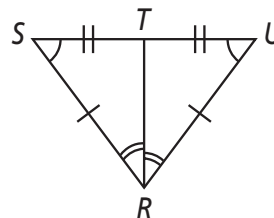
Are the triangles congruent? Justify your answer.



Problem 4

Given: $\overline{RS} \cong \overline{RU}$, $\overline{TS} \cong \overline{TU}$,
 $\angle S \cong \angle U$, $\angle SRT \cong \angle URT$

Prove: $\triangle RST \cong \triangle RUT$



ELL Support

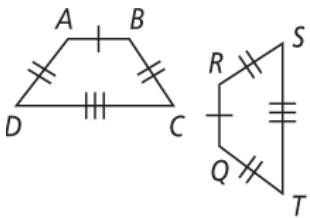
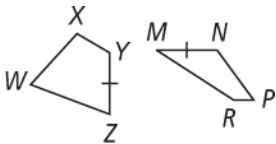
4-1

Congruent Figures

Concept List

algebraic equation	angle measure	congruency statement
congruent angles	congruent polygons	congruent segments
congruent triangles	proof	segment measure

Choose the concept from the list above that best represents the item in each box.

<p>1. $\overline{GH} \cong \overline{ST}$</p>	<p>2. $m\angle A = 45$</p>	<p>3. </p>
<p>4. $YZ = MN$</p> 	<p>5. $\triangle ABC \cong \triangle XYZ$</p>	<p>6. Given: \overline{BD} is the angle bisector of $\angle ABC$, and \overline{BD} is the perpendicular bisector of \overline{AC}.</p> <p>Prove: $\triangle ADB \cong \triangle CDB$</p>
<p>7. $m\angle H = 5x$ $m\angle W = x + 28$ Solve $5x = x + 28$ to find the measures of $\angle H$ and $\angle W$.</p>	<p>8. $BC = 3$ cm</p>	<p>9. $\angle ADB$ and $\angle SDT$ are vertical angles. So, $\angle ADB \cong \angle SDT$.</p>

4-2

Triangle Congruence
by SSS and SAS

Vocabulary

● Review

1. Use the diagram at the right. Find each.

included angle between \overline{AB} and \overline{CA}



included side between $\angle A$ and $\angle C$



included angle between \overline{BC} and \overline{CA}



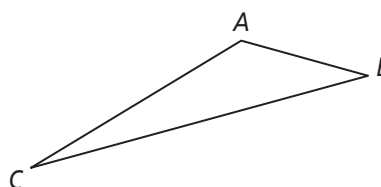
included side between $\angle B$ and $\angle C$



included angle between \overline{BC} and \overline{AB}



included side between $\angle B$ and $\angle A$



● Vocabulary Builder

postulate (noun) PAHS chuh lit

Definition: A **postulate** is a statement that is accepted as true without being proven true.

Main Idea: In geometry, you use what you know to be true to prove new things true. The statements that you accept as true without proof are called **postulates** or axioms.

● Use Your Vocabulary

2. Underline the correct word to complete the sentence.

You can use properties, *postulates*, and previously proven theorems as reasons / statements in a proof.

3. **Multiple Choice** What is a *postulate*?

- (A) a convincing argument using deductive reasoning
 (B) a conjecture or statement that you can prove true
 (C) a statement accepted as true without being proven true
 (D) a conclusion reached by using inductive reasoning

Take note

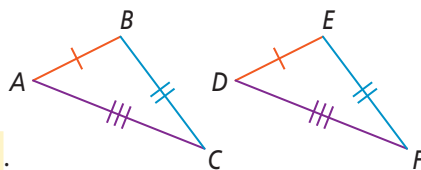
Postulate 4-1 Side-Side-Side (SSS) Postulate

Postulate 4-1 Side-Side-Side (SSS) Postulate

If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

4. Use the figures at the right to complete the sentence.

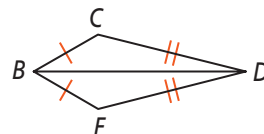
If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong$, then $\triangle ABC \cong \triangle$.



Problem 1 Using SSS

Got It? Given: $\overline{BC} \cong \overline{BF}$, $\overline{CD} \cong \overline{FD}$

Prove: $\triangle BCD \cong \triangle BFD$



5. You know two pairs of sides that are congruent. What else do you need to prove the triangles congruent by SSS?

6. The triangles share side .

7. Complete the steps of the proof.

Statement	Reason
1) $\overline{BC} \cong$ <input type="text"/>	1) Given
2) $\overline{CD} \cong$ <input type="text"/>	2) Given
3) $\overline{BD} \cong$ <input type="text"/>	3) Reflexive Property of \cong
4) $\triangle BCD \cong$ <input type="text"/>	4) SSS

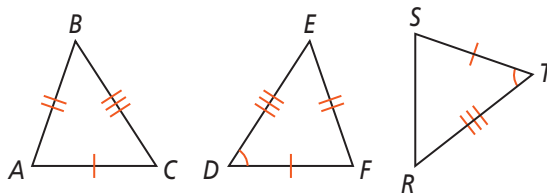
Take note

Postulate 4-2 Side-Angle-Side (SAS) Postulate

Postulate 4-2 Side-Angle-Side (SAS) Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

Use the figures below to complete each statement.



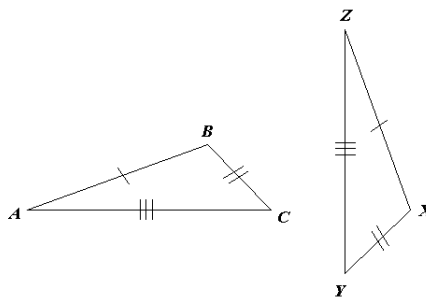
8. $\triangle DEF \cong$ by SAS.

9. $\triangle ABC \cong$ by SSS.

Section 4 - 4: Proving Congruence - SSS, SAS
Notes

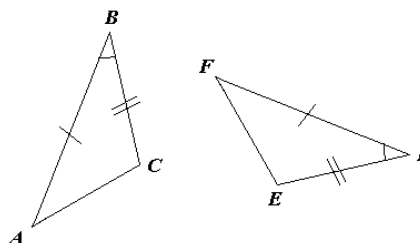
Side-Side-Side Congruence: If the _____ of one triangle are congruent to the sides of a second triangle, then the triangles are _____.

Abbreviation:



Side-Angle-Side Congruence: If two sides and the included _____ of one triangle are congruent to two _____ and the included angle of another triangle, then the triangles are _____.

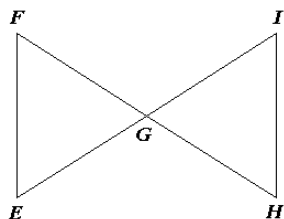
Abbreviation:



Example #1: Write a proof.

Given: $\overline{EI} \cong \overline{FH}$, $\overline{FE} \cong \overline{HI}$, and G is the midpoint of both \overline{EI} and \overline{FH} .

Prove: $\triangle FEG \cong \triangle HIG$

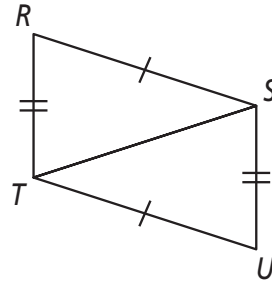


Example #2: Write a proof.

4-2 Additional Problems

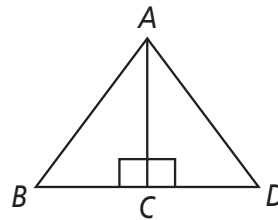
Triangle Congruence by SSS and SAS

Problem 1

Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{SU}$ **Prove:** $\triangle RST \cong \triangle UTS$ 

Problem 2

What other information do you need to prove $\triangle ABC \cong \triangle ADC$ by SAS? Explain your answer.

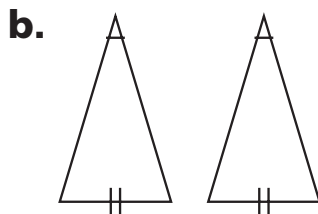
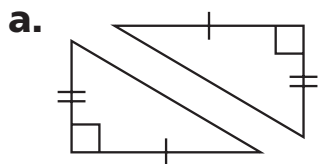


4-2 Additional Problems (continued)

Triangle Congruence by SSS and SAS

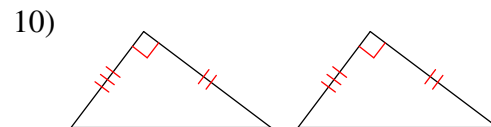
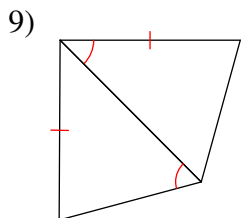
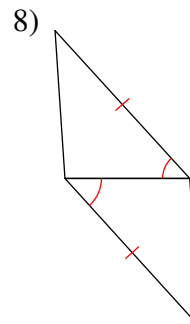
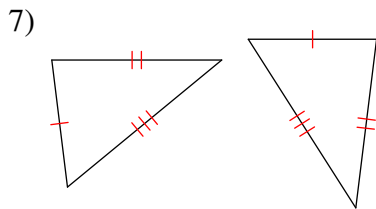
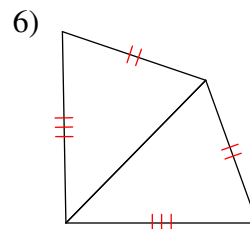
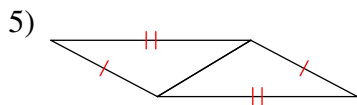
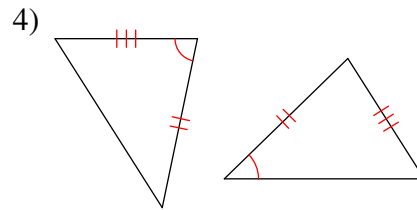
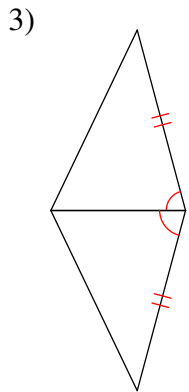
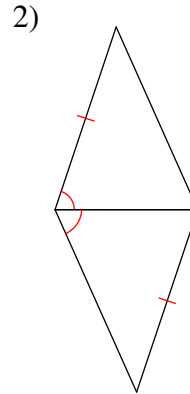
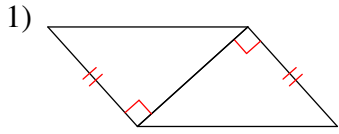
Problem 3

Would you use SSS or SAS to prove the triangles congruent?
 If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information*.
 Explain your answer.



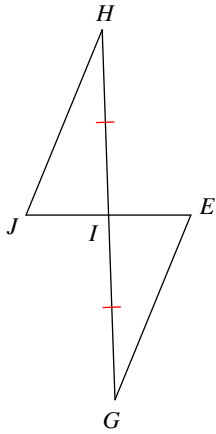
SSS and SAS Congruence

State if the two triangles are congruent. If they are, state how you know.

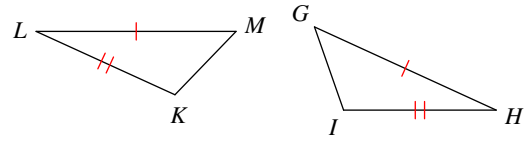


State what additional information is required in order to know that the triangles are congruent for the reason given.

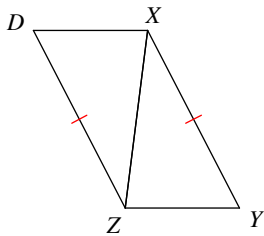
11) SAS



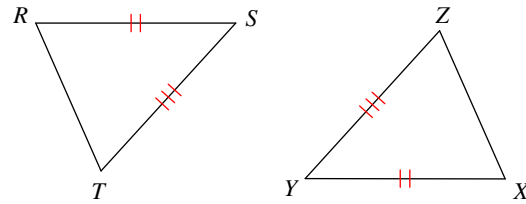
12) SAS



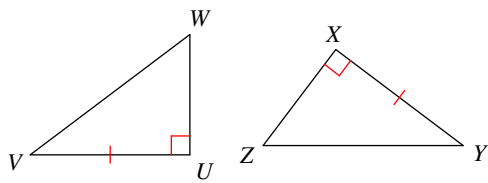
13) SSS



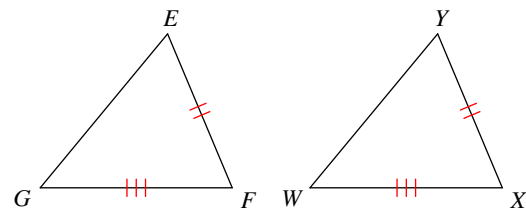
14) SSS



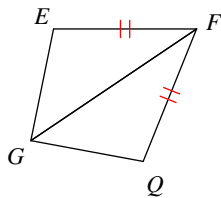
15) SAS



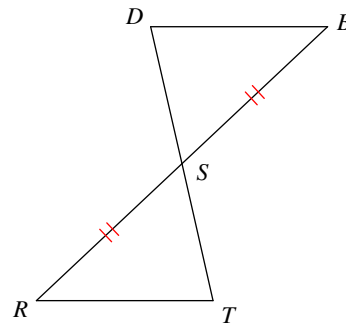
16) SSS



17) SAS



18) SAS



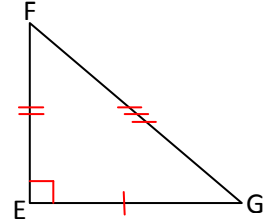
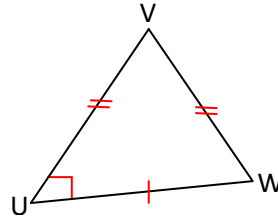
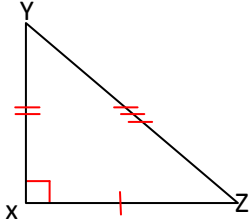
Name _____

Date _____

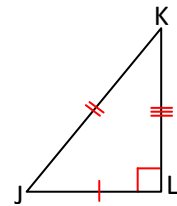
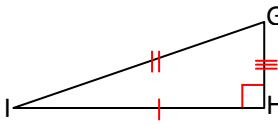
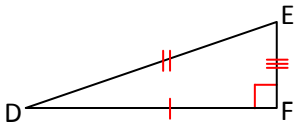
Congruent Triangles: SSS and SAS Theorems - Independent Practice Worksheet

Using either SSS or SAS determine which triangles are congruent.

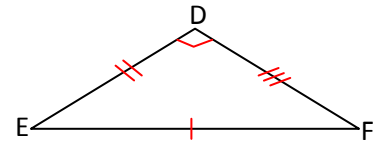
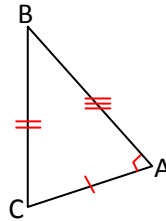
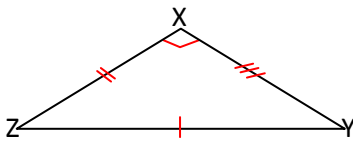
1.



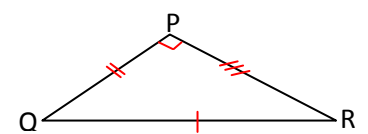
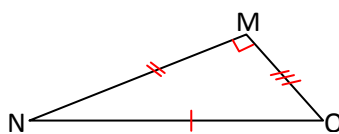
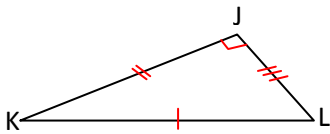
2.



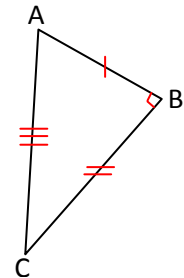
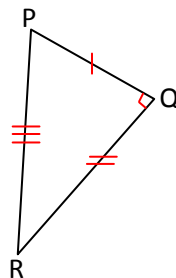
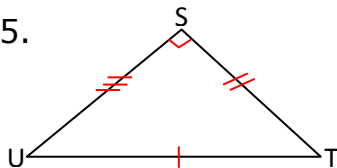
3.



4.



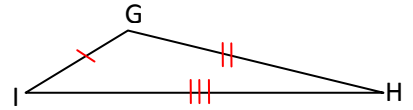
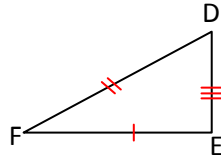
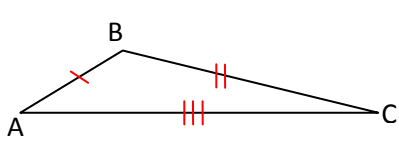
5.



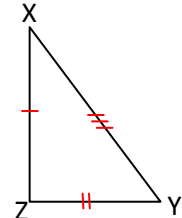
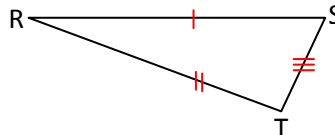
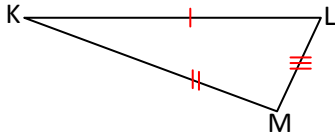
Name _____

Date _____

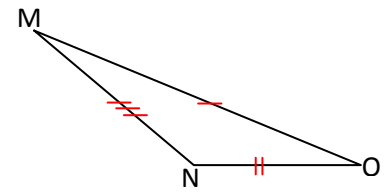
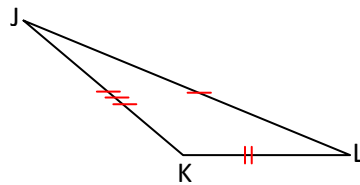
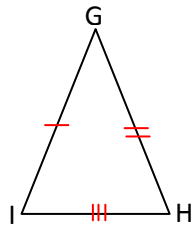
6.



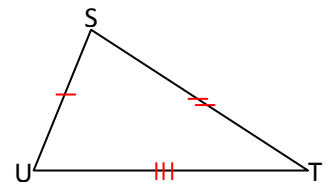
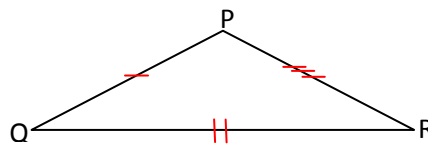
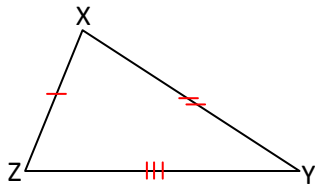
7.



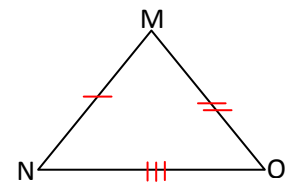
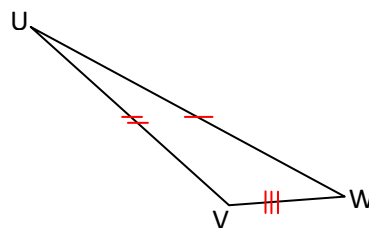
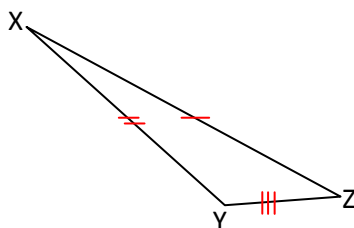
8.



9.



10.



4-3

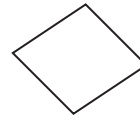
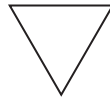
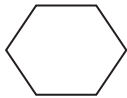
Triangle Congruence by ASA and AAS



Vocabulary

Review

1. Cross out the figure(s) that are NOT *triangle(s)*.



2. A *triangle* is a polygon with sides.

3. A *triangle* with a right angle is called a(n) obtuse / right / scalene *triangle*.

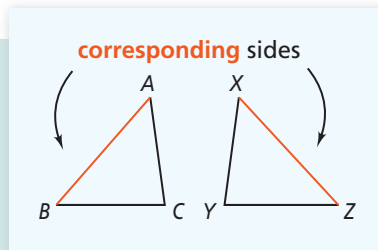
Vocabulary Builder

corresponding (adjective) kawr uh SPAHN ding

Other Word Forms: correspond (verb); correspondence (noun)

Definition: **Corresponding** means similar in position, purpose, or form.

Math Usage: Congruent figures have congruent **corresponding** parts.



Use Your Vocabulary

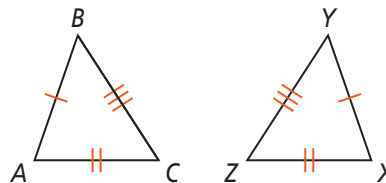
Draw a line from each part of $\triangle ABC$ in Column A to the *corresponding* part of $\triangle XYZ$ in Column B.

Column A

4. \overline{BC}
5. $\angle A$
6. \overline{AB}
7. $\angle C$
8. \overline{AC}
9. $\angle B$

Column B

- $\angle Z$
- $\angle Y$
- \overline{YZ}
- $\angle X$
- \overline{XY}
- \overline{XZ}



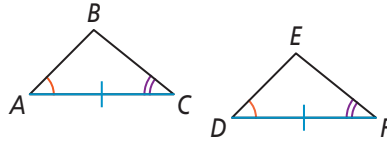
Postulate 4-3 Angle-Side-Angle (ASA) Postulate

Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

If ...

$$\angle A \cong \angle D, \overline{AC} \cong \overline{DF}, \angle C \cong \angle F$$



Then ...

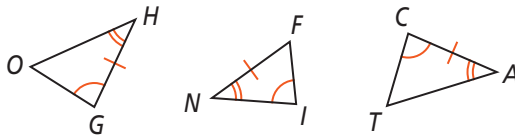
$$\triangle ABC \cong \triangle DEF$$

10. Explain how the ASA Postulate is different from the SAS Postulate.



Problem 1 Using ASA

Got It? Which two triangles are congruent by ASA? Explain.



11. Name the triangles. List the vertices in corresponding order: list the vertex with the one arc first, the vertex with the two arcs second, and the third vertex last.

12. $\angle G \cong \angle$ $\cong \angle$

13. $\angle H \cong \angle$ $\cong \angle$

14. $\overline{HG} \cong$ \cong

15. The congruent sides that are included between congruent angles are

and .

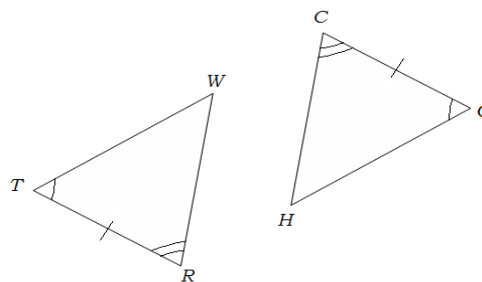
16. Write a congruence statement. Justify your reasoning.

\triangle $\cong \triangle$

Section 4 - 5: Proving Congruence - ASA, AAS

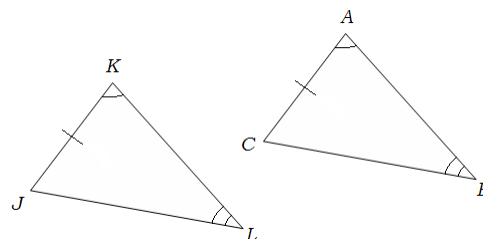
Notes

Angle-Side-Angle Congruence: If two _____ and the included _____ of one triangle are congruent to two angles and the included side of another triangle, then the triangles are _____.



Abbreviation:

Angle-Angle-Side Congruence: If two angles and a non-included side of one triangle are congruent to the corresponding two _____ and a side of a second triangle, then the two triangles are _____.

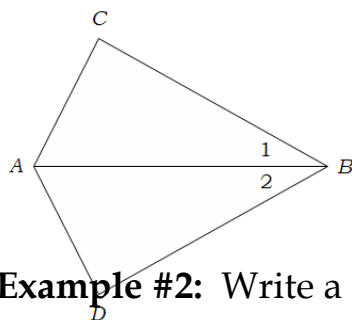


Abbreviation:

Example #1: Write a two-column proof.

Given: \overline{AB} bisects $\angle CAD$
 $\angle 1 \cong \angle 2$

Prove: $\triangle CAB \cong \triangle DAB$



Example #2: Write a two-column proof.

Given: $\overline{AD} \parallel \overline{CB}$
 $\angle A \cong \angle C$

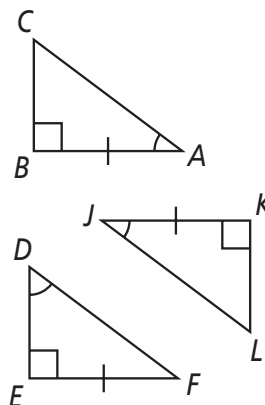
4-3

Additional Problems

Triangle Congruence by ASA and AAS

Problem 1

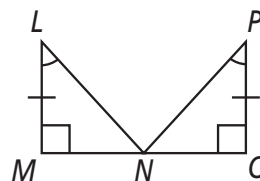
Which two triangles are congruent by ASA?



Problem 2

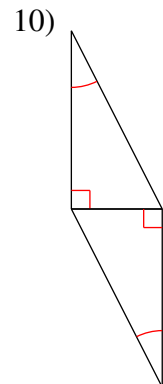
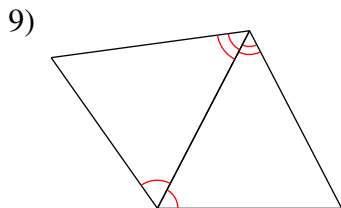
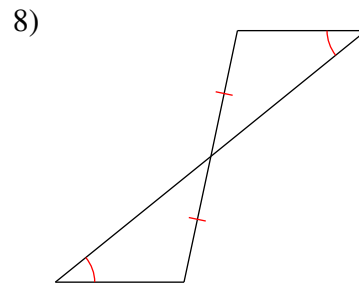
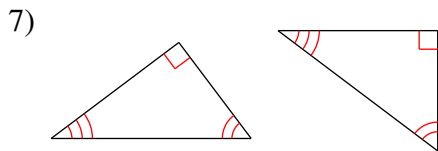
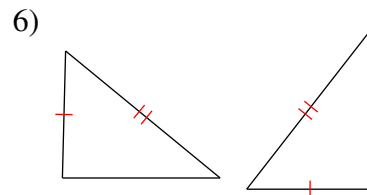
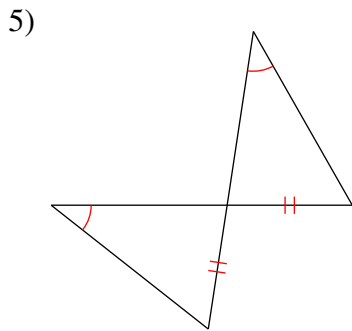
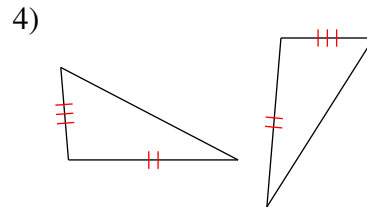
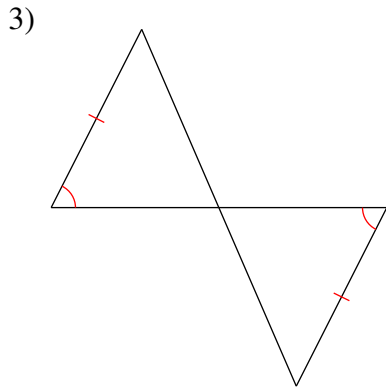
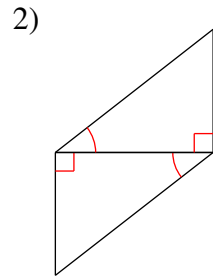
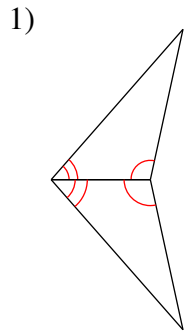
Given: $\overline{LM} \cong \overline{PO}$, $\angle L \cong \angle P$, $\angle M$ and $\angle O$ are both right angles.

Prove: $\triangle LMN \cong \triangle PON$



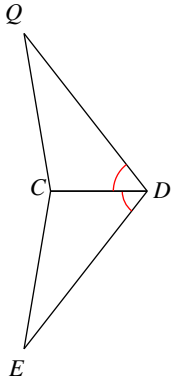
ASA and AAS Congruence

State if the two triangles are congruent. If they are, state how you know.

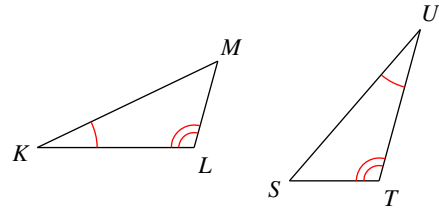


State what additional information is required in order to know that the triangles are congruent for the reason given.

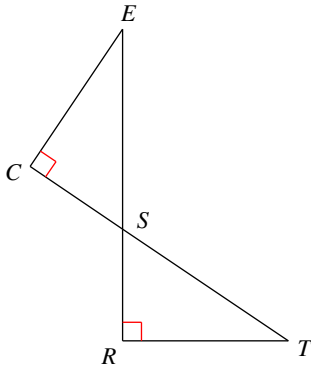
11) ASA



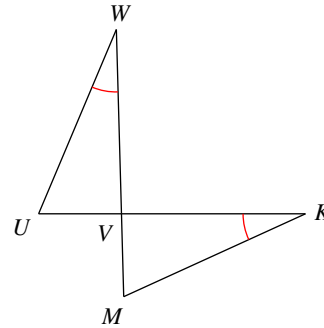
12) ASA



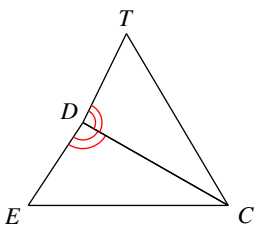
13) ASA



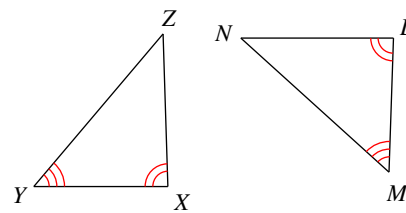
14) ASA



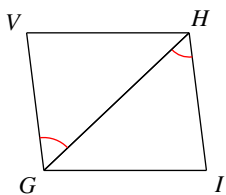
15) AAS



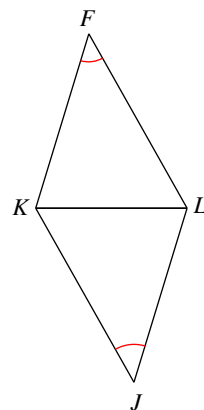
16) AAS



17) ASA



18) AAS



4-4

Using Corresponding Parts of Congruent Triangles



Vocabulary

Review

Underline the correct word(s) to complete each sentence.

- The *Reflexive* Property of Congruence states that any geometric figure is congruent / similar to itself.
- The *Reflexive* Property of Equality states that any quantity is equal to / greater than / less than itself.
- Circle the expressions that illustrate the *Reflexive* Property of Equality.

$$a = a$$

$$\text{If } AB = 2, \text{ then } 2 = AB.$$

$$3(x + y) = 3x + 3y$$

$$5 + c = 5 + c$$

- Circle the expressions that illustrate the *Reflexive* Property of Congruence.

$$\text{If } \angle A \cong \angle B, \text{ then } \angle B \cong \angle A.$$

$$\text{If } \overline{CD} \cong \overline{LM} \text{ and } \overline{LM} \cong \overline{XY}, \text{ then } \overline{CD} \cong \overline{XY}.$$

$$\angle ABC \cong \angle ABC$$

$$\overline{CD} \cong \overline{CD}$$

Vocabulary Builder

proof (noun) proof

Related Word: prove (verb)

Definition: A **proof** is convincing evidence that a statement or theory is true.

Math Usage: A **proof** is a convincing argument that uses deductive reasoning.

Use Your Vocabulary

Complete each statement with *proof* or *prove*.

- In geometry, a ? uses definitions, postulates, and theorems to prove theorems.

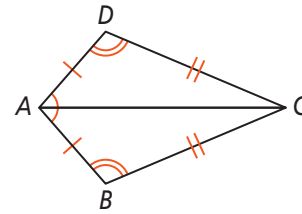
- No one can ? how our universe started.

- He can ? when he bought the computer because he has a receipt.

8. Complete the steps in the *proof*.

Given: $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{DC}$,
 $\angle D \cong \angle B$, $\angle DAC \cong \angle BAC$

Prove: $\triangle ABC \cong \triangle ADC$



Statements

- 1) $\overline{AB} \cong$ $\overline{BC} \cong$
- 2) $\overline{AC} \cong$
- 3) $\angle D \cong$ $\angle DAC \cong$
- 4) $\angle DCA \cong$
- 5) $\triangle ABC \cong$

Reasons

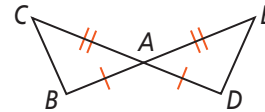
- 1) Given
- 2) Reflexive Property of \cong
- 3) Given
- 4) Third Angles Theorem
- 5) Definition of \cong triangles



Problem 1 Proving Parts of Triangles Congruent

Got It? Given: $\overline{BA} \cong \overline{DA}$, $\overline{CA} \cong \overline{EA}$

Prove: $\angle C \cong \angle E$



9. Name four ways you can use congruent parts of two triangles to prove that the triangles are congruent.

<input type="text"/>	<input type="text"/>
<input type="text"/>	<input type="text"/>

10. To prove triangles are congruent when you know two pairs of congruent corresponding sides, you can use or .

Underline the correct word to complete the sentence.

11. The *Given* states and the diagram shows that there are pairs of congruent sides.

12. Give a reason for each statement of the proof.

Statements

- 1) $\overline{BA} \cong \overline{DA}$
- 2) $\overline{CA} \cong \overline{EA}$
- 3) $\angle CAB \cong \angle EAD$
- 4) $\triangle CAB \cong \triangle EAD$
- 5) $\angle C \cong \angle E$

Reasons

- 1)
- 2)
- 3)
- 4)
- 5)

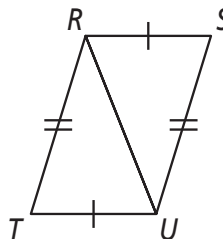
4-4 Additional Problems

Using Corresponding Parts of Congruent Triangles

Problem 1

Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$

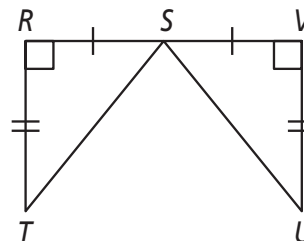
Prove: $\angle T \cong \angle S$



Problem 2

Given: $\overline{RS} \cong \overline{VS}$, $\overline{RT} \cong \overline{VU}$, $\angle R$ and $\angle V$ are both right angles.

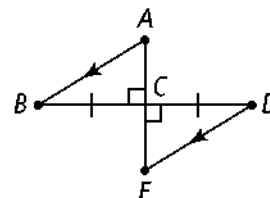
Prove: $\angle T \cong \angle U$



4-4 ELL Support

Using Corresponding Parts of Congruent Triangles

There are two sets of note cards below that show how to prove \overline{BD} is the perpendicular bisector of \overline{AE} . The set on the left has the statements and the set on the right has the reasons. Write the statements and the reasons in the correct order.



Statements

$\angle BAC \cong \angle DEC$

$\overline{AC} \cong \overline{EC}$

$\triangle ACB \cong \triangle ECD$

\overline{BD} is the perpendicular bisector of \overline{AE} .

$\overline{BC} \cong \overline{DC}$; $\angle ACB$ and $\angle ECD$ are right angles;
 $\overline{AB} \parallel \overline{DE}$

$\angle ACB \cong \angle ECD$

Reasons

Definition of the perpendicular bisector

Angle-Angle-Side (AAS) Theorem

When parallel lines are cut by a transversal, alternate interior angles are congruent.

Corresponding parts of congruent triangles are congruent.

Given

All right angles are congruent.

Statements

1)
2)
3)
4)
5)
6)

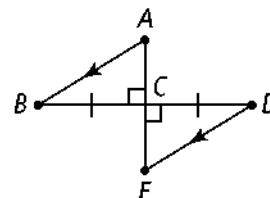
Reasons

1)
2)
3)
4)
5)
6)

4-4 ELL Support

Using Corresponding Parts of Congruent Triangles

There are two sets of note cards below that show how to prove \overline{BD} is the perpendicular bisector of \overline{AE} . The set on the left has the statements and the set on the right has the reasons. Write the statements and the reasons in the correct order.



Statements

$\angle BAC \cong \angle DEC$

$\overline{AC} \cong \overline{EC}$

$\triangle ACB \cong \triangle ECD$

\overline{BD} is the perpendicular bisector of \overline{AE} .

$\overline{BC} \cong \overline{DC}$; $\angle ACB$ and $\angle ECD$ are right angles;
 $\overline{AB} \parallel \overline{DE}$

$\angle ACB \cong \angle ECD$

Reasons

Definition of the perpendicular bisector

Angle-Angle-Side (AAS) Theorem

When parallel lines are cut by a transversal, alternate interior angles are congruent.

Corresponding parts of congruent triangles are congruent.

Given

All right angles are congruent.

Statements

1)
2)
3)
4)
5)
6)

Reasons

1)
2)
3)
4)
5)
6)

4-6

Congruence in Right Triangles



Vocabulary

Review

Write T for *true* or F for *false*.

- Segments that are *congruent* have the same length.
- Polygons that are *congruent* have the same shape but are not always the same size.
- In *congruent* figures, corresponding angles have the same measure.

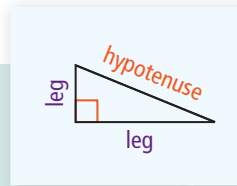
Vocabulary Builder

hypotenuse (noun) hy PAH tuh noos

Related Word: leg

Definition: The **hypotenuse** is the side opposite the right angle in a right triangle.

Main Idea: The **hypotenuse** is the longest side in a right triangle.



Use Your Vocabulary

Underline the correct word(s) to complete each sentence.

- One side of a right triangle is / is not a *hypotenuse*.
- A right triangle has one / two / three *legs*.
- The length of the *hypotenuse* is always equal to / greater than / less than the lengths of the *legs*.

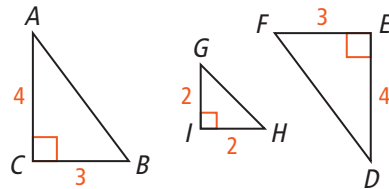
Use the triangles at the right for Exercises 7 and 8.

- Cross out the side that is NOT a *hypotenuse*.

\overline{BC} \overline{AB} \overline{GH} \overline{FD}

- Circle the *leg(s)*.

\overline{AC} \overline{AB} \overline{HI} \overline{ED}



You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove right triangles congruent by using one pair of right angles, a pair of hypotenuses, and a pair of legs.

Take note

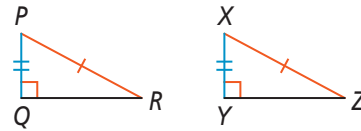
Theorem 4-6 Hypotenuse-Leg (HL) Theorem and Conditions

Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.

If . . .

$\triangle PQR$ and $\triangle XYZ$ are right triangles,
 $\overline{PR} \cong \overline{XZ}$, and $\overline{PQ} \cong \overline{XY}$



Then . . .

$\triangle PQR \cong \triangle XYZ$

9. To use the HL Theorem, the triangles must meet three conditions. Complete each sentence with *right* or *congruent*.

There are two ? triangles.

The triangles have ? hypotenuses.

There is one pair of ? legs.

Use the information in the Take Note for Exercises 10–12.

10. How do the triangles in the Take Note meet the first condition in Exercise 9? Explain.

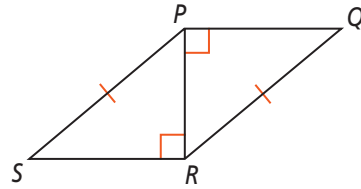
11. How do the triangles in the Take Note meet the second condition in Exercise 9? Explain.

12. How do the triangles in the Take Note meet the third condition in Exercise 9? Explain.



Problem 1 Using the HL Theorem

Got It? Given: $\angle PRS$ and $\angle RPQ$ are right angles, $\overline{SP} \cong \overline{QR}$
Prove: $\triangle PRS \cong \triangle RPQ$



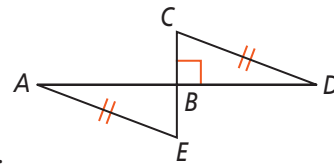
13. Complete each step of the proof.

Given $\angle PRS$ and \angle are right angles.	Given $\overline{SP} \cong$ 	Reflexive Prop. of \cong $\overline{PR} \cong$
↓	↓	↓
Definition of right triangle $\triangle PRS$ and \triangle are right triangles.	HL Theorem $\triangle PRS \cong \triangle$ 	



Problem 2 Writing a Proof Using the HL Theorem

Got It? Given: $\overline{CD} \cong \overline{EA}$, \overline{AD} is the perpendicular bisector of \overline{CE}
Prove: $\triangle CBD \cong \triangle EBA$



14. Circle what you know because \overline{AD} is the perpendicular bisector of \overline{CE} .

$\angle CBD$ and $\angle EBA$ are right angles.	$\angle CBD$ and $\angle EBA$ are acute angles.
B is the midpoint of \overline{AD} .	B is the midpoint of \overline{CE} .

15. Circle the congruent legs.

\overline{AB}	\overline{CB}	\overline{DB}	\overline{EB}
-----------------	-----------------	-----------------	-----------------

16. Write the hypotenuse of each triangle.

$\triangle CBD$ 	$\triangle EBA$
--	--

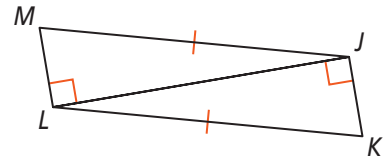
17. Complete the proof.

Statements	Reasons
1) $\overline{CD} \cong$ 	1) Given
2) $\angle CBD$ and \angle are right \angle s.	2) Definition of \perp bisector
3) $\triangle CBD$ and \triangle are right \triangle s.	3) Definition of right \triangle
4) $\overline{CB} \cong$ 	4) Definition of \perp bisector
5) $\triangle CBD \cong$ 	5) HL Theorem



Lesson Check • Do you UNDERSTAND?

Error Analysis Your classmate says that there is not enough information to determine whether the two triangles at the right are congruent. Is your classmate correct? Explain.



Write T for *true* or F for *false*.

18. There are three right angles.
19. There are two right triangles.
20. There are two congruent hypotenuses.
21. There are no congruent legs.
22. You need to use the Reflexive Property of Congruence.
23. $\overline{LJ} \cong \overline{LJ}$ is given.
24. Do you always need three congruent corresponding parts to prove triangles congruent? Explain.

25. Is your classmate correct? Explain.

Copyright © by Pearson Education, Inc. or its affiliates. All Rights Reserved.

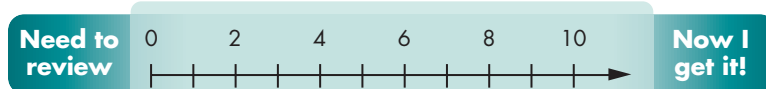


Math Success

Check off the vocabulary words that you understand.

- hypotenuse legs of a right triangle

Rate how well you can use *the Hypotenuse-Leg (HL) Theorem*.



4-6 Additional Problems

Congruence in Right Triangles

Problem 1

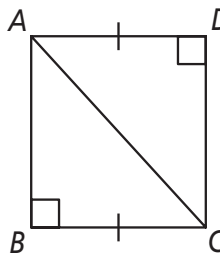
Raymond built two triangular supports for a picnic table with the dimensions shown below. Are the supports congruent? Explain.



Problem 2

Given: $\overline{AD} \cong \overline{CB}$, $\angle B$ and $\angle D$ are right angles

Prove: $\triangle ABC \cong \triangle CDA$



4-6 Standardized Test Prep

Congruence in Right Triangles

Multiple Choice

For Exercises 1-4, choose the correct letter.

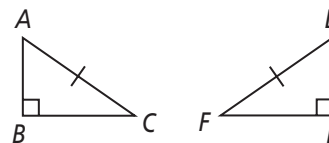
1. Which additional piece of information would allow you to prove that the triangles are congruent by the HL theorem?

(A) $m\angle DFE = 40$

(C) $\overline{AB} \cong \overline{DE}$

(B) $m\angle F = m\angle ABC$

(D) $\overline{AC} \cong \overline{DF}$



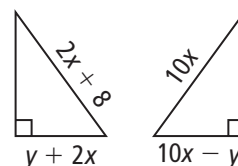
2. For what values of x and y are the triangles shown congruent?

(F) $x = 1, y = 4$

(H) $x = 4, y = 1$

(G) $x = 2, y = 4$

(I) $x = 1, y = 3$



3. Two triangles have two pairs of corresponding sides that are congruent. What else must be true for the triangles to be congruent by the HL Theorem?

(A) The included angles must be right angles.

(B) They have one pair of congruent angles.

(C) Both triangles must be isosceles.

(D) There are right angles adjacent to just one pair of congruent sides.

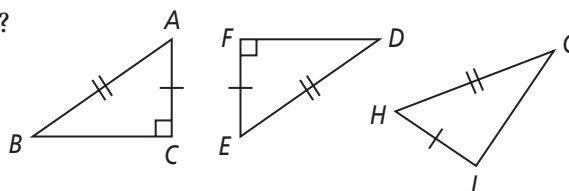
4. Which of the following statements is true?

(F) $\triangle BAC \cong \triangle GHI$ by SAS.

(G) $\triangle DEF \cong \triangle GHI$ by SAS.

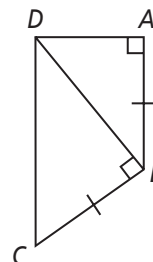
(H) $\triangle BAC \cong \triangle DEF$ by HL.

(I) $\triangle DEF \cong \triangle GHI$ by HL.



Extended Response

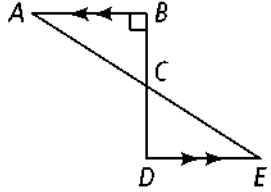
5. Are the given triangles congruent by the HL Theorem? Explain.



4-6 ELL Support

Congruence in Right Triangles

The column on the left shows the steps used to prove that $\overline{AB} \cong \overline{ED}$. Use the column on the left to answer each question in the column on the right.

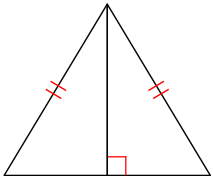
<p>Problem \cong in Right Triangles</p> <p>Given: C is the midpoint of \overline{AE} and \overline{BD}</p> <p>Prove: $\overline{AB} \cong \overline{ED}$</p> 	<p>1. What is the definition of the midpoint of a line segment?</p> <p>_____</p> <p>_____</p> <p>_____</p>
<p>1) C is the midpoint of $\overline{AB} \parallel \overline{DE}$ and \overline{BD}; $\overline{AB} \parallel \overline{DE}$; $m\angle B = 90$</p> <p>Given</p>	<p>2. How do you know that $\overline{AB} \parallel \overline{DE}$ and $m\angle B = 90$?</p> <p>_____</p>
<p>2) $\overline{AC} \cong \overline{EC}$; $\overline{BC} \cong \overline{DC}$</p> <p>Definition of midpoint</p>	<p>3. What does the symbol \cong between two line segments mean?</p> <p>_____</p>
<p>3) $\angle B = \angle D$</p> <p>Alternate Interior Angles Theorem</p>	<p>4. What does the word <i>interior</i> mean?</p> <p>_____</p>
<p>4) $\angle B$ and $\angle D$ are right angles.</p> <p>Definition of a right angle</p>	<p>5. What is the measure of $\angle D$?</p> <p>_____</p>
<p>5) $\triangle ABC \cong \triangle EDC$</p> <p>Hypotenuse-Leg (HL) Postulate</p>	<p>6. What information is necessary to apply the HL Postulate?</p> <p>_____</p> <p>_____</p> <p>_____</p>
<p>6) $\overline{AB} \cong \overline{ED}$</p> <p>Corresponding parts of congruent triangles are congruent.</p>	<p>7. What are the corresponding angles and sides for $\triangle ABC$ and $\triangle EDC$?</p> <p>_____</p> <p>_____</p> <p>_____</p>

Congruence Postulates: SSS, ASA, SAS, AAS, and HL

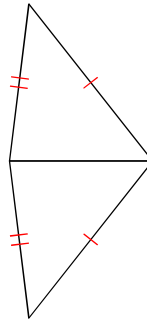
© 2013 Kuta Software LLC. All rights reserved.

State if the two triangles are congruent. If they are, state how you know.

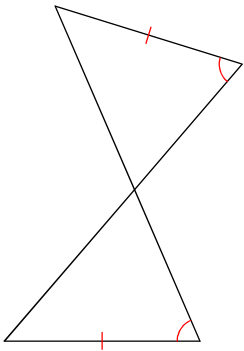
1)



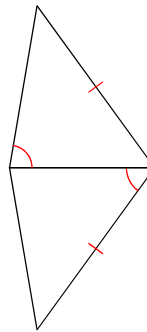
2)



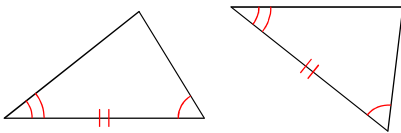
3)



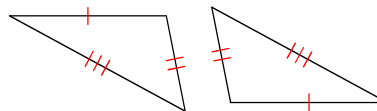
4)



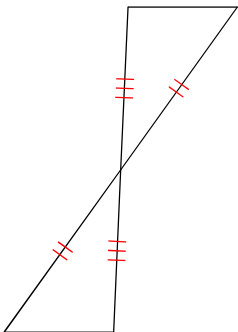
5)



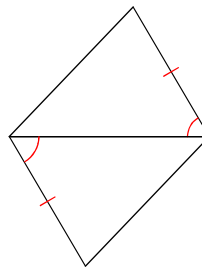
6)



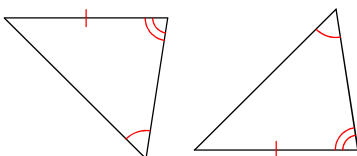
7)



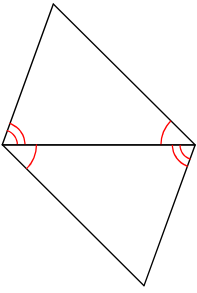
8)



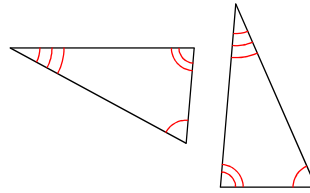
9)



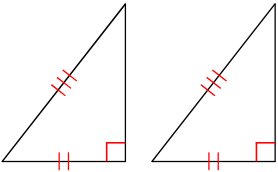
10)



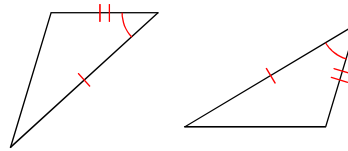
11)



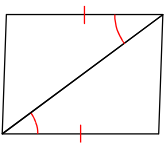
12)



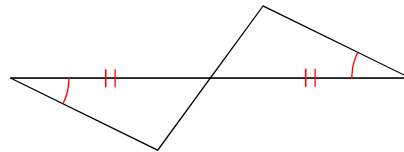
13)



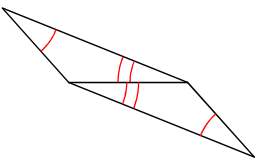
14)



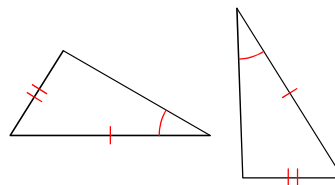
15)



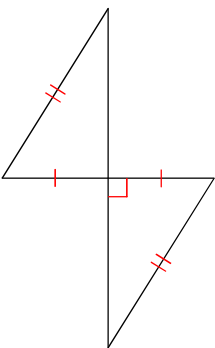
16)



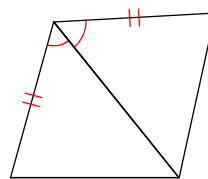
17)



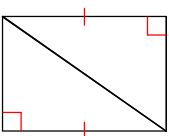
18)



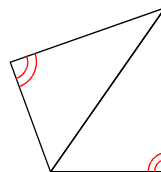
19)



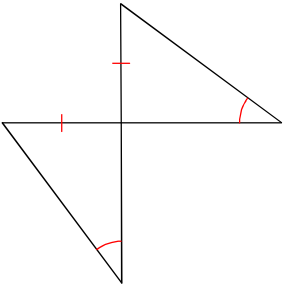
20)



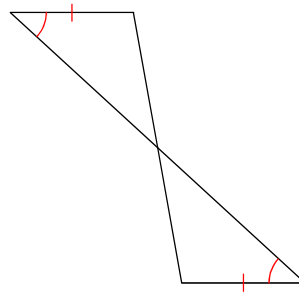
21)



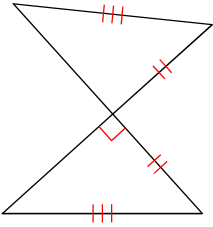
22)



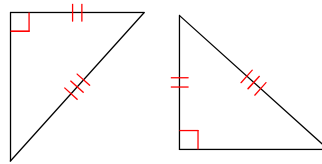
23)



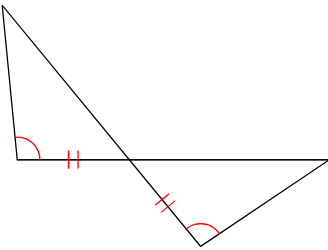
24)



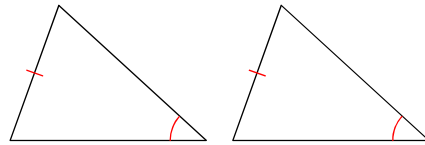
25)



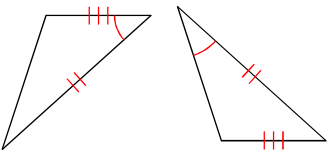
26)



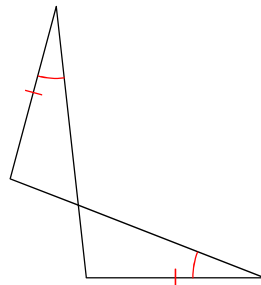
27)



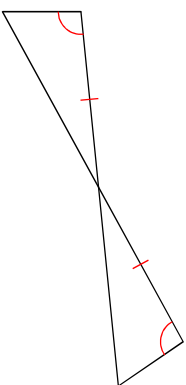
28)



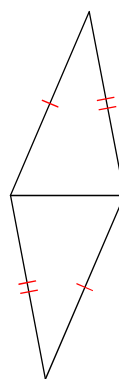
29)

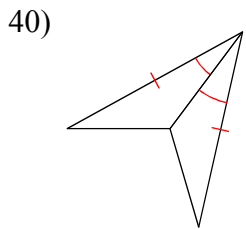
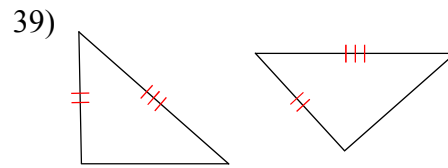
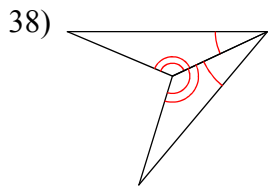
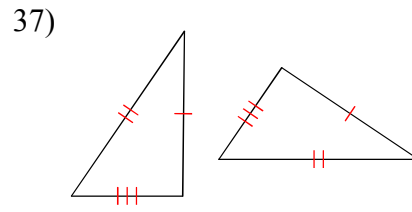
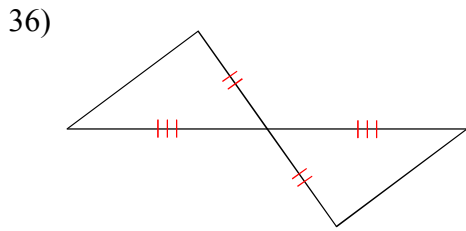
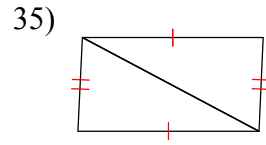
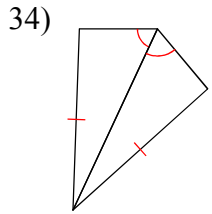
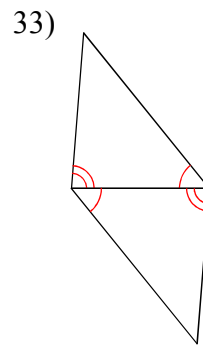
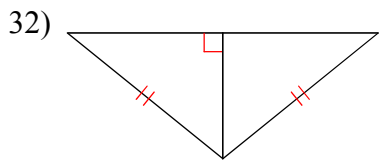


30)



31)





4-7

Congruence in Overlapping Triangles



Vocabulary

Review

1. Circle the *common* side of $\triangle ABC$ and $\triangle ADC$.

\overline{AB} \overline{AC} \overline{AD} \overline{BC}

2. Circle the *common* side of $\triangle XWZ$ and $\triangle YWZ$.

\overline{WZ} \overline{WX} \overline{WY} \overline{ZY}

3. Circle the *common* side of $\triangle RST$ and $\triangle RPT$.

\overline{RP} \overline{RS} \overline{RT} \overline{ST}

Vocabulary Builder

overlapping (adjective) oh vur LAP ing

Other Word Form: overlap (noun)

Definition: **Overlapping** events or figures have parts in common.

Math Usage: Two or more figures with common regions are **overlapping** figures.

Use Your Vocabulary

Circle the common regions of the *overlapping* figures in the diagram at the right.

4. $\triangle FGD$ and $\triangle CBE$

$\triangle ABG$ $\triangle ACF$ $\triangle EHD$ $\triangle GHB$

5. $\triangle BEC$ and $\triangle HED$

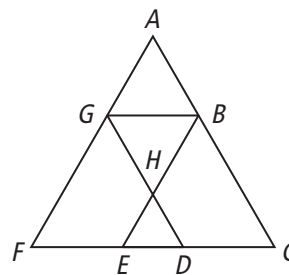
$\triangle BEC$ $\triangle GBH$ $\triangle GDF$ $\triangle HED$

6. $\triangle ACF$ and $\triangle ABG$

$\triangle ABG$ $\triangle ACF$ $\triangle GBH$ $\triangle EHD$

7. $\triangle ACF$ and $\triangle GBH$

$\triangle ABG$ $\triangle ACF$ $\triangle GBH$ $\triangle HED$

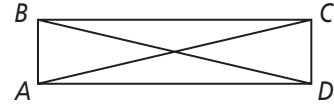




Problem 1 Identifying Common Parts

Got It? What is the common side in $\triangle ABD$ and $\triangle DCA$?

8. Separate and redraw $\triangle ABD$ and $\triangle DCA$.



9. You drew twice, so the common side is .

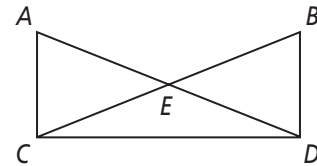


Problem 2 Using Common Parts

Got It? Given: $\triangle ACD \cong \triangle BDC$

Prove: $\overline{CE} \cong \overline{DE}$

10. Use the information in the problem to complete the problem-solving model below.



Know	Need	Plan

11. Use the justifications below to complete each statement.

Statements

Reasons

1) $\triangle ACD \cong$

1) Given

2) $\overline{AC} \cong$

2) Corresponding parts of \cong triangles are \cong .

3) $\angle A \cong$

3) Corresponding parts of \cong triangles are \cong .

4) $\cong \angle BED$

4) Vertical angles are congruent.

5) $\cong \triangle BED$

5) Angle-Angle-Side (AAS) Theorem

6) $\overline{CE} \cong$

6) Corresponding parts of \cong triangles are \cong .

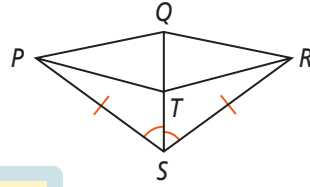
12. How could you use the Converse of the Isosceles Triangle Theorem to prove $\overline{CE} \cong \overline{DE}$?



Problem 3 Using Two Pairs of Triangles

Got It? Given: $\overline{PS} \cong \overline{RS}$, $\angle PSQ \cong \angle RSQ$

Prove: $\triangle QPT \cong \triangle QRT$



13. Give the reason for each statement in the proof.

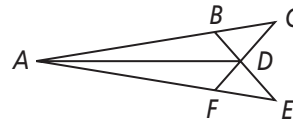
$\overline{PS} \cong \overline{RS}$	$\angle PSQ \cong \angle RSQ$	$\overline{ST} \cong \overline{ST}$ and $\overline{QS} \cong \overline{QS}$
$\triangle PST \cong \triangle RST$ and $\triangle QPS \cong \triangle QRS$		
$\overline{PT} \cong \overline{RT}$	$\overline{PQ} \cong \overline{RQ}$	
$\overline{QT} \cong \overline{QT}$	$\triangle QPT \cong \triangle QRT$	



Problem 4 Separating Overlapping Triangles

Got It? Given: $\angle CAD \cong \angle EAD$, $\angle C \cong \angle E$

Prove: $\overline{BD} \cong \overline{FD}$



14. Circle the angles that are vertical angles.

- $\angle ADB$ $\angle ADC$ $\angle ADE$ $\angle ADF$ $\angle BDC$ $\angle FDE$

15. Mark the angles that you know are congruent in each pair of separated triangles below.

--	--	--	--

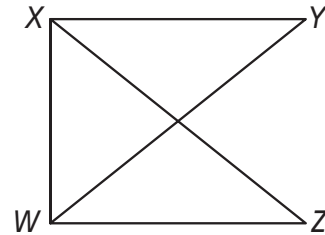
16. Which triangles are congruent by AAS? Explain.

4-7 Additional Problems

Congruence in Overlapping Triangles

Problem 1

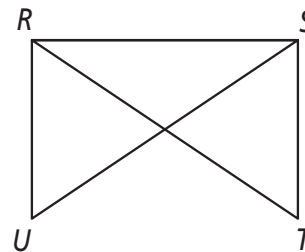
What is the common side in $\triangle WXZ$ and $\triangle XWY$?



Problem 2

Given: $\overline{RT} \cong \overline{US}$, $\angle URS$ and $\angle TSR$ are right angles.

Prove: $\angle T \cong \angle U$



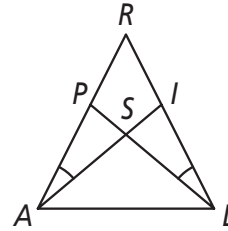
4-7 Additional Problems (continued)

Congruence in Overlapping Triangles

Problem 3

Given: $\overline{AR} \cong \overline{LR}$, $\overline{PR} \cong \overline{IR}$

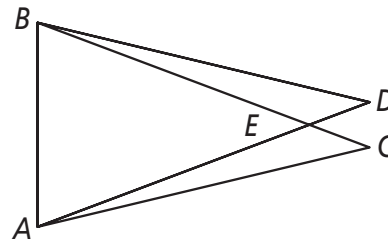
Prove: $\overline{AS} \cong \overline{LS}$



Problem 4

Given: $\overline{AE} \cong \overline{BE}$, $\angle C \cong \angle D$

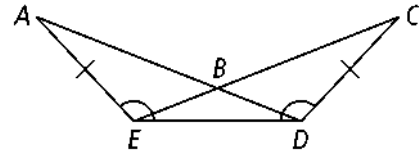
Prove: $\triangle ABD \cong \triangle BAC$



4-7 ELL Support

Congruence in Overlapping Triangles

A student wanted to prove $\overline{EB} \cong \overline{DB}$ given $\angle AED \cong \angle CDE$. She wrote the statements and reasons on note cards, but they got mixed up.



$\overline{EB} \cong \overline{DB}$ because corresponding parts of congruent triangles are congruent.	$\angle ABE \cong \angle CBD$ because vertical angles are congruent.	$\overline{ED} \cong \overline{DE}$ by the Reflexive Property of Congruence.	
$\triangle ABE \cong \triangle CBD$ by the Angle-Angle-Side (AAS) Theorem.	$\angle AED \cong \angle CDE$ and $\overline{AE} \cong \overline{CD}$ are given.	$\angle A \cong \angle C$ because corresponding parts of congruent triangles are congruent.	$\triangle AED \cong \triangle CDE$ by the Side-Angle-Side (SAS) Theorem.

Use the note cards to write the steps in order.

1. First, _____
2. Second, _____
3. Third, _____
4. Next, _____
5. Then, _____
6. Then, _____
7. Finally, _____

4-7

Standardized Test Prep

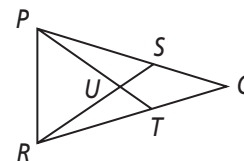
Congruence in Overlapping Triangles

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. What is the common angle of $\triangle PQT$ and $\triangle RSQ$?

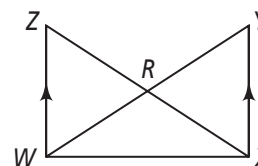
- (A) $\angle PQT$ (C) $\angle SRQ$
 (B) $\angle SPT$ (D) $\angle SUT$



Use the following information for Exercises 2–5.

Given: $\triangle ZWX \cong \triangle YXW$, $\overline{ZW} \parallel \overline{YX}$

Prove: $\triangle ZWR \cong \triangle YRX$



2. Which corresponding parts statement is needed to prove $\triangle ZWR \cong \triangle YRX$?

- (F) $\angle ZWR \cong \angle YXR$ (H) $ZW = YX$
 (G) $\angle Z \cong \angle R$ (I) $WX = WX$

3. A classmate writes the statement $\angle ZRW \cong \angle YRX$ to help prove the congruence of the triangles. What reason should the classmate give?

- (A) Given
 (B) Angles cut by a bisector are congruent.
 (C) Base angles of an isosceles triangle are congruent.
 (D) Vertical angles are congruent.

4. After using the congruence statements from Exercises 2 and 3, which statement can be used to prove the triangles congruent?

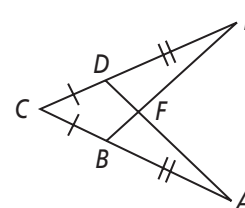
- (F) $\angle Z \cong \angle Y$ (H) $\overline{WX} \cong \overline{WX}$
 (G) $\angle ZWR \cong \angle RYX$ (I) $\overline{WR} \cong \overline{RX}$

5. Which theorem or postulate will prove $\triangle ZWR \cong \triangle YRX$?

- (A) SAS (B) SSS (C) ASA (D) AAS

Short Response

6. In the diagram at the right, which two triangles should be proved congruent first to help prove $\triangle ABF \cong \triangle EDF$?

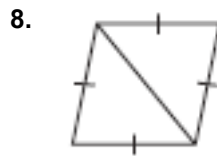
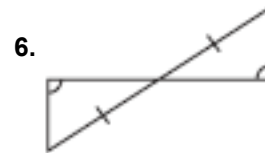
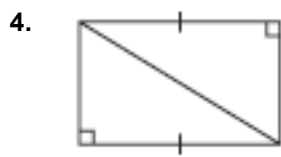
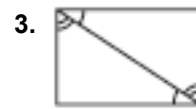


Chapter 4 Test

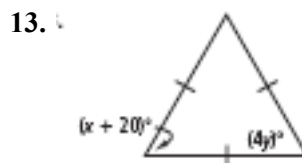
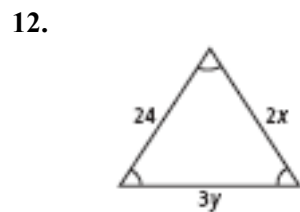
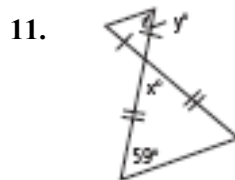
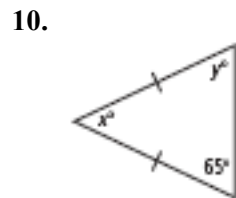
Form G

Do you know HOW?

State the postulate or theorem you would use to prove each pair of triangles congruent. If the triangles cannot be proven congruent, write *not enough information*.



Find the value of x and y .



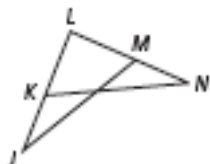
14. $\triangle CGI \cong \triangle MPR$. Name all of the pairs of corresponding congruent parts.

Chapter 4 Test (continued)

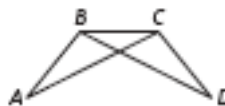
Form G

Name a pair of overlapping congruent triangles in each diagram. State whether the triangles are congruent by SSS, SAS, ASA, AAS, or HL.

15. Given: $\overline{LM} \cong \overline{LK}$; $\overline{LN} \cong \overline{LJ}$



16. Given: $\angle ABC \cong \angle DCB$; $\angle DBC \cong \angle ACB$



17. Given: $\angle E \cong \angle D \cong \angle DCF \cong \angle EFC$



18. Given: $\overline{HI} \cong \overline{JG}$

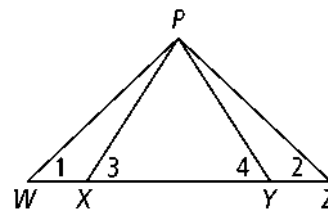


Do you UNDERSTAND?

19. **Reasoning** Complete the following proof by providing the reason for each statement.

Given: $\angle 1 \cong \angle 2$; $\overline{WX} \cong \overline{ZY}$

Prove: $\angle 3 \cong \angle 4$

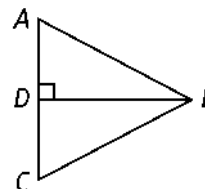


Statements	Reasons
1) $\angle 1 \cong \angle 2$; $\overline{WX} \cong \overline{ZY}$	1) ?
2) $\overline{WP} \cong \overline{ZP}$	2) ?
3) $\triangle WXP \cong \triangle ZYP$	3) ?
4) $\overline{XP} \cong \overline{YP}$	4) ?
5) $\angle 3 \cong \angle 4$	5) ?

20. **Reasoning** Write a proof for the following:

Given: $\overline{BD} \perp \overline{AC}$, D is the midpoint of \overline{AC} .

Prove: $\overline{BC} \cong \overline{BA}$



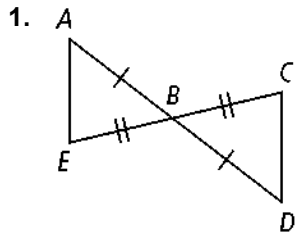
Chapter 4 Part B Test

Form K

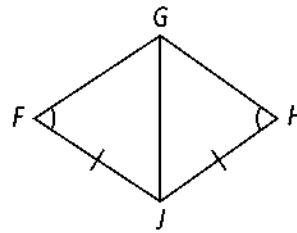
Lessons 4-4 through 4-7

Do you know HOW?

State the postulate or theorem you can use to prove each pair of triangles congruent. If the triangles cannot be proven congruent, write *not enough information*.

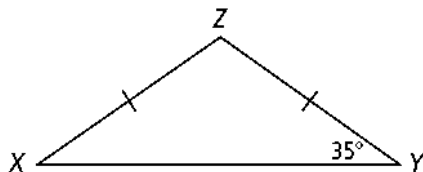


2.

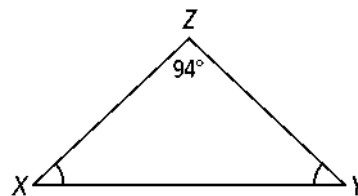


3. What is $m\angle X$?

a.

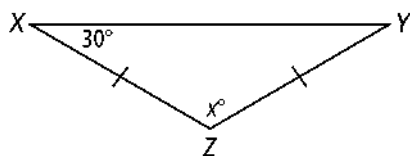


b.

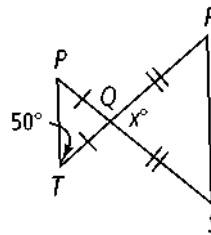


4. What is the value of x ?

a.

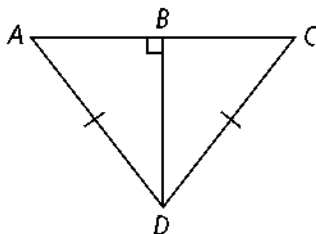


b.

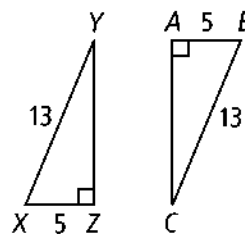


Write a congruence statement for each pair of triangles. If the triangles cannot be proven congruent, write *not enough information*.

5.



6.

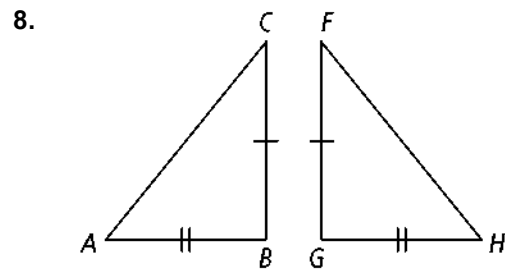
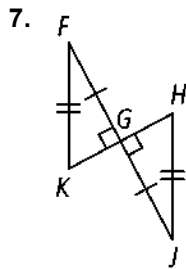


Chapter 4 Part B Test (continued)

Form K

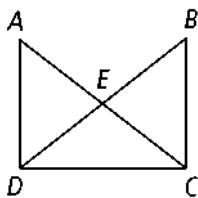
Lessons 4-4 through 4-7

Write a congruence statement for each pair of triangles. If the triangles cannot be proven congruent, write *not enough information*.

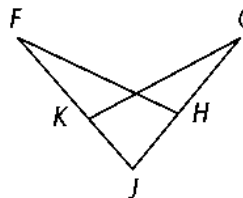


Identify any common angles or sides for the indicated triangles.

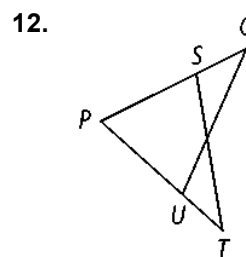
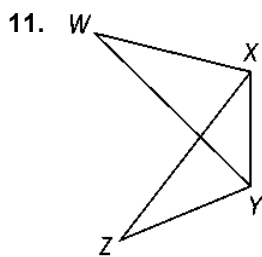
9. $\triangle ADC$ and $\triangle BDC$



10. $\triangle FHH$ and $\triangle GKJ$



Separate and redraw the indicated triangles.



Do you UNDERSTAND?

13. **Error Analysis** Your friend claims isosceles triangles are congruent if two corresponding sides are congruent. He explains there are only two different lengths of sides, so the third side must always be congruent. Explain the error in his reasoning.

14. **Compare and Contrast** How can you use the Isosceles Triangle Theorem to prove that all equilateral triangles are also equiangular?