GEOMETRY

Chapter 2: LOGIC

Name:_____

Teacher:_____

Pd: _____

Table of Contents

DAY 1

Warm – Up

$\overrightarrow{RT}$ bisects $\angle QRS$. If $m\angle QRS = 10x$ and $m\angle SRT = 3x + 30$, what is $m\angle QRS$?

Introduction to Logic

Logic is the study of reasoning. All reasoning, mathematical or verbal, is based on how we put sentences together.

A *mathematical sentence* is a sentence that states a fact or contains a complete idea. A sentence that can be judged true or false is called a *statement*. In the study of logic, each statement is designated by a letter (p, q, r, etc.) and is assigned a true value (*T* for true or *F* for false.)

Two types of mathematical sentences are

    a) *open sentence* – contains a variable and cannot assign a truth value.
        Ex. She is at the park.

        Ex. It is fun.

    b) *closed sentence (statement)* - a sentence that can be judged to be either true or false.
        Ex. The degree measure of a right angle is 90^0.

        Ex. There are eight days in a week.

Example/Practice 1:

Identify each of the following as a true sentence, a false sentence, an open sentence, or not a mathematical sentence at all.

a. Football is a water sport.

b. Football is a team sport.

c. He is a football player.

d. Do you like football?

e. Read this book.

f. $3x - 7 = 11$

g. $3x - 7$

Negation   *Symbol:* ~

The **negation** of a statement always has the opposite truth value of the original statement. It is usually formed by adding the word **not** to the original statement.

Ex.   p : There are 31 days in January.   (True)

    ~p : There are not 31 days in January.   (False)

- A statement and its negation have opposite truth values.

p	~p
T	F
F	T

Negation of symbols:

=	
>	
<	
≥	
≤	

Example/Practice 2:

Write the negations for each of the following true sentences.

1. Today's weather is sunny.   _____

2. Alice is not going to the play.   _____

3. x > 2   _____

Example 3:

In this example, symbols are used to represent statements. The truth value of each statement is given.

> k: Oatmeal is a cereal.   (True)
> m: Massachusetts is a city.   (False)

For each sentence given in symbolic form:

a. Write a complete sentence in words to show what the symbols represent.

b. Tell whether the statement is true or false.

(1) $\sim k$
(2) $\sim m$

Answers:

a. _____   b. _____

a. _____   b. _____

EXAMPLE 4

> Let p represent: January has 31 days
> Let q represent: Christmas is in December

For each given sentence:

a. Write the sentence in symbolic form using the symbols below.
b. Tell whether the sentence is true or false.

1.   It is not the case that January has 31 days   _____

2.   It is not true that Christmas is not in December   _____

3.   January has 31 days.   _____

Truth Table

To study sentences where we wish to consider all truth values that could be assigned, we use a device called a truth table. A <u>truth table</u> is a compact way of listing symbols to show all possible truth values for a set of sentences.

Example 5: Fill in all missing symbols.

p	~p
	F
F	

s	~s
	T
	F

Translating Statements from Words to Symbols

Words	Symbols
Not	~
And	∧
Or	∨
If.....then	→
If and only if	↔

Example 6:

Let *p* represents "The bird has wings"
Let *q* represents "The bird can fly"

Write each of the following sentences in symbolic form.

1. The bird has wings and the bird cannot fly.   _____

2. If the bird can fly, then the bird has wings.   _____

3. The bird cannot fly or the bird has wings.   _____

4. The bird can fly if and only if the bird has wings.   _____

Challenge Problem

Copy the truth table and fill in all missing symbols.

q	~q	~(~q)	~(~(~q))
T			
F			

Summary/Closure

There are two types of mathematical sentences:

An open sentence is a sentence which contains a variable.

- "$x + 2 = 8$" is an open sentence -- the variable is "x."
- "It is my favorite color." is an open sentence-- the variable is "It."
- The truth value of theses sentences depends upon the value replacing the variable.

A closed sentence, or statement, is a mathematical sentence which can be judged to be true or false. A closed sentence, or statement, has no variables.

- "Garfield is a cartoon character." is a true closed sentence, or statement.
- "A pentagon has exactly 4 sides." is a false closed sentence, or statement.

A compound sentence is formed when two or more thoughts are connected in one sentence. Words such as **and, or, if...then** and **if and only if** allow for the formation of compound sentences, or statements. Notice that more than one truth value is involved in working with a compound sentence.

- "Today is a vacation day *and* I sleep late."

- "You can call me at 10 o'clock *or* you can call me at 2 o'clock."

- "*If* you are going to the beach, *then* you should take your sunscreen."

- "A triangle is isosceles *if and only if* it has two congruent sides."

Mathematicians often use symbols and tables to represent concepts in logic. The use of these variables, symbols and tables creates a shorthand method for discussing logical sentences.

Truth table for negation (not):
(notice the symbol used for "not" in the table below)

A truth table is a pictorial representation of all of the possible outcomes of the truth value of a sentence. A letter such as p is used to represent the sentence or statement.

p	$\sim p$
T	F
F	T

Exit Ticket

What is the negation of the statement "The Sun is shining"?
1) It is cloudy.
2) It is daytime.
3) It is not raining.
4) The Sun is not shining.

What is the negation of the statement "I am not going to eat ice cream"?
1) I like ice cream.
2) I am going to eat ice cream.
3) If I eat ice cream, then I like ice cream.
4) If I don't like ice cream, then I don't eat ice cream.

In 1–8, write the negation of each sentence.
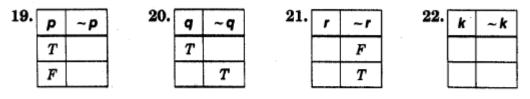
1. The school has a cafeteria.
2. Georgia is not a city.
3. A school bus is painted yellow.
4. $18 + 20 \div 2 = 28$
5. The measure of a right angle is 90°.
6. $1 + 2 + 3 \neq 4$
7. There are 100 centimeters in a meter.
8. Today is not Saturday.

In 9–18, for each given sentence: **a.** Write the sentence in symbolic form, using the symbols shown below. **b.** Then tell if the sentence is true, false, or open.

Let p represent "A cat is an animal."

Let q represent "A poodle is a cat."

Let r represent "His cat is gray."

9. A cat is an animal.
10. A poodle is a cat.
11. A poodle is not a cat.
12. A cat is not an animal.
13. His cat is gray.
14. His cat is not gray.
15. It is not true that a poodle is a cat.
16. It is not the case that a cat is an animal.
17. It is not the case that a cat is not an animal.
18. It is not the case that a poodle is not a cat.

In 19–22, copy the truth table for negation and fill in all missing symbols.

19.

p	$\sim p$
T	
F	

20.

q	$\sim q$
T	
	T

21.

r	$\sim r$
	F
	T

22.

k	$\sim k$

In 25–32, the symbols represent sentences.

> p: Summer follows spring. r: Baseball is a summer sport.
>
> q: Baseball is a sport. s: He likes baseball.

For each sentence given in symbolic form: **a.** Write a complete sentence in words to show what the symbols represent. **b.** Tell if the sentence is true, false, or open.

25. ~p **26.** ~q **27.** ~r **28.** ~s

29. ~(~q) **30.** ~(~p) **31.** ~(~r) **32.** ~(~s)

a. _____ b. _____

a. _____ b. _____

a. _____ b. _____

a. _____ b. _____

a. _____ b. _____

a. _____ b. _____

a. _____ b. _____

a. _____ b. _____

Translating Statements From Words To Symbols

In 1–8, use p and q to write the given sentence in symbolic form.

1. p represents "You are correct" and q represents "I agree." Write: "You are correct and I agree."

2. If p represents "I have studied" and q represents "I will pass this test," write: "I have studied or I will not pass this test."

3. If p represents "He is tall" and q represents "He is handsome," write: "He is not tall and he is handsome."

4. Let p represent "It is summer" and q represent "I go swimming." Write: "If I go swimming, then it is summer."

5. Let p represent "I have studied" and q represent "I will pass this test." Write: "If I have not studied, then I will not pass this test."

6. If p represents "Today is Monday" and q represents "I am tired," write: "Today is Monday and I am not tired."

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7. p represents "It is cold" and q represents "I will go skiing." Write: "If it is cold, then I will not go skiing."

8. If p represents "Two sides of a triangle are congruent" and q represents "Two angles of a triangle are congruent," write: "Two sides of a triangle are congruent if and only if two angles of a triangle are congruent."

9. If p represents "It is sunny" and q represents "It is warm," then which is a correct translation of $p \wedge \sim q$?
   (1) It is sunny and it is warm.
   (2) If it is sunny, then it is not warm.
   (3) It is sunny and it is not warm.
   (4) It is sunny or it is not warm.

10. If p represents "I do try" and q represents "I do succeed," then which is a correct translation of $\sim p \wedge q$?
    (1) I do try and I do succeed.
    (2) I do not try and I do succeed.
    (3) If I do not try, then I do succeed.
    (4) If I do not try, then I do not succeed.

11. If p represents "x is even" and q represents "x^2 is even," then which is a correct translation of $p \rightarrow \sim q$?
    (1) If x is even, then x^2 is odd.
    (2) If x is odd, then x^2 is even.
    (3) If x is even, then x^2 is even.
    (4) If x is odd, then x^2 is odd.

12. If p represents "Three sides of a triangle are congruent" and q represents "Three angles of a triangle are congruent," then which is a correct translation of $\sim p \leftrightarrow \sim q$?
    (1) Three sides of a triangle are not congruent if and only if three angles are congruent.
    (2) Three sides of a triangle are not congruent if three angles are congruent.
    (3) Three sides of a triangle are not congruent if three angles are not congruent.
    (4) Three sides of a triangle are not congruent if and only if three angles are not congruent.

13. If p represents "Today is Saturday" and q represents "Tomorrow is Sunday," then which is a correct translation of $q \rightarrow \sim p$?
    (1) If tomorrow is not Sunday, then today is not Saturday.
    (2) If tomorrow is not Sunday, then today is Saturday.
    (3) If tomorrow is Sunday, then today is Saturday.
    (4) If tomorrow is Sunday, then today is not Saturday.

DAY 2

SWBAT: Write and analyze the truth values of Conjunctions and Disjunctions

Warm – Up

Write each sentence in symbolic form, using the given symbols.

Let *p* represent "It is cold."
Let *q* represent "It is snowing."
Let *r* represent "The sun is shining."

1. It is not cold and it is snowing.
2. It is snowing or the sun is not shining.
3. If it is snowing, then the sun is not shining.
4. It is not the case that it is snowing if and only if it is not cold.

One of the goals of studying mathematics is to develop the ability to think critically. The study of critical thinking, or reasoning, is called _____.

A _____ is formed when two or more thoughts are connected in one sentence.

Example 1:

In logic, a _____ is a compound sentence formed by combining two sentences (or facts) using the word _____

p	q	p ^ q

p: Wash the dishes.

q: Vacuum the house.

p ^ q: Wash the dishes and vacuum the house.

For a conjunction (and) to be true, _____ facts must be true.

Practice:

In 1 – 4, For each given statement: A. Write the statement in symbolic form, using the symbols shown below. B. Tell whether the statement is true or false.

Let *b* represent "Water boils at 100°C."   (True)
Let *f* represent "Water freezes at 0°C."   (True)
Let *t* represent "Normal body temperature is 37°C."   (True)
Let *r* represent "Room temperature is 60°C."   (False)

1. Normal body temperature is 37°C and water boils at 100°C.

2. Normal body temperature is 37°C and room temperature is 60°C.

3. Water does not boil at 100°C and water does not freeze at 0°C.

4. Room temperature is not 60°C and water freezes at 0°C.

Example 2:

In logic, a _____ is a compound sentence formed by combining two sentences (or facts) using the word _____

p	q	p ∨ q

p: Wash the dishes.

q: Vacuum the house.

p v q: Wash the dishes and vacuum the house.

For a disjunction (or) to be true, _____ facts must be true.

Practice:

In 1 – 4, For each given statement: A. Write the statement in symbolic form, using the symbols shown below. B. Tell whether the statement is true or false.
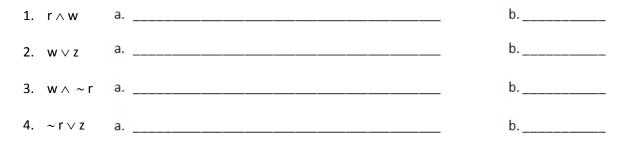
Let *c* represent "A meter contains 100 centimeters."   (True)
Let *m* represent "A meter contains 1,000 millimeters."   (True)
Let *k* represent "A kilometer is 1,000 meters."   (True)
Let *l* represent "A meter is a liquid measure."   (False)

1. A meter contains 1,000 millimeters or a kilometer is 1,000 meters.

2. A meter contains 100 centimeters or a meter is a liquid measure.

3. A kilometer is not 1,000 meters or a meter does not contain 100 centimeters.

4. It is not the case that a meter contains 100 centimeters or 1,000 millimeters.

Practice Problems : Conjunctions &Disjunctions

For each sentence given in symbolic form: **a.** Write a complete sentence in words to show what the symbols represent. **b.** Tell whether the sentence is true or false.

Let r represent: "January has 28 days."
Let w represent: "Saturday lies on the weekend."
Let z represent: "July is in the Winter."

1. r ∧ w   a. _____   b. _____

2. w ∨ z   a. _____   b. _____

3. w ∧ ~r   a. _____   b. _____

4. ~r ∨ z   a. _____   b. _____

5. Determine the truth value of: "21 is divisible by 3 *and* 21 is not prime."

6. Determine the truth value of: "45 is a multiple of 9 *or* 13 - 20 = 7."

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<u>Summary</u>

A *conjunction* is a compound sentence formed by using the word *and* to combine two simple sentences.

p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

A *disjunction* is a compound sentence formed by using the word *or* to combine two simple sentences.

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

<u>Challenge Problem:</u>

Directions: Complete the truth table

p	q	~p	~q	~p V ~q

<u>Exit Ticket:</u>

The statement "*x* is a multiple of 3, and *x* is an even integer" is true when *x* is equal to
1) 9
2) 8
3) 3
4) 6

Conjunction "∧"

In 3–12, write each sentence in symbolic form, using the given symbols.

Let p represent "It is hot."
Let q represent "It is raining."
Let r represent "The sky is cloudy."

3. It is hot and it is raining.

4. It is hot and the sky is cloudy.

5. It is not hot.

6. It is not hot and the sky is cloudy.

7. It is raining and the sky is not cloudy.

8. It is not hot and it is not raining.

9. The sky is not cloudy and it is not hot.

10. The sky is not cloudy and it is hot.

11. It is not the case that it is hot and it is raining.

12. It is not the case that it is raining and it is not hot.

In 13–20, using the truth value for each given statement, tell if the conjunction is true or false.

A piano is a percussion instrument.	(True)
A piano has 88 keys.	(True)
A flute is a percussion instrument.	(False)
A trumpet is a brass instrument.	(True)

13. A flute is a percussion instrument and a piano is a percussion instrument.

14. A flute is a percussion instrument and a trumpet is a brass instrument.

15. A piano has 88 keys and is a percussion instrument.

16. A piano has 88 keys and a trumpet is a brass instrument.

17. A piano is not a percussion instrument and a piano does not have 88 keys.

18. A trumpet is not a brass instrument and a piano is a percussion instrument.

19. A flute is not a percussion instrument and a trumpet is a brass instrument.

20. It is not true that a piano is a percussion instrument and has 88 keys.

In 21–28, complete each sentence with "true" or "false" to make a correct statement.

21. When p is true and q is true, then $p \wedge q$ is _____.

22. When p is false, then $p \wedge q$ is _____.

23. If p is true, or q is true, but not both, then $p \wedge q$ is _____.

24. When $p \wedge q$ is true, then p is _____ and q is _____.

25. When $p \wedge \sim q$ is true, then p is _____ and q is _____.

26. When $\sim p \wedge q$ is true, then p is _____ and q is _____.

27. When p is false and q is true, then $\sim(p \wedge q)$ is _____.

28. If both p and q are false, then $\sim p \wedge \sim q$ is _____.

Disjunction "∨"

In 3–12, for each given statement: **a.** Write the statement in symbolic form, using the symbols given below. **b.** Tell whether the statement is true or false.

Let *c* represent "A gram is 100 centigrams."	(True)
Let *m* represent "A gram is 1,000 milligrams."	(True)
Let *k* represent "A kilogram is 1,000 grams."	(True)
Let *l* represent "A gram is a measure of length."	(False)

3. A gram is 1,000 milligrams or a kilogram is 1,000 grams.

4. A gram is 100 centigrams or a gram is a measure of length.

5. A gram is 100 centigrams or 1,000 milligrams.

6. A kilogram is not 1,000 grams or a gram is not 100 centigrams.

7. A gram is a measure of length or a kilogram is 1,000 grams.

8. A gram is a measure of length and a gram is 100 centigrams.

9. It is not the case that a gram is 100 centigrams or 1,000 milligrams.

10. It is false that a kilogram is not 1,000 grams or a gram is a measure of length.

11. A gram is 100 centigrams and a kilogram is 1,000 grams.

12. A gram is not 100 centigrams or is not 1,000 milligrams, and a gram is a measure of length.

In 13–20, symbols are assigned to represent sentences.

Let *b* represent "Breakfast is a meal."
Let *s* represent "Spring is a season."
Let *h* represent "Halloween is a season."

For each sentence given in symbolic form: **a.** Write a complete sentence in words to show what the symbols represent. **b.** Tell whether the sentence is true or false.

13. $s \lor h$ **14.** $b \land s$ **15.** $\sim s \lor h$ **16.** $b \land \sim h$

17. $\sim b \lor \sim s$ **18.** $\sim(s \land h)$ **19.** $\sim(b \lor \sim s)$ **20.** $\sim b \land \sim s$

Warm – Up

Directions: Complete the truth table

1.

p	q	~p	~q	(~p ∨ q)

2. Find two supplement angles in the ratio of 7:5

In logic, a _____ is a compound statement formed by combining _____ sentences (or facts) using the words _____

p   q

p is known as the _____.

q is known as the _____.

Conditionals: $p \rightarrow q$

p: (fact 1): If you come to school.

q: (fact 2): Ms. Williams will give me an A.

p → q: If you come to school, then Ms. Williams will give me an A.

p	q	$p \rightarrow q$

A _____ is _____ when a true hypothesis leads to a
_____ conclusion. In all other cases, the conditional is _____. Think of it
as a broken promise.

Example 1:

There are different ways to write the conditional.

$p \rightarrow q$: If it is January, then it is winter.
          hypothesis   conclusion

$p \rightarrow q$: It is January implies that it is winter.
       hypothesis   conclusion

$p \rightarrow q$: It is January only if it is winter.
       hypothesis   conclusion

Practice:

For each given sentence:

a. Identify the hypothesis p.   b. Identify the conclusion q.

(1) If Mrs. Shuda teaches our class, then we will learn.

(2) The assignment will be completed if I work at it every day.

(3) The task is easy when we all work together and do our best.

For each given statement:

  a. Write the statement in symbolic form using the symbols given below.

  b. Tell whether the statement is true or false.

     Let *m* represent "Monday is the first day of the week."   (True)
     Let *w* represent "There are 52 weeks in a year."   (True)
     Let *h* represent "An hour has 75 minutes."   (False)

                                       Answers

(1) If Monday is the first day of the week, then there are 52 weeks in a year.
     a.
     b.

(2) If there are 52 weeks in a year, then an hour has 75 minutes.
     a.
     b.

(3) If there are not 52 weeks in a year then Monday is the first day of the week.
     a.
     b.

(4) If Monday is the first day of the week and there are 52 weeks in a year, then an hour has 75 minutes.
     a.
     b.

Biconditionals: $p \leftrightarrow q$

In logic, a _____is a compound statement formed by combining the conditional and the converse using the word _____.

p	q	$p \leftrightarrow q$

In 1 - 4, for each given statement:
a. Write the statement in symbolic form, using the symbols shown below.
b. Tell whether the statement is true or false.

Let p represent "It is cold."   (True)
Let q represent "it is snowing."   (False)
Let r represent "The sun is shining." (True)

Answers

1. It is cold if and only if it is snowing.

    a.

    b.

2. The sun is shining if and only if it is snowing.

    a.

    b.

3. It is not cold if and only if the sun is shining.

    a.

    b.

4. It is not cold if and only if the sun is not shining.

    a.

    b.

Challenge Problem:

p	q	p → q	q → p	(p → q) ^ (q → p)

A *conditional* is a compound sentence usually formed by using the words *if...then* to combine two simple sentences.

p	q	p → q
T	T	
T	F	
F	T	
F	F	

A *biconditional* is a compound sentence formed by combining the two conditionals p → q and q → p.

p	q	p ↔ q
T	T	
T	F	
F	T	
F	F	

Exit Ticket

The statement "If x is divisible by 8, then it is divisible by 6" is false if x equals
1) 6
2) 14
3) 32
4) 48

Which statement is an example of a biconditional statement?
1) If Craig has money, he buys a car.
2) Craig buys a car if and only if he has money.
3) Craig has money or he buys a car.
4) Craig has money and he buys a car.

In 1–8, for each given sentence: a. Identify the hypothesis p.   b. Identify the conclusion q.

1. If it rains, then the game is canceled.
2. If it is 9:05 A.M., then I'm late to class.
3. When it rains, then I do not have to water the lawn.
4. You can get to the stadium if you take the Third Avenue bus.
5. The perimeter of a square is $4x + 8$ if one side of the square is $x + 2$.
6. If the shoe fits, wear it.   7. If a polygon has exactly three sides, it is a triangle.
8. When you have a headache, you should take time out and get some rest.

In 9–14, write each sentence in symbolic form, using the given symbols.

p: The test is easy.
q: Sam studies.
r: Sam passes the test.

9. If the test is easy, then Sam will pass the test.
10. If Sam studies, then Sam will pass the test.
11. If the test is not easy, then Sam will not pass the test.
12. Sam will not pass the test if Sam doesn't study.
13. The test is easy if Sam studies.   14. Sam passes the test if the test is easy

In 15–22, for each given statement: a. Write the statement in symbolic form, using the sy᠁ given below.   b. Tell whether the conditional statement is true or false, based upon the truth ᠁ given.

r: The race is difficult.   (True)
p: Karen practices.   (False)
w: Karen wins the race.   (True)

15. If Karen practices, then Karen will win the race.
16. If Karen wins the race, then Karen has practiced.
17. If Karen wins the race, the race is difficult.
18. Karen wins the race if the race is not difficult.
19. Karen will not win the race if Karen does not practice.
20. Karen practices if the race is difficult.
21. If the race is not difficult and Karen practices, then Karen will win the race.
22. If the race is difficult and Karen does not practice, then Karen will not win the race.

In 25–34, for each given statement:

a. Write the statement in symbolic form, using the symbols given below.
b. Tell whether the statement is true or false.

Let M represent "May has 31 days."   (True)
Let J represent "June has 31 days."   (False)
Let F represent "June follows May."   (True)

25. May does not have 31 days.
26. May has 31 days and June has 31 days.
27. May or June has 31 days.
28. If June follows May, then May has 31 days.
29. If May has 31 days, then June has 31 days.
30. June follows May if and only if June has 31 days.
31. If June does not follow May, then June does not have 31 days.
32. June has 31 days if June follows May.
33. If May and June have 31 days, then June does not follow May.
34. June does not have 31 days implies that May has 31 days.

In 35–46, use the symbols assigned to represent the four true statements.

Let m represent "A segment is bisected at its midpoint."   (True)
Let s represent "Congruent segments are equal in length."   (True)
Let b represent "An angle bisector forms two congruent angles." (True)
Let a represent "Congruent angles are equal in measure."   (True)

For the compound sentences in symbolic form:

a. Write a complete sentence in words to show what the symbols represent.
b. Tell whether the compound sentence is true or false.

35. $m \wedge s$

36. $a \wedge s$

37. $m \rightarrow s$

38. $\sim m \vee \sim s$

39. $s \leftrightarrow a$

40. $b \rightarrow \sim a$

41. $\sim a \rightarrow \sim s$

42. $(m \wedge b) \rightarrow (s \wedge a)$

43. $m \rightarrow \sim s$

44. $\sim b \vee m$

45. $\sim m \leftrightarrow \sim s$

46. $(b \vee \sim a) \rightarrow \sim s$

DAY 5

Warm – Up

Which statement is expressed as a biconditional?
(1) Two angles are congruent if they have the same measure.
(2) If two angles are both right angles, then they are congruent.
(3) Two angles are congruent if and only if they have the same measure.
(4) If two angles are congruent, then they are both right angles.

Review:

In logic, a conditional is a compound statement formed by combining two sentences (or facts) using the words "if ... then."

$$p \rightarrow q$$

p is known as the premise, hypothesis or antecedent.

q is known as the conclusion or consequent.

The _____ of a conditional statement is formed by _____ the _____ and _____ of the original statement. In other words, the parts of the sentence change places. The words "if" and "then" do not move.

Conditional: If you study **, then** you will get an A

Converse: If _____ **, then** _____   Symbolic Form

p: you study.

q: you will get an A.

p → q: If you study, then you will get an A.

p	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$

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Practice:
1. Which statement is the converse of "If the sum of two angles is 180°, then the angles are supplementary"?
(1) If two angles are supplementary, then their sum is 180°.
(2) If the sum of two angles is not 180°, then the angles are not supplementary.
(3) If two angles are not supplementary, then their sum is not 180°.
(4) If the sum of two angles is not 180°, then the angles are supplementary.

2. What is the converse of the statement "If it is Sunday, then I do not go to school"?
(1) If I do not go to school, then it is Sunday.
(2) If it is not Sunday, then I do not go to school.
(3) If I go to school, then it is not Sunday.
(4) If it is not Sunday, then I go to school.

The _____ of a conditional statement is formed by _____ the hypothesis and
_____ the conclusion of the original statement. In other words, the word "not" is added to both parts of
the sentence.

Conditional: If   you study   , then you will get an A

Inverse: If   , then   Symbolic Form

p	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	~p	~q	Inverse $\sim p \rightarrow \sim q$
T	T	T	T			
T	F	F	T			
F	T	T	F			
F	F	T	T			

An interesting fact: The inverse has the same truth value as the converse of the original statement. The INVERSE and
the CONVERSE of the original statement are _____. ("equivalent" means "the same")

Practice:
1. What is the inverse of the statement "If Mike did his homework, then he will pass this test"?
(1) If Mike passes this test, then he did his homework.
(2) If Mike does not pass this test, then he did not do his homework.
(3) If Mike does not pass this test, then he only did half his homework.
(4) If Mike did not do his homework, then he will not pass this test.

2. What is the inverse of the statement "If I do not buy a ticket, then I do not go to the concert"?
(1) If I buy a ticket, then I do not go to the concert.
(2) If I buy a ticket, then I go to the concert.
(3) If I go to the concert, then I buy a ticket.
(4) If I do not go to the concert, then I do not buy a ticket.

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The _____ of a conditional statement is formed by negating both the hypothesis and the conclusion, and then interchanging the resulting negations.

In other words, the _____ negates and switches the parts of the sentence. It does BOTH the jobs of the INVERSE and the CONVERSE.

Conditional: If   you study , then you will get an A

Contrapositive: If   , then   Symbolic Form

p	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	~p	~q	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	T	T	F	F	T	
T	F	F	T	F	T	T	
F	T	T	F	T	F	F	
F	F	T	T	T	T	T	

**If the original statement is TRUE, the contrapositive is TRUE.
  If the original statement is FALSE, the contrapositive is FALSE.
  They are said to be logically equivalent. ("equivalent" means "the same")

Practice:
1. What is the contrapositive of the statement "If I study, then I pass the test"?
(1) I pass the test if I study.
(2) If I do not study, then I do not pass the test.
(3) If I do not pass the test, then I do not study.
(4) If I pass the test, then I study.

2. Which statement is logically equivalent to "If it is Saturday, then I am not in school"?
(1) If I am not in school, then it is Saturday.
(2) If it is not Saturday, then I am in school.
(3) If I am in school, then it is not Saturday.
(4) If it is Saturday, then I am in school.

Challenge Problem:

p	q	~p	~q	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$	$(\sim p \rightarrow \sim q) \vee (\sim q \rightarrow \sim p)$

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A conditional and its inverse can have the same truth value.

> Let *r* represent "Twice Talia's age is 10."
> Let *q* represent "Talia is 5 years old."

Conditional If twice Talia's age is 10, then Talia is 5 years old.
(r → s):   *r*   *s*

   Inverse If twice Talia's age is *not* 10, then Talia is *not* 5 years old.
(~r → ~s):   *~r*   *~s*

A conditional and its converse can have the same truth value.

> Let *p* represent "Today is Friday."
> Let *q* represent "Tomorrow is Saturday."

Conditional (p → q): If today is Friday, then tomorrow is Saturday.
                                  p   *q*

Converse (q → p): If tomorrow is Saturday, then today is Friday.
                              q   *p*

A true conditional can have a true contrapositive.

> Let *p* represent "Gary arrives late to class."
> Let *q* represent "Gary is marked tardy."

Conditional If Gary arrives late to class, then Gary is marked tardy.
(p → q):   *p*   *q*

Contrapositive If Gary is *not* marked tardy,
(~q → ~p):   *~q*

       then Gary does *not* arrive late to class.
                                  ~p

Exit Ticket:

A conditional statement is always logically equivalent to its
1)   contrapositive
2)   converse
3)   conjunction
4)   inverse

DAY 5 – Homework

1. What is the converse of the statement "If it is sunny, I will go swimming"?
(1) If it is not sunny, I will not go swimming.
(2) If I do not go swimming, then it is not sunny.
(3) If I go swimming, it is sunny.
(4) I will go swimming if and only if it is sunny.

2. Which statement is the converse of "If it is a 300 ZX, then it is a car"?
(1) If it is not a 300 ZX, then it is not a car.
(2) If it is not a car, then it is not a 300 ZX.
(3) If it is a car, then it is a 300 ZX.
(4) If it is a car, then it is not a 300 ZX.

3. What is the inverse of the statement "If Julie works hard, then she succeeds"?
(1) If Julie succeeds, then she works hard.
(2) If Julie does not succeed, then she does not work hard.
(3) If Julie works hard, then she does not succeed.
(4) If Julie does not work hard, then she does not succeed.

4. What is the inverse of the statement "If it is sunny, I will play baseball"?
(1) If I play baseball, then it is sunny.
(2) If it is not sunny, I will not play baseball.
(3) If I do not play baseball, then it is not sunny.
(4) I will play baseball if and only if it is sunny.

5. Which statement is the inverse of "If the waves are small, I do not go surfing"?
(1) If the waves are not small, I do not go surfing.
(2) If I do not go surfing, the waves are small.
(3) If I go surfing, the waves are not small.
(4) If the waves are not small, I go surfing.

6. Which statement is logically equivalent to "If I did not eat, then I am hungry"?
(1) If I am not hungry, then I did not eat.
(2) If I did not eat, then I am not hungry.
(3) If I am not hungry, then I did eat.
(4) If I am hungry, then I did eat.

7. Which statement is logically equivalent to "If I eat, then I live"?
(1) If I live, then I eat.
(2) If I eat, then I do not live.
(3) I live if and only if I eat.
(4) If I do not live, then I do not eat.

8. Which statement is logically equivalent to "If the team has a good pitcher, then the team has a good season"?
(1) If the team does not have a good season, then the team does not have a good pitcher.
(2) If the team does not have a good pitcher, then the team does not have a good season.
(3) If the team has a good season, then the team has a good pitcher.
(4) The team has a good pitcher and the team does not have a good season.

Given the statement: "If I live in Albany, then I am a New Yorker."
In the spaces provided below, write the inverse, the converse, and the contrapositive of this statement.

Inverse:_____
_____
_____

Converse:_____
_____
_____

Contrapositive:_____
_____
_____

Which conditional is logically equivalent to its original statement?

inverse   converse
contrapositive

a. Write the inverse of each conditional statement in words. b. Give the truth value of the conditional. c. Give the truth value of the inverse.

   9. If $6 > 3$, then $-6 > -3$.

   10. If a polygon is a parallelogram, then the polygon has two pairs of parallel sides.

write the converse of each statement in words.

11. If you lower your cholesterol, then you eat Quirky oatmeal.

12. If you enter the Grand Prize drawing, then you will get rich.

a. Write the converse of each conditional statement in words. b. Give the truth value of the conditional. c. Give the truth value of the converse.

   13. If a number is even, then the number is exactly divisible by 2.

   14. If 0.75 is an integer, then it is rational.

a. Write the contrapositive of each statement in words. b. Give the truth value of the conditional. c. Give the truth value of the contrapositive.

   15. If Rochester is a city, then Rochester is the capital of New York.

   16. If two angles form a linear pair, then they are supplementary.