Geometry Workbook 4:

Angle Relationships, Parallel Lines & Transversals, Coordinate Formulas, and Equations of Parallel & Perpendicular Lines

Student Name ____

STANDARDS:

G.CO.C.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
G.GPE.B.4	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, sqrt(3)) lies on the circle centered at the origin and containing the point (0, 2).
G.GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

SKILLS:

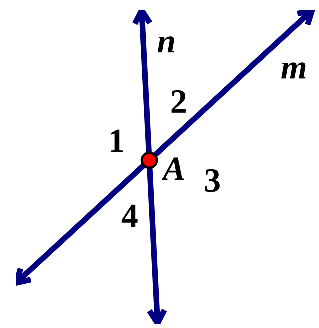
- ☐ I will be able to prove and apply that vertical angles are congruent.
- ☐ I will be able to prove and apply the angle relationships formed when two parallel lines are cut by a transversal.
- ☐ I will be able to prove that all points on a perpendicular bisector of a segment are equidistant from the segment endpoints.
- ☐ I will be able to know all of the relationships between pairs of angles.
- ☐ I will be able to establish relationships and /or characteristics of geometric shapes using coordinate geometry.
- ☐ I will be able to classify a quadrilateral through use of coordinate analysis.
- ☐ I will be able to prove that parallel lines have congruent slopes and its converse.
- ☐ I will be able to prove that perpendicular lines have negative reciprocal slopes and its converse.
- ☐ I will be able to determine the slope of a line.
- ☐ I will be able to determine whether two slopes represent parallel or perpendicular relationships.
- ☐ I will be able to classify geometric shapes using slopes and/or distances.

Notes:

Proving things to be true is a common task for geometry students. To prove something is to logically establish the connections from what you know to what you want to prove, all the while giving accurate reasoning for each conclusion. This process is often difficult for new geometry students – it is hard to clearly explain what you know and why you know it. One format for a proof is to provide it in a paragraph form: to simply write it as you would say it. This can be a comfortable style for many students. The key is to, after each conclusion or deduction, state the reason for knowing it. If you do this, the proof will flow naturally and correctly.

PAIRS OF ANGLES

It is very common for two lines to intersect in the plane. When two lines intersect, a point is formed and also a number of angles. In the diagram to the right, the intersection of line m and line n is point A. The angles formed have many different names and relationships.



The diagram to the right has some Adjacent Angles.

ADJACENT ANGLES are angles that share a vertex and a ray and no interior points. So in the diagram to the right $\angle 1$ & $\angle 2$ are adjacent angles. There are other examples of adjacent angles in the diagram such as $\angle 4$ & $\angle 1$.

The diagram to the right has some Linear Pairs.

A LINEAR PAIR is two angles that are adjacent and sum to 180°. In this particular diagram $\angle 1$ & $\angle 2$ are more specifically called a linear pair. $\angle 2$ & $\angle 3$, $\angle 3$ & $\angle 4$, and $\angle 4$ & $\angle 1$ are also a linear pairs.

The diagram to the right has some Vertical Angles.

VERTICAL ANGLES are a pair of non-adjacent angles formed by the intersection of two lines. The angles labeled $\angle 1$ & $\angle 3$ and $\angle 2$ & $\angle 4$ are vertical angles.

SUPPLEMENTARY ANGLES – Two angles are supplementary if the sum of their measures is 180°.

COMPLEMENTARY ANGLES – Two angles are complementary if the sum of their measures is 90°.

PROVING RELATIONSHIPS

Prove Vertical Angles are Congruent.

Our knowledge of rotations will help us here, so first I want to look back at how we defined a 180° rotation. When we defined a rotation, we looked at the properties of the special rotation of 180°.

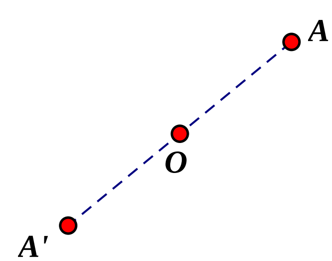
A rotation of 180° maps A to A' such that:

a) m$\angle$AOA' = 180° (from definition of rotation)

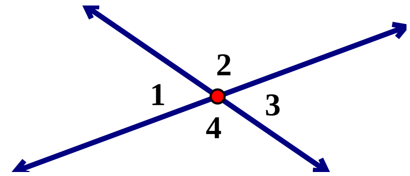
b) OA = OA' (from definition of rotation)

c) Ray $\overrightarrow{OA}$ and Ray $\overrightarrow{OA'}$ are opposite rays. (They form a line.)

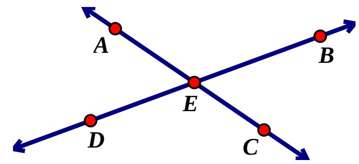
$\overleftrightarrow{AO}$ is the same line as $\overleftrightarrow{AA'}$



This will help us prove the relationship between two vertical angles. First of all, **vertical angles are the two non-adjacent angles formed by intersecting lines.** So in the diagram, ∠1 and ∠3 are vertical angles, and ∠2 and ∠4 are vertical angles as well.

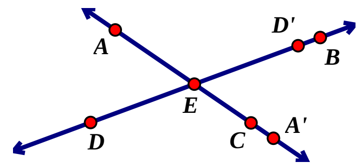


To Prove that Vertical Angles are Congruent we use the properties of a 180° rotation.



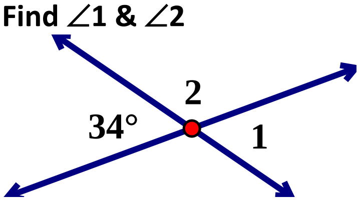
Prove: ∠DEA ≅ ∠BEC

A rotation of 180° about point E, maps D onto opposite ray $\overrightarrow{EB}$. D' lies on $\overrightarrow{EB}$. A rotation of 180° about point E maps A onto opposite ray $\overrightarrow{EC}$. A' lies on $\overrightarrow{EC}$. ∠D'EA' ≅ ∠BEC because the angles use the same rays and vertex. Thus, using the transitive property, ∠DEA ≅ ∠BEC.



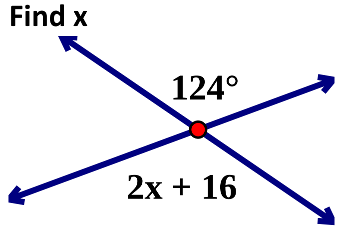
Using a similar argument we could also prove, ∠DEC ≅ ∠BEA.

Find ∠1 & ∠2



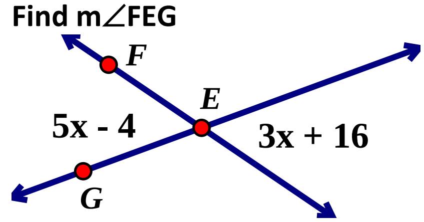
m∠1 = 34° (vertical ∠ =)
m∠2 = 180 – 34 (linear pair)
m∠2 = 146°

Find x



2x + 16 = 124 (vertical ∠ =)
2x = 108
x = 54

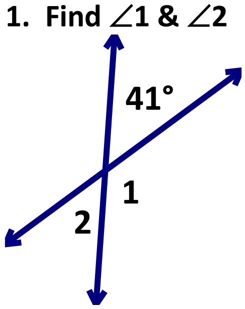
Find m∠FEG



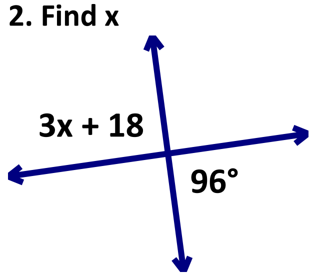
5x – 4 = 3x + 16 (vertical ∠ =)
2x = 20
x = 10
5(10) – 4 = **46° = m∠FEG**

NYTS (Now You Try Some)

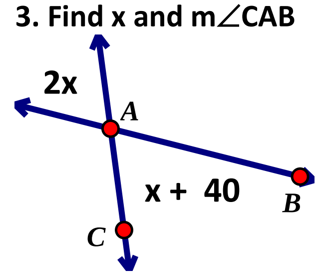
1. Find ∠1 & ∠2



2. Find x



3. Find x and m∠CAB

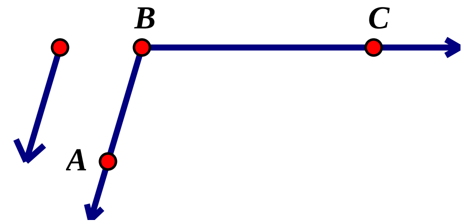


Prove when a transversal crosses parallel lines, alternate interior/exterior angles are congruent and corresponding angles are congruent.

To prove this relationship we are also going to go back to the properties of a translation of an angle along one of its rays.

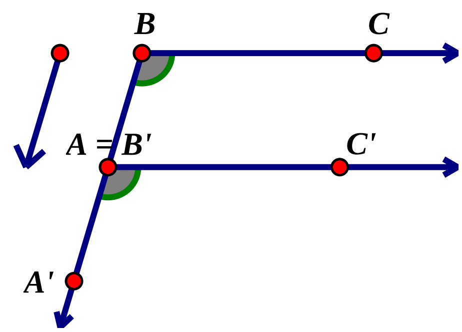
A translation of ∠ABC by vector $\overrightarrow{BA}$ maps all points such that

1. ∠ABC ≅ ∠A'B'C' (Isometry)
2. B, A, B' and A' are collinear (translation on angle ray)



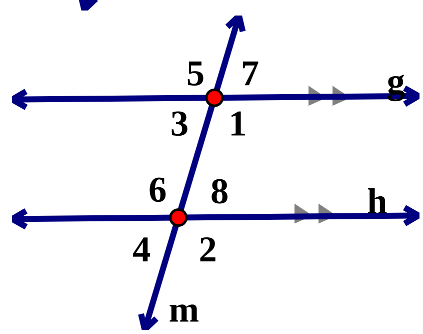
Because the two angles are equal and formed on the same ray, then:

$$\overrightarrow{BC} \;||\; \overrightarrow{B'C'}$$



Parallel lines are formed when we translate an angle along one of its rays. If we extend those rays into lines, we form a few more angles. When lines are parallel, we use arrowheads to denote which lines are parallel to each other. So in the diagram, line g || line h.

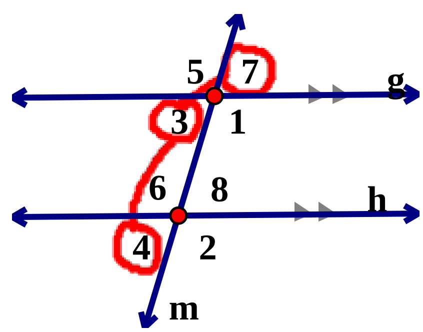
The translation of angles ∠1, ∠3, ∠5 & ∠7 along the transversal line m gives us congruent corresponding angles, ∠2, ∠4, ∠6 & ∠8.



This angle relationship is called **CORRESPONDING ANGLES** and because of the properties of the isometric translation, **CORRESPONDING ANGLES MUST BE CONGRUENT**.

$$\angle 1 \cong \angle 2,\ \angle 3 \cong \angle 4,\ \angle 5 \cong \angle 6\ \&\ \angle 7 \cong \angle 8$$

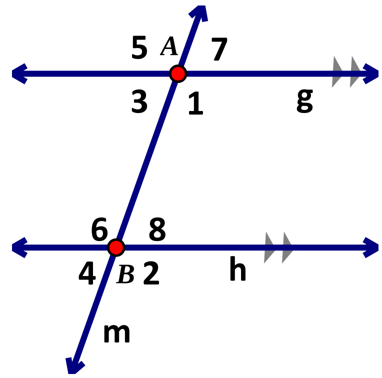
∠5 & ∠2 and ∠7 & ∠4 are called **ALTERNATE EXTERIOR ANGLES**. Alternate because they are on alternating sides of the transversal and exterior because they are on the outside of the parallel lines.



PROVE: **ALTERNATE EXTERIOR ANGLES ARE CONGRUENT**
PROVE: **∠4 ≅ ∠7 & ∠2 ≅ ∠5**

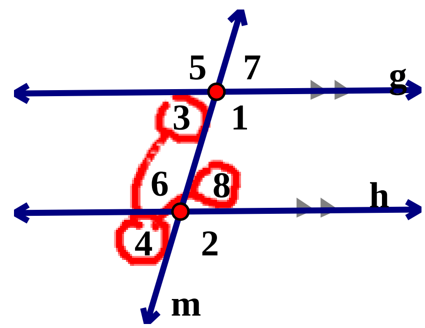
∠4 ≅ ∠3 because corresponding angles are congruent and ∠3 ≅ ∠7 because vertical angles are congruent. Thus using the transitive property, ∠4 ≅ ∠7. We could use a similar argument to prove ∠2 ≅ ∠5.

An alternate way of writing it…..
PROVE: $\angle\mathbf{4} \cong \angle\mathbf{7}$ **&** $\angle\mathbf{2} \cong \angle\mathbf{5}$



Earlier we established that opposite angles are equal due to the rotation of 180°… thus $\angle 7 \cong \angle 3$ because they are opposite angles. $\angle 3 \cong \angle 4$ because we established that corresponding angles are congruent due to the translation $\overrightarrow{AB}$. Using the transitive property, $\angle 4 \cong \angle 7$. We could use a similar argument to prove $\angle 2 \cong \angle 5$.

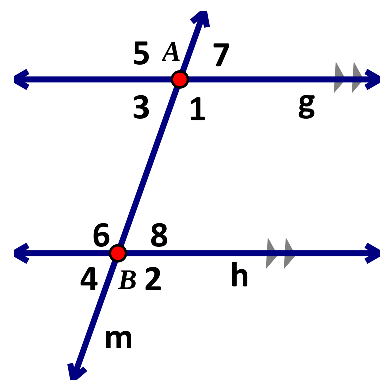
$\angle 3$ & $\angle 8$ and $\angle 6$ & $\angle 1$ are called **ALTERNATE INTERIOR ANGLES**. Alternate because they are on alternating sides of the transversal and interior because they are on the interior of the parallel lines.



PROVE: **ALTERNATE INTERIOR ANGLES ARE CONGRUENT**
PROVE: $\angle\mathbf{3} \cong \angle\mathbf{8}$ **&** $\angle\mathbf{6} \cong \angle\mathbf{1}$

$\angle 3 \cong \angle 4$ because corresponding angles are congruent and $\angle 4 \cong \angle 8$ because vertical angles are congruent. Thus using the transitive property, $\angle 3 \cong \angle 8$. We could use a similar argument to prove $\angle 6 \cong \angle 1$.

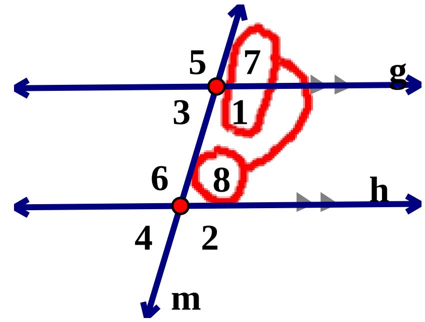
An alternate way of writing it…..
PROVE: $\angle\mathbf{3} \cong \angle\mathbf{8}$ **&** $\angle\mathbf{6} \cong \angle\mathbf{1}$



Earlier we established that opposite angles are equal due to the rotation of 180°… thus $\angle 3 \cong \angle 7$ because they are opposite angles. $\angle 7 \cong \angle 8$ because we established that corresponding angles are congruent due to the translation $\overrightarrow{AB}$. Using the transitive property, then $\angle 3 \cong \angle 8$. We could use a similar argument to prove $\angle 6 \cong \angle 1$.

Prove when a transversal crosses parallel lines, consecutive (same side) interior/exterior angles are supplementary.

$\angle 3$ & $\angle 6$ and $\angle 1$ & $\angle 8$ are called **CONSECUTIVE INTERIOR ANGLES (OR SAME SIDE INTERIOR ANGLES)**. I prefer same side…. Same Side because they are on the same side of the transversal and interior because they are on the interior of the parallel lines.

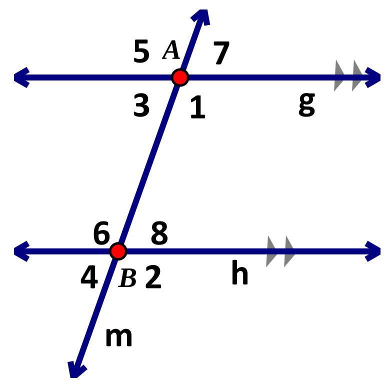


PROVE: **SAME SIDE INTERIOR ANGLES ARE SUPPLEMENTARY**
PROVE: **$m\angle 1 + m\angle 8 = 180°$ & $m\angle 3 + m\angle 6 = 180°$**

$m\angle 1 + m\angle 7 = 180°$ because they are a linear pair and $m\angle 7 = m\angle 8$ because corresponding angles are congruent. If we substitute, we get $m\angle 1 + m\angle 8 = 180°$.

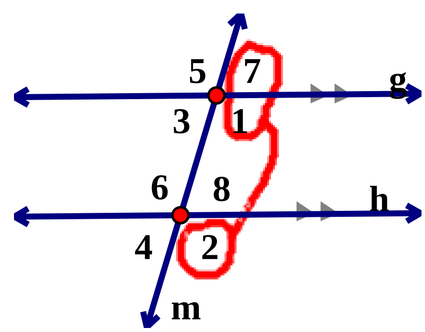
We could use a similar argument to prove $m\angle 3 + m\angle 6 = 180°$.

An alternate proof using transformations.
PROVE: **$m\angle 1 + m\angle 8 = 180°$ & $m\angle 3 + m\angle 6 = 180°$**



$\angle 2$ and $\angle 8$ are a linear pair. Thus $m\angle 2 + m\angle 8 = 180°$ by definition of linear pair. It is also true that $\angle 2 \cong \angle 1$ ($m\angle 2 = m\angle 1$) because a translation of $\overrightarrow{BA}$ maps $\angle 2$ onto $\angle 1$. So if we substitute these values we get $m\angle 1 + m\angle 8 = 180°$. We could use a similar argument to prove $m\angle 3 + m\angle 6 = 180°$.

$\angle 3$ & $\angle 6$ and $\angle 1$ & $\angle 8$ are called **CONSECUTIVE EXTERIOR ANGLES (OR SAME SIDE EXTERIOR ANGLES)**. I prefer same side…. Same Side because they are on the same side of the transversal and exterior because they are on the exterior of the parallel lines.



PROVE: **SAME SIDE EXTERIOR ANGLES ARE SUPPLEMENTARY**
PROVE: **$m\angle 2 + m\angle 7 = 180°$ & $m\angle 4 + m\angle 5 = 180°$**

$m\angle 1 + m\angle 7 = 180°$ because they are a linear pair and $m\angle 1 = m\angle 2$ because corresponding angles are congruent. If we substitute, we get $m\angle 2 + m\angle 7 = 180°$.

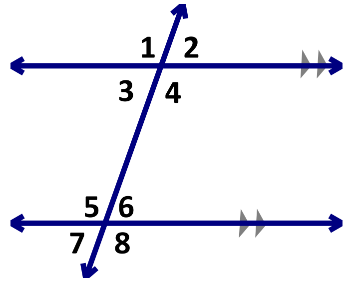
We could use a similar argument to prove $m\angle 4 + m\angle 5 = 180°$.

CONGRUENT	SUPPLEMENTARY
Corresponding angles are congruent. Alternate interior angles are congruent. Alternate exterior angles are congruent.	Consecutive (Same Side) interior angles are supplementary. Consecutive (Same Side) exterior angles are supplementary.

4. Provide the name of the following relationships.

a) $\angle 1$ & $\angle 5$ ________________ b) $\angle 2$ & $\angle 7$ ________________

c) $\angle 5$ & $\angle 4$ ________________ d) $\angle 4$ & $\angle 6$ _______________



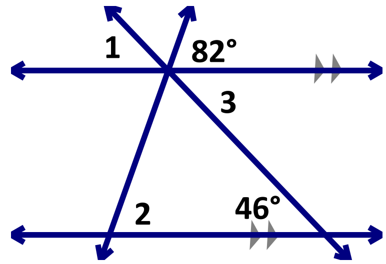
5. Find the measure of the angle and give a reason for knowing it.

(measure) (reason)

a) $m\angle 1$ = ___________ ________________________

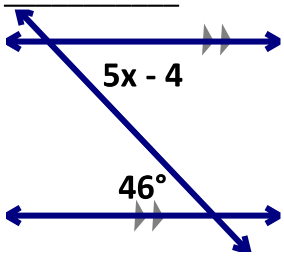
b) $m\angle 2$ = ___________ ________________________

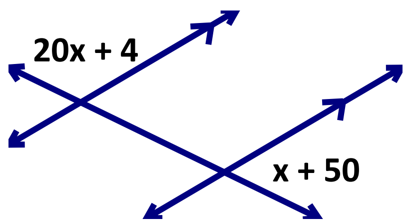
c) $m\angle 3$ = ___________ ________________________

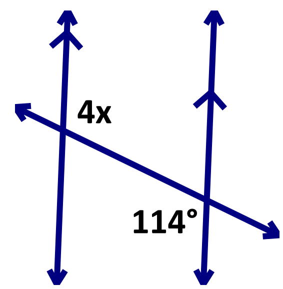


6. Solve for the unknown values.

a) x = ___________ b) x = ___________ c) x = ___________



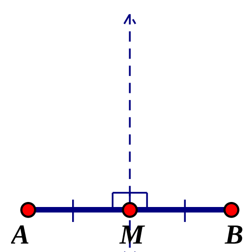


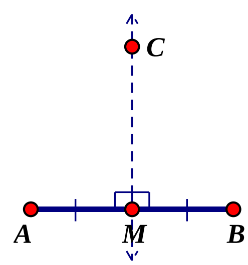


C) Points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

As defined, a perpendicular bisector is the perpendicular line that passes through the midpoint of a segment.

We have also learned that the perpendicular bisector is the line of reflection for $\overline{AB}$.

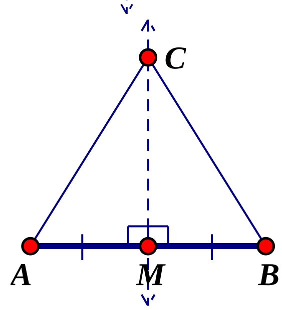




PROVE: $\overline{AC} \cong \overline{BC}$

A reflection over $\overline{MC}$ maps A onto B because of the definition of a reflection and C onto C and M onto M because they are on the line of reflection.

Because $\overline{AC}$ maps onto $\overline{BC}$ by the isometric transformation reflection, $\overline{AC} \cong \overline{BC}$.

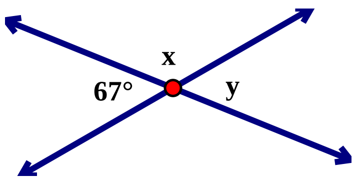


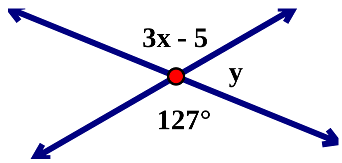
1. Solve the following.

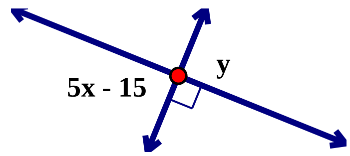
a) x = _________ y = _________

b) x = _________ y = _________

c) x = _________ y = _________







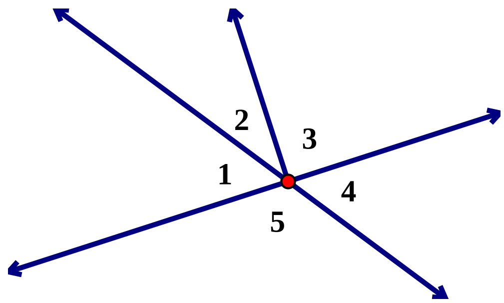
2. ∠5 and ∠3 are vertical angles. T or F

3. ∠1 and ∠5 are a linear pair. T or F

4. ∠4 and ∠3 are adjacent angles. T or F

5. ∠4 and ∠1 are vertical angles. T or F

6. ∠3 and ∠4 are a linear pair. T or F



7. If ∠A and ∠B are supplements and m∠A = 150°, what is m∠B? ____________

8. If ∠A and ∠B are complements and m∠A = 27°, what is m∠B? ____________

9. If ∠A and ∠B are vertical angles and m∠A = 36°, what is m∠B? ____________

10. If ∠A and ∠B are a linear pair and m∠A = 2x + 8 and m∠B = 3x + 2, what is the value of x? x = ________

11. If ∠A and ∠B are vertical angles and m∠A = 7x -5 and m∠B = 4x + 10, what is the value of x? x = ________

12. Provide the name of the following relationships.

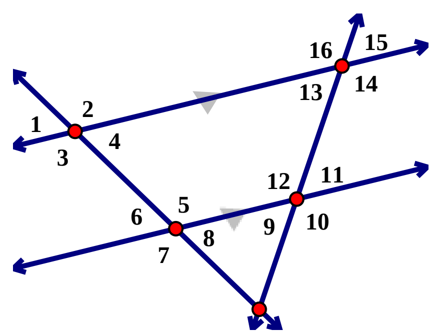
a) ∠1 & ∠6 _________________ b) ∠2 & ∠7 _________________

c) ∠16 & ∠14 _________________ d) ∠14 & ∠11 _________________

e) ∠1 & ∠7 _________________ f) ∠6 & ∠5 _________________

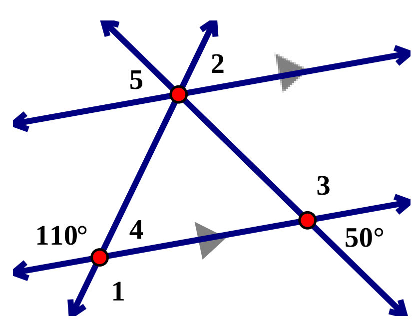
g) ∠15 & ∠10 _________________ h) ∠1 & ∠2 _________________

i) ∠13 & ∠12 _________________ j) ∠16 & ∠9 _________________

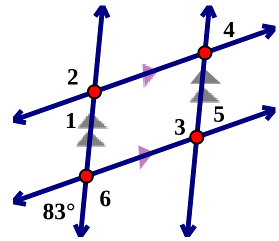


13. Find the measure of the angle and give a reason for knowing it.

	(measure)	(reason)
a) m∠1 =	____________	_________________________
b) m∠2 =	____________	_________________________
c) m∠3 =	____________	_________________________
d) m∠4 =	____________	_________________________
e) m∠5 =	____________	_________________________



14. Find the measure of the angle.

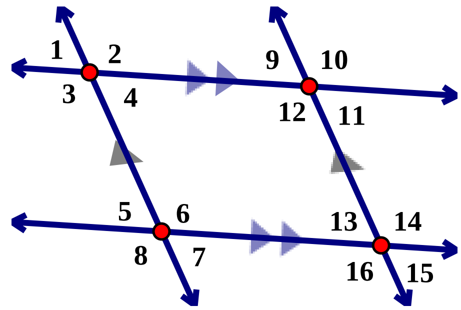


a) m∠1 = ____________ b) m∠2 = ____________

c) m∠3 = ____________ d) m∠4 = ____________

e) m∠5 = ____________ f) m∠6 = ____________

15. Circle (T)rue or (F)alse.



a) ∠1 ≅ ∠4 T or F b) ∠6 ≅ ∠16 T or F

c) ∠3 ≅ ∠5 T or F d) ∠4 ≅ ∠5 T or F

e) ∠2 ≅ ∠10 T or F f) ∠9 ≅ ∠15 T or F

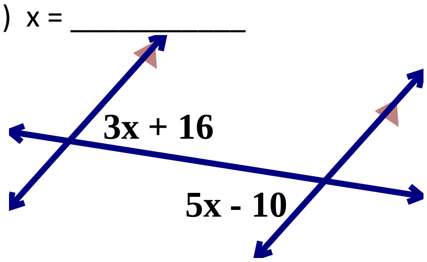
g) ∠12 ≅ ∠14 T or F h) ∠9 ≅ ∠11 T or F

i) m∠11 + m∠15 = 180° T or F

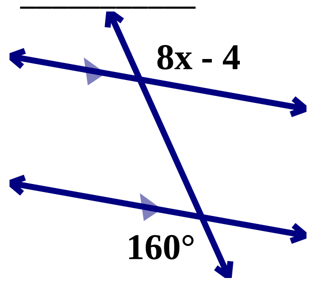
j) m∠1 + m∠8 = 180° T or F

16. Solve for the unknown values.

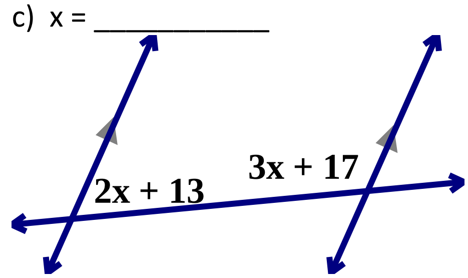
a) x = ____________



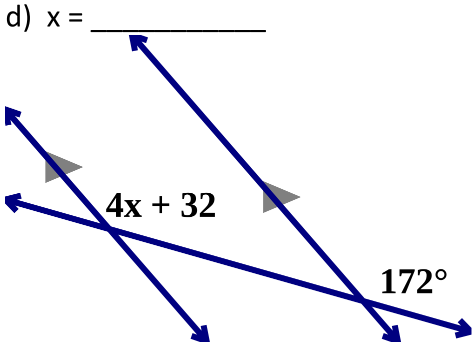
b) x = ____________



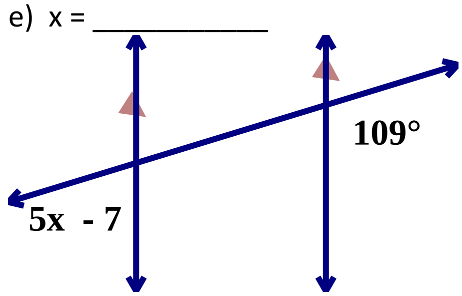
c) x = ____________



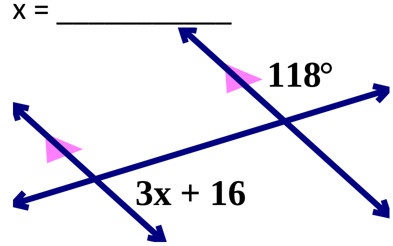
d) x = ____________



e) x = ____________



f) x = ____________



1. Solve the following.

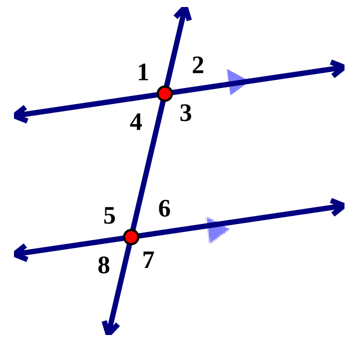
a) if m∠7 = 100°, find m∠3 = ________ b) if m∠7 = 95°, find m∠6 = ________

c) if m∠1 = 120°, find m∠5 = ________ d) if m∠4 = 20°, find m∠7 = ________

e) if m∠3 = 140°, find m∠5 = ________ f) if m∠4 = 30°, find m∠1 = ________

g) if m∠4 = 40°, find m∠2 = ________ h) if m∠3 = 125°, find m∠8 = ________

i) if m∠1 + m∠5 = 234°, find m∠6 = ________

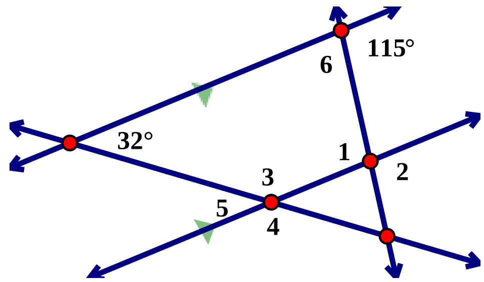


2. Solve the following.

a) m∠ 1 = ______________ b) m∠ 2 = ______________

c) m∠ 3 = ______________ d) m∠ 4 = ______________

e) m∠ 5 = ______________ f) m∠ 6 = ______________

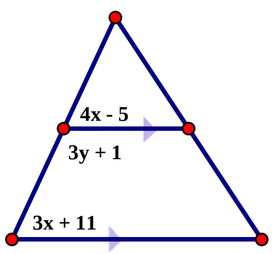


3. Solve the following.

a)

x = _________

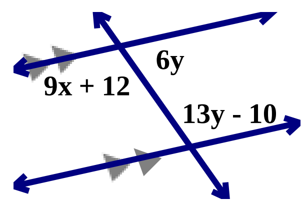
y = _________



b)

x = _________

y = _________



4. Solve

a) m∠ 6 = ________

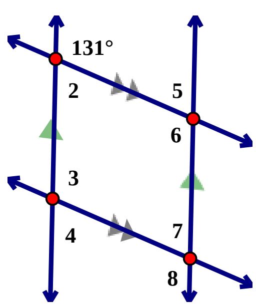
b) m∠ 7 = ________

c) m∠ 4 = ________

d) m∠ 2 = ________

e) m∠ 5 = ________

f) m∠ 8 = ________



5. Solve

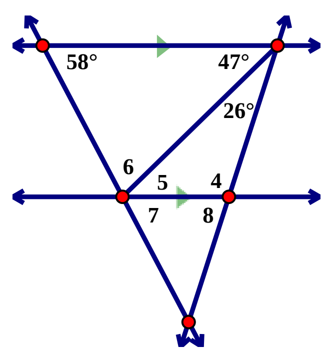
a) m∠ 7 = ________

b) m∠ 5 = ________

c) m∠ 6 = ________

d) m∠ 4 = ________

e) m∠ 8 = ________



6. Solve

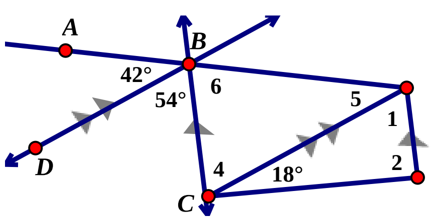
a) m∠ 5 = ________

b) m∠ 1 = ________

c) m∠ 4 = ________

d) m∠ 6 = ________

e) m∠ 2 = ________



7. Solve

a) m∠ 3 = ________

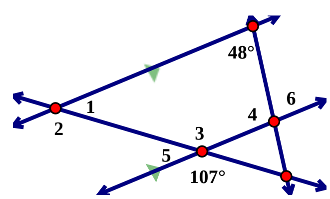
b) m∠ 5 = ________

c) m∠ 1 = ________

d) m∠ 4 = ________

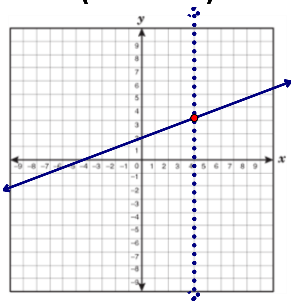
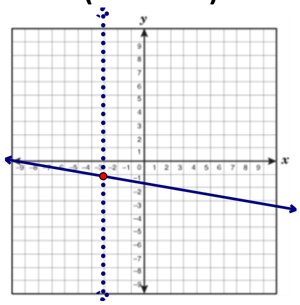
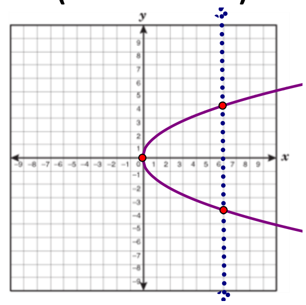
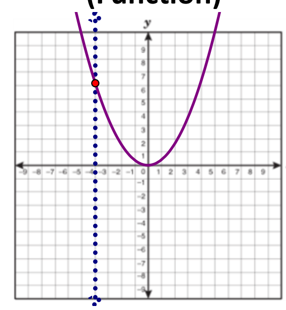
e) m∠ 6 = ________

f) m∠ 2 = ________

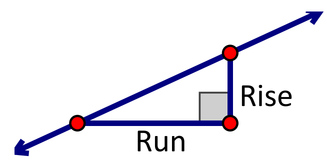


Linear Equations

Linear equations are functions. What that means is that for each x value there is exactly one y value. The vertical line test is used to determine if something is a function or not because if the vertical line intersects the relation exactly once for all values of the domain, then it must be a function. All non-vertical linear equations are functions.

Linear Equation (Function)	Linear Equation (Function)	Parabolic Equation (Not Function)	Parabolic Equation (Function)
			

One of the characteristics of a linear equation is that it has a constant slope. The slope of a line is defined as the rise/run or the change of y over the change is x. Universally, we use m as the variable that represents slope.



Slope Formula $slope = m = \frac{Rise}{Run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

A (-3, 6) B (4, 3)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-6}{4-(-3)} = \frac{-3}{7}$$

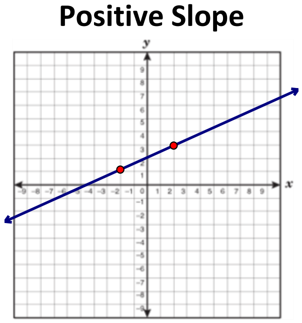
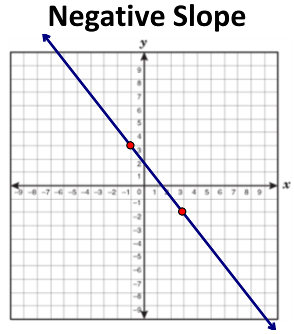
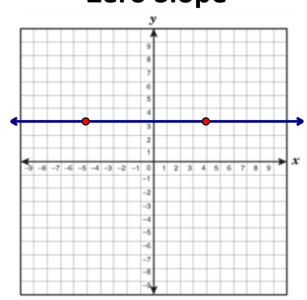
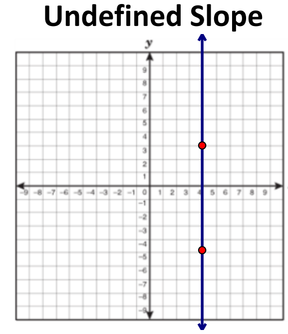
A (0, 3) B (5, 0)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{5-0} = \frac{-3}{5}$$

A (7, 3) B (5, -2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-2)-3}{5-7} = \frac{-5}{-2} = \frac{5}{2}$$

Positive & Negative Slopes

Positive Slope	Negative Slope	Zero Slope	Undefined Slope
			
Rises to the right	Falls to the right	Horizontal line	Vertical line

NYTS (Now You Try Some)

1. Determine the slope.

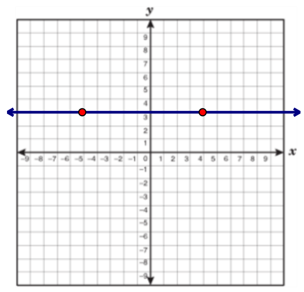
a) A (3, 5) B (8, 3) b) A (-5, 1) B (-3, -7) c) A (0, -9) B (2, 7) d) A (-5, 3) B (-1, -5)

<u>Special Lines</u> $y = b$ $x = a$

<u>HORIZONTAL LINE</u>

Horizontal lines have a slope of zero and are written y = b, where b is a constant. The line y = 3 is a horizontal line where every point on the line has a y coordinate of 3: (x, 3).

$y = b$

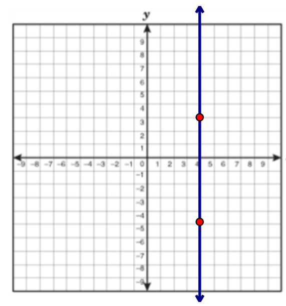


$y = 3$

<u>VERTICAL LINE</u>

Vertical lines have undefined slopes and are written x = a, where a is a constant. The line x = 4 is a vertical line where every point on the line has an x coordinate of 4: (4, y).

$x = a$



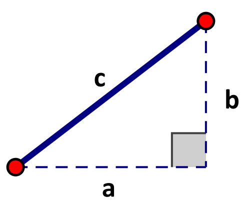
$x = 4$

Students struggle with these two equations. They want to relate the x-axis, which is a horizontal line, to x = a, but THIS IS INCORRECT. The x-axis has an equation of y = 0 and the y-axis has an equation of x = 0.

2. Determine if the line is vertical or horizontal.

a) $y = -7$	b) $x = 0$	c) $x = 4$	d) $y = 3$
Vertical or Horizontal	Vertical or Horizontal	Vertical or Horizontal	Vertical or Horizontal

<u>The Distance Formula</u> – In an earlier objective we discussed how the distance formula is actually the Pythagorean Theorem. When not on the coordinate plane we refer to it as the Pythagorean Theorem, and when we are on the coordinate grid, we refer to it as the distance formula, but ultimately, they are the same relationship. Once on the coordinate grid we are able to determine the length of the legs of the triangle by calculating the difference between x values and the difference between the y values.



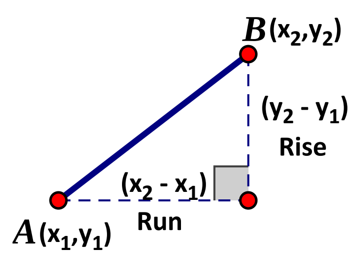
$a^2 + b^2 = c^2$

$$a^2 + b^2 = c^2$$

$$(run)^2 + (rise)^2 = (\text{distance})^2$$

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = (\text{distance})^2$$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \text{distance}$$



Using this formula, we are able to easily calculate any distances found on the coordinate plane.

A (1, -2) **B (5, 1)**

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \text{distance}$$

$$\sqrt{(5-1)^2 + (1-(-2))^2} = \text{distance}$$

$$\sqrt{(4)^2 + (3)^2} = \text{distance}$$

$$\sqrt{25} = 5 = \text{distance}$$

A (4, -2) **B (1, -3)**

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \text{distance}$$

$$\sqrt{(1-4)^2 + ((-3)-(-2))^2} = \text{distance}$$

$$\sqrt{(-3)^2 + (-1)^2} = \text{distance}$$

$$\sqrt{10} = \text{distance}$$

3. Determine the distance between the two points.

a) A (8, -1) B (0, 14) **b) A (1, -3) B (4, 8)**

Midpoint Formula – The midpoint formula comes in handy in lots of different circumstances. Many shapes and relationships use the midpoint. It is important to note that the result of the formula is not a distance or length; it is a point – a location.

$$\left(\frac{(x_1+x_2)}{2},\frac{(y_1+y_2)}{2}\right)=(x_m,y_m)$$

$$\frac{(x_1+x_2)}{2}=x_m \qquad \frac{(y_1+y_2)}{2}=y_m$$

a) A (1, 4) B (6, -5)

$$\left(\frac{(x_1+x_2)}{2},\frac{(y_1+y_2)}{2}\right)=(x_m,y_m)$$

$$\left(\frac{(1+6)}{2},\frac{(4+(-5))}{2}\right)=(x_m,y_m)$$

$$(3.5,-0.5)=(x_m,y_m)$$

b) A (4, -1) B (-8, -5)

$$\left(\frac{(x_1+x_2)}{2},\frac{(y_1+y_2)}{2}\right)=(x_m,y_m)$$

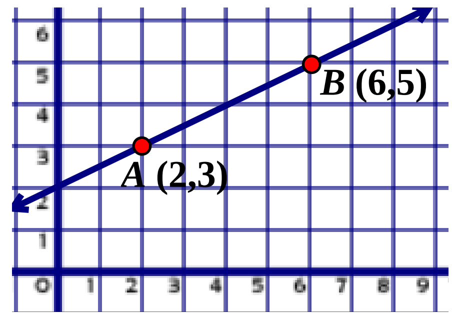
$$\left(\frac{(4+(-8))}{2},\frac{((-1)+(-5))}{2}\right)=(x_m,y_m)$$

$$(-2,-3)=(x_m,y_m)$$

4. Determine the midpoint between the two points.

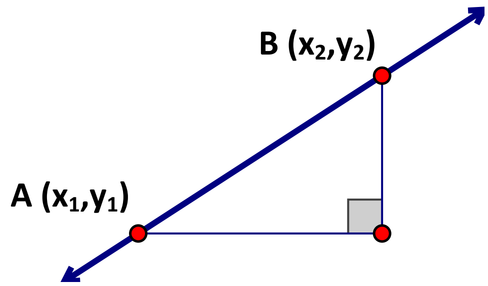
a) A (4, -1) B (2, 10) **b) A (5, -3) B (7, 9)**

1. On the graph draw where the rise and the run are, and then determine the rise, run, and slope of the line.



Rise = ______ Run = ______ Slope = ____________

2. Using the general points A and B, explain where the slope formula comes from. Use the diagram as a part of your explanation.



3. Determine the slope of the lines.

a) A (-4, 6) B (2, -3) **b) A (2, -1) B (-12, 4)** **c) A (-1, -8) B (-5, 4)**

m = _________ m = _________ m = _________

d) y = 2x – 7 **e) y = ½ x + 3** **f) y + 6 = -4(x – 5)**

m = _________ m = _________ m = _________

g) y = -5 **h) 6x – 2y = 4** **i) 2x – 7y = 14**

m = _________ m = _________ m = _________

4. Determine whether the slope is (P)ositive, (N)egative, (Z)ero, or (U)ndefined.

a) y = -4x + 4 **b) x = -3.5** **c) 2x – 3y = 12** **d) y – 2 = -3(x – 5)**

P or N or Z or U P or N or Z or U P or N or Z or U P or N or Z or U

a) y = 4 **b) y + 7 = 7(x + 2)** **c) 5x + 2y = 15** **d) -5x = 12**

P or N or Z or U P or N or Z or U P or N or Z or U P or N or Z or U

5. Write the equation of the line from the given description.

a) a horizontal line through (2, -5)

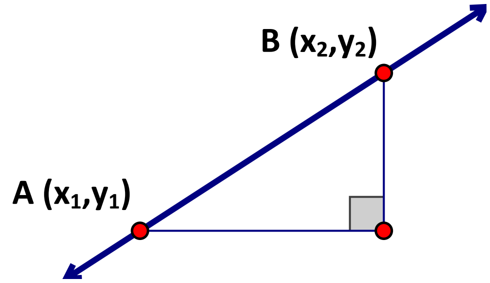
b) a vertical line through (0, 5)

c) a vertical line through (-2, -4)

d) a horizontal line through (5, 4)

_____________________ _____________________ _____________________ _____________________

6. Using the general points A and B, label and explain where the midpoint formula comes from.



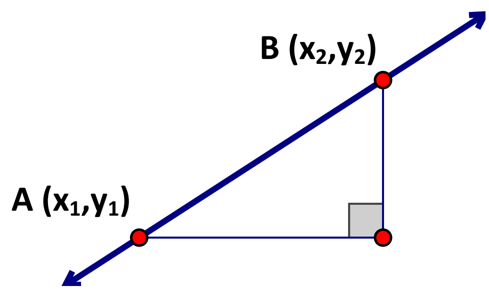
7. Determine the midpoint of the following segments.

a) A (-4, 6) B (2, -3)

b) A (2, -1) B (-12, 4)

c) A $\left(\frac{2}{5}, -3\right)$ B $\left(\frac{-1}{3}, \frac{1}{2}\right)$

8. Demonstrate how the Pythagorean Theorem turns into the distance formula.



9. Determine the distance from A to B.

a) A (-4, 6) B (2, -3)

b) A (2, -1) B (-12, 4)

c) A (-3, 8) B (-3, -4)

Equations of Lines

Writing, graphing, and analyzing linear equations is also another essential foundational concept. Linear equations other than vertical and horizontal lines have multiple ways that they can be written. Each of the different forms has its own purpose - Its own strengths. The linear form that you will use will depend on what you are being asked to do or determine.

<u>FORM -- Slope Intercept Form</u> $y = mx + b$

Strengths of this form: Graphing a line; determining the y-intercept; Determining the slope

This is probably the most popular form of a line because of its ease in graphing and finding slope.

$y = mx + b$ -- where m is the slope and b is the y intercept.

Examples: Determine the equation of the line in slope intercept form.

a) m = 3 y intercept = -6

$$y = mx + b$$
$$y = 3x - 6$$

b) m = 5 A (4, -1)

$$y = mx + b$$
$$-1 = 5(4) + b$$
$$-1 = 20 + b$$
$$-21 = b$$
$$y = 5x - 21$$

c) A (4, -2) B (1, 4)

$$\frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{4-(-2)}{1-4}$$
$$\frac{6}{-3} = -2$$

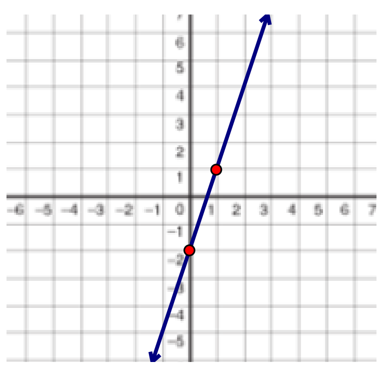
$$y = mx + b$$
$$-2 = -2(4) + b$$
$$-2 = -8 + b$$
$$6 = b$$
$$y = mx + b$$
$$y = -2x + 6$$

Examples: Graph the line

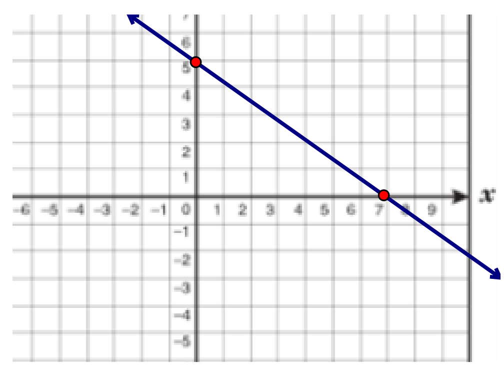
Because the slope intercept form provides a point (the y-intercept) and a slope, it is very easy to graph lines.

$$y = mx + b$$
$$y = 3x - 2$$

m = 3 y int. (0, -2)



$$y = mx + b$$
$$y = -\frac{5}{7}x + 5$$
$$m = \frac{-5}{7}$$
y int. $(0, 5)$



NYTS (Now You Try Some)

1. Determine the equation of the line in slope intercept form.

a) m = -5 B (2, -12)

b) A (-3, 4) B (0, -5)

FORM -- Point Slope Form $(y_2 - y_1) = m(x_2 - x_1)$

Strengths of this form: Easy to create an equation.

Point Slope Form is actually just a simple transformation of the slope formula,

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{to} \quad m(x_2 - x_1) = y_2 - y_1 .$$

Ultimately this form is popular when creating an equation of a line quickly. Most often the known values of a line are its slope and a point, and so an equation can be determined quickly.

Examples: Determine the equation of the line.

a) m = 3 y intercept = -6

$m(x_2 - x_1) = y_2 - y_1$
$3(x_2 - 0) = y_2 - (-6)$
$3x = y + 6$

b) m = 5 A (4, -1)

$m(x_2 - x_1) = y_2 - y_1$
$5(x_2 - 4) = y_2 - (-1)$
$5(x - 4) = y + 1$

c) m = -2 A (-8, -5)

$m(x_2 - x_1) = y_2 - y_1$
$-2(x_2 - (-8)) = y_2 - (-5)$
$-2(x + 8) = y + 5$

2. Determine the equation of the line in point slope form.

a) m = -9 A (-5, 3)

b) m = $\frac{1}{3}$ A (4, -1)

FORM -- Standard Form $Ax + By = C$

Strengths of this form: Determining x and y intercepts; Determining the slope.

This form simplifies the process of solving for intercepts because by placing in zero values for x or y, the equation is ready to be solved for the y intercept or x intercept.

Examples: Determine the x and y intercepts of the line.

$2x + 3y = 6$

$2x + 3y = 6$
$2(0) + 3y = 6$
$y = 2$
(0, 2)

$2x + 3y = 6$
$2x + 3(0) = 6$
$x = 3$
(3, 0)

$5x - 3y = 30$

$5x - 3y = 30$
$5(0) - 3y = 30$
$y = -10$
(0, -10)

$5x - 3y = 30$
$5x - 3(0) = 30$
$x = 6$
(6, 0)

Find the Slope

$Ax + By = C$
$By = -Ax + C$
$y = \frac{-A}{B}x + \frac{C}{B}$
$m = \frac{-A}{B}$

Examples: Determine the slope of the line.

$2x + 3y = 6$

$m = \frac{-A}{B}$

$m = \frac{-2}{3}$

$5x - 3y = 30$

$m = \frac{-A}{B}$

$m = \frac{-5}{-3} = \frac{5}{3}$

$x - 8y = 12$

$m = \frac{-A}{B}$

$m = \frac{-1}{-8} = \frac{1}{8}$

3. Find the intercepts

a) $8x - 1y = 24$

b) $4x - 6y = 24$

4. Find the slope.

a) $4x - 2y = 8$

b) $10x - 4y = 8$

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1. Write the equation of a line in slope intercept form.

a) m = 6   y int. = -4

b) m = -2 (2, 6)

c) m = $\frac{2}{3}$ (12, -1)

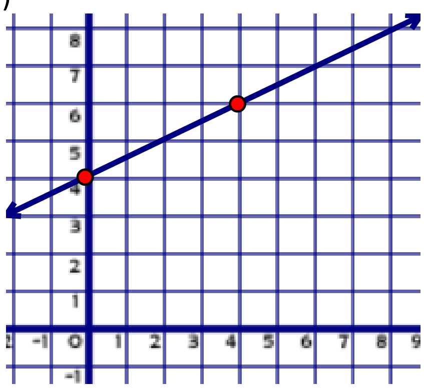
d) (0, 3)   (-5, 0)

e) (2, 5)   (-1, -4)

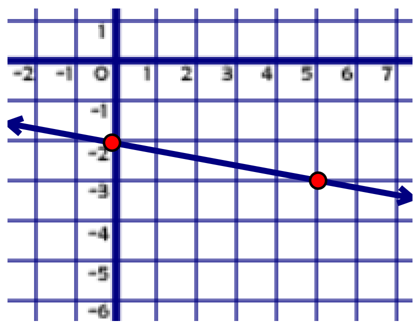
f) 3x – 2y = 8

2. Write the equation of a line in slope intercept form.

a)



b)



_________________________   _________________________   _________________________

3. Write the equation of a line in point slope form.

a) m = -3   (-3, 8)

b) m = $\frac{5}{7}$ (2, 7)

c) m = 5   (7, -1)

_________________________   _________________________   _________________________

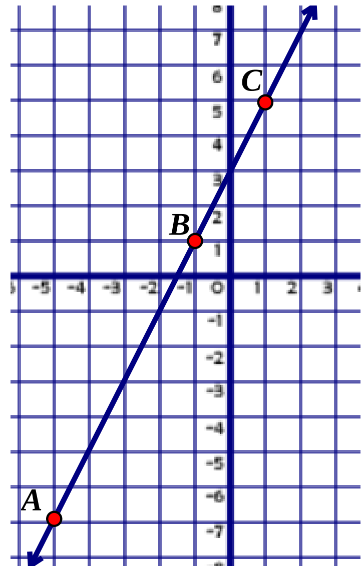
4. Determine the x and y intercepts of the line.

a) 5x + 2y = 30

b) y = 3x - 12

c) 4x - 3y = 12

(_____ , 0)   (0, ______)   (_____ , 0)   (0, ______)   (_____ , 0)   (0, ______)



5. Determine the equation of the line in point slope form (PSF) using the given point and then transform the equation into slope intercept form (SIF).

a) Point A (-5, -7)

PSF ____________________________

SIF____________________________

b) Point C (1, 5)

PSF ____________________________

SIF____________________________

c) In a) and b) you used different points, but the equation simplified to the same thing. Why didn't the point that we used matter?

6. Determine if the given point is a solution for the given line.

a) $y = 5x - 1$ (-3, -14)

Yes or No

b) $y = -3x + 6$ (3, -3)

Yes or No

c) $3x - y = -1$ (2, -7)

Yes or No

7. Create the equation of a line given the following information.

a) $m = 2$ (2, 5)

b) $m = -4$ (5, 0)

c) $m = \frac{1}{4}$ (0, -4)

d) (-2, 4) (8, -4)

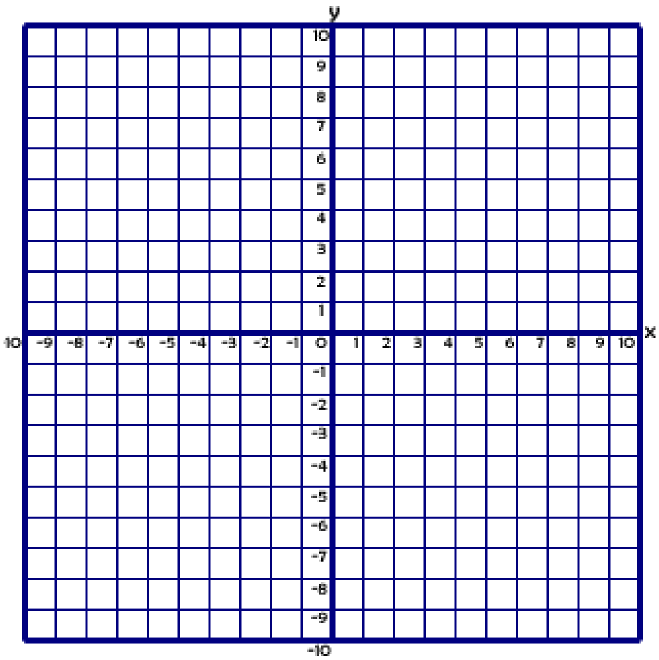
e) (5, 0) (0, -3)

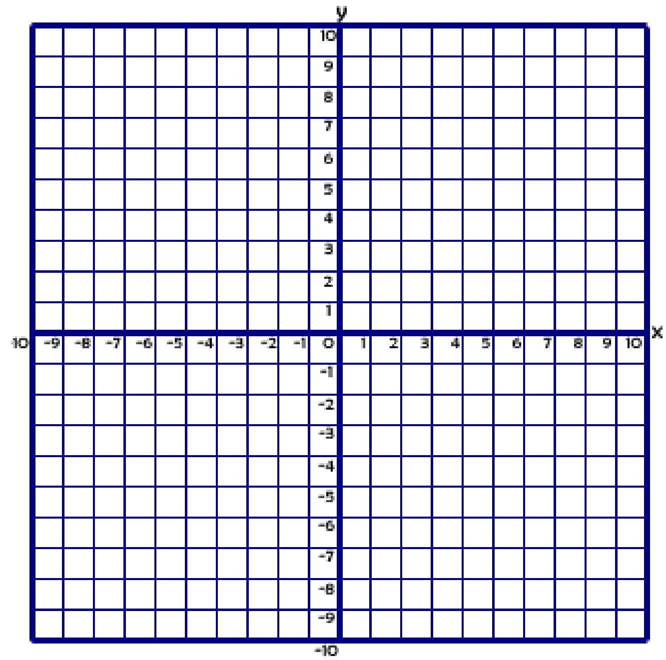
f) y int. = 3 $m = -11$

8. Graph the following lines

a) y = -3x + 7   and y – 3 = 2 (x – 4)

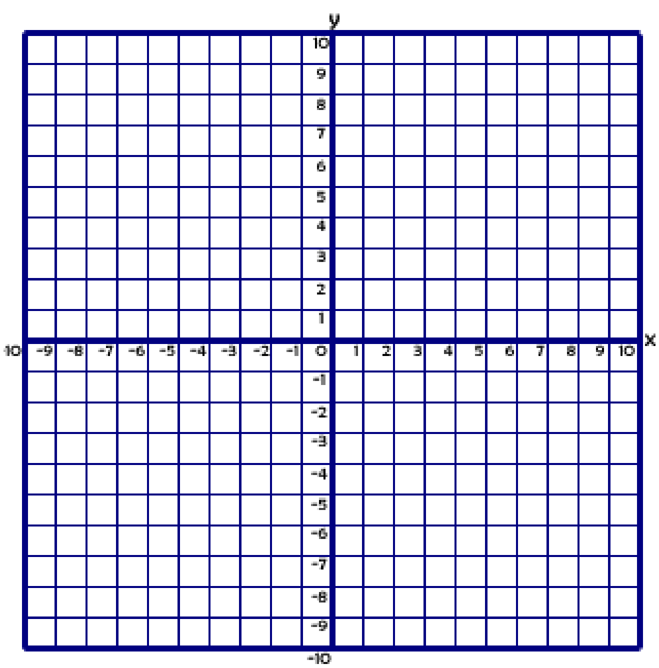
b) 3x + y = 2   and y + 3 = -1 (x + 1)

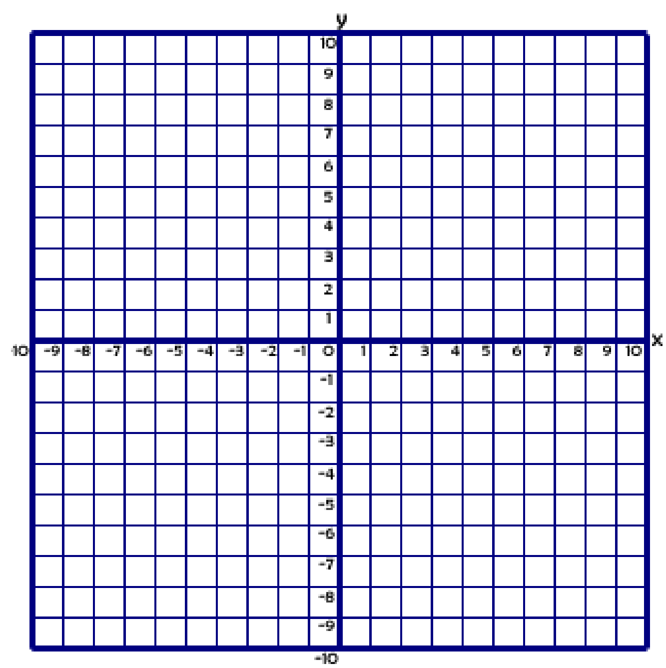




c) $y = \frac{2}{5}x - 3$ and $y = -\frac{1}{3}x + 5$

d) 5x + 3y = 15 and 2x - 6y = 12

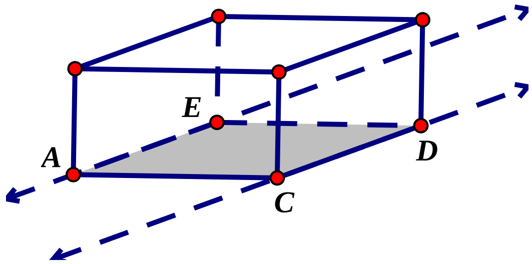




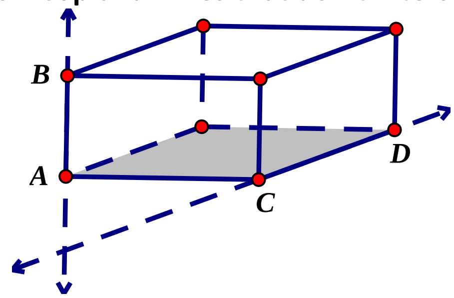
Parallel Lines and Skew Lines

Parallel lines are two **COPLANAR** lines that never intersect. Most definitions that students (and some teachers) use leave out the word coplanar. Coplanar is an essential part of the definition because there are other types of lines that never intersect but are not parallel; these types of lines are called Skew Lines.

Parallel Lines
(Coplanar lines that don't intersect)



Skew Lines
(Non-Coplanar lines that don't intersect)



It appears that these lines do intersect but it is a distortion in the 2-D drawing of the 3-D relationship - $\overline{AB}$ and $\overline{CD}$ will never intersect.

NYTS (Now You Try Some)

1. **Determine the relationship between the lines.**

a) $\overline{AB}$ and $\overline{CD}$

Skew Parallel Intersect

b) $\overline{EF}$ and $\overline{DG}$

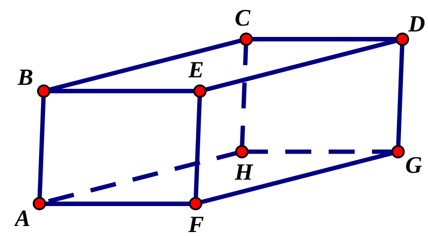
Skew Parallel Intersect

c) $\overline{AF}$ and $\overline{EF}$

Skew Parallel Intersect

d) $\overline{CG}$ and $\overline{AH}$

Skew Parallel Intersect



Parallel Lines and Slope

When we move from the general plane to the coordinate plane, we notice that lines that never intersect have the same slope. This relationship can be proven without even using the coordinate plane, which is very powerful because then we know that it works in all cases.

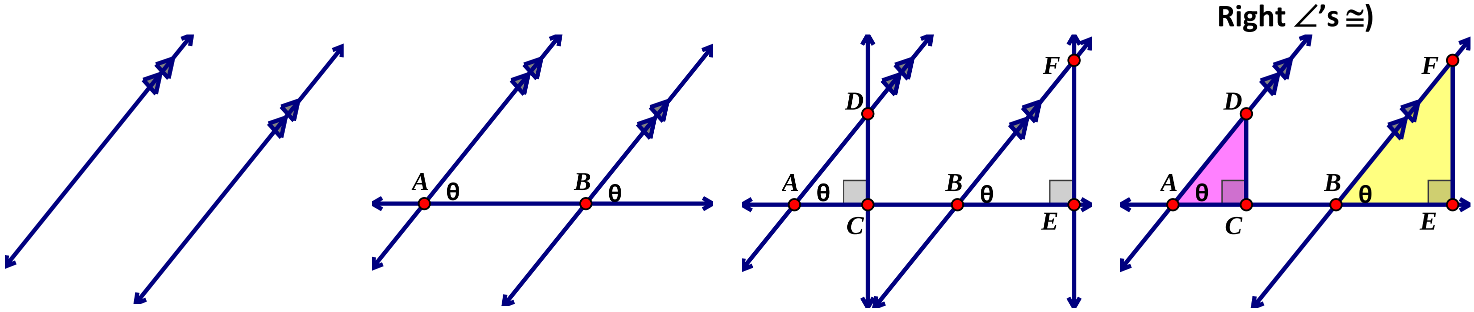
Given: Two lines are parallel. **Prove**: The slopes are equal.

Given parallel lines

Draw in transversal; Corresponding $\angle$'s $\cong$

Draw in $\perp$ lines; Right $\angle$'s $\cong$

ΔACD ~ ΔBEF by AA (Corr. $\angle$'s $\cong$ & Right $\angle$'s $\cong$)



Because the triangles are similar, the sides are proportional.

$$\frac{DC(rise)}{AC(run)} = \frac{FE(rise)}{BE(run)}$$

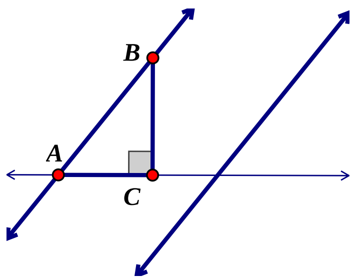
The slopes are equal because of this proportional relationship.

This is true for all cases.

It is important to establish the converse relationship as well: that if lines have the same slope, then they are parallel.

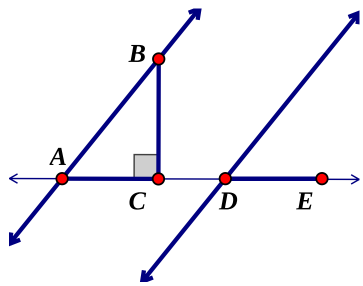
Given: The slopes are equal. **Prove**: Two lines are parallel.

Given the slopes are equal; Extend $\overline{AC}$



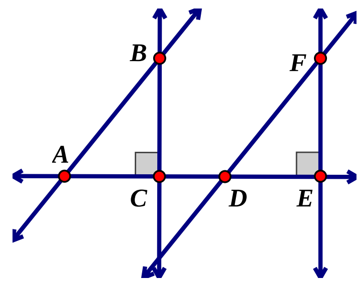
Because the triangles are congruent, ∠A ≅ ∠D

Determine E, such that $\overline{AC} \cong \overline{DE}$



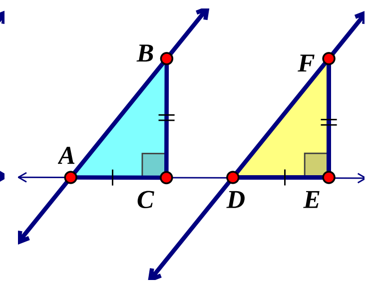
If ∠A ≅ ∠D, then $\overline{AB} \parallel \overline{DF}$

Construct ⊥ through E; $\overline{BC} \cong \overline{EF}$ because given equal slopes



Because Corresponding ∠'s are congruent.

ΔACB ≅ ΔDEF by SAS ($\overline{AC} \cong \overline{DE}$, ∠C ≅ ∠E & $\overline{BC} \cong \overline{EF}$)



This is true for all cases.

Some examples of parallel lines would be:

a) $y = 3x + 11$

$y = 3x - \frac{2}{3}$

Slopes =, m = 3

b) $y = \frac{3}{6}x + 5$

$y = \frac{1}{2}x - 1$

Reduced Slopes = , m = $\frac{1}{2}$

c) $4x - 5y = 8$

$4x - 5y = -1$

m = $\frac{-A}{B} = \frac{-4}{-5} = \frac{4}{5}$

d) $5y = -10x - 8$

$2x + 1y = 15$

Slopes =, m = -2

2. Determine if the following lines are parallel to each other or not.

a) $y = -1x + 5$

$-3y = 3x + 9$

Parallel or Not Parallel

b) $3x + 5y = 15$

$3y + 5x = 15$

Parallel or Not Parallel

c) $y = 3x + 5$

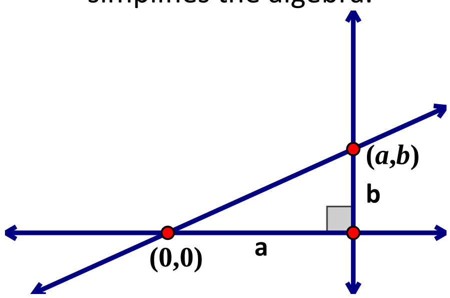
$y = \frac{15}{5}x - 2$

Parallel or Not Parallel

Perpendicular Lines and Slope

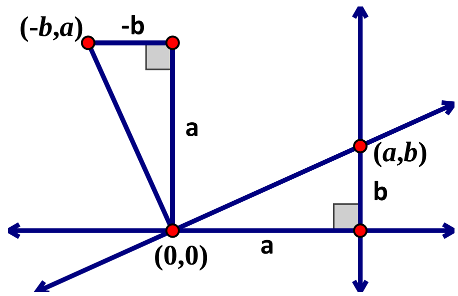
Perpendicular lines also have a slope relationship. Two perpendicular slopes have negative reciprocal slopes, or in other words, the product of two perpendicular slopes is -1. I will prove this below.

For simplicity, I am going to use the origin as our center of rotation to demonstrate the concept. Any point would work (and provide a more general proof), but using the origin simplifies the algebra.



Rotate point (a, b) 90°. A rotation of 90° takes (a, b) to (-b, a).

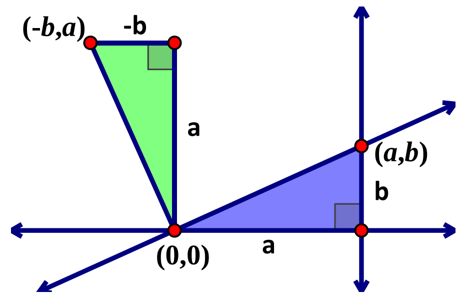
The slope of the new line is $\frac{a}{-b}$



Slope of original line, $\frac{b}{a}$

Slope of 90° rotated line, $\frac{a}{-b}$

$$\left(\frac{b}{a}\right)\left(\frac{a}{-b}\right) = \frac{ab}{-ab} = -1$$



Perpendicular slopes are negative reciprocals, and their products will always equal -1.

Some examples of parallel lines would be:

a) $y = 3x + 1$
$y = -\frac{1}{3}x - 5$

Slopes neg. reciprocal
m = 3 & m = $-\frac{1}{3}$

b) $y = \frac{3}{5}x$
$y = -\frac{5}{3}x + 7$

Slopes neg. reciprocal
m = $\frac{3}{5}$ & m = $-\frac{5}{3}$

c) $4x - 5y = 8$
$5x + 4y = -1$

m = $\frac{-A}{B} = \frac{-4}{-5} = \frac{4}{5}$
m = $\frac{-A}{B} = \frac{-5}{4}$

d) $5y = 10x - 8$
$1x + 2y = 15$

Slopes neg. reciprocal
m = 2 & m = $-\frac{1}{2}$

3. Determine whether the following are perpendicular or not.

a) $y = -1x + 5$
$y = x + 9$

$\perp$ or Not $\perp$

b) $3x + 5y = 15$
$3y - 5x = 15$

$\perp$ or Not $\perp$

c) $y = -3x + 5$
$y = \frac{15}{5}x - 2$

$\perp$ or Not $\perp$

Equations of Parallel and Perpendicular Lines

Knowing the slope relationships of parallel and perpendicular lines helps us determine equations of these types of lines quite easily. Usually to generate the equation of a line we need a slope and a point, knowing that the relationship is either parallel or perpendicular ultimately provides us slope information.

What is the equation of a parallel line to y = 3x + 2 that goes through A (-4,6)?	Slopes of parallel lines are equal. Thus, the new line has a slope of 3. m = 3 and goes through A (-4, 6)	$y-6=3(x-(-4))$ $y-6=3(x+4)$ $y-6=3x+12$ $y=3x+18$
What is the equation of a perpendicular line to y = 3x + 2 that goes through A (6,2)?	Slopes of perpendicular lines are negative reciprocals. Thus the new line has a slope of $-\frac{1}{3}$. m = $-\frac{1}{3}$ and goes through A (6, 2)	$y-2=-\frac{1}{3}(x-6)$ $y-2=-\frac{1}{3}x+\left(\frac{1}{3}\right)(6)$ $y-2=-\frac{1}{3}x+2$ $y=-\frac{1}{3}x+4$

4. Determine the equation.

a) What is the equation of a line parallel to y = -7x - 5 that goes through A (1, 3)?

b) What is the equation of a line perpendicular to $y=\frac{1}{4}x-2$ that goes through A (-12, 4)?

1. Why do we need to include the word COPLANAR when we define parallel lines?

2. Determine whether the given lines would be (P)arallel, (S)kew or (I)ntersecting.

a) $\overleftrightarrow{AB}$ and $\overleftrightarrow{EH}$ P or S or I

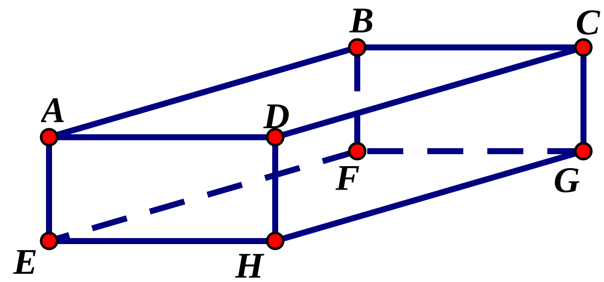
b) $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ P or S or I

c) $\overleftrightarrow{HG}$ and $\overleftrightarrow{FG}$ P or S or I

d) $\overleftrightarrow{FG}$ and $\overleftrightarrow{CD}$ P or S or I

e) $\overleftrightarrow{AB}$ and $\overleftrightarrow{HG}$ P or S or I

f) $\overleftrightarrow{CE}$ and $\overleftrightarrow{HG}$ P or S or I



3. Determine whether the given equations of lines are Parallel (||), Perpendicular (⊥) or Intersecting (×).

a) $2x+4=y$, $y=-2x-3$ || or ⊥ or ×

b) $y=\frac{5}{4}x$, $y=-\frac{4}{5}x+4$ || or ⊥ or ×

c) $3x+5y=15$, $3x+5y=10$ || or ⊥ or ×

d) $y=4x-3$, $2y+12=8x$ || or ⊥ or ×

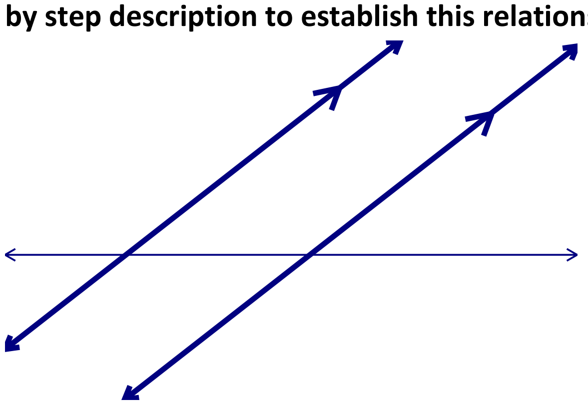
e) $y=\frac{1}{9}x-2$, $y=9x+4$ || or ⊥ or ×

f) $7x-1y=14$, $7x+1y=14$ || or ⊥ or ×

g) $4x+8y=10$, $y=-2x-3$ || or ⊥ or ×

h) $2x-7y=12$, $7x+2y=-4$ || or ⊥ or ×

4. Prove that if you have two lines that are parallel, then the slopes are equal. Use the diagram and a step by step description to establish this relationship.



5. $y = \frac{2}{3}x - 4$ **and** $y = -\frac{3}{2}x + 1$ **are perpendicular lines. Jack notices that the product of the two perpendicular slopes is -1. If a line has a slope of** $\frac{a}{b}$**, determine the perpendicular slope, and then show that the product is -1.**

6. Determine the equation of the line that is:

a) parallel to y = -3x + 2 and goes through (1, 5) in slope intercept form.

b) parallel to $y = \frac{1}{5}x - 4$ and goes through (10, -2) in point slope form.

c) perpendicular to y = 5x + 4 through (-2, -3) in point slope form.

d) perpendicular to y = -2x – 1 through (-5, 2) in the slope intercept form.

e) parallel to y = -6x + 4 through $\left(\frac{2}{3}, 2\right)$ in slope intercept form.

f) perpendicular to $y = -\frac{7}{3}x - 1$ through $\left(14, \frac{5}{2}\right)$ in point slope form.

g) parallel to x = 5 through (-3, 9) in slope intercept form.

h) perpendicular to $y = \frac{3}{10}x - 9$ through (5, 2) in slope intercept form.