Geometry Workbook 7:

Dilations

Student Name ____

STANDARDS:

G.SRT.A.1	a) Verify experimentally the properties of dilations given by a center and a scale factor: a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
	b) Verify experimentally the properties of dilations given by a center and a scale factor: the dilation of a line segment is longer or shorter in the ratio given by the scale factor.
G.GPE.B.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

SKILLS:

- ☐ I will determine the properties of dilation.
- ☐ I will be able to dilate when the center of dilation is in, on and out of the shape.
- ☐ I will be able to dilate when given a center of dilation and a scale factor.
- ☐ I will be able to determine the center of dilation and the scale factor from a diagram.
- ☐ I will be able to dilate using both positive and negative scale factors.
- ☐ I will be able to construct a dilation.
- ☐ I will be able to use the dilation coordinate rules for dilations using any center of dilation.
- ☐ I will be able to partition a line segment based on a provided ratio.

Notes:

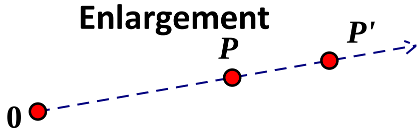
So what is a dilation? How is it defined?

A dilation with center O and a scale factor of k is a transformation that maps every point P in the plane to point P' so that the following properties are true:

1. **If P is NOT the center O**, then P' lies on $\overrightarrow{OP}$. The scale factor k is a **positive** number such that

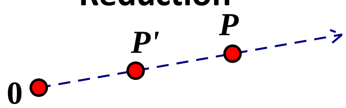
$$k = \frac{OP'}{OP} \text{ and } k \neq 1.$$

Enlargement



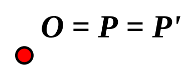
$$\frac{OP'}{OP} > 1$$

Reduction



$$0 < \frac{OP'}{OP} < 1$$

2. **If P is the center point O**, then P = P'. The center of dilation is the only point in the plane that does not move.



NOTATION

$$D_{O,k}(x, y)$$

O is the center of dilation.
k is the value of the scale factor.

What are the properties for dilation?

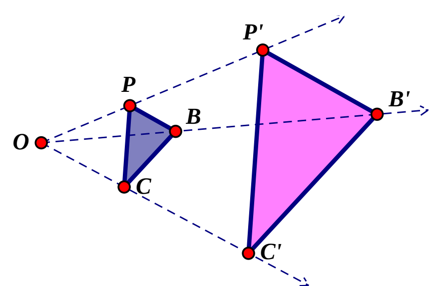
DILATION PROPERTIES - Dilation is NOT an isometric transformation so its properties differ from the ones we saw with reflection, rotation and translation. The following properties are preserved between the pre-image and its image when dilating:

- **Angle measure** (angles stay the same)
- **Parallelism** (things that were parallel are still parallel)
- **Collinearity** (points on a line remain on the line)
- **Distance IS NOT preserved!!!**

After a dilation, the pre-image and image have the same shape but not the same size.

TRANSFORMATION PROPERTIES – The following properties are present in dilation:

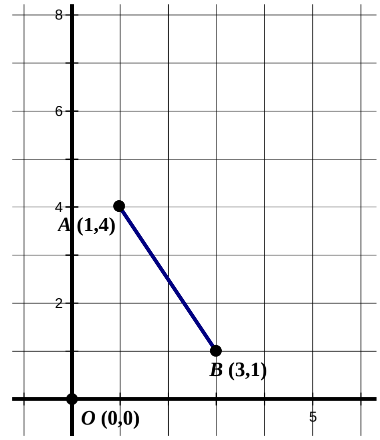
- **DISTANCES ARE DIFFERENT (PROPORTIONAL)** – The distance points move during dilation depends on their distance from the center of dilation - points closer to the center of dilation will move a shorter distance than those further away. In our example $PP' \neq BB' \neq CC'$ and point B is farther away from the center of dilation O than point P, thus $BB' > PP'$.



- **ORIENTATION IS THE SAME** – The orientation of the shape is maintained.
- **SPECIAL POINTS** – The center of dilation is an invariant point and does not move in a dilation. If the pre-image (P) = image (P') after a dilation, then point P was the center of dilation.

1. Investigating the length and slope properties of dilation.

GRAPH #1



a) Calculate the length AB. (reduced radical)

A (1,4) B (3,1)

$$dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

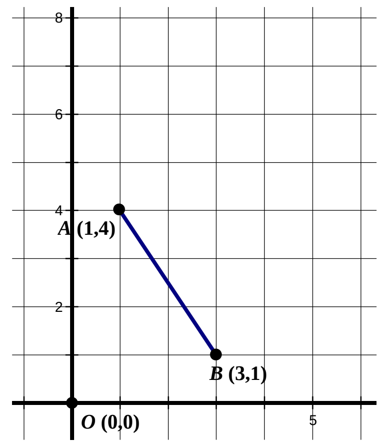
b) Calculate the slope of $\overline{AB}$.

A (1,4) B (3,1)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

c) Dilate $\overline{AB}$ about center O with scale factor of 2. (Graph it on graph #2)

GRAPH #2



d) Calculate the length A'B'. (reduced radical)

A' (_____,_____) B' (_____,_____)

$$dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

How does this compare to AB?

e) Calculate the slope of $\overline{A'B'}$.

A' (_____,_____) B' (_____,_____)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

How does this compare to the slope of $\overline{AB}$?

Using graph #2, calculate the following.

f) Calculate the length OA. (reduced radical)

0 (0,0) A (1,4)

$$dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

g) Calculate the length OA'. (reduced radical)

0 (0,0) A' (_____, _____)

$$dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

What is the relationship between OA' and OA?

h) Calculate the length OB. (reduced radical)

0 (0,0) B (3,1)

$$dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

i) Calculate the length OB'. (reduced radical)

0 (0,0) B' (_____, _____)

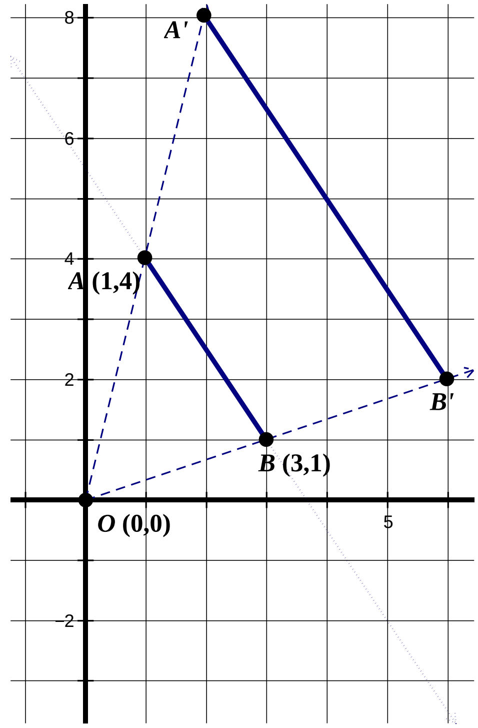
$$dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

What is the relationship between OB' and OB?

2. Investigating the angle properties of dilation.

a) Use a protractor to determine the following angle measures. (round to nearest degree)	b) Use a protractor to determine the following angle measures. (round to nearest degree)
In ΔAOB	In ΔA'OB'
m∠OAB = _________	**m∠OA'B' = _________**
m∠OBA = _________	**m∠OB'A' = _________**

GRAPH #3



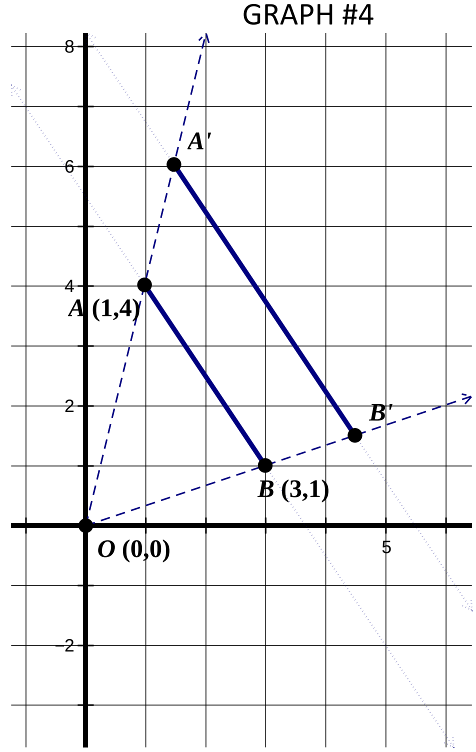
c) What do you notice about these angles?

d) What do these angles tell you about $\overline{AB}$ and $\overline{A'B'}$?

Do these relationships change when we dilate by a different value?

e) Use a protractor to determine the following angle measures. (round to nearest degree)	f) Use a protractor to determine the following angle measures. (round to nearest degree)
In ΔAOB	In ΔA'OB'
m∠OAB = _________	**m∠OA'B' = _________**
m∠OBA = _________	**m∠OB'A' = _________**

GRAPH #4



g) Did the scale factor change the angle relationships you found in a – d? Why did/didn't it affect it?

1. What does it mean to dilate?

2. Where do we see dilations in the world?

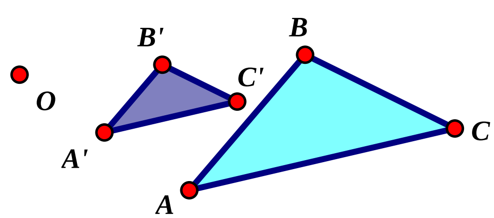
3. What do we mean when we say that the dilation was an enlargement?

4. What do we mean when we say that the dilation was a reduction?

5. Determine whether the following situations are REDUCTIONS OR ENLARGEMENTS.

Scale Factor of 5:2
(image : pre-image)

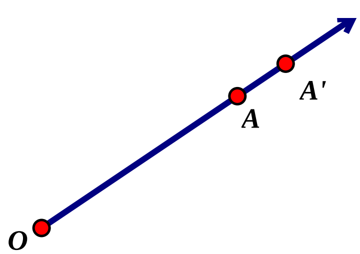
Reduction or Enlargement



Reduction or Enlargement

$$D_{O,5}(A) = A'$$

Reduction or Enlargement



Reduction or Enlargement

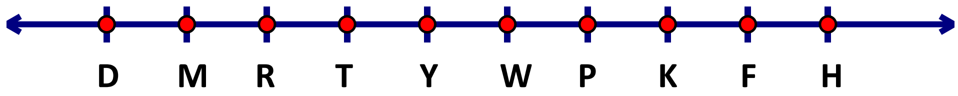
$$D_{O,0.5}(H) = H'$$

Reduction or Enlargement

Scale Factor of $\frac{2}{3}$

Reduction or Enlargement

6. Determine the missing point.



a) $D_{T,3}(Y)=(\underline{\hspace{2cm}})$ b) $D_{R,\frac{1}{2}}(F)=(\underline{\hspace{2cm}})$ c) $D_{P,4}(\underline{\hspace{2cm}})=(R)$

d) $D_{H,3}(K)=(\underline{\hspace{2cm}})$ e) $D_{T,-2}(M)=(\underline{\hspace{2cm}})$ f) $D_{F,\frac{1}{8}}(\underline{\hspace{2cm}})=(K)$

7. The dilation centered at O with a scale factor of 3. OA = 6 cm, OB = 7 cm and AB = 8 cm

a) A'B' = ______________

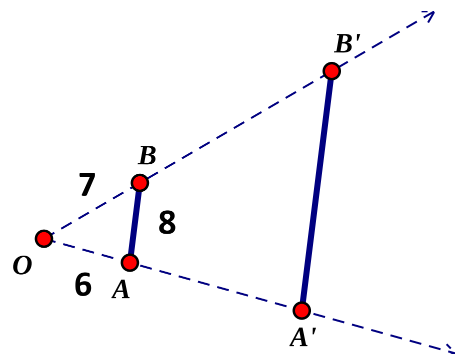
b) OB' = ______________

c) OA' = ______________

d) AA' = ______________

e) BB' = ______________

f) What is the ratio of OA:AA'? ______________



8. T the dilation centered at O with a scale factor of $\frac{1}{2}$. OA = 6 cm, OB = 8 cm and AB = 4 cm.

a) A'B' = ______________

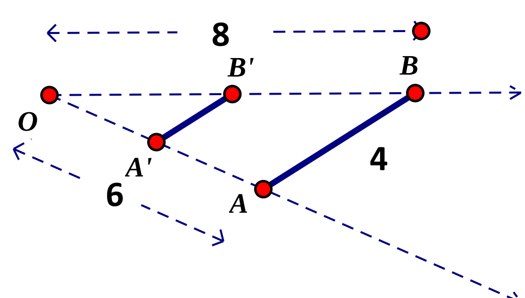
b) OB' = ______________

c) OA' = ______________

d) AA' = ______________

e) BB' = ______________

f) What is the ratio of OA':AA'? _____________



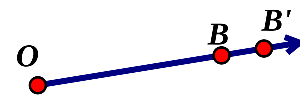
1. Circle whether the following situations are REDUCTIONS OR ENLARGEMENTS.

a) Scale Factor of 7:1
(image : pre-image)

Reduction or Enlargement

b) $D_{O,3}(H) = H'$

Reduction or Enlargement

c) 

Reduction or Enlargement

d) $D_{O,1.75}(A) = A'$

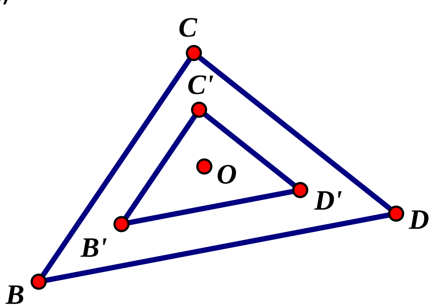
Reduction or Enlargement

e) Scale Factor of 2:3
(image : pre-image)

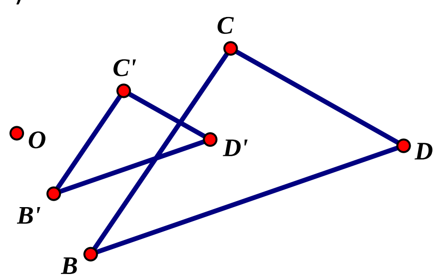
Reduction or Enlargement

f) $D_{O,\frac{5}{3}}(G) = G'$

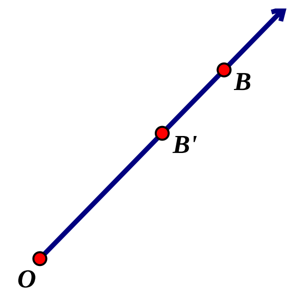
Reduction or Enlargement

g)


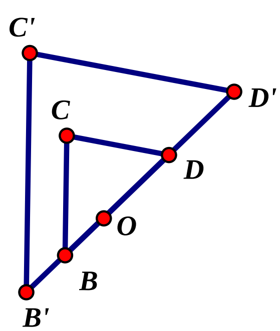
Reduction or Enlargement

h)


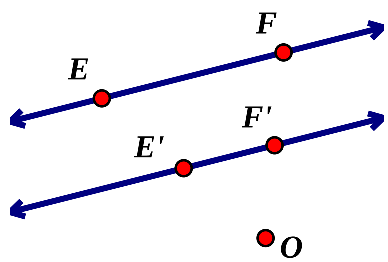
Reduction or Enlargement

i)


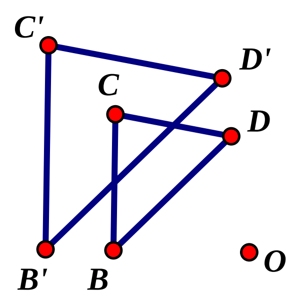
Reduction or Enlargement

J)


Reduction or Enlargement

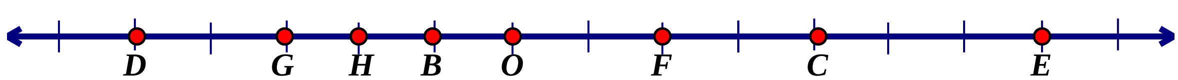
k)


Reduction or Enlargement

l)


Reduction or Enlargement

2. Determine the missing point.



a) $D_{O,3}(B) = (\_\_\_\_\_\_)$

b) $D_{O,-2}(H) = (\_\_\_\_\_\_)$

c) $D_{G,-2}(H) = (\_\_\_\_\_\_)$

d) $D_{E,3}(C) = (\_\_\_\_\_\_)$

e) $D_{H,4}(\_\_\_\_\_\_) = (F)$

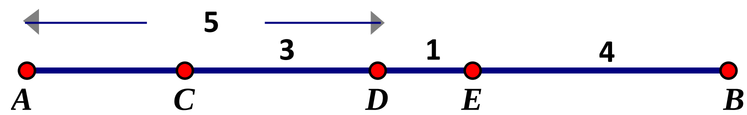
f) $D_{H,-9}(\_\_\_\_\_\_) = (E)$

g) $D_{H,3}(\_\_\_\_) = (C)$

h) $D_{C,2.5}(F) = (\_\_\_\_)$

i) $D_{G,\frac{7}{5}}(F) = (\_\_\_\_\_\_)$

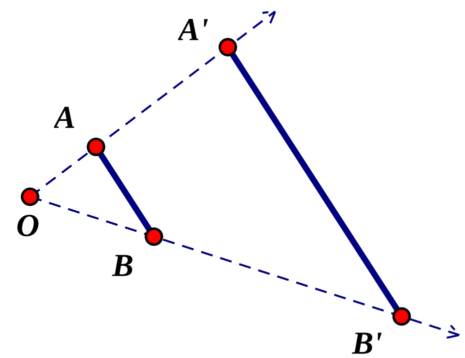
3. Determine the ratio. (Reduce the ratio)



a) CD : DE _______ : _______ b) EB : BD _______ : _______ c) CD : DA _______ : _______

d) AC : CD _______ : _______ e) CE : CD _______ : _______ f) AC : AB _______ : _______

4. Answer the following questions about the dilation, centered at O.

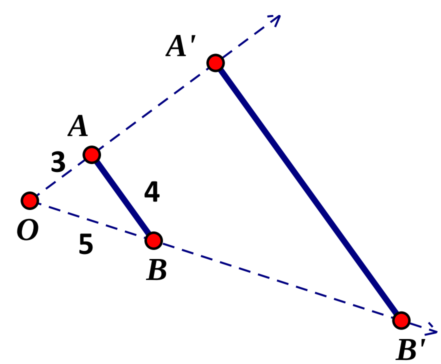


a) Is this an enlargement or a reduction? ____________________
Explain how you determined your answer.

b) What scale factor do you think this is? _____________
Explain how you determined your answer.

c) What angle is the same size as ∠OBA? _____________
Explain how you determined your answer.

5. Answer the following questions about the dilation centered at O with a scale factor of 3.
OA = 3, OB = 5 and AB = 4



a) A'B' = ______________

b) OB' = ______________

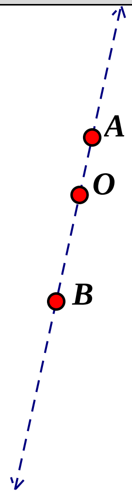
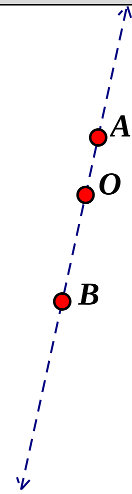
c) OA' = ______________

d) AA' = ______________ (be careful)

e) BB' = ______________ (be careful)

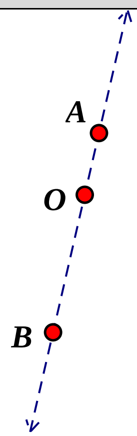
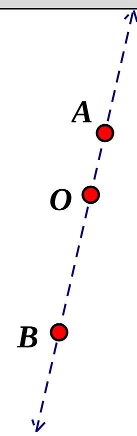
f) What is the ratio of OA:AA'? _______________

WHEN THE CENTER OF DILATION IS ON THE LINE

PRE-IMAGE	IMAGE
	

The center of dilation is point O and the scale factor is k > 1, say 2.

$$D_{O,2}(\overleftrightarrow{AB})$$

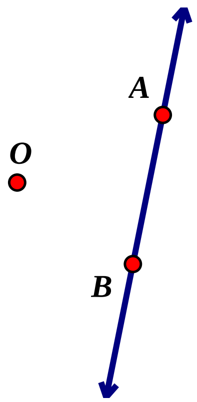
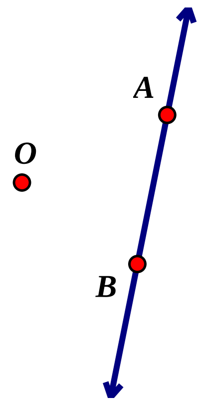
PRE-IMAGE	IMAGE
	

The center of dilation is point O and the scale factor is 0 > k > 1, say $\frac{1}{2}$.

$$D_{O,\frac{1}{2}}(\overleftrightarrow{AB})$$

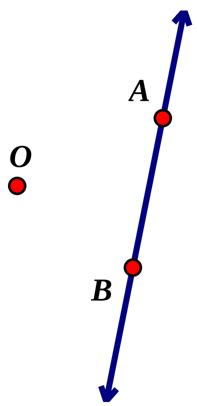
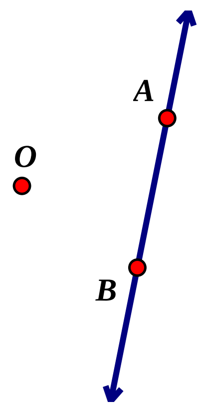
So what do we notice when you dilate a line with the center of dilation on that line?

WHEN THE CENTER OF DILATION IS OFF THE LINE

PRE-IMAGE ($\overleftrightarrow{AB}$)	IMAGE ($\overleftrightarrow{A'B'}$)
	

The center of dilation is point O (not on $\overleftrightarrow{AB}$) and the scale factor is k > 1, say 3.

$D_{O,3}(\overleftrightarrow{AB})$

PRE-IMAGE ($\overleftrightarrow{AB}$)	IMAGE ($\overleftrightarrow{A'B'}$)
	

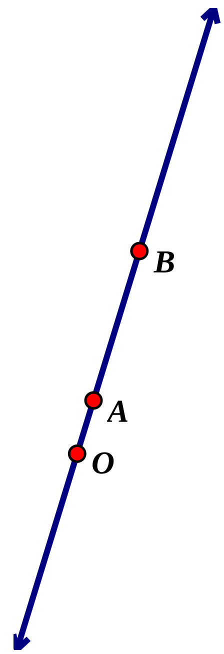
The center of dilation is point O (not on $\overleftrightarrow{AB}$) and the scale factor is 0 > k > 1, say $\frac{1}{2}$.

$D_{O,\frac{1}{2}}(\overleftrightarrow{AB})$

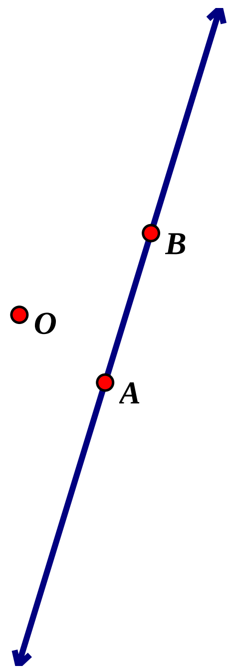
So what do we notice when you dilate a line with the center of dilation NOT on the line?

1. Draw the result of the dilation, centered at O, with a scale factor of 2. Completely label the diagram.

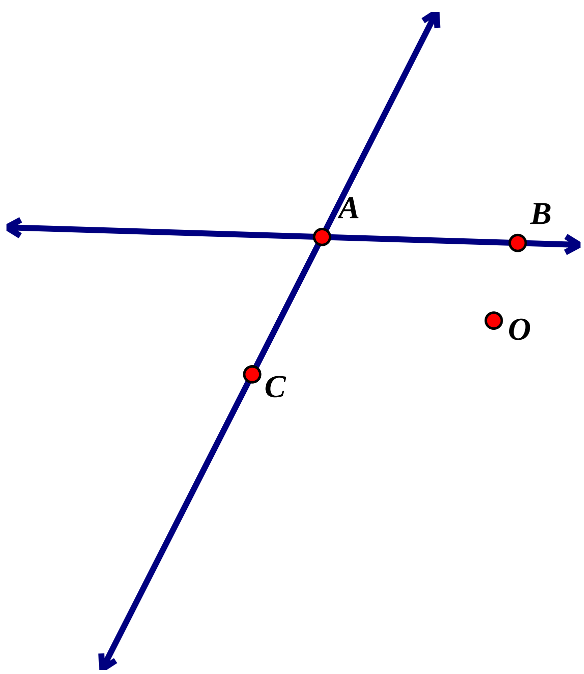
a)



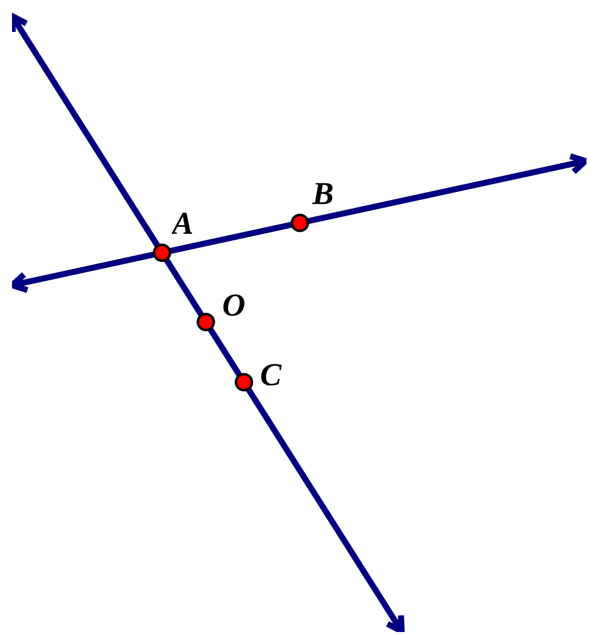
b)



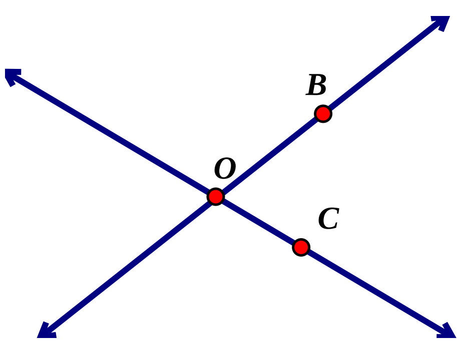
c)



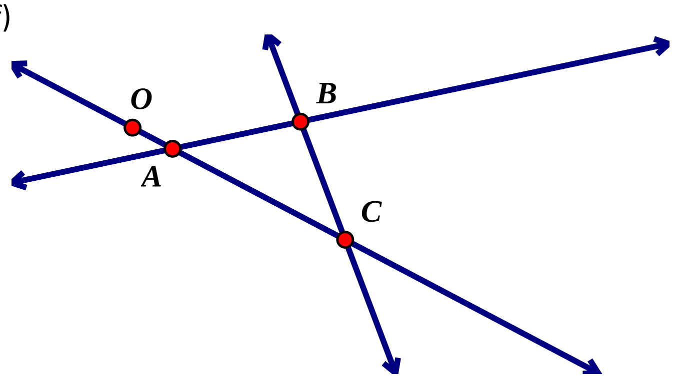
d)



e)



f)

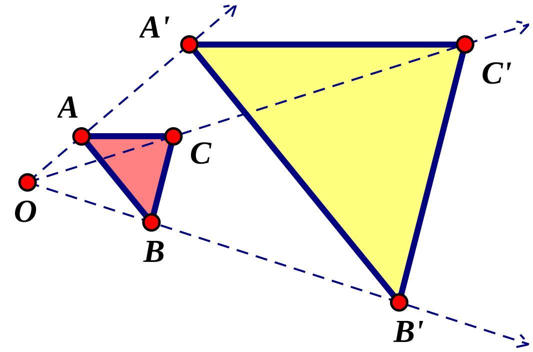


2. Complete the missing information.

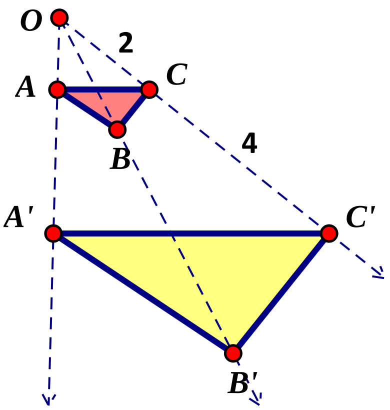
a) $\overline{AB} \parallel$ ________ because $\angle OAB \cong$ __________

b) $\overline{A'C'} \parallel$ ________ because $\angle OAC \cong$ __________

c) $\overline{BC} \parallel$ ________ because $\angle OBC \cong$ __________



3. Tiffany sees this given dilation and claims that the scale factor is 2 because 4 is twice as big as 2. Is this a scale factor of 2? Explain.



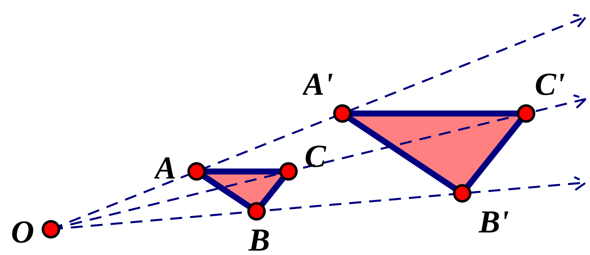
4. An invariant point is a point that is unaffected by the transformation. In other words, A = A'. With the transformations that you know to this point, describe and diagram all invariant point situations.

5. Jeff is describing a transformation of $\triangle ABC$ to Jennifer. He says that $AA' \neq BB' \neq CC'$ and the orientation is the same. Before he could give his final clue out of his mouth, Jennifer says "I know what it is!! – It is a rotation!!" Jeff smiles and says "Nope… you needed my last clue." If the transformation was a dilation, what might have been his last clue?

6. Sally draws $\overleftrightarrow{AB}$ on a piece of paper and then performs a dilation, centered at O, with a scale factor of 5. Where was point O if $\overleftrightarrow{AB}$ and $\overleftrightarrow{A'B'}$ are the same line? Draw a diagram to help clarify your answer.

7. Sarah draws $\overleftrightarrow{AB}$ on a piece of paper and then performs a dilation, centered at O with a positive scale factor on $\overleftrightarrow{AB}$. When she is done, she has two parallel lines and $\overleftrightarrow{A'B'}$ is closer to O than $\overleftrightarrow{AB}$. What does this tell you about the scale factor used?

8. DEEPER THOUGHT -- The teacher creates a dilation on the board and states that it is a scale factor of 2. Tony, an observant student, looks at the two shapes and says, "The area of the image is much bigger than 2 times bigger. Look how much area it takes up!!!" He is actually correct. ∆A'C'B' does not have double the area of ∆ACB. How much more area do you think it has? Why?

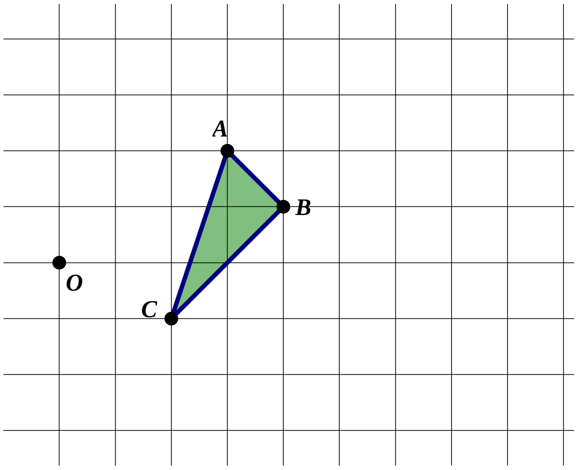


9. DEEPER THOUGHT -- Is a dilation of scale factor -5 an enlargement or a reduction? Explain your answer.

What happens when the center of dilation is in, on, and out of a figure?

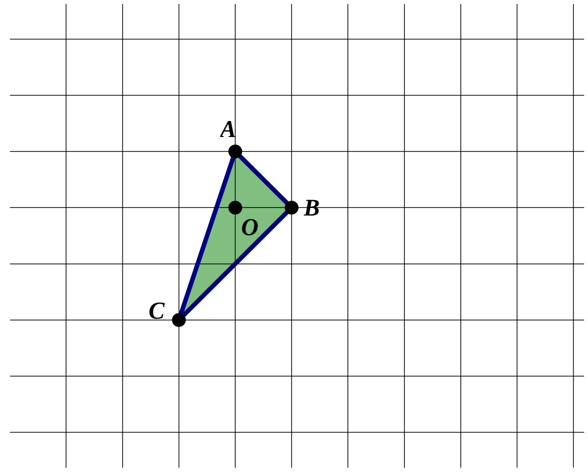
OUTSIDE

$$D_{0,2}(\Delta ABC) = \Delta A'B'C'$$



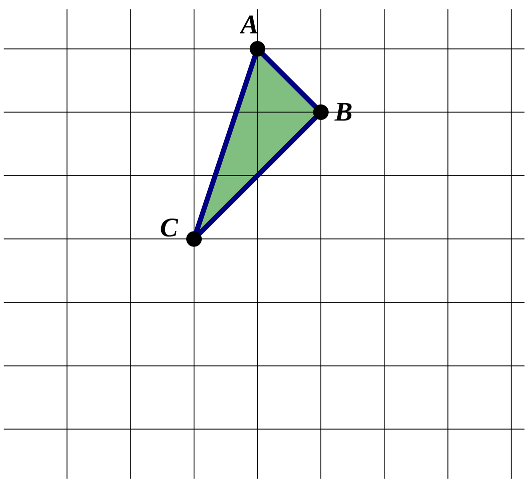
INSIDE

$$D_{0,2}(\Delta ABC) = \Delta A'B'C'$$



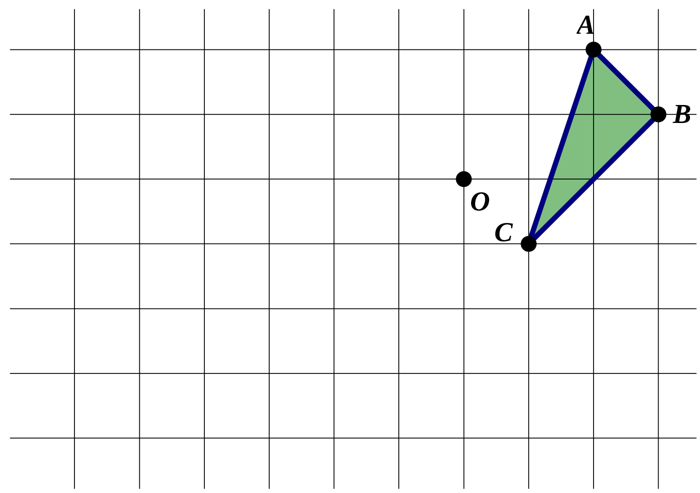
ON

$$D_{A,2}(\Delta ABC) = \Delta A'B'C'$$

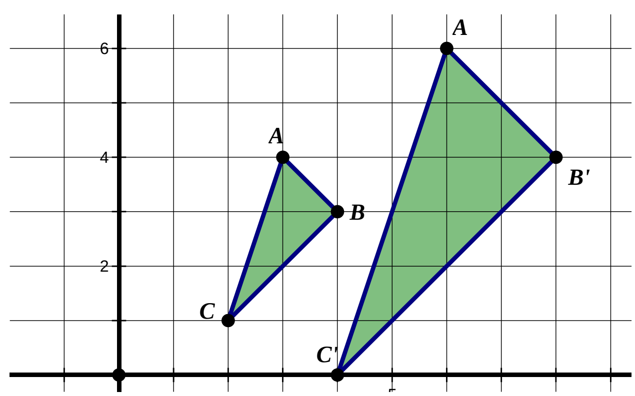


NEGATIVE SCALE FACTOR

$$D_{A,-2}(\Delta ABC) = \Delta A'B'C'$$



DETERMINING THE CENTER OF DILATION AND THE SCALE FACTOR



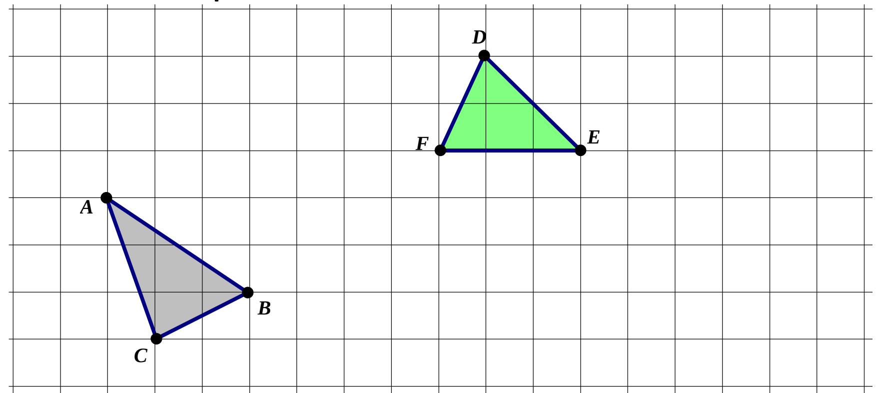
1. What happens when the center of dilation is a vertex of the shape?

a) Dilate ΔABC from C using a scale factor of 2

$D_{C,2}(\Delta ABC)$

b) Dilate ΔDEF from D using a scale factor of 3

$D_{D,3}(\Delta DEF)$

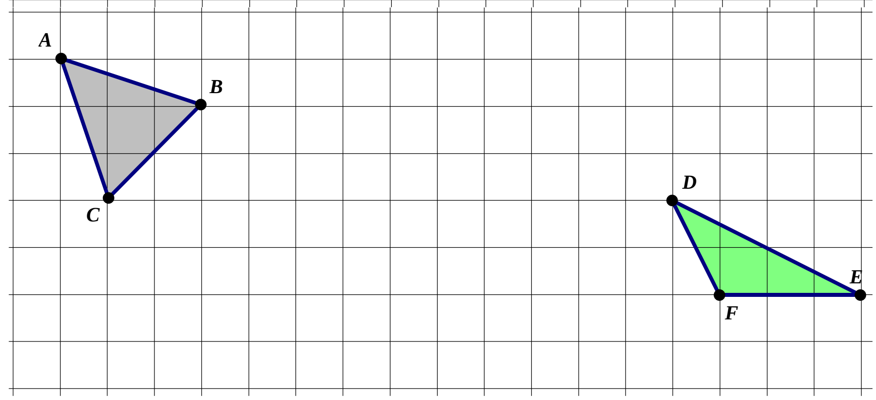


c) Dilate ΔABC from A using a scale factor of 2

$D_{A,2}(\Delta ABC)$

d) Dilate ΔDEF from E using a scale factor of 3

$D_{E,3}(\Delta DEF)$



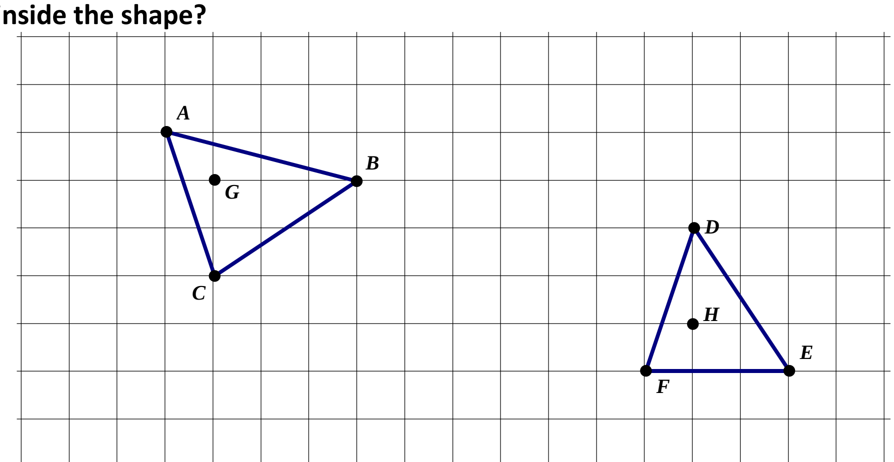
2. What happens when the center of dilation is inside the shape?

a) Dilate ΔABC from G using a scale factor of 3

$D_{G,3}(\Delta ABC)$

b) Dilate ΔDEF from H using a scale factor of 2

$D_{H,2}(\Delta DEF)$

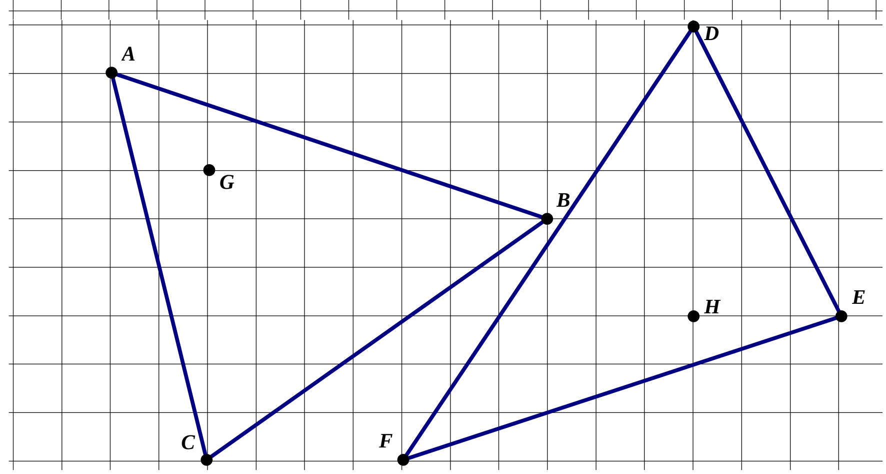


c) Dilate ΔABC from G using a scale factor of $\frac{1}{2}$

$D_{G,\frac{1}{2}}(\Delta ABC)$

d) Dilate ΔDEF from H using a scale factor of $\frac{1}{3}$

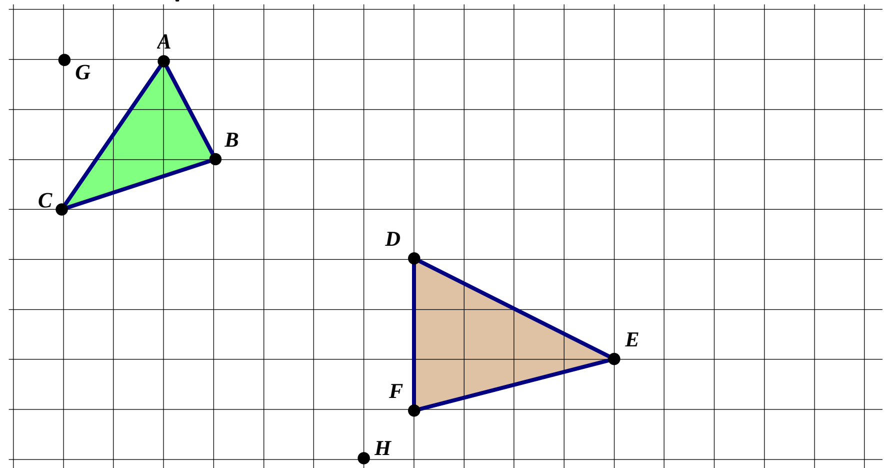
$D_{H,\frac{1}{3}}(\Delta DEF)$



3. What happens when the center of dilation is outside the shape?

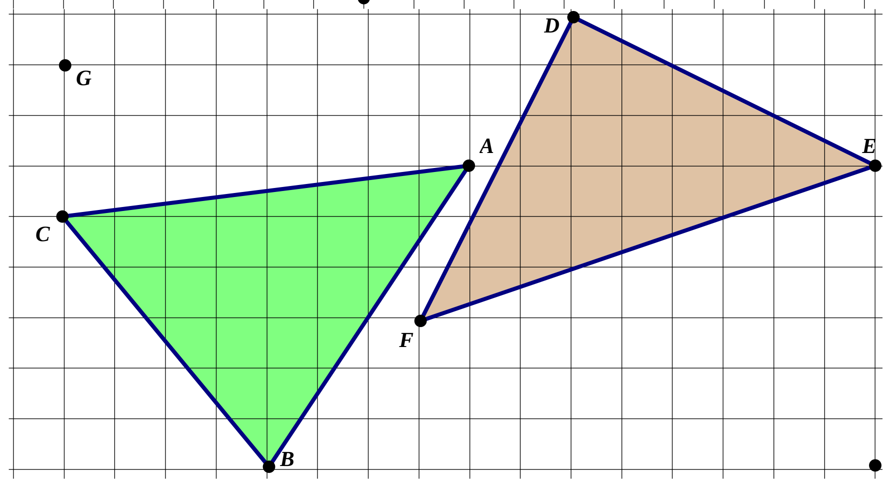
a) Dilate ΔABC from G using a scale factor of 2
$D_{G,2}(\Delta ABC)$

b) Dilate ΔDEF from H using a scale factor of 2
$D_{H,2}(\Delta DEF)$



c) Dilate ΔABC from G using a scale factor of $\frac{1}{2}$
$D_{G,\frac{1}{2}}(\Delta ABC)$

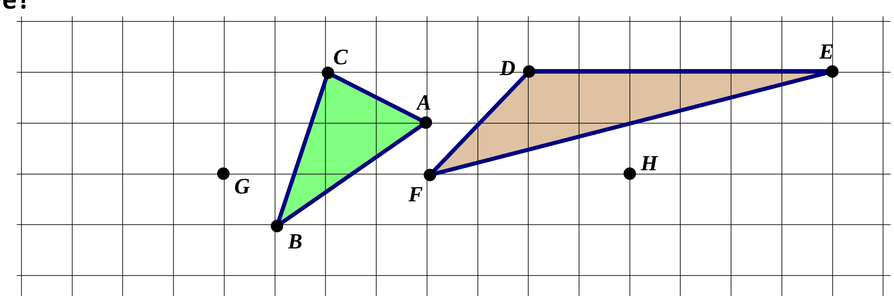
d) Dilate ΔDEF from H using a scale factor of $\frac{1}{3}$
$D_{H,\frac{1}{3}}(\Delta DEF)$



4. What happens when the scale factor is negative?

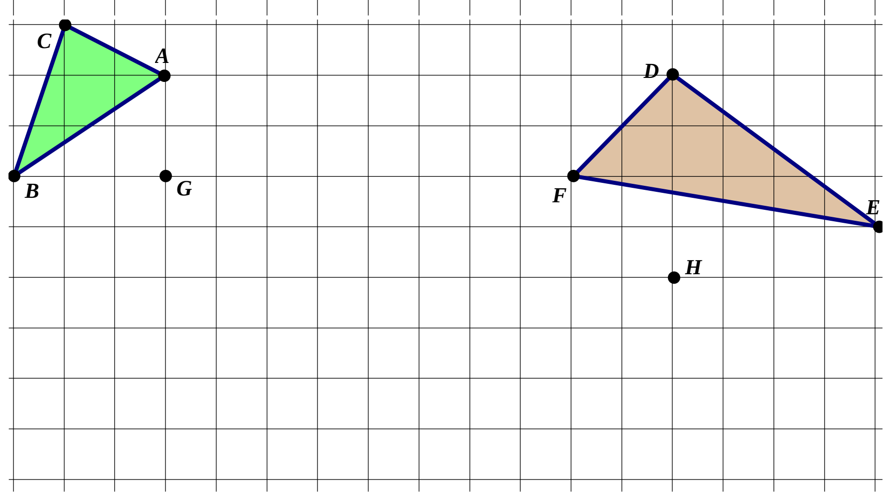
a) Dilate ΔABC from G using a scale factor of -1
$D_{G,-1}(\Delta ABC)$

b) Dilate ΔDEF from H using a scale factor of -$\frac{1}{2}$
$D_{H,-\frac{1}{2}}(\Delta DEF)$



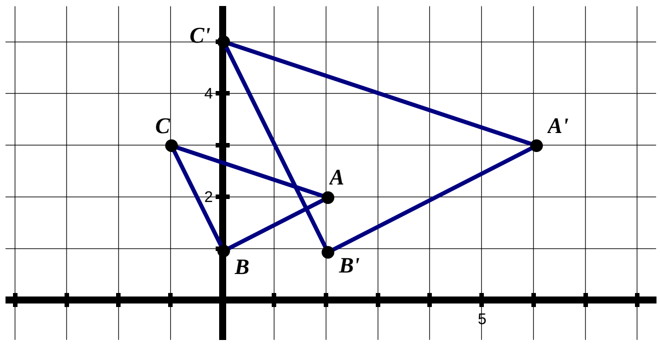
c) Dilate ΔABC from G using a scale factor of -2
$D_{G,-2}(\Delta ABC)$

d) Dilate ΔDEF from H using a scale factor of -1
$D_{H,-1}(\Delta DEF)$

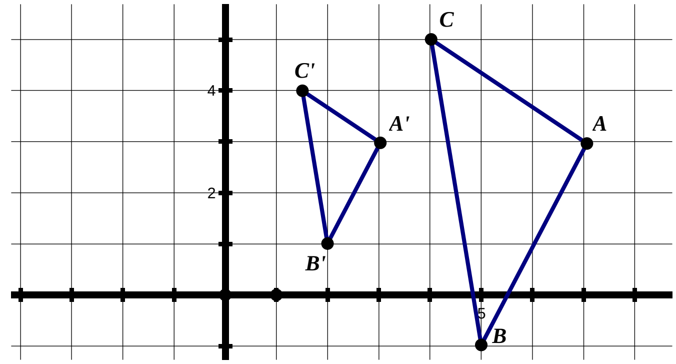


5. Work backwards to find the center of dilation and also determine the scale factor.

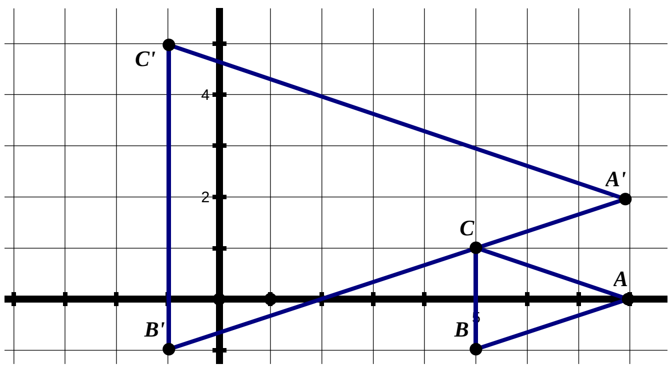
a) Center (______ , _______) Scale Factor = _____



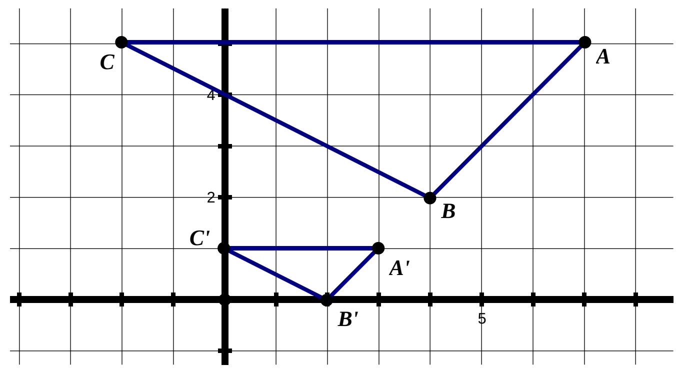
b) Center (______ , _______) Scale Factor = _____



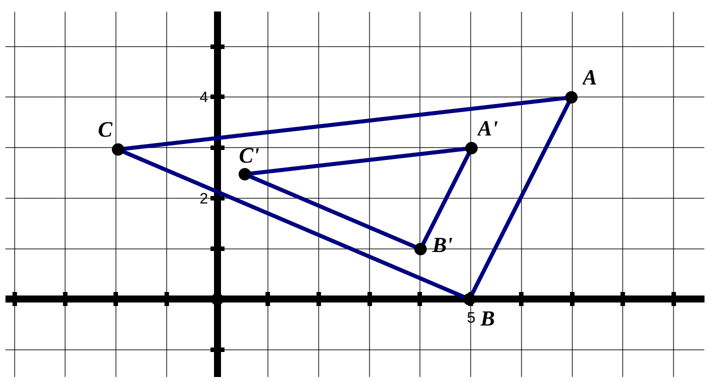
c) Center (______ , _______) Scale Factor = _____



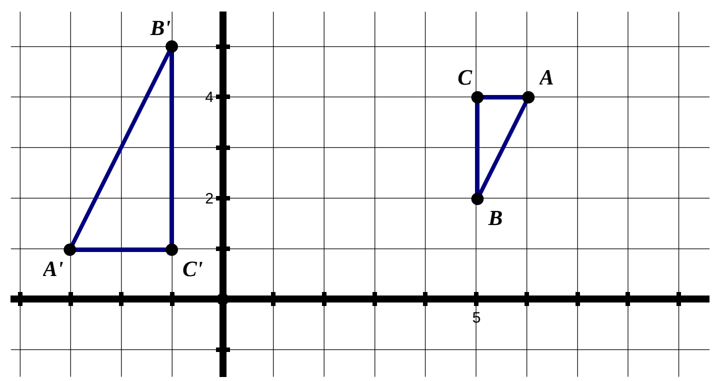
d) Center (______ , _______) Scale Factor = _____



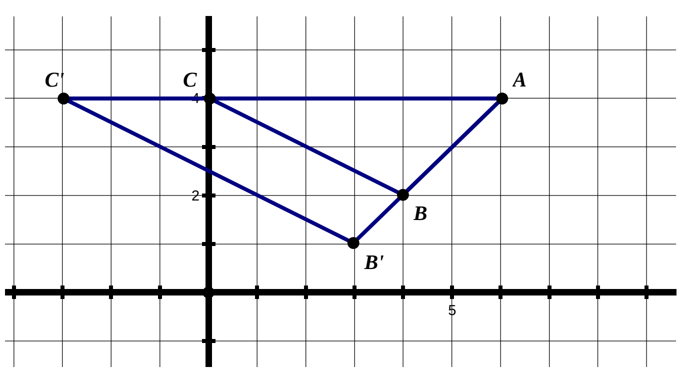
e) Center (______ , _______) Scale Factor = _____



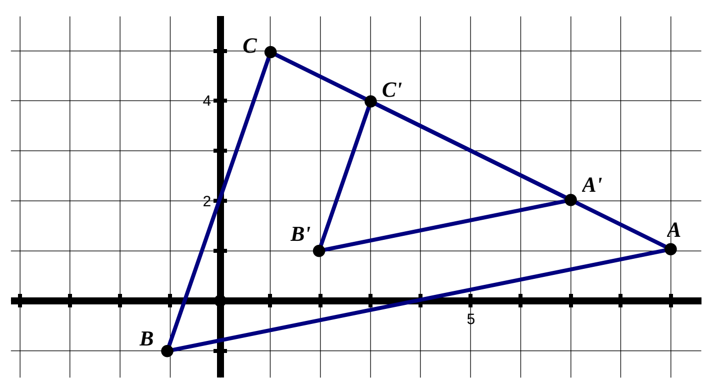
f) Center (______ , _______) Scale Factor = _____



g) Center (______ , _______) Scale Factor = _____



h) Center (______ , _______) Scale Factor = _____



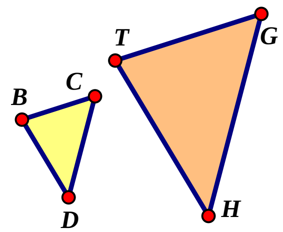
What is a stretch?

What is a dilation?

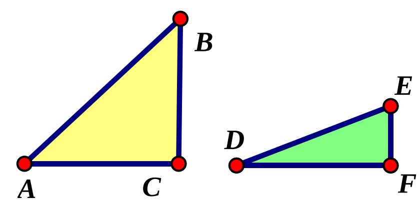
Determine whether the following are stretch or dilation transformations:

$R(x,y)--->(x,3y)$

Stretch or Dilation



Stretch or Dilation



Stretch or Dilation

$W(x,y)--->(\sqrt{5}x,\sqrt{5}y)$

Stretch or Dilation

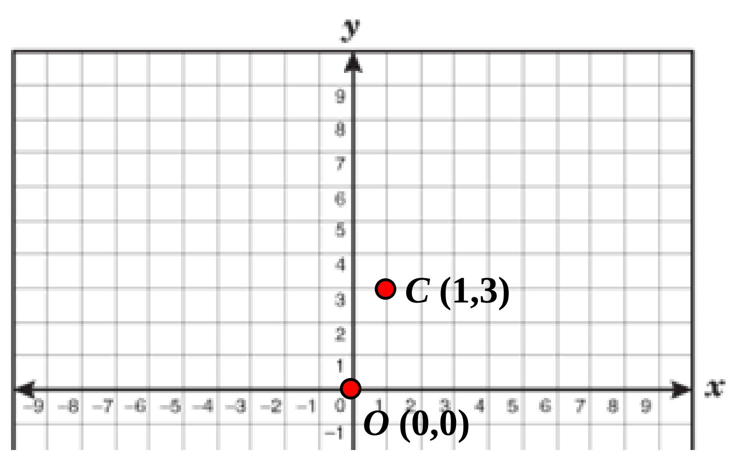
Stretch or Dilation

Stretch or Dilation

<u>The coordinate rule for a dilation with the center at the origin (0,0)</u>

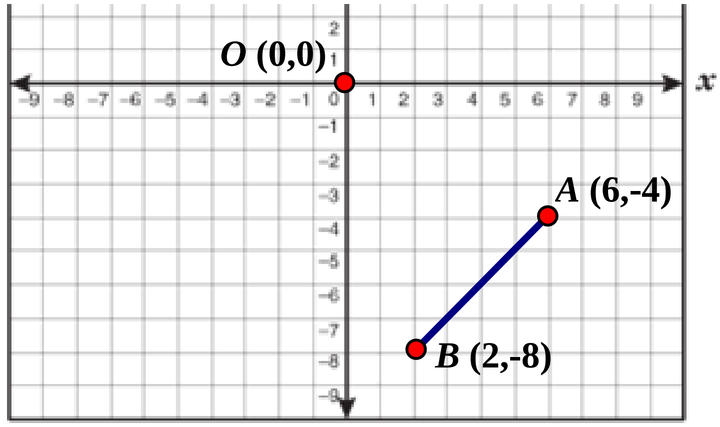
A dilation of 2 with center of dilation O, the origin.

$$D_{O,2}(x,y)=(2x,2y)$$



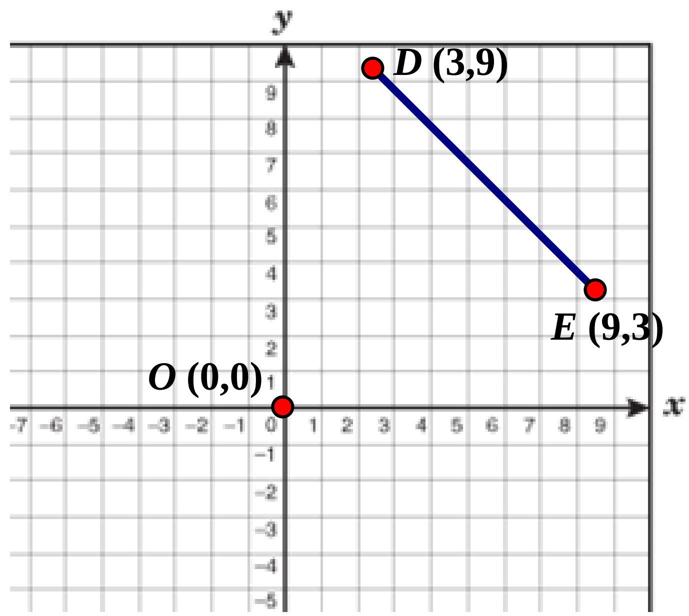
A dilation of ½ with center of dilation O, the origin.

$$D_{O,\frac{1}{2}}(x,y)=(\frac{1}{2}x,\frac{1}{2}y)$$



A dilation of -1/3 with center of dilation O, the origin.

$$D_{O,-\frac{1}{3}}(x,y)=(-\frac{1}{3}x,-\frac{1}{3}y)$$



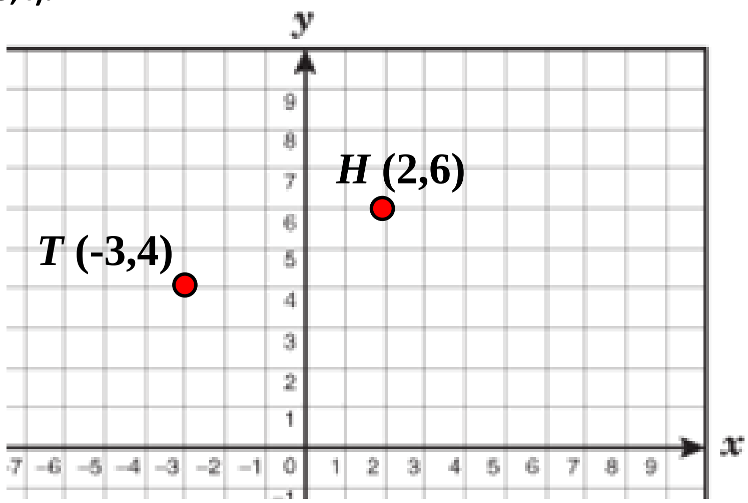
Dilate the following. (O is the origin).

a) $D_{O,2}(-3,5) = (\_\_\_\_,\_\_\_\_)$ b) $D_{O,5}(2,8) = (\_\_\_\_,\_\_\_\_)$ c) $D_{O,\frac{1}{3}}(6,24) = (\_\_\_\_,\_\_\_\_)$

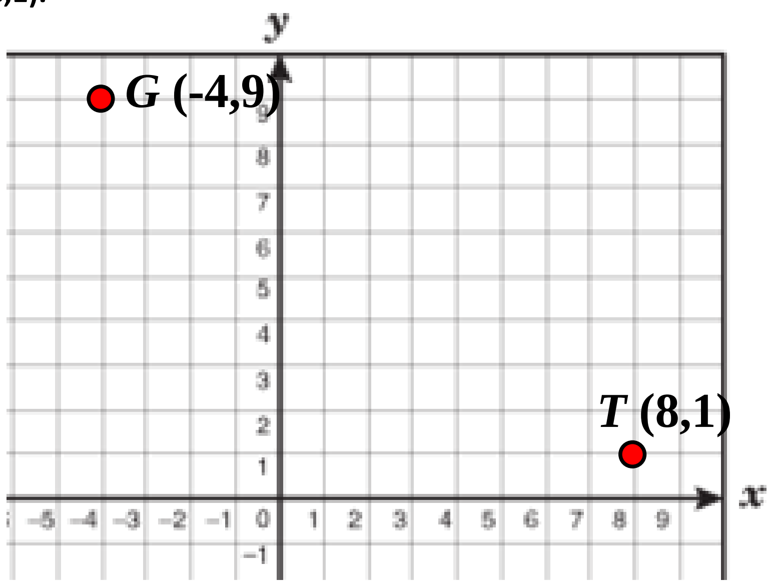
d) $D_{O,8}(\frac{1}{8},\frac{3}{4}) = (\_\_\_\_,\_\_\_\_)$ e) $D_{O,-2}(\_\_\_\_,\_\_\_\_) = (12,-10)$ f) $D_{O,2.5}(2,-4) = (\_\_\_\_,\_\_\_\_)$

COORDINATE RULE OF DILATION WHEN THE CENTER IS NOT AT THE ORIGIN (0,0)

A dilation of 2 with the center of dilation at T (-3,4).



A dilation of ¼ with the center of dilation at T (8,1).



Here is the general relationship for all dilations centered at (a,b) with a scale factor of k.

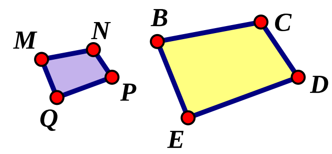
$$D_{(a,b),k}(x,y) = (a + k(x-a), b + k(y-b))$$

1. Determine whether the following are stretch or dilation transformations.

a) $H(x,y) ---> (2x,5y)$

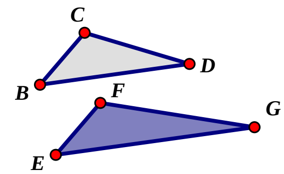
Stretch or Dilation

b)



Stretch or Dilation

c)

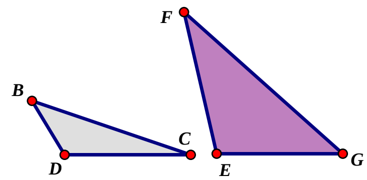


Stretch or Dilation

d) $W(x,y) ---> (\sqrt{5}x, \sqrt{5}y)$

Stretch or Dilation

e)



Stretch or Dilation

f) $L(x,y) ---> (.3x, .2y)$

Stretch or Dilation

2. Dilate the following. (O is the origin).

a) $D_{O,3}(5,3) = (\_\_\_\_, \_\_\_\_)$ b) $D_{O,7}(-2,0) = (\_\_\_\_, \_\_\_\_)$ c) $D_{O,\frac{1}{3}}(9,-6) = (\_\_\_\_, \_\_\_\_)$

d) $D_{O,4}(\frac{4}{5},\frac{3}{7}) = (\_\_\_\_, \_\_\_\_)$ e) $D_{O,-\frac{3}{4}}(-4,12) = (\_\_\_\_, \_\_\_\_)$ f) $D_{O,2.5}(10,-6) = (\_\_\_\_, \_\_\_\_)$

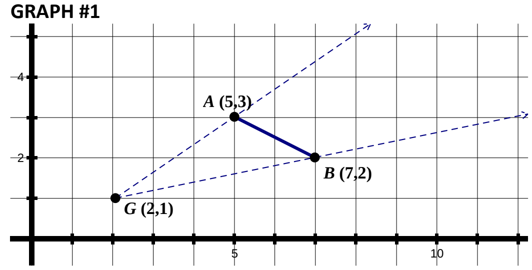
g) $D_{O,\frac{1}{2}}(5,-8) = (\_\_\_\_, \_\_\_\_)$ h) $D_{O,\frac{2}{3}}(8,5) = (\_\_\_\_, \_\_\_\_)$ i) $D_{O,-\frac{4}{3}}(3,-5) = (\_\_\_\_, \_\_\_\_)$

3. The center of dilation is G. G (2, 1) A (5, 3)

a) Determine the slope of $\overleftrightarrow{GA}$ from $G(x_1, y_1)$ to $A(x_2, y_2)$.

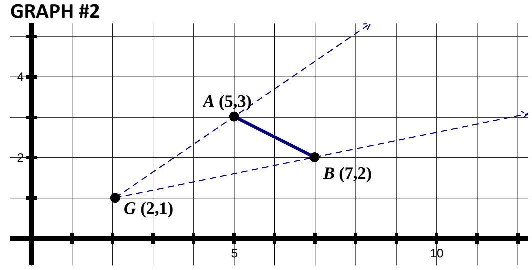
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

GRAPH #1



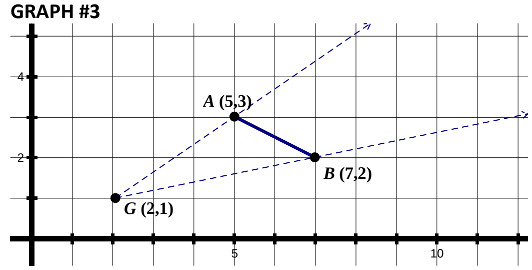
b) Using graph #2, how can this slope help you find A', if the scale factor is 2?

GRAPH #2



c) Using graph #3, how can this slope help you find A', if the scale factor is ½?

GRAPH #3



4. Complete the following. (When calculating the slope **do not** simplify it in any way!! The slope is actually a vector.)

a) Center of dilation is G. G (1, 5) A (5, 8)
Scale Factor 2

Determine the slope of $\overleftrightarrow{GA}$ from $G(x_1, y_1)$ to $A(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Determine A'.

(____ + (2)(____) , ____ + (2)(____)) = A' (____ ,____)

b) Center of dilation is G. G (-2, 5) A (0, 4)
Scale Factor 3

Determine the slope of $\overleftrightarrow{GA}$ from $G(x_1, y_1)$ to $A(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Determine A'.

(____ + (3)(____) , ____ + (3)(____)) = A' (____ ,____)

c) Center of dilation is G. G (-3, 1) A (-4, -5)
Scale Factor 2

Determine the slope of $\overleftrightarrow{GA}$ from $G(x_1, y_1)$ to $A(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Determine A'.

(____ + (2)(____) , ____ + (2)(____)) = A' (____ ,____)

d) Center of dilation is G. G (-2, -5) A (1, 13)
Scale Factor $\frac{1}{3}$

Determine the slope of $\overleftrightarrow{GA}$ from $G(x_1, y_1)$ to $A(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Determine A'.

(___ + ($\frac{1}{3}$)(____) , ____ + ($\frac{1}{3}$)(____)) = A' (____ ,____)

e) Center of dilation is G. G (2, 3) A (4, 7)
Scale Factor 5

Determine the slope of $\overleftrightarrow{GA}$ from $G(x_1, y_1)$ to $A(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Determine A'.

A' (____ ,____)

f) Center of dilation is G. G (8, 6) A (3, 2)
Scale Factor $\frac{1}{2}$

Determine the slope of $\overleftrightarrow{GA}$ from $G(x_1, y_1)$ to $A(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Determine A'.

A' (____ ,____)

5. What is wrong with this student's work?

Center of dilation is G. Scale Factor = 5 G (2, 3) A (1, 8)

$$m = \frac{3-8}{2-1} = \frac{-5}{1}$$

(2 + (5)(1) , 3 + (5)(-5)) = A' (7, -22)

Directed Line Segments

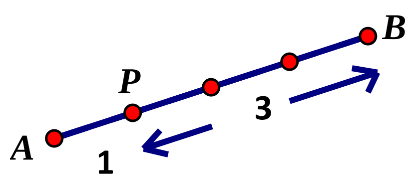
A directed line segment is a segment that ____.

The directed line segment $\overline{AB}$ implies that we are starting at _________ and going towards __________.

Partitioning (dividing) segment $\overline{AB}$ into a 1:3 ratio implies that we start at __________ and then have ____ for a total of __________.

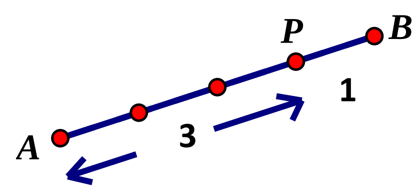
Given the directed line segment $\overline{AB}$, determine point P so that it divides the segment into the ratio of:

1:3



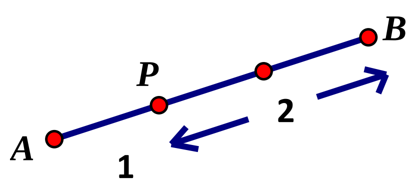
The ratio is 1:3 but the segment has 4 parts.

3:1



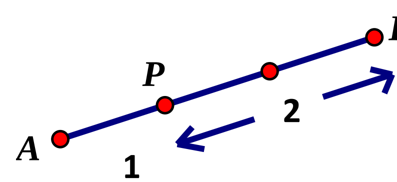
The ratio is 3:1 but the segment has 4 parts.

Directed line segment $\overline{AB}$
is divided into a ratio of 1:2



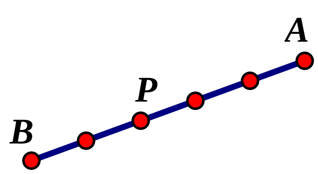
The ratio is 1:2 but the segment has 3 parts.

Directed line segment $\overline{BA}$
is divided into a ratio of 2:1

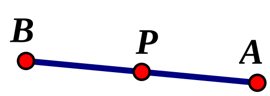


The ratio is 2:1 but the segment has 3 parts.

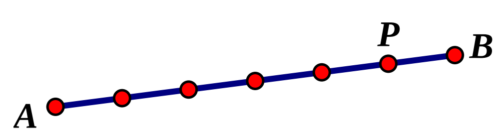
1. Determine the ratio of the directed line segment $\overline{BA}$ when partitioned by point P. (Hint: B is the initial point)



a) _____ : _____



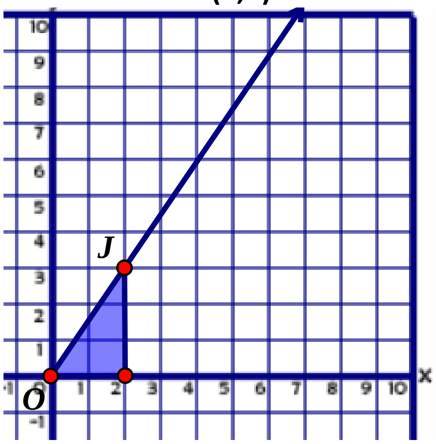
b) _____ : _____



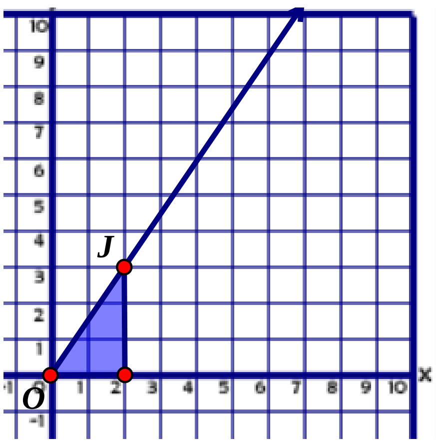
c) _____ : _____

Dilations

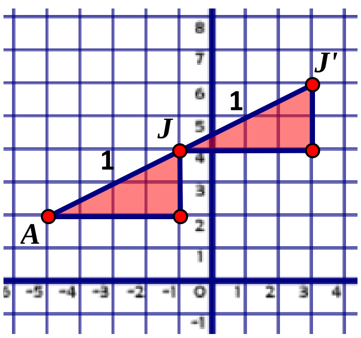
Dilation Center Origin (0,0)
J(2,3)



Dilate OJ from O with a scale factor of 2



Dilate OJ from O with a scale factor of 3



Initial Point A (-5,2)

Scale Factor = 2

Slope = $\frac{2}{4}$, run = 4, rise = 2

These dilations remind us that dilations use slope to transform an object.
If we move the initial point (the center of dilation), then we adjust our relationship to represent that change.

$$D_{(a,b),k}\left(a + k(run), b + k(rise)\right)$$

Partitioning a Directed Line Segment

To partition a line segment means to divide it up into pieces. To relate this to a dilation means that we will do a reduction so that the point will be on the segment.

We can convert ratios to scale factors:

Ratio	Scale Factor	Ratio	Scale Factor	Ratio	Scale Factor
a:b	$\boldsymbol{\frac{a}{a+b}}$	1:3	$\frac{1}{1+3}=\frac{1}{4}$	2:3	$\frac{2}{5}$
Ratio	**Scale Factor**	**Ratio**	**Scale Factor**	**Ratio**	**Scale Factor**
1:6	________	3:1	_______	4:5	_______

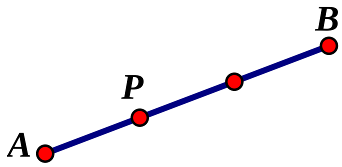
These scale factors can be used in a reduction to determine the point that partitions the segment to the correct ratio.

2. Determine the point P that partitions the directed line segment $\overline{AB}$ into a ratio of 1:1, where A (-5, 2) and B (3, 6).

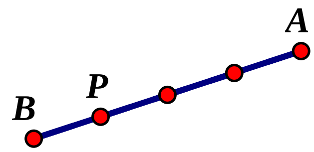
3. Determine the point P that partitions the directed line segment $\overline{AB}$ into a ratio of 2:3, where A (1, -5) and B (9, -1).

G.GPE.B.6 WORKSHEET #1 – geometrycommoncore.com *Name: ______________________________ 1*

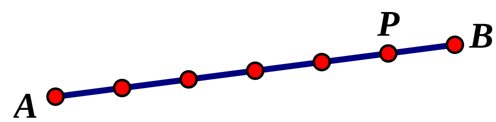
1. Determine the ratio of the directed line segment $\overline{AB}$ when partitioned by point P. (Hint: A is the initial point)



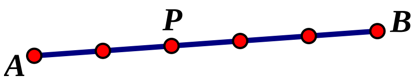
a) _____ : _____



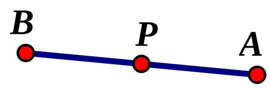
b) _____ : _____



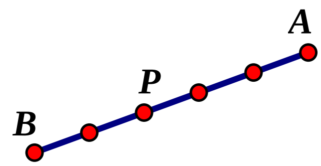
c) _____ : _____



d) _____ : _____

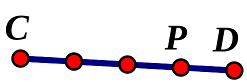


e) _____ : _____



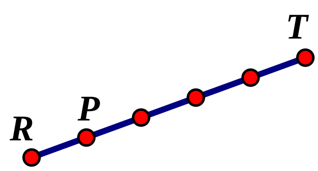
f) _____ : _____

2. Determine the ratio of the directed line segment when partitioned by point P. (The first stated point is the initial point.)



a) Directed Line Segment $\overline{DC}$

_____ : _____

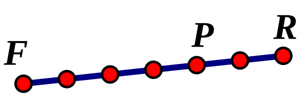


b) Directed Line Segment $\overline{RT}$

_____ : _____

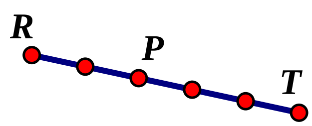
c) Directed Line Segment $\overline{HG}$

_____ : _____



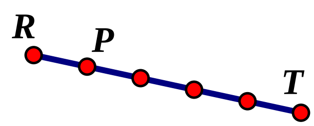
d) Directed Line Segment $\overline{RF}$

_____ : _____



e) Directed Line Segment $\overline{RT}$

_____ : _____

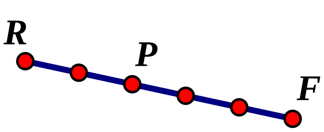


f) Directed Line Segment $\overline{TR}$

_____ : _____

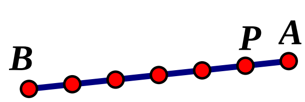
3. Create a partition that would be the same as the one provided.

a) Directed Line Segment $\overline{RF}$ is partitioned by point P into a ratio of 2:3



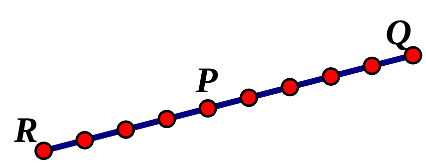
Directed Line Segment ______ is partitioned by point P into a ratio of _____ : _____.

a) Directed Line Segment $\overline{AB}$ is partitioned by point P into a ratio of 1:5



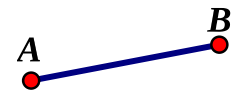
Directed Line Segment ______ is partitioned by point P into a ratio of _____ : _____.

a) Directed Line Segment $\overline{RQ}$ is partitioned by point P into a ratio of 4:5



Directed Line Segment ______ is partitioned by point P into a ratio of _____ : _____.

4. Determine the dilation scale factor that would partition the directed line segment $\overline{AB}$ into the given ratio.



a) 3:2 $D_{A,k}$ k = _______

b) 1:5 $D_{A,k}$ k = _______

c) 3:4 $D_{A,k}$ k = _______

d) 8:1 $D_{A,k}$ k = _______

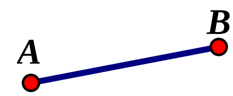
e) 1:1 $D_{A,k}$ k = _______

f) 3:1 $D_{A,k}$ k = _______

g) 2:1 $D_{A,k}$ k = _______

h) 5:4 $D_{A,k}$ k = _______

5. Determine the ratio that the given dilation would partition the directed line segment $\overline{AB}$ into.



a) $D_{A,\frac{1}{3}}$ _____ : _____

b) $D_{A,\frac{2}{5}}$ _____ : _____

c) $D_{A,\frac{3}{4}}$ _____ : _____

d) $D_{A,\frac{5}{9}}$ _____ : _____

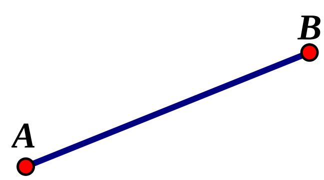
e) $D_{A,\frac{1}{2}}$ _____ : _____

f) $D_{A,\frac{6}{8}}$ _____ : _____

g) $D_{A,\frac{7}{8}}$ _____ : _____

h) $D_{A,\frac{8}{15}}$ _____ : _____

6. Explain why $D_{A,\frac{2}{3}}$ results in a partition ratio of 2:1.



7. Given the initial point A and a scale factor, determine the slope, the rise, the run, and the image of B'.

a) A (-2,3) B (1,7) Scale Factor = 3

Determine the slope.

Run = ______

Rise = ______

$$D_{(a,b),k}\left(a + k(run), b + k(rise)\right)$$

$$(\_\_\_ + \_\_\_(\_\_\_), \_\_\_ + \_\_\_(\_\_\_))$$

$$(\_\_\_ + \_\_\_, \_\_\_ + \_\_\_)$$

$$(\_\_\_, \_\_\_)$$

b) A (1,-4) B (3,-2) Scale Factor = 5

Determine the slope.

Run = ______

Rise = ______

$$D_{(a,b),k}\left(a + k(run), b + k(rise)\right)$$

$$(\_\_\_ + \_\_\_(\_\_\_), \_\_\_ + \_\_\_(\_\_\_))$$

$$(\_\_\_ + \_\_\_, \_\_\_ + \_\_\_)$$

$$(\_\_\_, \_\_\_)$$

c) A (0,1) B (11,4) Scale Factor = 2

Determine the slope.

Run = ______

Rise = ______

$$D_{(a,b),k}\left(a + k(run), b + k(rise)\right)$$

$$\left(\_\_\_ + \_\_\_(\_\_\_), \_\_\_ + \_\_\_(\_\_\_)\right)$$

$$\left(\_\_\_ + \_\_\_, \_\_\_ + \_\_\_\right)$$

$$(\_\_\_, \_\_\_)$$

d) A (-3,5) B (5,-3) Scale Factor = $\frac{1}{2}$

Determine the slope.

Run = ______

Rise = ______

e) A (4,-4) B (9,11) Scale Factor = $\frac{2}{5}$

Run = ______

Rise = ______

f) A (1,2) B (8,7) Scale Factor = $\frac{4}{5}$

Run = ______

Rise = ______

#1 Determine the point P that partitions the directed line segment $\overline{AB}$ into a ratio of 1:2, where A (1,4) and B (4,10).

Initial Point (___ , ___) $m = \frac{y_2 - y_1}{x_2 - x_1}$

Run = _____

Rise = _____

Scale Factor = _______

$D_{(a,b),k}\left(a + k(run), b + k(rise)\right)$

$(\_\_\_ + \_\_\_(\_\_\_), \_\_\_ + \_\_\_(\_\_\_))$

$(\_\_\_ + \_\_\_, \_\_\_ + \_\_\_)$

$(\_\_\_, \_\_\_)$

#2 Determine the point P that partitions the directed line segment $\overline{AB}$ into a ratio of 3:1, where A (-2,1) and B (-6,-15).

Initial Point (___ , ___) $m = \frac{y_2 - y_1}{x_2 - x_1}$

Run = _____

Rise = _____

Scale Factor = _______

$D_{(a,b),k}\left(a + k(run), b + k(rise)\right)$

$(\_\_\_ + \_\_\_(\_\_\_), \_\_\_ + \_\_\_(\_\_\_))$

$(\_\_\_ + \_\_\_, \_\_\_ + \_\_\_)$

$(\_\_\_, \_\_\_)$

#3 Determine the point P that partitions the directed line segment $\overline{AB}$ into a ratio of 2:3, where A (10,-3) and B (5,22).

Initial Point (___ , ___) $m = \frac{y_2 - y_1}{x_2 - x_1}$

Run = _____

Rise = _____

Scale Factor = _______

$D_{(a,b),k}\left(a + k(run), b + k(rise)\right)$

$(\_\_\_ + \_\_\_(\_\_\_), \_\_\_ + \_\_\_(\_\_\_))$

$(\_\_\_ + \_\_\_, \_\_\_ + \_\_\_)$

$(\_\_\_, \_\_\_)$

#4 Determine the point P that partitions the directed line segment $\overline{AB}$ into a ratio of 4:5, where A (5,-4) and B (14,5).

Initial Point (___ , ___) $m = \frac{y_2 - y_1}{x_2 - x_1}$

Run = _____

Rise = _____

Scale Factor = _______

$D_{(a,b),k}\left(a + k(run), b + k(rise)\right)$

#5 Determine the point P that partitions the directed line segment $\overline{AB}$ into a ratio of 1:3, where A (8,6) and B (1,10).

Initial Point (____ , ____)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Run = ______

Rise = ______

Scale Factor = ________

$$D_{(a,b),k}\left(a + k(run), b + k(rise)\right)$$

#6 Determine the point P that partitions the directed line segment $\overline{AB}$ into a ratio of 2:1, where A (0,5) and B (3,9).

Initial Point (____ , ____)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Run = ______

Rise = ______

Scale Factor = ________

#7 Determine the point P that partitions the directed line segment $\overline{AB}$ into a ratio of 2:3, where A (4,-5) and B (-3,8).

Initial Point (____ , ____)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Run = ______

Rise = ______

Scale Factor = ________

In the next problems be careful how the ratio is presented. The ratio is still comparing the two partitioned parts of segment but is presented as an equation.

If you are told that AP = 3(PB), then AP is the bigger portion and the ratio would be 3:1.

#8 Determine the point P that partitions the directed line segment $\overline{AB}$ so that AP = 5(PB), where A (-1,-11) and B (5,1).

Initial Point (___ , ___) $m = \frac{y_2 - y_1}{x_2 - x_1}$ Run = _____ Rise = _____ Scale Factor = _______

$D_{(a,b),k}\left(a + k(run), b + k(rise)\right)$

#9 Determine the point P that partitions the directed line segment $\overline{AB}$ so that AP = 2(PB), where A (2,5) and B (-1,17).

Initial Point (___ , ___) $m = \frac{y_2 - y_1}{x_2 - x_1}$ Run = _____ Rise = _____ Scale Factor = _______

$D_{(a,b),k}\left(a + k(run), b + k(rise)\right)$

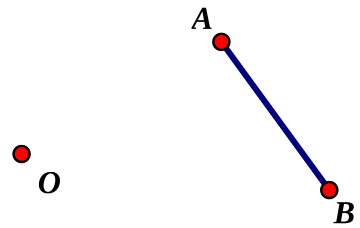
#10 Determine the point P that partitions the directed line segment $\overline{AB}$ so that 2(AP) = PB, where A (0,4) and B (12,1).

Initial Point (___ , ___) $m = \frac{y_2 - y_1}{x_2 - x_1}$ Run = _____ Rise = _____ Scale Factor = _______

$D_{(a,b),k}\left(a + k(run), b + k(rise)\right)$

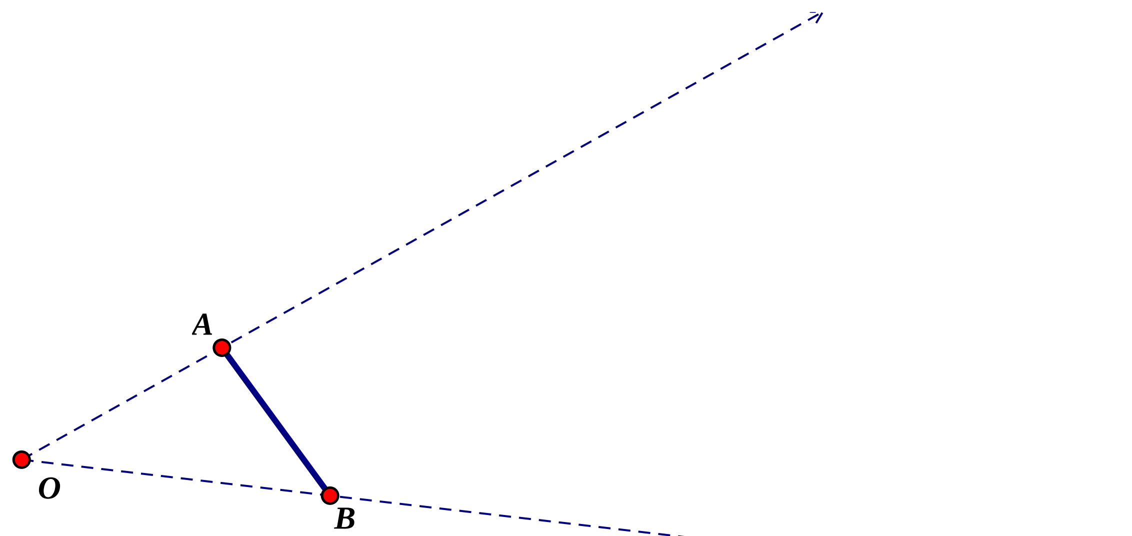
How to perform a dilation construction

$D_{O,3}(\overline{AB})$

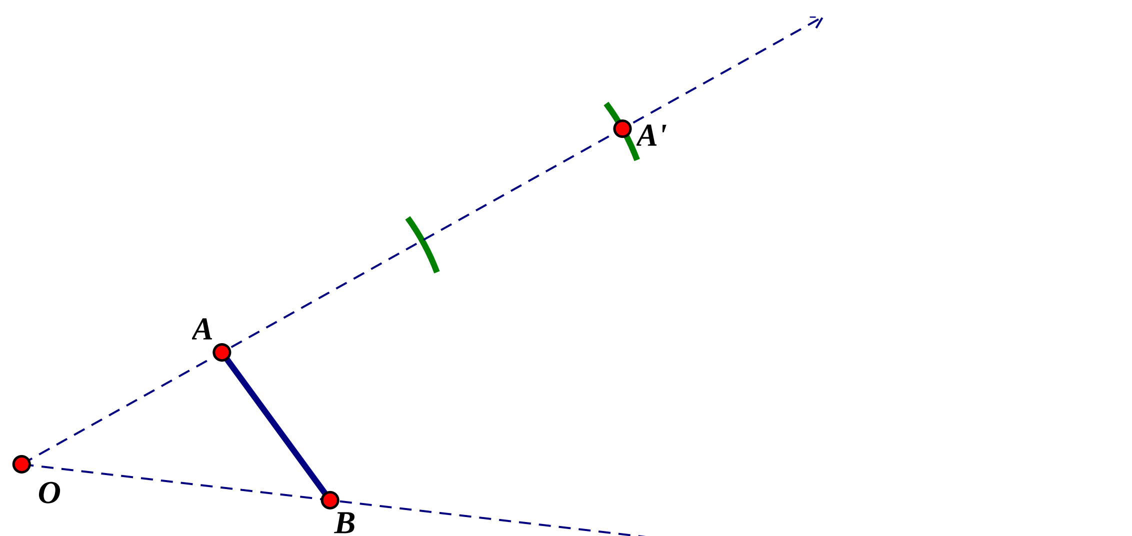


Create Rays from the center of dilation

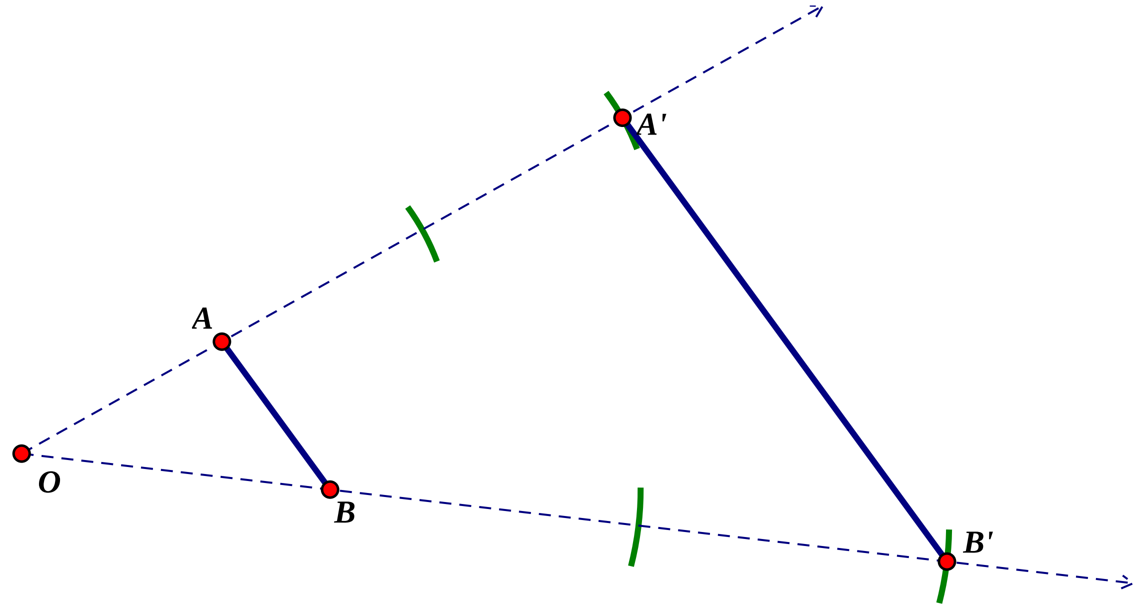
$\overrightarrow{OA}$ & $\overrightarrow{OB}$



Place the pointer of your compass at O, and then measure OA. Use that measurement to mark two more distances of OA making the total distance from O, 3(OA). This location is A'



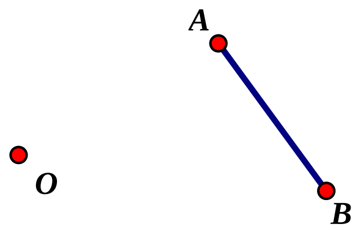
Place the pointer of your compass at O, and then measure OB. Use that measurement to mark two more distances of OB making the total distance from O, 3(OB). This location is B'. Finish the construction by forming $\overline{A'B'}$.



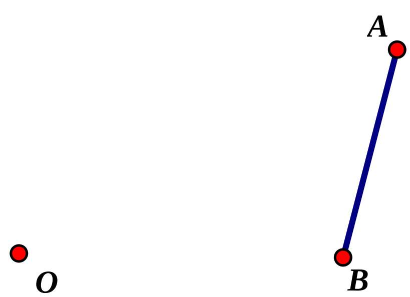
If scale factor was 3.5 or 5.25 you would follow the same steps but to get the half or the quarter you would use your midpoint construct once or twice to cut it up small enough.

How to perform a dilation construction

$D_{O,3}(\overline{AB})$



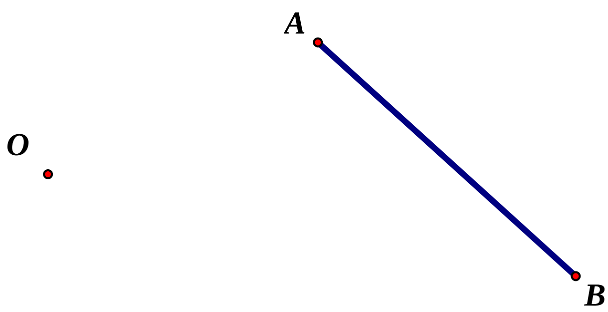
$D_{O,1.5}(\overline{AB})$



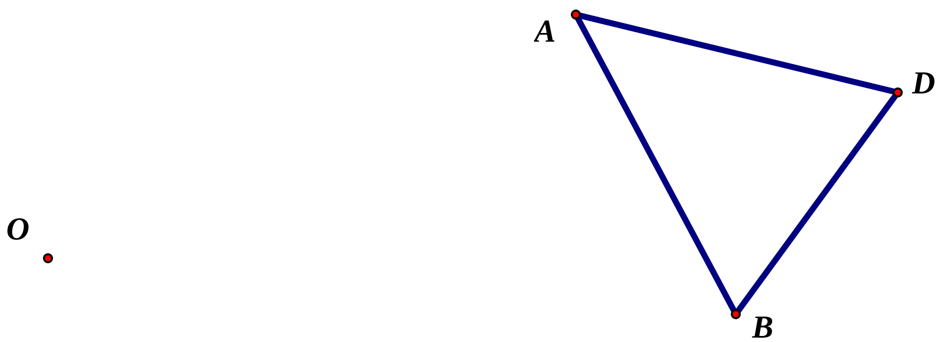
G.SRT.A.1 WORKSHEET #6 – geometrycommoncore.com NAME: ________________________________

1. Use a compass and a straightedge to construct the following dilations.

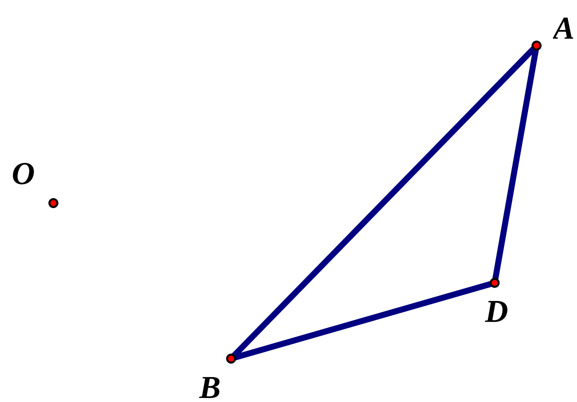
a) $D_{O,2}\left(\overline{AB}\right)$



b) $D_{O,\frac{1}{2}}\left(\Delta ADB\right)$

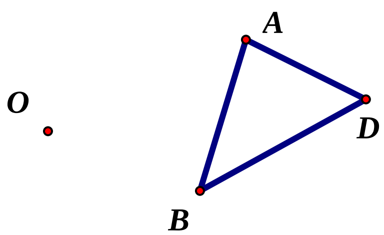


c) $D_{O,-1}\left(\Delta ADB\right)$

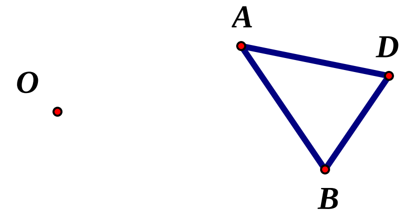


d) $D_{O,2.5}\left(A\right)$

e) $D_{O,3}\left(\Delta ADB\right)$



f) $D_{O,-2}\left(\Delta ADB\right)$



g) $D_{O,2}\left(\overleftrightarrow{AB}\right)$, $D_{O,2}\left(\overleftrightarrow{BC}\right)$