

# 8-5

# Trapezoids

### What You'll Learn

You'll learn to identify and use the properties of trapezoids and isosceles trapezoids.

### Why It's Important

**Art** Trapezoids are used in perspective drawings. See Example 1.

Many state flags use geometric shapes in their designs. Can you find a quadrilateral in the Maryland state flag that has exactly one pair of parallel sides?



Maryland state flag

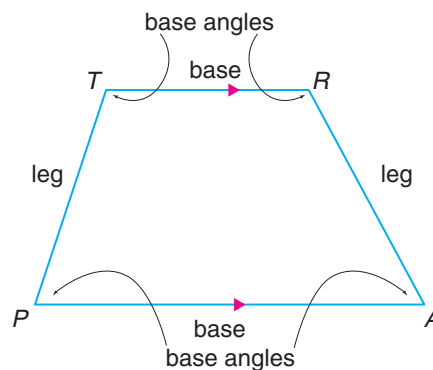
A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases**. The nonparallel sides are called **legs**.

Study trapezoid  $TRAP$ .

$\overline{TR} \parallel \overline{PA}$        $\overline{TR}$  and  $\overline{PA}$   
are the bases.

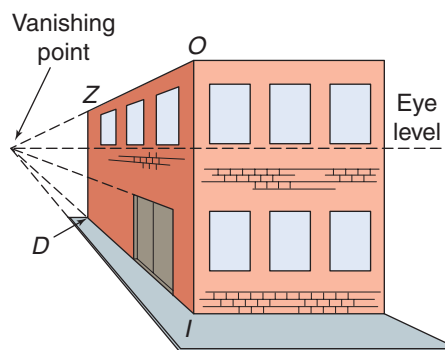
$\overline{TP} \nparallel \overline{RA}$        $\overline{TP}$  and  $\overline{RA}$   
are the legs.

Each trapezoid has two pairs of **base angles**. In trapezoid  $TRAP$ ,  $\angle T$  and  $\angle R$  are one pair of base angles;  $\angle P$  and  $\angle A$  are the other pair.



### Example Art Link

**1** Artists use *perspective* to give the illusion of depth to their drawings. In perspective drawings, vertical lines remain parallel, but horizontal lines gradually come together at a point. In trapezoid  $ZOID$ , name the bases, the legs, and the base angles.



**Bases**  $\overline{ZD}$  and  $\overline{OI}$  are parallel segments.

**Legs**  $\overline{ZO}$  and  $\overline{DI}$  are nonparallel segments.

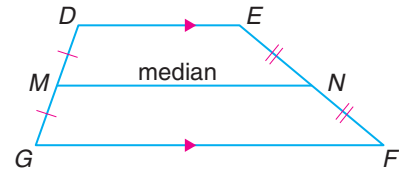
**Base Angles**  $\angle Z$  and  $\angle D$  are one pair of base angles;  
 $\angle O$  and  $\angle I$  are the other pair.



## Reading Geometry

Another name for the median of a trapezoid is the **midsegment** of the trapezoid.

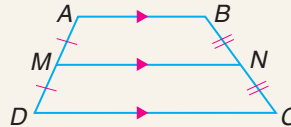
The **median** of a trapezoid is the segment that joins the midpoints of its legs. In the figure,  $\overline{MN}$  is the median.



### Theorem 8-13

**Words:** The median of a trapezoid is parallel to the bases, and the length of the median equals one-half the sum of the lengths of the bases.

**Model:**



**Symbols:**

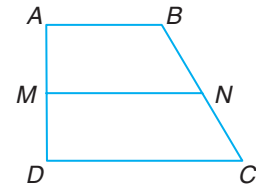
$$\overline{AB} \parallel \overline{MN}, \overline{DC} \parallel \overline{MN}$$

$$MN = \frac{1}{2}(AB + DC)$$

### Example

- 2 Find the length of median  $MN$  in trapezoid  $ABCD$  if  $AB = 12$  and  $DC = 18$ .

$$\begin{aligned} MN &= \frac{1}{2}(AB + DC) && \text{Theorem 8-13} \\ &= \frac{1}{2}(12 + 18) && \text{Replace } AB \text{ with } 12 \\ &&& \text{and } DC \text{ with } 18. \\ &= \frac{1}{2}(30) \text{ or } 15 && \text{Simplify.} \end{aligned}$$



The length of the median of trapezoid  $ABCD$  is 15 units.

### Your Turn

- a. Find the length of median  $MN$  in trapezoid  $ABCD$  if  $AB = 20$  and  $DC = 16$ .

### Look Back

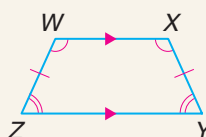
Isosceles Triangle:  
Lesson 6-4

If the legs of a trapezoid are congruent, the trapezoid is an **isosceles trapezoid**. In Lesson 6-4, you learned that the base angles of an isosceles triangle are congruent. There is a similar property for isosceles trapezoids.

### Theorem 8-14

**Words:** Each pair of base angles in an isosceles trapezoid is congruent.

**Model:**

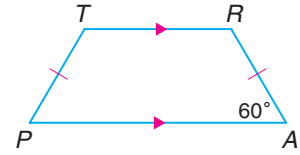


**Symbols:**

$$\angle W \cong \angle X, \angle Z \cong \angle Y$$

**Example****3**

Find the missing angle measures in isosceles trapezoid  $TRAP$ .



Find  $m\angle P$ .

$$\angle P \cong \angle A \quad \text{Theorem 8-14}$$

$$m\angle P = m\angle A \quad \text{Definition of congruent}$$

$$m\angle P = 60 \quad \text{Replace } m\angle A \text{ with } 60.$$

Find  $m\angle T$ . Since  $TRAP$  is a trapezoid,  $\overline{TR} \parallel \overline{PA}$ .

$$m\angle T + m\angle P = 180 \quad \text{Consecutive interior angles are supplementary.}$$

$$m\angle T + 60 = 180 \quad \text{Replace } m\angle P \text{ with } 60.$$

$$m\angle T + 60 - 60 = 180 - 60 \quad \text{Subtract } 60 \text{ from each side.}$$

$$m\angle T = 120 \quad \text{Simplify.}$$

Find  $m\angle R$ .

$$\angle R \cong \angle T \quad \text{Theorem 8-14}$$

$$m\angle R = m\angle T \quad \text{Definition of congruent}$$

$$m\angle R = 120 \quad \text{Replace } m\angle T \text{ with } 120.$$

**Look Back**

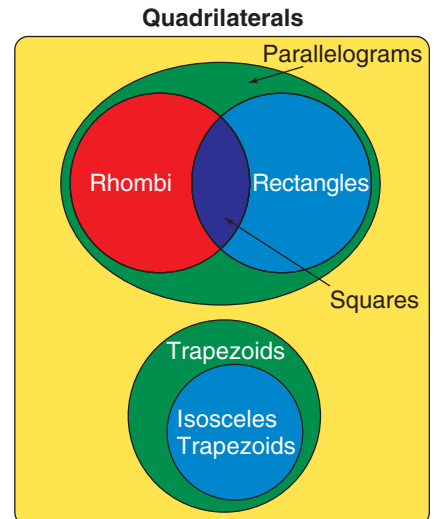
Consecutive Interior  
Angles:  
Lesson 4-2

**Your Turn**

- b. The measure of one angle in an isosceles trapezoid is 48. Find the measures of the other three angles.

In this chapter, you have studied quadrilaterals, parallelograms, rectangles, rhombi, squares, trapezoids, and isosceles trapezoids. The Venn diagram illustrates how these figures are related.

- The Venn diagram represents all quadrilaterals.
- Parallelograms and trapezoids do not share any characteristics except that they are both quadrilaterals. This is shown by the nonoverlapping regions in the Venn diagram.
- Every isosceles trapezoid is a trapezoid. In the Venn diagram, this is shown by the set of isosceles trapezoids contained in the set of trapezoids.
- All rectangles and rhombi are parallelograms. Since a square is both a rectangle and a rhombus, it is shown by overlapping regions.



# Check for Understanding

## Communicating Mathematics

1. Draw an isosceles trapezoid and label the legs and the bases.
2. Explain how the length of the median of a trapezoid is related to the lengths of the bases.
3. **Writing Math** Copy and complete the following table. Write *yes* or *no* to indicate whether each quadrilateral always has the given characteristics.

### Vocabulary

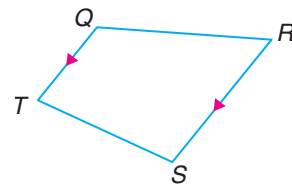
trapezoid  
bases  
legs  
base angles  
median  
isosceles trapezoid

Characteristics	Parallelogram	Rectangle	Rhombus	Square	Trapezoid
Opposite sides are parallel.					
Opposite sides are congruent.					
Opposite angles are congruent.					
Consecutive angles are supplementary.					
Diagonals bisect each other.					
Diagonals are congruent.					
Diagonals are perpendicular.					
Each diagonal bisects two angles.					

## Guided Practice

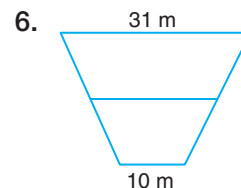
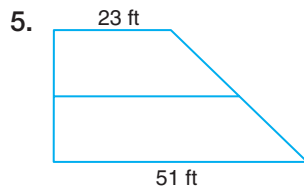
### Example 1

4. In trapezoid  $QRST$ , name the bases, the legs, and the base angles.



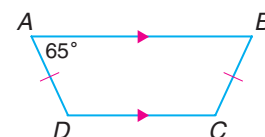
### Example 2

- Find the length of the median in each trapezoid.



### Example 3

7. Trapezoid  $ABCD$  is isosceles. Find the missing angle measures.



**Example 3**

8. **Construction** A hip roof slopes at the ends of the building as well as the front and back. The front of this hip roof is in the shape of an isosceles trapezoid. If one angle measures  $30^\circ$ , find the measures of the other three angles.



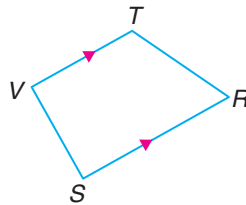
**Exercises**

**Practice**

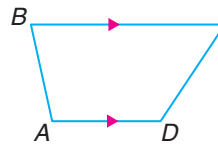
For each trapezoid, name the bases, the legs, and the base angles.

Homework Help	
For Exercises	See Examples
8–20, 22	3
9–11, 29	1
12–17, 21, 30	2
Extra Practice	
See page 741.	

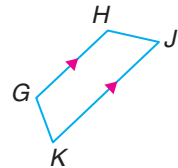
9.



10.

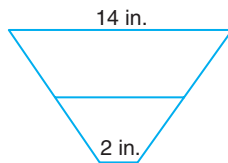


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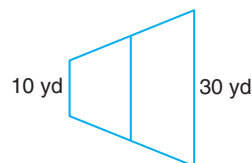


Find the length of the median in each trapezoid.

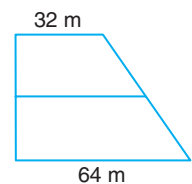
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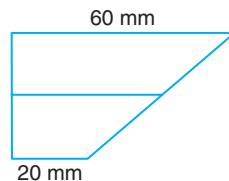
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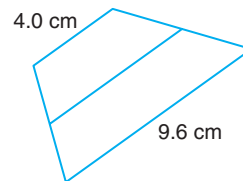
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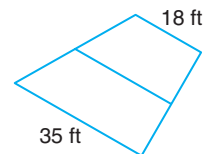
15.



16.

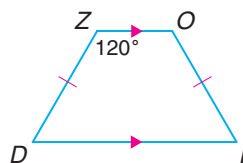


17.

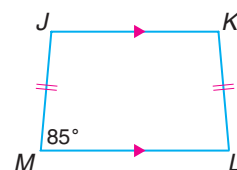


Find the missing angle measures in each isosceles trapezoid.

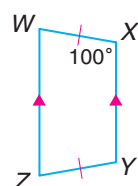
18.



19.



20.



21. Find the length of the shorter base of a trapezoid if the length of the median is 34 meters and the length of the longer base is 49 meters.
22. One base angle of an isosceles trapezoid is  $45^\circ$ . Find the measures of the other three angles.

**Determine whether it is possible for a trapezoid to have the following conditions. Write yes or no. If yes, draw the trapezoid.**

23. three congruent sides
24. congruent bases
25. four acute angles
26. two right angles
27. one leg longer than either base
28. two congruent sides, but not isosceles

### Applications and Problem Solving

29. **Bridges** Explain why the figure outlined on the Golden Gate Bridge is a trapezoid.



Photo Graphic

30. **Algebra** If the sum of the measures of the bases of a trapezoid is  $4x$ , find the measure of the median.
31. **Critical Thinking** A sequence of trapezoids is shown. The first three trapezoids in the sequence are formed by 3, 5, and 7 triangles.



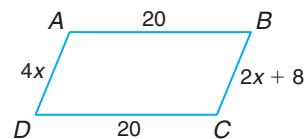
- a. How many triangles are needed for the 10th trapezoid?
- b. How many triangles are needed for the  $n$ th trapezoid?

### Mixed Review

**Name all quadrilaterals that have each property.** (Lesson 8-4)

32. four right angles
33. congruent diagonals

34. **Algebra** Find the value for  $x$  that will make quadrilateral  $ABCD$  a parallelogram. (Lesson 8-3)



### Standardized Test Practice

- (A) (B) (C) (D)

35. **Extended Response** Draw and label a figure to illustrate that  $\overline{JN}$  and  $\overline{LM}$  are medians of  $\triangle JKL$  and intersect at  $I$ . (Lesson 6-1)

36. **Multiple Choice** In the figure,  $AC = 60$ ,  $CD = 12$ , and  $B$  is the midpoint of  $\overline{AD}$ . Choose the correct statement. (Lesson 2-5)



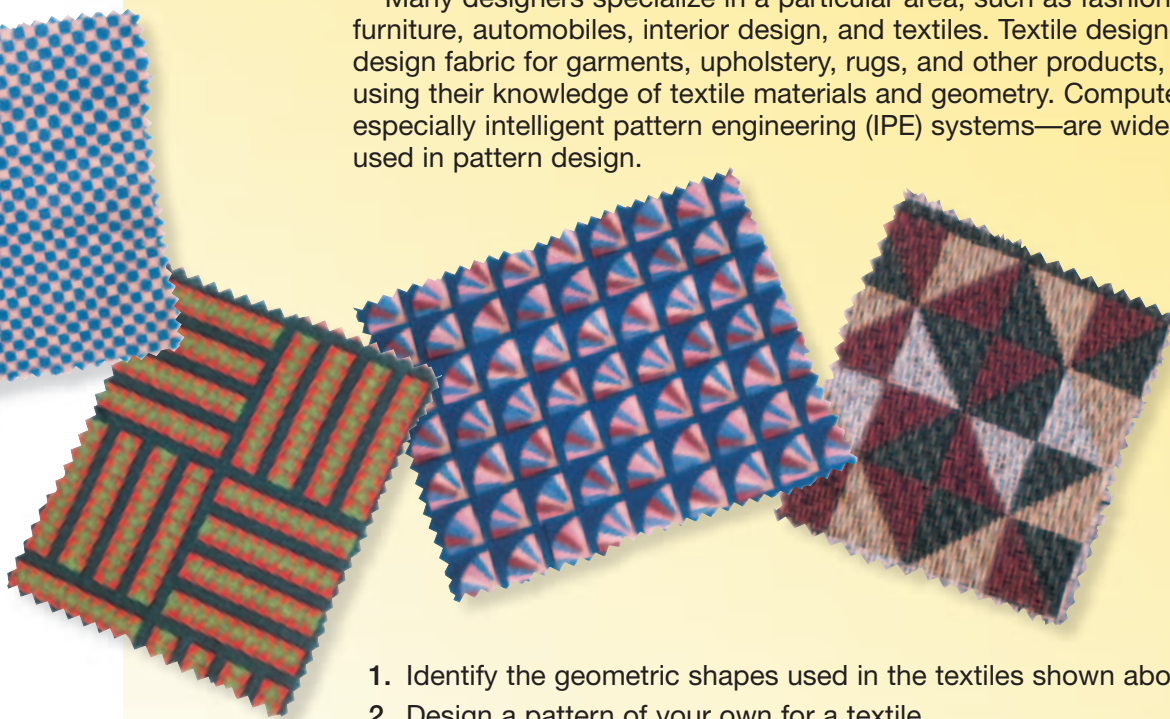
- (A)  $BC > CD$
- (B)  $BC < CD$
- (C)  $BC = CD$
- (D) There is not enough information.



## Designer

Are you creative? Do you find yourself sketching designs for new cars or the latest fashion trends? Then you may like a career as a designer. Designers organize and design products that are visually appealing and serve a specific purpose.

Many designers specialize in a particular area, such as fashion, furniture, automobiles, interior design, and textiles. Textile designers design fabric for garments, upholstery, rugs, and other products, using their knowledge of textile materials and geometry. Computers—especially intelligent pattern engineering (IPE) systems—are widely used in pattern design.



1. Identify the geometric shapes used in the textiles shown above.
2. Design a pattern of your own for a textile.

### FAST FACTS About Fashion Designers

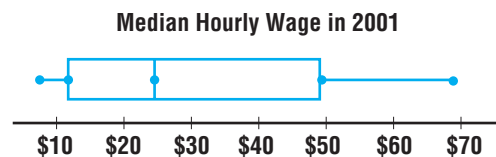
#### Working Conditions

- vary by places of employment
- overtime work sometimes required to meet deadlines
- keen competition for most jobs

#### Education

- a 2- or 4-year degree is usually needed
- computer-aided design (CAD) courses are very useful
- creativity is crucial

#### Earnings



Source: Bureau of Labor Statistics








**Career Data** For the latest information on a career as a designer, visit:

[www.geomconcepts.com](http://www.geomconcepts.com)



### Materials

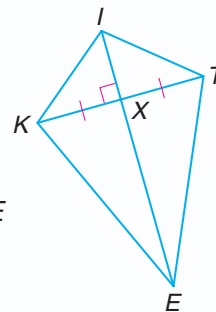
-  unlined paper
-  compass
-  straightedge
-  protractor
-  ruler

### Kites

A kite is more than just a toy to fly on a windy day. In geometry, a **kite** is a special quadrilateral that has its own properties.

### Investigate

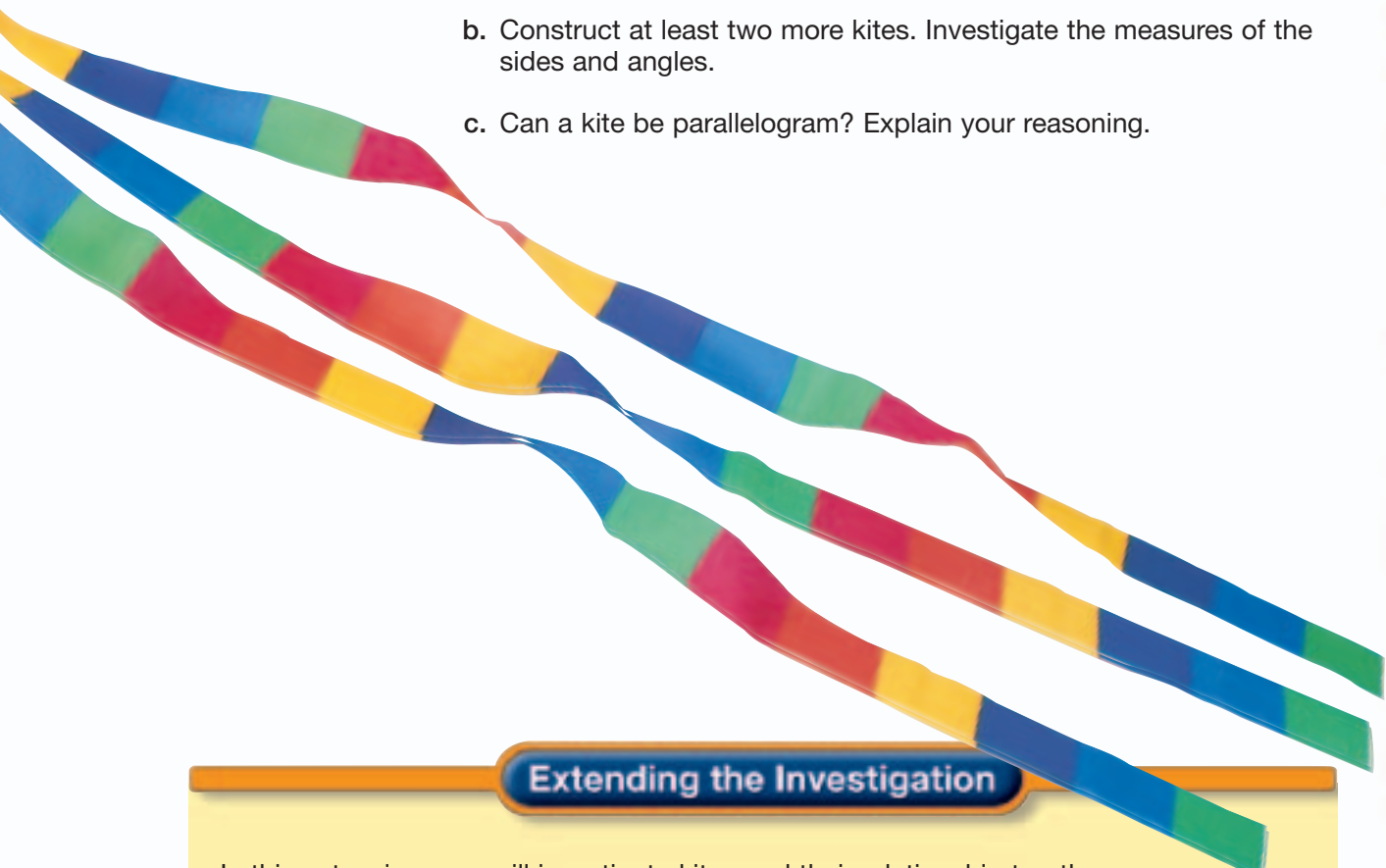
1. Use paper, compass, and straightedge to construct a kite.
  - a. Draw a segment about six inches in length. Label the endpoints  $I$  and  $E$ . Mark a point on the segment. The point should *not* be the midpoint of  $\overline{IE}$ . Label the point  $X$ .
  - b. Construct a line that is perpendicular to  $\overline{IE}$  through  $X$ . Mark point  $K$  about two inches to the left of  $X$  on the perpendicular line. Then mark another point,  $T$ , on the right side of  $X$  so that  $\overline{KX} \cong \overline{XT}$ .



- c. Connect points  $K$ ,  $I$ ,  $T$ , and  $E$  to form a quadrilateral.  $KITE$  is a kite. Use a ruler to measure the lengths of the sides of  $KITE$ . What do you notice?
- d. Write a definition for a kite. Compare your definition with others in the class.





- 
2. Use compass, straightedge, protractor, and ruler to investigate kites.
    - a. Use a protractor to measure the angles of *KITE*. What do you notice about the measures of opposite and consecutive angles?
    - b. Construct at least two more kites. Investigate the measures of the sides and angles.
    - c. Can a kite be parallelogram? Explain your reasoning.

### Extending the Investigation

In this extension, you will investigate kites and their relationship to other quadrilaterals. Here are some suggestions.

1. Rewrite Theorems 8–2 through 8–6 and 8–10 through 8–12 so they are true for kites.
2. Make a list of as many properties as possible for kites.
3. Build a kite using the properties you have studied.

### Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a booklet showing the differences and similarities among the quadrilaterals you have studied. Be sure to include kites.
- Make a video about quadrilaterals. Cast your actors as the different quadrilaterals. The script should help viewers understand the properties of quadrilaterals.



**Investigation** For more information on kites, visit: [www.geomconcepts.com](http://www.geomconcepts.com)

