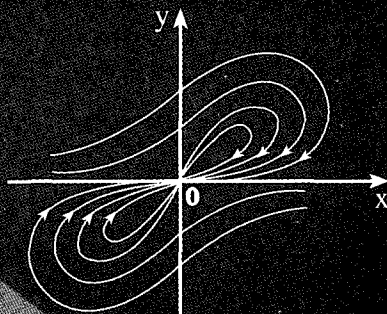


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Global Attractors of Non-Autonomous Dissipative Dynamical Systems

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