GLOBAL CIRCULATION

CONTENTS

11 Solar radiation absorbed in the tropics exceeds infrared loss, causing heat to accumulate. The opposite is true at the poles, where there is net radiative cooling. Such radiative **differential heating** between poles and equator creates an imbalance in the atmosphere/ocean system (Fig. 11.1a).

Le Chatelier's Principle states that an imbalanced system reacts in a way to partially undo the imbalance. For the atmosphere of Fig. 11.1a, warm tropical air rises due to buoyancy and flows toward the poles, and the cold polar air sinks and flows toward the equator (Fig. 11.1b).

Because the radiative forcings are unremitting, a ceaseless movement of wind and ocean currents results in what we call the **general circulation** or **global circulation**. The circulation cannot instantly undo the continued destabilization, resulting in tropics that remain slightly warmer than the poles.

But the real general circulation on Earth does not look like Fig. 11.1b. Instead, there are three bands of circulations in the Northern Hemisphere (Fig. 11.1c), and three in the Southern. In this chapter, we will identify characteristics of the general circulation, explain why they exist, and learn how they work.

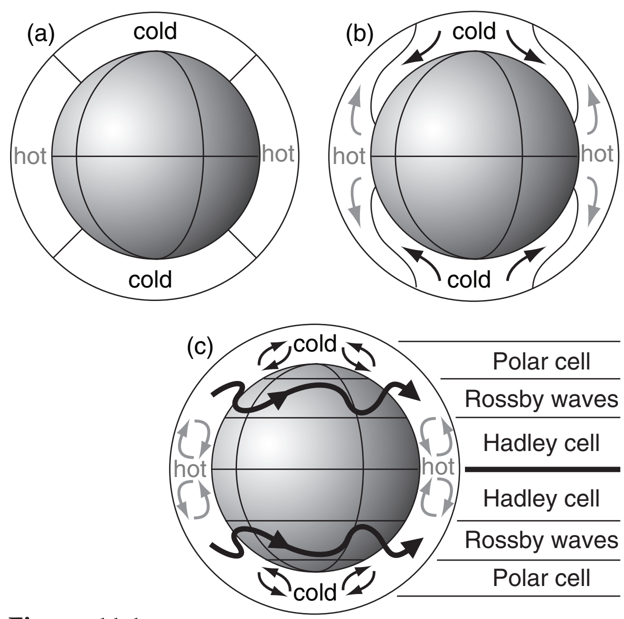


Figure 11.1
Earth/atmosphere system, showing that (a) differential heating by radiation causes (b) atmospheric circulations. Add Earth's rotation, and (c) 3 circulation bands form in each hemisphere.

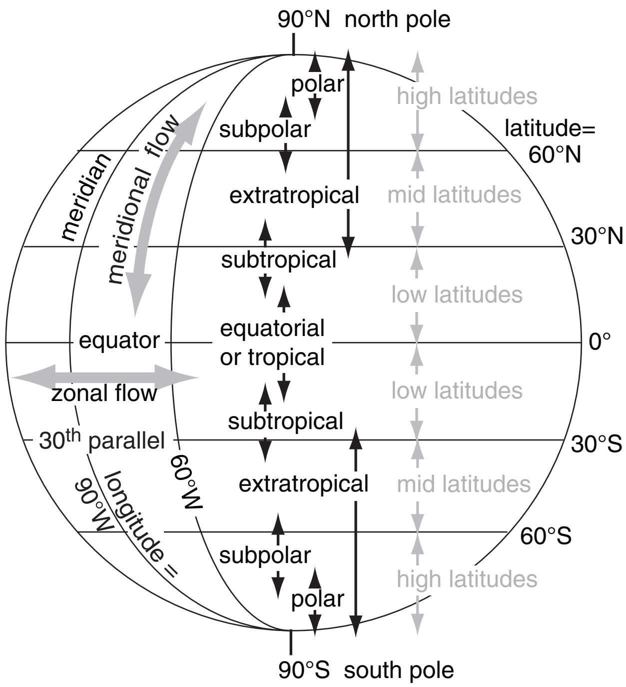


Figure 11.2
Global nomenclature.

NOMENCLATURE

Latitude lines are **parallels**, and east-west winds are called **zonal flow** (Fig. 11.2). Each 1° of latitude = 111 km. Longitude lines are **meridians**, and north-south winds are called **meridional flow**.

Mid-latitudes are the regions between about 30° and 60° latitude. **High latitudes** are 60° to 90°, and **low latitudes** are 0° to 30°.

The **subtropical** zone is at roughly 30° latitude, and the **subpolar** zone is at 60° latitude, both of which partially overlap mid-latitudes. **Tropics** span the **equator**, and **polar** regions are near the Earth's poles. **Extratropical** refers to everything outside of the tropics: poleward from roughly 30°N and 30°S.

For example, **extratropical cyclones** are low-pressure centers — typically called **lows** and labeled with $\mathbb{L}$ — that are found in mid- or high-latitudes. **Tropical cyclones** include **hurricanes** and **typhoons**, and other strong lows in tropical regions.

In many climate studies, data from the months of June, July, and August (**JJA**) are used to represent conditions in N. Hemisphere summer (and S. Hemisphere winter). Similarly, December, January, February (**DJF**) data are used to represent N. Hemisphere winter (and S. Hemisphere summer).

ON DOING SCIENCE • Toy Models

Some problems in meteorology are so complex, and involve so many interacting variables and processes, that they are intimidating if not impossible to solve. However, we can sometimes gain insight into fundamental aspects of the problem by using idealized, simplified physics. Such an approximation is sometimes called a "toy model".

A rotating spherical Earth with no oceans is one example of a toy model. The meridional variation of temperature given later in this chapter is another example. These models are designed to capture only the dominant processes. They are toy models, not complete models.

Toy models are used extensively to study climate change. For example, the greenhouse effect can be examined using toy models in the Climate chapter. Toy models capture only the dominant effects, and neglect the subtleties. You should never use toy models to infer the details of a process, particularly in situations where two large but opposite processes nearly cancel each other.

For other examples of toy models applied to the environment, see John Harte's 1988 book *"Consider a Spherical Cow"*, University Science Books. 283 pp.

A SIMPLIFIED DESCRIPTION OF THE GLOBAL CIRCULATION

Consider a hypothetical rotating planet with no contrast between continents and oceans. The **climatological average** (average over 30 years; see the Climate chapter) winds in such a simplified planet would have characteristics as sketched in Figs. 11.3. Actual winds on any day could differ from this climatological average due to transient weather systems that perturb the average flow. Also, monthly-average conditions tend to shift toward the summer hemisphere (e.g., the circulation bands shift northward during April through September).

Near-surface Conditions

Near-surface average winds are sketched in Fig. 11.3a. At low latitudes are broad bands of persistent easterly winds ($U \approx -7$ m/s) called **trade winds**, named because the **easterlies** allowed sailing ships to conduct transoceanic trade in the old days.

These trade winds also blow toward the equator from both hemispheres, and the equatorial belt of convergence is called the **intertropical conver-**

gence zone (**ITCZ**). On average, the air at the ITCZ is hot and humid, with low pressure, strong upward air motion, heavy convective (thunderstorm) precipitation, and light to calm winds except in thunderstorms. This **equatorial trough** (low-pressure belt) was called the **doldrums** by sailors whose sailing ships were becalmed there for many days.

At 30° latitude are belts of high surface pressure called **subtropical highs** (Fig. 11.3a). In these belts are hot, dry, cloud-free air descending from higher in the troposphere. Surface winds in these belts are also calm on average. In the old days, becalmed sailing ships would often run short of drinking water, causing horses on board to die and be thrown overboard. Hence, sailors called these miserable places the **horse latitudes**. On land, many of the world's deserts are near these latitudes.

In mid-latitudes are transient centers of low pressure (**mid-latitude cyclones, L**) and high pressure (**anticyclones, H**). Winds around lows **converge** (come together) and circulate **cyclonically** — counterclockwise in the N. Hemisphere, and clockwise in the S. Hemisphere. Winds around highs **diverge** (spread out) and rotate **anticyclonically** — clockwise in the N. Hemisphere, and counterclockwise in the S. Hemisphere. The cyclones are regions of bad weather (clouds, rain, high humidity, strong winds) and fronts. The anticyclones are regions of good weather (clear skies or fair-weather clouds, no precipitation, dry air, and light winds).

The high- and low-pressure centers move on average from west to east, driven by large-scale winds from the west. Although these **westerlies** dominate the general circulation at mid-latitudes, the surface winds are quite variable in time and space due to the sum of the westerlies plus the transient circulations around the highs and lows.

Near 60° latitude are belts of low surface pressure called **subpolar lows**. Along these belts are light to calm winds, upward air motion, clouds, cool temperatures, and precipitation (as snow in winter).

Near each pole is a climatological region of high pressure called a **polar high**. In these regions are often clear skies, cold dry descending air, light winds, and little snowfall. Between each polar high (at 90°) and the subpolar low (at 60°) is a belt of weak easterly winds, called the **polar easterlies**.

Upper-tropospheric Conditions

The stratosphere is strongly statically stable, and acts like a lid to the troposphere. Thus, vertical circulations associated with our weather are mostly trapped within the troposphere. These vertical circulations couple the average near-surface winds with the average upper-tropospheric (near the tropopause) winds described here (Fig. 11.3b).

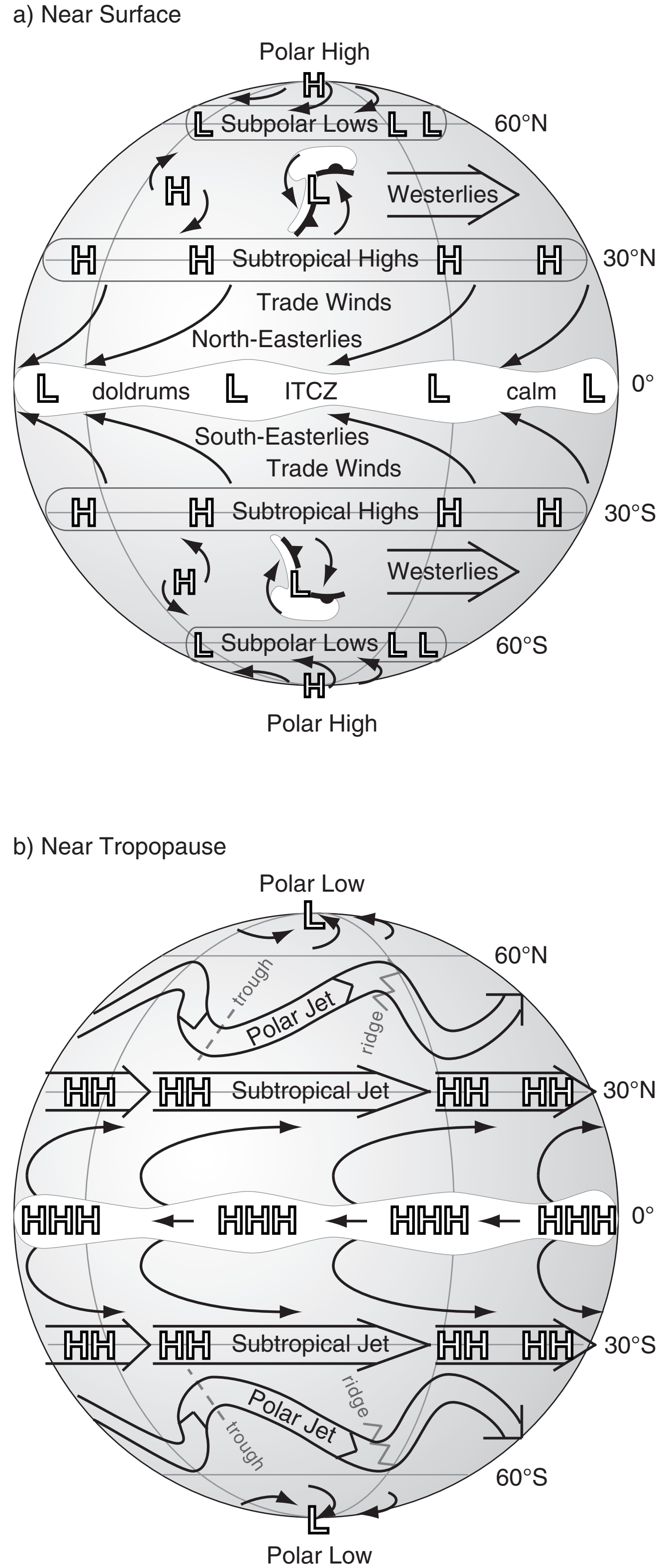
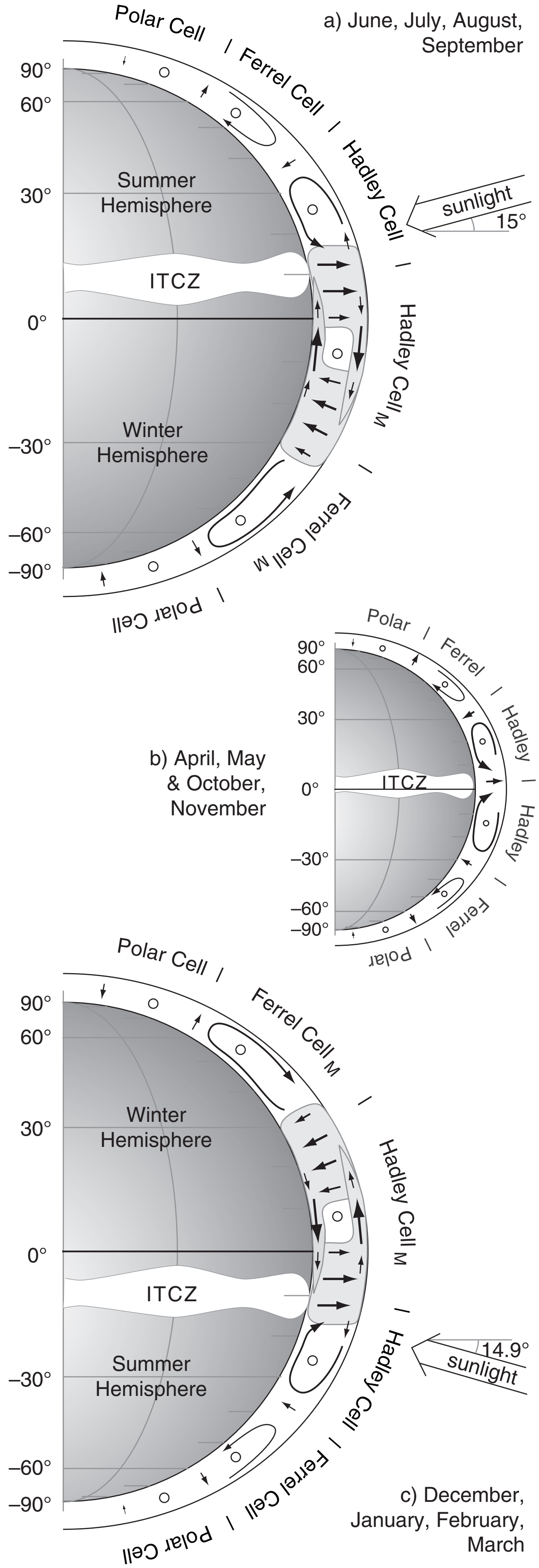


Figure 11.3
Simplified global circulation in the troposphere: (a) near the surface, and (b) near the tropopause. H and L indicate high and low pressures, and HHH means very strong high pressure. White indicates precipitating clouds.



In the tropics is a belt of very strong equatorial high pressure along the tops of the ITCZ thunderstorms. Air in this belt blows from the east, due to easterly inertia from the trade winds being carried upward in the thunderstorm convection. Diverging from this belt are winds that blow toward the north in the N. Hemisphere, and toward the south in the S. Hemisphere. As these winds move away from the equator, they turn to have an increasingly westerly component as they approach 30° latitude.

Near 30° latitude in each hemisphere is a persistent belt of strong westerly winds at the tropopause called the **subtropical jet**. This jet meanders north and south a bit. Pressure here is very high, but not as high as over the equator.

In mid-latitudes at the tropopause is another belt of strong westerly winds called the **polar jet**. The centerline of the polar jet meanders north and south, resulting in a wave-like shape called a **Rossby wave**, as sketched in Fig. 11.1c. The equatorward portions of the wave are known as low-pressure **troughs**, and poleward portions are known as high-pressure **ridges**. These ridges and troughs are very transient, and generally shift from west to east relative to the ground.

Near 60° at the tropopause is a belt of low to medium pressure. At each pole is a low-pressure center near the tropopause, with winds at high latitudes generally blowing from the west causing a cyclonic circulation around the **polar low**. Thus, contrary to near-surface conditions, the near-tropopause average winds blow from the west at all latitudes (except near the equator).

Vertical Circulations

Vertical circulations of warm rising air in the tropics and descending air in the subtropics are called **Hadley cells** or **Hadley circulations** (Fig. 11.4). At the bottom of the Hadley cell are the trade winds. At the top, near the tropopause, are divergent winds. The updraft portion of the Hadley circulation is often filled with thunderstorms and heavy precipitation at the ITCZ. This vigorous convection in the troposphere causes a high tropopause (15 - 18 km altitude) and a belt of heavy rain in the tropics.

The summer- and winter-hemisphere Hadley cells are strongly asymmetric (Fig. 11.4). The major Hadley circulation (denoted with subscript "M") crosses the equator, with rising air in the summer

Figure 11.4 *(at left)*
Vertical cross section of Earth's global circulation in the troposphere. (a) N. Hemisphere summer. (b) Transition months. (c) S. Hemisphere summer. The major (subscript $_M$) Hadley cell is shaded light grey. Minor circulations have no subscript.

hemisphere and descending air in the winter hemisphere. The updraft is often between 0° and 15° latitudes in the summer hemisphere, and has average core vertical velocities of 6 mm/s. The broader downdraft is often found between 10° and 30° latitudes in the winter hemisphere, with average velocity of about –4 mm/s in downdraft centers. Connecting the up- and downdrafts are meridional wind components of 3 m/s at the cell top and bottom.

The major Hadley cell changes direction and shifts position between summer and winter. During June-July-August-September, the average solar declination angle is 15°N, and the updraft is in the Northern Hemisphere (Fig. 11.4a). Out of these four months, the most well-defined circulation occurs in August and September. At this time, the ITCZ is centered at about 10°N.

During December-January-February-March, the average solar declination angle is 14.9°S, and the major updraft is in the Southern Hemisphere (Fig. 11.4c). Out of these four months, the strongest circulation is during February and March, and the ITCZ is centered at 10°S. The major Hadley cell transports significant heat away from the tropics, and also from the summer to the winter hemisphere.

During the transition months (April-May and October-November) between summer and winter, the Hadley circulation has nearly symmetric Hadley cells in both hemispheres (Fig. 11.4b). During this transition, the intensities of the Hadley circulations are weak.

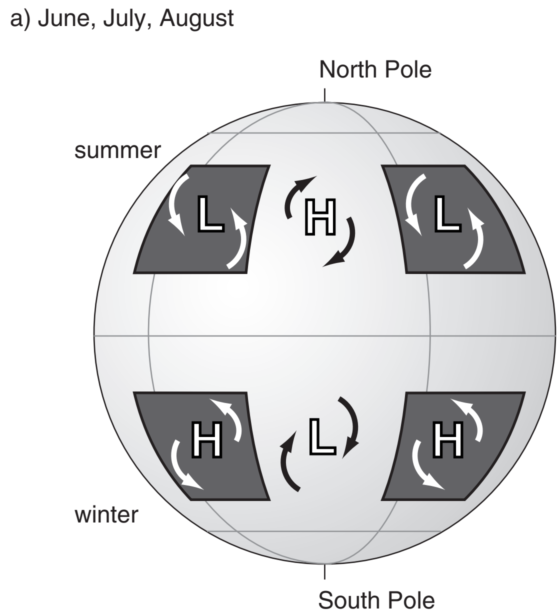
When averaged over the whole year, the strong but reversing major Hadley circulation partially cancels itself, resulting in an annual average circulation that is somewhat weak and looks like Fig. 11.4b. This weak annual average is deceiving, and does not reflect the true movement of heat, moisture, and momentum by the winds. Hence, climate experts prefer to look at months JJA and DJF separately to give **seasonal averages**.

In the winter hemisphere is a **Ferrel cell**, with a vertical circulation of descending air in the subtropics and rising air at high latitudes; namely, a circulation opposite to that of the major Hadley cell. In the winter hemisphere is a modest **polar cell**, with air circulating in the same sense as the Hadley cell.

In the summer hemisphere, all the circulations are weaker. There is a minor Hadley cell and a minor Ferrel cell (Fig. 11.4). Summer-hemisphere circulations are weaker because the temperature contrast between the tropics and poles are weaker.

Monsoonal Circulations

Monsoon circulations are continental-scale circulations driven by continent-ocean temperature contrasts, as sketched in Figs. 11.5. In summer, high-



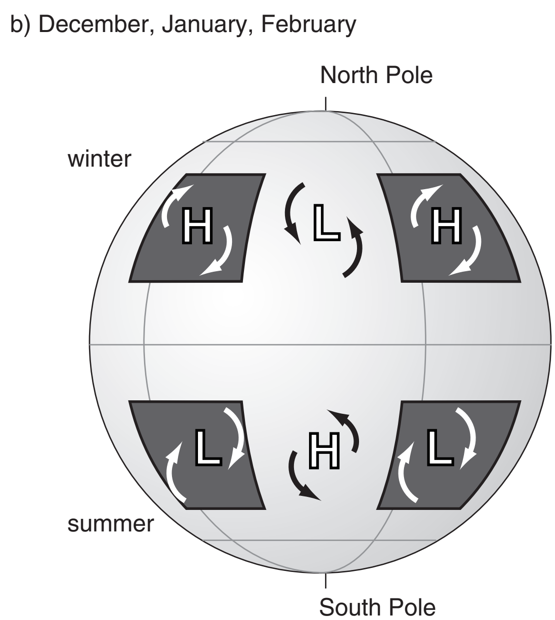


Figure 11.5
Idealized seasonal-average monsoon circulations near the surface. Continents are shaded dark grey; oceans are light grey. H and L are surface high- and low-pressure centers.

pressure centers (anticyclones) are over the relatively warm oceans, and low-pressure centers (cyclones) are over the hotter continents. In winter, low-pressure centers are over the cool oceans, and high-pressure centers are over the colder continents.

These monsoon circulations represent average conditions over a season. The actual weather on any given day can be variable, and can deviate from these seasonal averages.

Our Earth has a complex arrangement of continents and oceans. As a result, seasonally-varying monsoonal circulations are superimposed on the seasonally-varying planetary-scale circulation to yield a complex and varying global-circulation pattern.

At this point, you have a descriptive understanding of the global circulation. But what drives it?

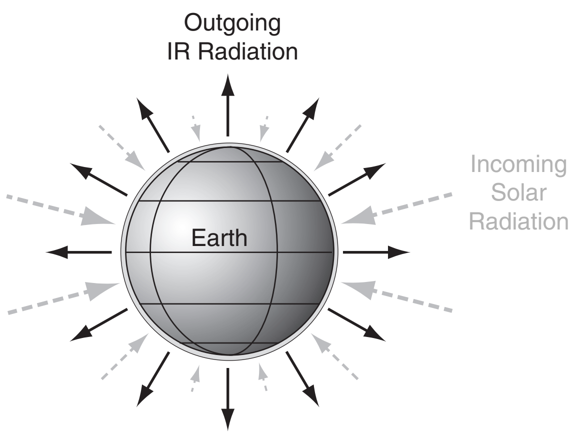


Figure 11.6
Annual average incoming solar radiation (grey dashed arrows) and of outgoing infrared (IR) radiation (solid black arrows), where arrow size indicates relative magnitude. [Because the Earth rotates and exposes all locations to the sun at one time or another, the incoming solar radiation is sketched as approaching all locations on the Earth's surface.]

DIFFERENTIAL HEATING

Differential heating drives the global circulation. Incoming solar radiation (**insolation**) nearly balances the outgoing infrared (IR) radiation when averaged over the whole globe. However, at different latitudes are significant imbalances (Fig. 11.6), which cause the differential heating.

Recall from the Radiation chapter that the flux of solar radiation incident on the top of the atmosphere depends more or less on the cosine of the latitude, as shown in Fig. 11.7. The component of the incident ray of sunlight that is perpendicular to the Earth's surface is small in polar regions, but larger toward the equator (grey dashed arrows in Figs. 11.6 and 11.7). The incoming energy adds heat to the Earth-atmosphere-ocean system.

Heat is lost due to infrared (IR) radiation emitted from the Earth-ocean-atmosphere system to space. Since all locations near the surface in the Earth-ocean-atmosphere system are relatively warm compared to absolute zero, the Stefan-Boltzmann law from the Radiation chapter tells us that the emission rates are also more or less uniform around the Earth. This is sketched by the solid black arrows in Fig. 11.6.

Thus, at low latitudes, more solar radiation is absorbed than leaves as IR, causing net warming. At high latitudes, the opposite is true: IR radiative losses exceed solar heating, causing net cooling. This differential heating drives the global circulation.

Because the global circulation cannot instantly eliminate the temperature disparity across the globe, there is a residual north-south temperature gradient that we will examine first.

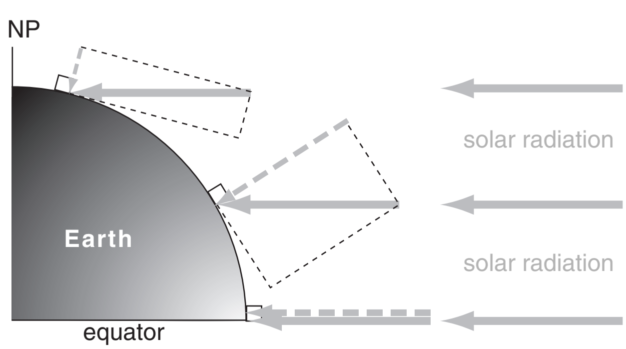


Figure 11.7
Of the solar radiation approaching the Earth (thick solid grey arrows), the component (dashed grey arrow) that is perpendicular to the top of the atmosphere is proportional to the cosine of the latitude (during the equinox).

Meridional Temperature Gradient

Sea-level temperatures are warmer near the equator than at the poles, when averaged over a whole year and averaged around latitude belts (Fig. 11.8a). Although the Northern Hemisphere is slightly cooler than the Southern at sea level, an idealization (toy model) of the variation of zonally-averaged temperature T with latitude ϕ is:

$$T \approx a + b \cdot \left[\frac{3}{2} \cdot \left(\frac{2}{3} + \sin^2 \phi \right) \cdot \cos^3 \phi \right] \tag{11.1}$$

For sea level, the parameters are: $a \approx -12°C$, $b \approx 40°C$. Parameter b represents a temperature difference between equator and pole, so it could also have been written as $b \approx 40$ K.

The magnitude of the equator-to-pole temperature gradient decreases with altitude, and changes sign in the stratosphere. Namely, in the stratosphere, it is cold in the tropics and warmer near the poles. For this reason, parameter b in eq. (11.1) can be generalized as:

$$b \approx b_1 \cdot \left(1 - \frac{z}{z_T} \right) \tag{11.2}$$

where $b_1 = 40°C$, z is height above sea level, and $z_T \approx$ 11 km is the average depth of the troposphere.

A similarly crude but useful generalization of parameter a can be made so that it too changes with altitude z above sea level:

$$a \approx a_1 - \gamma \cdot z \tag{11.3}$$

where $a_1 = -12°C$ and $\gamma = 3.14$ °C/km.

From Fig. 11.8, we see that in the tropics there is little temperature variation — it is hot everywhere. The reason for this is the very strong mixing and transport of heat by the Hadley circulations. However, in mid-latitudes, there is a significant north-south temperature gradient that supports a variety of storm systems. Although eq. (11.1) might seem unnecessarily complex, it is designed to give sufficient uniformity in the tropics to shift the large temperature gradients toward mid-latitudes.

The temperature gradient associated with eq. (11.1) is

$$\frac{\Delta T}{\Delta y} \approx -b \cdot c \cdot \left[\sin^3 \phi \cdot \cos^2 \phi \right] \tag{11.4}$$

where $c = 1.18\text{x}10^{-3}$ km^{-1} is a constant valid at all heights, b is given by eq. (11.2), and y is distance in the north-south direction. The meridional temperature gradients at sea level and at z = 15 km are plotted in Fig. 11.8b.

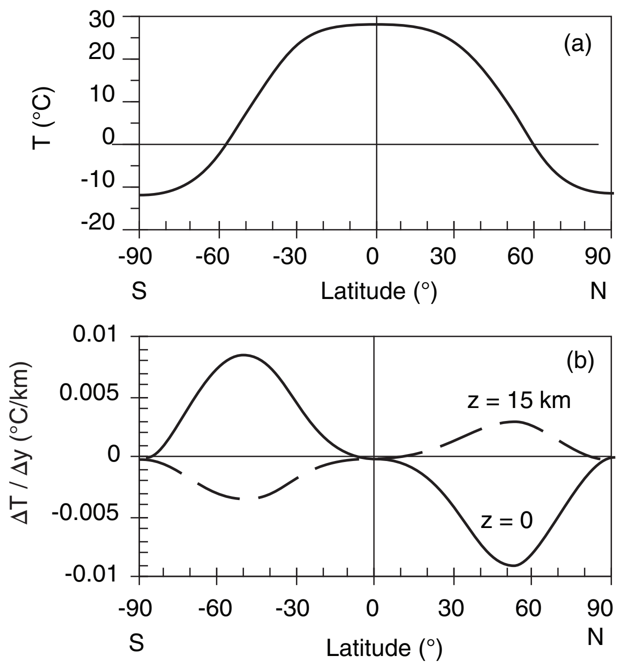


Figure 11.8
Idealized annually and zonally averaged (a) temperature at sea level, and (b) north-south temperature gradient at sea level and at 15 km altitude.

Solved Example

Find the meridional temperature and temperature gradient at 45°N latitude, at sea level (sl).

Solution

Given: $\phi = 45°$
Find: T_{sl} = ? °C, $\Delta T/\Delta y$ = ? °C/km

Use eq. (11.1) with $z = 0$ at sea level:

$$T_{sl} \approx -12°C + (40°C) \cdot \left[\frac{3}{2} \cdot \left(\frac{2}{3} + \sin^2 45° \right) \cdot \cos^3 45° \right]$$

= **12.75 °C**

Use eq. (11.4):

$$\frac{\Delta T}{\Delta y} \approx -(40°C) \cdot (1.18 \times 10^{-3}) \cdot \left[\sin^3 45° \cdot \cos^2 45° \right]$$

= **–0.0083 °C/km**

Check: Units OK. Physics OK. As a quick check, from Fig. 11.8a, the temperature decreases by about 9°C between 40°N to 50°N latitude. But each 1° of latitude equals 111 km of distance y. This temperature gradient of –9°C/(1110 km) from the figure agrees with the numerical answer above.

Discussion: Temperature decreases toward the north in the northern hemisphere, which gives the negative sign for the gradient.

BEYOND ALGEBRA • Temperature Gradient

Problem: Derive eq. (11.4) from eq. (11.1).

Solution: Given:

$$T \approx a + b \cdot \left[\frac{3}{2} \cdot \left(\frac{2}{3} + \sin^2 \phi \right) \cdot \cos^3 \phi \right]$$

Find: $$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial \phi} \cdot \frac{\partial \phi}{\partial y} \qquad \text{(a)}$$

The first factor on the right in (a) is: $\frac{\partial T}{\partial \phi} = b \cdot \left(\frac{3}{2}\right) \cdot$

$$\left[(2 \sin\phi \cdot \cos\phi) \cos^3 \phi - 3 \left(\frac{2}{3} + \sin^2 \phi \right) \cos^2 \phi \cdot \sin\phi \right]$$

Taking $\sin\phi \cdot \cos^2\phi$ out of the square brackets:

$$\frac{\partial T}{\partial \phi} = b \left(\frac{3}{2} \right) \sin\phi \cdot \cos^2 \phi \cdot \left[2\cos^2 \phi - 2 - 3 \sin^2 \phi \right]$$

But $\cos^2\phi = 1 - \sin^2\phi$, thus $2 \cos^2\phi = 2 - 2 \sin^2\phi$:

$$\frac{\partial T}{\partial \phi} = b \left(\frac{3}{2} \right) \sin\phi \cdot \cos^2 \phi \cdot \left[-5 \sin^2 \phi \right] \qquad \text{(b)}$$

Inserting eq. (b) into (a) gives:

$$\frac{\partial T}{\partial y} = -b \cdot \left(\frac{15}{2} \cdot \frac{\partial \phi}{\partial y} \right) \cdot \sin^3 \phi \cdot \cos^2 \phi \qquad \text{(c)}$$

Define: $\left(\frac{15}{2} \cdot \frac{\partial \phi}{\partial y} \right) = c$ Thus, the final answer is:

$$\boxed{\frac{\partial T}{\partial y} = -b \cdot c \cdot \sin^3 \phi \cdot \cos^2 \phi} \qquad \text{(11.4)}$$

All that remains is to find c. $\partial\phi/\partial y$ is the change of latitude per distance traveled north. The total change in latitude to circumnavigate the Earth from the north pole past the south pole and back to the north pole is 2π radians, and the circumference of the Earth is $2\pi R$ where R = 6371 km is the average radius of the Earth. Thus:

$$\frac{\partial \phi}{\partial y} = \frac{2\pi}{2\pi \cdot R} = \frac{1}{R}$$

The constant c is then:

$$c = \left(\frac{15}{2} \cdot \frac{1}{R} \right) = 1.18 \times 10^{-3} \text{ km}^{-1} \qquad \text{(e)}$$

Check: The solved example on the previous page verified that eq. (11.4) is a reasonable answer, given the temperature defined by eq. (11.1).

Caution: Eq. (11.1) is only a crude approximation to nature, designed to be convenient for analytical calculations of temperature gradients, thermal winds, IR emissions, etc. While it serves an education purpose here, more accurate models should be used for detailed studies.

Because the meridional temperature gradient results from the interplay of differential radiative heating and advection by the global circulation, let us now look at radiative forcings in more detail.

Radiative Forcings

Incoming Solar Radiation

Because of the tilt of the Earth's axis and the change of seasons, the actual flux of incoming solar radiation is not as simple as was sketched in Fig. 11.6. But this complication was already discussed in the Radiation chapter, where we saw an equation to calculate the incoming solar radiation (**insolation**) as a function of latitude and day. The resulting insolation figure is reproduced below (Fig. 11.9a).

If you take the spreadsheet data from the Radiation chapter that was used to make this figure, and average rows of data (i.e., average over all months for any one latitude), you can find the annual average insolation E_{insol} for each latitude (Fig. 11.9b). Insolation in polar regions is amazingly large.

The curve in Fig. 11.9b is simple, and in the spirit of a toy model can be nicely approximated by:

$$E_{insol} = E_o + E_1 \cdot \cos(2\,\phi) \qquad (11.5)$$

where the empirical parameters are E_o = 298 W/m^2, E_1 = 123 W/m^2, and ϕ is latitude. This curve and the data points it approximates are plotted in Fig. 11.10.

But not all the radiation incident on the top of the atmosphere is absorbed by the Earth-ocean-atmosphere system. Some is reflected back into space

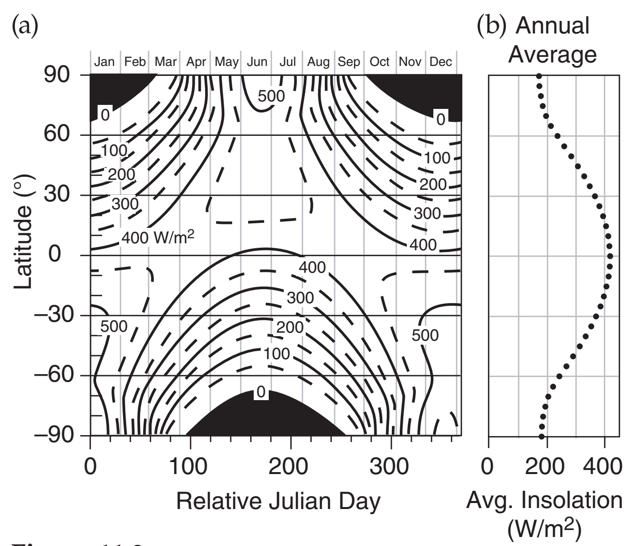


Figure 11.9
(a) Solar radiation (W/m^2) incident on the top of the atmosphere for different latitudes and months (copied from the Radiation chapter). (b) Meridional variation of insolation, found by averaging the data from the left figure over all months for each separate latitude (i.e., averages for each row of data).

from snow and ice on the surface, from the oceans, and from light-colored land. Some is reflected from cloud top. Some is scattered off of air molecules. The amount of insolation that is NOT absorbed is surprisingly constant with latitude at about $E_2 \approx 110$ W/m^2. Thus, the amount that IS absorbed is:

$$E_{in} = E_{insol} - E_2 \tag{11.6}$$

where E_{in} is the incoming flux (W/m^2) of solar radiation absorbed into the Earth-ocean-atmosphere system (Fig. 11.10).

Outgoing Terrestrial Radiation

As you learned in the Remote Sensing chapter, infrared radiation emission and absorption in the atmosphere are very complex. At some wavelengths the atmosphere is mostly transparent, while at others it is mostly opaque. Thus, some of the IR emissions to space are from the Earth's surface, some from cloud top, and some from air at middle altitudes in the atmosphere.

In the spirit of a toy model, suppose that the net IR emissions are characteristic of the absolute temperature T_m near the middle of the troposphere (at about $z_m = 5.5$ km). Next, idealize the zonally- and annually-averaged outgoing radiative flux E_{out} by the Stefan-Boltzmann law (see the Radiation chapter):

$$E_{out} \approx \varepsilon \cdot \sigma_{SB} \cdot T_m^4 \tag{11.7}$$

where σ_{SB} = 5.67x10^{-8} W·m^{-2}·K^{-4} is the Stefan-Boltzmann constant, and $\varepsilon \approx 0.9$ is effective emissivity (see the Climate chapter). When you use $z = z_m$ = 5.5 km in eqs. (11.1 - 11.3) to get T_m vs. latitude for use in eq. (11.7), the result is E_{out} vs. ϕ, as plotted in Fig. 11.10.

Net Radiation

The net radiative heat flux per vertical column of atmosphere is the input minus the output:

$$E_{net} = E_{in} - E_{out} \tag{11.8}$$

which is plotted in Fig. 11.10 for our toy model.

Solved Example

What is E_{net} at the Eiffel Tower latitude?

Solution

Given: ϕ = 48.8590°, z = 5.5 km
E_{in} = 171.5 W/m^2 from previous solved example.
Find: E_{net} = ? W/m^2

Use eq. (11.2): b = (40°C)·[1 – (5.5 km/11 km)] = 20°C
Use eq.(11.3): a=(-12°C)–(3.14°C/km)·(5.5 km)= -29.27°C
(continued in next column)

Solved Example *(continuation)*

Use eq. (11.1): T_m = –18.73 °C = 254.5 K
Use eq.(11.7): E_{out}=(0.9)·(5.67x10^{-8}W·m^{-2}·K^{-4})·(254.5 K)^4
= 213.8 W/m^2
Use eq. (11.8): E_{net} = (171.5 W/m^2) – (213.8 W/m^2)
= **<u>–42.3 W/m^2</u>**

Check: Units OK. Agrees with Fig. 11.10
Discussion: Net radiative heat loss at Paris latitude. Must be compensated by winds blowing heat in.

Solved Example

Estimate the annual average solar energy <u>absorbed</u> at the latitude of the Eiffel Tower in Paris, France.

Solution

Given: ϕ = 48.8590° (at the Eiffel tower)
Find: E_{in} = ? W/m^2

Use eq. (11.5):
E_{insol} = (298 W/m^2) + (123 W/m^2)·cos(2 · 48.8590°)
= (298 W/m^2) – (16.5 W/m^2) = 281.5 W/m^2
Use eq. (11.6):
E_{in} = 281.5 W/m^2 – 110.0 W/m^2 = **<u>171.5 W/m^2</u>**

Check: Units OK. Agrees with Fig. 11.10.
Discussion: The actual annual average E_{in} at the Eiffel Tower would probably differ from this zonal avg.

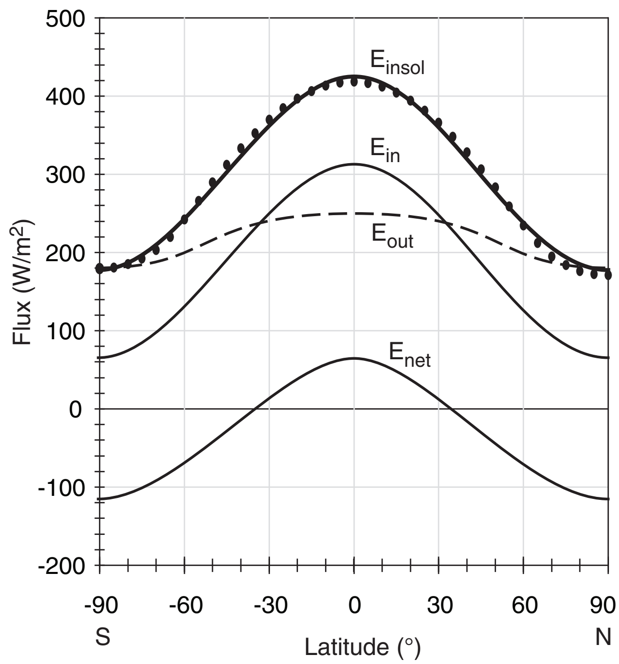


Figure 11.10
Data points are insolation vs. latitude from Fig. 11.9b. Eq. 11.5 approximates this insolation E_{insol} (thick black line). E_{in} is the solar radiation that is absorbed (thin solid line, from eq. 11.6). E_{out} is outgoing terrestrial (IR) radiation (dashed; from eq. 11.7). Net flux $E_{net} = E_{in} - E_{out}$. Positive E_{net} causes heating; negative causes cooling.

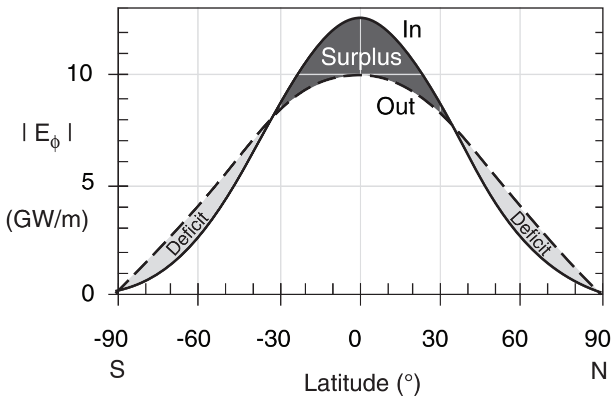


Figure 11.11
Zonally-integrated radiative forcings for absorbed incoming solar radiation (solid line) and emitted net outgoing terrestrial (IR) radiation (dashed line). The surplus balances the deficit.

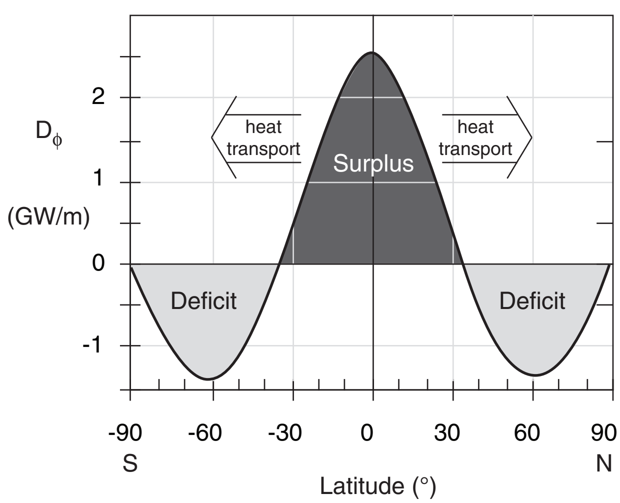


Figure 11.12
Difference between incoming and outgoing zonally-integrated radiative forcings on the Earth. Surplus balances deficits. This differential heating imposed on the Earth must be compensated by heat transport by the global circulation; otherwise, the tropics would keep getting hotter and the polar regions colder.

Solved Example
From the previous solved example, find the zonally-integrated differential heating at $\phi = 48.859°$.

Solution
Given: $E_{net} = -42.3$ W/m^2 , $\phi = 48.859°$ at Eiffel Tower
Find: $D_\phi = ?$ GW/m

Combine eqs. (11.8-11.10): $D_\phi = 2\pi \cdot R_{Earth} \cdot \cos(\phi) \cdot E_{net}$
$= 2(3.14159)\cdot(6.357\text{x}10^6\text{m})\cdot\cos(48.859°)\cdot(-42.3 \text{ W/m}^2)$
$= -1.11\text{x}10^9$ W/m = **-1.11 GW/m**

Check: Units OK. Agrees with Fig. 11.12.
Discussion: Net radiative heat loss at this latitude is compensated by warm Gulf stream and warm winds.

Radiative Forcing by Latitude Belt

Eq. (11.8) and Fig. 11.10 can be deceiving, because latitude belts have shorter circumference near the poles than near the equator. Namely, there are fewer square meters near the poles that experience the net deficit than there are near the equator that experience the net surplus.

To compensate for the shrinking latitude belts, multiply the radiative flux by the circumference of the belt $2\pi \cdot R_{Earth} \cdot \cos(\phi)$, to give a more appropriate zonally-integrated (i.e., summed over all x around the latitude circle) measure of radiative forcing E_ϕ vs. latitude ϕ:

$$E_\phi = 2\pi \cdot R_{Earth} \cdot \cos(\phi) \cdot E \qquad (11.9)$$

where E is the magnitude of outgoing or incoming radiative flux from eqs. (11.7 or 11.6), and R_{Earth} = 6371 km is the average Earth radius. These E_ϕ values are plotted in Fig. 11.11 in units of GW/m.

The difference D_ϕ between incoming and outgoing values of E_ϕ is plotted in Fig. 11.12.

$$D_\phi = E_{\phi\ in} - E_{\phi\ out} \qquad (11.10)$$

To interpret this curve, picture a sidewalk built around the world along a parallel. If this sidewalk is 1 m wide, then Fig. 11.12 gives the number of gigawatts of net radiative power absorbed by the sidewalk, for sidewalks at different latitudes.

This curve shows the radiative differential heating. The areas under the positive and negative portions of the curve in Fig. 11.12 balance, leaving the Earth in overall equilibrium (neglecting global warming for now).

Heat Transport by the Global Circulation

The radiative imbalance between equator and poles in Fig. 11.12 drives atmospheric and oceanic circulations. These circulations act to undo the imbalance by removing the excess heat from the equator and depositing it near the poles (as per Le Chatelier's Principle). First, we can use the radiative differential heating to find how much global-circulation heat transport is needed. Then, we can examine the actual heat transport by atmospheric and oceanic circulations.

Transport Needed

The net meridional transport is zero at the poles by definition. Starting at the north pole and summing D_ϕ over all the "sidewalks" to any latitude ϕ of interest gives the total transport Tr needed for the global circulation to compensate all the radiative imbalances north of that latitude:

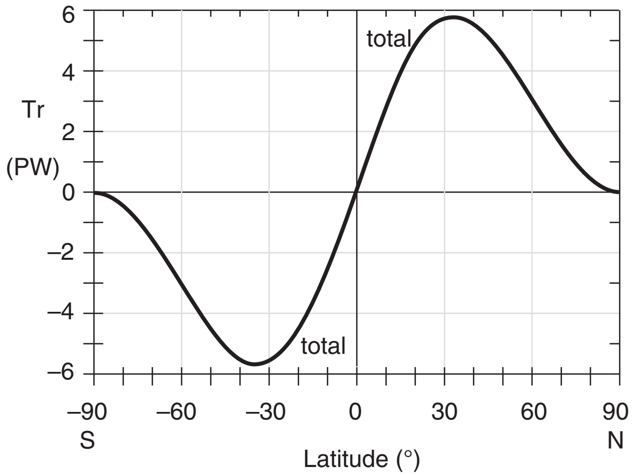


Figure 11.13
Needed heat transport Tr by the global circulation to compensate radiative differential heating, based on a simple "toy model". Agrees very well with achieved transport in Fig. 11.14.

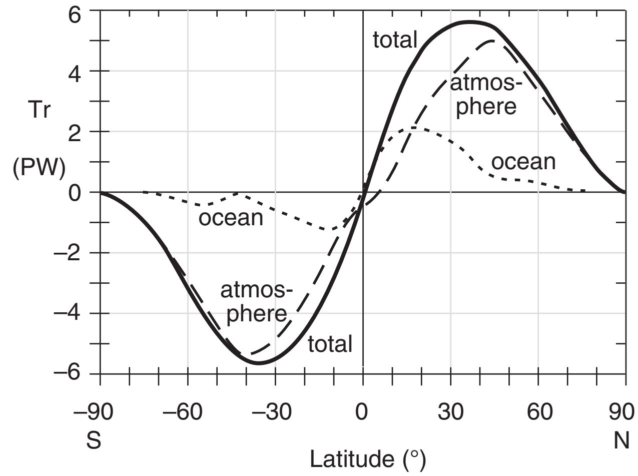


Figure 11.14
*Meridional heat transports: Satellite-observed total (solid line) & ocean estimates (dotted). Atmospheric (dashed) is found as a residual. 1 PW = 1 petaWatt = 10^{15} W. [Data from K. E. Trenberth and J. M. Caron, 2001: "J. Climate", **14**, 3433-3443.]*

$$Tr(\phi) = \sum_{\phi_o = 90°}^{\phi} (-D_\phi) \cdot \Delta y \qquad (11.11)$$

where Δy is the width of the sidewalk.

[Meridional distance Δy is related to latitude change $\Delta\phi$ by: Δy(km) = (111 km/°) · $\Delta\phi$ (°) .]

The resulting "needed transport" is shown in Fig. 11.13, based on the simple "toy model" temperature and radiation curves of the past few sections. The magnitude of this curve peaks at about 5.6 PW (1 petaWatt = 10^{15} W) at latitudes of about 35° North and South (positive *Tr* means northward transport).

Transport Achieved

Satellite observations of radiation to and from the Earth, estimates of heat fluxes to/from the ocean based on satellite observations of sea-surface temperature, and in-situ measurements of the atmosphere provide some of the transport data needed. Numerical forecast models are then used to tie the observations together and fill in the missing pieces. The resulting estimate of heat transport achieved by the atmosphere and ocean is plotted in Fig. 11.14.

Ocean currents dominate the total heat transport only at latitudes 0 to 17°N, and remain important up to latitudes of ± 40°. In the atmosphere, the Hadley circulation is a dominant contributor in the tropics and subtropics, while the Rossby waves dominate atmospheric transport at mid-latitudes.

Knowing that global circulations undo the heating imbalance raises another question. How does the differential heating in the atmosphere drive the winds in those circulations? That is the subject of the next three sections.

Solved Example

What total heat transports by the atmosphere and ocean circulations are needed at 50°N latitude to compensate for all the net radiative cooling between that latitude and the North Pole? The differential heating as a function of latitude is given in the following table (based on the toy model):

Lat (°)	D_ϕ (GW/m)	Lat (°)	D_ϕ (GW/m)
90	0	65	–1.380
85	–0.396	60	–1.403
80	–0.755	55	–1.331
75	–1.049	50	–1.164
70	–1.261	45	–0.905

Solution

Given: ϕ = 50°N. D_ϕ data in table above.
Find: *Tr* = ? PW

Use eq. (11.11). Use sidewalks (belts) each of width $\Delta\phi$ = 5°. Thus, Δy (m) = (111,000 m/°) · (5°) = 555,000 m is the sidewalk width. If one sidewalk spans 85 - 90°, and the next spans 80 - 85° etc, then the values in the table above give D_ϕ along the edges of the sidewalk, not along the middle. A better approximation is to average the D_ϕ values from each edge to get a value representative of the whole sidewalk. Using a bit of algebra, this works out to:

Tr = – (555000 m)· [(0.5)·0.0 – 0.396 – 0.755 – 1.049 – 1.261 – 1.38 – 1.403 – 1.331 – (0.5)·1.164] (GW/m)

Tr = (555000 m)·[8.157 GW/m] = **4.527 PW**

Check: Units OK (10^6 GW = 1 PW). Agrees with Fig. 11.14. **Discussion**: This northward heat transport warms all latitudes north of 50°N, not just one sidewalk. The warming per sidewalk is $\Delta Tr/\Delta\phi$.

ON DOING SCIENCE • Residuals

If something you cannot measure contributes to things you can measure, then you can estimate the unknown as the **residual** (i.e., difference) from all the knowns. This is a valid scientific approach. It was used in Fig. 11.14 to estimate the atmospheric portion of global heat transport.

CAUTION: When using this approach, your residual not only includes the desired signal, but it also includes the sums of all the errors from the items you measured. These errors can easily accumulate to cause a "noise" that is larger than the signal you are trying to estimate. For this reason, **error estimation** and **error propagation** (see Appendix A) should always be done when using the method of residuals.

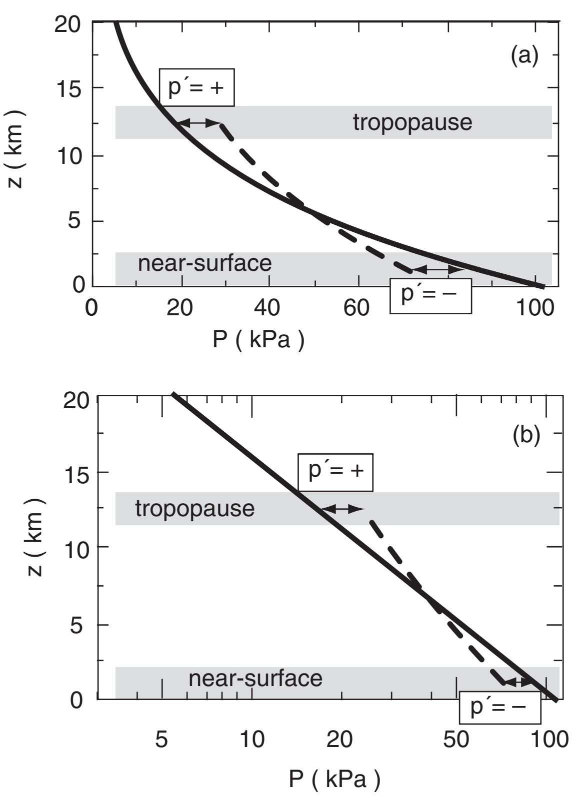


Figure 11.15
Background hydrostatic pressure (solid line), and non-hydrostatic column of air (dashed line) with pressure perturbation p′ that deviates from the hydrostatic pressure $P_{hydrostatic}$ at most heights z. In this example, even though p′ is positive (+) near the tropopause, the total pressure in the column ($P_{column} = P_{hydrostatic} + p'$) at the tropopause is still less than the surface pressure. The same curve is plotted as (a) linear and as (b) semilog.

PRESSURE PROFILES

The following fundamental concepts can help you understand how the global circulation works:

- non-hydrostatic pressure couplets due to horizontal winds and vertical buoyancy,
- hydrostatic thermal circulations,
- geostrophic adjustment, and
- the thermal wind.

The first two concepts are discussed in this section. The last two are discussed in subsequent sections.

Non-hydrostatic Pressure Couplets

Consider a background reference environment with no vertical acceleration (i.e., **hydrostatic**). Namely, the pressure-decrease with height causes an upward pressure-gradient force that exactly balances the downward pull of gravity, causing zero net vertical force on the air (see Fig. 1.12 and eq. 1.25).

Next, suppose that immersed in this environment is a column of air that might experience a different pressure decrease (Fig. 11.15); i.e., **non-hydrostatic pressures**. At any height, let $p' = P_{column} - P_{hydrostatic}$ be the deviation of the actual pressure in the column from the theoretical hydrostatic pressure in the environment. Often a positive p' in one part of the atmospheric column is associated with negative p' elsewhere. Taken together, the positive and negative p's form a **pressure couplet**.

Non-hydrostatic p' profiles are often associated with non-hydrostatic vertical motions through Newton's second law. These non-hydrostatic motions can be driven by horizontal convergence and divergence, or by buoyancy (a vertical force). These two effects create opposite pressure couplets, even though both can be associated with upward motion, as explained next.

Horizontal Convergence/Divergence

If external forcings cause air near the ground to converge horizontally, then air molecules accumulate. As density ρ increases according to eq. (10.60), the ideal gas law tells us that p' will also become positive (Fig. 11.16a).

Positive p' does two things: it (1) decelerates the air that was converging horizontally, and (2) accelerates air vertically in the column. Thus, the pressure perturbation causes **mass continuity** (horizontal inflow near the ground balances vertical outflow).

Similarly, an externally imposed horizontal divergence at the top of the troposphere would lower the air density and cause negative p', which would also accelerate air in the column upward. Hence, we expect upward motion (W = positive) to be driven by a p' couplet, as shown in Fig. 11.16a.

Buoyant Forcings

For a different scenario, suppose air in a column is positively buoyant, such as in a thunderstorm where water-vapor condensation releases lots of latent heat. This vertical buoyant force creates upward motion (i.e., warm air rises, as in Fig. 11.16b).

As air in the thunderstorm column moves away from the ground, it removes air molecules and lowers the density and the pressure; hence, p' is negative near the ground. This suction under the updraft causes air near the ground to horizontally converge, thereby conserving mass.

Conversely, at the top of the troposphere where the thunderstorm updraft encounters the even warmer environmental air in the stratosphere, the upward motion rapidly decelerates, causing air molecules to accumulate, making p' positive. This pressure perturbation drives air to diverge horizontally near the tropopause, causing the outflow in the anvil-shaped tops of thunderstorms.

The resulting pressure-perturbation p' couplet in Fig. 11.16b is opposite that in Fig. 11.16a, yet both are associated with upward vertical motion. The reason for this pressure-couplet difference is the difference in driving mechanism: imposed horizontal convergence/divergence vs. imposed vertical buoyancy.

Similar arguments can be made for downward motions. For either upward or downward motions, the pressure couplets that form depend on the type of forcing. We will use this process to help explain the pressure patterns at the top and bottom of the troposphere.

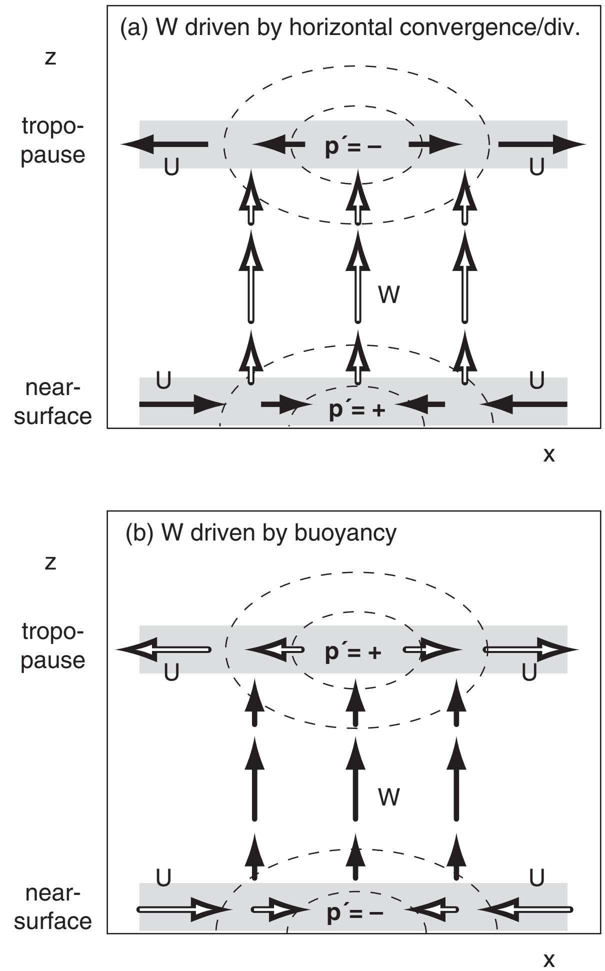


Figure 11.16
(a) Vertical motions driven by horizontal convergence or divergence. (b) Vertical motions driven by buoyancy. In both figures, black arrows indicate the cause (the driving force), and white arrows are the effect (the response). p′ is the pressure perturbation (deviation from hydrostatic), and thin dashed lines are isobars of p′. U and W are horizontal and vertical velocities. In both figures, the responding flow (white arrows) is driven down the pressure-perturbation gradient, away from positive p′ and toward negative p′.

Hydrostatic Thermal Circulations

Cold columns of air tend to have high surface pressures, while warm columns have low surface pressures. Figs. 11.17 illustrate how this happens.

Consider initial conditions (Fig. 11.17i) of two equal columns of air at the same temperature and with the same number of air molecules in each column. Since pressure is related to the mass of air above, this means that both columns A and B have the same initial pressure (100 kPa) at the surface.

Next, suppose that some process heats one column relative to the other (Fig. 11.17ii). Perhaps condensation in a thunderstorm cloud causes latent heating of column B, or infrared radiation cools column A. After both columns have finished expanding or contracting due to the temperature change, they will reach new hydrostatic equilibria for their respective temperatures.

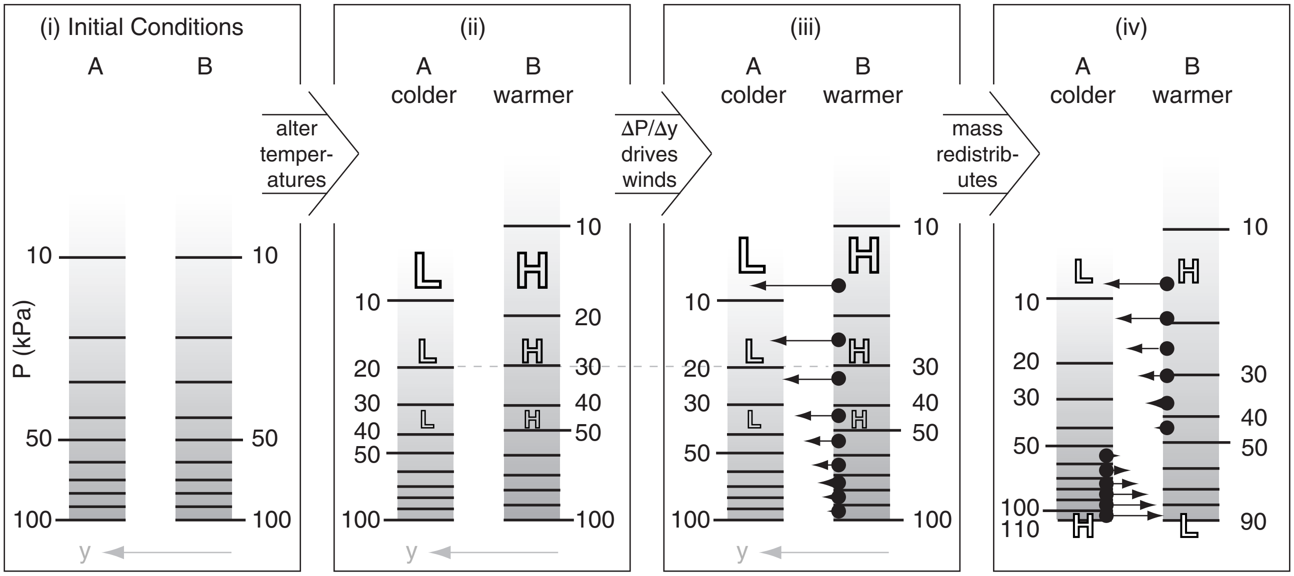


Figure 11.17
Formation of a thermal circulation. The response of two columns of air that are heated differently is that the warmer air column develops a low pressure perturbation at the surface and a high pressure perturbation aloft. Response of the cold column is the opposite. Notation: H = high pressure perturbation, L = low; black dots represents air parcels; thin arrows are winds.

The hypsometric equation (see Chapter 1) says that pressure decreases more rapidly with height in cold air than in warm air. Thus, although both columns have the same surface pressure because they contain the same number of molecules, the higher you go above the surface, the greater is the pressure difference between warm and cold air. In Fig. 11.17ii, the printed size of the "H" and "L" indicate the relative magnitudes of the high- and low-pressure perturbations p'.

The horizontal pressure gradient $\Delta P/\Delta y$ aloft between the warm and cold air columns drives horizontal winds from high toward low pressure (Fig. 11.17iii). Since winds are the movement of air molecules, this means that molecules leave the regions of high pressure-perturbation and accumulate in the regions of low. Namely, they leave the warm column, and move into the cold column.

Since there are now more molecules (i.e., more mass) in the cold column, it means that the surface pressure must be greater in the cold column (Fig. 11.17iv). Similarly, mass lost from the warm column results in lower surface pressure. This is called a **thermal low.**

A result (Fig. 11.17iv) is that, near the surface, high pressure in the cold air drives winds toward the low pressure in warm air. Aloft, high pressure-perturbation in the warm air drives winds towards low pressure-perturbation in the cold air. The resulting **thermal circulation** causes each column to gain as many air molecules as they lose; hence, they are in mass equilibrium.

This equilibrium circulation also transports heat. Air from the warm air column mixes into the cold column, and vice versa. This intermixing reduces the temperature contrast between the two columns, causing the corresponding equilibrium circulations to weaken. Continued destabilization (more latent heating or radiative cooling) is needed to maintain the circulation.

The circulations and mass exchange described above can be realized at the equator. At other latitudes, the exchange of mass is often slower (near the surface) or incomplete (aloft) because Coriolis force turns the winds to some angle away from the pressure-gradient direction. This added complication, due to geostrophic wind and geostrophic adjustment, is described next.

GEOSTROPHIC WIND & GEOSTROPHIC ADJUSTMENT

Ageostrophic Winds at the Equator

Air at the equator can move directly from high (H) to low (L) pressure (Fig. 11.18 - center part) under the influence of pressure-gradient force. Zero Coriolis force at the equator implies infinite geostrophic winds. But actual winds have finite speed, and are thus **ageostrophic** (not geostrophic).

Because such flows can happen very easily and quickly, equatorial air tends to quickly flow out of highs into lows, causing the pressure centers to neutralize each other. Indeed, weather maps at the equator show very little pressure variations zonally. One exception is at continent-ocean boundaries, where continental-scale differential heating can continually regenerate pressure gradients to compensate the pressure-equalizing action of the wind. Thus, very small pressure gradients can cause continental-scale (5000 km) monsoon circulations near the equator. Tropical forecasters focus on winds, not pressure.

If the large-scale pressure is uniform in the horizontal near the equator (away from monsoon circulations), then the horizontal pressure gradients disappear. With no horizontal pressure-gradient force, no large-scale winds can be driven there. However, winds can exist at the equator due to inertia — if the winds were first created geostrophically at nonzero latitude and then coast across the equator.

But at most other places on Earth, Coriolis force deflects the air and causes the wind to approach geostrophic or gradient values (see the Dynamics chapter). Geostrophic winds do not cross isobars, so they cannot transfer mass from highs to lows. Thus, significant pressure patterns (e.g., strong high and low centers, Fig. 11.18) can be maintained for long periods at mid-latitudes in the global circulation.

Definitions

The spatial distribution of wind speeds and directions is known as the **wind field**. Similarly, the spatial distribution of temperature is called the **temperature field**.

The **mass field** is the spatial distribution of air mass. As discussed in Chapter 1, pressure is a measure of the mass of air above. Thus, the term "mass field" generically means the spatial distribution of pressure (i.e., the **pressure field**) on a constant altitude chart, or of heights (the **height field**) on an isobaric surface.

In **baroclinic conditions** (where temperature changes in the horizontal), the hypsometric equation allows changes to the pressure field to be described

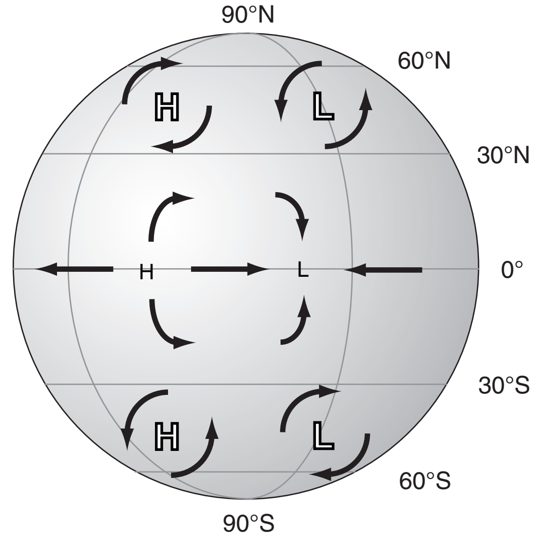


Figure 11.18
At the equator, winds flow directly from high (H) to low (L) pressure centers. At other latitudes, Coriolis force causes the winds to circulate around highs and lows. Smaller size font for H and L at the equator indicate weaker pressure gradients.

ON DOING SCIENCE • The Scientific Method Revisited

"Like other exploratory processes, [the scientific method] can be resolved into a dialogue between fact and fancy, the actual and the possible; between what could be true and what is in fact the case. The purpose of scientific enquiry is not to compile an inventory of factual information, nor to build up a totalitarian world picture of Natural Laws in which every event that is not compulsory is forbidden. We should think of it rather as a logically articulated structure of justifiable beliefs about a Possible World — a story which we invent and criticize and modify as we go along, so that it ends by being, as nearly as we can make it, a story about real life."

- by Nobel Laureate Sir Peter Medawar (1982) *Pluto's Republic*. Oxford Univ. Press.

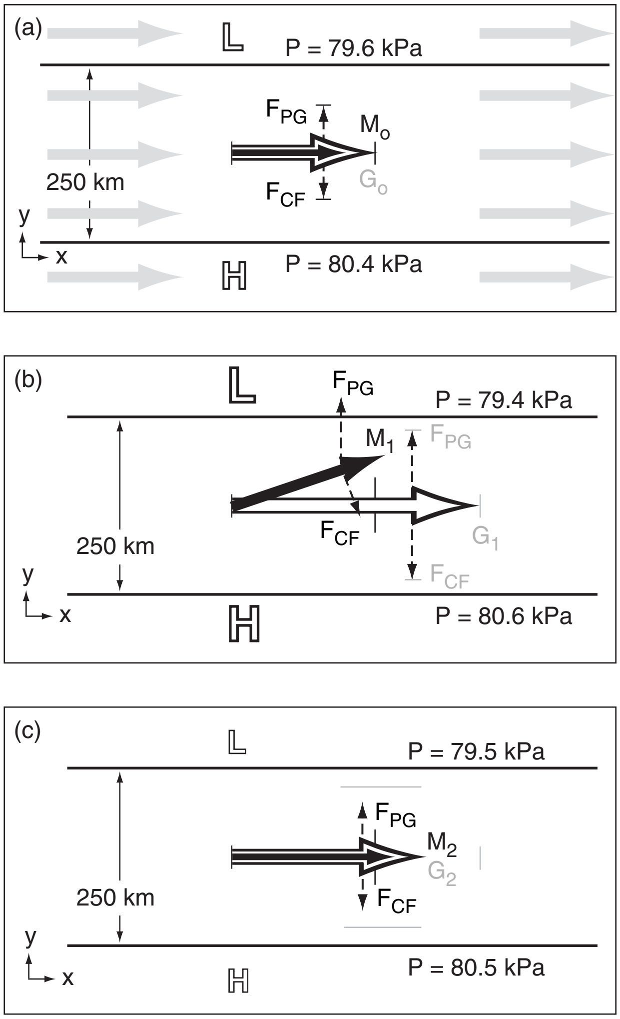


Figure 11.19
Example of geostrophic adjustment in the N. Hemisphere (not at equator). (a) Initial conditions, with the actual wind M (thick black arrow) in equilibrium with (equal to) the theoretical geostrophic value G (white arrow with black outline). (b) Transition. (c) End result at a new equilibrium. Dashed lines indicate forces F. Each frame focuses on the region of disturbance.

Solved Example
Find the internal Rossby radius of deformation in a standard atmosphere at 45°N.

Solution
Given: ϕ = 45°. Standard atmosphere from Chapter 1:
$T(z = Z_T = 11$ km$) = -56.5$°C, $T(z=0) = 15$°C.
Find: λ_R = ? km

First, find $f_c = (1.458\text{x}10^{-4}\ \text{s}^{-1})\cdot\sin(45°) = 1.031\text{x}10^{-4}\ \text{s}^{-1}$
Next, find the average temperature and temperature difference across the depth of the troposphere:
$T_{avg} = 0.5\cdot(-56.5 + 15.0)$°C $= -20.8$°C $= 252$ K
$\Delta T = (-56.5 - 15.0)$ °C $= -71.5$°C across $\Delta z = 11$ km
(continued on next page)

by changes in the temperature field. Similarly, the thermal wind relationship (described later in this chapter) relates changes in the wind field to changes in the temperature field.

Geostrophic Adjustment - Part 1

The tendency of <u>non-equatorial</u> winds to approach geostrophic values (or gradient values for curved isobars) is a very strong process in the Earth's atmosphere. If the actual winds are not in geostrophic balance with the pressure patterns, then both the winds and the pressure patterns tend to change to bring the winds back to geostrophic (another example of Le Chatelier's Principle). This process is called **geostrophic adjustment**.

Picture a wind field (grey arrows in Fig. 11.19a) initially in geostrophic equilibrium ($M_o = G_o$) at altitude 2 km above sea level (thus, no drag at ground). We will focus on just one of those arrows (the black arrow in the center), but all the wind vectors will march together, performing the same maneuvers.

Next, suppose an external process increases the horizontal pressure gradient to the value shown in Fig. 11.19b, with the associated faster geostrophic wind speed G_1. With pressure-gradient force F_{PG} greater than Coriolis force F_{CF}, the force imbalance turns the wind M_1 slightly toward low pressure and accelerates the air (Fig. 11.19b).

The component of wind M_1 from high to low redistributes air mass (moves air molecules) in the horizontal, weakening the pressure field and thereby reducing the theoretical geostrophic wind (G_2 in Fig. 11.19c). Thus, **the mass field adjusts to the wind field**. Simultaneously the actual wind accelerates to M_2. Thus, **the wind field adjusts to the mass field**. After both fields have adjusted, the result is $M_2 > M_o$ and $G_2 < G_1$, with $M_2 = G_2$.

These adjustments are strongest near the disturbance (i.e., the region forced out of equilibrium), and gradually weaken with distance. The e-folding distance, beyond which the disturbance is felt only a little, is called the **internal Rossby radius of deformation**, λ_R:

$$\lambda_R = \frac{N_{BV}\cdot Z_T}{f_c} \qquad \bullet(11.12)$$

where f_c is the Coriolis parameter, Z_T is the depth of the troposphere, and N_{BV} is the Brunt-Väisälä frequency. This radius relates buoyant and inertial forcings. It is on the order of 1300 km.

For large-scale disturbances (wavelength $\lambda > \lambda_R$), most of the adjustment is in the wind field. For small-scale disturbances ($\lambda < \lambda_R$), most of the adjustment is in the temperature or pressure fields. For mesoscales, all fields adjust a medium amount.

Solved Example *(continuation)*
Find the Brunt-Väisälä frequency (see the Stability chapter).

$$N_{BV} = \sqrt{\frac{(9.8\text{m/s})}{252\text{K}}\left[\frac{-71.5\text{K}}{11000\text{m}} + 0.0098\frac{\text{K}}{\text{m}}\right]} = 0.0113\text{s}^{-1}$$

where the temperature differences in square brackets can be expressed in either °C or Kelvin.

Finally, use eq. (11.12): λ_R = (0.0113 s^{-1})·(11 km) / (1.031x10^{-4} s^{-1}) = **1206 km**

Check: Units OK. Magnitude OK. Physics OK.
Discussion: When a cold-front over the Pacific approaches the steep mountains of western Canada, the front feels the influence of the mountains 1200 to 1300 km before reaching the coast, and begins to slow down.

THERMAL WIND RELATIONSHIP

Recall that horizontal temperature gradients cause vertically varying horizontal pressure gradients (Fig. 11.17), and that horizontal pressure gradients drive geostrophic winds. We can combine those concepts to see how horizontal temperature gradients drive vertically varying geostrophic winds. This is called the **thermal wind effect**.

This effect can be pictured via the slopes of isobaric surfaces (Fig. 11.20). If a horizontal temperature gradient is present, it changes the tilt of pressure surfaces with increasing altitude because the thickness between isobaric surfaces is greater in warmer air (see eq. 1.26, the hypsometric equation).

But geostrophic wind (U_g, V_g) is proportional to the tilt of the pressure surfaces (eq. 10.29). Thus, as shown in the Focus box on the next page, the **thermal wind relationship** is:

$$\frac{\Delta U_g}{\Delta z} \approx \frac{-|g|}{T_v \cdot f_c} \cdot \frac{\Delta T_v}{\Delta y} \qquad •(11.13a)$$

$$\frac{\Delta V_g}{\Delta z} \approx \frac{|g|}{T_v \cdot f_c} \cdot \frac{\Delta T_v}{\Delta x} \qquad •(11.13b)$$

where $|g|$ = 9.8 m·s^{-2} is gravitational acceleration magnitude, T_v is the virtual temperature (in Kelvins, and nearly equal to the actual temperature if the air is fairly dry), and f_c is the Coriolis parameter. The north-south temperature gradient alters the east-west geostrophic winds with height, and vice versa.

Because the atmosphere above the boundary layer is nearly in geostrophic equilibrium, the change of actual wind speed with height is nearly equal to the change of the geostrophic wind.

Thickness

The thickness between two different pressure surfaces is a measure of the average virtual temperature within that layer. For example, consider the two different pressure surfaces colored white in Fig.

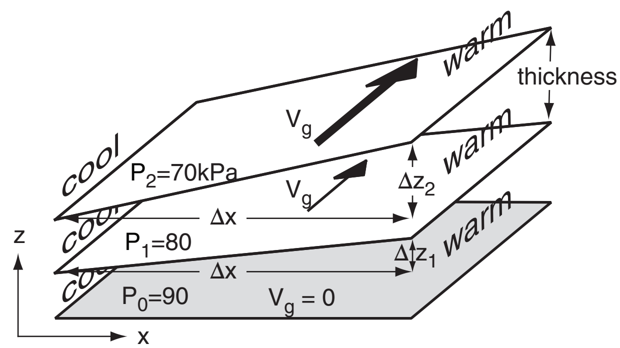


Figure 11.20
The three planes are surfaces of constant pressure (i.e., isobaric surfaces) in the N. Hemisphere. A horizontal temperature gradient tilts the pressure surfaces and causes the geostrophic wind to increase with height. Geostrophic winds are reversed in S. Hemisphere.

Solved Example
Temperature increases from 8°C to 12°C toward the east, across 100 km distance (Fig. 11.20). Find the vertical gradient of geostrophic wind, given f_c = 10^{-4} s^{-1}.

Solution
Assume: Dry air. Thus, $T_v = T$
Given: ΔT = 12 – 8°C = 4°C, Δx = 100 km,
T = 0.5·(8+12°C) = 10°C = 283 K, f_c = 10^{-4} s^{-1}.
Find: $\Delta U_g/\Delta z$ & $\Delta V_g/\Delta z$ = ? (m/s)/km

Use eq. (11.13a):

$$\frac{\Delta U_g}{\Delta z} \approx \frac{-(9.8\text{m/s}^2)}{(283K)\cdot(10^{-4}\text{s}^{-1})}\cdot(0°\text{C/km}) = 0 \text{ (m/s)/km}$$

Thus, U_g **= uniform with height**.
Use eq. (11.13b):

$$\frac{\Delta V_g}{\Delta z} \approx \frac{(9.8\text{m/s}^2)}{(283K)\cdot(10^{-4}\text{s}^{-1})}\cdot\frac{(4°\text{C})}{(100\text{km})}$$

= **13.9 (m/s)/km**

Check: Units OK. Physics OK. Agrees with Fig.
Discussion: Each kilometer gain in altitude gives a 13.9 m/s increase in northward geostrophic wind speed. For example, if the wind at the surface is –3.9 m/s (i.e., light from the north), then the wind at 1 km altitude is 10 m/s (strong from the south).

BEYOND ALGEBRA • Thermal Wind Effect

Problem: Derive Thermal Wind eq. (11.13a).

Solution: Start with the definitions of geostrophic wind (10.26a) and hydrostatic balance (1.25b):

$$U_g = -\frac{1}{\rho \cdot f_c}\frac{\partial P}{\partial y} \quad \text{and} \quad \rho \cdot |g| = -\frac{\partial P}{\partial z}$$

Replace the density in both eqs using the ideal gas law (1.20). Thus:

$$\frac{U_g \cdot f_c}{T_v} = -\frac{\Re_d}{P}\frac{\partial P}{\partial y} \quad \text{and} \quad \frac{|g|}{T_v} = -\frac{\Re_d}{P}\frac{\partial P}{\partial z}$$

Use $(1/P)\cdot\partial P = \partial \ln(P)$ from calculus to rewrite both:

$$\frac{U_g \cdot f_c}{T_v} = -\Re_d \frac{\partial \ln(P)}{\partial y} \quad \text{and} \quad \frac{|g|}{T_v} = -\Re_d \frac{\partial \ln(P)}{\partial z}$$

Differentiate the left eq. with respect to z:

$$\frac{\partial}{\partial z}\left(\frac{U_g \cdot f_c}{T_v}\right) = -\Re_d \frac{\partial \ln(P)}{\partial y\, \partial z}$$

and the right eq. with respect to y:

$$\frac{\partial}{\partial y}\left(\frac{|g|}{T_v}\right) = -\Re_d \frac{\partial \ln(P)}{\partial y\, \partial z}$$

But the right side of both eqs are identical, thus we can equate the left sides to each other:

$$\frac{\partial}{\partial z}\left(\frac{U_g \cdot f_c}{T_v}\right) = \frac{\partial}{\partial y}\left(\frac{|g|}{T_v}\right)$$

Next, do the indicated differentiations, and rearrange to get the exact relationship for thermal wind:

$$\boxed{\frac{\partial U_g}{\partial z} = -\frac{|g|}{T_v \cdot f_c}\frac{\partial T_v}{\partial y} + \frac{U_g}{T_v}\left(\frac{\partial T_v}{\partial z}\right)}$$

The last term depends on the geostrophic wind speed and the lapse rate, and has magnitude of 0 to 30% of the first term on the right. If we neglect the last term, we get the approximate thermal wind relationship:

$$\boxed{\frac{\partial U_g}{\partial z} \approx -\frac{|g|}{T_v \cdot f_c}\frac{\partial T_v}{\partial y}} \qquad (11.13a)$$

Discussion: A **barotropic** atmosphere is when the geostrophic wind does not vary with height. Using the exact equation above, we see that this is possible only when the two terms on the right balance.

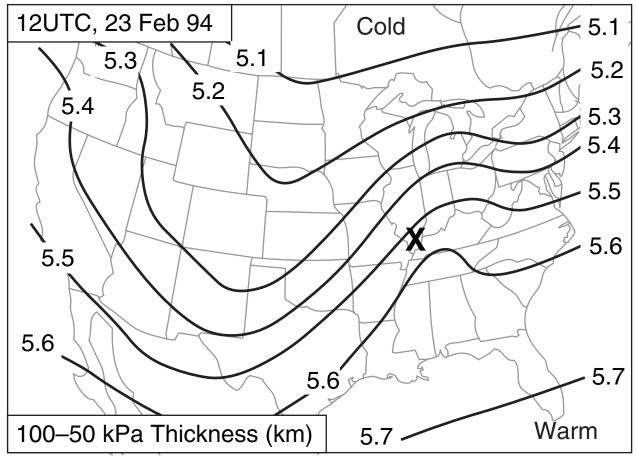


Figure 11.21
100–50 kPa thickness (km), valid at 12 UTC on 23 Feb 94. X marks the location of a surface low-pressure center.

11.20. The air is warmer in the east than in the west. Thus, the thickness of that layer is greater in the east than in the west.

Maps of thickness are used in weather forecasting. One example is the "100 to 50 kPa thickness" chart, such as shown in Fig. 11.21. This is a map with contours showing the thickness between the 100 kPa and 50 kPa isobaric surfaces. For such a map, regions of low thickness correspond to regions of cold temperature, and vice versa. It is a good indication of average temperature in the bottom half of the troposphere, and is useful for identifying airmasses and fronts (discussed in the next chapter).

Define **thickness** TH as

$$TH = z_{P2} - z_{P1} \qquad \bullet(11.14)$$

where z_{P2} and z_{P1} are the heights of the P_2 and P_1 isobaric surfaces.

Thermal Wind

The thermal wind relationship can be applied over the same layer of air bounded by isobaric surfaces as was used to define thickness. By manipulating eqs. (10.29), we find:

$$U_{TH} = U_{G2} - U_{G1} = -\frac{|g|}{f_c}\frac{\Delta TH}{\Delta y} \qquad \bullet(11.15a)$$

$$V_{TH} = V_{G2} - V_{G1} = +\frac{|g|}{f_c}\frac{\Delta TH}{\Delta x} \qquad \bullet(11.15b)$$

where subscripts $G2$ and $G1$ denote the geostrophic winds on the P_2 and P_1 pressure surfaces, $|g|$ is gravitational-acceleration magnitude, and f_c is the Coriolis parameter.

The variables U_{TH} and V_{TH} are known as the **thermal wind components**. Taken together (U_{TH}, V_{TH}) they represent the vector difference between the geostrophic winds at the top and bottom pressure surfaces. **Thermal wind magnitude** M_{TH} is:

$$M_{TH} = \sqrt{U_{TH}{}^2 + V_{TH}{}^2} \qquad (11.16)$$

In Fig. 11.20, this vector difference happened to be in the same direction as the geostrophic wind. But this is not usually the case, as illustrated in Fig. 11.22.

Given an isobaric surface P_1 (medium grey in Fig. 11.22), with a height contour shown as the dashed line. The geostrophic wind G_1 on the surface is parallel to the height contour, with low heights to its left (N. Hemisphere). Isobaric surface P_2 (for $P_2 < P_1$) is also plotted (light grey). Cold air to the west is associated with a thickness of TH = 5 km between the two pressure surfaces. To the east, warm air has thickness 7 km. These thicknesses are added to the bottom pressure surface, to give the corner altitudes of the upper surface. On that upper surface are shown a height contour (dashed line) and the geostrophic wind vector G_2. We see that $G_2 > G_1$ be-

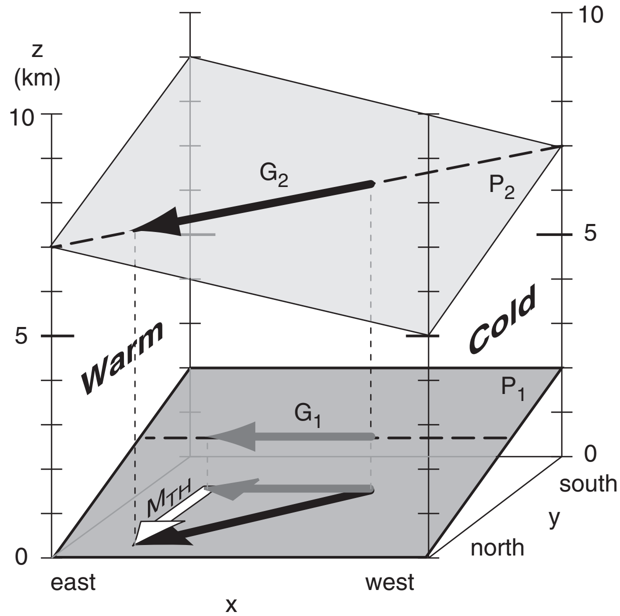


Figure 11.22
Relationship between the thermal wind M_{TH} and the geostrophic winds G on isobaric surfaces P. Viewpoint is from north of the air column.

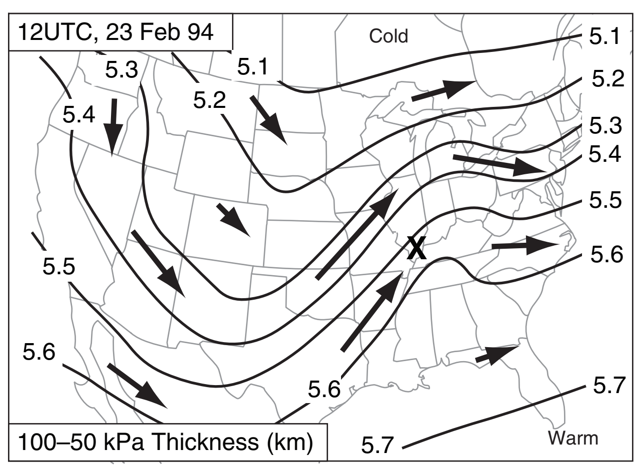


Figure 11.23
100–50 kPa thickness (smooth black curves, in units of km), with thermal wind vectors added. Larger vectors qualitatively denote stronger thermal winds.

Solved Example

Suppose the thickness of the 100 - 70 kPa layer is 2.9 km at one location, and 3.0 km at a site 500 km to the east. Find the components of the thermal wind vector, given $f_c = 10^{-4}$ s^{-1}.

Solution

Assume: No north-south thickness gradient.
Given: TH_1 = 2.9 km, TH_2 = 3.0 km,
Δx = 500 km, $f_c = 10^{-4}$ s^{-1}.
Find: U_{TH} = ? m/s, V_{TH} = ? m/s

Use eq. (11.15a): U_{TH} = **0 m/s**.
Use eq. (11.15b):

$$V_{TH} = \frac{g}{f_c}\cdot\frac{\Delta TH}{\Delta x} = \frac{(9.8\text{ms}^{-2})\cdot(3.0-2.9)\text{km}}{(10^{-4}\text{s}^{-1})\cdot(500\text{km})}$$

= **19.6 m/s**

Check: Units OK. Physics OK. Agrees with Fig. 11.22.
Discussion: There is no east-west thermal wind component because the thickness does not change in the north-south direction. The positive sign of V_{TH} means a wind from south to north, which agrees with the rule that the thermal wind is parallel to the thickness contours with cold air to its left (west, in this example).

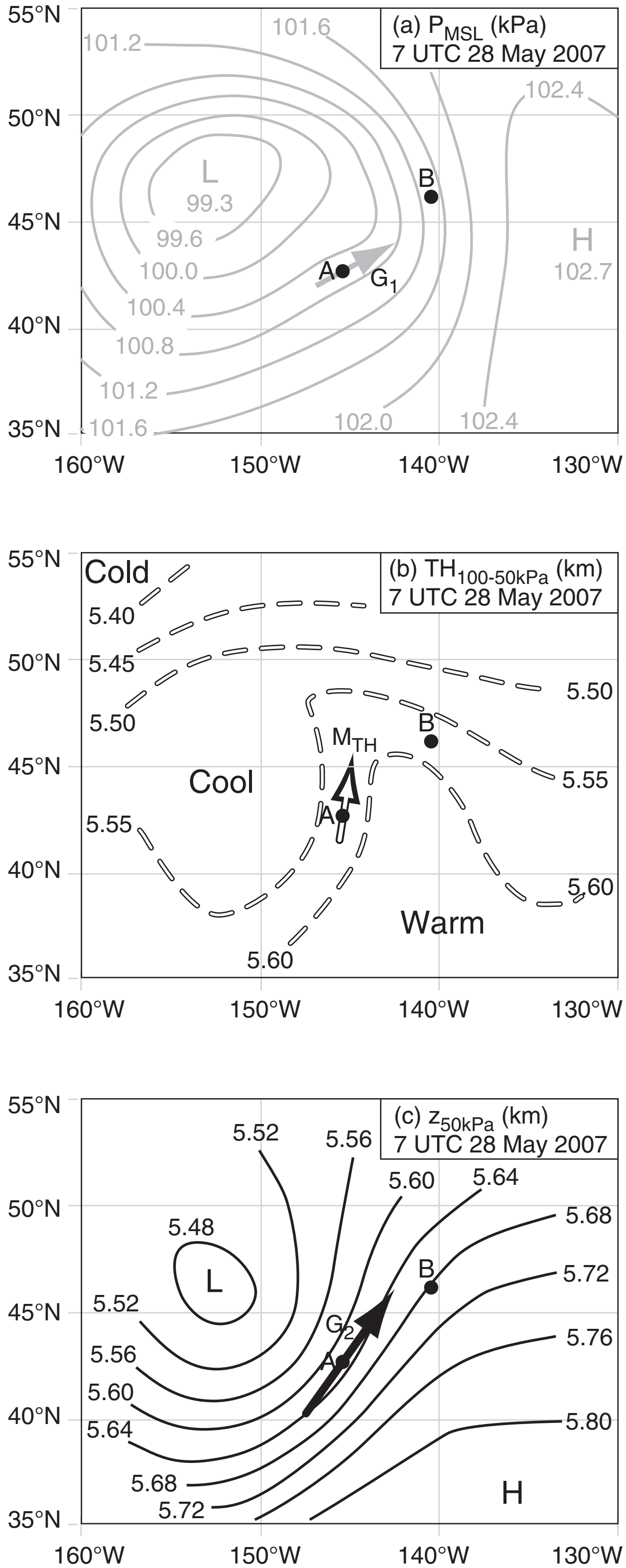


Figure 11.24
Weather maps for a thermal-wind case-study. (a) Mean sea-level pressure (kPa), as a surrogate for height of the 100 kPa surface. (b) Thickness (km) of the layer between 100 kPa to 50 kPa isobaric surfaces. (c) Geopotential heights (km) of the 50 kPa isobaric surface.

cause the top surface has greater slope than the bottom. The thermal wind is parallel to the thickness contours with cold to the left (white arrow). This thermal wind is the vector difference between the geostrophic winds at the two pressure surfaces, as shown in the projection on the ground.

Eqs. (11.15) imply that the thermal wind is parallel to the thickness contours, with cold temperatures (low thickness) to the left in the N. Hemisphere (Fig. 11.23). Closer packing of the thickness lines gives stronger thermal winds because the horizontal temperature gradient is larger there.

Thermal winds on a thickness map behave analogously to geostrophic winds on a constant pressure or height map, making their behavior a bit easier to remember. However, while it is possible for actual winds to equal the geostrophic wind, there is no real wind that equals the thermal wind. The thermal wind is just the vector difference between geostrophic winds at two different heights or pressures.

Case Study

Figs. 11.24 show how geostrophic winds and thermal winds can be found on weather maps, and how to interpret the results. These maps may be copied onto transparencies and overlain.

Fig. 11.24a is a weather map of pressure at sea level in the N. Hemisphere, at a location over the northeast Pacific Ocean. As usual, L and H indicate low- and high-pressure centers. At point A, we can qualitatively draw an arrow (grey) showing the theoretical geostrophic (G_1) wind direction; namely, it is parallel to the isobars with low pressure to its left.

Recall that pressures on a constant height surface (such as at height $z = 0$ at sea level) are closely related to geopotential heights on a constant pressure surface. So we can be confident that a map of 100 kPa heights would look very similar to Fig. 11.24a.

Fig. 11.24b shows the 100 - 50 kPa thickness map, valid at the same time and place. The thickness between the 100 and the 50 kPa isobaric surfaces is about 5.6 km in the warm air, and only 5.4 km in the cold air. The white arrow qualitatively shows the thermal wind M_{TH}, as being parallel to the thickness lines with cold temperatures to its left.

Fig. 11.24c is a weather map of geopotential heights of the 50 kPa isobaric surface. L and H indicate low and high heights. The black arrow at A shows the geostrophic wind (G_2), drawn parallel to the height contours with low heights to its left.

If we wished, we could have calculated quantitative values for G_1, G_2, and M_{TH}, utilizing the scale that 5° of latitude equals 555 km. [*CAUTION: This scale does not apply to longitude, because the meridians get closer together as they approach the poles. However,* *once you have determined the scale (map mm : real km) based on latitude, you can use it to good approximation in any direction on the map.*]

Back to the thermal wind: if you add the geostrophic vector from Fig. 11.24a with the thermal wind vector from Fig. 11.24b, the result should equal the geostrophic wind vector in Fig. 11.24c. This is shown in the solved example.

Although we will study much more about weather maps and fronts in the next few chapters, I will interpret these maps for you now.

Point A on the maps is near a cold front. From the thickness chart, we see cold air west and northwest of point A. Also, knowing that winds rotate counterclockwise around lows in the N. Hemisphere (see Fig. 11.24a), I can anticipate that the cool air will advance toward point A. Hence, this is a region of **cold-air advection**. Associated with this cold-air advection is **backing of the wind** (i.e., turning counterclockwise with increasing height), which we saw was fully explained by the thermal wind.

Point B is near a warm front. I inferred this from the weather maps because warmer air is south of point B (see the thickness chart) and that the counterclockwise winds around lows are causing this warm air to advance toward point B. **Warm air advection** is associated with **veering of the wind** (i.e., turning clockwise with increasing height), again as given by the thermal wind relationship. I will leave it to you to draw the vectors at point B to prove this to yourself.

Solved Example

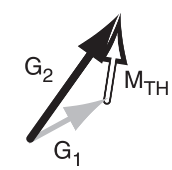
For Fig. 11.24, qualitatively verify that when vector M_{TH} is added to vector G_1, the result is vector G_2.

Solution

Given: the arrows from Fig. 11.24 for point A. These are copied and pasted here.

Find: The vector sum of $G_1 + M_{TH} = ?$

Recall that to do a vector sum, move the tail of the second vector (M_{TH}) to be at the arrow head of the first vector (G_1). The vector sum is then the vector drawn from the tail of the first vector to the tip (head) of the second vector.



Check: Sketch is reasonable.

Discussion: The vector sum indeed equals vector G_2, as predicted by the thermal wind.

Thermal Wind & Geostrophic Adjustment - Part 2

As geostrophic winds adjust to pressure gradients, they move mass to alter the pressure gradients. Eventually, an equilibrium is approached (Fig. 11.25) based on the combined effects of geostrophic adjustment and the thermal wind. This figure is much more realistic than Fig. 11.17 because Coriolis force prevents the winds from flowing directly from high to low pressure.

With these concepts of:

- differential heating,
- nonhydrostatic pressure couplets due to horizontal winds and vertical buoyancy,
- hydrostatic thermal circulations,
- geostrophic adjustment, and
- the thermal wind,

we can now go back and explain why the global circulation works the way it does.

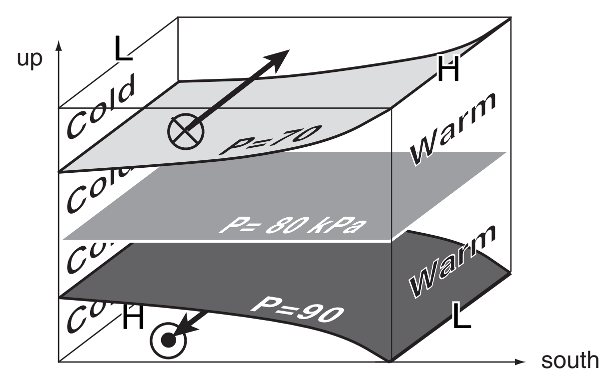


Figure 11.25
Typical equilibrium state after geostrophic adjustment. Isobaric surfaces are shaded, and recall that high heights of isobaric surfaces correspond to regions of high pressure on constant altitude surfaces. Arrows represent geostrophic wind.

Note: High (H) and low (L) pressure labels are relative to the average pressure at the same altitude where the labels are drawn. For example, the pressure at the top right is high relative to the pressure at the top left. Both of these top pressures are lower than both of the surface pressures, because pressure monotonically decreases with increasing height.

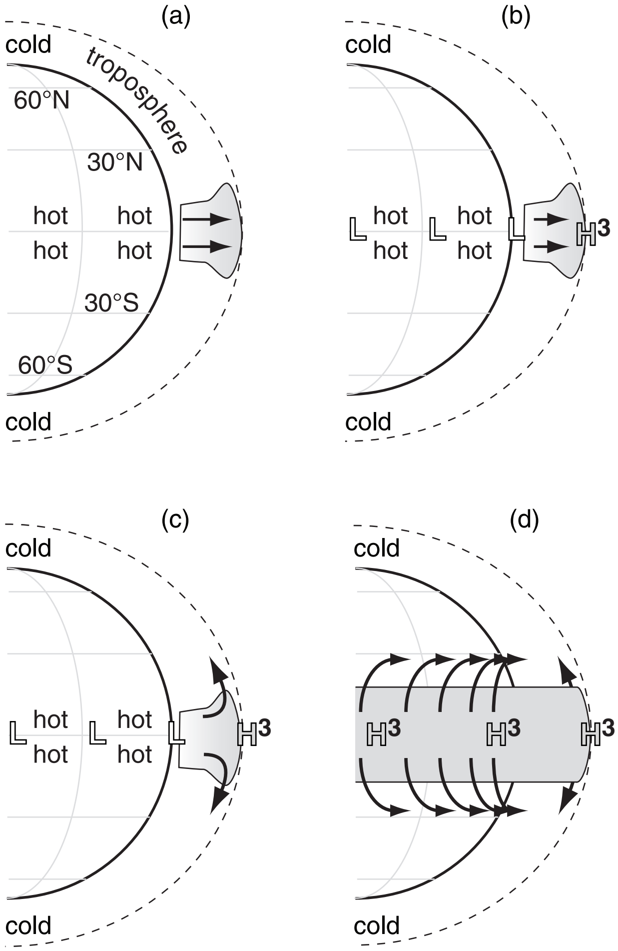


Figure 11.26
Application of physical concepts to explain the general circulation (see text). White-filled H and L indicate surface high and low pressure regions; grey-filled H and L are pressure regions near the tropopause. (continued on next pages)

> **ON DOING SCIENCE • Model Sensitivity**
>
> CAUTION: Whenever you find that a model has high **sensitivity** (i.e., the output result varies by large magnitude for small changes in the input parameter), you should be especially wary of the results. Small errors in the parameter could cause large errors in the result. Also, if the real atmosphere does not share the same sensitivity, then this is a clue that the model is poorly designed, and perhaps a better model should be considered.
>
> Models are used frequently in meteorology — for example: numerical weather prediction models (Chapter 20) or climate-change models (Chapter 21). Most researchers who utilize models will perform careful **sensitivity studies** (i.e., compute the output results for a wide range of parameter values) to help them gauge the potential weaknesses of the model.

EXPLAINING THE GLOBAL CIRCULATION

Low Latitudes

Differential heating of the Earth's surface warms the tropics and cools the poles (Fig. 11.26a). The warm air near the equator can hold large amounts of water vapor evaporated from the oceans. Buoyancy force causes the hot humid air to rise over the equator. As the air rises, it cools and water vapor condenses, causing a belt of thick thunderstorm clouds around the equator (Fig. 11.26a) with heavy tropical precipitation.

The buoyantly forced vertical motion removes air molecules from the lower troposphere in the tropics, and deposits the air near the top of the troposphere. The result is a pressure couplet (Fig. 11.26b) of very high perturbation pressure p' (indicated with HHH or H^3 on the figures) near the tropopause, and low perturbation pressure (L in the figures) at the surface.

The belt of tropical high pressure near the tropopause forces air to diverge horizontally, forcing some air into the Northern Hemisphere and some into the Southern (Fig. 11.26c). With no Coriolis force at the equator, these winds are driven directly away from the high-pressure belt.

But as these high-altitude winds move away from the equator, they are increasingly affected by Coriolis force (Fig. 11.26d). This causes winds moving into the Northern Hemisphere to turn to their right, and those moving into the Southern Hemisphere to turn to their left.

But as the winds move further and further away from the equator, they are turned more and more to the east, creating the subtropical jet (Fig. 11.26e) at about 30° latitude. Coriolis force prevents these upper-level winds from getting further away from the equator than about 30° latitude (north and south), so the air accumulates and the pressure increases in those belts. When simulations of the general circulation impose a larger Coriolis force (as if the Earth spun faster), the convergence bands occur closer to the equator. For weaker Coriolis force, the convergence is closer to the poles. But for our Earth, the air converges at 30° latitude.

This pressure perturbation p' is labeled as HH or H^2 in Fig. 11.26e, to show that it is a positive pressure perturbation, but not as strong as the H^3 perturbation over the equator. Namely, the horizontal pressure gradient between H^3 and H^2 drives the upper-level winds to diverge away from the equator.

The excess air accumulated at 30° latitudes cannot go up into the stratosphere in the face of very strong static stability there. The air cannot go further poleward because of Coriolis force. And the air cannot move equatorward in the face of the strong upper-level winds leaving the equator. The only remaining path is downward at 30° latitude (Fig. 11.26f) as a nonhydrostatic flow. As air accumulates near the ground, it causes a high-pressure perturbation there — the belt of subtropical highs labeled with H.

The descending air at 30° latitude warms dry-adiabatically, and does not contain much moisture because it was squeezed out earlier in the thunderstorm updrafts. These are the latitudes of the subtropical deserts (Fig. 11.26h), and the source of hot airmasses near the surface.

Finally, the horizontal pressure gradient between the surface subtropical highs near 30° latitude and the equatorial lows near 0° latitude drives the surface winds toward the equator. Coriolis force turns these winds toward the west in both hemispheres (Fig. 11.26g), resulting in the easterly trade winds (winds from the east) that converge at the ITCZ.

The total vertical circulation in the tropics and subtropics we recognize as the Hadley Cell (labeled h.c. in Fig. 11.26f). This vertical circulation (a **thermally-direct circulation**) is so vigorous in its vertical mixing and heat transport that it forces a deeper troposphere in the tropics than elsewhere (Fig. 11.4). Also, the vigorous circulation spreads and horizontally mixes the radiatively warmed air somewhat uniformly between ±30° latitude, as sketched by the temperature plateau in Fig. 11.8a.

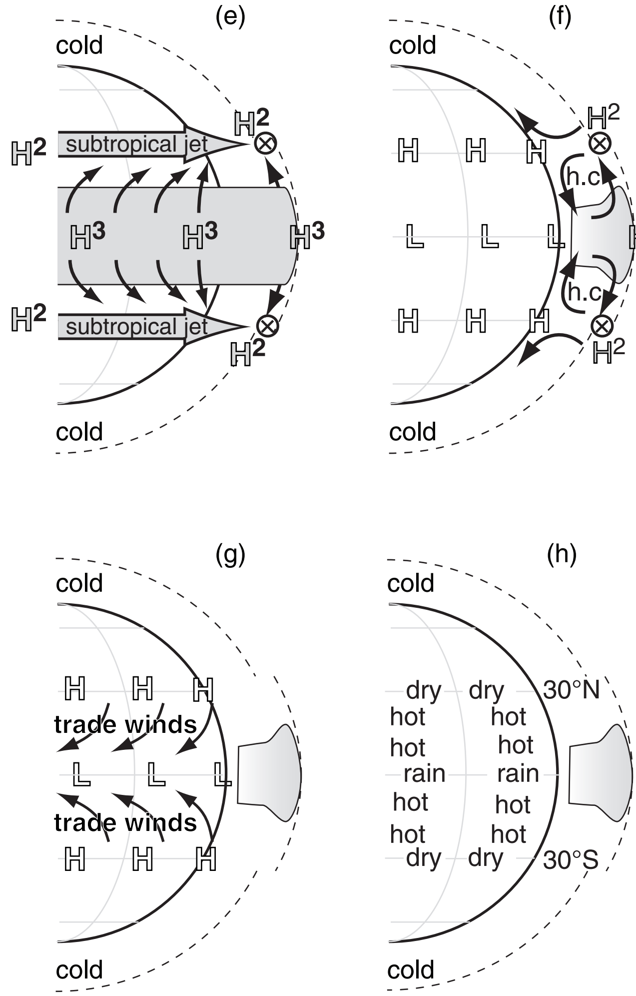
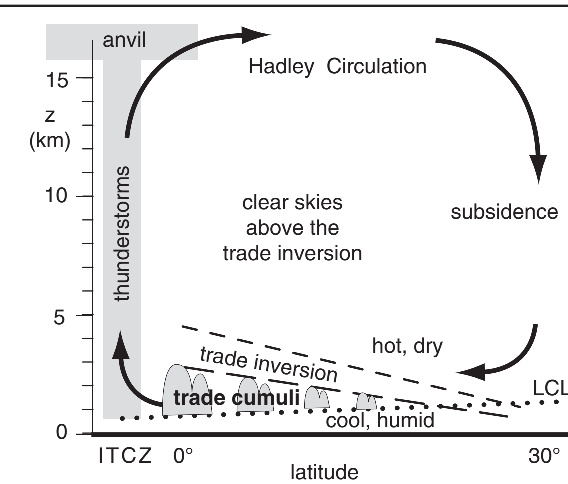


Figure 11.26 (continuation)
Explanation of low-latitude portion of the global circulation. The dashed line shows the tropopause. The "x" with a circle around it (representing the tail feathers of an arrow) indicates the axis of a jet stream that goes into the page.

FOCUS • The Trade Inversion

Descending air in the subtropical arm of the Hadley circulation is hot and dry. Air near the tropical sea surface is relatively cool and humid. Between these layers is a strong temperature inversion called the **trade inversion** or **passat inversion**. This statically stable layer (between the dashed lines in the Figure) creates a lid to the tropical convection below it. The inversion base is lowest (order of 500 m) in the subtropics, and is highest (order of 2,500 m) near the ITCZ. Fair-weather cumulus clouds (**trade cumuli**) between the lifting condensation level (LCL) and the trade inversion are shallowest in the subtropics and deeper closer to the ITCZ.

By capping the humid air below it, the trade inversion allows a latent-heat fuel supply to build up, which can be released in hurricanes and ITCZ thunderstorms.



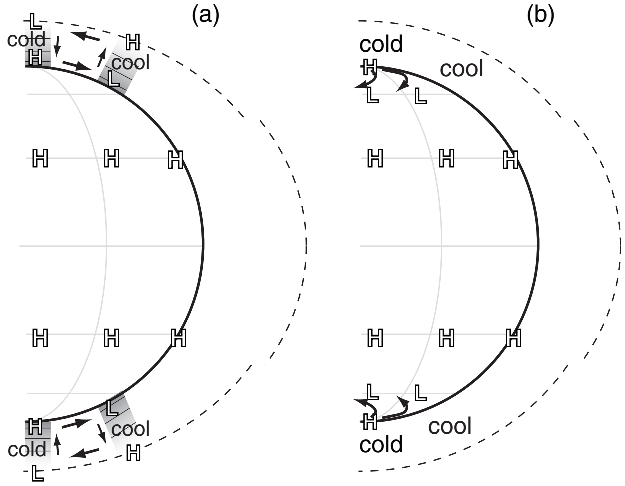


Figure 11.27
Explanation of high-latitude portion of general circulation.

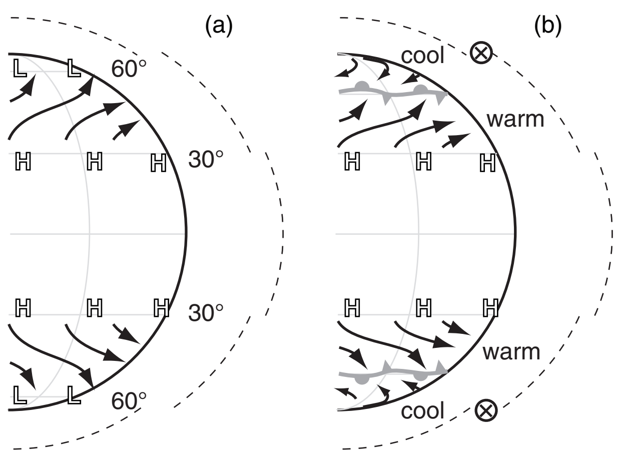


Figure 11.28
Explanation of mid-latitude flow near the Earth's surface. H and L indicate belts of high and low pressure, black arrows are average surface winds, and the polar front is shaded grey. The circle with "x" in it represents the tail feathers of the jet-stream wind vector blowing into the page.

High Latitudes

As sketched in Fig. 11.8a, air temperature is very cold at the poles, and is cool at 60° latitude. The temperature difference between 60° and 90° latitudes creates opposite north-south pressure gradients and winds at the top and bottom of the troposphere, due to the thermal circulation effect (Fig. 11.17). A vertical cross section of this thermal circulation (Fig. 11.27a) shows a weak polar cell. Air generally rises near 60°N and descends near the pole.

At the poles are surface high-pressure centers, and at 60° latitudes are belts of subpolar lows at the surface. This horizontal pressure gradient drives equatorward winds, which are turned toward the west in both hemispheres due to Coriolis force (Fig. 11.27b). Namely, the winds become geostrophic, and are known as **polar easterlies**.

At the top of the shallow (6 to 8 km thick) troposphere are poleward winds that are turned toward the east by Coriolis force. These result in an upper-level westerly flow that circulates around the upper-level polar low (Fig. 11.3b).

Mid-latitudes

Recall that the Hadley cell is unable to mix heat beyond about ±30° latitude. This leaves a very strong meridional temperature gradient in mid-latitudes (Fig. 11.8) throughout the depth of the troposphere. Namely, the temperature change between the equator and the poles has been compressed to a latitude band of about 30 to 60° in each hemisphere.

Between the subtropical high-pressure belt near 30° latitude and the subpolar low-pressure belt near 60° latitude is a weak meridional pressure gradient near the Earth's surface. This climatological-average pressure gradient drives weak boundary-layer winds from the west in both hemispheres (Fig. 11.28a), while the drag of the air against the surface causes the wind to turn slightly toward low pressure.

Near the subpolar belt of low pressure is a region of surface-air convergence, with easterly winds from the poles meeting westerly winds from mid-latitudes. The boundary between the warm subtropical air and the cool polar air at the Earth's surface is called the **polar front** (Fig. 11.28b) — a region of even stronger horizontal temperature gradient.

Recall from the hypsometric equation in Chapter 1 that the height difference (i.e., the thickness) between two isobaric surfaces increases with increasing temperature. As a result of the meridional temperature gradient, isobaric surfaces near the top of the troposphere in mid-latitudes are much more steeply sloped than near the ground (Figs. 11.29 & 11.32). This is related to the thermal-wind effect.

In the Northern Hemisphere this effect is greatest in winter (Fig. 11.32), because there is the greatest temperature contrast between pole and equator. In the Southern Hemisphere, the cold Antarctic continent maintains a strong meridional temperature contrast all year.

Larger pressure gradients at higher altitudes drive stronger winds. The core of fastest westerly winds near the tropopause (where the pressure-gradient is strongest) is called the **polar jet stream**, and is also discussed in more detail later in this chapter. Thus, the climatological average winds throughout the troposphere at mid-latitudes are from the west (Fig. 11.30a) in both hemispheres.

Although the climatological average polar-jet-stream winds are straight from west to east (as in Fig. 11.30a), the actual flow on any single day is unstable. Two factors cause this instability: the variation of Coriolis parameter with latitude (an effect that leads to **barotropic instability**), and the increase in static stability toward the poles (an effect that leads to **baroclinic instability**). Both of these instabilities are discussed in more detail later.

These instabilities cause the jet stream to meander meridionally (north-south) as it continues to blow from the west (Fig. 11.30b). The meanders that form in this flow are called **Rossby waves**. Regions near the tropopause where the jet stream meanders equatorward are called **troughs**, because the lower pressure on the north side of the jet stream is brought equatorward there. Poleward meanders of the jet stream are called **ridges**, where higher pressure extends poleward. The locations of Rossby-wave troughs and ridges usually propagate toward the east with time, as will be explained in detail later in this chapter.

Recall that there is a subtropical jet at roughly 30° latitude associated with the Hadley Cell. Thus, in each hemisphere are a somewhat-steady subtropical jet and an unsteady polar jet (Fig. 11.30b). Both of these jets are strongest in the winter hemisphere, where there is the greatest temperature gradient between cold poles and hot equator.

Troughs and ridges in the jet stream are crucial in creating and killing cyclones and anticyclones near the Earth's surface. Namely, they cause the extremely large weather variability that is normal for mid-latitudes. The field of **synoptic meteorology** comprises the study and forecasting of these variable systems, as discussed in the Airmasses, Fronts, and Extratropical Cyclone chapters.

Figs. 11.31 and 11.32 show actual global pressure patterns at the bottom and top of the troposphere. The next section explains why the patterns over the oceans and continents differ.

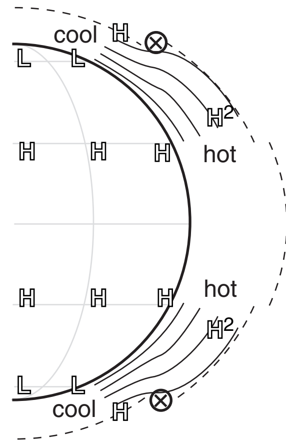


Figure 11.29
Isobaric surfaces (thin solid black lines) are spaced further apart in hot air than in cool air. Regions of steeper slope of the isobars have stronger pressure gradient, and drive faster winds. The jet-stream axis is shown with ⊗ .

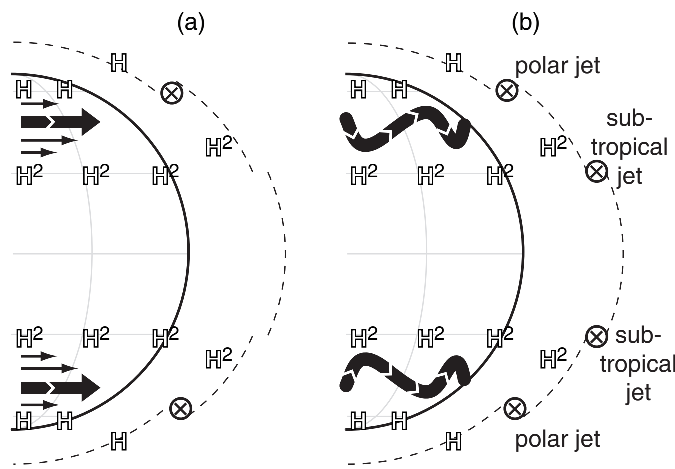


Figure 11.30
Mid-latitude flow near the top of the troposphere. The thick black arrow represents the core or axis of the jet stream: (a) average, (b) snapshot. (See caption in previous two figures for legend.)

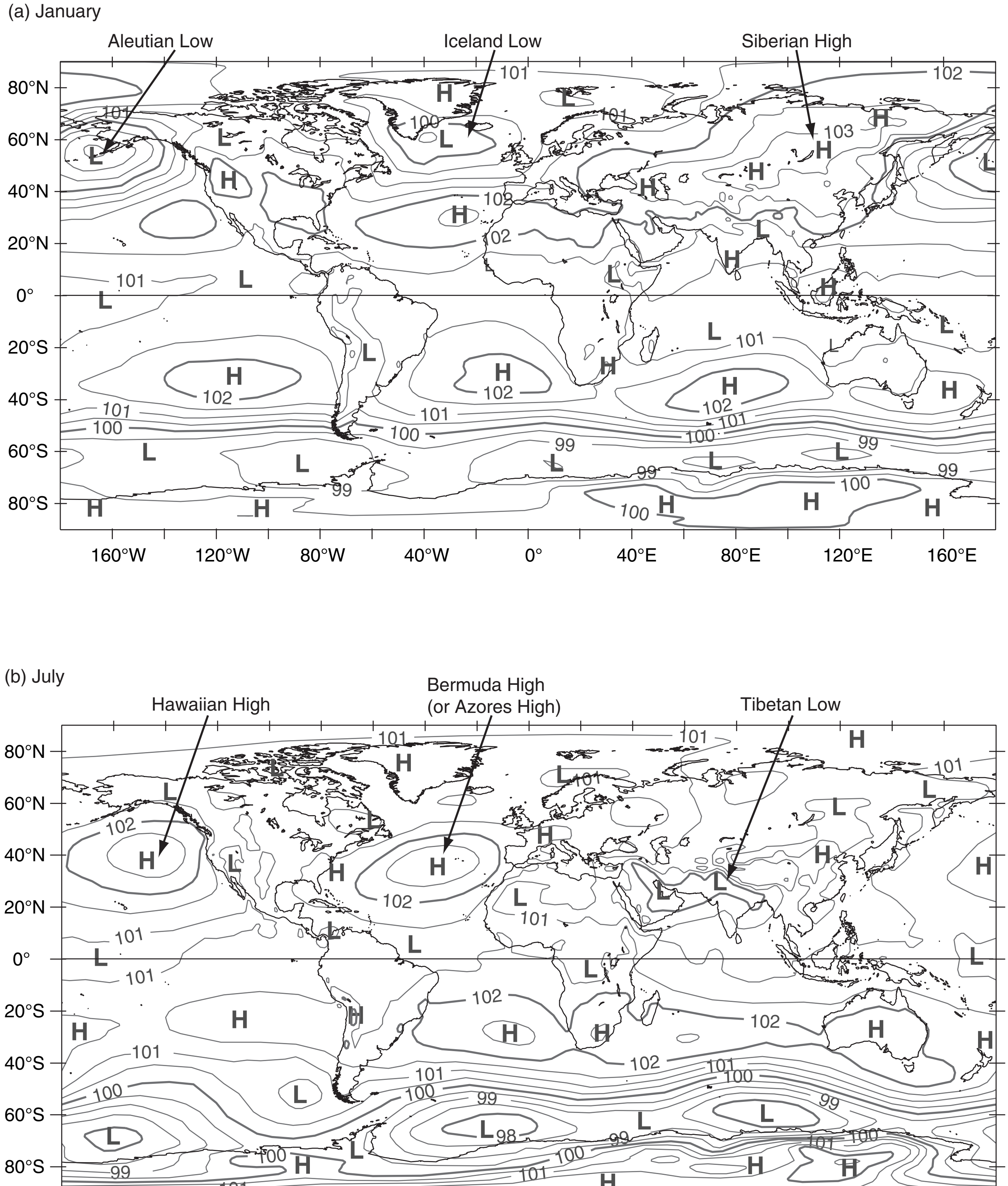


Figure 11.31
Mean sea-level pressure (kPa) for (a) January 2001, and (b) July 2001. [European Centre for Medium Range Weather forecasts (ECMWF) ERA-40 data used in this analysis have been provided by ECMWF, and have been obtained from the ECMWF data server.]

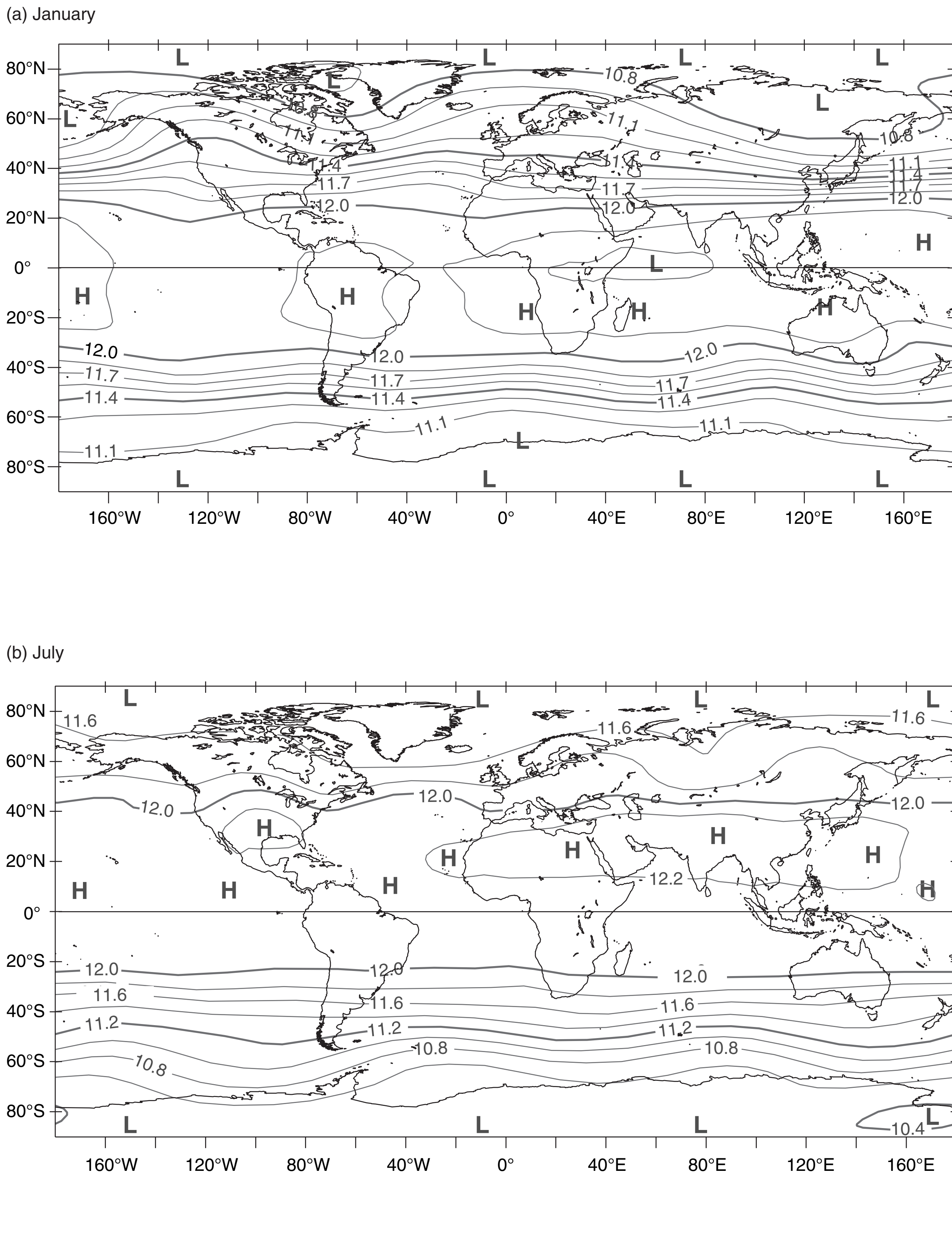


Figure 11.32
Geopotential height (km) of the 20 kPa isobaric surface (near the <u>*tropopause*</u>*) for (a) January 2001, and (b) July 2001. [ECMWF ERA-40 data used in this analysis have been provided by ECMWF, and have been obtained from the ECMWF data server.]*

Monsoon

Recall from the Heat chapter that the temperature change of an object depends on the mass of material being heated or cooled, and on the specific heat of the material. If you put the same amount of heat into objects of similar material but differing mass, the smaller masses will warm the most.

Rocks and soil on continents are opaque to sunlight, and are good insulators of heat. Thus, sunlight directly striking the land surface warms only a very thin top layer (mm) of molecules, causing this thin layer to get quite warm. Similarly, longwave (infrared) radiative heat loss at night causes the very thin top layer to become very cold. Namely, there is a large **diurnal** (daily) temperature contrast. Also, because there are more daylight hours in summer and more nighttime hours in winter, continental land surfaces tend to become hot in summer and cold in winter.

Water in the oceans is partially transparent and sometimes turbulent, allowing sunlight to be absorbed and spread over a thick layer (meters to tens of meters) of molecules. Also, water has a large specific heat (see the Heat chapter), hence a large input of heat causes only a small temperature change. Thus, ocean surfaces have very small diurnal temperature changes, and have only a medium amount of seasonal temperature changes.

The net result is that during summer, continents warm faster than the oceans. During winter, continents cool faster than the oceans.

Consider a cold region next to a warm region. Over the cold surface, the near-surface air cools and develops a high-pressure center with anticyclonically rotating winds, as explained by the thermal circulation sketched in Fig. 11.17. Over warm surfaces, the thermal circulation causes near-surface air to warm and develop a low-pressure center with cyclonic winds. As already mentioned, this is called a **thermal low**.

Combining the effects from the previous two paragraphs with the strong tendency of the winds to become geostrophic (or gradient) yields the near-surface monsoonal flows shown in Fig. 11.5. Opposite pressure patterns and circulations would occur near the top of the troposphere. The regions near surface lows tend to have rising air and abundant clouds and rain. Regions near surface highs tend to have dry fair weather with few clouds.

Monsoon circulations occur over every large continent and ocean (Fig. 11.31). Some are given names. Over the Atlantic in summer, winds on the south and west sides of the monsoonal **Bermuda High** (also called the **Azores High**) steer Atlantic hurricanes northward as they near North America. Over the North Pacific in summer is the **Hawaiian High** or **Pacific High** which provides cool northerly breezes and months of fair weather to the west coast of North America.

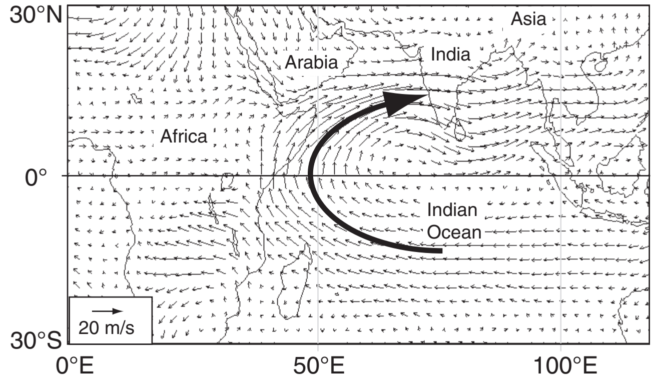


Figure 11.33
Monsoon winds near India on 85 kPa isobaric surface (at z ≈ 1.5 km), averaged over July 2001. Thin arrow length shows wind speed (see legend). Thick arrow shows cross-equatorial flow.

The summer low over northern India is called the **Tibetan Low**. It helps drive strong cross-equatorial flow (Fig. 11.33) that brings the much needed monsoon rains over India. Ghana in West Africa also receives a cross-equatorial monsoon flow.

Winter continental highs such as the **Siberian High** over Asia are formation locations for cold airmasses. Over the North Atlantic Ocean in winter is the **Icelandic Low**, with an average circulation on its south side that steers mid-latitude cyclones toward Great Britain and northern Europe. The south side of the winter **Aleutian Low** over the North Pacific brings strong onshore flow toward the west coast of North America, causing many days of clouds and rain.

The actual global circulation is a superposition of the zonally averaged flows and the monsoonal flows (Fig. 11.31). Also, a snapshot or satellite image of the Earth on any given day would likely deviate from the one-month averages presented here. Other important aspects of the global circulation were not discussed, such as conversion between available potential energy and kinetic energy. Also, monsoons and the whole global circulation are modulated by El Niño / La Niña events and other oscillations, discussed in the Climate chapter.

In the previous sections, we have described characteristics of the global circulation in simple terms, looked at what drives these motions, and explained dynamically why they exist. Some of the phenomena we encountered deserve more complete analysis, including the jet stream, Rossby waves with their troughs and ridges, and some aspects of the ocean currents. The next sections give details about how these phenomena work.

JET STREAMS

In the winter hemisphere there are often two strong jet streams of fast west-to-east moving air near the tropopause: the **polar jet stream** and the **subtropical jet stream** (Figs. 11.34 & 11.35).

The subtropical jet is centered near 30° latitude in the winter hemisphere. This jet: (1) is very steady; (2) meanders north and south a bit; (3) is about 10° latitude wide (width ≈1,000 km); and (4) has seasonal-average speeds of about 45 m/s over the Atlantic Ocean, 55 to 65 m/s over Africa and the Indian Ocean, and 60 to 80 m/s over the western Pacific Ocean. The **core** of fast winds near its center is at 12 km altitude (Fig. 11.35). It is driven by outflow from the top of the Hadley cell, and is affected by both Coriolis force and angular-momentum conservation.

The polar jet is centered near 50 to 60° latitude in the winter hemisphere. The polar jet: (1) is extremely variable; (2) meanders extensively north and south; (3) is about 5° latitude wide; and (4) has widely varying speeds (25 to 100 m/s). The core altitude is about 9 km. This jet forms over the polar front — driven by thermal-wind effects due to the strong horizontal temperature contrast across the front.

When meteorological data are averaged over a month, the subtropical jet shows up clearly in the data (e.g., Fig. 11.36) because it is so steady. However, the polar jet disappears because it meanders and shifts so extensively that it is washed out by the long-term average. Nonetheless, these transient meanders of the polar jet (troughs and ridges in the Rossby waves) are extremely important for mid-latitude cyclone formation and evolution (see the Extratropical Cyclone chapter).

In the summer hemisphere both jets are much weaker (Figs. 11.35b and 11.36), because of the much weaker temperature contrast between the equator and the warm pole. Core wind speeds in the subtropical jet are 0 to 10 m/s in N. Hemisphere summer, and 5 to 45 m/s in S. Hemisphere summer. This core shifts poleward to be centered near 40° to 45° latitude. The polar jet is also very weak (0 to 20 m/s) or non-existent in summer, and is displaced poleward to be centered near 60° to 75° latitude.

FOCUS • Jet Stream Aspect Ratio

Jet streams in the real atmosphere look very much like the thin ribbons of fast-moving air, as sketched in Fig. 11.34. Jet vertical thickness (order of 5 to 10 km) is much smaller than their horizontal width (order of 1000 to 2000 km). Namely, their aspect ratio (width/thickness) is large. Figures such as 11.35 are intentionally distorted to show vertical variations better.

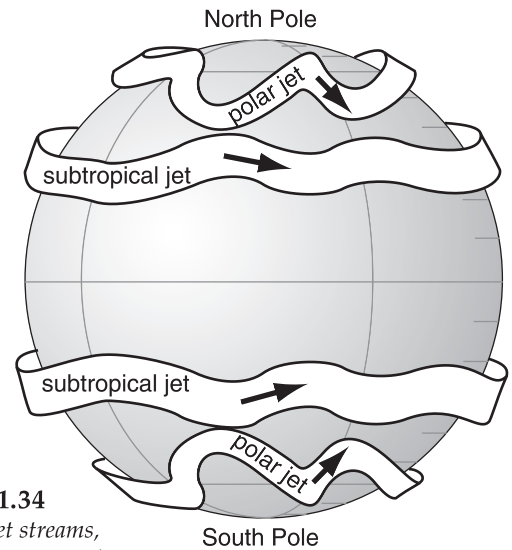


Figure 11.34
Sketch of jet streams, representing a snapshot.

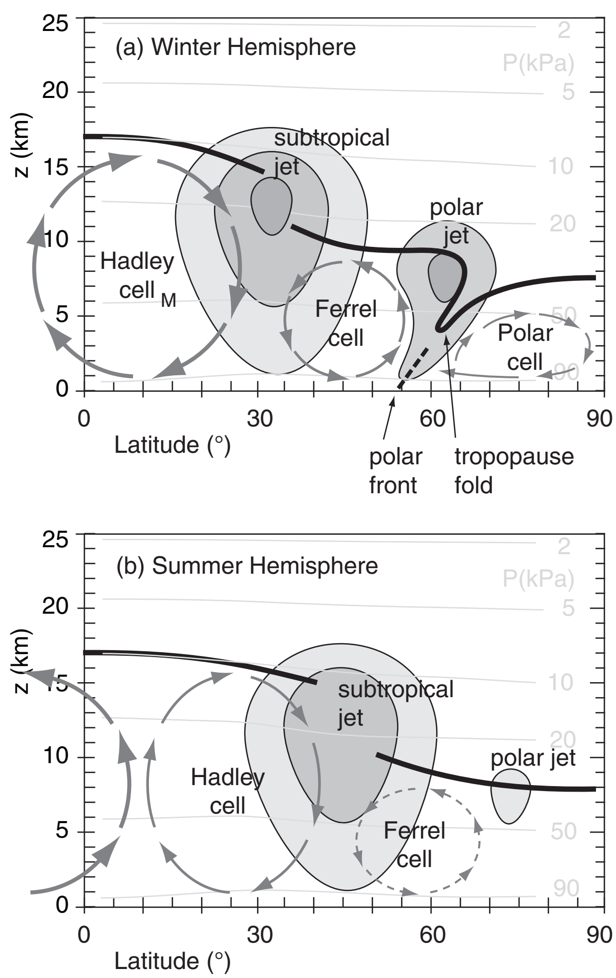


Figure 11.35
Simplified vertical cross-section. Thick solid line is the tropopause. Darker shading indicates faster winds from the west (perpendicular to the page). This is a snapshot, not a climatological average; hence, the polar jet and polar front can be seen.

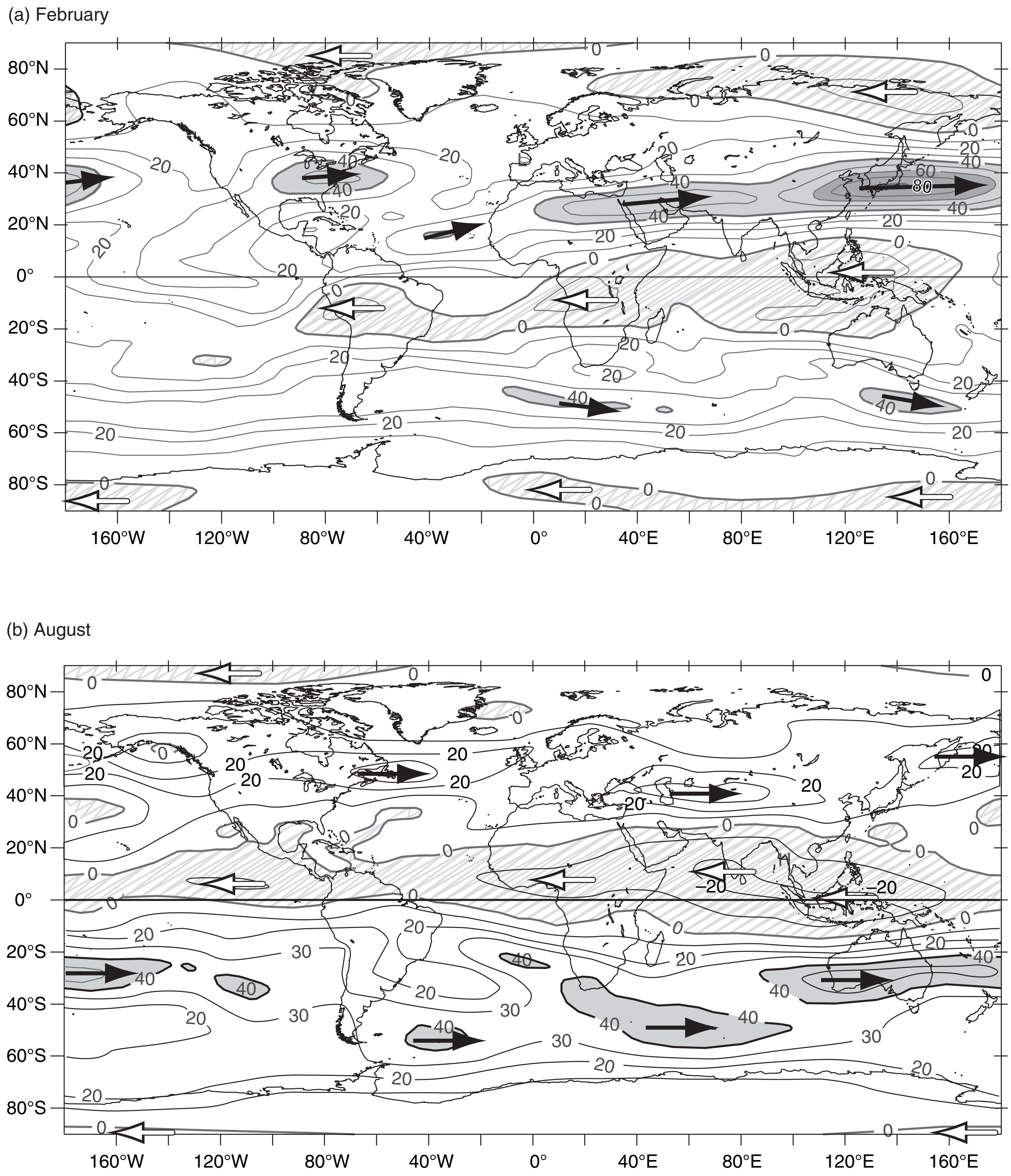


Figure 11.36

Zonal (U) component of winds (m/s) at 20 kPa isobaric surface (near the tropopause) for (a) February, and (b) August 2001. Contour interval is 10 m/s. All negative (east) winds are lightly hatched, and are indicated with white arrows. Positive (west) winds are indicated with black arrows, and are shaded at 40, 60, and 80 m/s. February and August are shown because the jets are stronger than during January and July, respectively. In 2001, polar easterlies were observed at 20 kPa, contrary to the longer-term climate average of polar westerlies as sketched in Fig. 11.3b. [ECMWF ERA-40 data used in this analysis have been provided by ECMWF, and have been obtained from the ECMWF data server.]

Baroclinicity & the Polar Jet

Baroclinicity associated with the meridional temperature gradient drives the west-to-east winds near the top of the troposphere, via the thermal-wind relationship. Fig. 11.37a shows a typical isotherm vertical cross-section for N. Hemisphere winter. Air near the ground is warmer near the equator and colder at the poles. This meridional temperature gradient exists throughout the troposphere.

The temperature at the tropopause at 60°N is warmer than the temperature at the higher-altitude tropopause near the equator. This is associated with a temperature reversal in the stratosphere, where the air is colder over the equator and warmer over 60° latitude. Recall that the spatial distribution of temperature is called the **temperature field**.

Apply the hypsometric equation to the temperature field to give the **pressure field** (the spatial distribution of pressures, Fig. 11.37b). In the troposphere, greater thickness between any two pressure surfaces in the warmer equatorial air than in the colder polar air causes the isobars to become more tilted at mid-latitudes as the tropopause is approached. Above the tropopause, tilt decreases because the meridional temperature gradient is reversed. Although the isobar tilt in Fig. 11.37b is subtle, it is significant.

Regions with the greatest tilt have the greatest meridional pressure gradient, which drives the fastest geostrophic wind (Fig. 11.37c). This maximum of westerly winds at mid-latitudes explains the polar jet stream at the tropopause in mid-latitudes. The center of the jet stream, known as the **jet-stream core**, occurs in a region that can be idealized as a break or fold in the tropopause, as will be discussed in the Fronts and Extratropical Cyclones chapters.

The toy model presented in eq. (11.1) and Fig. 11.8 is a starting point for quantifying the nature of the jet stream. Equations (11.2), (11.4) and (11.13) can be combined to give the wind speed as a function of latitude ϕ and altitude z, where it is assumed for simplicity that the winds near the ground are zero:

$$U_{jet} \approx \frac{|g| \cdot c \cdot b_1}{2\Omega \cdot T_v} \cdot z \cdot \left(1 - \frac{z}{2 \cdot z_T}\right) \cdot \cos^2(\phi) \cdot \sin^2(\phi) \tag{11.17}$$

where U_{jet} is used in place of U_g to indicate a specific application of geostrophic wind to the polar jet stream, and where $c = 1.18\text{x}10^{-3}$ km^{-1} and $b_1 \approx 40$ K as before. The definition of the Coriolis parameter eq. (10.16) $f_c = 2 \cdot \Omega \cdot \sin\phi$ was used, where $2 \cdot \Omega = 1.458\text{x}10^{-4}$ s^{-1}. The average troposphere depth is $z_T \approx 11$ km, gravitational acceleration is $|g| = 9.8$ m/s^2, and T_v is the average virtual temperature (in Kelvin).

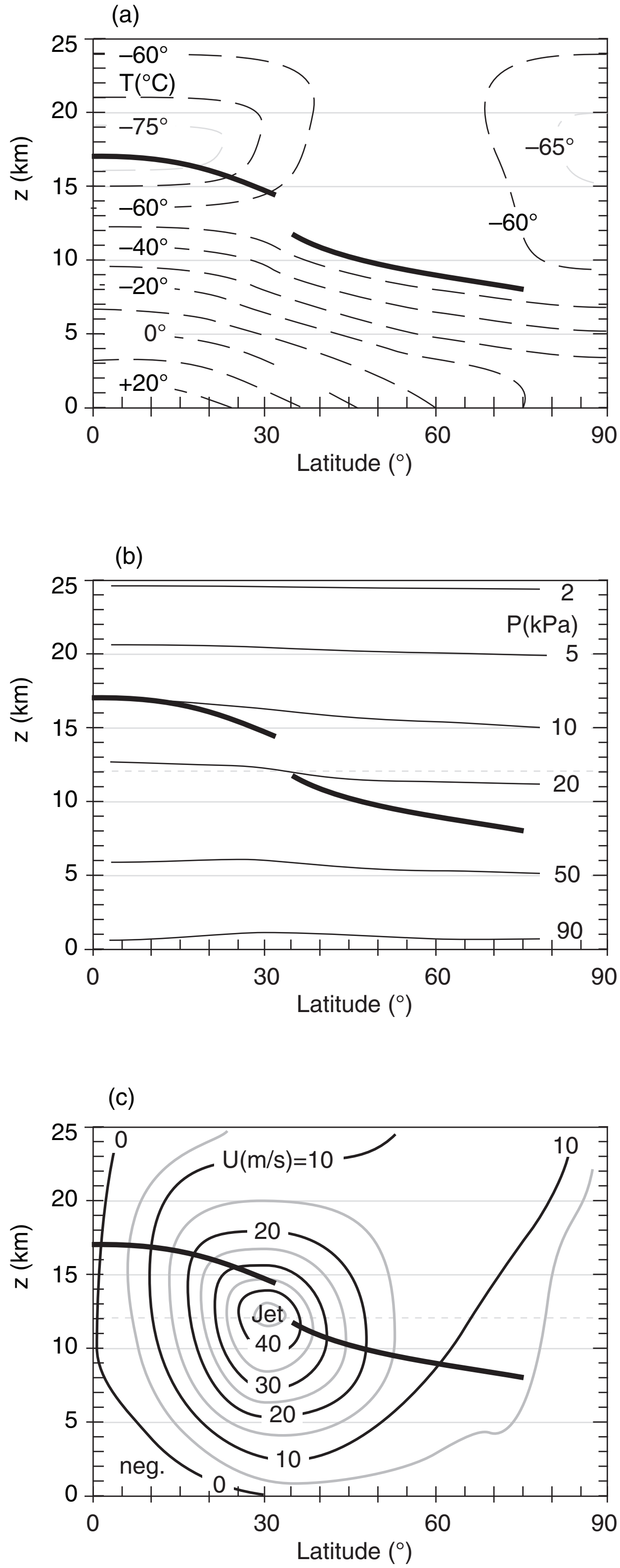


Figure 11.37
Example of vertical slices through the N. Hemisphere atmosphere for January 2003: (a) isotherms T (°C), (b) isobars P (kPa), (c) isotachs of the zonal wind U (m/s). Heavy solid line marks the tropopause. Wind direction of the jet in (c) is from the west (i.e., toward the reader). These figures can be overlain.

Solved Example

Find the geostrophic wind speed at the tropopause at 45° latitude, assuming an idealized temperature structure as plotted in Fig. 11.8. Assume an average temperature of 0°C, and neglect moisture.

Solution

Given: $z = z_T = 11$ km, $\phi = 45°$, $T_v = 273$ K.
Find: U_g = ? m/s

Use eq. (11.17):

$$U_{jet} \approx \frac{(9.8\text{m}\cdot\text{s}^{-2})\cdot(1.18\times10^{-3}\text{km}^{-1})\cdot(40\text{K})}{(1.458\times10^{-4}\text{s}^{-1})\cdot(273\text{K})}\cdot$$

$$(11\text{km})\cdot\left(1-\frac{1}{2}\right)\cdot\cos^2(45°)\cdot\sin^2(45°)$$

= <u>**16 m/s**</u>

Check: Units OK. Physics OK.
Discussion: Actual average wind speeds of about 40 m/s are observed in the jet stream of the winter hemisphere over a three-month average. Sometimes jet velocities as great as 100 m/s are observed on individual days. In the winter hemisphere, the temperature gradient is often greater than that given by eq. (11.4), which partially explains our low wind speed. Also, angular momentum can accelerate the wind.

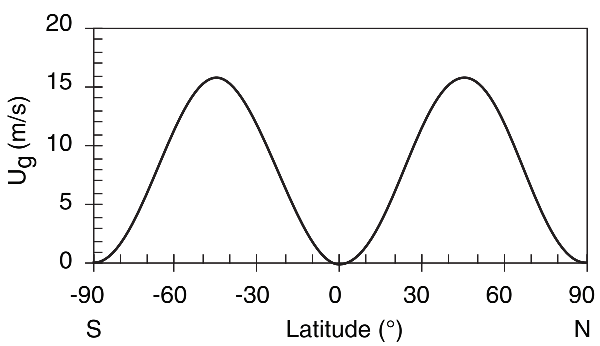
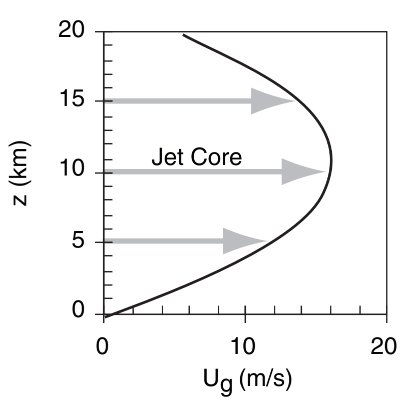


Figure 11.38
Geostrophic winds at 11 km altitude (from toy model).

Figure 11.39
Vertical profile of (toy model) geostrophic wind at 45° latitude.



From Fig. 11.8b we see there are two extremes of north-south temperature gradient, one in the northern hemisphere and one in the southern. Those are the latitudes (in this toy model) where we can anticipate to find the strongest jet velocities (Fig. 11.38). Although the temperature gradient in the southern hemisphere has a sign opposite to that in the north, the sign of the Coriolis parameter also changes. Thus, the jet stream velocity is positive (west to east) in both hemispheres.

A spreadsheet solution of eq. (11.17) is shown in Figs 11.38 and 11.39. At the tropopause (z = 11 km), Fig. 11.38 shows the jet-stream velocity, with maxima at 45° north and south latitudes. For 45° latitude, Fig. 11.39 shows the vertical profile of geostrophic wind speed, with the fastest speeds (i.e., the **jet core**) at the tropopause.

The gross features shown in this figure are indeed observed in the atmosphere. However, actual spacing between jets in the northern and southern hemispheres is less than 90°; it is roughly 70° or 75° latitude difference. Also, the centers of both jets shift slightly northward during northern-hemisphere summer, and southward in winter. Jet speeds are not zero at the surface. Nevertheless, the empirical toy model presented here via eq. (11.1) serves as an instructive first introduction.

Angular Momentum & Subtropical Jet

A factor affecting the <u>subtropical</u> jet stream is **angular momentum** (mass times velocity times radius of curvature).

Consider air initially moving at some zonal velocity U_s relative to the Earth's surface at some initial (source) latitude ϕ_s. Because the Earth is rotating, the Earth's surface at the source latitude is moving eastward at velocity U_{Es}. Thus, the total eastward speed of the air parcel relative to the Earth's axis is $(U_s + U_{Es})$.

If there were <u>no forces</u> acting on the air, then conservation of angular momentum as the air moves to a destination latitude ϕ_d (Fig. 11.40) is:

$$m\cdot(U_s + U_{Es})\cdot R_s = m\cdot(U_d + U_{Ed})\cdot R_d \qquad \bullet(11.18)$$

where m is the mass of air, U_d is the tangential velocity of the air relative to the Earth's surface at the destination latitude, and U_{Ed} is the tangential velocity of the Earth's surface at the destination.

The radius from the Earth's axis to the surface at latitude ϕ is $R_\phi = R_E\cdot\cos(\phi)$, where R_ϕ represents source or destination radius (R_s or R_d), and R_E = 6371 km is the average Earth radius (Fig. 11.40). Also, the tangential velocity of the Earth at latitude ϕ is $U_\phi = \Omega\cdot R_\phi = \Omega\cdot R_E\cdot\cos(\phi)$, where the rate of rotation is $\Omega = 0.729\text{x}10^{-4}\ \text{s}^{-1}$.

Solving these equations for the air velocity U_d relative to the Earth at the destination gives:

$$U_d = \left[\Omega \cdot R_E \cdot \cos(\phi_s) + U_s\right] \cdot \frac{\cos(\phi_s)}{\cos(\phi_d)} - \Omega \cdot R_E \cdot \cos(\phi_d) \tag{11.19}$$

As we already discussed, winds at the top of the Hadley cell diverge away from the equator, but cannot move beyond 30° latitude because Coriolis force turns the wind. When we use eq. (11.19) to predict the zonal wind speed for typical trade-wind air that starts at the equator with U_s = –7 m/s and ends at 30° latitude, we find unrealistically large wind speeds (125 m/s) for the subtropical jet (Fig. 11.41). Actual typical wind speeds in the subtropical jet are of order 40 to 80 m/s in the winter hemisphere, and slower in the summer hemisphere.

The discrepancy is because angular momentum is not conserved — due to forces acting on the air. Coriolis force turns the wind, causing air to accumulate and create a pressure-gradient force to oppose poleward motion in the Hadley cell. Turbulent drag force slows the wind a small amount. Also, the jet streams meander north and south, which helps to transport slow angular momentum southward and fast angular momentum northward. Namely, these meanders or synoptic-scale eddies cause mixing of zonal momentum.

Next, the concept of vorticity is discussed, which will be useful for explaining how troughs and ridges develop in the jet stream. Then, we will introduce a way to quantify circulation, to help understand the global circulation.

Solved Example

For air starting at the equatorial tropopause, what would be its zonal velocity at 20°N if angular momentum were conserved?

Solution

Given: ϕ_d = 20°N, ϕ_s = 0°, $\Omega \cdot R_E$ = 463 m/s
Find: U_d = ? m/s
Assume: no turbulence; U_s = –7 m/s easterlies.

Use eq. (11.19): U_d =

$$\left[(463\text{m/s})\cos(0°) + U_s\right]\frac{\cos(0°)}{\cos(20°)} - (463\text{m/s})\cos(20°)$$

U_d = = **50.2 m/s**

Check: Units OK. Physics OK. Agrees with Fig. 11.41
Discussion: Winds in the real atmosphere are much less than this, because forces and momentum mixing prevent conservation of angular momentum.

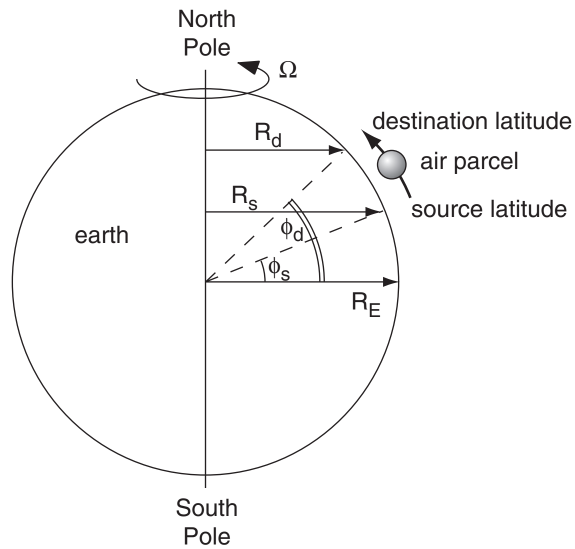


Figure 11.40
Geometry for northward air-parcel movement.

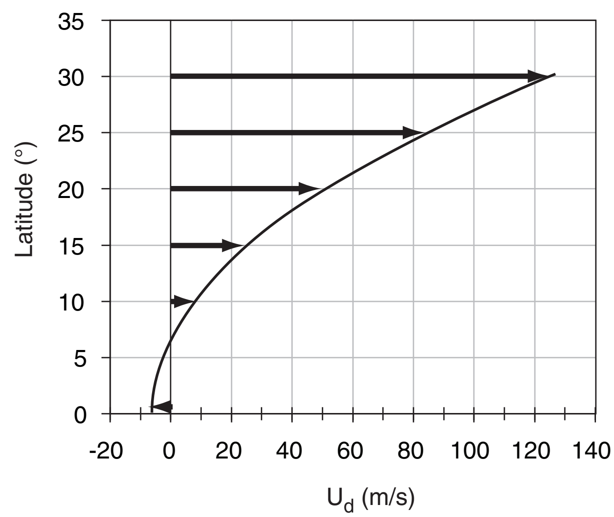


Figure 11.41
Zonal wind speed U_d at various destination latitudes for typical trade-wind air that starts at the equator with U_s = –7 m/s, for the unrealistic case of conservation of angular momentum.

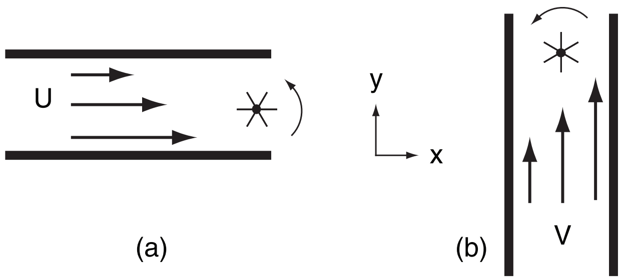


Figure 11.42
Shear-induced relative vorticity. Counter-clockwise turning of the paddle wheels indicates positive vorticity for both examples. (a) $\Delta U/\Delta y$ is negative. (b) $\Delta V/\Delta x$ is positive.

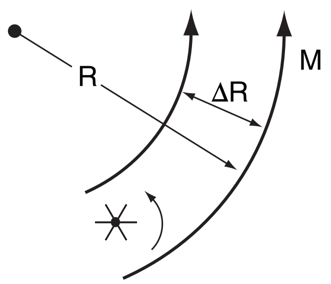


Figure 11.43
Vorticity for flow around a curve, where both the curvature and the shear contribute to positive vorticity in this example (i.e., the paddle wheel rotates counter-clockwise).

VORTICITY

Relative Vorticity

Relative vorticity ζ_r is a measure of the rotation of fluids about a vertical axis relative to the Earth's surface, as introduced in the Dynamics chapter. It is defined as positive in the counter-clockwise direction. The unit of measurement of vorticity is inverse seconds.

The following two definitions are equivalent:

$$\zeta_r = \frac{\Delta V}{\Delta x} - \frac{\Delta U}{\Delta y} \qquad \bullet(11.20)$$

$$\zeta_r = \frac{\Delta M}{\Delta R} + \frac{M}{R} \qquad \bullet(11.21)$$

where (U, V) are the (eastward, northward) components of the wind velocity, R is the radius of curvature traveled by a moving air parcel, and M is the tangential speed along that circumference in a counterclockwise direction.

Fig. 11.42 illustrates eq. (11.20). If fluid travels along a straight channel, but has shear, then it also has vorticity because a tiny paddle wheel carried by the fluid would be seen to rotate. Fig. 11.43 illustrates eq. (11.21), where fluid following a curved path also has vorticity, so long as radial shear of the tangential velocity does not cancel it. Fig. 11.44 shows both shear and rotational relative vorticity.

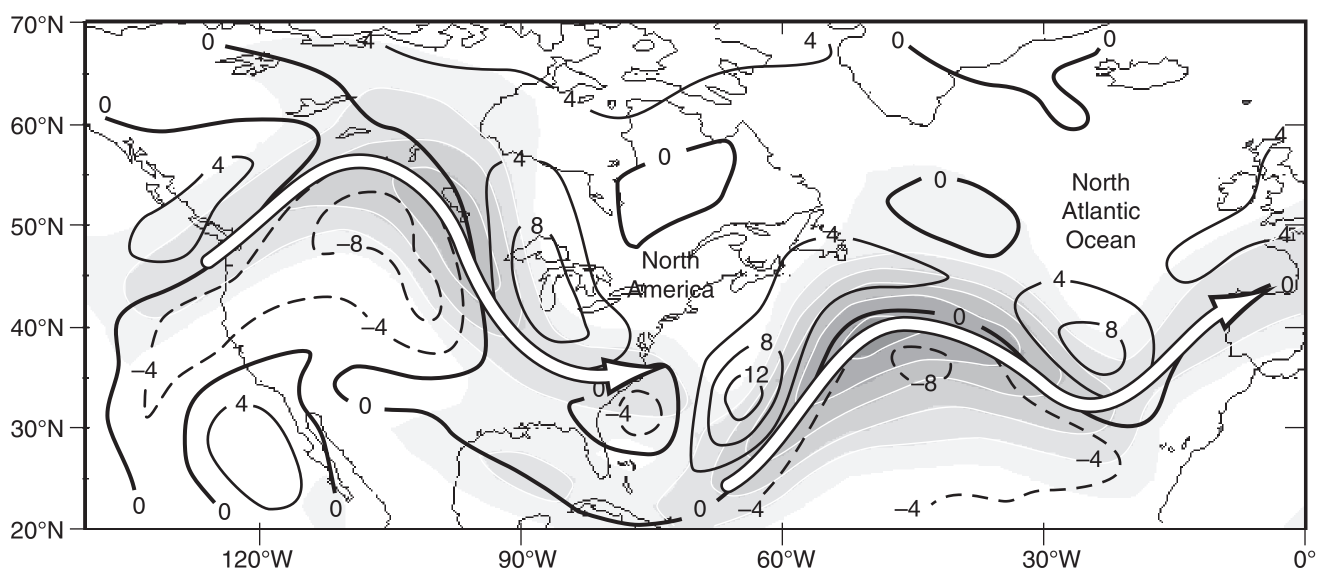
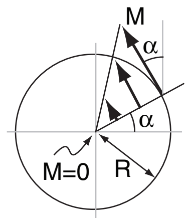


Figure 11.44
Weather map example of relative vorticity (contour lines, with units of 10^{-5} s^{-1}) near the tropopause (at the 20 kPa isobaric surface) at 12 UTC on 5 January 2001. White arrows show jet stream axis of fastest winds over North America and the Atlantic Ocean. Shading gives wind speeds every 10 m/s from 30 m/s (lightest grey) to over 80 m/s (darkest grey). (Based on NCEP/NCAR 40-year reanalysis data, with initial plots produced using the plotting tool by Christopher Godfrey, the University of Oklahoma School of Meteorology.)

FOCUS • Solid Body Relative Vorticity

One can derive eq. (11.22) from either eq. (11.20) or eq. (11.21). Consider the sketch at right, where speed M is tangent to the rotating disk at radius R.



Starting with eq. (11.21):

$$\zeta_r = \frac{\Delta M}{\Delta R} + \frac{M}{R}$$

Thus:

$$\zeta_r = \frac{(M-0)}{R} + \frac{M}{R} = \boxed{\frac{2M}{R}}$$

Or, starting with (11.20):

$$\zeta_r = \frac{\Delta V}{\Delta x} - \frac{\Delta U}{\Delta y} = \frac{(M\cos\alpha - 0)}{R\cos\alpha} - \frac{(-M\sin\alpha - 0)}{R\sin\alpha}$$

Thus:

$$\zeta_r = \frac{M}{R} + \frac{M}{R} = \boxed{\frac{2M}{R}}$$

A special case of the last equation is where the radial shear is just great enough so that winds at different radii sweep out identical angular velocities about the center of curvature. In other words, the fluid rotates as a solid body. For this case, the last equation reduces to

$$\zeta_r = \frac{2M}{R} \qquad \bullet(11.22)$$

Absolute Vorticity

Measured with respect to the "fixed" stars, the total vorticity must include the Earth's rotation in addition to the relative vorticity. This sum is called the absolute vorticity ζ_a :

$$\zeta_a = \zeta_r + f_c \qquad \bullet(11.23)$$

where the Coriolis parameter $f_c = 2\Omega\cdot\sin(\phi)$ is a measure of the vorticity of the planet, ϕ is latitude, and where $2\Omega = 1.458\text{x}10^{-4}\ s^{-1}$.

Potential Vorticity

Potential vorticity ζ_p is defined as the absolute vorticity divided by the depth Δz of the column of air that is rotating:

$$\zeta_p = \frac{\zeta_r + f_c}{\Delta z} = \text{constant} \qquad \bullet(11.24)$$

It has units of ($m^{-1}\cdot s^{-1}$). In the absence of turbulent drag and heating (latent, radiative, etc.), potential vorticity is conserved.

Solved Example

At 50°N is a west wind of 100 m/s. At 46°N is a west wind of 50 m/s. Find (a) relative, (b) absolute vorticity.

Solution

Given: U_2 =100 m/s, U_1 =50 m/s, $\Delta\phi$ =4°lat. V =0 m/s
Find: ζ_r = ? s^{-1} , ζ_a = ? s^{-1}

(a) Use eq. (11.20): Appendix A: 1°latitude = 111 km
ζ_r = – (100 – 50 m/s) / (4.4x10^5 m) = **–1.14x10^{-4} s^{-1}**

(b) Average ϕ = 48°N. Thus, f_c = (1.458x10^{-4} s^{-1})·sin(48°)
f_c = 1.08x10^{-4} s^{-1}. then, use eq. (11.23):
ζ_a = (–1.14x10^{-4}) + (1.08x10^{-4}) = **–6x10^{-6} s^{-1}** .

Check: Units OK. Physics OK.
Discussion: Shear vorticity from a strong jet stream, but its vorticity is opposite to the Earth's rotation.

Solved Example

If wind rotates as a solid body about the center of a low pressure system, and the tangential velocity is 10 m/s at radius 300 km, find the relative vorticity.

Solution

Given: M = 10 m/s (cyclonic), R = 300,000 m.
Find: ζ_r = ? s^{-1}

Use eq. (11.22):
ζ_r = 2 · (10 m/s) / (3x10^5 m) = **6.67x10^{-5} s^{-1}**

Check: Units OK. Physics OK.
Discussion: Relative vorticities of synoptic storms are typically about this order of magnitude.

Solved Example

An 11 km deep layer of air at 45°N latitude has no curvature, but has a shear of –10 m/s across horizontal distance 500 km. What is the potential vorticity?

Solution

Assume the shear is in the cyclonic direction.
Given: Δz = 11000 m, ϕ = 45°N, ΔM = –10 m/s,
ΔR = 500000 m.
Find: ζ_p = ? $m^{-1}\cdot s^{-1}$

Use eq. (11.24):

$$\zeta_p = \frac{\frac{(10\text{m/s})}{5\times10^5\text{m}} + (1.458\times10^{-4}\text{s}^{-1})\cdot\sin(45°)}{11000\text{m}}$$

= (2+10.31)x10^{-5} /11000 m = **1.12x10^{-8} m^{-1}·s^{-1}**

Check: Units OK. Physics OK.
Discussion: Planetary vorticity is 5 times greater than the relative vorticity for this case. It cannot be neglected when computing potential vorticity.

Solved Example
Find the IPV for the previous example ($\zeta_a = 1.23\text{x}10^{-4}$ s^{-1}), using ρ = 0.5 kg/m^3 and $\Delta\theta/\Delta z$ = 3.3 K/km.

Solution
Given: $\zeta_a = 1.23\text{x}10^{-4}$ s^{-1} , ρ = 0.5 kg/m^3 ,
$\Delta\theta/\Delta z$ = 3.3 K/km , Δz = 11 km
Find: ζ_{IPV} = ? PVU

Use eq. (11.26):

$$\zeta_{IPV} = \frac{(1.23\times10^{-4}\text{s}^{-1})}{(0.5\text{kg}\cdot\text{m}^{-3})}\cdot\left(3.3\text{K}\cdot\text{km}^{-1}\right)$$

$$= 8.12\text{x}10^{-7}\ \text{K·m}^2\text{·s}^{-1}\text{·kg}^{-1} = \underline{\textbf{0.812 PVU}}$$

Check: Units OK. Physics OK.
Discussion: This is a reasonable value (< 1.5 PVU) in troposphere.

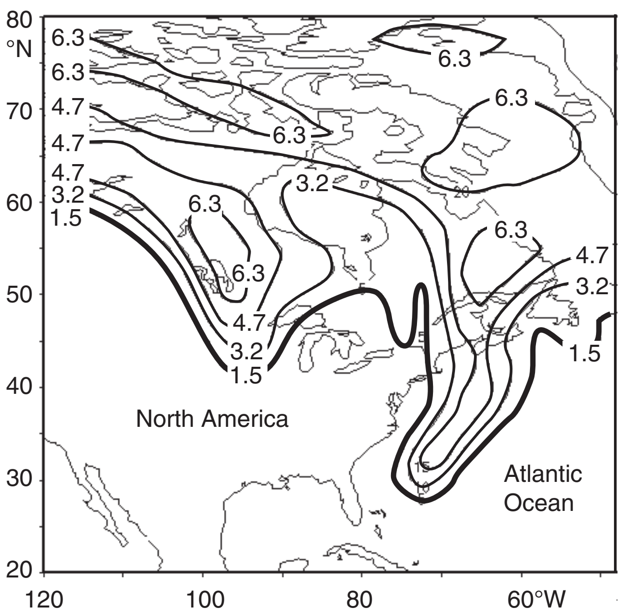


Figure 11.45
Example of isentropic potential vorticity on the 315 K isentropic surface, at 00 UTC on 5 January 2001. Units are PVU. Values greater than 1.5 are in stratospheric air. Because the tropopause is at lower altitude near the poles than at the equator, the 315 K potential temperature surface crosses the tropopause; so it is within the troposphere in the south part of the figure and in the stratosphere in the north part. Tropopause folds are evident by the high PVU values just west of the Great Lakes, and just east of the North American coastline. (Based on NCEP/NCAR 40-year reanalysis data, with initial plots produced using the plotting tool by Christopher Godfrey, the University of Oklahoma School of Meteorology.)

Combining the previous equations yields:

$$-\frac{\Delta M}{\Delta n} + \frac{M}{R} + f_c = \zeta_p \cdot \Delta z \qquad \bullet(11.25)$$

shear curvature planetary stretching

where ζ_p is a constant that depends on the initial conditions of the flow. The last term states that if the column of rotating air is stretched vertically, then its relative vorticity must increase or it must move further north where planetary vorticity is greater.

Isentropic Potential Vorticity

Isentropic potential vorticity (IPV) is

$$\zeta_{IPV} = \frac{\zeta_r + f_c}{\rho}\cdot\left(\frac{\Delta\theta}{\Delta z}\right) \qquad (11.26)$$

where ζ_r is the relative vorticity measured on an isentropic surface (i.e., a surface connecting points of equal potential temperature θ), and ρ is air density. Larger IPVs exist where the air is less dense and where the static stability $\Delta\theta/\Delta z$ is greater. For this reason, the IPV is two orders of magnitude greater in the stratosphere than in the troposphere.

Using the hydrostatic relation (Chapter 1), we can rewrite this as

$$\zeta_{IPV} = -|g|\cdot(\zeta_r + f_c)\cdot\frac{\Delta\theta}{\Delta p} \qquad (11.27)$$

IPV is measured in **potential vorticity units** (**PVU**), defined by 1 PVU = 10^{-6} K·m^2·s^{-1}·kg^{-1} . On average, ζ_{IPV} < 1.5 PVU for tropospheric air, and ζ_{IPV} > 1.5 PVU for stratospheric air (a good atmospheric cross-section example is shown in the Extratropical Cyclone chapter).

Isentropic potential vorticity is conserved for air moving adiabatically and frictionlessly along an isentropic surface (i.e., a surface of constant potential temperature). Thus, it can be used as a tracer of air motion.

Stratospheric air entrained into the troposphere retains its IPV > 1.5 PVU for a while before losing its identity due to turbulent mixing. Thus, IPV is useful for finding tropopause folds and the accompanying intrusions of stratospheric air into the troposphere (Fig. 11.45), which can bring down toward the ground the higher ozone concentrations and **radionuclides** (radioactive atoms from former atomic-bomb tests) from the stratosphere.

Because isentropic potential vorticity is conserved, if static stability ($\Delta\theta/\Delta z$) weakens, then eq. (11.26) says absolute vorticity must increase to maintain constant IPV. For example, Fig. 11.46 shows isentropes for flow from west to east across the

Rocky Mountains. Where isentropes are spread far apart, static stability is low. Because air tends to follow isentropes (for frictionless adiabatic flow), a column of air between two isentropes over the crest of the Rockies will remain between the same two isentropes as the air continues eastward.

Thus, the column of air shown in Fig. 11.46 becomes stretched on the lee side of the Rockies and its static stability decreases (same $\Delta\theta$, but spread over a larger Δz). Thus, absolute vorticity in the stretched region must increase. Such increased cyclonic vorticity encourages formation of low-pressure systems (extratropical cyclones) to the lee of the Rockies — a process called **lee cyclogenesis**.

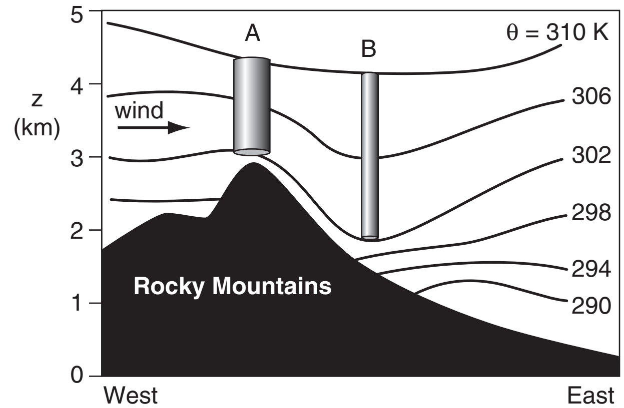


Figure 11.46
Wind blowing from west to east (A) over the Rocky Mountains creates mountain waves and downslope winds, which cause greater separation (B) between the 302 and 310 K isentropes to the lee of the mountains. This greater separation implies reduced static stability and vertical stretching.

Solved Example
Given Fig. 11.46. (a) Estimate $\Delta\theta/\Delta z$ at A and B. (b) if the initial absolute vorticity at A is 10^{-4} s^{-1}, find the absolute vorticity at B.

Solution
Given: $\zeta_a = 10^{-4}$ s^{-1} at A, θ_{top} = 310 K, θ_{bottom} = 302 K.
Find: (a) $\Delta\theta/\Delta z$ = ? K/km at A and B.
(b) ζ_a = ? s^{-1} at B.

$\Delta\theta$ = 310 K – 302 K = 8 K at A & B. Estimate the altitudes at the top and bottom of the cylinders in Fig. 11.46.
A: z_{top} = 4.4 km, z_{bottom} = 3.1 km. Thus Δz_A = 1.3 km.
B: z_{top} = 4.2 km, z_{bottom} = 1.9 km. Thus Δz_B = 2.3 km.

(a) $\Delta\theta/\Delta z$ = 8K / 1.3 km = **6.15 K/km** at A.
$\Delta\theta/\Delta z$ = 8K / 2.3 km = **3.48 K/km** at B.

(b) If initial (A) and final (B) IPV are equal, then rearranging eq. (11.26) and substituting $(\zeta_r + f_c) = \zeta_a$ gives:
$\zeta_{aB} = \zeta_{aA} \cdot (\Delta z_B/\Delta z_A)$
$= (10^{-4}$ s$^{-1}) \cdot [(2.3$ km$)/(1.3$ km$)]$ = **1.77x 10^{-4}** s^{-1}

Check: Units OK. Sign OK. Magnitude good.
Discussion: Static stability $\Delta\theta/\Delta z$ is much weaker at B than A. Thus absolute vorticity at B is much greater than at A. If the air flow directly from west to east and if the initial relative vorticity were zero, then the final relative vorticity is $\zeta_r = 0.77\times10^{-4}$ s^{-1}. Namely, to the lee of the mountains, cyclonic rotation forms in the air where none existed upwind. Namely, this implies cyclogenesis (birth of cyclones) to the lee (downwind) of the Rocky Mountains.

HORIZONTAL CIRCULATION

Consider a closed shape of finite area (Fig. 11.47a). Pick any starting point on the perimeter, and hypothetically travel counterclockwise around the perimeter until you return to the starting point. As you travel each increment of distance Δl, observe the local winds along that increment, and get the average tangential component of wind velocity M_t.

The horizontal **circulation** C is defined as the product of this tangential velocity times distance increment, summed over all the increments around the whole perimeter:

$$C = \sum_{i=1}^{n} \left(M_t \cdot \Delta l \right)_i \tag{11.28}$$

where i is the index of each increment, and n is the number of increments needed to complete one circuit around the perimeter. Take care that the sign of M_t is such that it is positive if the tangential wind is in the same direction as you are traveling, and negative if opposite. The units of circulation are m^2·s^{-1}.

If we approximate the perimeter by Cartesian line segments (Fig. 11.47b), then eq. (11.28) becomes:

$$C = \sum_{i=1}^{n} \left(U \cdot \Delta x + V \cdot \Delta y \right)_i \tag{11.29}$$

The sign of Δx is (+) if you travel in the positive x-direction (toward the East), and (–) if opposite. Similar rules apply for Δy (+ toward North).

To better understand circulation, consider idealized cases (Figs. 11.48a & b). For winds of tangential velocity M_t rotating counterclockwise around a circle of radius R, the circulation is $C = 2\pi R \cdot M_t$. For clockwise circular rotation, the circulation is $C = -2\pi R \cdot M_t$, namely, the circulation value is negative. From these two equations, we see that a fast speed around a small circle (such as a tornado) can give

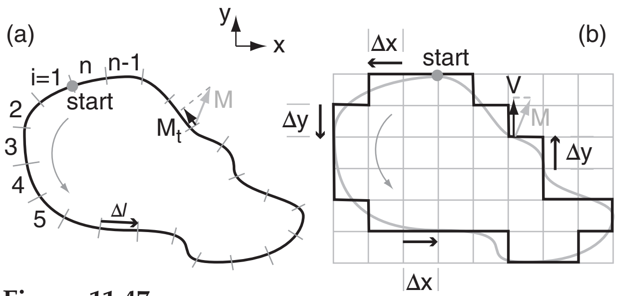


Figure 11.47
Method for finding the circulation. M is the wind vector. M_t and V are projections onto the perimeter. (a) Stepping in increments of Δl around an arbitrary shape. (b) Stepping around a Cartesian (gridded) approximation to the shape in (a).

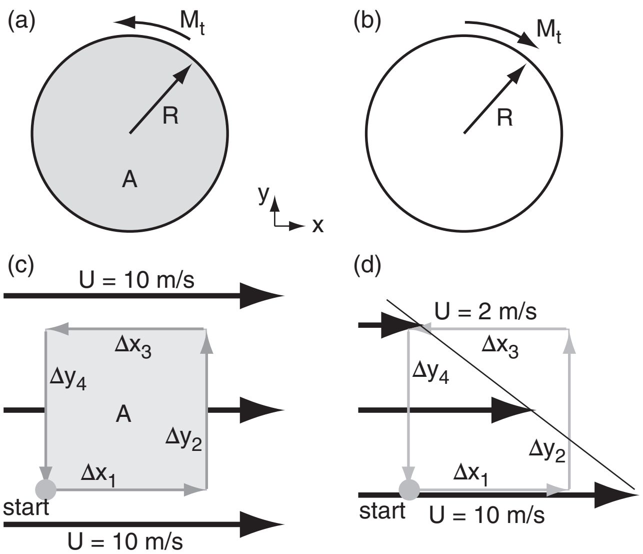


Figure 11.48
Circulation examples. (a) Counterclockwise rotation around a circle. (b) Clockwise rotation around a circle. (c) Uniform wind. (d) Uniform shear. Area A enclosed by circulation is shaded.

Solved Example
Find the horizontal circulation for Fig. 11.48d. Assume $\Delta x = \Delta y = 1$ km. Relate to shear and rel. vorticity.

Solution
Given: U_{bottom}= 10 m/s, U_{top}= 2 m/s, V=0, Δx=Δy=1 km
Find: C = ? m²/s, ζ_r = ? s⁻¹

Use eq. (11.29) from start point:
$C = (U\cdot\Delta x)_1 + (V\cdot\Delta y)_2 + (U\cdot\Delta x)_3 + (V\cdot\Delta x)_4$
(10m/s)·(1km) + 0 + (2m/s)·(–1km) + 0
C = 8 (m/s)·km = **<u>8000 m</u>**^{2}/s
Use eq. (11.30): with area $A = \Delta x\cdot\Delta y$ = 1 km^2
$U_{shear} = \Delta U/\Delta y = (U_{top} - U_{bottom})/\Delta y$
= [(2 – 10 m/s)/(1 km)] = –8 (m/s)/km
C = [0 – U_{shear}]·A = –[–8 (m/s)/km]· (1 km^2)
= 8 (m/s)·km = **<u>8000 m</u>**^{2}/s.
Use eq. (11.31): $\zeta_r = C/A$ = 8 (m/s)/km = **<u>0.008 s</u>**$^{-1}$.

Check: Units OK. Physics OK. Magnitude OK.
Discussion: Strong shear. Large circ. Large vorticity.

the same circulation magnitude as a slower speed around a larger circle (e.g., a mid-latitude cyclone).

Consider two more cases (Figs. 11.48c & d). For a circuit in a constant wind field of any speed, the circulation is C = 0. For a circuit within a region of uniform shear such as $\Delta U/\Delta y$, the circulation is $C = -(\Delta U/\Delta y)\cdot(\Delta y\cdot\Delta x)$. Comparing these last two cases, we see that the wind speed is irrelevant for the circulation, but the wind shear is very important.

In the last equation above, $(\Delta y\cdot\Delta x) = A$ is the area enclosed by the circulation. In general, for uniform U and V <u>shear</u> across a region, the horizontal circulation is:

$$C = \left(\frac{\Delta V}{\Delta x} - \frac{\Delta U}{\Delta y}\right)\cdot A \tag{11.30}$$

But the term in parentheses is the relative vorticity ζ_r. This gives an important relationship between horizontal circulation and vorticity:

$$C = \zeta_r \cdot A \tag{11.31}$$

Vorticity is defined at any one point in a fluid, while circulation is defined around a finite-size area. Thus, eq. (11.31) is valid only in the limit as A becomes small, or for the special case of a fluid having uniform vorticity within the whole circulation area.

The horizontal circulation C defined by eq. (11.30 & 11.31) is also known as the **relative circulation** C_r . An **absolute circulation** C_a can be defined as

$$C_a = (\zeta_r + f_c)\cdot A \tag{11.32}$$

where f_c is the Coriolis parameter. The absolute circulation is the circulation that would be seen from a fixed point in space looking down on the atmosphere rotating with the Earth.

For the special case of a frictionless **barotropic atmosphere** (where isopycnics are parallel to isobars), **Kelvin's circulation theorem** states that C_a is constant with time.

For a more realistic **baroclinic atmosphere** containing horizontal temperature gradients, the **Bjerknes circulation theorem:**

$$\frac{\Delta C_r}{\Delta t} = -\sum_{i=1}^{n}\left(\frac{\Delta P}{\rho}\right)_i - f_c\cdot\frac{\Delta A}{\Delta t} \tag{11.33}$$

says relative circulation varies with the torque applied to the fluid (via the component of pressure forces in the direction of travel, summed around the perimeter of the circulation area) minus the Earth's rotation effects in a changing circulation area. The units of $\Delta P/\rho$ are J/kg, which are equivalent to the m^2·s^{-2} units of $\Delta C_r/\Delta t$. The pressure term in eq. (11.33) is called the **solenoid term**.

MID-LATITUDE TROUGHS AND RIDGES

The jet streams do not follow parallels in zonal flow around the Earth to make perfect circles around the north and south poles. Instabilities in the atmosphere cause the jet stream to meander north and south in a wavy pattern as they encircle the globe (Fig. 11.49). As already mentioned, these waves are called **planetary waves** or **Rossby waves**, and have a wavelength of roughly 3000 to 4000 km (Fig. 11.44). The number of waves around the globe is called the **zonal wavenumber**, and is typically 7 or 8, although they can range from about 3 to 13.

In Fig. 11.49, jet stream winds (shaded) follow the height contours in a general counterclockwise (west to east) circulation as viewed looking down on the North Pole. The jet stream roughly demarcates the boundary between the cold polar air from the warmer tropical air, because this temperature difference generates the jet stream winds due to the thermal-wind relationship.

Regions of relatively low pressure or low height are called **troughs**. The center of the trough, called the **trough axis**, is indicated with a dashed line. Winds turn cyclonically (counterclockwise in the N. Hemisphere) around troughs; hence, troughs and low-pressure centers (lows) are similar.

Ridges of relatively high pressures or heights are between the troughs. **Ridge axes** are indicated with a zig-zag line. Air turns anticyclonically (clockwise in the N. Hemisphere) around ridges; hence, ridges and high-pressure centers (highs) are similar.

Two types of instabilities trigger these waves in the global circulation: barotropic instability and baroclinic instability. Barotropic instability, caused by the Earth's rotation, is described next. Baroclinic instability adds the effects of the north-south temperature gradient.

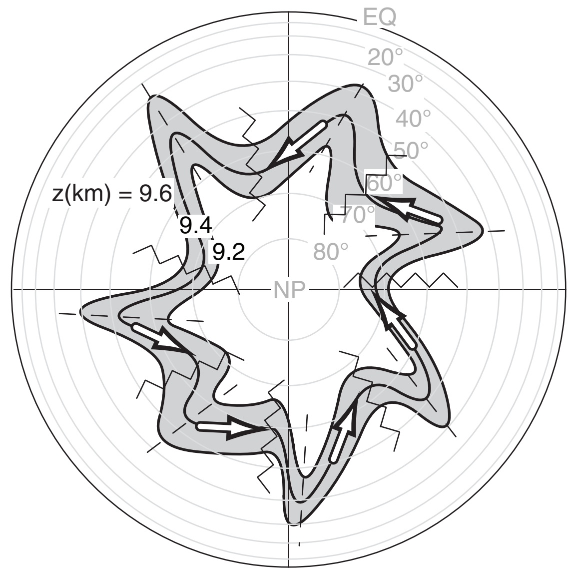


Figure 11.49
Height (z) contours of the 30 kPa pressure surface. View is looking down on the north pole (NP), and circles are parallels labeled with latitude. Dashed lines indicate trough axes; zig-zag lines indicate ridge axes. The jet stream is shaded, with white arrows showing wind direction. EQ is the equator.

Barotropic Instability & Rossby Waves

Picture the jet stream at mid-latitudes, blowing from west to east. If some small disturbance (such as flow over mountains, discussed in the Extratropical Cyclones chapter) causes the jet to turn slightly northward, then conservation of potential vorticity causes the jet to meander north and south. This meander is the **Rossby wave** or **planetary wave**.

To understand this process, picture initially-zonal flow at mid-latitudes, such as sketched in Fig. 11.50 at point 1. Straight zonal flow has no relative vorticity (no shear or curvature; i.e., $M/R = 0$), but there is the planetary vorticity term in eqs. (11.23 - 11.25) related to the latitude of the flow. For the

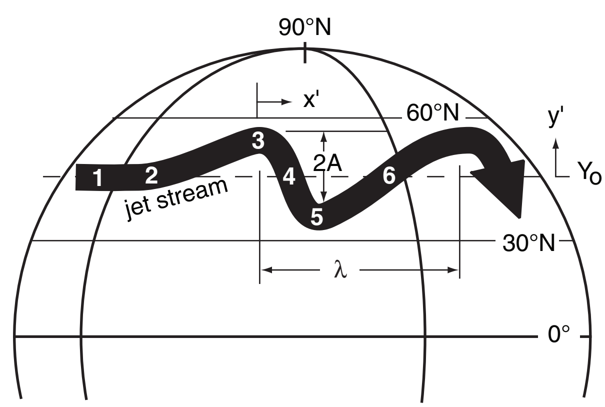


Figure 11.50
Initially zonal flow of the jet stream at point 1, if disturbed at point 2, will develop north-south meanders called Rossby waves. λ = wavelength. A = wave amplitude.

BEYOND ALGEBRA • Vorticity of a Wave

Problem: How does relative vorticity ζ_r vary with distance east (x) in a planetary wave?

Solution: Idealize the planetary wave as a sine wave

$$y = A \cdot \sin(2\pi \cdot x / \lambda) \tag{a}$$

where y is distance north of some reference parallel (such as 45°N), A is north-south amplitude, and λ is east-west wavelength. Assume constant wind speed M following the path of the sine wave. The U and V components can be found from the slope s of the curve. Combining $s = V/U$ and $U^2 + V^2 = M^2$ gives:

$$U = M \cdot (1+s^2)^{-1/2} \quad \text{and} \quad V = M \cdot s \cdot (1+s^2)^{-1/2} \tag{b}$$

But the slope is just the first derivative of eq. (a)

$$s = \partial y / \partial x = (2\pi A / \lambda) \cdot \cos(2\pi \cdot x / \lambda) \tag{c}$$

Write eq. (11.20) in terms of partial derivatives:

$$\zeta_r = \partial V / \partial x - \partial U / \partial y \tag{d}$$

which can be rewritten as:

$$\zeta_r = \partial V / \partial x - (\partial U / \partial x) \cdot (\partial x / \partial y)$$

or

$$\zeta_r = \partial V / \partial x - (\partial U / \partial x) \cdot (1 / s) \tag{e}$$

Plugging eqs. (b) and (c) into (e) gives the answer:

$$\zeta_r = \frac{-2 \cdot M \cdot A \cdot \left(\frac{2\pi}{\lambda}\right)^2 \cdot \sin\left(\frac{2\pi x}{\lambda}\right)}{\left[1 + \left(\frac{2\pi A}{\lambda}\right)^2 \cdot \cos^2\left(\frac{2\pi x}{\lambda}\right)\right]^{3/2}} \tag{f}$$

This is plotted in Fig. 11.a below, for an example where λ = 4000 km, A = 1000 km, and M = 50 m/s.

Discussion: $|\zeta_r| = 2M/R$, where R is the radius of curvature for a sine wave, as is given in many calculus textbooks. Notice the narrow, sharp peaks of vorticity. Increased wave amplitude narrows the vorticity peaks.

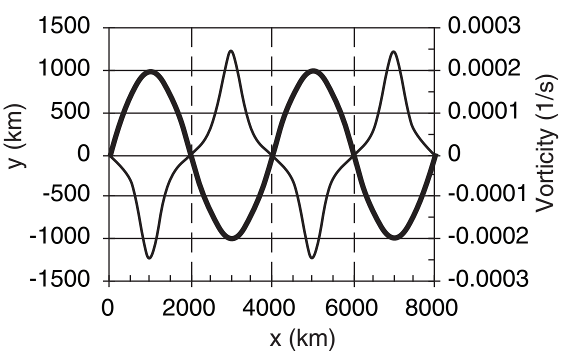


Figure 11.a.
Streamline of an idealized planetary wave (thick), and the corresponding vorticity (thin).

special case of a fluid of fixed depth Δz such as the troposphere (Δz = 11 km), the conservation of potential vorticity simplifies to

$$\left[\frac{M}{R} + f_c\right]_{initial} = \left[\frac{M}{R} + f_c\right]_{later} \tag{11.34}$$

where we will focus on the curvature term as a surrogate to the full relative vorticity.

If this flow is perturbed slightly (at point 2) to turn to the north, the air is now moving into higher latitudes where the Coriolis parameter and planetary vorticity are greater. Thus, a negative shear or curvature (negative R) must form in the flow to compensate the increased planetary shear, in order to keep potential vorticity constant. In plain words, the jet turns clockwise in the N. Hemisphere (anticyclonic) at point 3 until it points southeast.

As it proceeds southward toward its initial latitude (point 4), it has less curvature (i.e., less relative vorticity), but still points southeast. The jet then overshoots south of its initial latitude to a region where planetary vorticity is less (point 5). To preserve potential vorticity, it develops a cyclonic (counterclockwise in N. Hemisphere) curvature and heads back northeast. Thus, initially stable (zonal) flow from point 1 has become wavy, and is said to have become **unstable**.

This Rossby wave requires a variation of Coriolis parameter with latitude to create the instability — an effect called **barotropic instability**. Parameter $\beta = \Delta f_c / \Delta y$ gives the rate of change of Coriolis parameter f_c with distance north y:

$$\beta = \frac{\Delta f_c}{\Delta y} = \frac{2 \cdot \Omega}{R_{earth}} \cdot \cos\phi \qquad \bullet(11.35)$$

where R_{Earth} =6371 km is the average Earth radius. Thus, $2 \cdot \Omega / R_{Earth} = 2.29\text{x}10^{-11}\ \text{m}^{-1} \cdot \text{s}^{-1}$, and β is on the order of (1.5 to 2)x$10^{-11}\ \text{m}^{-1} \cdot \text{s}^{-1}$.

The path taken by the wave is approximately:

$$y' \approx A \cdot \cos\left[2\pi \cdot \left(\frac{x' - c \cdot t}{\lambda}\right)\right] \tag{11.36}$$

where y' is the north-south displacement distance from the center latitude Y_o of the wave, x' is the distance east from some arbitrary longitude, c is the phase speed (the speed at which the crest of the wave moves relative to the Earth), A is the north-south amplitude of the wave, λ is the wavelength (see Fig. 11.50), and the primes indicate the deviation from a mean background state.

Typical wavelengths are $\lambda \approx 6000$ km, although a wide range of wavelengths can occur. The circumference ($2\pi \cdot R_{Earth} \cdot \cos\phi$) along a parallel at mid-latitudes limits the total number of barotropic waves that can fit around the globe to about 4 to 5. The north-south domain of the wave roughly corresponds to the 30°-60° mid-latitudes where the jet stream is strongest, giving $A \approx 1665$ km.

These waves propagate <u>relative</u> to the mean zonal wind U_o at **intrinsic phase speed** c_o of about

$$c_o = -\beta \cdot \left(\frac{\lambda}{2\pi}\right)^2 \qquad \bullet(11.37)$$

where the negative sign indicates westward propagation relative to the mean background flow. However, when typical values for β and λ are used in eq. (11.37), the intrinsic phase speed is roughly half the magnitude of the eastward jet-stream speed.

A **phase speed** c relative to the ground is defined as:

$$c = U_o + c_o \qquad \bullet(11.38)$$

which gives the west-to-east movement of the wave crest. For typical values of c_o and background zonal wind speed U_o, the phase speed c is positive. Hence, the mean wind pushes the waves toward the east relative to observers on the Earth. Eastward moving waves are indeed observed.

Equation (11.37) is called a **dispersion relation** because waves of different wavelengths propagate at different phase speeds. Waves that initially coincide would tend to separate or disperse with time.

By combining eqs. (11.37) and (11.38), we see that waves of shorter wavelength (**short waves**) travel faster toward the east than **long waves** (Fig. 11.51). Thus, short waves ride the long wave analogous to a car driving along a hilly road (see solved example).

BEYOND ALGEBRA • The Beta Plane

We can derive β from the definition of f_c. Start with eq. (10.16):

$$f_c = 2\,\Omega \sin\phi$$

where ϕ is latitude.

Since y is the distance along the perimeter of a circle of radius R_{Earth}, recall from geometry that

$$y = R_{Earth} \cdot \phi$$

for ϕ in radians.

Rearrange this to solve for ϕ, and then plug into the first equation to give:

$$f_c = 2\,\Omega \sin(y/R_{Earth})$$

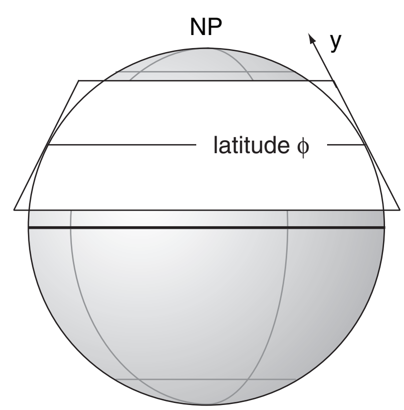
By definition of β, take the derivative to find

$$\beta = \frac{\partial f_c}{\partial y} = \frac{2 \cdot \Omega}{R_{earth}} \cdot \cos\left(\frac{y}{R_{earth}}\right)$$

Finally, use the second equation above to give:

$$\boxed{\beta = \frac{2 \cdot \Omega}{R_{earth}} \cdot \cos\phi} \qquad \bullet(11.35)$$

For midlatitude planetary waves confined to a latitude belt such as sketched in Fig. 11.50, it is often convenient to assume β = constant. This has the same effect as assuming that a portion of the Earth is shaped like a cone (as sketched below in white) rather than a sphere. The surface of the cone is called a **beta plane**, and looks like a lamp shade. Such an idealization allows barotropic effects to be described with slightly simpler math.



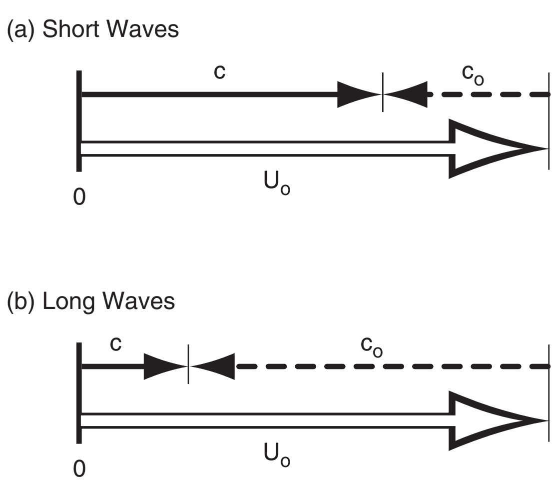


Figure 11.51 *(at left)*
Sum of large background west wind U_o with smaller intrinsic Rossby-wave phase speed c_o from the east gives propagation speed c of Rossby waves relative to the ground.

Solved Example (§)

A background jet stream of speed 50 m/s meanders with 6000 km wavelength and 1500 km amplitude, centered at 45°N. Find the beta parameter. Also, plot the path (i.e., the meridional displacement) of the barotropic wave between $0 \le x' \le 10000$ km at some initial time and 6 hours later, and find the phase speeds.

Solution

Given: $\phi = 45°$, $U_o = 50$ m/s, $\lambda = 6000$ km,
$A = 1500$ km, $t = 0$ and 6 h

Find: $\beta = ?\ m^{-1}{\cdot}s^{-1}$, $y'(x') = ?$ km, $c_o = ?$ m/s,
$c = ?$ m/s

First, use eq. (11.35):

$$\beta = \frac{1.458\times10^{-4}s^{-1}}{6371000m}\cdot\cos(45°) = \underline{\mathbf{1.62x10^{-11}\ m^{-1}{\cdot}s^{-1}}}.$$

Next, use eq. (11.37):

$$c_o = -(1.62\times10^{-11}m^{-1}s^{-1})\left(\frac{6\times10^6 m}{2\pi}\right)^2$$
$$= \underline{\mathbf{-14.7\ m/s}}$$

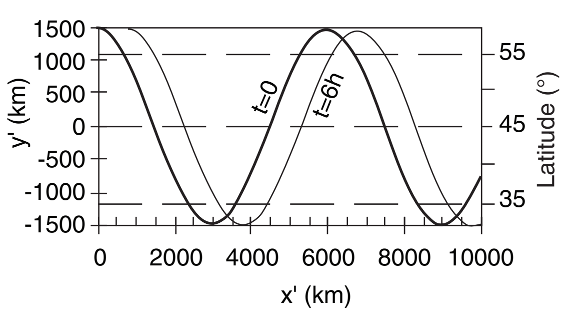
Use eq. (11.38):

$$c = (50 - 14.7)\ m/s = \underline{\mathbf{35.26\ m/s}}$$

Use eq. (11.36) on a spreadsheet for $t = 0$ & 6 h:

$$y' \approx (1500km)\cdot\cos\left[2\pi\cdot\left(\frac{x' - (35.26m/s)\cdot t}{6\times10^6 m}\right)\right]$$

The results are plotted below:



Check: Units OK. Physics OK.

Discussion: The barotropic long wave propagates east about 800 km during the 6 h interval. The jet stream blows along this wavy path at speed 50 m/s.

Although this Rossby-wave phase speed is much slower than a jet airliner, the wave does not need to land and refuel. Thus, during 24 hours, this long wave could travel over 3000 km — greater than the distance between San Francisco, CA and Chicago, IL, USA.

This is why weather systems can move so far in one day.

Solved Example (§)

Same as the previous solved example, except that in addition to the previous long wave, there is also a barotropic short wave of 1000 km wavelength with amplitude 300 km.

Solution

Given: Same, plus $\lambda = 1000$ km, $A = 300$ km.

Find: $c = ?$ m/s, $y'(x') = ?$ km

Use eq. (11.37):

$$c_o = -(1.62\times10^{-11}m^{-1}s^{-1})\left(\frac{10^6 m}{2\pi}\right)^2 = \underline{\mathbf{-0.41\ m/s}}$$

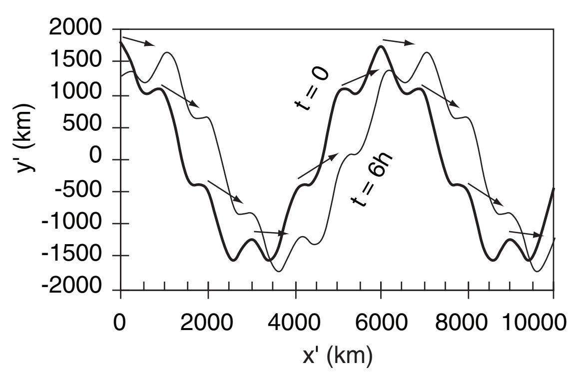
Use eq. (11.38):

$$c = (50 - 0.41)\ m/s = \underline{\mathbf{49.6\ m/s}}$$

Use eq. (11.36):

$$y' \approx (300km)\cdot\cos\left[2\pi\cdot\left(\frac{x' - (49.6m/s)\cdot t}{1\times10^6 m}\right)\right]$$

The results are plotted below:



Check: Units OK. Physics OK.

Discussion: The movement of the crest of each short wave is indicated with the arrows. Not only do they move with the propagating background long wave, but they also ride the long wave toward the east at a speed nearly equal to the wind speed in this example.

At this speed, a short wave could travel about 4285 km during 24 hours, more than the distance from San Francisco, CA to New York City, USA. Not only do these short waves travel very fast, but they are in and out of the city very quickly. Thus, they cause rapid changes in the weather.

For this reason, weather forecasters pay particular attention to short waves to avoid surprises. Although short waves are sometimes difficult to spot visually in a plot of geopotential height, you can see them more easily on plots of 50 kPa vorticity. Namely, each short wave has a vorticity that can be highlighted or colorized on a forecast map.

Baroclinic Instability & Rossby Waves

Recall from Figs. 11.1b and 11.37a that, at mid-latitudes, the cold polar air slides under the warmer tropical air. This causes the air to be statically stable, as can be quantified by a Brunt-Väisälä frequency N_{BV}. The development later in this section extends what we learned of barotropic flows to the more complete baroclinic case having both β and N_{BV} effects in an environment with north-south temperature gradient.

First, we take a heuristic approach, and idealize the atmosphere as being a two-layer fluid, with a north-south sloping density interface such as idealized in Fig. 11.52. This will give us a qualitative picture of baroclinic waves.

Qualitative Model

Initially-zonal flow of the jet stream (sketched as the dark-shaded ribbon of air at point 1) has no relative vorticity, but it does have planetary vorticity related to its latitude. The narrow white column in the front left of Fig. 11.52 represents air having such absolute vorticity.

If this jet stream is perturbed northward by some outside influence such as a mountain, it rides up on the density interface over the cold air. The stratosphere is so statically stable that it acts like a lid on the troposphere. Thus, as the ribbon of air meanders northward, it is squeezed between the tropopause and the rising density interface. Namely, Δz shrinks. For this situation, the potential vorticity equation can be written as:

$$\left[\frac{f_c + (M/R)}{\Delta z}\right]_{initial} = \left[\frac{f_c + (M/R)}{\Delta z}\right]_{later} \qquad (11.39)$$

where f_c is the Coriolis parameter, M is the wind speed, and R is the radius of curvature.

The column depth is less at point 3 than initially at point 2; hence, the absolute vorticity at 3 must also be less than that at 2, for the ratio of absolute vorticity to depth to remain constant. The planetary contribution does not help – in fact, it is larger at point 3 because it is at a higher latitude. Hence, the only way to conserve potential vorticity is for the relative vorticity to decrease substantially. As it decreases below its initial value of zero, the jet-stream path curves anticyclonically at point 3.

The jet stream overshoots to the south, develops cyclonic relative vorticity and turns back to the north (not sketched in Fig. 11.52). This breakdown of zonal flow into wavy flow is called **baroclinic instability**. The baroclinic Rossby waves look similar to those in Fig. 11.50, except with <u>shorter</u> wavelength λ than barotropic waves because now <u>both</u> f_c and Δz work together to cause the oscillation.

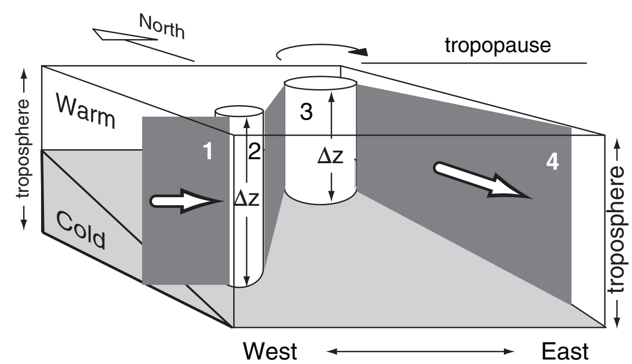


Figure 11.52
Some of the processes involved in baroclinic instability. Dark gray ribbon represents the jet-stream axis, while white columns indicate rotation associated with the absolute vorticity of the jet.

Solved Example(§)

Same as the previous solved example, except for a baroclinic wave in a standard atmosphere, with the one dominant wave. Wave amplitude is 1500 km. Plot results at $t = 0$ and 6 h for $z = 0$ and 11 km.

Solution

Given: Same, and use Table 1-5 for std. atmos:
$\Delta T = -56.5-15 = -71.5°C$ across $\Delta z = 11$ km.
$T_{avg} = 0.5\cdot(-56.5+15°C) = -20.8°C = 252$ K
Find: λ_R = ? km, λ = ? km, c = ? m/s, $y'(x')$ = ? km

Use eq. (5.4a):

$$N_{BV} = \sqrt{\frac{(9.8\text{m/s})}{252\text{K}}\left(\frac{-71.5\text{K}}{1.1\times10^4\text{m}}+0.0098\frac{\text{K}}{\text{m}}\right)}$$

$N_{BV} = 0.0113\ \text{s}^{-1}$

Use eq. (10.16):

$f_c = (1.458\times10^{-4}s^{-1})\cdot\sin(45°) = 1.031\text{x}10^{-4}\ \text{s}^{-1}$

Use eq. (11.12):

$$\lambda_R = \frac{(0.0113\text{s}^{-1})\cdot(11\text{km})}{1.03\times10^{-4}\text{s}^{-1}} = \underline{\textbf{1206 km}}$$

Use eq. (11.43):

$\lambda = \lambda_d = 2.38\ \lambda_R = \underline{\textbf{2870 km}}$

Use eq. (11.41):

$$c_o = \frac{-(1.62\times10^{-11}\text{m}^{-1}\text{s}^{-1})}{\pi^2\cdot\left[\frac{4}{(2870\text{km})^2}+\frac{1}{(1206\text{km})^2}\right]} = \underline{\textbf{-1.40 m/s}}$$

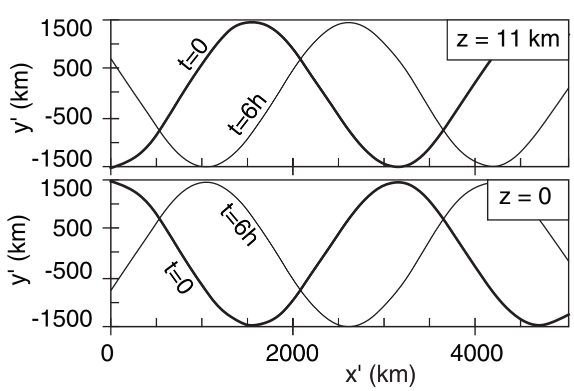
Use eq. (11.42):

$c = (50-1.40)\text{m/s} = \underline{\textbf{48.6 m/s}}$

Use eq. (11.40):

$$y' \approx (1500\text{km})\cos\left(\frac{\pi\cdot z}{11\text{km}}\right)\cos\left[2\pi\left(\frac{x'-48.6\text{m/s}\cdot t}{2870\text{km}}\right)\right]$$

The results are plotted below:



Check: Units OK. Physics OK.
Discussion: The Rossby wave at z = 11 km is indeed 180° out of phase from that at the surface.

Quantitative Approach

To extend this argument, consider the continuous stratification of Fig. 5.20 (reproduced on the next page). Between the 290 K and 340 K isentropes in that illustration, the air is statically stable. A column of air bounded at the bottom and top by these isentropes near the equator shortens as it moves poleward. Thus, we could use the Brunt-Väisälä frequency and the north-south temperature gradient (i.e., the baroclinicity) as a better representation of the physics than the two layer model of Fig. 11.52.

Skipping a long derivation, we end up with a north-south displacement y' of a baroclinic wave that is approximately:

$$y' \approx A\cdot\cos\left(\pi\cdot\frac{z}{Z_T}\right)\cdot\cos\left[2\pi\cdot\left(\frac{x'-c\cdot t}{\lambda}\right)\right] \qquad (11.40)$$

where $Z_T \approx 11$ km is the depth of the troposphere, A is north-south amplitude, c is phase speed, λ is wavelength, x is distance East, and t is time. The extra cosine term containing height z means that the north-south wave amplitude first decreases with height from the surface to the middle of the troposphere, and then increases with opposite sign toward the top of the troposphere. Namely, the planetary wave near the tropopause is 180° out of phase compared to that near the ground.

[*CAUTION: this is an oversimplification. Waves in the real atmosphere aren't always 180° out of phase between top and bottom of the troposphere. Nonetheless, this simple approach gives some insight into the workings of baroclinic waves.*]

The dispersion relation gives intrinsic phase speed c_o as:

$$c_o = \frac{-\beta}{\pi^2\cdot\left[\frac{4}{\lambda^2}+\frac{1}{\lambda_R^2}\right]} \qquad \bullet(11.41)$$

where eq. (11.35) gives β, wavelength is λ, and eq. (11.12) gives the internal Rossby deformation radius λ_R. As before, the phase speed relative to the ground is:

$$c = U_o + c_o \qquad \bullet(11.42)$$

Intrinsic phase speed c_o is negative, with magnitude less than the background jet velocity U_o, allowing the waves to move east relative to the ground.

As for barotropic waves, baroclinic waves have a range of wavelengths that can exist in superposition. However, some wavelengths grow faster than others, and tend to dominate the flow field. For baroclinic waves, this dominant wavelength λ_d is on the order of 3000 to 4000 km, and is given by:

$$\lambda_d \approx 2.38\cdot\lambda_R \qquad (11.43)$$

where λ_R is the internal Rossby radius of deformation (eq. 11.12).

North-south displacement y' is not the only characteristic that is wavy in baroclinic flow. Also wavy are the perturbation velocities (u', v', w'), pressure p', potential temperature θ', and vertical displacement η', where the prime denotes deviation from the mean background state.

To present the wave equations for these other variables, we will simplify the notation by using:

$$a = \pi \cdot z / Z_T \tag{11.44}$$

and

$$b = 2\pi \cdot (x' - c \cdot t) / \lambda \tag{11.45}$$

Thus:

$$\begin{aligned} y' &= \hat{Y} \cdot \cos(a) \cdot \cos(b) \\ \eta' &= \hat{\eta} \cdot \sin(a) \cdot \cos(b) \\ \theta' &= -\hat{\theta} \cdot \sin(a) \cdot \cos(b) \\ p' &= \hat{P} \cdot \cos(a) \cdot \cos(b) \\ u' &= \hat{U} \cdot \cos(a) \cdot \cos(b) \\ v' &= -\hat{V} \cdot \cos(a) \cdot \sin(b) \\ w' &= -\hat{W} \cdot \sin(a) \cdot \sin(b) \end{aligned} \tag{11.46}$$

Symbols wearing the caret hat (^) represent amplitude of the wave. These amplitudes are defined to be always positive (remember c_o is negative) in the Northern Hemisphere:

$$\begin{aligned} \hat{Y} &= A \\ \hat{\eta} &= \frac{A \cdot \pi \cdot f_c \cdot (-c_o)}{Z_T} \cdot \frac{1}{N_{BV}^2} \\ \hat{\theta} &= \frac{A \cdot \pi \cdot f_c \cdot (-c_o)}{Z_T} \cdot \frac{\theta_o}{g} \\ \hat{P} &= A \cdot \rho_o \cdot f_c \cdot (-c_o) \\ \hat{U} &= \left[\frac{A \cdot 2\pi \cdot (-c_o)}{\lambda} \right]^2 \cdot \frac{1}{A \cdot f_c} \\ \hat{V} &= \frac{A \cdot 2\pi \cdot (-c_o)}{\lambda} \\ \hat{W} &= \frac{A \cdot 2\pi \cdot (-c_o)}{\lambda} \cdot \frac{\pi \cdot (-c_o) \cdot f_c}{Z_T \cdot N_{BV}^2} \end{aligned} \tag{11.47}$$

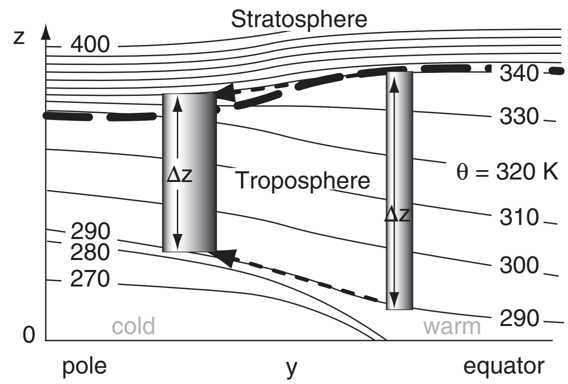


Figure 5.20 (again)
Vertical cross section through the atmosphere, showing isentropes (lines of constant potential temperature θ). The depth Δz of column of air on the right will shrink as the column moves poleward, because air tends to follow isentropes during adiabatic processes. Thus, the same Δθ between top and bottom of the air columns spans a shorter vertical distance for the poleward column, meaning that static stability and Brunt-Väisälä frequency are greater there.

Solved Example

Same as previous solved example, but find the pressure amplitudes at sea level.

Solution

Given: A = 1500 km, f_c = 1.031x10^{-4} s^{-1}, c_o = –1.40 m/s
Find: $\hat{P}$ = ? kPa

Use std. atmosphere for sea level: ρ_o = 1.225 kg·m^{-3} .
Use eq. (11.47):

$$\hat{P} = (1500\text{km})\left(1.225\frac{\text{kg}}{\text{m}^3}\right)(1.031 \times 10^{-4}\text{s}^{-1})\left(1.4\frac{\text{m}}{\text{s}}\right)$$

= **0.265 kPa**

Check: Units OK. Physics OK.
Discussion: Between high and low pressure is a distance of half a wavelength, which is 1435 km for this case (see previous solved example). The pressure difference between crest and trough of the wave is 2· $\hat{P}$ = 0.53 kPa. Thus, the pressure gradient is order of 0.53/1435 = 0.00037 kPa/km, which is sufficient to drive the winds.

In the real atmosphere, condensation in clouds (which was neglected in eqs. 11.47) contributes to the dynamics to make horizontal pressure gradients even stronger.

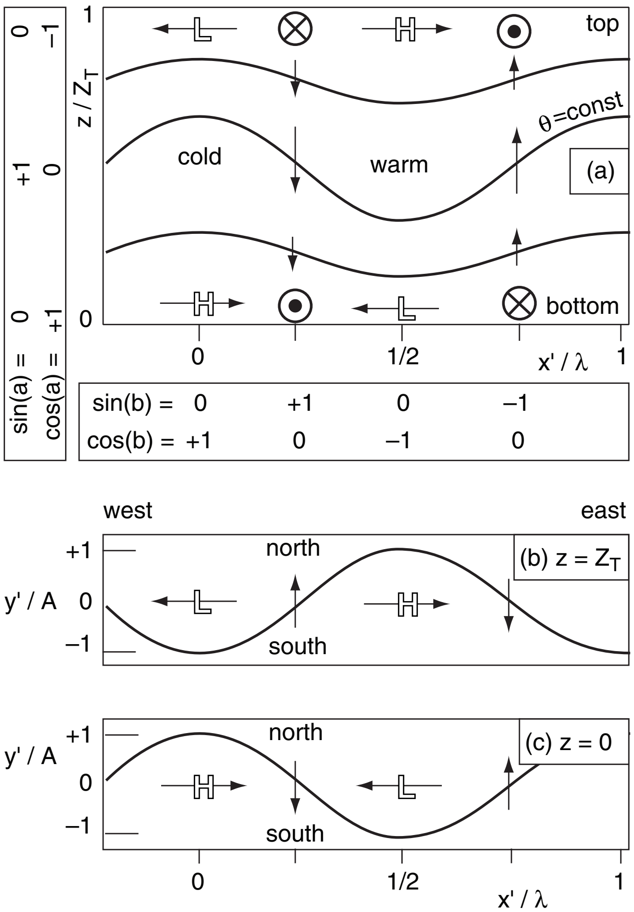


Figure 11.53
Idealized structure of baroclinic wave in N. Hem. (a) West-to-east vertical cross section. (b) Weather map (plan view) at tropopause. (c) Surface weather map. Legend: Arrows show wind. L & H are low & high pressures, dot-circle is southward-pointing vector, x-circle is northward vector. Wavy lines on the cross section are isentropes. Also shown are sine & cosine terms from baroclinic wave equations. (after Cushman-Roisin, 1994)

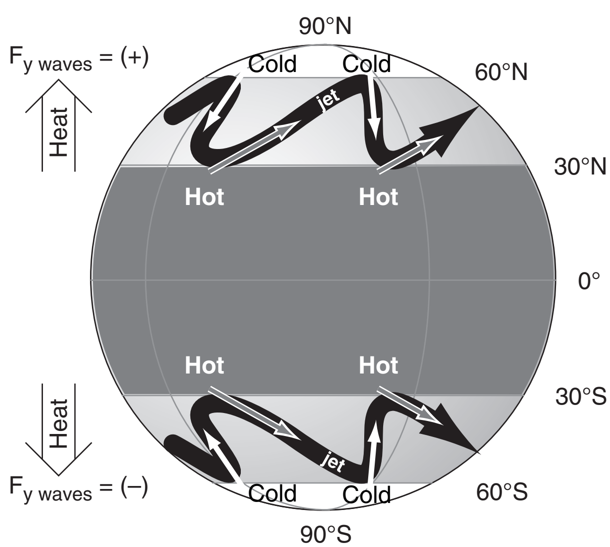


Figure 11.54
Heat transport by Rossby waves (black line) in midlatitudes.

where ρ_o is the average air density at wave altitude z. Each of these amplitudes depends on A, the amplitude of the meridional displacement, which is not fixed but depends on the initial disturbance and wave amplitude.

Fig. 11.53 illustrates the structure of this baroclinic wave in the Northern Hemisphere. Fig. 11.53c corresponds to the surface weather maps used in the previous solved examples. All the variables are interacting to produce the wave. The impact of all these variables on synoptic weather will be discussed in the Extratropical Cyclones chapter.

Actual variables are found by adding the perturbation variables (eqs. 11.46) to the background state. Mean background pressure P_o decreases hydrostatically with height. Thus, the actual pressure is $P = P_o + p'$ at any height.

Mean background potential temperature θ_o increases linearly with height (assuming constant N_{BV}), making the actual potential temperature $\theta = \theta_o + \theta'$. Background meridional and vertical winds are assumed to be zero, leaving $V = v'$ and $W = w'$. Background zonal wind U_o is assumed to be geostrophic and constant, leaving $U = U_o + u'$. Meridional displacement perturbations (y') are relative to the reference latitude (Y_o) used for the Coriolis-parameter calculation. Vertical displacement perturbations (η') are relative to the height variable z that appears in eq. (11.44).

Many other factors affect wave formation in the jet stream, including turbulent drag, clouds and latent heating, and nonlinear processes in large-amplitude waves. Also, the more complete solution to baroclinic instability includes waves propagating meridionally as well as zonally (we focused on only zonal propagation here). However, our description captures most of the important aspects of midlatitude flow that are observed in the real atmosphere.

Meridional Transport by Rossby Waves

Heat Transport

As the jet stream blows along the meandering planetary-wave path (Fig. 11.54), it picks up warm air (T' = deviation from mean temperature = positive) and carries it poleward (v' = + in N. Hem.). Similarly, cold air (T' = –) is carried equatorward (v' = – in N. Hem.). The average meridional kinematic heat flux ($F_{y\ waves}$) is the sum of advective transports divided by the number N of these transports. For the N. Hemisphere, this meridional heat flux is:

$$F_{y\ waves} = (1/N)\cdot\Sigma(v'\cdot T') = (+)\cdot(+) + (-)\cdot(-) = \text{positive.}$$

while it is negative in the Southern Hemisphere.

Thus, the north-south meandering jet stream causes a northward heat flux in the N. Hemisphere and a southward heat flux in the S. Hemisphere (Fig. 11.54) without requiring a vertical circulation cell.

The greatest flux is likely to occur where the waves have the most intense *v*-component (Fig. 11.54), and also where the north-south temperature gradient is greatest (Fig. 11.8). Both of these processes conspire together to create significant wave heat fluxes centered in mid-latitudes (Fig. 11.55 bottom).

Curvature (*Curv*) of a line is defined to be positive when the line bends concave up (shaped like a bowl). Likewise, curvature is negative for concave down (shaped like a hill). The signs of the curvatures of wave heat flux are indicated in the top of Fig. 11.55, which we will use in the next section.

Momentum Transport

Recall from Fig. 11.41 that tropical air is faster (u' = positive) when moved toward 45° latitude. Similarly, polar air is slower (u' = negative) when moved toward 45° latitude. This was because of angular-momentum conservation, where air near the equator is at a larger distance (larger radius) from the Earth's axis than air near the poles. Thus, the polar and tropical regions serve as reservoirs of slow and fast momentum, respectively, which can be tapped by the meandering jet stream.

Analogous to heat transport, we find that the poleward-moving (v' = + in N. Hem.) portions of Rossby waves carry fast zonal momentum (u'= +) air, and equatorward-moving (v' = – in N. Hem.) portions carry slow zonal momentum (u' = –) (Fig. 11.56). The net meridional transport of zonal momentum $\overline{u'v'}$ for the Northern Hemisphere is positive, and is largest at the center of mid-latitudes.

$$\overline{u'v'} = (1/N)\cdot\Sigma(v'\cdot u') = (+)\cdot(+) + (–)\cdot(–) = \text{positive.}$$

It is negative for the Southern Hemisphere.

However, the reservoir of tropical momentum is much larger than that of polar momentum because of the larger circumference of latitude lines in the tropics. Hence, tropical momentum can have more influence on midlatitude air. To account for such unequal influence, a weighting factor $a = \cos(\phi_s)/\cos(\phi_d)$ can be included in the angular momentum expression:

$$a \cdot u' \approx \Omega \cdot R_{earth} \cdot \left[\frac{\cos^2 \phi_s}{\cos \phi_d} - \cos \phi_d \right] \cdot \frac{\cos \phi_s}{\cos \phi_d} \qquad (11.48)$$

where subscript s represents source location, d is destination, and $\Omega \cdot R_{Earth}$ = 463.4 m/s as before.

Fig. 11.57 shows that the weighted zonal velocity is asymmetric about 45° latitude. Namely, from 60°

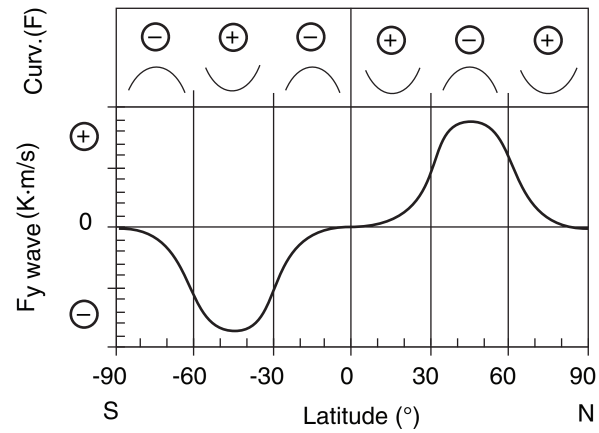


Figure 11.55
Bottom: Sketch of meridional wave flux of heat (F_y, positive northward, in kinematic units). Top: Sketch of sign of curvature of the wave flux.

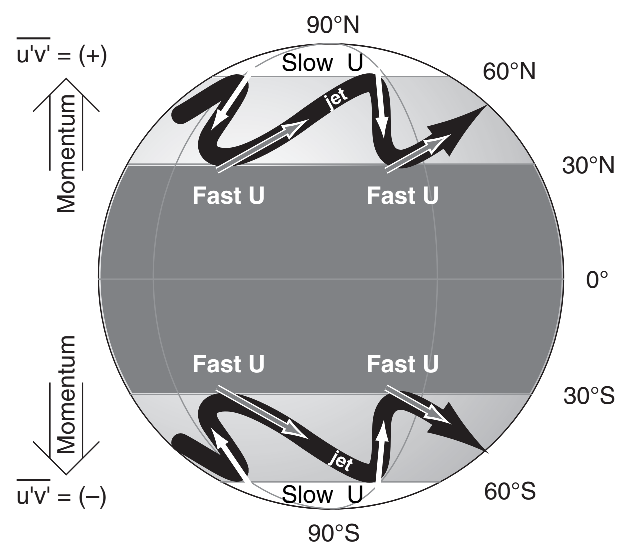


Figure 11.56
Meridional transport of eastward momentum by Rossby waves in the jet stream (black wavy line) in midlatitudes.

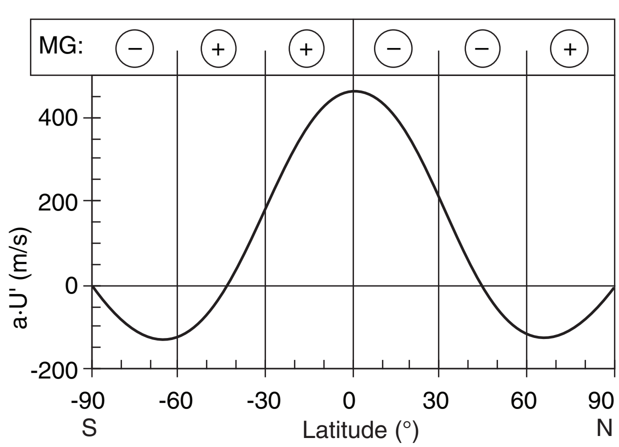


Figure 11.57
Zonal velocity from various source latitudes reaching destination 45°, weighted by the relative amounts of source-region air.

Solved Example

Find the weighted zonal velocity perturbation for air arriving at 50°N from 70°N.

Solution

Given: ϕ_s = 70°N, ϕ_d = 50°N
Find: $a \cdot u'$ = ? m/s

Use eq. (11.48):

$$a \cdot u' = (463.4\text{m/s}) \cdot \left[\frac{\cos^2(70°)}{\cos(50°)} - \cos(50°) \right] \frac{\cos(70°)}{\cos(50°)}$$

= **– 113.6 m/s**

Check: Physics OK. Magnitude too large.
Discussion: The negative sign means air from 70°N is moving slower from the west than any point at 50°N on the Earth's surface is moving. Thus, relative to the Earth, the wind is from the east.

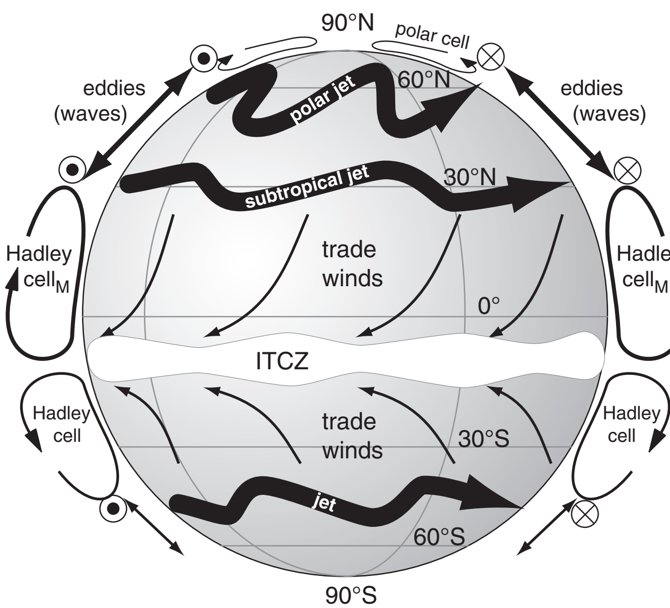


Figure 11.58
Sketch of three-band structure of general circulation for February (N. Hemisphere winter): 1) vertical Hadley cell in tropics, 2) horizontal Rossby waves at midlatitudes, and 3) a weak vertical circulation near the poles. For the jet axes, the dot-circle is wind out of the page, and x-circle is wind into the page.

FOCUS • Torques on the Earth

During high-wind episodes in one of the circulation bands, temporary changes in wind-drag torques are large enough to make measurable changes in the Earth's rotation rate — causing the length of a day to increase or decrease 1 - 3 µs over periods of months. In addition, external influences (lunar and solar tides, solar wind, geomagnetic effects, space dust) cause the Earth to spin ever more slowly, causing the length of a day to increase 1.4 ms/century at present.

latitude the velocity is –100 m/s, but from a source of 30° latitude the velocity is about +200 m/s. Although the momentum transport is positive everywhere, it decreases to the north. Hence, the meridional gradient *MG* of zonal momentum is negative in the Northern Hemisphere mid-latitudes:

$$MG = \Delta \overline{u'v'} / \Delta y = \text{negative.} \qquad (11.49)$$

Although quantitatively the wind magnitudes from angular-momentum arguments are unrealistically large, as discussed earlier, the qualitative picture of momentum transport is valid.

Rossby waves that transport *U* momentum poleward have a recognizable rounded-sawtooth shape, as was sketched in Fig. 11.56. Specifically, the equatorward-moving portions of the jet stream are aligned more north-south (i.e., are more meridional), and sometimes even tilt backwards (toward the west as it moves toward the equator). The poleward-moving portions of the jet are more zonal (west to east).

THREE-BAND GENERAL CIRCULATION

The preceding sections explained the assertion that was made in the opening paragraphs of this chapter. Namely, Coriolis force causes the thermally-driven planetary circulation to break down into three latitude-bands of circulation (Fig. 11.58) in each hemisphere. These bands are: a strong, asymmetric, **direct** vertical-circulation Hadley cell in the tropics (0° to 30°); a band of mostly horizontal Rossby waves at mid-latitudes (30° to 60°); and a weak direct vertical circulation cell in polar regions (60° to 90°). Fig. 11.58 includes more (but not all) of the details and asymmetries explained in this chapter.

The circulation bands work together to globally transport atmospheric heat (Fig. 11.14), helping undo the differential heating that was caused by solar and IR radiation. The Earth-atmosphere-ocean system is in near equilibrium thermally, with only extremely small trends over time related to global warming.

The circulation bands also work together in the meridional transport of zonal momentum. The trade winds, blowing opposite to the Earth's rotation, exert a **torque** (force times radius) that tends to slow the Earth's spin due to frictional drag against the land and ocean surface. However, in mid-latitudes, the westerlies dragging against the Earth's surface and against mountains apply an opposite torque, tending to accelerate the Earth's spin. On the long term, the opposite torques nearly cancel each other. Thus, the whole Earth-atmosphere-ocean system maintains a near-equilibrium spin rate.

A Measure of Vertical Circulation

Getting back to atmospheric circulations, a measure of vertical-cell circulation CC is:

$$CC = \left[\frac{f_c^{\,2}}{N_{BV}^2} \frac{\Delta V}{\Delta z} \right] - \frac{\Delta w}{\Delta y} \qquad (11.50)$$

CC is positive for **direct cells** in the N. Hemisphere, as illustrated in Fig. 11.59. Direct cells are ones where the vertical circulation is in the direction that would be expected if there were no Coriolis force. The units for circulation are s^{-1}.

In a direct cell, vertical velocity decreases and even changes sign toward the north, making $\Delta w/\Delta y$ negative. Also, northward velocity V increases with height, making $\Delta V/\Delta z$ positive for a direct cell. Both of these terms contribute to positive CC in a direct cell. Similarly, CC is negative for an **indirect cell** (one that circulates oppositely to a direct cell).

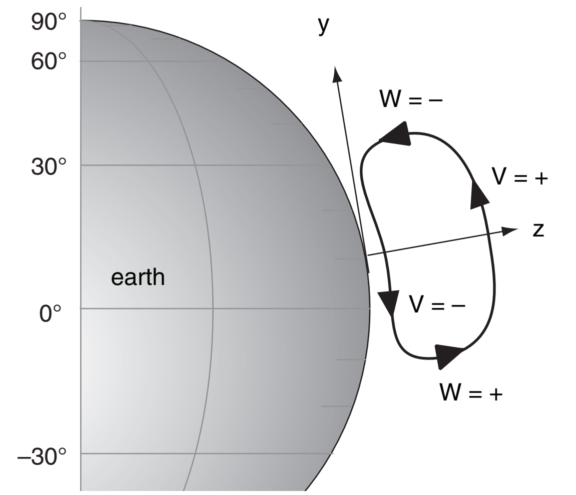


Figure 11.59
Example of a direct-circulation cell in the N. Hemisphere, for the major Hadley cell during N. Hem. winter.

Effective Vertical Circulation

The equations of motion can be combined to yield an equation for the vertical circulation CC that can be applied even where vertical cells do not dominate. This circulation equation is written in abbreviated form as eq. (11.51). Buoyancy associated with a heated surface can drive a vertical circulation. But if some of the heating difference is transported by planetary waves, then there will be less heat available to drive a direct circulation.

Based on the previous discussions of radiative differential heating E_{net} from Fig. 11.10, momentum gradient MG (proportional to $\Delta U/\Delta z$) from Fig. 11.57, and heat flux curvature $Curv(F_{y\ wave})$ from Fig. 11.55, the contributions of each term to the circulation are listed below the equation. The sign of $\Delta MG/\Delta z$ equals the sign of MG, assuming that northward momentum transport is weakest at the ground, and increases with height because the jet-stream winds increase with height.

$$CC \;\propto\; -\frac{\Delta E_{net}}{\Delta y} + Curv(F_{y\ wave}) + \frac{\Delta MG}{\Delta z} \qquad (11.51)$$

circulation *radiation* *wave-heat* *wave-momentum*

CC_{polar} ∝ positive + positive + positive = <u>positive</u>
CC_{midlat} ∝ positive + negative +negative = <u>negative</u>
$CC_{tropics}$ ∝ positive + positive + positive = <u>positive</u>

Thus, horizontal planetary waves at mid-latitudes are so effective that they reverse the effective vertical circulation — a result called the **Ferrel cell**.

Solved Example

Suppose the Hadley cell updraft and downdraft velocities are 6 and –4 mm/s, respectively, and the meridional wind speeds are 3 m/s at the top and bottom of the cell. The major Hadley cell is about 17 km high by 3900 km wide, and is centered at about 10° latitude. Temperature in the tropical atmosphere decreases from about 25°C near the surface to –77°C at 17 km altitude. Find the vertical cell circulation.

Solution

Given: Δw = –10 mm/s = –0.01 m/s , $\Delta y = 3.9\text{x}10^3$ km
ΔV = 6 m/s, Δz = 17 km, ϕ = 10°, ΔT = –102°C.
Find: CC = ? s^{-1}.

First, use eq. (10.16):
$f_c = (1.458\text{x}10^{-4}\ s^{-1})\cdot\sin(10°) = 2.53\text{x}10^{-5}\ s^{-1}$
Next, for the Brunt-Väisälä frequency, we first need:
T_{avg} = 0.5*(25 – 77)°C = –26°C = 247 K
In the tropics $\Delta T/\Delta z$ = –6 °C/km = –0.006 K/m
Then use eq. (5.4), and assume $T_v = T$:

$$N_{BV} = \sqrt{\frac{9.8\text{m/s}^2}{247\text{K}}(-0.006 + 0.0098)\frac{\text{K}}{\text{m}}} = 0.0123\ s^{-1}$$

Finally, use eq. (11.50):

$$CC = \left[\left(\frac{2.53\times10^{-5}\text{s}^{-1}}{0.0123\text{s}^{-1}}\right)^2 \cdot \frac{6\text{m/s}}{17000\text{m}}\right] - \frac{-0.01\text{m/s}}{3.9\times10^6\text{m}}$$

$CC = 1.493\text{x}10^{-9} + 2.564\text{x}10^{-9}\ s^{-1}$ = <u>**4.06x10**</u>$^{-9}\ s^{-1}$

Check: Units OK. Physics OK.
Discussion: Both terms contribute positively to the circulation of the major Hadley cell during Northern Hemisphere winter.

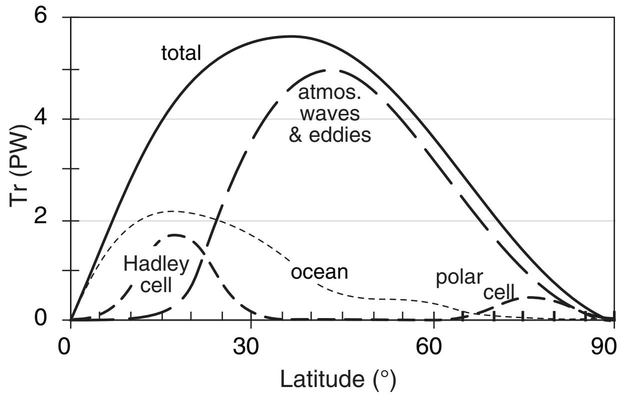


Figure 11.60
Contributions of direct circulations (medium dashed line, for Hadley cell and polar cell) and indirect circulations (long dashed line, for Rossby waves and eddies) to total meridional heat transport Tr in the N. Hemisphere

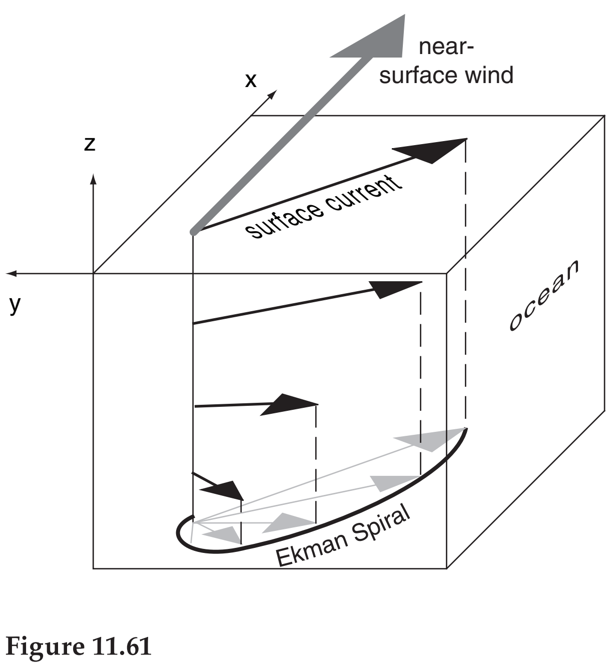


Figure 11.61
Ekman spiral of ocean currents. Black arrows show currents at different depths. Projecting these vectors onto an imaginary plane at the bottom gives the grey vectors, which show the spiral nature of the Ekman currents.

Fig. 11.60 redraws the N. Hemisphere portion of Fig. 11.14, qualitatively highlighting the relative contributions of the direct (Hadley and polar cells) and indirect (Ferrel cell/Rossby waves) atmospheric circulations to the total meridional heat transport. The Rossby wave circulation (and its associated high and low-pressure eddies) dominate at mid-latitudes. Vertically direct circulations dominate elsewhere.

Ocean currents also contribute to global heat redistribution. Although ocean-circulation details are not within the scope of this book, we will introduce one ocean topic here — the Ekman spiral. This describes how wind drag can drive some ocean currents, including hurricane storm surges.

EKMAN SPIRAL IN THE OCEAN

As the wind blows over the oceans, the air drags along some of the water. The resulting ocean currents turn under the influence of Coriolis force (Fig. 11.61), eventually reaching an equilibrium given by:

$$U = \left[\frac{u_{*water}^{\ 2}}{(K \cdot f_c)^{1/2}}\right] \cdot \left[e^{\gamma \cdot z} \cdot \cos\left(\gamma \cdot z - \frac{\pi}{4}\right)\right] \tag{11.52a}$$

$$V = \left[\frac{u_{*water}^{\ 2}}{(K \cdot f_c)^{1/2}}\right] \cdot \left[e^{\gamma \cdot z} \cdot \sin\left(\gamma \cdot z - \frac{\pi}{4}\right)\right] \tag{11.52b}$$

where the x-axis and U-current direction point in the direction of the surface wind. When these current vectors are plotted vs. depth (z is positive upward), the result is a spiral called the Ekman spiral.

The friction velocity u_{*water} for water is

$$u_{*water}^{\ 2} = \frac{\rho_{air}}{\rho_{water}} \cdot u_{*air}^{\ 2} \tag{11.53}$$

where the friction velocity for air u_{*air} is given in the Boundary Layer Chapter. The **depth parameter** is

$$\gamma = \sqrt{\frac{f_c}{2 \cdot K}} \tag{11.54}$$

where K is the eddy viscosity (see the Boundary-Layer chapter) in the ocean. The inverse of γ is called the **Ekman layer depth**. Transport of water by Ekman processes is discussed further in the Hurricane chapter.

SUMMARY

Radiation causes differential heating — net heating of the tropics and cooling at the poles. In response, a global circulation develops due to buoyancy, pressure, and geostrophic effects, which moves the excess heat from the equator to the poles.

Near the equator, warm air rises and creates a band of thunderstorms at the ITCZ. The updrafts are a part of the Hadley-cell direct vertical circulation, which moves heat away from the equator. This cell cannot extend beyond about 30° to 35° latitude because Coriolis force turns the upper-troposphere winds toward the east, creating a subtropical jet near 30° latitude at the tropopause. Near the surface are the trade-wind return flows from the east.

A strong meridional temperature gradient remains at mid-latitudes, which drives westerly winds via the thermal-wind effect, and creates a polar jet at the tropopause. This jet stream is unstable, and usually meanders north and south as Rossby waves. Vorticity can help explain these waves.

The waves transport so much heat and angular momentum in mid-latitudes that a reverse (indirect) circulation develops there, called the Ferrel cell. A weak direct cell also exists in polar regions. Atmospheric winds drive ocean currents. All these ceaseless global circulations transport sufficient heat and momentum to keep the Earth in near equilibrium.

Threads

Radiation (Chapter 2) causes the differential heating that drives the global circulation in the troposphere (Chapter 1). This circulation moves both sensible and latent heat (Chapter 3).

The jet stream, caused by various dynamic forces (Chapter 10), meanders north and south creating troughs and ridges. This pattern creates surface high pressure areas and airmasses (Chapter 12), and low pressure areas called cyclones (Chapter 13). Upward motion in troughs and lows causes moist (Chapter 4) and dry (Chapt 3) adiabatic cooling (Chapter 5), which create clouds (Chapter 6) and precipitation (Chapter 7). Downward motion in ridges and highs creates relatively shallow boundary layers (Chapter 18) that can trap air pollutants (Chapter 19) .

Numerical models (Chapter 20) of the general circulation are called **global climate models** (**GCMs**) when used to forecast climate change (Chapter 21). Hurricane tracks (Chapter 16) are steered by monsoon circulations such as the Bermuda high. The ITCZ consists of thunderstorms (Chapters 14 and 15). The Ekman spiral process causes sea level to rise ahead of landfalling hurricanes (Chapter 16), contributing to the destructive storm surge.

Solved Example

Plot the Ekman spiral in the ocean at 45° latitude, using an eddy viscosity of 2×10^{-3} m^2·s^{-1}, and $u_{*water}{}^2 = 4\times10^{-4}$ m^2/s^2.

Solution

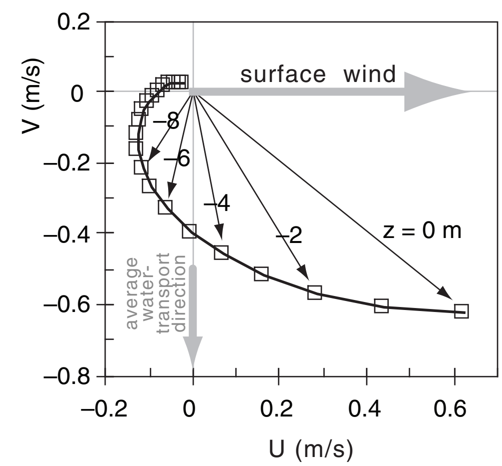
Given: $u_{*water}{}^2 = 4\times10^{-4}$ m^2/s^2, $K = 0.002$ m^2·s^{-1},
$\phi = 45°$

Find: U & V (m/s) vs. z (m)

First solve for Coriolis param.: $f_c = 0.0001031$ s^{-1} .
Use eq. (11.54):

$$\gamma = \sqrt{\frac{0.0001031\text{s}^{-1}}{2\cdot(0.002\text{m}^2\text{s}^{-1})}} = 0.1605 \text{ m}^{-1}$$

Solve eqs. (11.52) on a spreadsheet, resulting in:



Check: Units OK. Physics OK.

Discussion: The surface current is 45° to the right of the wind direction. Deeper currents decrease in speed and turn further to the right due to Coriolis force. The average water transport is in the $-V$ direction; namely, at right angles to the surface wind.

EXERCISES

Numerical Problems

N1(§). Plot the idealized zonally-averaged temperature (°C) vs. latitude at the following heights (km) above ground (AGL).
a. 0.5 b. 1 c. 1.5 d. 2 e. 2.5 f. 3 g. 3.5
h. 4 i. 4.5 j. 5 k. 5.5 l. 6 m. 6.6 n. 7
o. 8 p. 9 q. 10 r. 11 s. 12 t. 13 u. 14

N2(§). Plot the idealized zonally-averaged meridional temperature gradient (°C/km) vs. latitude at the following heights (km) above ground level.
a. 0.5 b. 1 c. 1.5 d. 2 e. 2.5 f. 3 g. 3.5
h. 4 i. 4.5 j. 5 k. 5.5 l. 6 m. 6.6 n. 7
o. 8 p. 9 q. 10 r. 11 s. 12 t. 13 u. 14

N3. Estimate the annual average insolation (W/m^2) at the following latitude:
a. 90° b. 85° c. 80° d. 75° e. 70° f. 65° g. 60°
h. 55° i. 50° j. 45° k. 40° l. 35° m. 30° n. 25°
o. 20° p. 15° q. 10° r. 5° s. equator

N4. Estimate the annual average amount of incoming solar radiation (W/m^2) that is absorbed in the Earth-ocean-atmosphere system at latitude:
a. 90° b. 85° c. 80° d. 75° e. 70° f. 65° g. 60°
h. 55° i. 50° j. 45° k. 40° l. 35° m. 30° n. 25°
o. 20° p. 15° q. 10° r. 5° s. equator

N5. Using the idealized temperature near the middle of the troposphere (at z = 5.5 km), estimate the outgoing infrared radiation (W/m^2) from the atmosphere at the following latitude:
a. 90° b. 85° c. 80° d. 75° e. 70° f. 65° g. 60°
h. 55° i. 50° j. 45° k. 40° l. 35° m. 30° n. 25°
o. 20° p. 15° q. 10° r. 5° s. equator

N6. Using the results from the previous two exercises, find the net radiation magnitude (W/m^2) that is input to the atmosphere at latitude:
a. 90° b. 85° c. 80° d. 75° e. 70° f. 65° g. 60°
h. 55° i. 50° j. 45° k. 40° l. 35° m. 30° n. 25°
o. 20° p. 15° q. 10° r. 5° s. equator

N7. Using the results from the previous exercise, find the latitude-compensated net radiation magnitude (W/m^2; i.e., the differential heating) at latitude:
a. 90° b. 85° c. 80° d. 75° e. 70° f. 65° g. 60°
h. 55° i. 50° j. 45° k. 40° l. 35° m. 30° n. 25°
o. 20° p. 15° q. 10° r. 5° s. equator

N8. Assuming a standard atmosphere, find the internal Rossby radius of deformation (km) at latitude:
a. 90° b. 85° c. 80° d. 75° e. 70° f. 65° g. 60°
h. 55° i. 50° j. 45° k. 40° l. 35° m. 30° n. 25°
o. 20° p. 15° q. 10° r. 5° s. equator

N9. Given the following virtual temperatures at your location (20°C) and at another location, find the change of geostrophic wind with height [(m/s)/km]. Relative to your location, the other locations are:

	Δx(km)	Δy(km)	T_v(°C)		Δx(km)	Δy(km)	T_v(°C)
a.	0	100	15	k.	100	0	15
b.	0	100	16	l.	100	0	16
c.	0	100	17	m.	100	0	17
d.	0	100	18	n.	100	0	18
e.	0	100	19	o.	100	0	19
f.	0	100	21	p.	100	0	21
g.	0	100	22	q.	100	0	22
h.	0	100	23	r.	100	0	23
i.	0	100	24	s.	100	0	24
j.	0	100	25	t.	100	0	25

N10. Find the thermal wind (m/s) components, given a 100 to 50 kPa thickness change of 0.1 km across the following distances:
Δx(km) = a. 200 b. 250 c. 300 d. 350
e. 400 f. 450 g. 550 h. 600 i. 650
Δy(km) = j. 200 k. 250 l. 300 m. 350
n. 400 o. 450 p. 550 q. 600 r. 650

N11. Find the magnitude of the thermal wind (m/s) for the following thickness gradients:

	ΔTH(km) / Δx(km)	&	ΔTH(km) / Δy(km)
a.	–0.2 / 600	and	– 0.1 / 400
b.	–0.2 / 400	and	– 0.1 / 400
c.	–0.2 / 600	and	+ 0.1 / 400
d.	–0.2 / 400	and	+ 0.1 / 400
e.	–0.2 / 600	and	– 0.1 / 400
f.	–0.2 / 400	and	– 0.1 / 400
g.	–0.2 / 600	and	+ 0.1 / 400
h.	–0.2 / 400	and	+ 0.1 / 400

N12. For the toy model temperature distribution, find the wind speed (m/s) of the jet stream at the following heights (km) for latitude 30°:
a. 0.5 b. 1 c. 1.5 d. 2 e. 2.5 f. 3 g. 3.5
h. 4 i. 4.5 j. 5 k. 5.5 l. 6 m. 6.6 n. 7
o. 8 p. 9 q. 10 r. 11 s. 12 t. 13 u. 14

N13. If an air parcel from the starting latitude 5° has zero initial velocity relative to the Earth, then find its U component of velocity (m/s) relative to the Earth when it reaches the following latitude, assuming conservation of angular momentum.
a. 0° b. 2° c. 4° d. 6° e. 8° f. 10° g. 12°
h. 14° i. 16° j. 18° k. 20° l. 22° m. 24° n. 26°

N14. Find the relative vorticity (s^{-1}) given the following changes of (U , V) wind speed (m/s), across distances of Δx = 300 km and Δy = 600 km respectively.
a. 50, 50 b. 50, 20 c. 50, 0 d. 50, –20 e. 50, –50
f. 20, 50 g. 20, 20 h. 20, 0 i. 20, –20 j. 20, –50
k. 0, 50 l. 0, 20 m. 0, 0 n. 0, –20 o. 0, –50
p. –20, 50 q. –20, 20 r. –20, 0 s. –20, –20 t. –20, –50
u. –50, 50 v. –50, 20 x. –50, 0 y. –50, –20 z. –50, –50

N15. Given the following radial shear ($\Delta M/\Delta R$) in [(m/s)/km] and tangential wind speed M (m/s) around radius R (km), find relative vorticity (s^{-1}):
a. 0.1, 30, 300 b. 0.1, 20, 300 c. 0.1, 10, 300
d. 0.1, 0, 300 e. 0, 30, 300 f. 0, 20, 300
g. 0, 10, 300 h. –0.1, 30, 300 i. –0.1, 20, 300
j. –0.1, 10, 300 k. –0.1, 0, 300

N16. If the air rotates as a solid body of radius 500 km, find the relative vorticity (s^{-1}) for tangential speeds (m/s) of:
a. 10 b. 20 c. 30 d. 40 e. 50 f. 60 g. 70
h. 80 i. 90 j. 100 k. 120 l. 140 m. 150

N17. If the relative vorticity is $5\text{x}10^{-5}$ s^{-1}, find the absolute vorticity at the following latitude:
a. 90° b. 85° c. 80° d. 75° e. 70° f. 65° g. 60°
h. 55° i. 50° j. 45° k. 40° l. 35° m. 30° n. 25°
o. 20° p. 15° q. 10° r. 5° s. equator

N18. If the absolute vorticity is $5\text{x}10^{-5}$ s^{-1}, find the potential vorticity ($m^{-1}{\cdot}s^{-1}$) for a layer of thickness (km) of:
a. 0.5 b. 1 c. 1.5 d. 2 e. 2.5 f. 3 g. 3.5
h. 4 i. 4.5 j. 5 k. 5.5 l. 6 m. 6.6 n. 7
o. 8 p. 9 q. 10 r. 11 s. 12 t. 13 u. 14

N19. The potential vorticity is $1\text{x}10^{-8}$ $m^{-1}{\cdot}s^{-1}$ for a 10 km thick layer of air at latitude 48°N. What is the change of relative vorticity (s^{-1}) if the thickness (km) of the rotating air changes to:
a. 9.5 b. 9 c. 8.5 d. 8 e. 7.5 f. 7 g. 6.5
h. 10.5 j. 11 k. 11.5 l. 12 m. 12.5 n. 13

N20. If the absolute vorticity is $3\text{x}10^{-5}$ s^{-1} at 12 km altitude, find the isentropic potential vorticity (PVU) for a potential temperature change of ___ °C across a height increase of 1 km.
a. 1 b. 2 c. 3 d. 4 e. 5 f. 6 g. 6.5
h. 7 i. 8 j. 9 k. 10 l. 11 m. 12 n. 13

N21. Find the horizontal circulation associated with average relative vorticity $5\text{x}10^{-5}$ s^{-1} over area (km^2):
a. 500 b. 1000 c. 2000 d. 5000 e. 10,000
f. 20,000 g. 50,000 h. 100,000 i. 200,000

N22. Find the beta parameter (m^{-1} s^{-1}) at latitude:
a. 90° b. 85° c. 80° d. 75° e. 70° f. 65° g. 60°
h. 55° i. 50° j. 45° k. 40° l. 35° m. 30° n. 25°
o. 20° p. 15° q. 10° r. 5° s. equator

N23. Suppose the average wind speed is 60 m/s from the west at the tropopause. For a <u>barotropic</u> Rossby wave at 50° latitude, find both the intrinsic phase speed (m/s) and the phase speed (m/s) relative to the ground for wavelength (km) of:
a. 1000 b. 1500 c. 2000 d. 2500 e. 3000
f. 3500 g. 4000 h. 4500 i. 5000 j. 5500
k. 6000 l. 6500 m. 7000 n. 7500 o. 8000

N24. Plot the barotropic wave (y' vs x') from the previous exercise, assuming amplitude 2000 km.

N25. Same as exercise N23, but for a <u>baroclinic</u> Rossby wave in an atmosphere where air temperature decreases with height at 4°C/km.

N26(§). Plot the baroclinic wave (y' vs x') from the previous exercise, assuming amplitude 2000 km and a height (km):
(i) 2 (ii) 4 (iii) 6 (iv) 8 (v) 10

N27. Find the dominant (fastest growing) wavelength for a baroclinic wave in a standard atmosphere at latitude:
a. 90° b. 85° c. 80° d. 75° e. 70° f. 65° g. 60°
h. 55° i. 50° j. 45° k. 40° l. 35° m. 30° n. 25°
o. 20° p. 15° q. 10° r. 5° s. equator

N28. For the baroclinic Rossby wave of exercise N25 with amplitude 2000 km, find the wave amplitudes of the:
(i). vertical-displacement perturbation
(ii). potential-temperature perturbation
(iii). pressure perturbation
(iv). U-wind perturbation
(v). V-wind perturbation
(vi). W-wind perturbation

N29(§). For the previous exercise, plot separate vertical cross sections showing perturbation values for those variables.

N30. Find the latitude-weighted $a{\cdot}u'$ momentum value (m/s) for air that reaches destination latitude 50° from source latitude: a. 80° b. 75° c. 70° d. 65° e. 60° f. 55° g. 45° h. 40° i. 35°

N31. Suppose the ____ cell updraft and downdraft velocities are ___ and ___ mm/s, respectively, and the meridional wind speeds are 3 m/s at the top and bottom of the cell. The cell is about __ km high by

___ km wide, and is centered at about ___ latitude. Temperature in the atmosphere decreases from about 15°C near the surface to –57°C at 11 km altitude. Find the vertical circulation.

cell	W_{up} (mm/s)	W_{down} (mm/s)	Δz (km)	Δy (km)	ϕ (°)
a. Hadley	6	–4	17	3900	10
b. Hadley	4	–4	15	3500	10
c. Hadley	3	–3	15	3500	5
d. Ferrel	3	–3	12	3000	45
e. Ferrel	2	–2	11	3000	45
f. Ferrel	2	–2	10	3000	50
g. polar	1	–1	9	2500	75
h. polar	1	–1	8	2500	75
i. polar	0.5	–0.5	7	2500	80

N32. Find the friction velocity at the water surface if the friction velocity (m/s) in the air (at sea level for a standard atmosphere) is:

a. 0.05 b. 0.1 c. 0.15 d. 0.2 e. 0.25
f. 0.3 g. 0.35 h. 0.4 i. 0.45 j. 0.5
k. 0.55 l. 0.6 m. 0.65 n. 0.7 o. 0.75

N33. Find the Ekman-spiral depth parameter at latitude 50° for eddy viscosity ($m^2 s^{-1}$) of:

a. 0.0002 b. 0.0004 c. 0.0006 d. 0.0008 e. 0.001
f. 0.0012 g. 0.0014 h. 0.0016 i. 0.0018 j. 0.002
k. 0.0025 l. 0.003 m. 0.0035 n. 0.004 o. 0.005

N34(§). For the previous exercise, plot the Ekman spiral (U, V) components for depths from the surface down to where the velocities are near zero, for water friction velocity of $u_* = 0.025$ m/s.

Understanding & Critical Evaluation

U1. During the months when the major Hadley cell exists, trade winds can cross the equator. If there are no forces at the equator, explain why this is possible.

U2. In regions of surface high pressure, descending air in the troposphere is associated with dry (non-rainy) weather. These high-pressure belts are where deserts form. In addition to the belts at ±30° latitude, semi-permanent surface highs also exist at the poles. Are polar regions deserts? Explain.

U3. The subtropical jet stream for Earth is located at about 30° latitude. Due to Coriolis force, this is the poleward limit of outflow air from the top of the ITCZ. If the Earth were to spin faster, numerical experiments suggest that the poleward limit (and thus the jet location) would be closer to the equator. Based on the spins of the other planets (get this info from the web or a textbook) compared to Earth, at what latitudes would you expect the subtropical jets to be on Jupiter? Do your predictions agree with photos of Jupiter?

U4. Horizontal divergence of air near the surface tends to reduce or eliminate horizontal temperature gradients. Horizontal convergence does the opposite. Fronts (as you will learn in the next chapter) are regions of strong local temperature gradients. Based on the general circulation of Earth, at what latitudes would you expect fronts to frequently exist, and at what other latitudes would you expect them to rarely exist? Explain.

U5. In the global circulation, what main features can cause mixing of air between the Northern and Southern Hemispheres? Based on typical velocities and cross sectional areas of these flows, over what length of time would be needed for the portion $1/e$ of all the air in the N. Hemisphere to be replaced by air that arrived from the S. Hemisphere?

U6. In Fig. 11.4, the average declination of the sun was listed as 14.9° to 15° for the 4-month periods listed in those figures. Confirm that those are the correct averages, based on the equations from the Radiation chapter for solar declination angle vs. day of the year.

U7. Thunderstorms are small-diameter (15 km) columns of cloudy air from near the ground to the tropopause. They are steered by the environmental winds at an altitude of roughly 1/4 to 1/3 the troposphere depth. With that information, in what direction would you expect thunderstorms to move as a function of latitude (do this for every 10° latitude)?

U8. The average meridional wind at each pole is zero. Why? Also, does your answer apply to instantaneous winds such as on a weather map? Why?

U9. Can you detect monsoonal (monthly or seasonal average) pressure centers on a normal (instantaneous) weather map analysis or forecast? Explain.

U10. Figs. 11.3a & 11.5a showed idealized surface wind & pressure patterns. Combine these and draw a sketch of the resulting <u>idealized</u> global circulation including <u>both</u> planetary and monsoon effects.

U11. Eqs. (11.1-11.3) represent an idealized ("toy model") meridional variation of zonally averaged temperature. Critically analyze this model and discuss. Is it reasonable at the ends (boundaries) of the curve; are the units correct; is it physically justifiable; does it satisfy any budget constraints (e.g., conservation of heat, if appropriate), etc. What aspects of it are too simplified, and what aspects are OK?

U12. (a) Eq. (11.4) has the 3rd power of the sine times the 2nd power of the cosine. If you could arbitrarily change these powers, what values would lead to reasonable temperature gradients ($\Delta T/\Delta y$) at the surface and which would not (Hint: use a spreadsheet and experiment with different powers)?

(b) Of the various powers that could be reasonable, which powers would you recommend as fitting the available data the best? (Hint: consider not only the temperature gradient, but the associated meridional temperature profile and the associated jet stream.) Also, speculate on why I chose the powers that I did for this toy model.

U13. Concerning differential heating, Fig. 11.9 shows the annual average insolation vs. latitude. Instead, compute the average insolation over the two-month period of June and July, and plot vs. latitude. Use the resulting graph to explain why the jet stream and weather patterns are very weak in the summer hemisphere, and strong in the winter hemisphere.

U14. At mid- and high-latitudes, Fig. 11.9 shows that each hemisphere has one full cycle of insolation annually (i.e., there is one maximum and one minimum each year).

But look at Fig. 11.9 near the equator.

a. Based on the data in this graph (or even better, based on the eqs. from the Radiation chapter), plot insolation vs. relative Julian day for the equator.

b. How many insolation cycles are there each year at the equator?

c. At the equator, speculate on when would be the hottest "seasons" and when would be the less-hot "seasons".

d. Within what range of latitudes near the equator is this behavior observed?

U15. Just before idealized eq. (11.6), I mentioned my surprise that E_2 was approximately constant with latitude. I had estimated E_2 by subtracting my toy-model values for E_{insol} from the actual observed values of E_{in}. Speculate about what physical processes could cause E_2 to be constant with latitude all the way from the equator to the poles.

U16. How sensitive is the toy model for E_{out} (i.e., eq. 11.7) to the choice of average emission altitude z_m? Recall that z_m, when used as the altitude z in eqs. (11.1-11.3), affects T_m. Hint: for your sensitivity analysis, use a spreadsheet to experiment with different z_m and see how the resulting plots of E_{out} vs. latitude change. (See the "On Doing Science" box about model sensitivity.)

U17(§). Solve the equations to reproduce the curves in figure: a. 11.10 b. 11.11 c. 11.12 d. 11.13

U18. We recognize the global circulation as a response of the atmosphere to the instability caused by differential heating, as suggested by **LeChatelier's Principle**. But the circulation does not totally undo the instability; namely, the tropics remain slightly warmer than the poles. Comment on why this remaining, unremoved instability is required to exist, for the global circulation to work.

U19. In Fig. 11.12, what would happen if the surplus area exceeded the deficit area? How would the global circulation change, and what would be the end result for Fig. 11.12?

U20. Check to see if the data in Fig. 11.12 does give zero net radiation when averaged from pole to pole.

U21. The observation data that was used in Fig. 11.14 was based on satellite-measured radiation and differential heating to get the total needed heat transport, and on estimates of heat transport by the oceans. The published "observations" for net atmospheric heat transport were, in fact, estimated as the difference (i.e., residual) between the total and the ocean curves. What could be some errors in this atmosphere curve? (Hint: see the On Doing Science box about Residuals.)

U22. Express the 5 PW of heat transport typically observed at 30° latitude in other units, such as

a. horsepower b. megatons of TNT

(Hint: 1 Megaton of TNT ≈ 4.2×10^{15} J.)

U23. For Fig. 11.15, explain why it is p' vs. z that drive vertical winds, and not P_{column} vs. z.

U24. a. Redraw Figs. 11.16 for situations that cause downdrafts.

b. Figs. 11.16 both show updraft situations, but they have opposite pressure couplets. As you already found from part (a) both pressure couplets can also be associated with downdrafts. What external information (in addition to the sign of the pressure couplet) do you always need to decide whether a pressure couplet causes an updraft or a downdraft? Explain.

U25. a. For the thermal circulation of Fig. 11.17(iv), what needs to happen for this circulation to be maintained? Namely, what prevents it from dying out?

b. For what situations in the real atmosphere can thermal circulations be maintained for several days or more?

U26. a. Study Fig. 11.18 closely, and explain why the wind vectors to/from the low- and high-pressure centers at the equator differ from the winds near pressure centers at mid-latitudes.

b. Redraw Fig. 11.5a, but with continents and oceans at the equator. Discuss what monsoonal pressures and winds might occur during winter and summer, and why.

U27. a. Redraw Fig. 11.19, but for the case of geostrophic wind decreasing from its initial equilibrium value. Discuss the resulting evolution of wind and pressure fields during this geostrophic adjustment.

b. Redraw Fig. 11.19, but for flow around a low-pressure center (i.e., look at the gradient winds rather than the geostrophic winds). Discuss how the wind and pressure fields adjust when the geostrophic wind is increased above its initial equilibrium value.

U28. How would the vertical potential temperature gradient need to vary with latitude for the "internal Rossby radius of deformation" to be invariant? Assume constant troposphere depth.

U29. In the Local Winds chapter, gap winds and coastally-trapped jets are explained. Discuss how these flows relate to geostrophic adjustment.

U30. At the top of **hurricanes** (see the Hurricanes chapter), so much air is being continuously pumped to the top of the troposphere that a high-pressure center is formed over the hurricane core there. This high is so intense and localized that it violates the conditions for gradient winds; namely, the pressure gradient around this high is too steep (see the Dynamics chapter).

Discuss the winds and pressure at the top of a hurricane, using what you know about geostrophic adjustment. Namely, what happens to the winds and air mass if the wind field is not in geostrophic or gradient balance with the pressure field?

U31. In the thermal-wind relationship (eqs. 11.13), which factors on the right side are constant or vary by only a small amount compared to their magnitude, and which factors vary more (and are thus more important in the equations)?

U32. In Fig. 11.20, how would it change if the bottom isobaric surface were tilted; namely, if there were already a horizontal pressure gradient at the bottom?

U33. Draw a sketch similar to Fig. 11.20 for the thermal-wind relationship for the Southern Hemisphere.

U34. In maps such as Fig. 11.21, explain why thickness is related to average temperature.

U35. Redraw Fig. 11.22 for the case cold air in the east and warm air in the west (i.e., opposite to what is in the existing figure). Assume no change to the bottom isobaric surface.

U36. Copy Fig. 11.24.

a. On your copy, draw the G_1 and G_2 vectors, and the M_{TH} vector at point B. Confirm that the thermal wind relationship is qualitatively satisfied via vector addition. Discuss why point B is an example of veering or backing.

b. Same as (a) but calculate the actual magnitude of each vector at point B based on the spacing between isobars, thickness contours, or height contours, and then using the appropriate equations from this and previous chapters. Again, confirm that the thermal wind relationship is satisfied. (1° latitude = 111 km)

U37. Using a spreadsheet, start with an air parcel at rest at the tropopause over the equator. Assume a realistic pressure gradient between the equator and 30° latitude. Use dynamics to solve for acceleration of the parcel over a short time step, and then iterate over many time steps to find parcel speed and position. How does the path of this parcel compare to the idealized paths drawn in Fig. 11.26d? Discuss.

U38. In the thunderstorms at the ITCZ, copious amounts of water vapor condense and release latent heat. Discuss how this condensation affects the average lapse rate in the tropics, the distribution of heat, and the strength of the equatorial high-pressure belt at the tropopause.

U39. Summarize in a list or an outline all the general-circulation factors that make the mid-latitude weather different from tropical weather.

U40. Explain the surface pressure patterns in Figs. 11.31 in terms of a combination of idealized monsoon and planetary circulations.

U41. Figs. 11.31 show mid-summer and mid-winter conditions in each hemisphere. Speculate on what the circulation would look like in April or October.

U42. Compare Figs. 11.32 with the idealized planetary and monsoon circulations, and discuss similarities and differences.

U43. Based on Figs. 11.32, which hemisphere would you expect to have strong subtropical jets in both summer and winter, and which would not. What factors might be responsible for this difference?

U44. For the Indian monsoon sketched in Fig. 11.33, where are the updraft and downdraft portions of the major Hadley cell for that month? Also, what is the relationship between the trade winds at that time, and the Indian monsoon winds?

U45. What are the dominant characteristics you see in Fig. 11.34, regarding jet streams in the Earth's atmosphere? Where don't jet streams go?

U46. In Figs. 11.35, indicate if the jet-stream winds would be coming out of the page or into the page, for the: a) N. Hemisphere, (b) S. Hemisphere.

U47. Although Figs. 11.36 are for different months than Figs. 11.32, they are close enough in months to still both describe summer and winter flows.

a. Do the near-tropopause winds in Figs. 11.36 agree with the pressure gradients (or height gradients) in Figs. 11.32?

b. Why are there easterly winds at the tropopause over/near the equator, even though there is negligible pressure gradient there?

U48. Describe the mechanism that drives the polar jet, and explain how it differs from the mechanism that drives the subtropical jet.

U49. In Fig. 11.37b, we see a very strong pressure gradient in the vertical (indicated by the different isobars), but only small pressure gradients in the horizontal (indicated by the slope of any one isobar). Yet the strongest average winds are horizontal, not vertical. Why?

U50. Why does the jet stream wind speed decrease with increasing height above the tropopause?

U51. Derive eq. (11.17) for jet stream wind speed, knowing the temperature field given by the toy model earlier in this chapter. Describe the physical meaning of each term in that equation. Also, describe the limitations of that equation.

U52. Why does an air parcel at rest (i.e., calm winds) near the equator possess large angular momentum? What about for air parcels that move from the east at typical trade wind speeds?

U53. At the equator, air at the bottom of the troposphere has a smaller radius of curvature about the Earth's axis than at the top of the troposphere. How significant is this difference? Can we neglect it?

U54. Although angular-momentum conservation is not a good explanation for the jet stream, can it explain the trade winds? Discuss with justification.

U55. Suppose water in your sink is in solid body rotation at one revolution per 5 seconds, and your sink is 1 m in diameter and 0.5 m deep.

a. Find the potential vorticity.

b. If you suddenly pull the stopper and the fluid stretches to depth 1 m in your drain, what is the new relative vorticity and new rotation rate?

U56. In eq. (11.20), why is there a negative sign on the last term? Hint: How does the rotation direction implied by the last term without a negative sign compare to the rotation direction of the first term?

U57. In the Thunderstorm chapters, you will learn that the winds in a portion of the tornado can be irrotational. This is surprising, because the winds are traveling so quickly around a very tight vortex. Explain what wind field is needed to gave **irrotational winds** (i.e., no relative vorticity) in air that is rotating around the tornado. Hint: Into the wall of a tornado, imagine dropping a neutrally-buoyant small paddle wheel the size of a flower. As this flower is translated around the perimeter of the tornado funnel, what must the local wind shear be at the flower to cause it to not spin relative to the ground? Redraw Fig. 11.43 to show what you propose.

U58. Eq. (11.25) gives names for the different terms that can contribute toward vorticity. For simplicity, assume Δz is constant (i.e., assume no stretching). On a copy of Fig. 11.44, write these names at appropriate locations to identify the dominant factors affecting the vorticity max and min centers.

U59. If you were standing at the equator, you would be rotating with the Earth about its axis. However, you would have zero vorticity about your vertical axis. Explain how that is possible.

U60. Eq. (11.26) looks like it has the <u>absolute</u> vorticity in the numerator, yet that is an equation for a form of <u>potential</u> vorticity. What other aspects of that equation make it like a potential vorticity?

U61. Compare the expression of horizontal circulation C with that for vertical circulation CC.

U62. Relate Kelvin's circulation theorem to the conservation of potential vorticity. Hint: Consider a constant Volume = $A \cdot \Delta z$.

U63. The jet stream sketched in Fig. 11.49 separates cold polar air near the pole from warmer air near the equator. What prevents the cold air from extending further away from the poles toward the equator?

U64. If the Coriolis force didn't vary with latitude, could there be Rossby waves? Discuss.

U65. Compare the barotropic and baroclinic relationships for phase speed of planetary waves. Which is fastest (and in what direction) in mid-latitudes?

U66. If the troposphere were isothermal everywhere, find the number of planetary waves that would encircle the Earth at 45°N, using baroclinic theory. How does this differ from barotropic theory?

U67. Once a Rossby wave is triggered, what mechanisms do you think could cause it to diminish (i.e., to reduce the waviness, and leave straight zonal flow).

U68. In Fig. 11.50 at point (4) in the jet stream, why doesn't the air just continue turning clockwise around toward points (2) and (3), instead of starting to turn the other way?

U69. How would you explain the difference between barotropic and baroclinic waves to non-scientists?

U70. What conditions are needed so that Rossby waves have zero phase speed relative to the ground? Can such conditions occur in the real atmosphere?

U71. How does static stability affect the phase speed of baroclinic Rossby waves?

U72. Using eq. (11.40) for the north-south displacement of a baroclinic wave, discuss (and/or plot) how the location of the crest of the wave changes with altitude z within the troposphere.

U73. Where, with respect to the ridges and troughs in a baroclinic wave, would you expect to find the greatest: (a) vertical displacement; (b) vertical velocity; (c) potential temperature perturbation?

U74. In Figs. 11.51 and 11.53 in the jet stream, there is just as much air going northward as there is air going southward across any latitude line, as required by mass conservation. If there is no net mass transport, how can there be heat or momentum transport?

U75. a. What is the sign of CC for a direct circulation in the S. Hemisphere (i.e., for eq. 11.50)?

b. Determine the signs of terms in eq. (11.51) for the S. Hemisphere.

U76. Why does the expression for cell circulation contain the Coriolis parameter and Brunt-Väisälä frequency?

U77. Would there be an Ekman spiral in the ocean if there was no Coriolis force? Explain.

U78. Consider a cyclonic air circulation over an ocean in your hemisphere. Knowing the relationship between ocean currents and surface winds, would you anticipate that the near-surface wind-driven ocean currents are diverging away from the center of the cyclone, or converging toward the center? Explain, and use drawings. Note: Due to mass conservations, horizontally diverging ocean surface waters cause upwelling of nutrient-rich water toward the surface, which can support ocean plants and animals, while downwelling does the opposite.

Web-Enhanced Questions

W1. Download a northern or southern hemispheric map of the winds near the tropopause (or pressures or heights from which winds can be inferred), and identify regions of near zonal flow, near meridional flow, monsoon circulations over the ocean and over continents, and the ITCZ.

W2. Download an animated loop of geostationary satellite photos (IR or water vapor) for the whole Earth disk, and identify regions of near zonal flow, near meridional flow, extratropical cyclones, ITCZ, and tropical cyclones (if any).

W3. From the web, find a rawinsonde sounding at a location in the trade-wind region, and confirm the wind reversal between low and high altitudes.

W4. Download a still, daytime, visible satellite image for the whole disk from a geostationary satellite, and estimate fractional cloudiness over the different latitude belts for that instant. Use this to estimate the incoming solar radiation reaching the ground in each of those belts, and plot the zonally-averaged results vs. latitude.

W5. Download a series of rawinsonde soundings for different latitudes between the equator and a pole. Find the tropopause from each sounding, and then plot the variation of tropopause height vs. latitude.

W6. Download a map of sea-surface temperature (SST), and discuss how SST varies with latitude.

W7. Download an IR satellite image showing the entire Earth (i.e., a whole disk image), and compute the effective radiation temperature averaged around latitude belts. Use this with the Stefan-Boltzmann equation to estimate and plot E_{out} vs. latitude due to IR radiation.

W8. Download satellite-derived images that show the climatological average incoming and outgoing radiation at the top of the atmosphere. How does it relate to the idealized descriptions in this chapter?

W9. Download satellite-derived or buoy & ship-derived ocean currents for the global oceans, and discuss how they transport heat meridionally, and why the oceanic transport of heat is relatively small at mid to high latitudes in the N. Hemisphere.

W10. Use a satellite image to locate a strong portion of the ITCZ over a rawinsonde site, and then download the rawinsonde data. Plot (compute if needed) the variation of pressure with altitude, and discuss how it does or doesn't deviate from hydrostatic.

W11. Search the web for sites where you can plot "reanalysis data", such as the NCEP/NCAR reanalysis or any of the ECMWF reanalyses. Pick a month during late summer from some past year in this data-base, and plot the surface pressure map. Explain how this "real" result is related to a combination of the "idealized" planetary and monsoonal circulations.

W12. Same as the previous exercise, but for monthly average vertical cross sections that can be looped as movies. Display fields such as zonal wind, meridional wind, and vertical velocity, and see how they vary over a year.

W13. Download weather maps of temperature for the surface, and for either the 85 kPa or 70 kPa isobaric surfaces. Over your location, or other location specified by your instructor, use the thermal wind relationship to find the change of geostrophic wind with height between the surface and the isobaric level you chose above the surface.

W14 Download a weather map for the 100-50 kPa thickness, and calculate the components of the thermal-wind vector for that surface. Also, what is the thermal-wind magnitude? Draw arrows on the thickness chart representing the thermal-wind vectors.

W15. Download data from a string of rawinsonde stations arranged north to south that cross the jet stream. Use the temperature, pressure, and wind speed data to produce vertical cross-section analyses of: (a) temperature; (b) pressure; and (c) wind speed. Discuss how these are related to the dynamics of the jet stream.

W16. Download weather maps of height contours for the 20 or 30 kPa isobaric surface, and draw a line within the region of most closely-spaced contours to indicate the location of the jet stream core. Compare this line to weather maps you can download from various TV and weather networks showing the jet stream location. Comment on both the location and the width of the jet stream.

W17. Download a weather forecast map that shows vorticity. What type of vorticity is it (relative, absolute, potential, isentropic)? What is the relationship between vorticity centers and fronts? What is the relationship between vorticity centers and bad weather (precipitation)?

W18. Download surface weather observations for many sites around your location (or alternately, download a weather map showing wind speed and direction), and calculate the following vorticities at your location: (a) relative; (b) absolute; (c) potential.

W19. Download a map of height contours for either the 50, 30, or 20 kPa surface. Measure the wavelength of a dominant planetary wave near your region, and calculate the theoretical phase speed of both barotropic and baroclinic waves for that wavelength. Compare and discuss.

W20. Download maps at a variety of altitudes and for a variety of fields (e.g., heights, temperature, vertical velocity, etc.), but all valid at the same time, and discuss how they are related to each other using baroclinic theory.

W21. View an animation of a loop of IR or water vapor satellite images for the whole disk of the Earth. Identify the three bands of the general circulation, and discuss how they deviate from the idealized picture sketched in this chapter. To hand a result in, print one of the frames from the satellite loop and sketch on it lines that demarcate the three zones. Speculate on how these zones vary with season.

W22. Download a map of recent ocean surface currents, and compare with a map showing the general circulation winds for the same time. Discuss how the two maps are related.

Synthesis Questions

S1. If the Earth did not rotate, how would the steady-state general circulation be different, if at all?

S2. If our current rotating Earth with current global circulation suddenly stopped rotating, describe the evolution of the global circulation over time. This is called the **spin-down** problem.

S3. If the Earth rotated twice as fast, how would the steady-state global circulation be different, if at all?

S4. If our current rotating Earth with current global circulation suddenly started rotating twice as fast, describe the evolution of the global circulation over time. This is a **spin-up** situation.

S5. If radiative heating was uniform between the equator and the poles, how would the steady-state global circulation be different, if at all?

S6. If the equator were snow covered and the poles were not, how would the global circulation change, if at all?

S7. Suppose that the sun caused radiative cooling of Earth, while IR radiation from space caused warming of Earth. How would the weather and climate be different, if at all?

S8. If an ice age caused the polar ice caps to expand to 50° latitude in both hemisphere, describe how global circulation would change, if at all?

S9. About 250 million years ago, all of the continents had moved together to become one big continent called **Pangaea**, before further plate tectonic movement caused the continents to drift apart. Pangaea spanned roughly 120° of longitude (1/3 of Earth's circumference) and extended roughly from pole to pole. Also, at that time, the Earth was spinning faster, with the solar day being only about 23 of our present-day hours long. Assuming no other changes to insolation, etc, how would the global circulation have differed compared to the current circulation?

S10. If the Earth was dry and no clouds could form, how would the global circulation differ, if at all? Would the tropopause height be different? Why?

S11. Suppose the troposphere were half as deep. How would the global circulation be different?

S12. If potential vorticity was not conserved, how would the global circulation be different, if at all?

S13. Suppose the average zonal winds were faster than the phase speed of short waves, but slower than the phase speed for long waves. Describe how the weather and weather forecasting would be different at mid-latitudes, if at all?

S14. Suppose that short waves moved faster than long waves, how would the weather at mid-latitudes be different, if at all?

S15. Suppose the troposphere was statically <u>un</u>stable everywhere. Could baroclinic waves exist? If so, how would they behave, and how would the weather and global circulation be different, if at all?

S16. If planetary waves did not transport momentum, heat, and moisture meridionally, how would the weather and climate be different, if at all?

S17. If the ocean currents did not transport any heat, how would the atmospheric global circulation and weather be different, if at all?

S18. What if the ocean surface were so slippery that there would be zero wind drag. Would any ocean currents develop? How? Why?

S19. Suppose there was an isolated small continent that was hot relative to the surrounding cooler ocean. Sketch a vertical cross section in the atmosphere across that continent, and use thickness concepts to draw the isobaric surfaces. Next, draw a plan-view map of heights of one of the mid-troposphere isobaric surfaces, and use thermal-wind effects to sketch wind vectors on this same map. Discuss how this approach does or doesn't explain some aspects of monsoon circulations.

S20. If the Rossby wave of Fig. 11.50 was displaced so that it is centered on the equator (i.e., point (1) starts at the equator), would it still oscillate as shown in that figure, or would the trough of the wave (which is now in the S. Hem.) behave differently? Discuss.

S21. If the Earth were shaped like a cylinder with its axis of rotation aligned with the axis of the real Earth, could Rossby waves exist? How would the global circulation be different, if at all?

S22. In the subtropics, low altitude winds are from the east, but high altitude winds are from the west. In mid-latitudes, winds at all altitudes are from the west. Why are the winds in these latitude bands different?