AP Calculus AB/Sem and AB/BC Summer Assignment

Please complete each problem **in order** on a separate sheet(s) of paper. This assignment is due about one week into next school year, **not** the first day of school. At that time, you will be asked to submit all of your work and copy several of your answers (**only** the answers) onto an Answer Sheet.

This assignment is tough – there's no other way to say it. You are going to be asked to review all of the algebra and geometry skills that you've ever been taught – and that's a lot! Please don't get too discouraged if you have some trouble; it is to be expected. Also, expect to work for a minimum of 10 hours on this assignment over the summer – **plan accordingly!**

A few words on our work expectations … Please know that AP Calculus is a tough and demanding course. Expect this because it is meant to be the equivalent of a college-level calculus course. Understand that we are expecting you to work, and to work hard. We need you to persevere through the times when it seems tough so that you can reap the sweet rewards that come from hard work. Life is all about choices – now that you have chosen to take this course, we expect you to choose to do the work required. And remember that the things in life that are the most rewarding are the things for which you have persevered and worked the most.

This assignment, however, is designed to assess if **you** are ready, not if your friends are ready for Calculus, so please understand that it is hoped and expected that you will "do the right thing" and work mostly by yourself. If you've already tried a problem, but couldn't figure it out, your Calculus teacher next year is another possible source of help.

Note on calculator use: Although you'll probably have your calculator handy as you work, please know that most of the first semester of AP Calculus is done **without** a calculator of any kind. In fact, over **half** of the AP Exam is done without a calculator. We ask that you show algebra work as you do most of these problems because we want you to understand what you'll be asked to do next year.

You are welcome to use any printed resource at your disposal (old notes, our textbook, Internet sites, etc.) and everyone will probably need to look up something (distance formula, quadratic formula, etc.) – please do so!

Suggested General Math Help Websites:

Practice problems and tutorials at: http://prep.math.lsa.umich.edu/pmc/
Algebra topics: http://www.purplemath.com/modules/index.htm
Trigonometric topics: https://www.khanacademy.org/math/trigonometry

Good luck! You can do it!!

AP Calculus (AB/Sem + AB/BC) – Summer Assignment Problems

1. On your work sheet, redraw the table below and then complete it with the appropriate notation or graph.

Inequality Notation	**Interval Notation**	**Graph**
$-2 < x \le 4$		
	$[-1,7)$	
$x \in R$ (all real numbers)		

2. Determine equations of the following lines (in **<u>point-slope form</u>**):

   (a) the slope is -4 and the y-intercept is 5.
   (b) the slope is 8 and the line passes through $(-6,123)$.
   (c) the line that passes through $(-1,3)$ and $(2,-4)$.
   (d) the line that passes through $(-1,2)$ and is perpendicular to the line $2x-3y+5=0$.
   (e) the line that passes through $(2,3)$ <u>and</u> the midpoint of the line segment from $(-1,4)$ to $(3,2)$.

3. Find the point of intersection of the lines <u>by hand</u>: $3x-y-7=0$ and $x+5y+3=0$.

4. Solve the following equations for the indicated variables:

   (a) $V=2(ab+bc+ca)$, solve for a (b) $A=P+nrP$, solve for P

   (c) $\frac{2x}{4\pi}+\frac{1-x}{2}=0$, solve for x (d) $2x-2y\frac{dy}{dx}=y+x\frac{dy}{dx}$, solve for $\frac{dy}{dx}$

   (e) $3y^2y'+2yy'=5y'+2x$, solve for y'

5. For the function $f(x)=2x^2-1$, find <u>and simplify</u> each of the following:
   (a) $f(-2)$ (b) $f(w)$ (c) $f(x+5)$ (d) $f(x+h)$ (e) $f(x+h)-f(x)$

6. Simplify $\frac{f(x+h)-f(x)}{h}$, where … (a) $f(x)=2x+3$ (b) $f(x)=x^2$ (c) $f(x)=\frac{1}{x+1}$

7. Find the domain of the functions <u>by hand</u>: (a) $g(x)=\frac{5x-3}{2x+1}$ (b) $f(x)=7$ (c) $f(x)=\frac{3x+1}{\sqrt{x+2}}$

8. Write each absolute-value function as a piecewise function:

(a) $f(x)=|x|$ (b) $f(x)=|x-5|$ (c) $f(x)=\dfrac{|x|}{x}$

9. Simplify each of the following expressions to the form $ca^p b^q$ where c, p, and q are real numbers:

(a) $\dfrac{\left(2a^2\right)^3}{b}$ (b) $\sqrt{9ab^3}$ (c) $\dfrac{a\left(\frac{2}{b}\right)}{\frac{3}{a}}$ (d) $\dfrac{ab-a}{b^2-b}$ (e) $\dfrac{a^{-1}}{\left(b^{-1}\right)\sqrt{a}}$ (f) $\left(\dfrac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\right)^2\left(\dfrac{b^{\frac{3}{2}}}{a^{\frac{1}{2}}}\right)$

10. Condense each of the following to an expression with a single base.

(a) $e^3 \cdot e^8$ (b) $e^{2x} \cdot e^{y^3}$

11. Expand each of the following to an expression with multiple, common bases.

(a) e^{x+y} (b) e^{x^2+4} (c) $e^{x+\ln 3}$

12. Write the conjugate of each expression. Look up "conjugate" if you don't recognize the term.

(a) $\left(3+2i\right)$ (b) $\left(-\sqrt{3}-\sqrt{7}\right)$ (c) $\left(\sqrt{x+h}-\sqrt{x}\right)$

13. Factor <u>completely</u>:

(a) $2x^2-7x+3$ (b) x^4-1 (c) x^6-16x^4

(d) $4x^3-8x^2-25x+50$ (e) $8x^3+27$

14. Find <u>all</u> real solutions to the following problems by factoring:

(a) $x^6-16x^4=0$ (b) $4x^3-8x^2-25x+50=0$ (c) $8x^3+27=0$ (d) $2x^2-7x=-3$

15. Solve the equations algebraically: (a) $4x^2+12x+3=0$ (b) $2x+1=\dfrac{5}{x+2}$ (c) $\dfrac{x+1}{x}-\dfrac{x}{x+1}=0$

16. Solve for x using a graphical approach: (a) $|-x+4|\le 1$ (b) $|5x-2|=8$

Hints: (a) for what value(s) of x is the graph of $y=|-x+4|$ below the graph of $y=1$?

(b) for what value(s) of x is the graph of $y=|5x-2|$ equivalent to the graph of $y=8$?

17. Use the table to evaluate.

(a) $f(2) =$

(b) $f^{-1}(6) =$

(c) $g(\pi) =$

(d) $g^{-1}(3) =$

x		**f(x)**	**g(x)**
2		9	10
π		6	7
6		4	3

18. Simplify as much as possible:

(a) ln 0 (b) ln 1 (c) $\ln e$ (d) $\log_{10}\left(10^{\frac{1}{2}}\right)$ (e) $\log_{10}\left(\frac{1}{10^x}\right)$

(f) $3^{2\log_3 5}$ (g) $2\log_{10}\sqrt{x} + 3\log_{10} x^{\frac{1}{3}}$ (h) $\log_2 5 + \log_2\left(x^2 - 1\right) - \log_2\left(x - 1\right)$

<u>Rational Expressions and Functions</u>

19. Simplify each rational expression: (a) $\frac{x^3 - 9x}{x^2 - 7x + 12}$ (b) $\frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$

20. Consider the rational function, $f(x) = \frac{x^2 - 5x + 4}{x - 1}$ and <u>describe</u> how to determine where the graph of a rational functional has a vertical asymptote.

21. <u>Describe</u> the three-part rule for determining if the graph of a rational function has a horizontal asymptote.

22. <u>Describe</u> how to determine if the graph of a rational function has a "hole" in the graph.

23. For the function $f(x) = \frac{x^2 - 5x + 4}{x - 1}$, <u>describe</u> what happens to the y-values as the x-values get …

(a) really close to $x = 1$.

(b) really large in the positive direction.

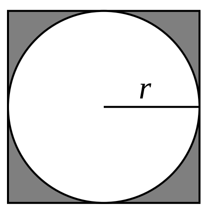
(c) really large in the negative direction.

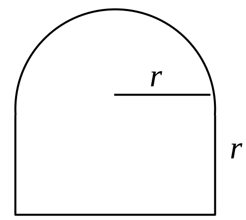
Geometry

24. Label and write the following formulas from geometry:

(a) Area of a circle.
(b) Circumference of a circle.
(c) Area of a triangle.
(d) Volume of a cube.
(e) Surface area of a cube.
(f) Volume of a right rectangular prism.
(g) Surface area of a right rectangular prism.
(h) Volume of a cone.
(i) Volume of a sphere.
(j) Surface area of a sphere.
(k) Area of an equilateral triangle.
(l) Area of a cross-section of a sphere.

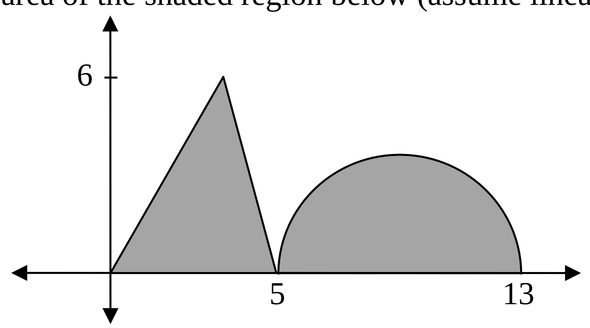
25. Find a formula (in terms of *r*) for the shaded area (below left) that is inside the square but outside the circle.





26. Find a formula (in terms of *r*) for the perimeter of a window in the shape pictured (above right).

27. Find the area of the shaded region below (assume linear segments and a semicircle).



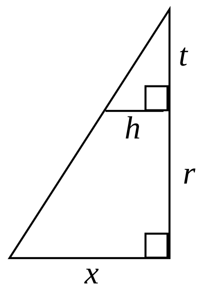
28. A water tank has the shape of a cone (like an ice cream cone without the ice cream – boo!). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle), what is the area of the surface (top) of the water?

29. Two cars start moving from the same point. One travels south at 100km/hr, the other west at 50 km/hr. How far apart are they two hours later?

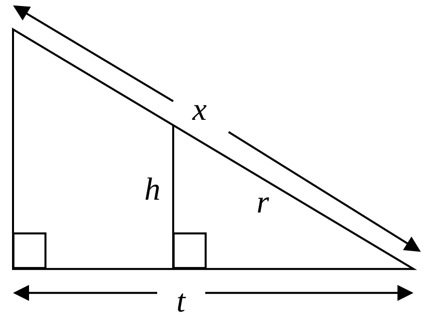
30. A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal? (Assume that the string is perfectly straight.)

31. Express x in terms of the other variables in the picture. Set up a proportion and solve for x.

(a)



(b)



32. (a) Sketch a 45-45-90 special right triangle with a leg of length 10 cm. Label the other side lengths.
(b) Sketch a 30-60-90 special right triangle with a short leg of length 10 cm. Label the other side lengths.

33. Fill in the Unit Circle provided on the last page of this packet by naming the special angles in both degrees and radians, and then labeling the coordinates in exact form at every "special" angle. Staple the completed unit circle to your work sheets following question number 45. Note: You must know the Unit Circle without hesitation.

34. Use the unit circle to answer the following questions. Give an <u>exact</u> answer (no decimals).

(a) $\cos 210^0$ (b) $\sin\frac{9\pi}{4}$ (c) $\cos\frac{5\pi}{4}$ (d) $\tan\frac{7\pi}{6}$

(e) $\sin^{-1}\frac{\sqrt{3}}{2}$ (f) $\cos^{-1}(-1)$ (g) $\sin^{-1}(-1)$ (h) $\tan^{-1}(-1)$

35. Solve for x algebraically in each of the following trigonometric equations. **Hints:** Isolate the variable; sketch a reference triangle or use your completed unit circle; find all the solutions within the domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle.

(a) $\sin x = \frac{-1}{2}$ (b) $2\cos x = \sqrt{3}$ (c) $\cos 2x = \frac{1}{\sqrt{2}}$

(d) $\sin^2 x = \frac{1}{2}$ (e) $\sin 2x = -\frac{\sqrt{3}}{2}$ (f) $4\cos^2 x - 3 = 0$

36. List the following Fundamental Trigonometric Identities. Look them up if you don't remember.
(a) There are six (6) Reciprocal Identities.
(b) There are two (2) Ratio/Quotient Identities.
(c) There are three (3) Pythagorean Identities.
(d) The Double-Angle Identities for sine and cosine.

Mini-Lesson #1: Complex Fractions

When simplifying complex fractions, one technique is to multiply by a fraction equal to "1" which has a numerator and denominator composed of the least common denominator (LCD) of all the denominators in the complex fraction. This is an example of a very common math "trick" where one multiplies by "1" written the way one desires (in this case, as $\frac{\text{LCD}}{\text{LCD}}$).

Examples:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Now that you have read and worked through the above notes and examples above about simplifying complex fractions, complete question 37.

37. Simplify each complex fraction: (a) $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$ (b) $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$ (c) $\dfrac{9-x^{-2}}{3+x^{-1}}$

Mini-Lesson #2: Evaluating and Rational Roots of Higher Ordered Polynomials

As we stated at the beginning of this assignment, about half of the AP Calculus test is done without a calculator. The use of synthetic division will enable us to find roots and evaluate higher degree polynomial equations without a calculator. There are two theorems that are the premises of this skill.

1) Remainder Theorem
2) The Rational Root Theorem

Remainder Theorem

"If a polynomial $f(x)$ is divided by $x-c$, then the remainder is $f(c)$."

Simply put, the Remainder Theorem says if you want to evaluate a polynomial at some value *c*, then the answer is the remainder of the polynomial long-division or synthetic division process.

Example: Use synthetic division and the Remainder Theorem to find $f(2)$ if $f(x)=-x^3+5x-7$.

Step 1: Step up the synthetic division problem. In this case, we are trying to evaluate $f(2)$, so put 2 in the division box.	$\begin{array}{r\|rrrr} 2 & -1 & 0 & 5 & -7 \\ & & & & \\ \hline \end{array}$
Step 2: Use the synthetic division process.	$\begin{array}{r\|rrrr} 2 & -1 & 0 & 5 & -7 \\ & \downarrow & -2 & -4 & 2 \\ \hline & -1 & -2 & 1 & -5 \end{array}$
Step 3: Apply the Remainder Theorem. In this case, the remainder is –5, so the answer to $f(2)$ is:	$f(2) = -5$

Useful websites:

1. http://www.mesacc.edu/~scotz47781/mat120/notes/divide_poly/remainder/remainder_thm.html
2. https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-div/x2ec2f6f830c9fb89:remainder-theorem/v/polynomial-remainder-theorem

Now that you have read and worked through the above notes and examples above and reviewed some web sites about evaluating higher degree polynomial equations, complete question 38 below.

38. Evaluate the following polynomials using synthetic division being sure to show your work.

(a)Find $f(3)$ if $f(x) = 2x^5 - 3x^4 - x^3 - 6x + 5$

(b) Find $g(-4)$ if $g(x) = -x^5 + 2x^3 - x^2 + 5x + 1$

<u>Mini-Lesson #3: Equations of Circles</u>

Important! <u>Read</u> and <u>work</u> through the following notes and examples about the equations of circles, as this information is REQUIRED for Calculus.

DEFINITION

In geometry, we define a circle as the set of all points $P(x,y)$ in the plane that are equidistant from a given point (center), $C(h,k)$.

We can derive the general equation of a circle using the distance formula. We need to find the distance between $P(x,y)$ and $C(h,k)$ and we call this distance the radius, *r*, of the circle.

$$r = PC$$
$$r = \sqrt{(x-h)^2+(y-k)^2}$$
$$r^2 = (x-h)^2+(y-k)^2$$

> The general equation of a circle with center at (h,k) and radius *r* is
> $$(x-h)^2+(y-k)^2 = r^2$$

NOTE: If the center of the circle is at the origin, the equation becomes $x^2+y^2=r^2$.

<u>Examples</u>

Ex 1) Find the center and radius of the circle: $(x-3)^2+(y+7)^2=19$

Solution:

The center is $(3,-7)$ and the radius is $r=\sqrt{19}$.

Ex 2) Find the equation of the circle with center $(7,-3)$ and the circle is tangent to the *y*-axis.

Solution:

The distance from the center point $(7,-3)$ to the *y*-axis on the circles edge is 7; this is the radius.

Therefore, the equation of this circle is $(x-7)^2+(y+3)^2=49$

Side note ➔Remember how to "**<u>complete the square</u>**" from your study of quadratics?

Let's review:

$x^2+6x=4$ (always make sure the constant is on the right)

$x^2+6x+\_\_=4+\_\_$ (balance the equation)

$x^2+6x+\underline{9}=4+\underline{9}$ (take $\frac{1}{2}$ the coefficient of *x*, square it, and add it to both sides)

$(x+3)^2 \quad =13$ (factor the left side and simplify the right side)

(notes and examples continued)

Now, try this one!

$x^2+6x-1=14$ You should be able to get: $(x+3)^2=24$

OK, now we need to use the "complete the square" technique (twice) to transform the original equation into the general form of the equation of a circle in the following example.

Ex 3) Find the center and radius of the circle: $x^2+y^2-6x+4y-12=0$

Solution:

Given: $x^2+y^2-6x+4y-12=0$

$$x^2-6x \qquad +y^2+4y \qquad =12$$
$$x^2-6x+\_\ +y^2+4y+\_\ =12$$
$$x^2-6x+9+y^2+4y+4=12+9+4$$
$$(x^2-6x+9)+(y^2+4y+4)=25$$
$$(x-3)^2+(y+2)^2=25$$

center $(3,-2)$

$\text{radius}=5$

Now that you have read and worked through the above notes and examples about the equations of circles, complete questions 39 through 43 below.

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39. Find the equation of the circle with center $(1,2)$ that passes through the point $(-2,-1)$.

40. For the circle $x^2+y^2+6x-4y+3=0$, find:
    (a) the center and radius
    (b) the equation of the line that is tangent to the circle at the point $(-2,5)$.

41. Graph the circle: $x^2+y^2=16$

42. Graph the circle: $y=\pm\sqrt{25-x^2}$

43. Graph the semicircle: $y=-\sqrt{81-x^2}$

Do not graph here.

Draw neat, labeled graphs on graph paper as a continuation of your work.

Graphs of basic functions (Toolkit of Functions)

44. Sketch the graphs of the following functions by hand**, labeling important features** such as intercepts (both *x*- and *y*-), vertex, asymptotes (dashed lines), etc.

(a) $f(x) = -x + 6$ (b) $g(x) = \frac{1}{3}x^2$ (c) $h(x) = 5x^3$ (d) $j(x) = \sqrt{x}$

(e) $k(x) = \sqrt[3]{x}$ (f) $l(x) = \dfrac{1}{x}$ (g) $m(x) = e^x$ (h) $n(x) = e^{-x}$

(i) $p(x) = \ln x$ (j) $q(x) = \log_5 x$ (k) $r(x) = -2(x+3)^2 - 1$

(l) $s(x) = \sin x$ (m) $t(x) = \cos x$ (n) $w(x) = \tan x$

Do not graph here.

You're almost done!!

Draw neat, labeled graphs on graph paper as a continuation of your work.

Unit Circle