‘And the whole is greater
than the part.’

-Euclid

MATHEMATICS

LESSON PLAN
GRADE 10 TERM 3

MESSAGE FROM NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE). We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

WHAT IS NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

WHAT ARE THE LEARNING PROGRAMMES?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Universalisation Programme and in its Provincialisation Programme.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

CONTENTS

Message from NECT ii

Contents iii

Programme Orientation iv

Topic 1 Analytical Geometry 1

Topic 1, Lesson 1: Distance between two points 5

Topic 1, Lesson 2: Gradient of a line segment 13

Topic 1, Lesson 3: Midpoint of a line segment 18

Topic 1, Lesson 4: Revision and Consolidation 23

Topic 2 Finance and Growth 31

Topic 2, Lesson 1: Simple Interest 35

Topic 2, Lesson 2: Compound Interest 42

Topic 2, Lesson 3: Hire Purchase 47

Topic 2, Lesson 4: Inflation and Growth 53

Topic 2, Lesson 5: Foreign Exchange Rates 59

Topic 2, Lesson 6: Revision and Consolidation 63

Topic 3 Statistics 69

Topic 3, Lesson 1: Measures of central tendency 75

Topic 3, Lesson 2: Measures of dispersion 83

Topic 3, Lesson 3: Five-number summary and box & whisker plots 90

Topic 3, Lesson 4: Interpretation 94

Topic 3, Lesson 5: Revision and Consolidation 99

Topic 4 Trigonometry 103

Topic 4, Lesson 1: Revision of trigonometric ratios 107

Topic 4, Lesson 2: Angle of elevation and depression 112

Topic 4, Lesson 3: Solving problems in 2 dimensions 115

Topic 5 Euclidean Geometry 125

Topic 5, Lesson 1: Solving geometry riders 130

Topic 6 Measurement 138

Topic 6, Lesson 1: Revision 142

Topic 6, Lesson 2: The effect on volume and surface area when multiplying any dimension by a constant factor k. 153

Topic 6, Lesson 3: Volume and surface areas of spheres, right pyramids and cones 156

PROGRAMME ORIENTATION

Welcome!

The NECT FET Mathematics Learning Programme is designed to support teachers by providing:

- Lesson Plans
- Trackers
- Resource Packs
- Assessments and Memoranda
- Posters.

This Mathematics Learning Programme provides most of the planning required to teach FET Mathematics. However, it is important to remember that although the planning has been done for you, preparation is key to successful teaching. Set aside adequate time to properly prepare to teach each topic.

Also remember that the most important part of preparation is ensuring that you develop your own deep conceptual understanding of the topic. Do this by:

- working through the lesson plans for the topic
- watching the recommended video clips at the end of the topic
- completing all the worked examples in the lesson plans
- completing all activities and exercises in the textbook.

If, after this, a concept is still not clear to you, read through the section in the textbook or related teacher's guide, or ask a colleague for assistance. You may also wish to search for additional teaching videos and materials online.

Orientate yourself to this Learning Programme by looking at each component, and by taking note of the points that follow.

TERM 3 TEACHING PROGRAMME

1. In line with CAPS, the following teaching programme has been planned for FET Mathematics for Term 3:

Grade 10		Grade 11		Grade 12	
Topic	**No. of weeks**	**Topic**	**No. of weeks**	**Topic**	**No. of weeks**
Analytical Geometry	2	Measurement	1	Euclidean Geometry	2
Finance and Growth	2	Euclidean Geometry	3	Statistics	2
Statistics	2	Trigonometry	2	Counting and probability	2
Trigonometry	1.5	Finance, growth and decay	2		
Euclidean Geometry	1	Probability	2		
Measurement	1.5				

2. Term 3 lesson plans and assessments are provided for ten weeks for Grades 10 and 11.
3. Term 3 lesson plans and assessments are provided for six weeks for Grade 12
4. Each week includes 4,5 hours of teaching time, as per CAPS.
5. You may need to adjust the lesson breakdown to fit in with your school's timetable.

LESSON PLAN STRUCTURE

The Lesson Plan for each term is divided into topics. Each topic is presented in exactly the same way:

TOPIC OVERVIEW

1. Each topic begins with a brief **Topic Overview**. The topic overview locates the topic within the term, and gives a clear idea of the time that should be spent on the topic. It also indicates the percentage value of this topic in the final examination, and gives an overview of the important skills and content that will be covered.
2. The **Lesson Breakdown Table** is essentially the teaching plan for the topic. This table lists the title of each lesson in the topic, as well as a suggested time allocation.
   For example:

	Lesson title	Suggested time (hours)
1.	Revision	1,5
2.	Gradient and average gradient	1

3. The **Sequential Table** shows the prior knowledge required for this topic, the current knowledge and skills to be covered, and how this topic will be built on in future years.
   - Use this table to think about the topic conceptually:
     - Looking back, what conceptual understanding should learners have already mastered?
     - Looking forward, what further conceptual understanding must you develop in learners, in order for them to move on successfully?
   - If learners are not equipped with the knowledge and skills required for you to continue teaching, try to ensure that they have some understanding of the key concepts before moving on.
   - In some topics, a revision lesson has been provided.
4. The **NCS Diagnostic Reports**. This section is potentially very useful. It lists common problems and misconceptions that are evident in learners' NSC examination scripts. The Lesson Plans aim to address these problem areas, but it is also a good idea for you to keep these in mind as you teach a topic.
5. The **Assessment of the Topic** section outlines the formal assessment requirements as prescribed by CAPS for Term 3.

Grade	Assessment requirements for Term 3 (as prescribed in CAPS)
10	Two tests
11	Two tests
12	One test and one preliminary examination

6. The glossary of **Mathematical Vocabulary** provides an explanation of each word or phrase relevant to the topic. In some cases, an explanatory sketch is also provided. It is a good idea to display these words and their definitions or sketches somewhere in the classroom for the duration of the topic. It is also a good idea to encourage learners to copy down this table in their free time, or alternately, to photocopy the Mathematical Vocabulary for learners at the start of the topic. You should explicitly teach the words and their meanings as and when you encounter these words in the topic.

INDIVIDUAL LESSONS

1.. Following the **Topic Overview**, you will find the **Individual Lessons.** Each lesson is structured in exactly the same way. The routine within the individual lessons helps to improve time on task, and therefore, curriculum coverage.

2. In addition to the lesson title and time allocation, each lesson plan includes the following:

   A. Policy and Outcomes*.* This provides the CAPS reference, and an overview of the objectives that will be covered in the lesson.

   B. Classroom Management*.* This provides guidance and support as you plan and prepare for the lesson.

   - Make sure that you are ready to begin your lesson, have all your resources ready (including resources from the Resource Pack), have notes written up on the chalk-board, and are fully prepared to begin.
   - Classroom management also suggests that you plan which textbook activities and exercises will be done at which point in the lesson, and that you work through all exercises prior to the lesson.
   - In some cases, classroom management will also require you to photocopy an item for learners prior to the lesson, or to ensure that you have manipulatives such as boxes and tins available.

   The Learner Practice Table**.** This lists the relevant practice exercises that are available in each of the approved textbooks.

   - It is important to note that the textbooks deal with topics in different ways, and therefore provide a range of learner activities and exercises. Because of this, you will need to plan when you will get learners to do the textbook activities and exercises.
   - If you feel that the textbook used by your learners does not provide sufficient practice activities and exercises, you may need to consult other textbooks or references, including on0line references.
   - The *Siyavula* Open Source Mathematics textbooks are offered to anyone wishing to learn mathematics and can be accessed on the following website: https://www.everythingmaths.co.za/read

   C. Conceptual Development*:*

   This section provides support for the actual teaching stages of the lesson.

   Introduction: This gives a brief overview of the lesson and how to approach it. Wherever possible, make links to prior knowledge and to everyday contexts.

   Direct Instruction: Direct instruction forms the bulk of the lesson. This section describes the teaching steps that should be followed to ensure that learners develop conceptual understanding. It is important to note the following:

- Grey blocks talk directly to the teacher. These blocks include teaching tips or suggestions.
- Teaching is often done by working through an example on the chalkboard. These worked examples are always presented in a table. This table may include grey cells that are teaching notes. The teaching notes help the teacher to explain and demonstrate the working process to learners.
- As you work through the direct instruction section, and as you complete worked examples on the chalkboard, ensure that learners copy down:
  - formulae, reference notes or explanations
  - the worked examples, together with the learner's own annotations.
- These notes then become a reference for learners when completing examples on their own, or when preparing for examinations.
- At relevant points during the lesson, ensure that learners do some of the Learner Practice activities as outlined at the beginning of each lesson plan. Also, give learners additional practice exercises and questions from past papers as homework. Ensure that learners are fully aware of your expectations in this respect.

D. Additional Activities / Reading*.* This section provides you with web links related to the topic. Get into the habit of visiting these links as part of your lesson preparation. As teacher, it is always a good idea to be more informed than your learners. If possible, organise for learners to view video clips that you find particularly useful.

TRACKER

1. A Tracker is provided for each grade for each term. The Trackers are CAPS compliant in terms of content and time.
2. You can use the Tracker to document your progress. This helps you to monitor your pacing and curriculum coverage. If you fall behind, make a plan to catch up.
3. Fill in the Tracker on a daily or weekly basis.
4. At the end of each week, try to reflect on your teaching progress. This can be done with the HoD, with a subject head, with a colleague, or on your own. Make meaningful notes about what went well and what didn't. Use the reflection section to reflect on your teaching, the learners' learning and to note anything you would do differently next time.
   These notes can become an important part of your preparation in the following year.

RESOURCE PACK, ASSESSMENT AND POSTERS

1. A Resource Pack with printable resources has been provided for each term.
2. These resources are referenced in the lesson plans, in the Classroom Management section.
3. Two posters have been provided as part of the FET Mathematics Learning Programme for Term 3.
4. Ensure that the posters are displayed in the classroom.
5. Try to ensure that the posters are durable and long-lasting by laminating it, or by covering it in contract adhesive.
6. Note that you will only be given these resources once. It is important for you to manage and store these resources properly. You can do this by
   - Writing your school's name on all resources
   - Sticking resource pages onto cardboard or paper
   - Laminating all resources, or covering them in contact paper
   - Filing the resource papers in plastic sleeves once you have completed a topic.
7. Add other resources to your resource file as you go along.
8. Note that these resources remain the property of the school to which they were issued.

ASSESSMENT AND MEMORANDUM

In the Resource Pack you are provided with assessment exemplars and memoranda as per CAPS requirements for the term. For Term 3, the Resource Pack contains two tests and memoranda for Grade 10, and contains two tests and memoranda for Grade 11. One test, with memorandum, is provided for Grade 12. If your learners write a common examination, you could use the examinations provided for revision or as trial examinations.

CONCLUSION

Teacher support and development is a complex process. For successful Mathematics teachers, certain aspects of this Learning Programme may strengthen your teaching approach. For emerging Mathematics teachers, we hope that this Learning Programme offers you meaningful support as you develop improved structure and routine in your classroom, develop deeper conceptual understanding in your learners and increase curriculum coverage.

Term 3, Topic 1: Topic Overview

ANALYTICAL GEOMETRY

A. TOPIC OVERVIEW

A

- This topic is the first of six topics in Term 1.
- This topic runs for two weeks (9 hours).
- It is presented over four lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 9 hours). For example, one lesson in this topic could take three school lessons. Plan according to your school's timetable.
- Analytical geometry counts 15% of the final Paper 2 examination.
- The time allocated to this topic is generous. Learners can spend the time gaining a good understanding of these concepts.
- The 4th lesson (Revision and Consolidation) has been allocated the most time so that time can be spent combining the three basic concepts and working through as many different types of questions as possible.

Breakdown of topic into 4 lessons:

	Lesson title	**Suggested time (hours)**		**Lesson title**	**Suggested time (hours)**
1	Distance between two points	2,5	3	Midpoint of a line segment	2
2	Gradient of a line segment	1,5	4	Revision and Consolidation	3

B SEQUENTIAL TABLE

GRADE 8 & 9	GRADE 10	GRADE 11 & 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Plotting points on a Cartesian plane.	Derive and apply the: • distance formula • gradient of a line segment • mid-point of a line segment.	Derive and apply the: • equation of a line through two points • equation of a line through one point and parallel or perpendicular to another • inclination of a line • equation of a circle • equation of a tangent to a circle at a given point.

C WHAT THE NSC DIAGNOSTIC REPORTS TELL US

According to **NSC Diagnostic Reports** there are a number of issues pertaining to Analytical Geometry.

These include:

- basic errors with signs and computation
- copying formulae from the information sheet incorrectly
- lack of knowledge of Euclidean Geometry in general (needed to answer Analytical Geometry questions)
- not giving reasons for statements
- confusing perpendicular lines with parallel lines.

It is important that you keep these issues in mind when teaching this section.

While teaching Analytical Geometry, it is important to remind learners that a knowledge of other aspects of the curriculum is important. For example, knowing the properties of quadrilaterals from Euclidean Geometry and finding the equation of a straight line from Functions are both skills required in this section.

ASSESSMENT OF THE TOPIC

D

- CAPS formal assessment requirements for Term 3:
  - Two tests
- Two tests of 50 marks each are provided in Resources 27 and 29 in the Resource Pack. The tests are aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53). The tests are also divided in the ratio of the allocated teaching time. Memoranda for the tests are provided in Resources 28 and 30 in the Resource Pack.
- This topic is assessed in the first of the two tests.
- The questions usually take the form of finding the distance, gradient or midpoint of two points within the Cartesian plane.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

MATHEMATICAL VOCABULARY

E

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
distance	Length (in units) from one point to another. Found by using the distance formula using two points given
gradient	How steep a line is. Found by using the gradient formula using two points given
mid-point	The co-ordinate that represents the middle of a line segment. Found by using the mid-point formula using two points given
parallel	Lines that have equal gradients are parallel to each other
perpendicular	At a right angle (90º)
x**-intercept**	The point at which a graph cuts the x-axis
y**-intercept**	The point at which a graph cuts the y-axis
point of intersection	The co-ordinate where two graphs intersect each other

diagonal	The line segment joining opposite corners of a quadrilateral.
rectangle	A 4-sided shape (quadrilateral) where both pairs of opposite sides are equal in length and each of the four angles are 90º.
square	A 4-sided shape (quadrilateral) where all four sides are equal in length and each of the four angles are equal to 90º.
kite	A 4-sided shape (quadrilateral) where the two pairs of adjacent sides are equal in length. The diagonals are perpendicular to each other.
rhombus	A parallelogram with four sides of equal length
parallelogram	A 4-sided shape (quadrilateral) that has two pairs of parallel sides
equilateral triangle	A triangle with three equal sides and three equal angles
isosceles triangle	A triangle with two equal sides and two equal angles
collinear	Points that lie on the same line
origin	The point where the x and y axis meet on a Cartesian plane
line segment	All points between two given points 
perimeter	The distance around the outside of a shape (the length of the outline of the shape)
equidistant	Exactly the same distance

TERM 3, TOPIC 1, LESSON 1

DISTANCE BETWEEN TWO POINTS

Suggested lesson duration: 2,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	26

Lesson Objectives

By the end of the lesson, learners should be able to:

- explain how the distance formula was derived
- find the distance between two points
- complete an exercise on various types of distance questions.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. You will need Resource 1 from the Resource Pack.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson draw a Cartesian plane.
6. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1	210	1 2	188 189	11.1	192	11.1 11.2	241 244	8.2	292

C

CONCEPTUAL DEVELOPMENT

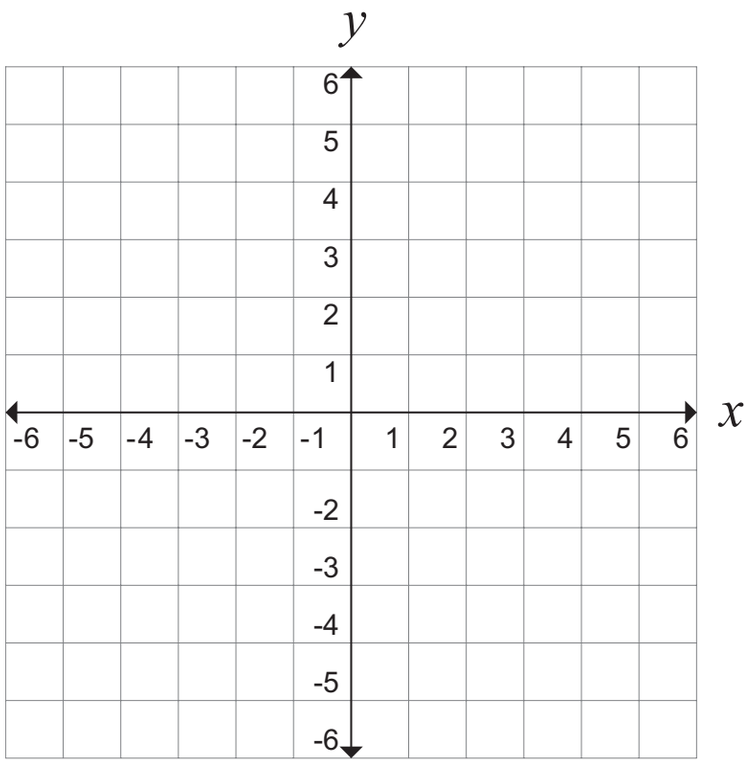
INTRODUCTION

1. This is an exciting new concept for learners. Analytical Geometry is new for learners. Even though they have already covered many skills required for this topic, learners have not yet encountered what they are about to learn.
2. Tell learners that there is plenty of time allocated to this topic so they will have time to investigate ideas and gain a good understanding of the concepts they are about to learn.
3. This lesson will follow an investigative approach. By using this approach, you give learners the opportunity to make meaning of the concept before they receive information from you.

DIRECT INSTRUCTION

1. Ask learners to draw a Cartesian plane in their books and to plot the following points:

A (1;2) *B* (5;2) *C* (5;4)



2. Learners should work with a partner to find the length of AC. If anyone is struggling to start, give them a clue: *Think of a theorem you learned in Grade 8*. Give learners a few minutes to do this task.

3. Learners should have used the theorem of Pythagoras to find that the distance of $AC = \sqrt{20}$ (or $4,5$).

4. If necessary, do this in full on the board like this:

$$AC^2 = 4^2 + 2^2$$

$$AC^2 = 16 + 4$$

$$AC^2 = 20$$

$$\therefore AC = \sqrt{20} = 2\sqrt{5}$$

5. Ask learners these directed questions regarding how they went about finding the length of AC:
   - *What did you need to do to find the horizontal length, AB?*
   - *What did you need to do to find the vertical length, BC?*

6. Listen carefully to learners' responses.
   For example, some may say they counted the blocks/units. This is acceptable. However, guide learners to think a little more deeply about what that actually means.

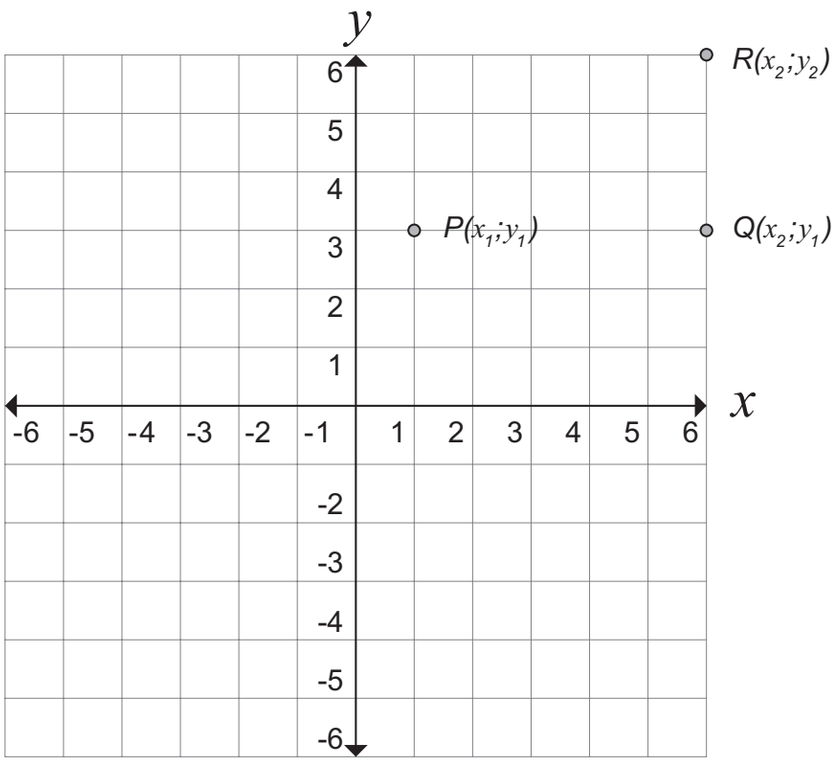
7. Ideally, you want learners to come up with the fact that they had to find the difference between the two *x*-values to find AB and to find the difference between the two *y*-values to find BC. This is the key to understanding how the distance formula is derived.

8. Once the discussion has produced the desired outcome, plot the following points on the same Cartesian plane on the board:

   P (1;3) Q (6;3) R (6;6)

   BUT, as you plot them label them as follows:

   $P(x_1;y_1)$ $Q(x_2;y_1)$ $R(x_2;y_2)$



9. Ask learners: *Is it clear why P and Q share the same y-value and why Q and R share the same x-value?* (It is the same value as they are in line with each other).

10. Learners should work with a partner to:
    - find the distance PQ, using the co-ordinates and not counting
    - find the distance QR, using the co-ordinates and not counting
    - using these two distances and the theorem of Pythagoras, find a formula that would give the length of PR.

11. Walk around the class. Guide learners as they work. Praise learners who are moving in the right direction (most should manage the first two).

12. After about 10 minutes, move back to the board. Work through the ideas with the class.

    - $PQ = x_2 - x_1$
    - $QR = y_2 - y_1$

13. Use the above distances to write a Pythagoras statement to find PR.

$$PR^2 = PQ^2 + QR^2$$

$$PR^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Ask: *What we can do to get PR on its own?* (Square root both sides)

$$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Remind learners that you cannot square root across a plus/minus sign (2 terms) so this is now in its simplest form. If learners need more explanation, do the following:

$\sqrt{16+9}$
$= \sqrt{25}$
$= 5$
NOT
$\sqrt{16+9}$
$\neq 4+3$

14. Tell learners: *We have derived the distance formula. This can be used to find the distance between any two points on a Cartesian plane.*

15. Point out: *There is no need to 'see' the entire right-angled triangle. If two points are given, one could draw in the horizontal and vertical lines to see the right-angled triangle but this is not necessary because we can use the formula.*

16. Tell learners: *You do not need to learn the formula as it will always be given in an assessment. It is given in one of the following forms.* Ask learners to write it down.

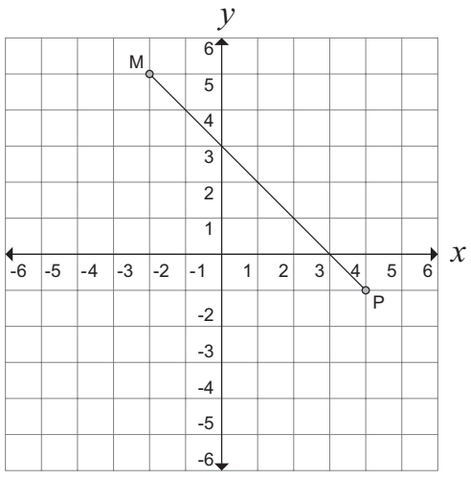
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

17. Do the following fully worked example with learners now. Tell learners to write the worked example in their books.

a) Plot the following points on a Cartesian plane and find the distance from M to P, rounded to two decimal places. *M* (–2 ;5) *P* (4 ; –1)	Tell learners: *This is the easier type of question that you will get. It is a straightforward plotting then substituting into the formula.* Advise learners: *Write the points in your book and label the first co-ordinate:* $x_1;y_1$ *and the second co-ordinate:* $x_2;y_2$. The most common error made by learners in assessments is copying the wrong numbers into the formula. Labelling them clearly in the beginning should help. Tell learners: *Distance is always units.*

Solution:



$M(-2;5)$ $P(4;-1)$

$x_1;y_1$ $x_2;y_2$

$$MP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$MP = \sqrt{(4 - (-2))^2 + (-1 - 5)^2}$$

$$MP = \sqrt{(4 + 2)^2 + (-1 - 5)^2}$$

$$MP = \sqrt{6^2 + (-6)^2}$$

$$MP = \sqrt{36 + 36}$$

$$MP = \sqrt{72} = 8,49 \text{ units}$$

Point out the different way the following distance question has been asked.
Learners need to look out for an ordinary distance question being asked in another way.
Share this with learners before doing the next example.
Other possibilities:

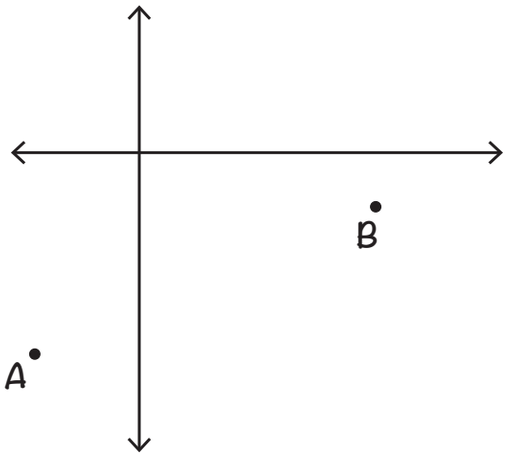
- Finding the perimeter of a triangle or quadrilateral (learners need to find the distance of all the line segments and add them).
- Proving that a triangle is scalene, isosceles or equilateral (learners need to find the distance of all the line segments in order to assess what type of triangle it is).
- Proving a certain type of quadrilateral – learners need to know the properties of a quadrilateral from Euclidean geometry.

b) The point A (-3,-6) lies on a circle. What is the length of the radius of this circle if the centre is located at B (9,-2)?

Point out the different way this distance question has been asked.
Give learners the following tip regarding ANY question in Analytical Geometry:
If a diagram is not provided, always make a sketch yourself.
This is good practice as it helps to gauge whether the answer looks reasonable.
Remind learners of this idea throughout the lessons.

Solution:

Sketch:



Notice how basic the sketch is — it is just a reference.

A (-3 ;-6) B (9 ; -1)

$x_1;y_1$ $x_2;y_2$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(9 - (-3))^2 + (-1 - (-6))^2}$

$AB = \sqrt{(9 + 3)^2 + (-1 + 6)^2}$

$AB = \sqrt{(12)^2 + (5)^2}$

$AB = \sqrt{144 + 25}$

$AB = \sqrt{169} = 13$ units

The radius of the circle is 13 units.

b) Show that the triangle ABC with co-ordinates
$A\ (a;a)$, $B\ (m;-a)$ and
$C\ (-a;m)$ is isosceles.

Many learners feel unsure of how to go about a question that only has variables as values.
If this is the case, ask: *How would you do this question if there were constants as values in the co-ordinates?*
(Find the distances and expect two of them to be the same).
Advise learners: *Do exactly the same using the variables.*

Solution:

$A\ (a;a)$ $\quad$ $B\ (m;-a)$

$x_1;y_1$ $\quad$ $x_2;y_2$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(m - a)^2 + (-a - a)^2}$$

$$B = \sqrt{(m - a)^2 + (-2a)^2}$$

$A\ (a;a)$ $\quad$ $C\ (-a;m)$

$x_1;y_1$ $\quad$ $x_2;y_2$

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AC = \sqrt{(-a - a)^2 + (m - a)^2}$$

$$AC = \sqrt{(-2a)^2 + (m - a)^2}$$

$B\ (m;-a)$ $\quad$ $C\ (-a;m)$

$x_1;y_1$ $\quad$ $x_2;y_2$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$BC = \sqrt{(-a - m)^2 + (m - (-a))^2}$$

$$BC = \sqrt{(-a - m)^2 + (m + a)^2}$$

Without simplifying further, we can see that AB = AC.

$\therefore \Delta ABC$ is isosceles.

18. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

19. Give learners an exercise to complete on their own.

20. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=0C_OJjK1VQw

https://www.mathsisfun.com/algebra/distance-2-points.html

http://www.mathwarehouse.com/algebra/distance_formula/index.php

TERM 3, TOPIC 1, LESSON 2

GRADIENT OF A LINE SEGMENT

Suggested lesson duration: 1,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	26

Lesson Objectives

By the end of the lesson, learners should be able to:

- find the gradient of a line segment
- complete an exercise on various types of gradient questions.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson draw a Cartesian plane on the chalkboard.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
3	220	3 4	193 195	11.2	195	11.4	257	8.3 8.4	296 304

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners have found the gradient in the Functions section and should therefore find this section relatively straightforward. Tell learners: *I know you have already encountered this concept but we are going to focus on using it within the topic: Analytical Geometry.*

2. Remind learners what you told them in the last lesson concerning the importance of knowing the properties of quadrilaterals. This will be extended further in this lesson.

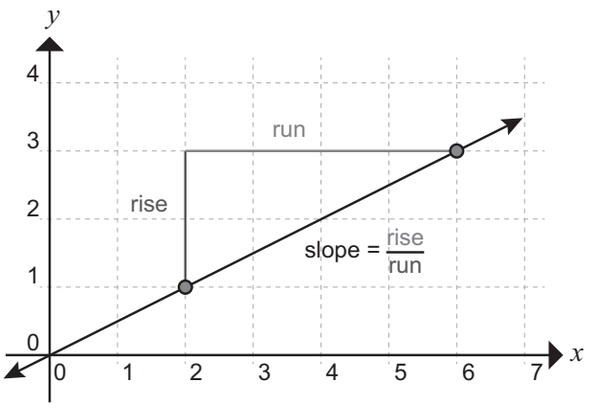
DIRECT INSTRUCTION

1. Ask: *Who remembers the formula for gradient?* ($m = \frac{y_2 - y_1}{x_2 - x_1}$). Tell learners to write this in their books.

2. Remind learners: *The gradient of a line segment is the vertical change in ratio to the horizontal change.*

3. Remember: *The up or down movement is the vertical change (y) and the left or right movement is the horizontal change (x).*

4. Sketch the diagram below to show how the 'rise' (vertical change) links directly to the y-values and that the 'run' (horizontal change) links directly to the x-values.



5. Ask: *Does this line have a positive gradient or a negative gradient?* (Positive).
   Tell learners to draw the sketch in their books. Tell learners the numbers on each axes are not important – the line with the rise over run is the essential idea.

6. Tell learners: *Draw a quick sketch of a line with a negative gradient on your sketch and hold it up to show me.*

7. Ask learners: *What do you remember about the gradient of parallel lines and perpendicular lines?*
   (Parallel lines have equal gradients; the gradients of perpendicular lines have a product of -1).

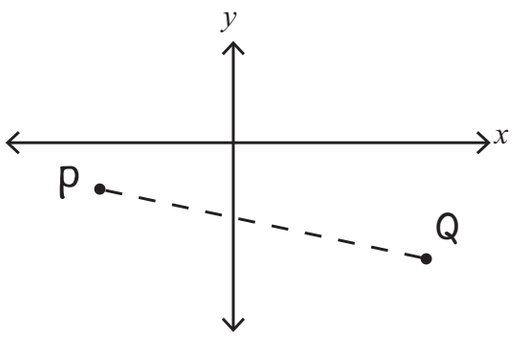
8. Tell learners to write these two points in their exercise book.

9. Tell learners: *I am going to do a few fully worked examples with you now.*
   Show how finding gradient can be asked in this section. Learners should write the examples in their books and make notes as they do so.

a) Find the gradient of PQ if P (-3 ;-2) and Q (6 ;-4)	Remind learners: *If there isn't a diagram, sketch one!* Remind learners: *Continue labelling points to ensure you don't make careless errors when substituting.*

Solution:

Sketch:



$P(-3; -2) \quad Q(6; -4)$

$x_1; y_1 \qquad x_2; y_2$

$$mPQ = \frac{y_2 - y_1}{x_2 - x_1}$$

$$mPQ = \frac{-4 - (-2)}{6 - (-3)}$$

$$mPQ = \frac{-4 + 2}{6 + 3}$$

$$mPQ = \frac{-2}{9}$$

Say: *Check your sketch and confirm you were expecting a negative answer.*
(Is the line sloping down?)

b) The line AB has a gradient of 2. Find x if A is the point $(x ; -1)$ and B is the point (1 ;7)	Tell learners: *Although this is still a gradient question, it is not being asked in the regular way. Here you are given gradient and one of the co-ordinates is missing.* Say: *We will have to work in reverse.* Tip: *The word gradient implies needing to use the gradient formula so that should be your starting point then you should be able to fill in what is given and work from there. It will require solving an equation.*

Solution:

$A\ (x\ ;-1)$ $B\ (1\ ;7)$

$x_1;y_1$ $x_2;y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$2 = \frac{7 - (-1)}{1 - x}$$

$$2 = \frac{7 + 1}{1 - x}$$

$$2 = \frac{8}{1 - x}$$

$$2(1 - x) = 8$$

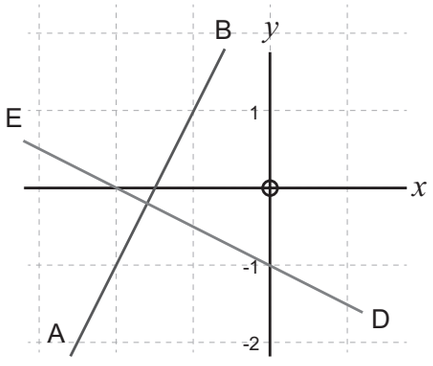
$$2 - 2x = 8$$

$$-2x = 6$$

$$x = -3$$

Tell learners: *Plot the points* on a sketch now and check that the line is sloping up as it has a positive gradient.

c) In the following diagram, AB and DE are lines that intersect.
Line AB passes through the points $(-2; -1)$ and $(-1; 1)$
Line DE passes through the points $(-2; 0)$ and $(0; -1)$
Find the gradient of each line.
What can you say about lines AB and DE?



This question starts out as an ordinary question but then goes on to test learner's understanding of gradient from a different perspective.

Solution:

$(-2;-1)$ and $(-1;1)$ $\qquad$ $(-2;0)$ and $(0;-1)$

$x_1;y_1 \qquad x_2;y_2$ $\qquad$ $x_1;y_1 \qquad x_2;y_2$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{1-(-1)}{-1-(-2)} \qquad m_2 = \frac{-1-0}{0-(-2)}$$

$$m_1 = \frac{1+1}{-1+2} \qquad m_2 = \frac{-1}{0+2}$$

$$m_1 = \frac{2}{1} = 2 \qquad m_2 = -\frac{1}{2}$$

The two lines are perpendicular because $m_1 \times m_2 = -1$

10. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

11. Give learners an exercise to complete on their own.

12. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

http://www.teacherschoice.com.au/Maths_Library/Gradient/gradient_-_two_fixed_points.htm

http://www.coolmath.com/algebra/08-lines/06-finding-slope-line-given-two-points-01

https://www.youtube.com/watch?v=QW2yT-AtsA0

TERM 3, TOPIC 1, LESSON 3

MIDPOINT OF A LINE SEGMENT

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	26

Lesson Objectives

By the end of the lesson, learners should be able to:

- find the mid-point of a line segment
- complete an exercise on various types of mid-point questions.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson draw a Cartesian plane.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
2	212	5	197	11.3	197	11.3	251	8.5	313

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. This is the third and final Analytical Geometry concept learners need to know for Grade 10. After this lesson all three concepts will need to be incorporated in order to answer the type of questions asked in an assessment.

DIRECT INSTRUCTION

1. Tell learners: *We are going to look at the third aspect needed for Analytical Geometry – finding the mid-point of a line segment.*

2. Ask learners: *How do you find the middle number between two other numbers, for example, the middle number between 12 and 30?* (Learners might answer that they would add the two and divide by 2).

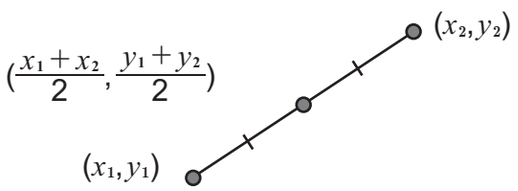
3. Ask: *What is this called?* (The average or more specifically, the mean). Praise any learner that remembered it was called the mean.
   Tell the class: *We will be doing data handling later in the term where this knowledge will be required.*

4. Conclude by telling learners: *We find the mid-point of a line segment the same way – by looking for the middle value.*

5. Say: *To find the mid-point of a line segment, we add the two x-values and divide by 2, then we add the two y-values and divide by 2.*

6. Teach them a little rhyme they can repeat after you: *"Midpoint, midpoint, what do you do? Add them together and divide by two!"*

7. Write the following formula and sketch on the board. Ask learners to write it in their books.



8. Tell learners: *We are going to do a few fully worked examples together.*

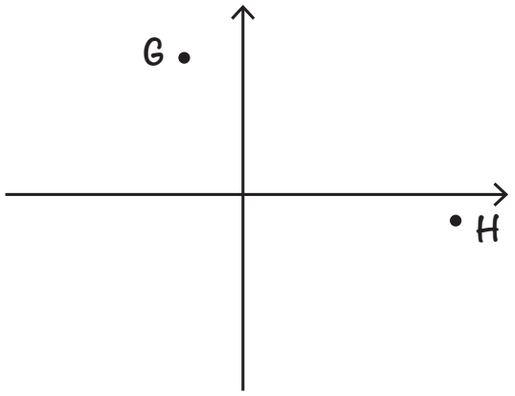
9. Show how the skill of finding the mid-point of a line segment can be asked in this section. Learners write the examples in their books and make notes as they do so.

a) Find the mid-point of the following two points: G (–2;8) and H (8;–1)

Tell learners: *This is the most straightforward way to ask mid-point.*
Remind learners: *Label the co-ordinates to help you not make careless mistakes. Draw a sketch to help you check your answer.*

Solution:

Sketch:



Tell learners again: *This does not need to be a very neat sketch but it should 'look' fairly accurate.*

G (–2;8) and H (8;-1)

$x_1;y_1 \qquad x_2;y_2$

$$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

$$= (\frac{-2 + 8}{2}, \frac{8 + (-1)}{2})$$

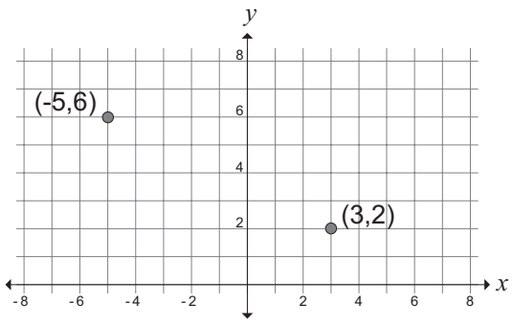
$$= (\frac{-2 + 8}{2}, \frac{8 - 1}{2})$$

$$= (\frac{6}{2}, \frac{7}{2})$$

$$= (3; \frac{7}{2})$$

Ask learners: *Plot this point on your sketch and check if it looks like it would be the mid-point of the two points.*

b) Find the mid-point, distance and gradient of the following two points:



Tell learners: *This is a fairly common question once all three concepts have been covered.*

Solution:

(–5;6) (3;2)

$x_1;y_1$ $x_2;y_2$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-5 + 3}{2}, \frac{6 + 2}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{8}{2}\right)$$

$$= (-1;4)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \left(\frac{2 - 6}{3 - (5)}\right)$$

$$m = \left(\frac{2 - 6}{3 + 8}\right)$$

$$m = \left(\frac{-4}{8}\right)$$

$$m = \frac{-1}{2}$$

(–5;6) (3;2)

$x_1;y_1$ $x_2;y_2$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(3 - (-5))^2 + (2 - 6)^2}$$

$$d = \sqrt{(3 + 5)^2 + (2 - 6)^2}$$

$$d = \sqrt{(8)^2 + (-4)^2}$$

$$d = \sqrt{64 + 16}$$

$$d = \sqrt{80} = 4\sqrt{5} = 8,9 \text{ units}$$

Say: *Plot the mid-point and check whether it looks correct.*

Say: *Check that the gradient is meant to be negative.*

Ask: *Does it look like the distance is correct?* (Horizontally the distance from one point to the other is 8 so 8,9 seems reasonable).

c) If (x ;1) is the mid-point of points *A* (–1;–2) and *B* (7 ;y).
Find x and y.

Tell: *Although this is still a mid-point question, it is not being asked in the regular way. Here we are given mid-point and one of the co-ordinates is missing. There is also a co-ordinate missing from one of the points.*

We will have to work in reverse.

Tip: *The word mid-point implies the need to use the mid-point formula so that should be your starting point then you should be able to fill in what is given and work from there.*

Tell: *Write the points down so the mid-point lies between the two points. This helps visually.*

It will require solving an equation.

Solution:

$A(-1;-2)$ $(x;1)$ $B(7;y)$

$x_1;y_1$ $x_2;y_2$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Each of these points will need to be dealt with separately:

$$x = \frac{x_1+x_2}{2} \qquad y = \frac{y_1+y_2}{2}$$

$$x = \frac{-1+7}{2} \qquad 1 = \frac{-2+y}{2}$$

$$x = \frac{6}{2} \qquad 2 = -2+y$$

$$x = 3 \qquad 4 = y$$

Say: *Plot these points and check if they look correct.*

10. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

11. Give learners an exercise to complete on their own.

12. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=ZXoJSzmaZ4E

http://virtualnerd.com/algebra-1/radical-expressions-equations/distance-midpoint-formulas/midpoint-formula/midpoint-between-coordinates

TERM 3, TOPIC 1, LESSON 4

REVISION AND CONSOLIDATION

Suggested lesson duration: 3 hours

POLICY AND OUTCOMES

A

CAPS Page Number	26

Lesson Objectives

By the end of the lesson, learners should be able to:

- apply and combine the three skills learned in the previous lessons.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. You will need Resource 2 from the Resource Pack.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson write the eight questions for the introductory task (point 1) and draw a Cartesian plane.
6. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev S Ch	221 223	6 Rev	200 203	w/sh	200	11.5 11.6	258 259	8.6	315

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. One point made in the diagnostic reports was that learners struggled to answer questions if topics were combined within one question. This lesson starts with a task where learners will be guided to come to the realisation that they can be working in one topic (Analytical Geometry) but find that they are also working within another topic already covered this year (Euclidean Geometry).

2. This task has two benefits – it is a quick recap of the three aspects learned in Analytical Geometry so far and will also remind learners of the new theorem they learned in Grade 10.

DIRECT INSTRUCTION

1. Say: *Draw a Cartesian plane in your exercise book and plot the following points: A* (–6;–2); *B* (–2;4) and *C* (4 ; – 6). Join the points to form ΔABC.

2. Once that is complete, ask: *Write the questions from the board and answer in your books.* Learners must work alone.

   (1) Find the mid-point of AB. Call it D and plot it on the sketch.

   (2) Find the mid-point of BC. Call it E and plot it on the sketch.

   (3) Find the length of AC. Leave your answer in simplest surd form.

   (4) Find the length of DE. Leave your answer in simplest surd form.

   (5) Find the gradient of AC.

   (6) Find the gradient of DE.

   (7) What conclusion can you draw from your answers to (3) and (4)?

   (8) What conclusion can you draw from your answers to (5) and (6)?

3. Walk around the class and assist where necessary.

4. Once learners have had a chance to complete the task, go back to the board and sketch points A, B and C on your Cartesian plane. Go through each question from (1) to (6) – learners check their own answers. Fill in the points, as well as the lengths and the gradients.

5. Ask: *What are your conclusions?* Confirm that they found $DE = \frac{1}{2}AC$ or that $2DE = AC$. Confirm too that they found that $DE // AC$.

6. Ask: *What theorem have you proved from the Euclidean Geometry section studied last term?* (The mid-point theorem).

7. Tell: *There are often connections made between different sections of mathematics. You should not always think that topics will be assessed on their own.*

8. Now that learners have revised the three main concepts in this topic, and they have revised some of their Euclidean geometry as well, tell them: *We are going to consolidate this topic by combining all the knowledge into one question. This is the way it is most often assessed.*

Tell learners the following two points concerning Analytical geometry questions in assessments:

(1) *You are always given the formulae so there is no need to lose marks because you couldn't remember it.*

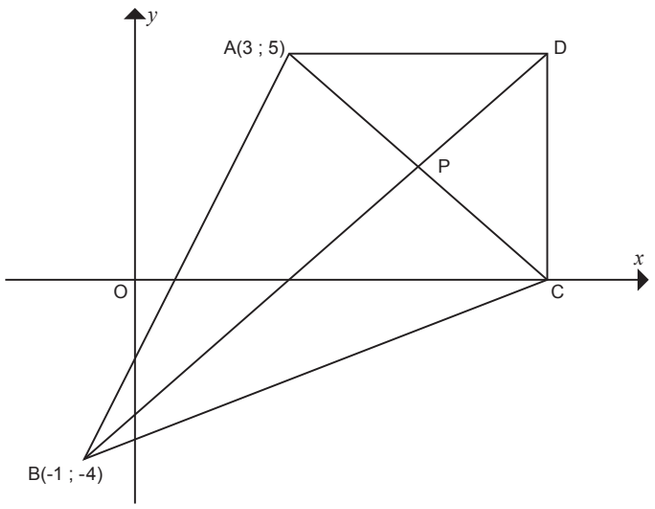
(2) *If a question is set out differently and you are unsure where to start, ask yourself:*

- *Would knowing the length of a line, help me answer the question?* (Distance)
- *Would knowing how steep the line is, help me answer the question?* (Gradient)
- *Would knowing where the middle of the line is help me answer the question?* (Mid-point).

9. Say: *I am going to do two examples in full from past exam papers with you. You should write them down as I do them, taking notes at the same time.*

Example 1

In the diagram below, ADCB is a kite with *A* (3;5) and *B*(-1;-4).
AD=DC and *AB=BC*. D is a point such that AD is parallel to the x-axis and
AD = 5 units. CD is perpendicular to the x-axis. The diagonals intersect at P.



(Nov 2016)

Question	Teaching notes:
a) Show that the co-ordinates of C are (8;0)	Point out: The fact that AD is parallel to the x-axis tells us that the y-co-ordinate of D must be the same as A. We are told that AD is 5 units long – this is enough information to find the x-co-ordinate. If CD is perpendicular to the x-axis, C and D will share the same x-co-ordinate.
b) Write down the co-ordinates of P.	The longer diagonal of a kite bisects the shorter diagonal. Therefore, P is the midpoint of AC.
c) Calculate the gradient of line BD.	Both points are known, therefore the formula for gradient can be used directly. Remind learners: *Use the diagram to check whether the answer looks correct according to a positive or negative slope.*
d) Calculate the length of line AC.	Both points are known, therefore the formula for distance can be used directly. Remind learners: *Check whether the length looks reasonable by approximating on the diagram.*
Solutions	
a) D (8 ;5) $\therefore C(8;0)$	b) A (3;5) and C (8;0) $x_1;y_1 \qquad x_2;y_2$ $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ $P\left(\frac{3+8}{2},\frac{5+0}{2}\right)$ $P\left(\frac{11}{2},\frac{5}{2}\right)$

c) B (–1 ;– 4) D (8 ;5)

$x_1;y_2$ $x_2;y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$mBD = \frac{5 - (-4)}{8 - (-1)}$$

$$mBD = \frac{5 + 4}{8 + 1}$$

$$mBD = \frac{9}{9}$$

$$mBD = 1$$

d) A (3 ;5) C (8 ; 0)

$x_1;y_1$ $x_2;y_2$

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AC = \sqrt{(8 - 3)^2 + (0 - 5)^2}$$

$$AC = \sqrt{5^2 + (-5)^2}$$

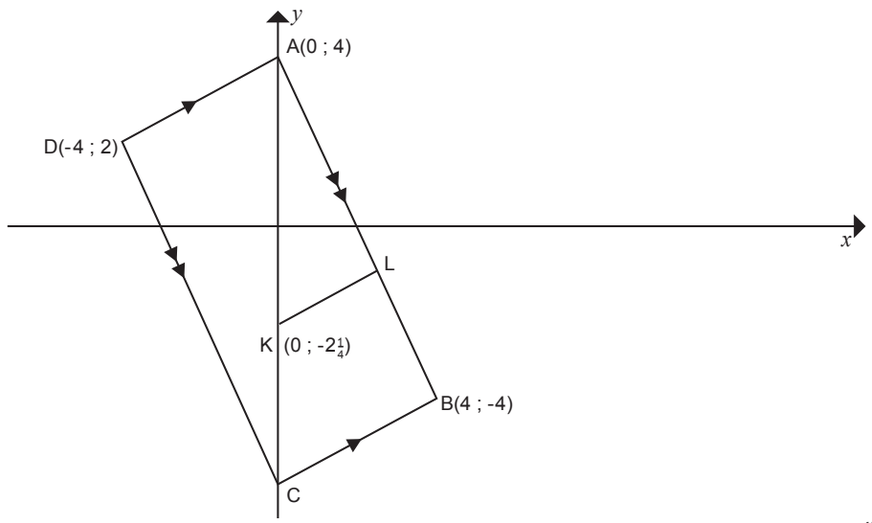
$$AC = \sqrt{25 + 25}$$

$$AC = \sqrt{50}$$

$$AC = 5\sqrt{2}$$

Example 2

In the diagram, C is a point on the y-axis such that A (0;4), B (4;– 4), C and D (– 4;2) are vertices of a parallelogram ABCD. K is the point (0;–2¼) and L is a point on AB such that $KL//CB$.



(Nov 2015)

Question	Teaching notes:
a) Calculate the length of diagonal DB.	Both points are known, therefore the formula for distance can be used directly. Remind learners: *Check whether the length looks reasonable by approximating on the diagram.*
b) Calculate the co-ordinates of M, the mid-point of DB.	Both points are known, therefore the formula for midpoint can be used directly. Remind learners: *Use the diagram to check whether the point looks correct.*

c) Calculate the gradient of AD.	Both points are known, therefore the formula for gradient can be used directly. Remind learners: *Use the diagram to check whether the answer looks correct according to a positive or negative slope.*
d) Prove that $AD \perp AB$.	The gradient of AD is known from (c). Find the gradient of AB. The product of the gradients should be equal to -1. Remind learners: *If the question states 'prove that...' then it is already definitely true.* Should learners not be able to prove it, advise them to go back and have another look as they have made an error somewhere.
e) Give a reason why parallelogram ABCD is a rectangle.	If one angle of a parallelogram is 90º, then the other angles should also be 90º due to co-interior angles.
f) Determine the equation of KL in the form $y = mx + c$	The gradient of KL is known as it is parallel to AD. Once a point and a gradient are known, the equation of the straight line can be found.
g) Write down, with reasons the co-ordinates of C.	The diagonals of a kite are equal. Therefore, AC = BD (distance found in (a)).
Solutions:	
a) $D(-4;2)$ $B(4;-4)$ $x_1;y_1$ $x_2;y_2$ $DB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $DB = \sqrt{(4 - (-4))^2 + (-4 - 2)^2}$ $DB = \sqrt{(4 + 4)^2 + (-4 - 2)^2}$ $DB = \sqrt{(8)^2 + (-6)^2}$ $DB = \sqrt{64 + 36}$ $DB = \sqrt{100}$ $DB = 10$ *units*	b) $D(-4;2)$ and $B(4;-4)$ $x_1;y_1$ $x_2;y_2$ $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $= \left(\frac{-4 + 4}{2}, \frac{2 - 4}{2}\right)$ $= \left(\frac{0}{2}, \frac{-2}{2}\right)$ $= (0; -1)$

c) $A(0;4)$ $D(-4;2)$ $x_1;y_1$ $x_2;y_2$ $mAD = \frac{y_2 - y_1}{x_2 - x_1}$ $mAD = \frac{2-4}{-4-0}$ $mAD = \frac{-2}{-4}$ $mAD = \frac{1}{2}$	d) $A(0;4)$ $B(4;-4)$ $x_1;y_1$ $x_2;y_2$ $mAB = \frac{y_2 - y_1}{x_2 - x_1}$ $mAB = \frac{-4-4}{4-0}$ $mAB = \frac{-8}{4}$ $mAB = -2$ and $mAD = \frac{1}{2}$ $-2 \times \frac{1}{2} = -1$ $\therefore AD \perp AB$
e) $AD \perp AB$ $\therefore \hat{A} = 90^0$ $\therefore B = 90^o$ (co - int < 's) $\therefore$ ABCD is a rectangle	f) $K(0;-2\frac{1}{4})$ $mAD = \frac{1}{2}$ $\therefore mKL = \frac{1}{2}$ $y = mx + c$ $y = \frac{1}{2}x + c$ $(0;-2\frac{1}{4})$ $-\frac{9}{4} = \frac{1}{2}(0) + c$ $-\frac{9}{4} = c$ $\therefore y = \frac{1}{2}x - \frac{9}{4}$
g) BD = 10 units and $A(0;4)$ $\therefore C(0;-6)$ (10 units from A)	

10. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

11. Give learners an exercise to complete on their own.

12. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

http://learn.mindset.co.za/resources/mathematics/grade-10/analytical-geometry

https://www.youtube.com/watch?v=1V_vB8lJQ1w

Term 3, Topic 1: Topic Overview

FINANCE AND GROWTH

A. TOPIC OVERVIEW

A

- This topic is the 2nd of six topics in Term 3.
- This topic runs for two weeks (9 hours).
- It is presented over six lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 9 hours). For example, one lesson in this topic could take three school lessons. Plan according to your school's timetable.
- Finance and Growth counts 10% of the final Paper 1 examination.
- This section covers interest (simple and compound), hire purchase, inflation, and foreign exchange rates.
- An understanding of financial matters is an important skill required by learners. This is a learned skill which is not taught by all parents – this makes the teacher's role at school even more important.

Breakdown of topic into 6 lessons:

	Lesson title	**Suggested time (hours)**		**Lesson title**	**Suggested time (hours)**
1	Simple Interest	1,5	4	Inflation (Including population growth)	1,5
2	Compound Interest	1,5	5	Foreign exchange rates	1,5
3	Hire Purchase	1,5	6	Revision and Consolidation	1,5

B SEQUENTIAL TABLE

GRADE 8 & 9	GRADE 10	GRADE 11 & 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as: ● profit, loss, discount and VAT ● budgets ● accounts ● loans ● simple interest ● hire purchase ● exchange rates ● commission ● rentals ● compound interest.	● Use the simple and compound growth formulae to solve problems These problems must include: ● hire purchase ● inflation ● population growth ● exchange rates (implications of fluctuating rates).	● Use simple and compound decay formulae to solve problems, including: • Straight line depreciation • Reducing balance depreciation • Nominal and effective interest rates ● Solve problems involving present and future value annuities ● Calculate the value of *n* (time period) using logarithms ● Critically analyse different loan options.

C WHAT THE NSC DIAGNOSTIC REPORTS TELL US

According to NSC Diagnostic Reports there are several issues pertaining to Financial mathematics. None of these issues relate directly to the Grade 10 curriculum.
General comments:

- Finance needs to be taught with more insight – it is not merely a substitution of values into a formula.
- Basic algebraic skills, such as multiplication and exponential laws, need to be taught properly then reinforced throughout the FET phase.
- Learners should be encouraged to check whether their solutions seem 'reasonable'.
- Rounding off must only be done at the end of a process – not in the middle of a problem.

Keep these issues in mind when teaching this section.

While teaching Financial mathematics, it is important to make it real for the learners.
Where possible, use examples from daily life which make sense to learners.

ASSESSMENT OF THE TOPIC

D

- CAPS formal assessment requirements for Term 3:
  - Two tests
- Two tests of 50 marks each are provided in Resources 27 and 29 in the Resource Pack. The tests are aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53). The tests are also divided in the ratio of the allocated teaching time. Memoranda for the tests are provided in Resources 28 and 30 in the Resource Pack.
- Finance is included in the first test to be written approximately half way through the term.
- The questions usually take the form of real life situations that require, for example finding the total amount saved, interest and exchange rates.
- During lessons, monitor each learner's progress to assess (informally) their grasp of the concepts. This inforation can form the basis of feedback to the learners and will provide valuable information regarding support and interventons required.

MATHEMATICAL VOCABULARY

E

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
principal amount	The initial or capital sum of money. This amount can represent a borrowed or invested amount of money
profit	Money made when income exceeds (is bigger than) expenditure. (You make more money than you spent)
loss	Money lost when expenditure exceeds income (You spent more than you made)
discount	Paying less than the usual price. A percentage of the original price is taken off
budget	A plan to manage money
loan	Borrowing of money
interest	The extra money paid back after taking a loan
interest rate	The percentage of interest charged (loan) or received (investment). Rate is often quoted per annum

simple interest	The interest on a loan that is calculated on a yearly basis. Interest on a loan amount is charged with only the principal amount taken into consideration
instalment	A sum of money due as one of several equal payments for something, spread over an agreed period of time
exchange rate	The rate of one country's money against another country's money
VAT	Value Added Tax is paid on goods or services. In South Africa VAT is 15% of the price
hire purchase	When an item is purchased but only a deposit is paid and then the item is paid off monthly over a specified period of time. The interest charged is always simple interest. Officially, the goods bought are being hired until the payments are complete
deposit	A sum of money paid as a first instalment on an item with the understanding that the balance will be paid at a later stage
compound interest	Interest is calculated not only on the principal amount but will also include any accumulated interest that has been added at certain time intervals
inflation	A general increase in prices and fall in the purchasing value of money. Inflation is always calculated using compound interest
fixed deposit	A single deposit invested for a certain period of time at a fixed interest rate

TERM 3, TOPIC 2, LESSON 1

SIMPLE INTEREST

Suggested lesson duration: 1,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	26

Lesson Objectives

By the end of the lesson, learners will have:

- revised the concept, simple interest
- answered questions related to simple interest.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the section heading on the board before learners arrive – Finance and Growth.
4. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1	227	1 2	207 208	12.1 (1a & 6)	204	12.1 (2a; 3a; 5a)	266	9.1	334

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Lessons in this section should relate directly to everyday issues (for learners or their families) as often as possible.
2. Tell learners that this is an exciting section – it has a direct link to their lives even if they do not do mathematics again beyond Grade 12.
3. Ensure learners are given the opportunity to recall words and concepts from Grades 8 and 9.

DIRECT INSTRUCTION

1. Start the lesson by asking learners: *What are the two types of interest you learned about last year?* (Simple interest and compound interest).
2. Tell learners that: *The focus of this lesson is going to be simple interest.*
3. Remind learners what the key issues are relating to simple interest.
4. Interest is calculated at the end of each year using the original amount only.
5. This simple explanation may not be entirely accurate depending on who is 'making the rules'. Loan sharks or money lenders are unfortunately common in most countries including our own. Some loan sharks or money lenders lend money to someone in need then charge exorbitant rates which the person lending the money can often not afford.
6. Share the story below with your learners. Start by explaining to learners that the British pound (£) is the currency used in Britain, in the same way that the rand is the currency used in South Africa.

'I borrowed £500 to buy a computer and ended up paying £88,000!'

One woman's sad story began with borrowing £500 for a Christmas present from a 'friend'. Then the lender demanded large repayments. When she couldn't make these, he charged her and added it to her debt. This spiralled until by the time she went to the police she had paid £88,000 back for the £500 loan.

To avoid getting yourself into trouble like this, it's really useful to understand interest - the extra money borrowers pay to lenders in return for using the money. That way you can choose for yourself what to do based on the figures.

http://www.mathscareers.org.uk/article/loan-sharks-loan-stars/

7. Although this happened in the United Kingdom, it can be a typical story from any country.

8. Loan sharks often do not give details of what rate they will be charging and can therefore change their mind at any time and start charging 'what they like'.

9. In general, banks deal with compound interest rather than simple interest. The most common situation that simple interest is used is when purchasing an item on hire purchase. This is dealt with in a later lesson in more detail.

10. Do the following simple example with learners now. It will lead to the use of the formula to be used from now on. Ask learners to take it down in their books.

11. Thembi borrowed R1 000 from a neighbour and promised to pay it back in two years with 2 equal annual payments. The neighbour told her she will be charged 15% interest per year. Calculate what Thembi will pay back for the loan of R1 000.

Solution:	Teaching notes
First, find the interest rate using the formula: $\text{Simple Interest} = P \times i \times n$ $P \times i \times n = 1000 \times 15\% \times 2$ $= 1000 \times 0,15 \times 2$ $= 300$	Where, P = principal amount, i = interest rate and n = the time period in years Remind learners that although the interest rate is given as a percentage, it can be changed to a decimal. Percentage means to divide by 100.
Once the interest has been calculated it needs to be added to the amount borrowed to find what Thembi will have to pay in total (the accumulated amount). $R1\,000 + R300 = R1\,300$	
In total, Thembi will pay back R1 300. Each payment will be R650.	Explain what this means: Thembi will be paying R150 interest per year. In other words, if she was willing to pay the loan back after 1 year instead of 2, her total interest would be R150.

12. Point out that although Thembi has already paid one instalment of R650 at the end of the first year, the interest charged did not change. Essentially, in the second year, she is still paying interest on the original amount (principal amount) when she doesn't really owe that anymore. This is how simple interest works.

13. Point out that in the above example, two calculations were necessary. First, we had to calculate the interest and then we had to add it on to the original amount.

14. There is a formula which will do both calculations in one step. In other words, it finds the accumulated amount (A) in one calculation.

15. Ask learners to write the following formula in their books:

$$A = P(1 + i.n)$$

16. Label what each of the four variables represent:

    A = accumulated amount

    P = principal amount

    i = interest rate

    n = time period

17. Ask learners: why this formula will calculate the interest AND add it on to the principal amount (point out that the distributive law ensures this; $\mathrm{P} \times 1$ will give us the principal amount added to $\mathrm{P} \times i \times n$ which will be the interest).

18. Use the formula to calculate the total amount that Thembi will need to pay back.

$$A = P(1 + i.n)$$

$$A = 1\,000(1 + (0,15)(2))$$

$$A = 1\,300$$

19. Give learners the following example to try on their own:
    Mary starts a business and borrows R20 000 for 5 years to get the business going.
    Mary agrees to pay it back at 18% p.a. Calculate the total amount that she will have to pay back.
    Point out the p.a. to learners. Ask: *What does the abbreviation stand for?*
    (Per annum, in other words – per year).

Solution:	Teaching notes
$A = P(1 + i.n)$ $A = 20\,000\,(1 + (0,18)(5))$ $A = 38\,000$	Have a discussion with learners about this large sum of money. Point out that she will be paying back almost double the amount she borrowed. Ask: *What suggestions do you have to help Mary not to pay so much back?* (Suggestions like paying off more quickly if possible – each year she is paying back R3 600 or rather saving for a while first, so she didn't need to borrow such a large sum). Even if none of these ideas were possible for Mary in her situation, it is important that learners try to think of other ways that may have worked in order to save money.

20. Tell learners that they will also need to be able to find any one of the other three variables in the equation.

    For example, a question could give the accumulated amount, the interest rate and the time period and ask what the initial (principal) amount was. This will require solving equations.

21. Do the following three examples with learners. Learners need to write them down in their books.

Example:	Teaching notes	Solution:
Calculate how long it will take an investment of R5 000 to grow to a value of R7 000 with a SI rate of 8% p.a.	For each of these questions, tell learners to start with writing the formula down. Then they should read the question again carefully and replace each of the 3 variables with the values given. Finally, learners need to solve for the unknown variable using their knowledge of solving algebraic equations.	$A = P(1 + i.n)$ $7\,000 = 5\,000(1 + (0,08)n)$ $\frac{7}{5} = 1 + (0,08)n$ $\frac{7}{5} - 1 = (0,08)n$ $\frac{2}{5} = (0,08)n$ $n = 5$ It will take 5 years
Steven wants to save money to buy a computer worth R6 000 in 3 years' time. The interest rate offered is 5% p.a. simple interest. How much money does he need to invest now?		$A = P(1 + i.n)$ $6\,000 = P(1 + (0,05)(3))$ $\frac{6\,000}{1 + (0,05)(3)} = P$ $5\,911,33 = P$ He will need R5 911,33 to invest now.
Your uncle borrows R8 000 and pays it back over a 9-year period. In total, he must pay back R18 000. What simple interest rate was he charged? Round your answer to two decimal places.		$A = P(1 + i.n)$ $18\,000 = 8\,000(1 + i.(9))$ $18\,000 = 8\,000(1 + 9i)$ $\frac{9}{4} = 1 + 9i$ $\frac{9}{4} - 1 = 9i$ $\frac{5}{4} = 9i$ $\frac{5}{36} = i$ $i = 0,138888...$ $\therefore i = 13,89\%$

22. An important issue to point out to learners is that answers should only ever be rounded AT THE END of a calculation. If money is the answer, it should ALWAYS be rounded to two decimal places.

23. Point out what happened at the end of the 3^{rd} example when the interest rate was being found. Due to the formula, the solution for the interest rate was in decimal form. To conclude and answer the question, this needed to be converted into a percentage.

24. Learners should be aware of the difference between 'interest' and 'interest rate'. Interest is the amount of money earned (when saving) or the extra money paid back (when borrowing). Interest rate is the percentage given per annum.

25. In the example concerning Mary, the interest rate was 18% whereas the interest paid back was R18 000.

26. Ask directed questions so that you can ascertain learners' level of understanding.

27. Ask learners if they have any questions.

28. Give learners an exercise to complete on their own.

29. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=UeZaNfBcdKs

https://www.youtube.com/watch?v=fpAAeAwF71Q

(Part 1 & Part 2 Grade 10 lesson – Simple Interest)

TERM 3, TOPIC 2, LESSON 2

COMPOUND INTEREST

Suggested lesson duration: 1,5 hours

A POLICY AND OUTCOMES

CAPS Page Number	26

Lesson Objectives

By the end of the lesson, learners will have:

- revised the concept, compound interest
- answered questions related to compound interest.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Advance preparation: Give learners the investigation (Resource 3 in the Resource Pack) one week before you plan to start this topic.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson, write 'Simple interest vs compound interest'.
6. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
3	234	3 4	210 211	12.1 (1b & 2-5) 12.2	204 205	12.1 (1; 2b; 3b; 4; 5b; 6-9) 12.2	266 267	9.2	339

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Compound interest is used more than simple interest. Simple interest is always used when hire purchase is involved (to be covered in the next lesson). There are few other situations where simple interest is used.

2. Ensure learners understand the key difference between compound interest and simple interest.

3. Debt is a slippery slope – it is important that you have discussions with learners reinforcing the idea of building good financial habits from early on. Avoiding debt where possible is the key to sound financial living.

4. Discuss credit cards with learners and how quickly debt can accumulate. Even though they will not own a credit card, they will still be interested in how it works.

5. Learners complete the investigation (Resource 3 in the Resource Pack). We recommend that learners do the investigation for homework so that the lesson can begin with a discussion of their findings.

6. The time allocated for this lesson excludes the investigation.

DIRECT INSTRUCTION

1. Ask learners: *What is the key difference between simple interest and compound interest?* [Answer appears in Point 2 below].

2. Once a few learners have verbalised their understanding of the difference, summarise by confirming the following key difference and asking learners to write it in their books:

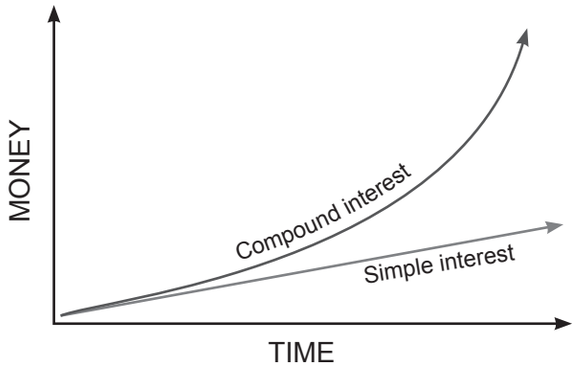
   Simple interest is based on the principal amount of a loan or deposit, while compound interest is based on the principal amount and the interest that accumulates on it in every period.

   Simple interest growth is represented by a straight line graph, whereas compound interest growth is represented by an exponential graph.

3. Spend time discussing the results of the investigation. Some learners may notice that although the money saved using compound interest was higher than that saved using simple interest, it didn't seem like a particularly large amount (R128,89), especially as it was over a rather long period of time (10 years). Agree with learners that this does seem to be

the case, but to keep in mind that R1000 is not a particularly large sum of money – and to imagine what the difference may have been on R10 000 or even R100 000.

4. Ask learners to summarise the difference graphically by drawing the following into their books:



5. Tell learners that although they did not use a formula in the investigation, there is one which will be looked at now.

6. Ask learners to write the formula in their books:

$$A = P(1 + i)^n$$

7. Label each variable. Learners should note the exponent in the formula which ensures that graphically this will form an exponential graph.

   A = accumulated amount

   P = principal amount

   i = interest rate

   n = time period

8. Learners can use this formula to confirm that their final amount in the investigation is indeed correct:

$$A = P(1 + i)^n$$

$$A = 1000(1 + 0,05)^{10}$$

$$A = 1628,89$$

9. Give learners the following example to try on their own:
   Katleho starts a business and borrows R28 000 for 3 years to get the business going. Katleho agrees to pay it back at 12% p.a. compounded annually.
   Calculate the total amount that Katleho will have to pay back.

Solution	*Teaching notes*
$A = P(1+i)^n$ $A = 28\,000(1+0,12)^3$ $A = 39\,337,98$ $\therefore$ R39 337,98	Ask: *What does this amount include?* (The initial loan of R28 000 with the interest added each year on what is still owing).

10. Tell learners that they will also need to be able to find the principal amount when given the other information as well as the interest rate. In Grade 10, learners will not need to calculate the time period (the exponent) unless the information leads to a simple exponential equation. Tell them that in Grade 12, they will learn about logarithms which will assist them in finding an exponent.

11. These questions (to find the principal amount or the interest rate) will involve solving equations.

12. Do the following examples with learners. They need to write the examples in their exercise books.

Example:	Teaching notes	Solution:
Steven wants to save money to buy a computer worth R6 000 in 3 years' time. The interest rate offered is 5% p.a. compounded. How much money does he need to invest now?	For each of these questions, tell learners to start with writing down the formula. Learners should read the question again carefully and replace each of the three variables with the values given. Finally, learners need to use their knowledge of solving algebraic equations to solve for the unknown variable.	$A = P(1+i)^n$ $6\,000 = P(1+0,05)^3$ $\frac{6\,000}{(1+0,05)^3} = P$ $\frac{6000}{(1+0,05)^3} = P$ Steven will need to invest R5 183,03 now.
Your uncle borrows R8 000 and pays it back over a 9-year period. In total, he must pay back R18 000. What compound interest rate was he charged? Round your answer to 2 decimal places.		$A = P(1+i)^n$ $18\,000 = 8\,000(1+i)^9$ $\frac{9}{4} = (1+i)^9$ $1,094287381 = 1+i$ $0,094287381 = i$ $i = 9,43\%$

13. Discuss example 1 by asking learners to turn back in their books and look at a similar example for simple interest. Ask: *What is the big difference?*
(He doesn't need as much money in the beginning for him to save R6 000 in 3 years).

14. Discuss example 2 by asking learners to look back at the similar example for simple interest.
    Ask: *What is the big difference?*
    (The interest rate is lower but accumulates to the same total over the 9-year period).

15. Confirm the following with learners:

    Compound interest is interest earned on top of interest already received ☺.

    OR

    Compound interest is interest charged on interest still outstanding ☹.

16. Ask directed questions so that you can ascertain learners' level of understanding.

17. Ask learners if they have any questions.

18. Give learners an exercise to complete on their own.

19. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=wf91rEGw88Q
(Although this is American and uses dollars it is a good explanation of compound interest)

https://www.youtube.com/watch?v=blEgAfSdsbs
(Simple and compound interest combined)

TERM 3, TOPIC 2, LESSON 3

HIRE PURCHASE

Suggested lesson duration: 1,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	26

Lesson Objectives

By the end of the lesson, learners should be able to:

- answer questions involving hire purchase which involve deposits and monthly instalments.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson the title is Hire Purchase.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
2	229	7	216	12.3	207	12.3	269	9.3	343

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners should be familiar with the concept of hire purchase from Grade 9.
2. Many families use Hire Purchase to buy items that they may not be able to afford otherwise.
3. Use this lesson to ensure learners understand the consequences of buying on credit.

DIRECT INSTRUCTION

1. Ask: *Give an example of hire purchase by naming an item that could be bought from a shop.* (Examples: a fridge from Game or a television from House and Home).
2. Ask: *Why is the hire purchase method of buying items appealing to some people?* (They don't need the full amount available upfront – they only need a deposit. They get the item and then pay it off in smaller instalments).
3. Point out that although it may sometimes be necessary for families to use this method of buying essential items due to financial circumstances and the need for the item, it is not really a financially sensible option. If a family's fridge needs to be replaced, then it is understandable that they may need to buy a new one on hire purchase. However, using the hire purchase method to buy a new surround sound entertainment system is more a 'want' than a 'need'.
4. Allow learners to ask questions and to take part in this discussion – it will give them the opportunity to think about situations that they may not have thought about before.
5. Once you are ready to continue with the lesson, tell learners: *When buying an item on credit (hire purchase), simple interest is ALWAYS used.*
6. Discuss why buying this way is called hire purchase. Essentially, the buyer is hiring the item until it is finally purchased (with the final instalment). If the buyer misses any payments it is within the seller's (the shop's) rights to take back the item (repossess the item). If this is the case, the buyer would have nothing to show for the money he/she has already paid.
7. Talk the learners through a purchase. Ask learners to close their eyes and visualise themselves doing it as you go through the steps.

You walk into a shop and start browsing. You notice a lovely set of garden furniture which would look perfect at your house. You see a sign that gives the cash price (R3 500). There is another sign advertising that it can be paid off in small monthly instalments. As you don't have that kind of cash on you or in your bank account this sounds like an excellent plan. You call over the sales person in the shop. He takes you over to his desk and starts filling out a contract for you. He tells you that you will have to pay a 10% deposit today but after that you will pay instalments of less than R150 per month. That sounds very easy! You sign the contract and organise delivery. You are very excited about your purchase.

8. Tell learners to open their eyes. You are now going to work through the mathematics of the purchase mentioned in the story.

9. The question would be written as follows in an assessment:
   Richard buys a set of garden furniture on hire purchase. The purchase price is R3 500. He will pay for it over 36 months. A 10% deposit is required, and interest will be charged at 18% p.a.
   Calculate how much Richard really paid for the garden furniture.

	Teaching notes
Deposit: 10% of R3 500 $= 0,1 \times 3\,500$ = R350	Remind learners that the first time a payment is to be made is today – the 10% deposit. We need to calculate this and subtract it from the original price.
New amount owing: R3 500 - R350 = R3 150	This is the amount that is now owing to the shop which will be used for the calculating of the monthly instalments.
Total owing (including interest): $A = P(1 + i.n)$ $A = 3\,150(1 + (0,18)(3)$ A = R4 851	Tell learners to be careful when finding the interest rate being charged and to not confuse it with the percentage deposit. Also, note that the time period was given in months, but years are required for simple interest.
Monthly instalment: $\frac{4\,851}{36}$ = R134,75	Once the total amount owing has been calculated, the monthly instalment can now be worked out. The amount owing will be divided equally over the number of months in the time period.
Total amount actually paid: R4 851 + R350 = R5 201	Remind learners that Richard has paid the deposit then 36 payments of R134,50 for the garden furniture to finally be his.

10. Tell learners that sometimes a question might mention insurance. For example, someone may buy a washing machine on hire purchase and the shop offers insurance so that if the washing machine breaks down, it will be repaired and the person buying will not need to pay. If the insurance is a percentage (and not a set amount), the original cost of the washing machine (before a deposit was paid) will need to be used to calculate the insurance.

11. Do one final example with learners. This time, give them time to get ahead of you. This is a good way to give learners a chance to check if they have understood the steps covered in the previous example. You may also want to give a hint as they go. Keep stopping them and doing one more step so learners can fix any problems as they go instead of getting to the end and finding that they made an error near the beginning.

12. Question:

A computer has been marked down from R10 999 to R9 999. The shop requires a 15% deposit and the rest will be paid in equal monthly instalments over a five-year period at an interest rate of 22% p.a.

a) Find the monthly instalments.

b) How much was paid for the computer in total after 5 years?

c) Do you think this is a good buy?

Solution	Teaching notes
a) Deposit: 15% of R9 999 $= 0,15 \times 9\,999$ $= \text{R}1\,499,85$ New amount owing: R9 999 - R1 499,85 $= \text{R}8\,509,15$ Total owing (including interest): $A = P(1 + i.n)$ $A = 8\,509,15(1 + (0,22)(5)$ $A = 17\,869,22$ Monthly instalment: $\frac{17\,869,22}{60} = \text{R}297,82$ b) Total amount paid: $\text{R}17\,869,22 + \text{R}1\,499,85 = \text{R}19\,369,07$	a) Remind learners to find the deposit and take it from the amount owed. Give them a chance to complete it before showing the full solution. After a few minutes tell them to use their simple interest formula to find the new amount owing and to read the question again carefully to ensure they use the correct percentage. Give them a chance to complete it before showing the full solution. Ask who can tell you how to find the monthly instalment. Let the learners do the calculation before showing the solution on the board. b) Remind learners to think of all the payments that have been made – from the day of purchase and for all the months over the 5-year period.

c)

Ask learners their opinion. Ensure at least the following points are noted:

- the computer cost almost double in the end
- after 5 years of paying all that money it may already be time to get a new one
- Unless the item was essential (and not a luxury) this is not a financially viable way to purchase items.

13. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

14. Give learners an exercise to complete with a partner.

15. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=Qsu-12MSp4Q
(Simple Interest Part 3 which deals with hire purchase)

TERM 3, TOPIC 2, LESSON 4

INFLATION AND GROWTH

Suggested lesson duration: 1,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	26

Lesson Objectives

By the end of the lesson, learners should be able to:

- answer questions related to inflation and population growth.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. You will need Resources 4 -12 from the Resource Pack. As you have been provided with a variety of resources, you could get different groups to work with different resources. For this lesson have the 'posters' stuck on the chalkboard and surrounding walls from the Resource Pack.
4. Write the lesson heading on the board before learners arrive.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
4	235	5 6	213 214	12.4	208	12.4 12.5 12.6	272 273 275	9.4 9.5	345 347

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Inflation is an aspect of financial mathematics that affects everyone. Inflation has the power to change market behaviour and even destroy economies if not managed properly.

2. The growth of a population is a special kind of inflation. Learners need to understand population growth.

3. The focus of this lesson is to give learners a better understanding of how the world around them works. There are examples of questions at the end, but the lesson's main objective is to improve learners' understanding of financial matters.

DIRECT INSTRUCTION

1. Start the lesson by asking the learners to look at the prices around them. Ask what they are thinking when looking at these prices from previous years (many before they were born).

2. Many learners will probably say that they wish they could pay those prices today! We all think that. There is also a good chance items were cheaper in those days.
   Ask: *What is the word that describes a general increase in the price of goods and fall in the purchasing value of money?* (Inflation).

3. Start a discussion by asking: *'Do you think everything was cheaper in years gone by?'* Hopefully this should lead to learners to realise that maybe not – and that surely it depends on what people were earning then.

4. Try to think of a personal story to tell learners about how items go up in price and how it may have affected you. Here is an example (for someone born in the 1960s):

   > When I was about 10 years old (mid-seventies), my mother gave me 10c to buy two packets of chips (crisps). When I got there, I was very disappointed to find that they had gone up to 6c each, so I could only buy one packet of chips.

After telling the story, ask what a small packet of crisps cost now. For the sake of showing an example, R6,59 will be used, but rather use the amount given at the time. Together with the learners, use the compound formula and find the average rate of inflation over the time period (use 43 years).

$$A = P(1 + i)^n$$

$$6{,}59 = 0{,}06(1 + i)^{43}$$

$$\frac{6{,}59}{0{,}06} = (1 + i)^{43}$$

$$1{,}11547267\ldots = 1 + i$$

$$0{,}11547267\ldots = i$$

$\therefore$ the average rate of inflation in those 43 years (according to a packet of chips!) was 11,55%

5. Ask learners to write the heading: Inflation and definition in their books now.

6. Inflation is a general increase in prices and fall in the purchasing value of money. Inflation is always calculated using compound interest.

7. Ask: *Why do you think only compound interest is used to calculate inflation?* (Because it is added onto the price from year to year then the new price is used the following year to calculate the next price with the current rate of inflation).

8. Tell learners what the average rate of inflation is for 2018 (4,05%).

9. Discuss what this means – that on average all goods will increase in price by 4,05%. Some items may increase by more and some may increase by less.

10. Ask: *What does an inflation rate of 4,05% mean to the average worker?* (He/she needs a salary increase of 4,05% to have the same lifestyle as he/she had last year. To improve their quality of life they would need a salary increase of more than 4,05%).

11. In the mid-eighties the rate of inflation peaked at around 20%.

12. Discuss the minimum wage in our country. A bill was passed recently that declared the minimum wage of R20 per hour. This would equate to about R3 500 per month if a person was working full time and for 8 hours per day.

13. However, some workers have a minimum wage less than R20 per hour.
    The information below gives learners some information which could lead to a better understanding of the difficulties that many of the workforce in our country face.

The minimum wage for

- farm workers will be 90% of R20 per hour (R18 per hour)
- domestic workers will be 75% of R20 per hour (R15 per hour)
- workers on an expanded public works programme will be R11 per hour.

Domestic worker salaries increased at an annual (nominal*) rate of 8,5% since 2007 – but in real terms, domestic worker wages have only seen an increase of just under 3% per year.

Domestic worker (27+ hours a week, Area A)	
2007 minimum wage	R1 066.83
2007 minimum wage (adjusted for inflation, 6.2% p.a)	R1 840.97
2017 minimum wage	R2 422.54
Annual change (nominal)	8.55%
Annual change (real)	2.78%

Put into a broader context, the National Agricultural Food Council's basic food basket came to **R287.97 per person** in 2007. By the start of 2017, this same basket cost **R668.50**.

https://businesstech.co.za/news/lifestyle/177153/how-much-domestic-workers-get-paid-in-2017-vs-2007/

14. * An understanding of nominal rate and effective rate is not required in Grade 10. If any of these words come up, tell learners that nominal rate is the official rate given before taking inflation into account. This concept will be covered in more detail in Grade 11.

15. Do these fully worked examples with learners. Learners should write them in their books.

Example	Teaching notes
A tub of margarine cost R6,99 in 2003. If the average rate of inflation was 4,88% find what the price of margarine was likely to be in 2010 and 2017.	Tell learners: *You will need to calculate each year separately.* Remind learners: *Inflation is always compound interest.*

Solution:

2010

$A = P(1 + i)^n$

$A = 6,99(1 + 0,0488)^7$

$A = 9,76$

2017

$A = P(1 + i)^n$

$A = 6,99(1 + 0,0488)^{14}$

$A = 13,62$

If a house was bought for R150 000 in 2012 and sold for R350 000 in 2017, what was the average rate of inflation?	Ask learners: *What makes this question different from the last question?* (The interest rate (i), and not the accumulated amount (A) is asked) This requires filling in the known values then solving an equation.

Solution:

$$A = P(1 + i)^n$$
$$350\,000 = 150\,000(1 + i)^5$$
$$\frac{7}{3} = (1 + i)^5$$
$$1,184664453 = 1 + i$$
$$0,184664453 = i$$

$\therefore$ the average rate of inflation in those 5 years was 18,47%

16. Discuss the last example with learners. Even though the rate of inflation did once climb as high as 20%, it is very unlikely for the average rate of inflation to be so high. Tell learners that property does not always link directly to the rate of inflation.
17. Ask learners: *What else could have affected why the house sold for such a high price?* (Among other possible reasons the following are plausible:
    - The people who owned the house spent a lot of money on it and it was greatly improved over the 5-year period
    - The area where the house was may have not been very popular when the house was first bought but during the 5-year period it improved and was now an area of interest to many people).
18. Discuss population growth with learners. Population growth is a special case of inflation. Each year most populations increase by a certain percentage.
19. Do the following worked example with learners. They should take it down in their books.

Example	Teaching notes
In 2003, the population of Zimbabwe was about 12,6 million people and in 2018 it is estimated at 16,9 million. Find the average rate of population growth over the 15-year period. Round your answer to two decimal places.	Show learners how there is little difference between this question and a question on inflation. Both population and inflation grow exponentially.

Solution:

$$A = P(1 + i)^n$$
$$16\,900\,000 = 12\,600\,000(1 + i)^{15}$$
$$\frac{169}{126} = (1 + i)^{15}$$
$$1,01976729 = 1 + i$$
$$0,01976729 = i$$

$\therefore$ the average rate of growth in those 15 years was 1,98%

20. Before giving learners work to complete, remind them again not to round any number off during the calculation. Rounding must only occur at the end of the process. If money is involved, remember to round off to two decimal places.

21. Ask directed questions so that you can ascertain learners' level of understanding.

22. Ask learners if they have any questions.

23. Give learners an exercise to complete with a partner.

24. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

http://www.lrs.org.za/docs/National%20Minimum%20Wage%20Booklet.pdf
(A booklet with information on minimum wages – for the teacher's information rather than being directly used in a lesson).

TERM 3, TOPIC 2, LESSON 5

FOREIGN EXCHANGE RATES

Suggested lesson duration: 1,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	26

Lesson Objectives

By the end of the lesson, learners will have revised:

- convert from one country's currency to another
- answer questions related to foreign exchange.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write the three ratios ready for learners to simplify.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
5	237	8	218	12.6	212	12.7	278	9.6	348

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Exchange rates are determined by the economy. Learners need to understand why this is so.

2. Exchange rates can be very volatile and change more than once per day.

3. Learners generally enjoy learning about financial issues – try to make it exciting for them.

DIRECT INSTRUCTION

1. Start the lesson by reminding learners how to work with ratios. Do the following examples: Find the missing values:

1 : 4	3 : 8	10 : 2
= ? : 20	= 12 : ?	= ? : 1

2. If learners struggle with these ratios, go through each one reminding them what they learnt in Grade 8 & 9. Ratios are like fractions. When finding a missing value, we treat it the same way as when finding a missing numerator or denominator in equivalent fractions.

   For example: $\frac{2}{3} = \frac{?}{12}$

   Ask: *What have we multiplied 3 by to get 12?* (4)

   Therefore, we need to multiply 2 by 4.

   $\frac{2}{3} = \frac{8}{12}$

3. Complete the three examples. Praise learners who did them correctly.

1 : 4	3 : 8	10 : 2
= **5** : 20	= 12 : **32**	= 5 : 1
(x 5)	(x 4)	(÷2)

4. Say: *Name currencies from other countries.* (America uses the dollar, will probably be a popular answer. Praise learners who named some currencies).

5. Ask: *Would you like to travel to another country? Where would you like to go and why?* Spend a few minutes allowing learners to talk about travelling to another country.

6. Continue the discussion by asking learners: *What will you need to do with your money when you travel to a new country?* (They would need to make sure they changed their rands into the currency of the country they are visiting).

7. Another possibility is that learners may want to order an item from another country. To do this they would need to calculate what the item would really be costing them in their own currency.

8. Both situations will be dealt with during this lesson.

9. Do the following examples on the board now. Learners should write them in their books.

Example:	Teaching notes:
In South Africa a Big Mac costs R35. In America it costs $3,99. Using the exchange rate below, calculate where the burger is cheaper. $1 = R12,06	Ask learners: *How are you going to start?* They could find the rand price of the American Big Mac using the exchange rate given. Remind learners to deal with the exchange rates the same way they deal with equivalent fractions.
Solution: $1 = R12,06 $3,99 = ? Working: $1 \times 3,99 = 3,99$ $\therefore 12,06 \times 3,99 = 48,12$ The Big Mac is cheaper in South Africa. South Africans would have to pay the equivalent of R48,12 for a Big Mac in America.	
The gold price is $\$1\,336,85$ per ounce. The exchange rate is $R1 = \$0,083$. Calculate the price of 3 000 ounces of gold in rands.	Point out to learners that this is not a direct conversion as in the previous example. This question will be a two-step process. First, we will need to calculate the price per ounce in rands, then we will need to calculate the price of 3 000 ounces.
Solution: R1 = $0,083 ? = $1 336,85 $\therefore \$1336,85 = R16\,106,63$ If one ounce costs R16 106,63, then 3 000 ounces will cost $3\,000 \times R16\,106,63 = R48\,319\,890$ Ask: *Who would like to read this number for us?* (48 million, 319 thousand, eight hundred and ninety rand)	Working: $1\,336,85 \div 0,083 = 16\,106,63$

On a certain day the exchange rate between the US dollar and South African rand is $\$1 = R12,91$. At the same time the exchange rate between the British pound and the South African rand is $£1 = R16,52$. Calculate the exchange rate between the British pound and US dollar on that day. DBE Nov 2017	As the rate has been given in one unit of both the dollar and the pound, we can compare the two-rand values to find the ratio. Learners may prefer to get both exchange rates given to R1. The dollar and pound can then be compared.
Solution: Option 1: $\frac{pound}{dollar} = \frac{16,52}{12,91}$ $\therefore £1 = \$1,28$	Option 2: $\$1 = R12,91$ $\therefore \$0,0775 = R1$ $£1 = R16,52$ $\therefore £0,0605 = R1$ $\therefore \$0,0775 = £0,0605$ $\$1,28 = £1$ (x 16,5289...)

10. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

11. Give learners an exercise to complete with a partner.

12. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=WDRhdtR96E0

TERM 3, TOPIC 2, LESSON 6

REVISION AND CONSOLIDATION

Suggested lesson duration: 1,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	26

Lesson Objectives

By the end of the lesson, learners will have revised:

- all the concepts from the 5 previous lessons by means of fully worked examples from past papers.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev S Ch	239 240	Rev	220	w/sh	216	12.9	285	9.7	350

Note that Survival and Classroom Maths both have an exercise on timelines. These are worth doing.

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Ask learners to recap what they have learned in this section. Spend time pointing out issues that you know are important as well as problems that you encountered from your own learners.

2. If learners want you to explain a concept again, do that now.

DIRECT INSTRUCTION

This lesson is made up of three fully worked examples from past papers covering all the concepts in this topic. As you work through these with the learners, it is important to frequently talk about as many concepts as possible.
For example, use the words simple interest, compound interest, hire purchase, inflation and exchange rates wherever possible, constantly reminding learners what they have already learnt.

Work through the three fully worked examples with learners now. Learners should write them in full in their books.

Example:	Teaching notes:
Mary wants to buy a fridge that costs R15 550. She must pay a deposit of 15% of the cost, and the balance by means of a hire-purchase agreement. The rate of interest on the loan 16,25% p.a. simple interest. The repayment period of the loan is 54 months. In addition to the hire-purchase agreement, an annual insurance premium of 1,5% of the total cost of the fridge is added. The annual insurance premium should be paid in monthly instalments. a) Calculate the value of the loan that Mary will take. b) Calculate the total amount that must be repaid on the hire-purchase agreement. c) Calculate the monthly repayment which includes the monthly insurance premium. EC 2016	Point out to learners that the time period has been given in months, but that years are required when using the simple interest formula. Ask them what they will need to do to change the months into years (divide by 12). Tell learners that they will need to be careful when dealing with the insurance. Once they have calculated the annual rate they will need to convert it to a monthly rate to add it onto the monthly instalment.

Solution:

a) Deposit:

$15\%\text{ of }R15\,550 = R2\,332,50$

New amount owing:

$R15\,550 - R2\,332,50 = R13\,217,50$

b) $A = P(1 + i.n)$

$A = 13\,217,50(1 + (0,1625)(4.5))$

$A = 22\,882,80$

c) Annual insurance:

$1,5\%\text{ of }15\,550$

$= 0,015 \times 15\,550$

$= 233,25$ (per annum)

Monthly repayments:

$$= \frac{22\,882,}{54} + \frac{233,25}{12}$$

$= R443,19$

The table below shows the rand equivalent of one British pound and one US dollar.

COUNTRY	CURRENCY	RATE OF EXCHANGE OF THE RAND
Britain (United Kingdom)	Pound (£)	21,41
United States of America	Dollar ($)	13,45

A South African nurse works in the United States of America.

a) The nurse saves the equivalent of R4 800 per month. Calculate the amount in US dollars that she saves per month.

b) The nurse ordered a book from the United Kingdom and paid $85 for it. Calculate the price of the book in pounds.

EC 2016

a) Is a straightforward conversion.

b) Will require two calculations. Since 'rands' is the common currency in the table, we will first need to find what the book cost in rands then a second calculation will be required to change that into pounds.

Solution:

a) $1 = R13,45
? = R4 800
∴ She saves $356,88

Working: $4\,800 \div 13,45 = 356,88$

b) $1 = R13,45
$85 = ?
The book costs R1143,25

Working: $13,45 \times 85 = 1143,25$

£1 = R21,41
? = R1143,25
∴ She pays £53,40 for the book.

Working: $1143,25 \div 21,41 = 53,40$

1. Seven years ago, Mrs Grey decided to invest R18 000 in a bank account that paid simple interest at 4,5% p.a. a) Calculate how much interest Mrs Grey has earned over the 7 years. b) Mrs Grey wants to buy a television set that costs R27 660.00 now. If the average rate of inflation over the last 5 years was 6,7% p.a., calculate the cost of the television set 5 years ago. c) At what rate of simple interest should Mrs Grey have invested her money 7 years ago if she intends buying the television set now using only her original investment of R18 000 and the interest carried over the last 7 years? DBE Nov 2017	Note: *Interest is asked for and not the interest rate. The interest is an amount of money.* Remind learners: *Inflation is always compound interest.* Point out that (c) will require finding the rate of interest. This will require substituting the known values then solving an equation.
2. A sum of money doubles in 5 years when the interest is compounded annually. Calculate the rate of interest. EC 2016	Ask learners: *What can you use for 'A' and 'P' as you have not been given the amount that is being saved?* Variables are preferable but even 2 and 1 could be used or 20 and 10 etc. Any of these could be used.

Solution:

1a) $A = P(1 + i.n)$

$A = 18\,000(1 + (0,045)(7))$

$A = 23\,670$

$\therefore$ Interest earned is: $R23\,670 - R18\,000 = R5\,670$

1b) $A = P(1 + i)^n$

$27\,660 = P(1 + 0,067)^5$

$$\frac{27\,600}{(1 + 0,067)^5} = P$$

$20\,000 = P$

The television cost R20 000 five years ago.

1c) $A = P(1 + i.n)$

$27\,660 = 18\,000(1 + i(7))$

$$\frac{27\,660}{18\,000} = 1 + 7i$$

$$\frac{27\,660}{18\,000} - 1 = 7i$$

$$\frac{\frac{27\,660}{18\,000} - 1}{7} = i$$

$i = 0,076666...$

$\therefore$ Mrs Grey needed an interest rate of 7,67%.

2. $2x = x(1 + i)^5$

$2 = (1 + i)^5$

$\sqrt[5]{2} = 1 + i$

$\sqrt[5]{2} - 1 = i$

$0,148698 = i$

$\therefore$ an interest rate of 14,87% is required for your money to double in 5 years.

1. Ask directed questions so that you can ascertain learners' level of understanding.
2. Ask learners if they have any questions.
3. Give learners an exercise to complete on their own.
4. Walk around the classroom as learners do the exercise. Support learners where necessary.

Term 3, Topic 3: Topic Overview

STATISTICS

A. TOPIC OVERVIEW

A

- This topic is the third of six topics in Term 3.
- This topic runs for two weeks (9 hours).
- It is presented over five lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 9 hours). For example, one lesson in this topic could take three school lessons. Plan according to your school's timetable.
- Statistics counts 15% of the final Paper 2 examination.
- An understanding of statistics is an important life skill. Studying statistics develops critical thinking and analytic skills.
- For any learners planning tertiary education, research will form part of their course. This would be difficult without the use of statistics.

Breakdown of topic into 5 lessons:

	Lesson title	Suggested time (hours)		Lesson title	Suggested time (hours)
1	Measures of central tendency	2	4	Interpretation	1,5
2	Measures of dispersion	2	5	Revision and consolidation	1,5
3	Five-number summary and box & whisker plots	2			

B SEQUENTIAL TABLE

GRADE 8 & 9	GRADE 10	GRADE 11 & 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Collect data • Pose questions relating to social, economic, and environmental issues • Select and justify appropriate sources for the collection of data • Distinguish between samples and populations • Organise numerical data in different ways to summarize by determining measures of central tendency, measures of dispersion, including extremes and outliers • Organize data according to more than one criteria • Critically read, interpret and analyse data • Critically compare two sets of data related to the same issue.	Collect, organise and interpret univariate numerical data to determine: • Measures of central tendency in grouped and ungrouped data • Estimated mean of grouped data • Modal interval and interval in which median lies • Five number summary • Box and whisker diagrams • Measures of dispersion to include range, percentiles, quartiles, interquartile range and semi-interquartile range • Use the statistical summaries to interpret, analyse and make meaningful comments on the context associated with the given data.	• Histograms • Frequency polygons • Ogives • Variance and standard deviation of ungrouped data • Symmetric and skewed data • Identification of outliers • Represent bivariate numerical data as a scatterplot • Calculate the linear regression line which best fits the given data • Calculate the correlation coefficient and make relevant deductions which include interpolation, extrapolation and discussions on skewness.

C WHAT THE NSC DIAGNOSTIC REPORTS TELL US

According to **NSC Diagnostic Reports** there are a number of issues pertaining to Statistics. It is important that you keep these issues in mind when teaching this section.

The issues below link directly to Grade 10 work:

- a lack of understanding of basic concepts such as mean and median as well as not knowing that arithmetic mean and average are the same concept
- not relating range and interquartile range to the five-number summary

- not showing an understanding that each quartile represents 25% of the data regardless of the length of the line
- a lack of understanding of the concept modal class – some learners thinking that modal class is just one point
- confusing the position of the median with the value of the median
- not understanding the concept of an outlier.

While teaching Statistics, ensure learners understand all the vocabulary and concepts. For example, the curriculum states that grouped and ungrouped data are both covered – learners need to understand the difference between these.

ASSESSMENT OF THE TOPIC

D

- CAPS formal assessment requirements for Term 3:
  - Two tests
- Two tests of 50 marks each are provided in Resources 27 and 29 in the Resource Pack. The tests are aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53). The tests are also divided in the ratio of the allocated teaching time. Memoranda for the tests are provided in Resources 28 and 30 in the Resource Pack.
- The questions usually take the form of being given a set of data and answering questions related to measures of central tendency and measures of dispersion.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

MATHEMATICAL VOCABULARY

E

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
data	Facts or information collected from people or objects. Data is plural for datum
population	The entire group of people or objects that data is being collected from
sample	A smaller part of the population. We study a sample if the population is large

random sampling	A method of choosing a smaller sample of the population that tries to ensure that each item has an equal probability of being chosen (not biased)
questionnaire	A set of questions with a choice of answers used in the data collection process
survey	The collecting of data from a group of people
discrete data	Data that can only take certain values. For example, the number of learners in a class (there can't be half a learner)
continuous data	Data that can take on any value within a certain range. For example, the heights of a group of learners (heights could be measured in decimals)
tally	A way of keeping count by drawing marks. Every fifth mark is drawn across the previous four (to form a gate-like diagram) so you can easily see groups of five
frequency table	A table that lists a set of scores and their frequency. Often used with tallies. Summarises the totals and shows how often something has occurred
measures of central tendency	A measure of central tendency is a single value that describes the way in which a group of data cluster around a central value. Central tendency is a way to describe the centre of a data set. There are three measures of central tendency: mean, median, and mode
mean	The average of a set of numbers. Calculated by adding all the values then dividing by how many numbers there are
median	The middle number in a sorted list of numbers. To find the median, place all numbers in order from smallest to biggest (or biggest to smallest) and find the middle number
mode	The number that appears the most often in a set of data. There can be two modes. There could also be no mode in a set of data
modal class	The class interval that has the most results

estimated mean	An estimated mean can be determined for grouped data. Unlike listed data, the individual values for grouped data are not available, and you are not able to calculate their sum. To calculate the mean of grouped data, the first step is to determine the midpoint of each interval, or class. These midpoints must then be multiplied by the frequencies of the corresponding classes. The sum of the products divided by the total number of values will be the value of the mean
measures of dispersion	Measures of dispersion indicate the spread of scores in a data set. Examples of measures of dispersion: range, percentiles and quartiles Like central tendency, measures of dispersion help to summarise a set of data with one, or just a few, numbers
range	The difference between the highest and lowest value in a set of data
percentiles	Each of the 100 equal groups into which a population can be divided according to the distribution of values of a variable The value below which a percentage of data falls
quartiles	Each of four equal groups into which a population can be divided according to the distribution of values of a variable The values that divide a list of numbers into quarters
interquartile range (IQR)	A measure of variability, based on dividing a data set into quartiles. Quartiles divide a rank-ordered data set into four equal parts The values that divide each part are called the first, second, and third quartiles; and they are denoted by Q1, Q2, and Q3 respectively
histogram	A graph representing grouped data. The data set is continuous. For example, one bar could represent how many learners got a mark from 40-49 and the next bar would then represent the marks from 50-59
class interval	The size of each class into which a range of a variable is divided, as represented by the divisions on a histogram or bar chart
broken line graph	A graph that uses points connected by lines to show how something changes in value as time goes by or as something else happens
scatter plots	A graph in which the values of two variables are plotted along two axes. The pattern of the resulting points reveals whether there is any correlation between the two sets of values

outlier	An extreme value which is much higher or much lower than the other values
ungrouped data	Data that has not been classified or has not been subdivided in the form of groups. Ungrouped data is raw data. Ungrouped data is in the form of a list of numbers
grouped data	Data that has been ordered and sorted into groups called classes. Data that has been bundled together in categories. Histograms and frequency tables can be used to show this type of data
modal class	The modal class is the class with the highest frequency from a set of grouped data The interval with the most “members”
five-number summary	Lowest value, lower quartile, median, upper quartile and highest value from a set of data The five numbers are used to draw a box and whisker plot
box and whisker plot	A simple way of representing statistical data on a plot in which a rectangle is drawn to represent the second and third quartiles, usually with a vertical line inside to indicate the median value. The lower and upper quartiles are shown as horizontal lines on either side of the rectangle. The lowest value and highest value in the data set are represented at each end

TERM 3, TOPIC 3, LESSON 1

MEASURES OF CENTRAL TENDENCY

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	27

Lesson Objectives

By the end of the lesson, learners should be able to find:

- measures of central tendency for ungrouped and grouped data
- the estimated mean of grouped data
- the modal interval and the interval in which the median lies.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Prepare Resource 13 for use during this lesson.
4. Write the lesson heading on the board before learners arrive: Measures of central tendency.
5. Write work on the chalkboard before the learners arrive. For this lesson write the list of data for the first example.
6. Note: when you first start the table (point 15), ensure you leave space on the right to add two more columns later (points 32 & 34). When learners start their table, tell them that columns will be added later, and they must leave space to allow for this.
7. The table on the next page provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
3	254	1	226	13.1	225	13.1	289	10.2	364
4	256	2	229			13.2	294	10.3	366
						13.3	297	10.4	370
						13.5	304		

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners covered measures of central tendency in the Senior Phase.
2. Learners should know what mean, median and mode are as well as what ungrouped data and grouped data are.
3. These concepts will be revised before moving on to new work involving measures of central tendency.

DIRECT INSTRUCTION

1. Start the lesson by asking: *How can we find the different averages for a set of data?* Praise learners that can tell you about mean, median and mode.
2. If anyone said ‘range’, tell them that although range is a way of knowing something about the set of data, it will be covered in a later lesson.
3. Tell learners: *The mean, median and mode are three types of average. They are measures of central tendency.* Point to the heading you wrote on the chalkboard before the lesson started. Explain why they are called measures of central tendency – *We are looking for one value that could represent the whole set of data. It is therefore a measure (the value) that takes all the data into account but goes towards one value only (the data is tending towards one central value).*

4. Instruct learners: *Copy this set of data and find the mean, median and the mode of:*

   2 ; 5 ; 5 ; 6 ; 7 ; 10 ; 14

   These are the ages of Ugogo's grandchildren.

5. Give learners a chance to remember how to find these values on their own before going through each solution in full. As you find each solution, describe the process.

   Mean – we find the sum of all the data (49) then divide by the number of values (7)

   Median – ensure the data is in order then look for the middle value. If there is an even set of values, add the middle two and divide by 2.

   Mode – look for the number that appears the most often. It is easy to do this in a ranked set of data.

6. Solutions:

   Mean $= \frac{49}{7} = 7$

   Median = 6

   Mode = 5

7. Remind learners that sometimes there may be no mode but there can also be two or more modes:
   - Add another 10 to the original set of data; now there would be two modes – 5 and 10.

8. Tell learners that this type of data is called ungrouped data. It is just a list of values.

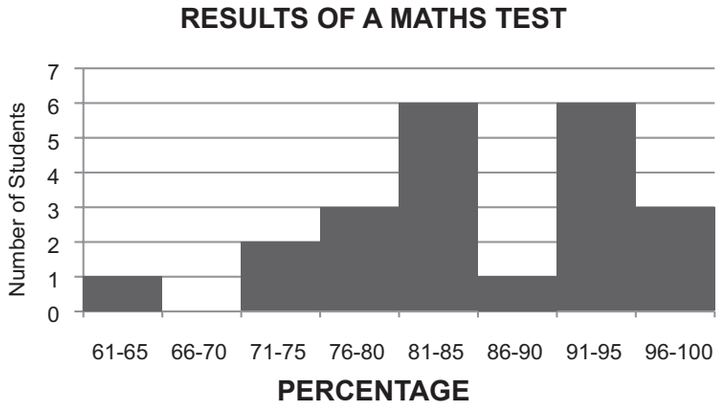
9. Discuss which measure of dispersion is the best one to use:
   The mean is the most common form of central tendency used to find one value to represent a set of data. The mean is considered a good measure as it takes all the data into account. The mean is the average used on report cards at school – all the subjects marks are added together and divided by the number of subjects to get an overall average. However, the mean has limitations – if there is an outlier (an extreme value which is much higher or much lower than the other values) then the mean isn't a good measure. In this case, the median would be a better measure to use. The mean also can't be used if the data is not numerical – for example, if your data is a list of colours.

10. Learners may have heard: *The average number of children in a family is 2,4.* This may be accurate if the number of children in each of the families used in the survey were added together and divided by the number of families. However, there is no such thing as a family with 2,4 children. In this case it would be better to use the mode as a measure of central tendency.

11. Ask learners: *What is grouped data? Give an example of grouped data.*

12. Grouped data is data that has been ordered and sorted into groups called classes. Histograms and frequency tables can be used to show grouped data.

13. Draw this example of grouped data represented on a histogram on the chalkboard. Ask learners to sketch it in their books.



14. Remind learners what a histogram is (it is not a bar graph with the bars joined together) – it is a representation of grouped and continuous data. In this example, the height of each bar represents the number of students who have achieved a certain result for a test. The first bar represents 1 student who got in the range 61 to 65. The next bar (second bar) continues directly from the previous bar and starts at 66; although in this case, there were no learners represented within the range.

15. Tell learners: *Let's look at what this information would look like in table form.*
Draw up a table with 2 columns labelled: Class interval and Frequency.

Class Interval	Frequency

16. Ask: *What is a class interval?* (The size of each class into which a range of a variable is divided).
Ask: *What are the class intervals in the given situation?*

(61 -65; 66 -70; 71 -75 etc)

17. Ask: *What do the class intervals represent in the histogram?* (Percentage achieved).
Fill this in on the table. Learners must add to their table as you do.

Class Interval (percentage)	Frequency

18. Complete the class intervals:

Class Interval (percentage)	Frequency
61 – 65	
66 – 70	
71 – 75	
76 – 80	
81 – 85	
86 – 90	
91 – 95	
96 – 100	

19. Point out: *These could also (and often are) be written as inequalities.*
    We present an example where this will be discussed in more detail later in the topic.

20. Ask: *Complete the rest of the table by completing the frequency column.*
    Remind learners: *Frequency means 'how many'.*

21. The final table look like this:

Class Interval (percentage)	Frequency
61 – 65	1
66 – 70	0
71 – 75	2
76 – 80	3
81 – 85	6
86 – 90	1
91 – 95	6
96 – 100	3

22. Ask: *Which range of marks was the most common?*
    (The 81 - 85 and the 91 - 95 each had six learners achieving these results).

23. Ask: *What type of measure of central tendency would this be?*
    (The mode). However, we don't know what the mode really is because we don't have the set of ungrouped data. We do not know the exact mark achieved by those 6 learners, we just know they all got a mark somewhere in the range 81 – 85; and 91 – 95.

24. To alleviate this problem, we state the modal class. The modal class is the class interval that has the most results. Our data has two modal classes: 81 - 85 and 91 – 95.

25. Ask: *How would we find where the median value lies?*
    (We would need to know the total number of learners represented and find where the middle value would be).

26. There are 22 learners in total represented here. The median position would be between 11 and 12 (11,5). Ask: *Which class interval would the median be in?* (81 – 85).

27. Ensure that all learners have understood how to work this out. Learners will need to accumulate the totals until they get to where the 11^{th} and 12^{th} value would be $(1+0=1;1+2=3;3+3=6;6+6=12)$.

28. Confirm that learners can answer the question: *In which interval would the median lie?*

29. Now that two of the measures of central tendency have been discussed for grouped data, we need to calculate the mean. Ask: *Why can't we add all the data up and divide by the number of learners?* (We don't have the raw set of data).

30. Although we don't have the actual set of data, we still have a reasonable idea what kind of results were in that set of data. For example, we know that three learners got a mark in the class interval 76 to 80.
    Ask: *What is the middle (median) number between 76 and 80?* (78). If we assumed that those three learners got 78, the total of their results would be 234. This could be very close to what the total of the three actual scores may be. In other words, the three results could have been 76, 79 and 80 (a total of 235 – very close to our estimate).

31. This method of estimating will be used to find an estimate of the mean as the actual mean is impossible to find. A question in an assessment will specifically state, find the estimated mean.

32. Add another column to the table:

Class Interval (percentage)	Frequency	Midpoint of class interval
61 – 65	1	63
66 – 70	0	68
71 – 75	2	73
76 – 80	3	78
81 – 85	6	83
86 – 90	1	88
91 – 95	6	93
96 – 100	3	98

33. Remind learners: *We need the midpoint of the class interval because we are going to assume that the number of learners represented in the class interval actually achieved the midpoint value.*

34. Add another column to the table: midpoint $\times$ frequency.
    Ask: *Why do we multiply the midpoint by frequency?* (We assume that each learner represented (frequency) achieved the middle score (midpoint), and therefore need to total up how many scores we have).

Class Interval (percentage)	Frequency	Midpoint of class interval	Midpoint × frequency
61 – 65	1	63	63
66 – 70	0	68	0
71 – 75	2	73	146
76 – 80	3	78	234
81 – 85	6	83	498
86 – 90	1	88	88
91 – 95	6	93	558
96 – 100	3	98	294

35. To continue finding the estimated mean, the totals still need to be added and divided by the number of learners (22).

36. Estimated mean: $\frac{1881}{22} = 85,5$

37. Learners should look at this value (estimated mean) and ask themselves if the answer is within the data range (61 to 100) and if it looks reasonable. It is within the range of data given.
    Tell learners: *None of the measures of central tendency could ever lie outside the set of data.* Do learners think it looks reasonable? (Yes – it is very high but 10 learners, which is almost half of the class; got higher than that and 6 more learners also had results in the 80s).

38. Confirm that learners understand the difference between ungrouped data and grouped data.
    Ask: *Give examples of other situations that would call for grouped data.*
    (Heights of learners, mass of babies, pocket money of a group of learners).

39. Ask directed questions so that you can ascertain learners' level of understanding.

40. Ask learners if they have any questions.

41. Give learners an exercise to complete on their own.

42. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://study.com/academy/lesson/difference-between-grouped-and-ungrouped-data.html
(Difference between grouped and ungrouped data)

https://www.youtube.com/watch?v=gYTwioS4mbo
https://www.youtube.com/watch?v=XlURkIXeC8E
(When to use which measure of central tendency)

https://www.youtube.com/watch?v=Vf4IyXC5KXk
https://www.youtube.com/watch?v=KwpcKCX51ro
(Estimated mean)

TERM 3, TOPIC 3, LESSON 2

MEASURES OF DISPERSION

Suggested lesson duration: 2 hours

POLICY AND OUTCOMES

A

CAPS Page Number	27

Lesson Objectives

By the end of the lesson, learners should be able to:

- measures of dispersion including range, percentiles, quartiles, interquartile range and semi-interquartile range.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson write Ugogo's grandchildren's ages from the previous lesson again.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
5	261	3 4	233 235	13.1	225	13.6	311	10.5	378

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners have covered measures of dispersion in the Senior Phase.
2. Measures of dispersion will be revised before moving on to new work involving other measures of dispersion including quartiles and percentiles.

DIRECT INSTRUCTION

1. Ask: *What is the range in ages of the grandchildren?* $(14 - 2 = 12)$.
   What does this value (12) tell us about the data?
   (How spread out the data is).
2. Tell learners that this a measure of dispersion. To disperse means to spread out.
3. Range is a simple measure of the spread of data. There are many more measures of dispersion. In Grade 10 we deal with quartiles (dividing the data into four equal parts) and percentiles (dividing the data into 100 equal parts).
4. The first example to demonstrate quartiles is a simple set of data to demonstrate quartiles without needing to perform calculations.
5. Tell learners: *Write down the following data representing marks out of 20 for a test:*

   2 5 5 6 9 11 11 11 14 15 19

6. Tell learners: *Circle the median.*

   2 5 5 6 9 (11) 11 11 14 15 19

   Remind learners: *The median represents the middle of the data. The data has been divided into two parts.*
   Consider the set of data on each side of the median and circle the median again:

   2 5 (5) 6 9 (11) 11 11 (14) 15 19

   Tell learners: *The circled numbers represent the quartiles – Quartile 1* (Q_1)*, Quartile 2* (Q_2)*, and Quartile 3* (Q_3)*. These values split the set of data into quarters.*
7. Ask: *What can you say about Quartile 2?*
   (It can also be called the median).

8. The interquartile range (IQR) can be calculated by subtracting quartile 1 from quartile 3 $(Q_3 - Q_1)$. The IQR of this set of data is $14 - 5 = 9$.

9. If the data set is bigger or has a more difficult number of values to work with, there are other methods to find the quartiles.

10. For example, consider the following set of data:

Example:	Teaching notes
Nineteen girls were required to complete a puzzle as quickly as possible. Their times (in seconds) were recorded: 14 15 16 16 17 17 18 18 19 19 19 20 21 21 22 23 24 24 29 a) Identify the median time taken by the girls to complete the puzzle. b) Determine the lower and upper quartiles for the data.	Remind learners that although they may be able to count out where the median and other quartiles lie, this example is being used to show another method. These formulae will be used to find the position in which the median (or Q_2) lies as well as the positions of Q_1 and Q_3 lie. $Q_1 = \frac{1}{4}(n + 1)$ $Q_2 = \frac{1}{2}(n + 1)$ $Q_3 = \frac{3}{4}(n + 1)$ n represents the number of values in the data set. It is important to note that the result of these calculations will give position and not the actual quartiles. Remind learners that the quartiles must be values within the data set.

Solutions:

a) $Q_2 = \frac{1}{2}(n + 1)$

$Q_2 = \frac{1}{2}(19 + 1)$

$Q_2 = \frac{1}{2}(20)$

$Q_2 = 10$

The median is in the 10^{th} position.
The median is 19.

b) $Q_1 = \frac{1}{4}(n + 1)$

$Q_1 = \frac{1}{4}(19 + 1)$

$Q_1 = \frac{1}{4}(20)$

$Q_1 = 5$

Quartile 1 is in the 5^{th} position.
Quartile 1 is 17

$Q_3 = \frac{3}{4}(n + 1)$

$Q_3 = \frac{3}{4}(19 + 1)$

$Q_3 = \frac{3}{4}(20)$

$Q_3 = 15$

Quartile 3 is in the 15^{th} position.
Quartile 3 is 22

Tell learners that if the result of any of the calculations gives a:

- ,5 – they need to find a value halfway between the positions given.
  For example, 4,5 would be the value halfway between the 4^{th} and 5^{th} position (in other words the mean of the two values)
- ,25 – they can round down.
  For example, 6,25 can be rounded to the 6^{th} position
- ,75 – they can round up. For example, 6,75 can be rounded to the 7^{th} position.

11. Ensure learners have written down the three formulae as well as explanations of how to round if necessary.

12. Measures of dispersion:
    We have spent some time on quartiles but there other measures of dispersion.
    Another measure of dispersion studied in Grade 10 is percentiles.
    Ask: *How many equal parts do you think a percentile would divide the data into?* (100).

13. Imagine the following situation:

A competition on general knowledge is to be held at your school. All learners will take part. Once learners have submitted their answers to the questions and they have been marked each learner will receive a slip of paper reporting in which percentile they have been placed. Unfortunately, many learners do not understand what this means.
Luckily a Grade 10 learner has just learned about percentiles in mathematics and offers to explain it to the 600 learners in the school.
Note to teacher: you can adjust this total and the numbers that follow to match the number of learners in your own school to make it more realistic. However, for the sake of the explanation a multiple of 100 would be best.
The learner explains as follows:
The group has been divided into 100 equal parts. In the case of our school, that means that there are 6 learners in each of these groups. These groups have been ordered from lowest results to top results. In other words, the 6 learners in the first group got the lowest marks – these learners are in the 1st percentile. The learners in the last group got the highest marks – these learners are in the 100th percentile.
If you are in the 65th percentile, it means there are $384\,(64 \times 6)$ *learners who scored lower than you and* $396\,(66 \times 4)$ *learners who scored higher than you. You could also say that you scored as well as, or better than, 65% of the group. The 50th percentile is the same as the median.*

14. Give learners another example and ask questions to check their level of understanding:
In a 400m race, your time was recorded as 70 seconds. Your coach tells you that you are in the 10th percentile. What conclusions can be drawn from this?
(My time was as good as or better than 10% of the group; 90% of the group had a better time than me).

15. Do the following example with learners. They must write the example in their books.

Example:	Teaching notes
The percentages achieved by learners in a mathematics test: 35 36 36 38 40 42 46 48 50 51 60 62 65 70 72 72 74 76 76 78 80 82 84 84 85 86 88 88 90 94 96 98	

a) Find the 20^{th} and 70^{th} percentile result.	a) Tell learners: *As we did when looking for quartiles, our aim is to first find the position of the percentiles before looking at the data and finding the actual percentile.* $\frac{k}{100} \times n$ Where *k* is the percentile required and n is the number of values in the data set.
b) Find the percentile rank for the result 50%.	b) This requires finding the position that the required result is in, then finding the percentage of that position out of the total number of values.
c) What would you tell the learner who got 50%?	c) Ask learners for their own ideas once the percentile of this result has been found.
Solutions: a) 20^{th} percentile: $\frac{20}{100} \times 32$ $= 6,4$ $\therefore$ position 6 The 20^{th} percentile is 42% 70^{th} percentile: $\frac{70}{100} \times 32$ $= 22,4$ $\therefore$ position 22 The 70^{th} percentile is 82%	b) 50% is in the 9^{th} position. $\frac{9}{22} \times 100$ $= 40,9\%$ The result 50% is in the 40^{th} percentile. c) Even though this learner got a reasonable result of 50%, he/she is only in the 40^{th} percentile which means he/she scored as well as or better than 40% of the learners. This may not seem very good but without knowing more about the learner's ability it is difficult to make a judgement.

16. Discuss the effect outliers may have on both measures of central tendency and measures of dispersion:
    Extremes (or outliers) need to be considered when making any of the above calculations. If one, or even two, set/s of data are very different to the rest of the data, they can make a significant difference to whether a measure of central tendency (specifically the mean) or the measure of dispersion is considered reliable or not. The affected mean or range incorrectly displays a bias toward the outlier value. The median and mode values, which express other measures of central tendency, are largely unaffected by an outlier.

17. Ask directed questions so that you can ascertain learners' level of understanding.

18. Ask learners if they have any questions.

19. Give learners an exercise to complete on their own.

20. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=wNamjO-JzUg

TERM 3, TOPIC 3, LESSON 3

FIVE-NUMBER SUMMARY AND BOX & WHISKER PLOTS

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	27

Lesson Objectives

By the end of the lesson, learners should be able to:

- find the five-number summary
- draw a box and whisker plot.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
   Write the lesson heading on the board before learners arrive: Five-number summary and box & whisker plots
3. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
6	266	5	237	13.1	225	13.7 13.8 13.9	316 318 321	10.6	380

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. The ground work has already been done for the skills required in this lesson.

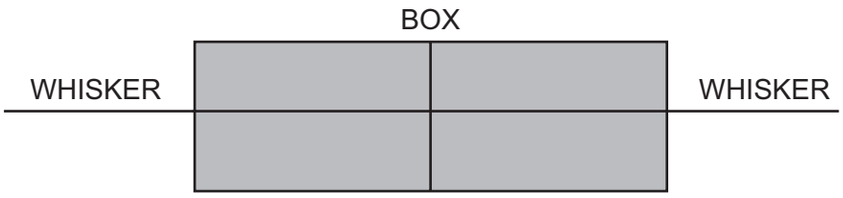
2. Being able to find the median and quartiles is essential.

DIRECT INSTRUCTION

1. Introduce the lesson by telling learners that all sets of data can be summarised using five numbers. This is called the five-number summary and will be useful to show a set of data in a more visual way.

2. Tell learners to write the following in their books:
   A five-number summary of a set of data is made up of:
   - The lowest value
   - The lower quartile (Q_1)
   - The median
   - The upper quartile (Q_3)
   - The highest value

3. Once these values have been found they can be represented in a box and whisker plot. Draw a basic box and whisker plot on the board which learners can copy into their books:



4. Ask learners if they can see where the 5 numbers from the five-number summary may be represented (at the beginning of the line, the beginning of the box, the middle of the box, the end of the box and the end of the line).

5. Point out that although it may not show on this diagram, it is important that these box and whisker plots are drawn to scale. This will become evident in the following example:

Example	Teaching notes
Nineteen girls were required to complete a puzzle as quickly as possible. Their times (in seconds) were recorded and shown below: 14 15 16 16 17 17 18 18 19 19 19 20 21 21 22 23 24 24 29 Draw a box and whisker diagram to represent the data. NSC Nov 2016	Do a five-number summary first. Decide on a scale. To do this, find the range, then choose an appropriate scale. For example, if the scale is 10 or less, $1cm$ per unit can be used. If the range is around 20 or 30, $0{,}5cm$ per unit can be used. Once the scale has been decided, draw a number line marking off a few values. Mark off the five numbers from the summary, draw in the lines and draw the box and whisker diagram.

Solution:

Five-number summary

Lowest value: 14

Highest value: 29

Lower quartile:

$Q_1 = \frac{1}{4}(n+1)$

$Q_1 = \frac{1}{4}(19+1)$

$Q_1 = \frac{1}{4}(20)$

$Q_1 = 5$

Quartile 1 is in the 5^{th} position.

Quartile 1 is 17

Median:

$Q_2 = \frac{1}{2}(n+1)$

$Q_2 = \frac{1}{2}(19+1)$

$Q_2 = \frac{1}{2}(20)$

$Q_2 = 10$

The median is in the 10^{th} position.

The median is 19.

Upper quartile:

$Q_3 = \frac{3}{4}(n+1)$

$Q_3 = \frac{3}{4}(19+1)$

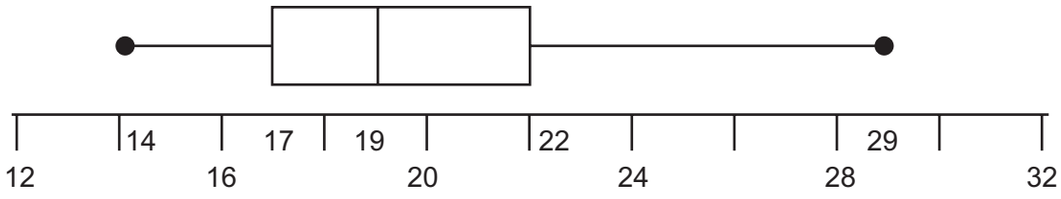
$Q_3 = \frac{3}{4}(20)$

$Q_3 = 15$

Quartile 3 is in the 15^{th} position.

Quartile 3 is 22

A range of 15 means we should either make 1cm per unit or 1cm for every 2 units



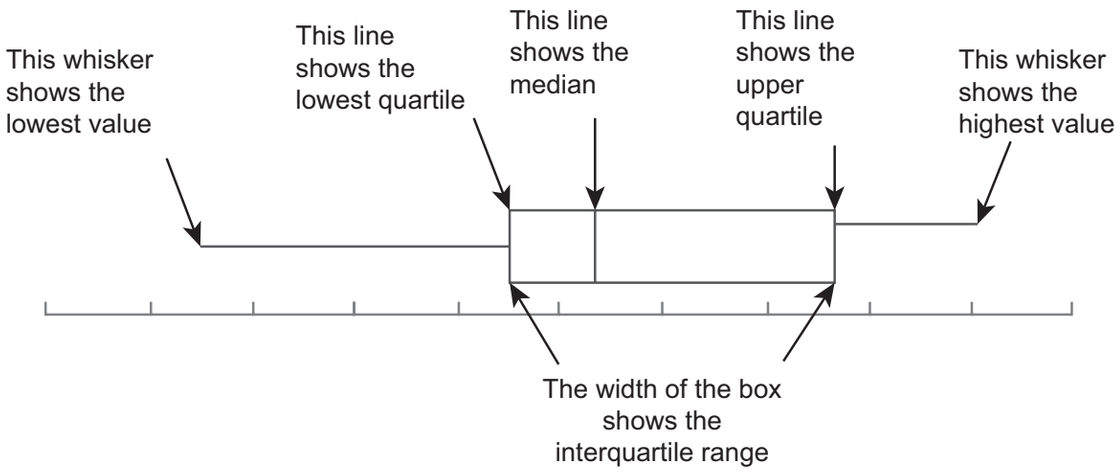
6. Point out what this means and why the scale is important.

7. Point out to learners that each of the 4 sections of this box and whisker plot (the two whiskers and the two parts of the box) represents a quarter of the data. This is important.

8. Ask learners to notice how long the right-hand whisker is – this does not mean that there is more data represented here. This means that the quarter of the data represented (4^{th} quarter) is more spread out than the data in the other 3 quarters. The range of the values is quite large.

   Notice how short the left-hand part of the box is - this means that the data represented in this quarter (2nd quarter) is very tightly packed together. The range of the value is small.

9. End the lesson by drawing this box and whisker diagram on the chalkboard for the learners to write in their exercise books:



10. Ask directed questions so that you can ascertain learners' level of understanding.

11. Ask learners if they have any questions.

12. Give learners an exercise to complete with a partner.

13. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=omOSu7_Z22o
https://www.youtube.com/watch?v=XDS5TgZ4CJA
(Five number summary)

https://www.youtube.com/watch?v=fJZv9YeQ-qQ
https://www.youtube.com/watch?v=QI0oaZ-WTvU
https://www.youtube.com/watch?v=CoVf1jLxgj4
(Box and whisker plots)

TERM 3, TOPIC 3, LESSON 4

INTERPRETATION

Suggested lesson duration: 1,5 hours

A POLICY AND OUTCOMES

CAPS Page Number	27

Lesson Objectives

By the end of the lesson, learners should be able to:

- use statistical summaries to interpret, analyse and make meaningful comments on the context associated with the given data.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. You will need Resources 14 and 15 from the Resource Pack.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson draw copies of the two graphs from point 4.
6. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
7	267	6	239	13.2	228	13.10	323		

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Being able to interpret data is an important life skill.

2. The following summary is adapted from an article called, '10 everyday reasons why statistics are important' (http://www.mathworksheetscenter.com/mathtips/statsareimportant.html)

   Share some or all these points with your learners. Have a class discussion where learners must be encouraged to discuss something they know about statistics or ask questions.

Statistics are sets of mathematical equations that are used to analyse what is happening in the world around us. Today we live in the Information Age where we understand a great deal about the world around us. Much of this information was determined mathematically by using statistics. When used correctly, statistics tell us any trends in what happened in the past and can be useful in predicting what may happen in the future.

Examples of how statistics shape your life when you don't even know it:

1. Weather Forecasts

Do you watch the weather forecast sometime during the day? How do you use that information? Have you ever heard the forecaster talk about weather models? These computer models are built using statistics that compare prior weather conditions with current weather to predict future weather.

2. Emergency Preparedness

What happens if the forecast indicates that a hurricane is imminent or that flooding may occur? Emergency management agencies move into high gear to be ready to rescue people. Emergency teams rely on statistics to tell them when danger may occur.

3. Predicting Disease

Statistics about a disease are often on given news reports. If the reporter simply reports the number of people who either have the disease or who have died from it, it's an interesting fact but it might not mean much in your life. However, when statistics become involved, you have a better idea of how that disease may affect you.

For example, studies have shown that 85 to 95 percent of lung cancers are smoking related. This statistic tells you that almost all lung cancers are related to smoking and that if you want to have a good chance of avoiding lung cancer, you shouldn't smoke.

4. Medical Studies

Scientists must show a statistically valid rate of effectiveness before any drug can be prescribed. Statistics are behind every medical study you hear about.

5. Genetics
Many people are afflicted with diseases that come from their genetic make-up and these diseases can potentially be passed on to their children. Statistics are critical in determining the chances of a new baby being affected by the disease.
6. Political Campaigns
Whenever there is an election, the news organisations consult their models when they try to predict who the winner is. Candidates consult voter polls to determine where and how they campaign. Statistics play a part in who your elected government officials will be.
7. Insurance
If you have a bond on your house, the house needs to be insured. The rate that an insurance company charges is based upon statistics from homeowners in your area. In some countries, car insurance is compulsory too.
8. Consumer Goods
Wal-Mart (a worldwide retailer), keeps track of everything they sell. Wal-Mart uses statistics to calculate what to ship to each store and when. For example: An analysis of their data store of information showed that people buy strawberry Pop Tarts when a hurricane is predicted in Florida! So, Wal-Mart ships this product to Florida stores based upon the weather forecast.
9. Quality Testing
Companies make thousands of products every day. Each company must make sure that a good quality item is sold. But a company can't test each item that they ship to you, the consumer. So, the company uses statistics to test just a few, called a sample, of what they make. If the sample passes quality tests, then the company assumes that all the items made in the group, called a batch, are good.
10. Stock Market
Another topic that you hear a lot about in the news is the stock market. Stock analysts also use statistical computer models to forecast what is happening in the economy.

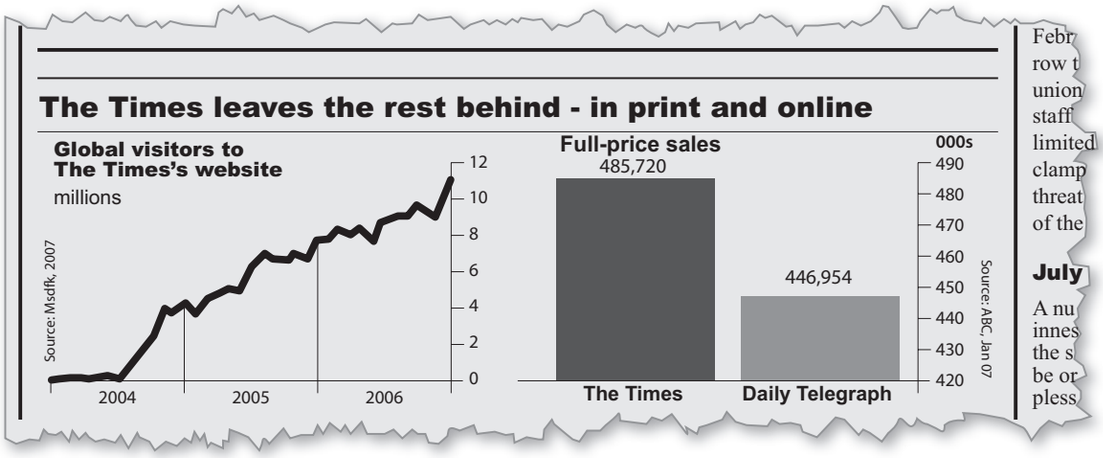
DIRECT INSTRUCTION

1. Introduce the lesson with a discussion sharing some or all the points from the article in the introduction. Encourage learners to discuss something they know about statistics or to ask questions.

2. Discuss the misuse of statistics. There are many misuses of statistics, but scale is probably the most common one at school level.

3. If the vertical scale is too big or too small or numbers are left out it could make the graph misleading.
   If the graph is not labelled correctly it could also mislead the reader.

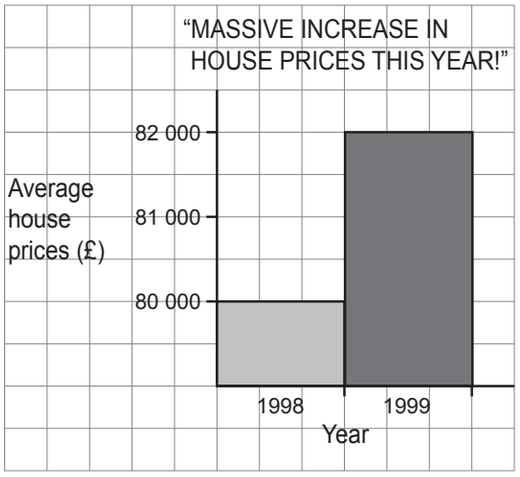
4. Below is an example of misleading graphs:
   (Refer to the sketches drawn on the chalkboard before the lesson)

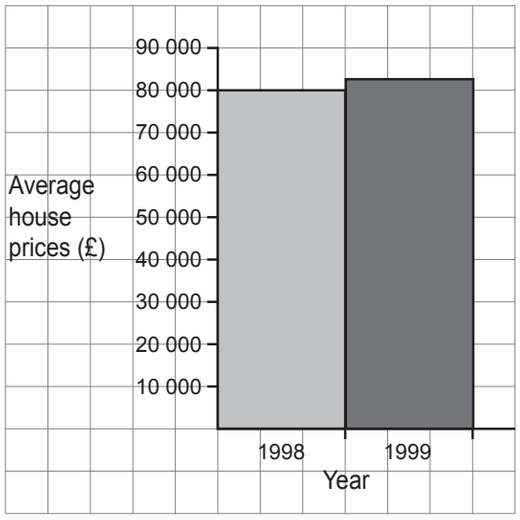


http://www.statisticshowto.com/wp-content/uploads/2014/01/misleading-graph-2.jpg

The line graph on the left looks fine. The vertical axis starts at zero and goes up in regular multiples. The horizontal axis starts in one year and shows the next 2 years.
Note the following from the bar graph on the right:
At a glance, it looks like The Times sells more than double the amount of newspapers as the Daily Telegraph. A closer look at the scale on the right shows that although The Times may well be making more sales it certainly isn't double the amount.

5. Use one more example to illustrate the idea of changing the scale:





The bar chart on the left could have been drawn by someone who wanted people to think that house prices have increased dramatically.
The scale starts at 80 000 then continues in increments of 1 000.
The chart on the right starts at 10 000 and continues in increments of 10 000 making it more accurate – but not as dramatic!

6. Ask directed questions so that you can ascertain learners' level of understanding.
7. Ask learners if they have any questions.
8. Give learners an exercise to complete in a small group. Encourage learners to verbalise/ talk about their ideas.
9. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://simplicable.com/new/misuse-of-statistics

An interesting infographic on guns and violence in our country:
http://www.saferspaces.org.za/understand/entry/gun-violence

TERM 3, TOPIC 3, LESSON 5

REVISION AND CONSOLIDATION

Suggested lesson duration: 1,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	27

Lesson Objectives

By the end of the lesson, learners will have revised:

- the main concepts covered in statistics.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive: Revision.
4. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev	270	Rev	242	W/sh	232	13.11 13.12	324 326	10.7	382

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Start the lesson by asking learners to recap what they have learned in this section. Spend time pointing out issues that you know are important as well as problems that you encountered from your own learners.

2. If learners want you to explain a concept again, do that now.

DIRECT INSTRUCTION

This lesson is made up of two fully worked examples from past papers covering all the concepts in this topic. As you work through these examples with the learners, talk about as many concepts as possible.
For example, use the words measures of central tendency, measures of dispersion, mean, median, mode, range, quartiles, percentiles, five-number summary and box and whisker plots.

Work through the two fully worked examples with learners. Learners should write them in full in their exercise books.

Example 1:	Teaching notes:
The data below shows the number of laptops sold by 15 sales agents during the last financial year. 43 48 62 52 46 90 58 37 48 73 84 68 54 34 78 a) Determine the median number of laptops sold.	a) Remind learners that to find the median, the data needs to be ordered.
b) Calculate the range of data.	b) Largest value – smallest value.
c) Calculate the interquartile range.	c) Find the upper quartile and lower quartile and subtract the lower quartile from the upper quartile.
d) Draw a box and whisker diagram for the data above. NSC NOV 2017	d) Use the five-number summary and remember to make sure the scale is accurate.

Solution:

Rearrange the data: 34 37 43 46 48 48 52 54 58 62 68 73 78 84 90

a) Median

$\frac{1}{2}(n+1)$

$= \frac{1}{2}(15+1)$

$= \frac{1}{2}(16)$

$= 8$

The median is in the 8^{th} position. $\therefore$ the median is 54

Note: Learners are welcome to count to find the median.

b) $90 - 34 = 56$

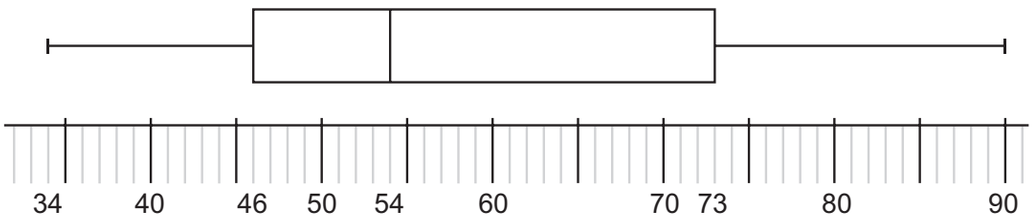
c) $Q_3 - Q_1$

$= 73 - 46$

$= 27$

d) Five-number summary:

34 46 54 73 90 ($90 - 34 = 56$. Suggested scale: 1cm = 5 units)



Example 2:	Teaching notes:
The table below shows information about the number of hours 120 learners spent on their cell phones in the last week.	

Number of hours (h)	Frequency
$0 < h \leq 2$	10
$2 < h \leq 4$	15
$4 < h \leq 6$	30
$6 < h \leq 8$	35
$8 < h \leq 10$	25
$10 < h \leq 12$	5

a) Identify the modal class for the data.

a)
Find the class that has the most number of values

b) Estimate the mean number of hours that these learners spent on their cell phones in the last week. NSC NOV 2015	b) Find the midpoint of the class intervals and multiply by the frequency. Find the total of the products and divide by the number in the data set. Remind learners why they are doing this.

Solution:

a) $6 < h \leq 8$

b)

Number of hours	Frequency	Midpoint of class intervals	Midpoint x frequency
$0 < h \leq 2$	10	1	10
$2 < h \leq 4$	15	3	45
$4 < h \leq 6$	30	5	150
$6 < h \leq 8$	35	7	245
$8 < h \leq 10$	25	9	225
$10 < h \leq 12$	5	11	55

Estimated mean

$$\frac{10 + 45 + 150 + 245 + 225 + 55}{120} = \frac{730}{120} = 6,08$$

$\therefore$ 6,08 hours

3. Ask directed questions so that you can ascertain learners' level of understanding.
4. Ask learners if they have any questions.
5. Give learners an exercise to complete on their own.
6. Walk around the classroom as learners do the exercise. Support learners where necessary.

Term 3, Topic 4: Topic Overview

TRIGONOMETRY

A. TOPIC OVERVIEW

A

- This topic is the fourth of six topics in Term 3.
- This topic will run for approximately one and a half weeks (6,5 hours).
- It is presented over three lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 6,5 hours). For example, one lesson in this topic could take three school lessons. Plan according to your school's timetable.
- Trigonometry counts 40% of the final Paper 2 examination.
- This is a very short topic. For this reason, 1 hour can be allocated for a test (with 30 minutes to go over the test once it has been marked) on the previous three topics and only 6,5 hours will be used on the teaching of this topic.
- One hour of the time gained has been allocated to Measurement due to the volume of work that is required.
- This topic leads directly into solving 2-dimensional problems in Grade 11 and 3-dimensional problems in Grade 12. In general, many learners struggle with these topics. Take the time to ensure learners feel confident in this topic.

Breakdown of topic into 3 lessons:

	Lesson title	**Suggested time (hours)**		**Lesson title**	**Suggested time (hours)**
1	Revision of trig ratios	1,5	3	Problems in 2 dimensions	3
2	Angles of elevation and depression	2			

B SEQUENTIAL TABLE

GRADE 8 & 9	GRADE 10	GRADE 11 & 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Right-angled triangles (with an understanding of where the hypotenuse is) • Trigonometric ratios (from earlier this year).	• Solving problems in 2 dimensions.	• Solving problems in 2 dimensions • Sine, cosine and area rules • Solving problems in 3 dimensions.

C WHAT THE NSC DIAGNOSTIC REPORTS TELL US

According to **NSC Diagnostic Reports** there are several issues pertaining to solving triangles in Trigonometry. However, few of them relate directly to the solving of right-angled triangles at Grade 10 level.

These include:

- applying the theorem of Pythagoras
- poor algebraic manipulation skills.

Ensure that learners have a deep understanding of solving problems in 2 dimensions. This will assist them in Grade 11 when they need to use the sine and cosine rules to solve 2-dimensional problems which in turn will assist them in Grade 12 when they will need to solve 3-dimensional problems.

D ASSESSMENT OF THE TOPIC

- CAPS formal assessment requirements for Term 3:
  - Two tests
- Two tests of 50 marks each are provided in Resources 27 and 29 in the Resource Pack. The tests are aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53). The tests are also divided in the ratio of the allocated teaching time. Memoranda for the tests are provided in Resources 28 and 30 in the Resource Pack.
- This topic is assessed in the 2^{nd} of the two tests.
- The questions usually take the form of being given a diagram or a question in words which links directly to a right-angled triabgle. Learners are usually required to find distance or the size of an angle.

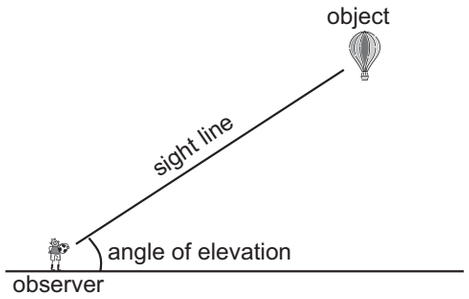
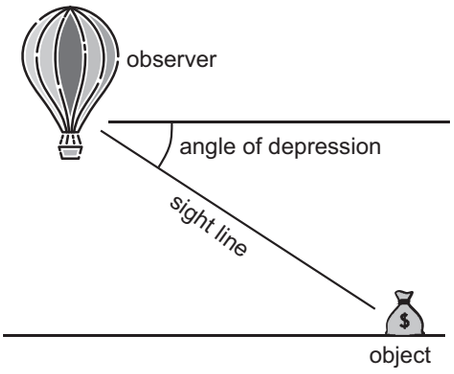
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

MATHEMATICAL VOCABULARY

E

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
right-angled triangle	A triangle with one angle of 90^o
opposite	In a triangle: the side opposite the angle of interest/the named angle
adjacent	In a triangle: the side next to the angle of interest/the named angle
hypotenuse	The side, in right-angled triangle, opposite the right angle (always the longest side)
alternate angles	Two angles, formed when a line (called the transversal) crosses two other lines. Alternate angles lie on opposite sides of the transversal line and on opposite relative sides of the other lines. If the two lines crossed are parallel, the alternate angles are equal
equidistant	The same distance
included angle	The angle between two sides. The angle formed by two sides
perpendicular	At a right angle
vertical	Upright. Perpendicular (at a right angle) to the ground
2-dimensional	Flat. Has two dimensions: usually length and breadth
3-dimensional	Not flat. Has three dimensions: usually length, height and width

angle of elevation	 The angle that a person must look UP to see an object. The angle is formed between the horizontal and the line of sight
angle of depression	 The angle that a person must look DOWN to see an object. The angle is formed between the horizontal and the line of sight

TERM 3, TOPIC 4, LESSON 1

REVISION OF TRIGONOMETRIC RATIOS

Suggested lesson duration: 1,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	28
Lesson Objectives By the end of the lesson, learners will have revised: ● trigonometric ratios.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. You will need Resource 14 from the Resource Pack.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson draw three right-angled triangles with θ in place. See the diagrams in point 1.

LEARNER PRACTICE

As this is revision of work done earlier in the year, you could re-do an exercise from the textbook that was done in Term 1. There is also a worksheet in the Resource Pack. The questions can be written on the chalkboard if you do not have photocopying facilities.

C

CONCEPTUAL DEVELOPMENT

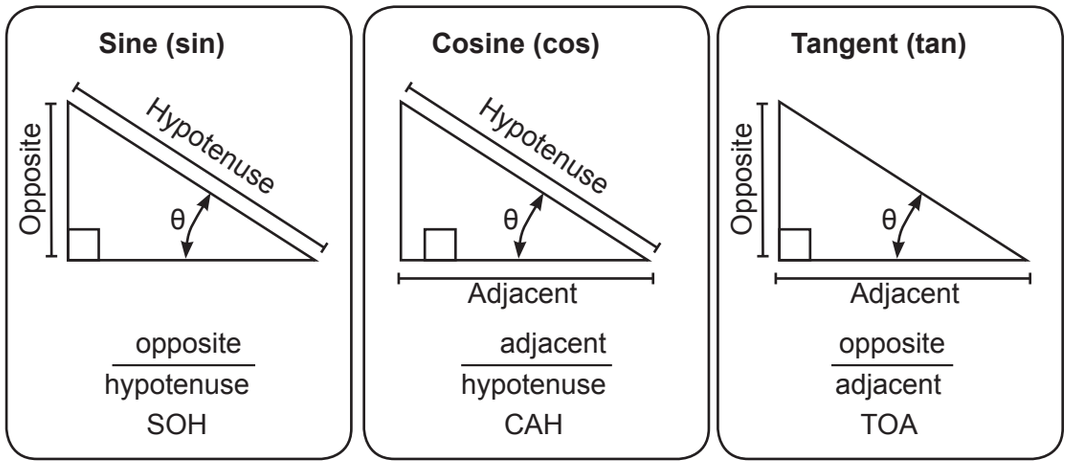
INTRODUCTION

1. Learners need to be proficient in the three main trigonometric ratios. This will help them in Grade 11 and 12.

2. Quiz the learners on a regular basis to establish their knowledge and to push them to improve where necessary.

DIRECT INSTRUCTION

1. Sketch the following on the chalk board. Ask learners to copy it into their books.



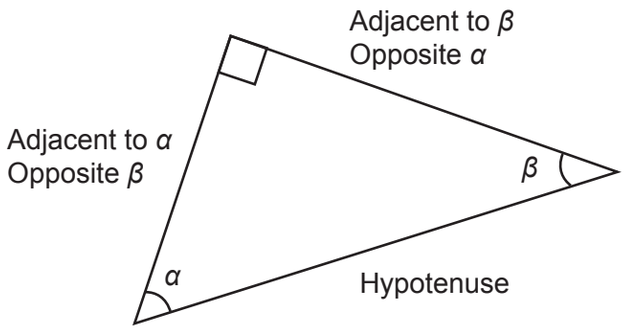
2. Discuss each one as you complete the labels on the diagrams.

3. The mnemonic at the bottom of the diagrams is to assist learners in remembering that:
   - **s**in is **o**pposite over **h**ypotenuse (SOH)
   - **c**os is **a**djacent over **h**ypotenuse (CAH)
   - **t**an is **o**pposite over **a**djacent (TOA)

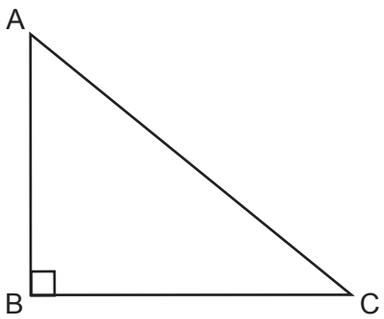
4. Stress the importance of knowing these ratios well.

5. The hypotenuse is always across from the right angle.

6. Remind learners that they always need to look at the angle that has been given a variable or an actual numerical size (the angle 'named') to decide where the opposite and adjacent is. Use this diagram to reiterate this:



7. Draw a right-angled triangle on the chalkboard and label it ABC.

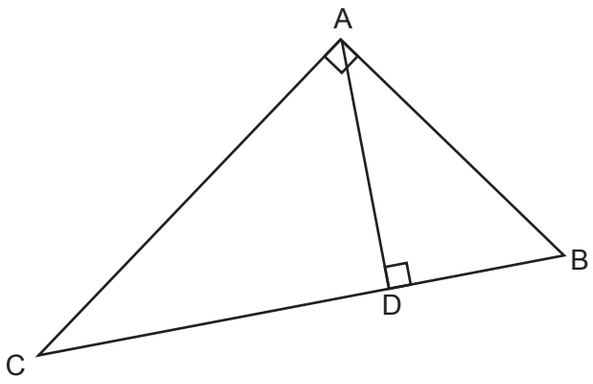


8. Use this triangle to quiz learners on the trig ratios. Ask different learners for different ratios. Each one can be used 3 or 4 times and aimed at different learners.

$sin\ C$	$\frac{AB}{AC}$	$sin\ A$	$\frac{BC}{AC}$
$cos\ C$	$\frac{BC}{AC}$	$cos\ A$	$\frac{AB}{AC}$
$tan\ C$	$\frac{AB}{BC}$	$tan\ A$	$\frac{BC}{AB}$

If learners struggled, draw another right-angled triangle labelled PQR and repeat the process before moving on to the more complex activity below.

9. Repeat the activity using the following diagram. Note that there are three right angled triangles.



Learners should note that $\hat{C}$, for example, is in two right-angled triangles (ΔACD and ΔABC). This means that when finding a ratio with the focus on $\hat{C}$, there will be two possible solutions.

	ΔACD	ΔABC		ΔABD	ΔABC
$\sin C$	$\frac{AD}{AC}$	$\frac{AB}{BC}$	$\sin B$	$\frac{AD}{AB}$	$\frac{AC}{BC}$
$\cos C$	$\frac{CD}{AC}$	$\frac{AC}{BC}$	$\cos B$	$\frac{BD}{AB}$	$\frac{AB}{BC}$
$\tan C$	$\frac{AD}{CD}$	$\frac{AB}{AC}$	$\tan B$	$\frac{AD}{BD}$	$\frac{AC}{AB}$

10. Ask the same questions more than once, getting as many learners to answer as possible.

11. If learners have coped well with the exercise, ask them similar questions about $C\hat{A}D$ and $B\hat{A}D$.

$\sin C\hat{A}D$	$\frac{CD}{AC}$	$\sin B\hat{A}D$	$\frac{BD}{AB}$
$\cos C\hat{A}D$	$\frac{AD}{AC}$	$\cos B\hat{A}D$	$\frac{AD}{AB}$
$\tan C\hat{A}D$	$\frac{CD}{AD}$	$\tan B\hat{A}D$	$\frac{BD}{AD}$

12. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

13. Give learners the worksheet to complete on their own.

14. Walk around the classroom as learners complete the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=PIWJo5uK3Fo
(SOHCAHTOA song)

https://www.youtube.com/watch?v=2cM0NBnWBMg

TERM 3, TOPIC 4, LESSON 2

ANGLE OF ELEVATION AND DEPRESSION

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	28

Lesson Objectives

By the end of the lesson, learners should be able to:

- explain what angles of elevation and depression are and represent it in a diagram.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan.
3. Write the lesson heading on the board before learners arrive: Angles of elevation and depression.
4. Ask learners prior to the lesson to ensure they have a long ruler at school.

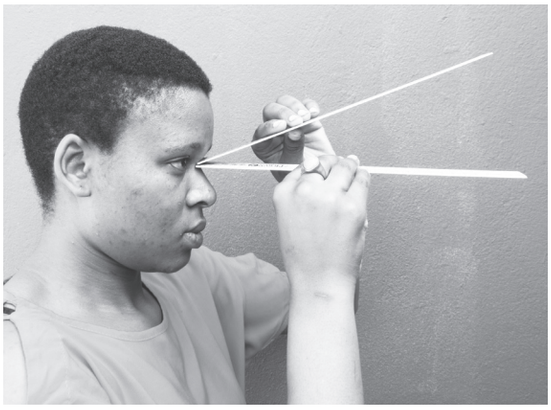
C CONCEPTUAL DEVELOPMENT

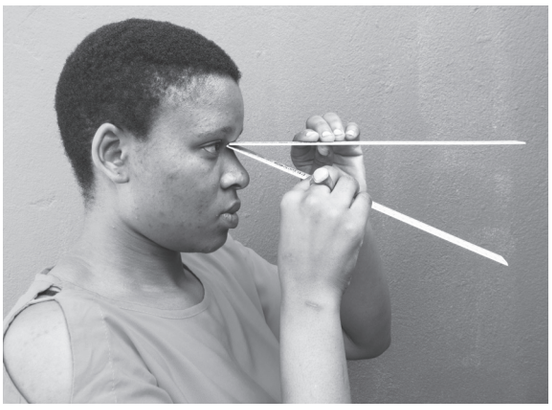
INTRODUCTION

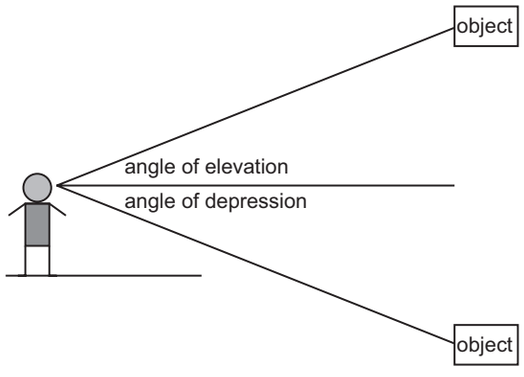
1. The aim of this lesson is to ensure that learners have a clear understanding of elevation and depression.
2. No formal exercises will be completed.
3. Assist learners with gaining an understanding of angles of elevation and depression. This includes time to take the learners outside. Allow learners to enjoy the opportunity of learning something new outside the classroom.

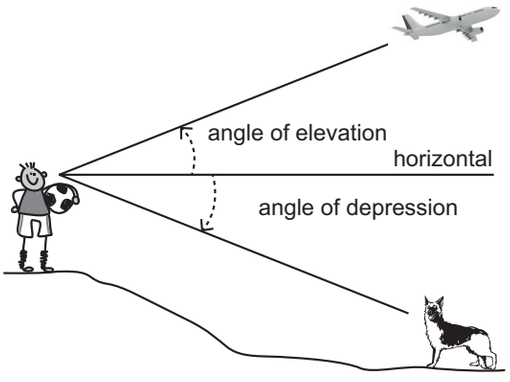
DIRECT INSTRUCTION

1. To assist the learners in experiencing angles of elevation and depression, it is ideal to take them outside.
2. First demonstrate the following in the classroom so all learners have had the opportunity to see easily before taking them outside to demonstrate again.
3. You will need two rulers.
4. Hold both rulers (on top of each other) horizontally from the bridge of your nose between the eyes, facing outwards. Ensure the rulers are parallel to the floor.
5. Keeping the rulers touching on your nose, slowly move the top ruler upwards.
6. The angle formed between the two rulers is an angle of elevation.
7. Explain to learners that the angle is formed when moving from the horizontal in an upwards direction. Tell the learners that you are looking UP at a mark on the ceiling.
8. Repeat, but this time move the bottom ruler downwards for the angle of depression.
9. Explain to learners that the angle is formed when moving from the horizontal in a downwards direction. Tell the learners that you are looking DOWN at a mark on the floor.









10. Take the learners outside. Ask learners to bring their rulers with them.

11. Start the lesson outside your own classroom:
    - If your classroom is on the ground floor demonstrate the angle of elevation first. Perhaps look up at a tree or if the building has more than one floor, look up at another classroom.
    - If your classroom is higher up, demonstrate the angle of depression first.

12. Once the first type of angle has been demonstrated, take the learners to a suitable place to demonstrate the second type of angle.

13. Demonstrate the angle of depression by asking learners for examples of what object or item they could be looking at on the ground (perhaps a dustbin or a goalpost).

14. Demonstrate the angle of elevation by asking learners for examples of what they could be looking at when looking up (the roof, a window).

15. In both instances allow an opportunity for learners to repeat the motion with the rulers that you have demonstrated. Learners can share rulers and take turns trying it themselves. As learners try it themselves, walk around and help them and answer questions.

16. Once learners have all had an opportunity to experience both angle of elevation and angle of depression, return to the classroom.

17. Ask directed questions so that you can ascertain learners' level of understanding.

18. Ask learners if they have any questions.

19. Tell learners to write the heading: Angles of elevation and depression in their books.
    Tell learners to:
    - draw diagrams that demonstrate their understanding of an angle of elevation and an angle of depression
    - write an explanation of the two in words
    - give an example of each.

20. Walk around the classroom as learners complete the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=bmU40g_mIwc

https://www.youtube.com/watch?v=Yxnzy5r2JMg
(Explanation of angle of elevation and depression)

TERM 3, TOPIC 4, LESSON 3

SOLVING PROBLEMS IN 2 DIMENSIONS

Suggested lesson duration: 3 hours

POLICY AND OUTCOMES

A

CAPS Page Number	28

Lesson Objectives

By the end of the lesson, learners should be able to:

- apply their knowledge of trigonometric ratios to solve problems in 2 dimensions.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive: Solving problems in 2 dimensions.
4. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
6 7 8 Rev (6-9)	91 92 95 112	2 Rev	261 265	8.7	101	15.2 15.4 15.5	347 352 354	11.1 11.2	396 398

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. This part of trigonometry is the basis of solving word problems as well as understanding 2-dimensional and 3-dimensional trigonometry in Grade 11 and 12.

2. The first two parts of the lesson focus on solving triangles already drawn. The second part of the lesson focuses on changing a word problem into a diagram and then solving the problem.

3. Use this information, as well as your own textbook/s, to plan carefully where you will give learners exercises to complete.

4. There is sufficient time allocated to ensure each learner can practice. Encourage learners to use their time wisely and aim at becoming experts!

DIRECT INSTRUCTION

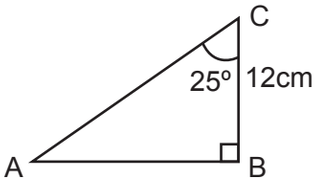
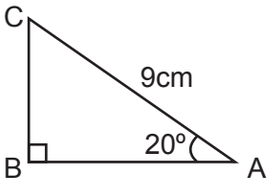
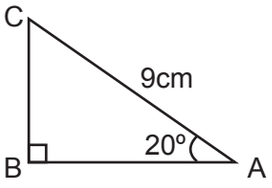
FINDING THE LENGTH OF A SIDE IN A RIGHT-ANGLED TRIANGLE

1. Start by reminding learners how important it is to look at a right-angled triangle and be able to decide which trig ratio will help you to find the length of a side.

2. Tell learners to write the following steps into their books:
   - According to the angle 'named', I have been given the … (opposite, adjacent/ hypotenuse)
   - According to the angle 'named', I am looking for the … (opposite, adjacent/hypotenuse)
   - Therefore, I will use …. (sine/cosine/tangent)

3. These three steps should be followed every time learners are required to solve a right-angled triangle.

4. Do the following examples with learners, referring to these steps in each one. Learners need to write the examples in their books.

	Thought process to choose correct trigonometric ratio	**Solution**
Find AB  	According to the angle 'named' (25^o), I have been given the <u>adjacent</u> According to the angle 'named' (25^o), I am looking for the <u>opposite</u> (AB) Therefore, I will use ***<u>tan</u>***	$\tan 25^o = \frac{opp}{adj}$ $\tan 25^o = \frac{AB}{12}$ $12 \tan 25^o = AB$ $\therefore AB = 5,6\text{ cm}$
Find AB  	According to the angle 'named' (20^o), I have been given the <u>hypotenuse</u> According to the angle 'named' (20^o), I am looking for the <u>adjacent</u> (AB) Therefore, I will use ***<u>cos</u>***	$\cos 20^o = \frac{adj}{hyp}$ $\cos 20^o = \frac{AB}{9}$ $9 \cos 20^o = AB$ $\therefore AB = 8,46\text{ cm}$
Find BC  	According to the angle 'named' (20^o), I have been given the <u>hypotenuse</u> According to the angle 'named' (20^o), I am looking for the <u>opposite</u> (BC) Therefore, I will use ***<u>sin</u>***	$\sin 20^o = \frac{opp}{hyp}$ $\sin 20^o = \frac{BC}{9}$ $9 \sin 20^o = BC$ $\therefore BC = 3,08\text{ cm}$

5. Ask directed questions so that you can ascertain learners' level of understanding.

6. Ask learners if they have any questions.

FINDING THE SIZE OF AN ANGLE IN A RIGHT-ANGLED TRIANGLE

1. Spend time reminding learners how to use their calculators to find the size of an angle.

> This will always involve using the second function key on the calculator.
> The following equation is used to show the order of buttons used on the calculator:
> $\sin x = 0.349$
> CALCULATOR: shift/2nd function; sin; 0,349
> The result will be the angle size.

2. Ask learners to copy the following three questions down then find the unknown angle:

$\sin\alpha = 0,125$	$\tan\beta = 1,317$	$\cos x = 0,841$
Solutions:		
$\therefore \alpha = 7,18^o$	$\therefore \beta = 52,79^o$	$x = 32,75^o$

3. This is a good time to remind learners why trigonometry works the way it does:

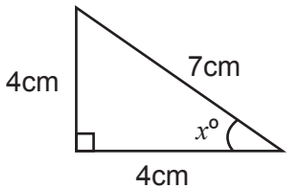
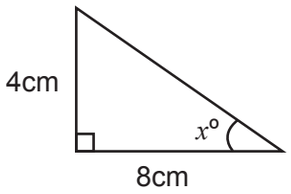
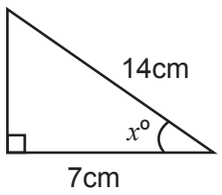
WHY DOES TRIGONOMETRY WORK LIKE IT DOES?

4. Remind learners why all these ratios in trigonometry actually work.

5. Ask: *What does the answer mean when you put sin 50^o into your calculator?*
   Praise learners who can tell you it is a ratio.
   Explain that it is the fraction/decimal (ratio) found when the length of the opposite side to the 50^o angle has been divided by the length of the hypotenuse side in any right-angled triangle.

6. Ask: *How does the calculator 'know' that the answer is always the same? Why doesn't the calculator need to know the size of the triangle?*
   Remind learners of similar triangles where all sides are always in proportion. Draw two different sized right-angled triangles on the board with one of the other angles marked as 50^o. Tell learners that you could even draw a right-angled triangle on the field outside and its sides would still be in proportion to the two you have drawn on the board.
   Explain that this is why the trigonometric ratios work – because similar triangles are always in proportion.

7. Do the following fully worked examples with learners now. They should take them down in their books.

Find the size of the unknown angle (x)	**Thought process to choose correct trigonometric ratio**	**Solution**
	According to the angle 'named' (x), I have been given the <u>hypotenuse</u> and the <u>opposite</u> Therefore, I will use <u>sin</u>	$\sin x = \frac{opp}{hyp}$ $\sin x = \frac{5}{7}$ $\therefore x = 45,58^o$ (shift ; sin ; $\left(\frac{5}{7}\right)$)
	According to the angle 'named' (x), I have been given the <u>adjacent</u> and the <u>opposite</u> Therefore, I will use <u>tan</u>	$\tan x = \frac{opp}{adj}$ $\tan x = \frac{4}{8}$ $\therefore x = 26,57^o$ (shift ; tan ; $\left(\frac{4}{8}\right)$)
	According to the angle 'named' (x), I have been given the <u>hypotenuse</u> and the <u>adjacent</u> Therefore, I will use <u>cos</u>	$\cos x = \frac{adj}{hyp}$ $\cos x = \frac{7}{14}$ $\therefore x = 60^o$ (shift ; cos ; $\left(\frac{7}{14}\right)$)

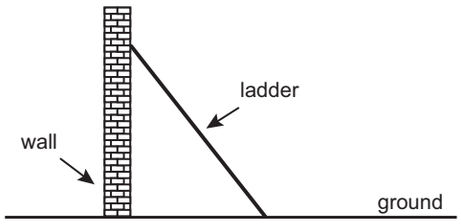
8. Ask directed questions so that you can ascertain learners' level of understanding.

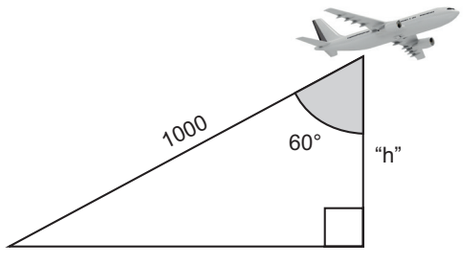
9. Ask learners if they have any questions.

SOLVING WORD PROBLEMS

1. Tell learners: *We are going to use the skills covered in the first part of the lesson to solve word problems. Each of these problems will lead to the possibility of representing the situation in a right-angled triangle.*

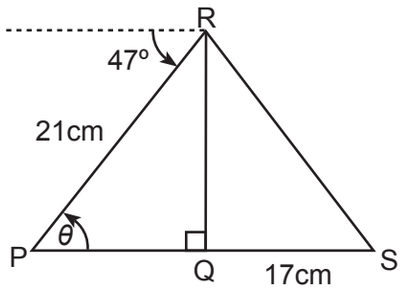
2. For example, there may be a ladder leaning against a wall or an airplane in the sky and questions can be asked about distance or the size of an angle.





3. Point out to learners that their knowledge of the angles of elevation and depression will be useful now.

4. Do the following examples now with learners. They should write them in their books.

Example	Teaching notes
RQ is a vertical pole. The foot of the pole, Q, is on the same horizontal plane as P and S. The pole is anchored with wire cables, RS and RP. The angle of depression from the top of the pole top point P is 47^o. PR is 21m and QS is 17m. *$R\hat{P}Q=\theta$.*  a) Write down the size of θ. b) Calculate the length of RQ. c) Hence, calculate the size of $\hat{S}$. d) If P, Q and S lie in a straight line, how far apart are the anchors of the wire cables? NSC NOV 2017	It is important to look at each piece of information carefully and to link the information back to the diagram. RQ is a vertical pole – point the pole out to learners noting that there is a right angle at ground level. If the right angle had not been filled in, learners must know that a pole will stand at a aright angle to the ground and should mark the angle as a right angle themselves. Same horizontal plane means they are on the same level which is the ground or the floor. Read about the wire cables and point them out. Some learners may want to label them. Discuss the angle of depression – imagine a cat is sitting at the top of the pole looking down. Ensure all measurements mentioned in the question are marked on the diagram.
Solutions: a) $\theta = 47^o$	Angle of depression is formed from the horizontal which will in turn be parallel to the ground. Therefore, alternate angles are equal. Tell learners to fill the answer in on the diagram.

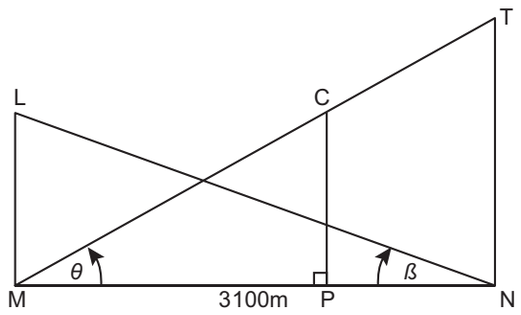
b) $\sin 47^o = \frac{QR}{21}$ $21 \sin 47^o = QR$ $\therefore QR = 15,36\,\text{m}$	Although QR lies in two right-angled triangles, only one of these (ΔPQR) has sufficient information to find the length of QR According to the angle named $(\theta/47^o)$, we are looking for the opposite and are given the hypotenuse. This requires us to use the trig ratio, sine. Tell learners to fill the answer in on the diagram.
c) $\tan \hat{S} = \frac{QR}{17}$ $\tan \hat{S} = \frac{15,36}{17}$ $\therefore \hat{S} = 42,1^o$	$\hat{S}$ lies in the right-angled triangle QRS. According to the angle required, we have the opposite and adjacent. This requires us to use the trig ratio, tan. Tell learners to fill the answer in on the diagram.
d) $\cos 47^o = \frac{PQ}{21}$ $21 \cos 47^0 = PQ$ $PQ = 14,32\,\text{m}$ $\therefore PS = 14,32\,\text{m} + 17\,\text{m} = 31,32\,\text{m}$	To calculate this, we require the length of PQ. According to the angle named $(\theta/47^o)$, we are looking for the adjacent and are given the hypotenuse. This requires us to use the trig ratio, cosine. Point out that learners could also use sine by using the 3^{rd} angle in the triangle OR they could use the length of QR but using an answer that has been calculated during the question when a measurement given can just as easily be used is not ideal in case an error was made in the calculation. The theorem of Pythagoras could also have been used.

The diagram below represents a cross-section of the peaks of Table Mountain, T, and Lions Head, L, above sea level. Points M and N are directly below peaks L and T respectively, such that MPN lies on the same horizontal plane at sea level and P is directly below C.

MN = 3 100 m

The angle of elevation of L from N is β and the angle of elevation of T from M is θ.

It is given that $tan\ \theta=0,35$ and $tan\ \beta=0,21$.



a) Calculate the ratio of LM:TN

b) A cable car, C, travelling from the top of Table Mountain, T, follows a path along TCM.

   (i) Calculate the angle formed ($M\hat{T}N$) between the cable and the vertical height TN.

   (ii) If the cable car, C, travels along the cable such that *TC*= 400m, calculate the height of the cable car above sea level at that instant.

It is important to look at each piece of information carefully and to link it back to the diagram.

Point out the top of Table Mountain and Lions Head.

Remind learners what the 'same horizontal plane' means.

If P is directly below C, this means there is a right angle formed with the horizontal (ground).

Discuss the angles of elevation given.

Tell learners to imagine someone is at N looking up at L and someone is at M looking up at T.

Discuss what is being given if $tan\ \theta=0,35$ and $tan\ \beta=0,21$.

(the ratio of $\frac{TN}{MN}$ is 0,35 and the ratio of $\frac{LM}{MN}$ is 0,21)

Ensure all measurements mentioned in the question are marked on the diagram.

Solutions:	
a) $\tan\beta = \frac{LM}{MN}$ but $\tan\beta = 0,21$ $\therefore \frac{LM}{MN} = 0,21$ $\frac{LM}{3100} = 0,21$ $LM = (0,21)(3100)$ $LM = 651\text{m}$ $\tan\theta = \frac{TN}{MN}$ but $\tan\theta = 0,35$ $\therefore \frac{TN}{MN} = 0,35$ $\frac{TN}{3100} = 0,35$ $TN = (0,35)(3100)$ $TN = 1085\text{m}$ $\therefore LN:TN = 651:1085$ $= 3:5$	a) To find a ratio of two lengths, the lengths are required. Note that there is not enough information in the triangles for this BUT remind learners that the ratios for the angles in the triangles were given.

b) (i) $tan\theta = 0,35$ $\therefore \theta = 19,29^o$ $\therefore$ MTN $= 70,71^o$ (ii) In ΔMNT: $\cos\theta = \frac{MN}{MT}$ $\cos 19,29^o = \frac{3100}{MT}$ $\therefore MT = 3284,39$ m $\therefore CM = 3284,39$ m $- 400$ m $CM = 2884,39$ m In ΔCPM: $\sin\theta = \frac{CP}{CM}$ $\sin 19,29^o = \frac{CP}{2884,39}$ $\therefore CP = 952,86$ m	b) (i) This could be found either by using the given information $tan\ \theta$=0,35, finding θ, then using the angles of a triangle to find $M\hat{T}N$ or since TN and MN are both known, use these to find $M\hat{T}N$ directly. ($\tan T = \frac{3100}{1085}$) Tell learners to fill these two angle sizes onto the diagram. (ii) Tell learners to fill the 400m measurement onto their diagram. Ask: *What needs to be found according to the diagram?* (CP). Ask: *Is it in a right-angled triangle?* (Yes - ΔCPM). Ask: *Is there enough information in this triangle to answer the question?* (No). This means we will have to look further to find measurements so that we can work in this triangle. Show learners that if we work in ΔMNT and find MT, we could subtract 400m from MT to find CM which is in the triangle we need to work in.

5. Ask directed questions so that you can ascertain learners' level of understanding.
6. Ask learners if they have any questions.
7. Give learners an exercise to complete on their own.
8. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=3umBUlrUCPQ

https://www.youtube.com/watch?v=e_QPK-pOPqY
(Although these videos do not use metric measurements the explanation is still useful)

Term 3, Topic 5: Topic Overview

EUCLIDEAN GEOMETRY

A. TOPIC OVERVIEW

A

- This topic is the 5^{th} of six topics in Term 3.
- This topic runs for one week (4,5 hours).
- It is presented over 1 lesson of 1,5 hours – the rest of the time will be used to practise as many riders as possible.
- Euclidean Geometry and Measurement counts 30% of the final Paper 2 examination.
- When Euclidean Geometry was covered in Term 2, riders were already covered. This week will be used for learners to become more confident in their ability to solve geometry riders.
- If there is one part of mathematics in which the mind is free to use all its resources, it is rider-work in Geometry. The successful solver must be able to explore his figure with an observant and roving eye, must be able to imagine how it may be varied, must spot relationships, must jump instinctively for the right construction, must argue backwards as well as forwards, before finally arranging his ideas in logical sequence. Rider solving is an art. (The Mathematical Gazette Vol 20, No 238, May 1936, pp 93-109)

Breakdown of topic into 1 lesson:

	Lesson title	**Suggested time (hours)**
1	Solving geometry riders	4.5

B SEQUENTIAL TABLE

GRADE 8 & 9	GRADE 10	GRADE 11 & 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Classifying 2D shapes • Geometry of straight lines • Geometry of 2D shapes • Theorem of Pythagoras • Similarity and Congruency From Term 2 this year: • Investigate line segments joining the mid-points of two sides of a triangle • Properties of quadrilaterals.	• Solve problems and prove riders using the properties of parallel lines, triangles and quadrilaterals.	• Circle Geometry • Proportionality theorems • Similar triangles • Theorem of Pythagoras (proof).

C WHAT THE NSC DIAGNOSTIC REPORTS TELL US

According to **NSC Diagnostic Reports** there are several issues pertaining to Euclidean Geometry.

These include:

- giving incorrect or incomplete reasons
- naming angles incorrectly
- making many irrelevant statements.

It is important to keep these issues in mind when teaching this section.

While teaching Euclidean Geometry, it is important to remind learners that this section requires logical reasoning. Learners need to be encouraged to scrutinise the given information for clues and then plan a way forward.

ASSESSMENT OF THE TOPIC

D

- CAPS formal assessment requirements for Term 3:
  - Two tests
- Two tests of 50 marks each are provided in Resources 27 and 29 in the Resource Pack. The tests are aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53). The tests are also divided in the ratio of the allocated teaching time. Memoranda for the tests are provided in Resources 28 and 30 in the Resource Pack.
- The questions usually take the form of given information and diagrams related to one or more of the theorems learned.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

MATHEMATICAL VOCABULARY

E

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
Euclidean Geometry	Geometry based on the postulates of Euclid. Euclidean geometry deals with space and shape using a system of logical deductions
theorem	A statement that has been proved based on previously established statements
converse	A statement formed by interchanging what is given in a theorem and what is to be proved
Theorem of Pythagoras	In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides
rider	A problem of more than usual difficulty added to another on an examination paper.
hypotenuse	The longest side in a right-angled triangle. It is opposite the right angle

complementary angles	Angles that add up to 90^o
supplementary angles	Angles that add up to 180^o
vertically opposite angles	Non-adjacent opposite angles formed by intersecting lines
intersecting lines	Lines that cross each other
perpendicular lines	Lines that intersect each other at a right angle
parallel lines	Lines the same distance apart at all points. Two or more lines are parallel if they have the same slope (gradient)
transversal	A line that cuts across a set of lines (usually parallel)
corresponding angles	Angles that sit in the same position on each of the parallel lines in the position where the transversal crosses each line
alternate angles	Angles that lie on different parallel lines and on opposite sides of the transversal
co-interior angles	Angles that lie on different parallel lines and on the same side of the transversal
polygon	A closed 2D shape in which all the sides are made up of line segments. A polygon is given a name depending on the number of sides it has. A circle is not a polygon as although it is a closed 2D shape it is not made up of line segments
quadrilateral	A 4-sided closed shape (polygon)
parallelogram	A quadrilateral with two pairs of parallel sides
square	A four-sided polygon with all four sides equal in length and all four angles are right angles
rectangle	A four-sided polygon with both pairs of opposite sides equal in length and all four angles are right angles

rhombus	A quadrilateral with two pairs of parallel sides and all four sides equal length
trapezium	A quadrilateral with one pair of parallel sides
kite	A quadrilateral with two pairs of adjacent sides equal
diagonal	A straight line joining two opposite vertices (corners) of a straight sided shape. It goes from one corner to another but is not an edge
triangle	A three-sided polygon
equilateral triangle	A triangle with all three sides and all three angles equal
isosceles triangle	A triangle with two equal sides and two equal angles
scalene triangle	A triangle with no equal sides
right-angled triangle	A triangle with one right angle
acute-angled triangle	A triangle with three acute angles
obtuse-angled triangle	A triangle with one obtuse angle
congruent	The same. Identical. Equal sides and equal angles
similar	Looks the same. Equal angles and sides in proportion

TERM 3, TOPIC 5, LESSON 1

SOLVING GEOMETRY RIDERS

Suggested lesson duration: 4,5 hours

A POLICY AND OUTCOMES

CAPS Page Number	28

Lesson Objectives

By the end of the lesson, learners should:

- feel confident in approaching a geometry rider
- have practiced many examples of geometry riders.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev	205	1	248	10.5*	178*	14.1	331	12.1	405
Some	206	2	249			14.2	335	12.2	409
Ch		Rev	251			14.3	337		
						14.4	338		
						14.5	341		

* This exercise was done in Term 2 but could be redone now if no other exercise is available

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. In the research paper, 'Levels of thought in geometry of pre-service mathematics educators according to the van Hiele model' by Sonja van Putten, the following observation of how FET learners deal with geometry is noted: 'Learn the theorems really well (off by heart); leave the riders until the rest of the exam has been completed; try to do them if time allows.'
   This is unfortunate but a reality. It is important that teachers assist their learners to develop strategies that will help them overcome this fear of riders (and geometry in general) and perhaps even get to the point that riders become enjoyable.
2. This lesson includes recommendations on how to help learners to improve their ability to solve a geometry rider.
3. There are also several worked examples to do with learners.
4. Plan your use of allocated time with care and according to your own class' ability. You may want to do one or two examples with the learners, then give them the opportunity to practice on their own before doing another example. Use the examples in your own textbook if more are required.
5. Whenever learners are working on their own, make a concerted effort to assist them by asking directed questions, rather than just giving them the solution.

DIRECT INSTRUCTION

1. Start the lesson by asking learners how to prove that a quadrilateral is a parallelogram. Remind learners that proving statements is a skill necessary in both Analytical Geometry and Euclidean Geometry.
2. Tell learners to write the heading 'How to prove a quadrilateral is a parallelogram' in their books. Ask: *Name any of the five possibilities for proving that a quadrilateral is a parallelogram.* If all five possibilities are not given by learners, top up the list and ensure that learners write all five possibilities in their exercise books.

Possibilities for proving that a quadrilateral is a parallelogram:
- Both pairs of opposite sides are equal
- Both pairs of opposite sides are parallel
- One pair of opposite sides are equal and parallel
- Diagonals bisect each other
- Opposite angles are equal

3. Remind learners that they do not need to prove all five. If any one of the above points are true, then the quadrilateral is a parallelogram.

4. Logical and deductive reasoning is a skill required when solving riders. Choose two learners to demonstrate the basics of this to the class. Here the learners are named A and B.
   The teacher says: *I live in the same street as A. I live in the same street as B.*
   What other piece of information can you tell me that I have not told you?'
   (That A and B live in the same street).
   Tell learners that this type of connection is regularly used in geometry.

5. Write the following on the chalkboard:
   $\hat{P} = \hat{Q}$ *and* $\hat{R} = \hat{Q}$; Ask learners: *What can you conclude? ($\hat{P} = \hat{R}$)*

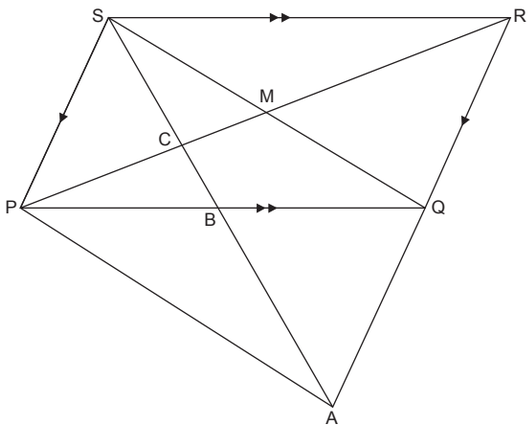
6. Tell learners: *Making connections is an important part of dealing with more complex questions in geometry.*

7. Learners should complete the following exercise with a partner. Encourage them to discuss their thoughts. There is a copy of the questions and a memorandum in the Resource Pack. If you do not have access to photocopying facilities, write the questions on the chalkboard for to copy into their exercise books.

A = B and B = C ∴ _____	A + B = 180º and _____ ∴ B = C
A = B + C and B = C ∴ _____	A = B + C and A = P + C ∴ _____
P = Q + R and _____ ∴ Q = S	C = F and _____ ∴ C = D
$A + B = 180^0$ and C = A and D = B ∴ _____	$C + D = 90^0$ and _____ $\therefore A + D = 90^0$
$A + C = 180^0$ and _____ ∴ A = B	A = B + C B = Q + R and C = Q ∴ _____

8. Discuss answers with the learners. Listen to their explanations on why they thought an answer was correct when it wasn't. Talk learners through any of the questions that they found difficult. The last one was particularly difficult, and a full explanation is given in the memorandum in the Resource Pack.

9. Do the following worked example with the learners. Tell them to look out for any of the types of deductive reasoning that they used in the exercise.

Example:	Teaching notes:
In the diagram below, *PQRS* is a parallelogram having diagonals PQ and RS intersecting at M. B is a point on PQ such that SBA and RQA are straight lines and *SB* = *BA*. SA cuts PR in C and PA is drawn.  a) Prove that SP = QA b) Prove that SPAQ is a parallelogram. c) Prove that AR = 4MB NSC NOV 2015	Point out to learners that if they are told a quadrilateral is a parallelogram that they are given a large amount of information. Ask learners to tell you what they know about *PQRS*. Recommend marking some of this information on the diagram. *(SR=PQ; PS=QR; PM=MR; SM=MQ)* (sometimes the diagram doesn't lend itself to marking all the information – for example, opposite angles equal in this case – the diagonals limit the marking of equal angles). Learners should confirm that all the information in the question is marked on the diagram. If not, they should mark it. Remind learners that information will not be given if it isn't useful. (*SB=BA* was not marked on the diagram) Tell learners to read through the questions and consider each one while looking at the diagram. Give learners the opportunity to talk through their ideas. If they are stuck, remind them to consider this year's work and be on the lookout for using those theorems.

Solutions: a) In ΔSAR: $SB=BA$ (given) $BQ//SR$ (given (opp sides parm)) $\therefore AQ=QR$ (line through midpoint // to 2^{nd} side) but, $SP=QR$ (opp sides parm) $\therefore SP=QA$	a) Tell learners to note ΔSAR. Note the parallel lines and the equal lengths. (Some learners may want to turn the page to see the triangle in its more 'common' format – this may help them to 'see' the midpoint theorem diagram more easily).
b) $SP=QA$ (proved above) $SP//QA$ (opp sides of parm) $\therefore SPAQ$ is a parm (one pair of opp sides equal and //)	b) Ask: *Without referring to your list, name the five possible ways of proving that a quadrilateral is a parallelogram.* Ask: *Considering the information we already have, which one would make most sense in this situation?* (One pair of opposite sides equal and parallel) Point out how the information proved in a) is useful to answer b).
c) In ΔPQR M is the midpoint of PR and B is the midpoint of PQ (diags of parm bisect) $\therefore MB = \frac{1}{2}QR$ (midpoint theorem) but $QR=QA$ $\therefore QR = \frac{1}{2}AR$ $\therefore MB = \frac{1}{2}(\frac{1}{2}AR)$ $MB = \frac{1}{4}AR$ $\therefore 4MB = AR$	c) The fact that MB is not drawn in can confuse learners. Ask them to draw it in and ask if they see any connections to MB and AR. (MB and QR form part of a triangle that could involve the midpoint theorem; $QR = AQ$ and $QR + AQ = AR$ so that could be a link to follow). Encourage learners to suggest ways to prove this.

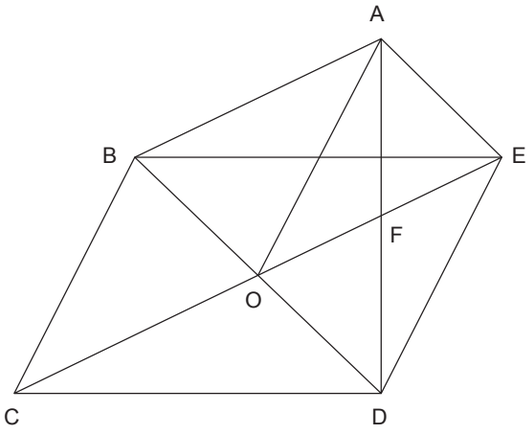
10. Ask learners to list what they needed to know to solve this rider.
    - Midpoint theorem and its converse
    - How to prove a quadrilateral is a parallelogram
    - Deductive reasoning
    - Diagonals bisect each other in a parallelogram.

11. Key issue: An important aspect when solving a geometry problem – learners must know their theory.

12. Another key issue: Remember that information is not given if it is not required. Tell learners that when they are stuck they should go back to the question and go through each piece of information again carefully. For each piece of information, they should ask themselves: *Could this help me?*

13. Tell learners that you will now do another example and they should keep these points in mind. They should also be on the lookout for the new theory that they have learned this year. Although geometry always uses theory from previous years, learners should always expect this year's work to be assessed.

14. Recommendation: Use colour to mark sides or angles equal. If learners have highlighters or crayons they should make use of them. It is a good visual aid.

Example:	Teaching notes:
In the diagram, BCDE and AODE are parallelograms.  a) Prove that OF//AB. b) Prove that ABOE is a parallelogram. c) Prove that $\Delta ABO \equiv \Delta EOD$ Exemplar 2012	Ask: *What have you been given?* (2 parallelograms) Ask: *What can be deduced from this information?* (Opposite sides equal, opposite sides parallel, diagonals bisect each other). Tell learners to mark these on the diagram. Recommendation: As there are two parallelograms shade each one a different colour to help them stand out.

Solutions: a) In ΔABD, O is the midpoint of BD and F is the midpoint of AD (diags of parm bisect) $\therefore OF // AB$ (midpoint theorem)	Learners should use their finger to draw over OF and AB and ask if they notice anything. (Hopefully they will see that it is part of ΔABD and looks like it could be linked to the midpoint theorem). Ask: Are *F* and *O* midpoints of the respective lines they lie on?' (Yes – diagonals of a parallelogram bisect each other – these should have been marked equal in the beginning).
b) OF//AB (proved above) $\therefore$ OE//AB AE//OD (opp sides parm) $\therefore$ AE//BO $\therefore$ ABOE is a parm (both pairs opp sides parallel)	Say: *Without referring to your list, name the five possible ways of proving that a quadrilateral is a parallelogram.* Ask: *Considering the information we already have, which one would make most sense in this situation?* (Both pairs of opposite sides parallel). Point out to learners how the information proved in a) is useful to answer b)).
c) In ΔABO and ΔEOD: $AB=EO$ (opp sides of parm ABOE) $BO=OD$ (diags of parm BCDE) $AO=ED$ (opp sides of parm AODE) $\therefore \Delta ABO \equiv \Delta EOD$ (SSS)	Point out to learners how geometry from previous grades can be asked. Ask: *What are the four conditions of congruency?* (SSS, SAS, AAS, RHS) Tell learners to shade the two triangles mentioned. Ask: *Which condition do you think you should use?* (SSS)

15. One final tip for when learners are writing a formal assessment is to look at the mark allocation. In general:
    - 1 or 2 marks would imply it is very straightforward and should be fairly obvious
    - 3 or 4 marks implies it may take a step or two to get to the answer (and there is a good chance there is one link of deductive reasoning)
    - 4 or more marks can imply that there is more effort required and multiple steps are probably required to arrive at the solution.

    The total of 4 marks was used twice intentionally – it could belong in any category. Emphasise that this is a guide and not a rule.

16. Ask directed questions so that you can ascertain learners' level of understanding.

17. Ask learners if they have any questions.

18. Give learners an exercise to complete. Early in the week it may be a good idea to let them work with a partner to encourage discussion of ideas. Towards the end of the week they should be working on their own.

19. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=2az9sa0j_RE
(Geometry proofs riders)

https://www.youtube.com/watch?v=pM_Lf1kHQho
https://www.youtube.com/watch?v=6bVN0DeEO2w
(Five tips to solving geometry proofs – two parts)

http://dipmat.math.unipa.it/~grim/EMogari9.PDF
(Research paper, 'Attitude and Achievement in Euclidean Geometry' by David Mogari – an excellent read)

Term 3, Topic 6: Topic Overview

MEASUREMENT

A

A. TOPIC OVERVIEW

- This topic is the sixth of six topics in Term 3.
- This topic will run for approximately one and a half weeks (5,5 hours).
- It is presented over three lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 5,5 hours). For example, one lesson in this topic could take two school lessons. Plan according to your school's timetable.
- Euclidean Geometry and Measurement counts 30% of the final Paper 2 examination.
- Although Measurement does not form a large part of the final assessment, do not allow learners to deem it unimportant. An understanding of both surface area and volume is essential in our daily lives.

Breakdown of topic into 3 lessons:

	Lesson title	**Suggested time (hours)**		**Lesson title**	**Suggested time (hours)**
1	Revision	2	3	Volume and surface areas of spheres, right pyramids and cones	2,5
2	The effect on volume and surface area when multiplying any dimension by a constant factor k.	1			

SEQUENTIAL TABLE

B

GRADE 8 & 9	GRADE 10	GRADE 11 & 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
● Solve problems involving volume and surface area of right prisms, spheres, pyramids and cones and combinations of those objects. ● Solve problems using appropriate formulae and conversions between SI units. ● Calculate the surface area, volume and capacity of cubes, rectangular prisms, triangular prisms and cylinders. ● Investigate how doubling any or all the dimensions of right prisms and cylinders affects their volume.	● Revise the volume and surface areas of right-prisms and cylinders. ● Study the effect on volume and surface area when multiplying any dimension by a constant factor *k*. ● Calculate the volume and surface areas of spheres, right pyramids and cones.	Measurement is not a stand-alone topic in Grade 11 & 12. The skills acquired up to Grade 10 will be used to: ● Solve problems involving volume and surface area of solids. These problems are often linked to Trigonometry.

WHAT THE NSC DIAGNOSTIC REPORTS TELL US

C

According to **NSC Diagnostic Reports** there are few issues pertaining directly to Measurement as it is incorporated into other areas in Grade 11 and 12.

It is important that learners are given the opportunity to use manipulatives where possible. This can assist them in their understanding of the topic and of the formulae used.

D ASSESSMENT OF THE TOPIC

- CAPS formal assessment requirements for Term 3:
  - Two Tests
- Two tests of 50 marks each are provided in Resources 27 and 29 in the Resource Pack. The tests are aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53). The tests are also divided in the ratio of the allocated teaching time. Memoranda for the tests are provided in Resources 28 and 30 in the Resource Pack.
- The questions usually take the form of diagrams of 3-dimensional shapes related to a word problem and learners need to find surface area and/or volume of the solids.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

E MATHEMATICAL VOCABULARY

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
2D	2-dimensional
3D	3-dimensional
polygon	A 2D shape in which all the sides are made up of line segments. A polygon is given a name depending on the number of sides it has. For example: A 5-sided polygon is called a pentagon
solid	An object that occupies space (3-dimensional)
prism	A solid object with two identical ends and flat sides. The shape of the ends gives the prism a name, example: triangular prism ● The cross section is the same all along its length ● The sides are parallelograms

right prism	A right prism is a geometric solid that has a polygon as its base and vertical sides perpendicular to the base. The base and top surface are the same shape and size. It is called a “right” prism because the angles between the base and sides are right angles
face	A flat surface of a prism
cube	A solid with six equal square faces
rectangular prism	A solid with six rectangular faces
triangular prism	A solid with two equal triangular faces (one is the base) and three rectangular faces
cylinder	A solid with two equal circular faces (one is the base) and one rectangle (curved) faces
sphere	A round solid figure with every point on its surface equidistant from its centre
cone	A solid or hollow object which tapers from a circular base to a point
pyramid	A structure with a square or triangular base and sloping sides formed by triangles that meet in a point at the top
net	A 2D shape that, when folded, forms a 3D object
surface area	The area taken up by the net of a 3D solid. The sum of the area of all the faces
volume	The space taken up by a 3D object. To find volume, the area of the base is multiplied by the perpendicular height. This only works for right prisms
capacity	The amount of liquid a 3D object can hold. Capacity is directly linked to volume

TERM 3, TOPIC 6, LESSON 1

REVISION

Suggested lesson duration: 2 hours

A POLICY AND OUTCOMES

CAPS Page Number	28

Lesson Objectives

By the end of the lesson, learners will have revised:

- volume and surface area of right prisms and pyramids.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. You will need Resource 17 from the Resource Pack.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson sketch the cube and rectangular prism (point 8).
6. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1 (1) 2 (1 & 2)	279 283	1 2	269 270	14.1	237	16.1 (1 a-e) 16.2 (1 a &,b)	360 367	13.2 13.3	418 425

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. This lesson has two parts: Surface Area and Volume.

2. Ensure the learners understand the concepts. Use manipulatives where possible. Ask learners to bring boxes and an empty can from home. Ask learners in the week or two leading up to this topic to ensure you have several different prisms and cylinders.

3. There are pictures of four 3D objects in the Resource Pack. Use these if manipulatives are not available.

DIRECT INSTRUCTION

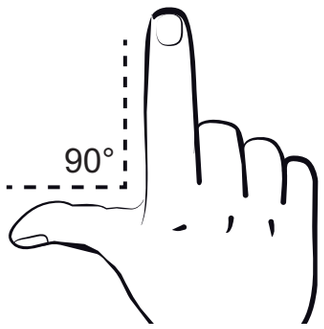
PART A: SURFACE AREA

1. Hold up a box (a rectangular prism). Ask: *What is the surface area of a 3D object?* (The area taken up by the net of the box).

2. Ask: *What 2D shapes make up this box?* (6 rectangles – unless there is a square base – accept answers according to the box you are holding).

3. Show learners that in a rectangular prism, the rectangles are always paired:
   - the front and back rectangles are the same size
   - the top and bottom rectangles are the same size
   - the two rectangles on the sides are the same size.

4. Remind learners why this is a right prism. The base is always at a right angle to the side. Show this by holding your thumb underneath the base and showing the right angle formed with your index finger going up the height.



If you have a cube and a triangular prism available, repeat the process with each of these right prisms.

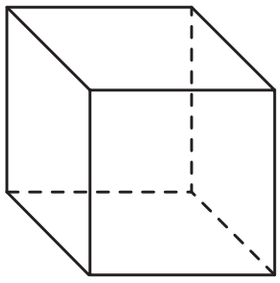
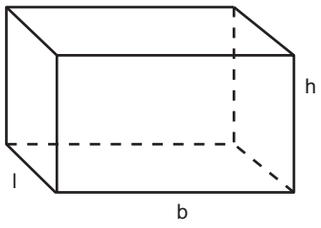
A cylinder could also be shown to have a side at a right angle to its base but by definition, a cylinder is not a prism. The term right cylinder can be used.

5. Another important concept: the base is generally the shape that gives the prism its name. The prism does not always stand on its base (imagine a triangular prism lying on one of the rectangles – this does not take away the fact that a triangle is the base of the triangular prism). There are two identical bases parallel to each other in any right prism. Note that a cube is a square prism.

6. Discuss how to find surface area of each of the four main solids, namely, the cube, rectangular prism, triangular prism and cylinder. Keep these explanations general, not linking them to a formula.

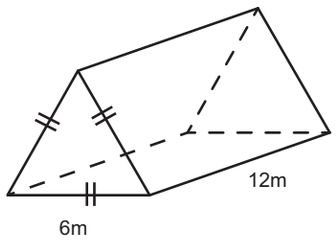
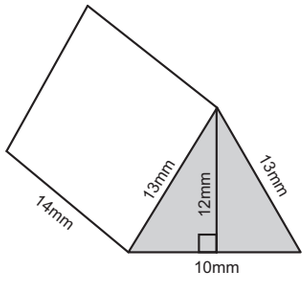
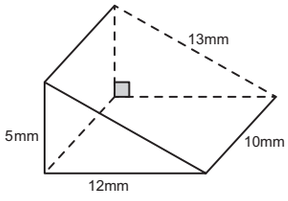
7. Cube: Find the area of six squares and add them together to give total surface area.
   Rectangular prism: Find the area of six rectangles and add them together to give total surface area.
   Triangular prism: Find the area of the three rectangles and two triangles and add them together to give total surface area.
   Cylinder: Find the area of two circles and the curved part and add them together to give total surface area.

8. Discuss the formula for finding the surface area of a cube and a rectangular prism. The cube and the rectangular prism are standard and do not usually cause any confusion. Ask learners to copy the two formulae, with the sketch and name of the shape, into their books.

Cube		Surface Area = $6l^2$ or $6s^2$ (where l or s represent the length of the sides)
Rectangular prism		Surface Area = $2lb + 2lh + 2bh$ (where l, b or h represent the length, breadth and height)

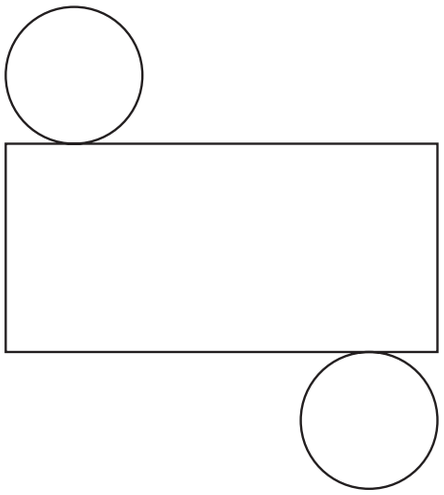
9. Remind learners that although the formula to find the surface area of a rectangular prism can be factorised $(2(lb + lh + bh))$, it is better to think of it as given above so it is easy to leave one rectangle out when calculating the surface area of a box without a lid.

10. Discuss the triangular prism and the issues that may arise from finding the surface area. Learners should write the information into their books as you go through all three possible types.
    First ask learners what a triangular prism is made up of (as asked previously) – 3 rectangles and 2 triangles.

Equilateral triangle as a base		Tell learners that this is the easiest triangular prism to find the surface area of: because the triangle has three equal sides, the rectangles will all be equal in size. The rectangles all have the same length (which is always the height of the prism). In this case, the length of the rectangles is 12m.
Isosceles triangle as a base		Ask: *Can you see that with an isosceles base, there will be two rectangles the same size (which link to the lengths of the two equal sides in the triangle) and one rectangle which will have a dimension matching the length of the* 3^{rd} *side of the triangle?* Note that all the rectangles share one dimension which is the height of the prism. In this case 14 mm.
Scalene triangle as a base		By now learners should know that the three rectangles will all be different sizes due to the scalene triangle at the base. Ask: *Which measurement do all the rectangles share?* (The height of the prism, in this case 10 cm).

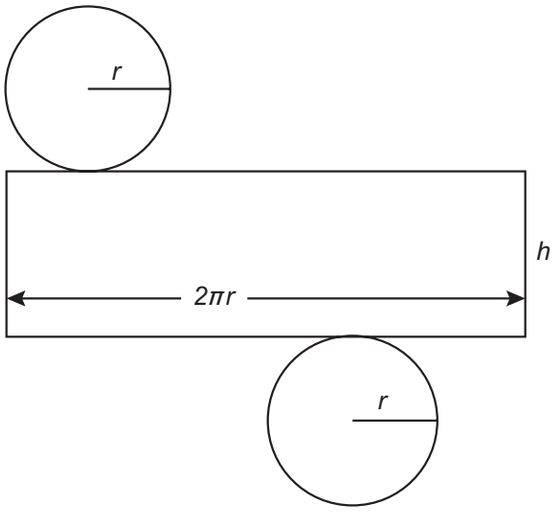
11. Discuss the surface area of a cylinder. Although the formula does not have the difficulties that the formula for the triangular prism has, it is important that learners understand how the formula is derived.

12. Ask for a volunteer to draw the net of a cylinder on the chalkboard.
    If it is drawn correctly (as on next page), ask all the learners to copy it into their books.



(The circles do not need to be in a certain position)

13. Discuss how we would find surface area of the cylinder – by finding the area of the two circles and the rectangle then adding them together.
Finding the combined area of the two circles should be straightforward: $\pi r^2 \times 2 = 2\pi r^2$.
Finding the area of the rectangle should be easy: $l \times b$. However, point out to learners if they were given a cylinder and asked to 'measure it', they are not likely to find a length and a breadth.
Ask: *In a cylinder, what does the breadth of the rectangle link directly to?* (Height).
Ask: *In a cylinder, what does the length of the rectangle link directly to?*
(The circumference of the circle).
Write these measurements on the diagram now and ask learners to do the same.



Write down the formula for finding the surface area of a cylinder. Learners should copy the formula their exercise books:
Surface Area of a cylinder $= 2\pi r^2 + 2\pi rh$

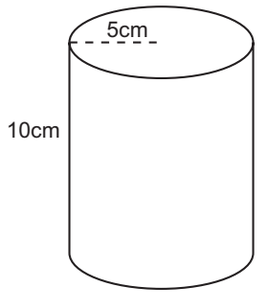
14. Ask: *What formula would you use to find the surface area of a cylinder without a lid?* $(\pi r^2 + 2\pi rh)$
    Ask: *What formula would you use to find the surface area of a cylinder with no lids, example a toilet roll?* $(2\pi rh)$.

15. Encourage learners not learn the formulae for surface area by heart – they need to look at the diagram and think through what 2D shapes form the 3D object, find the area of each and add the areas together to find the total surface area.
    The only formulae learners need to know are those of the four basic 2D shapes:

SHAPE	AREA FORMULA
Square	$l \times l = l^2$
Rectangle	$l \times b$
Triangle	$\frac{1}{2} b \times \perp height$
Circle	πr^2

16. Before doing an example of a triangular prism and a cylinder, ask learners to remind you what is actually being found when surface area is found:
    Surface area is only connected to 3D objects. If we are working with 2D shapes we find the area.
    To find the surface area of a 3D object, we find the area of the net. A more visual idea to assist understanding is, you are finding the area of the surface that could be painted or you are finding the area of cardboard needed to make the 3D object. You could deconstruct and flatten a 3D object to show learners the net of the 3D object.

17. Do some worked examples with learners. Learners should write the worked examples in their exercise books.

Example:	Teaching notes:
1. Find the surface area of this cylinder:  	This is a simple question that should pose no problems for Grade 10 learners. Learners simply need to use the formula which they have just derived and written down.

Solution:

Surface Area

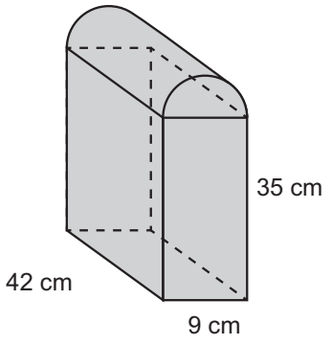
$= 2\pi r^2 + (2\pi rh)$

$= 2\pi(5\text{cm})^2 + (2\pi(5\text{cm})(10\text{cm}))$

$= 157,08\text{cm}^2 + 314,16\text{cm}^2$

$= 471,24\text{cm}^2$

2. Find the surface area of this 3D object:



This object needs to be analysed carefully. Remind learners that:

- the surface area is the area of all the 2D shapes that could be painted
- they cannot paint a shape that is not on the outside. For example, the rectangle at the bottom of the half cylinder is not part of the surface area.

Ask: *What 3D objects make up this complex object?*

(A rectangular prism and half a cylinder).

Ask: *What 2D shapes make up this 3D object?*

Tick the shapes off on the diagram as learners point them out. Encourage learners to mention if there is a pair of the same shape and size.

The list should include:

- 1 rectangle at the base.
- 2 rectangles at the back and front.
- 2 rectangles at each side (left and right)
- A curved piece (rectangle) at the top
- 2 semi-circles.

Ensure all the 2D shapes found are accounted for in the formula

Surface Area

$= lb + 2bh + 2lh + \frac{1}{2}(2\pi rh) + \frac{1}{2}(2\pi r^2)$

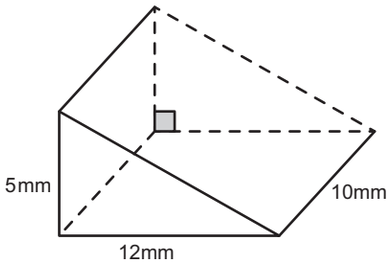
$= (42)(9) + 2(9)(35) + 2(42)(35) + \pi(4,5)(42) + \pi(4,5)^2$

$= 378 + 630 + 2940 + 593,76 + 63,62$

$= 4605,38\text{cm}^2$

Note that the radius measurement was not obvious – learners need to notice that the 9cm measurement at the base is the same as the diameter measurement of the half cylinder at the top.

3. Find the surface area of the following triangular prism:



This is a simple question that should pose no problems for Grade 10 learners.
Remind learners to think about what 2D shapes make up the 3D object.
Ask: *Is there enough information to find the area of the two triangles?* (Yes, the base and height are the two sides that make the right angle in a right-angled triangle)
Ask: *Which measurement is shared by all three rectangles?* (10cm)
Ask: *Is there enough information to find the area of all three rectangles?* (No, one dimension of the rectangle on top is not available).
Ask: *How can you find this information?* (By using the theorem of Pythagoras – the hypotenuse of the front triangle will be identical to the hypotenuse of the triangle at the back)

Solution:

$5^2 + 12^2 = h^2$

$25 + 144 = h^2$

$169 = h^2$

$13 = h$

Surface Area

$= 2\,\text{triangles} + 3\,\text{rectangles}$

$= 2(\frac{1}{2}(5\text{cm}) \times\!\perp (12\text{cm})) + (5\text{cm})(10\text{cm}) + (12\text{cm})(10\text{cm}) + (13\text{cm})(10\text{cm})$

$= 2(630) + 50\text{cm}^2 + 120\text{cm}^2 + 130\text{cm}^2$

$= 1260\text{cm}^2 + 50\text{cm}^2 + 120\text{cm}^2 + 130\text{cm}^2$

$= 1560\text{cm}^2$

18. Ask directed questions so that you can ascertain learners' level of understanding.

19. Ask learners if they have any questions.

PART B: VOLUME

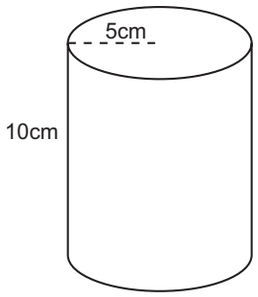
1. Ask: *What does it mean to find the volume of a shape?*
   (To find the amount of space a 3D object takes up).

2. Remind learners that the term 'volume' is linked to 3-dimensional objects only. Therefore, there is a 'cubed' in their answer – 3 dimensions are being multiplied together.

3. To find the volume of any right prism, the basic formula is:
   Area of base $\times$ perpendicular height
   Point out how important it is to know how to find area of the four basic shapes – these form the base of the right prisms.

VOLUME OF:	AREA OF BASE x HEIGHT
Cube	$(l \times l) \times ht$ $= l \times l \times l = l^3$
Rectangular prism	$(l \times b) \times h$ $= lbh$
Triangular prism	$(\frac{1}{2} b \times h) \times H$ Note: The first h represents the height of the triangle which is required to find the area of the base. The second H represents the height of the prism.
Cylinder	$\pi r^2 \times ht$ $= \pi r^2 ht$

4. Do some worked examples with learners. Learners should write the worked examples in their exercise books.

Example:	Teaching notes:
1. Find the volume of this cylinder:  	This is a simple question that should pose no problems for Grade 10 learners. Learners simply need to use the formula – area of base x height of cylinder

Solution:

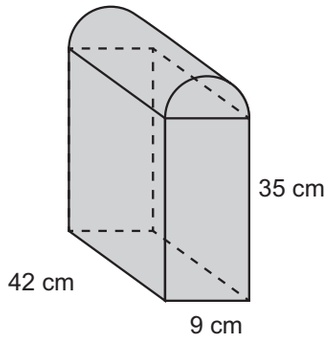
Volume

$= \pi r^2 h$

$= \pi (5\text{cm})^2 (10\text{cm})$

$= 785,4\text{cm}^3$

2. Find the volume of this 3-D object:



Finding the volume of a complex object is simpler than finding the surface area of a complex object.
When finding volume, the space the complex 3D object takes up is the sum of the space that each of the basic 3D objects takes up.
(A rectangular prism and half a cylinder).

Solution:

Volume

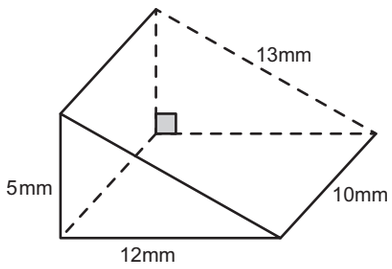
$= \frac{1}{2}\pi r^2 h + lbh$

$= \frac{1}{2}\pi (4,5\text{cm})^2 (42\text{cm}) + (42\text{cm})(9\text{cm})(35\text{cm})$

$= 14\,565,96\text{cm}^3$

Note that the radius measurement was not an obvious one – learners need to notice that the 9cm measurement at the base is the same as the diameter measurement of the half cylinder at the top.

3. Find the volume of the triangular prism:



This is a simple question that should pose no problems for Grade 10 learners.
Learners need to remember that they need the area of the base (the triangle) multiplied by the height of the prism.
Ask: *Do you have all the information required?*
(Yes – the base and height of the triangle, as well as the height of the prism is given)
Note – the hypotenuse of the right-angled triangle is not required to find the area of the triangle.

Solution:

Volume

$= \frac{1}{2}bh \times H$

$= \frac{1}{2}(12\text{cm})(5\text{cm}) \times (10\text{cm})$

$= 300\text{cm}^3$

5. Ask directed questions so that you can ascertain learners' level of understanding.

6. Ask learners if they have any questions.

7. Give learners an exercise to complete on their own.

8. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=wxDTrgnyu28
(Definition of a prism)

https://www.youtube.com/watch?v=WPyNjbI4_G8
(Volume of 3D shapes)

https://www.youtube.com/watch?v=Aigefhb-1NM
https://www.youtube.com/watch?v=T6VxjB7bS5o
(Surface area of a triangular prism – Part 1 & 2)

https://www.youtube.com/watch?v=LXBPJnlrEIQ
(Surface area of a cylinder)

TERM 3, TOPIC 6, LESSON 2

THE EFFECT ON VOLUME AND SURFACE AREA WHEN MULTIPLYING ANY DIMENSION BY A CONSTANT FACTOR *k*.

Suggested lesson duration: 1 hour

POLICY AND OUTCOMES

A

CAPS Page Number	28

Lesson Objectives

By the end of the lesson, learners should be able to:

- explain the effect on volume and surface area when any dimension is multiplied by a constant *k*.
- complete an exercise on their own.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. You will need Resources 18 and 19 (Investigation and memorandum) from the Resource Pack. If possible, make one copy of Resource 18 for each group (group size = 4).
4. Write the lesson heading on the board before learners arrive.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1 (2 & 3) 2 (3)	279 283			14.2	239	16.3	371	13.6	455

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. This lesson will take the form of an investigation. It is important that learners work out for themselves what happens to the surface area and volume of a shape when dimensions are changed.
2. While learners are working, walk around the classroom assisting where necessary. Use leading questions to aim struggling learners in the correct direction rather than tell them what to do.

DIRECT INSTRUCTION

1. Hand one copy of the investigation (Resource 22) to each group. The groups should have no more than four learners.
2. Give learners about 30 minutes to complete the investigation. Use your discretion as to whether more or less time is required.
3. Walk around the classroom as learners work through the investigation. Support learners where necessary.
4. Once time is up, summarise the conclusions that learners came to in their investigation. Allow learners to share their conclusions with you.
5. Ensure that the learners reached the following conclusions. Learners should write these conclusions in their exercise books if they haven't written them during their group discussion.

TOPIC 6, LESSON 2: THE EFFECT ON VOLUME AND SURFACE AREA WHEN MULTIPLYING ANY DIMENSION BY A CONSTANT FACTOR *K*.

The effect on volume when multiplying any dimension by a constant factor *k*:

- If only one dimension is changed by a value of *k*, the volume will be *k* times bigger.
- If only two dimensions are changed by a value of *k*, the volume will be k^2 times bigger.
- If all three dimensions are changed by a value of *k*, the volume will be k^3 times bigger.

6. Point out that when the measurement was halved that it was multiplied by $\frac{1}{2}$ and that $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ and $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

   Tell learners that if they said that the volume was 8 times smaller it is the same as saying it is $\frac{1}{8}$ of the original volume.

7. Complete a table with learners now looking at the effect of changing the dimensions on the surface area of the same rectangular prism. Calculations can be done in rough in learners' books. Once the table is complete, learners should take it down in their books.

	LENGTH	BREADTH	HEIGHT	SURFACE AREA
ORIGINAL	$10cm$	$5cm$	$2cm$	$160cm^2$
Each dimension is 2 times bigger	$20cm$	$10cm$	$4cm$	$640cm^2$
Each dimension is 3 times bigger	$30cm$	$15cm$	$6cm$	$1440cm^2$
Each dimension is $\frac{1}{2}$ the length	$5cm$	$2{,}5cm$	$1cm$	$40cm^2$

8. Give learners a few minutes to calculate how each of these answers compares with the original shape's surface area. Ask for volunteers to explain what they have found. Ensure that the following is noted in the learners' books.

The effect on surface area when multiplying all dimensions by a constant factor *k*:

- If all the dimensions are multiplied by a factor *k*, the surface area will be k^2 bigger.

9. Learners should do the textbook exercise for homework.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=0C_OJjK1VQw

https://www.mathsisfun.com/algebra/distance-2-points.html

http://www.mathwarehouse.com/algebra/distance_formula/index.php

TERM 3, TOPIC 6, LESSON 3

VOLUME AND SURFACE AREAS OF SPHERES, RIGHT PYRAMIDS AND CONES

Suggested lesson duration: 2,5 hours

A POLICY AND OUTCOMES

CAPS Page Number	28

Lesson Objectives

By the end of the lesson, learners should be able to:

- find the volume and surface area of a sphere, a cone and a pyramid.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. You will need Resources 19 and 20 from the Resource Pack. There are sketches of all three objects, as well as real-life examples in the Resource Pack. Use the sketches to make your sketches on the board. If possible display the real-life examples for the learners.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive. For this lesson draw sketches of the three solids (a cone, a sphere and a pyramid, but do not label them by name).
6. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
3	286	3 4	273 275	14.4	243	11.1 11.2	16.1 (1 f-h; 2-6) 16.2 (1 c-l; 2-5)	13.4 13.5	438 448

CONCEPTUAL DEVELOPMENT

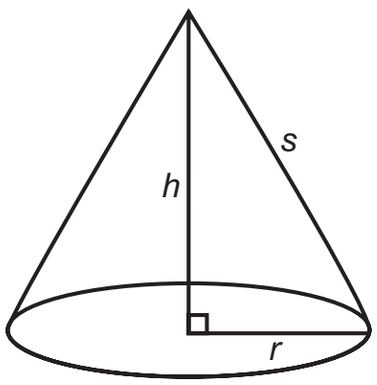
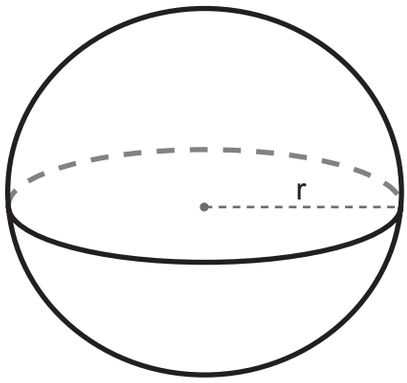
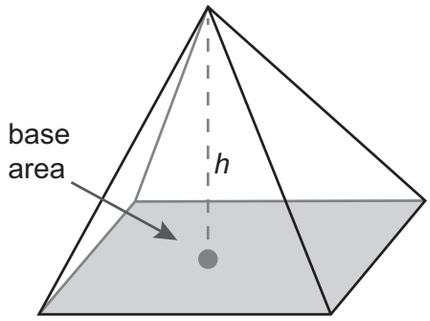
C

INTRODUCTION

1. This lesson is an extension of finding the volume and surface area of the 4 basic shapes covered in the senior phase as well as the revision lesson at the beginning of this section.
2. It is important that learners remember what they are finding when asked to find volume or surface area of a 3D shape.

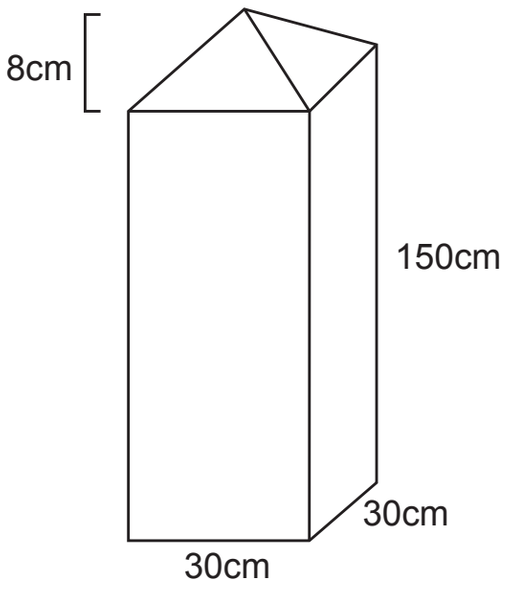
DIRECT INSTRUCTION

1. Start the lesson by asking: *What is the difference between surface area and volume of a 3D shape?*
   (Surface area: The area taken up by the net of a 3D object. The sum of the area of all the faces.
   Volume: The space taken up by a 3D object.)
2. Tell learners: *Today you are going to learn how to find the surface area and volume of more unusual shapes than the four you have already worked with extensively.*
3. Show learners the three sketches of the three objects on the chalkboard.
   Ask: *What are the names of the objects?* Fill them in as they are given.
4. Tell learners: *The formulae will always be given when questioned on one of these shapes.*
5. Give learners a summary of the formulae. Learners should write the summary in their books for reference purposes.

Shape	Surface Area	Volume
Cone 	$\pi rs + \pi r^2$ (the slant height is sometimes named l)	$\frac{1}{3}\pi r^2 h$
Sphere 	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Pyramid 	Sum of the areas of: ● the base and ● the triangles* * the number of triangles depends on the type of base	$\frac{1}{3}$(area of base)×h (remember that the base could be any polygon but generally the square, rectangle and triangle would be used)

6. Learners should not need you to do an example of each of the above as essentially the skill they require is that of substitution into a formula. If you feel that your learners require a basic example of each shape, do that now.

7. Do two worked examples from past exams with learners. Learners should write the worked examples in their exercise books.

Example 1:	Teaching notes:
A concrete gate post comprises a right rectangular prism with a square base and a pyramid at the top as shown in the diagram below. The length of the sides of the base is 30cm and the height of the rectangular section is 150cm. The perpendicular height of the pyramid section is 8cm. (Note: Volume and surface area formulae for a pyramid were provided)  a) Calculate the volume of concrete required to make one post.	Tell learners to check that they understand all the information in the question and whether each piece of information is represented on the diagram. If not, they should add to the diagram. Ask: *How will you find volume of the complex shape?* (Volume of the rectangular prism added to volume of the pyramid) Ask: What is the formula for finding the volume of a rectangular prism? $(l \times b \times h)$

b) Calculate the surface area of the pyramid section of the post.	Ask: *What should you always be careful of in a surface area question?* (Is the 3D object closed or not – there may be a face missing) Ask: *Is there an open/ closed issue in this question?* (Yes- the base cannot be counted because it is part of the inside of the post) Ask: *What is the surface area of the pyramid part of the complex 3D object made up of?* (4 triangles) Ask: *Do we have the base and height of the triangles?* (We have the base – 30cm – but not the height) Ask: *How will we calculate the height of the triangle?* (Find the slant height of the pyramid using the theorem of Pythagoras as that is the height of the triangle).
c) If the length of the sides of the base is halved, how many posts, having the same design as the original, can be made with the same volume of concrete as the original post? NSC NOV 2015	Ask: *How many dimensions are being changed?* (Two) Ask: *How much smaller will the volume be?* $(4\,(2^2)$ times smaller or $\frac{1}{4}$ of the original size)

Solutions:

a) Volume $= lbh + \frac{1}{3}(\text{area of base}) \times h$

$= (30)(30)(150) + \frac{1}{3}(30)(30)(8)$

$= 137\,400\text{cm}^3$

b) Slant height:

$s^2 = 15^2 + 8^2$

$s^2 = 289$

$s = 17$

Surface area

$= 4(\frac{1}{2}bh)$

$= 4(\frac{1}{2}(30)(17))$

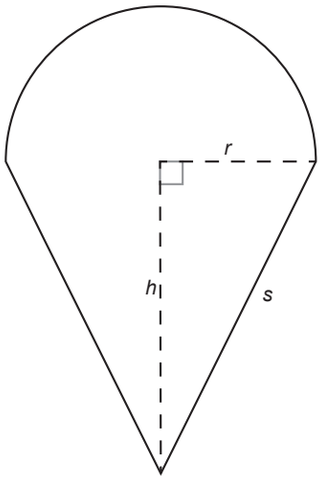
$= 1021\text{cm}^2$

c) Volume of new post = $\frac{1}{4}$ the volume of the original post

$= \frac{1}{4}(137\,400\text{cm}^3)$

$= 34\,350\text{cm}^2$

Number of smaller posts that can be made: $\frac{137\,400}{34\,350} = 4$

Example 2:	Teaching notes:
The diagram below shows the cross-section of a solid made up of a right circular cone with radius r and slant height s. The perpendicular height of the cone, h is $6,5cm$ and the volume of the cone is $83,38cm^3$.  (Note: Volume and surface area formulae for a sphere and a cone were provided) Calculate, correct to two decimal places: a) The radius, r, of the cone	Tell learners to check that they understand all the information in the question and whether each piece of information is represented on the diagram. If not, they should add to the diagram. Ask: What will you need to do to find the radius? (Volume and height are provided, so using the formula for volume, you can work in reverse to find the radius).
b) The slant height, s, of the cone	Learners should recognise that this is a theorem of Pythagoras question.
c) The surface area of the solid. NSC NOV 2015	Ask: *How can you find the volume of the complex shape?* (Volume of the cone added to volume of the hemisphere – half a sphere) Ask: *Do you have all the measurements available for this?* (Yes – you need radius and the slant height) Confirm that learners can see that they won't need the base circle of the cone's formula as it is 'inside' the solid.

Solutions:

a) Volume of cone $= \frac{1}{3}\pi r^2 h$

$$83,38 = \frac{1}{3}\pi r^2 (6,5)$$

$$250,14 = 6,5\pi r^2$$

$$\frac{250,14}{6,5\pi} = r^2$$

$$r = 3,5\,\text{cm}$$

b) $r^2 + h^2 = s^2$

$$(3,5)^2 + (6,5)^2 = s^2$$

$$54,5 = s^2$$

$$s = 7,38\,\text{cm}$$

c) Surface area

= cone (without base) + hemisphere

$$= \pi rs + \frac{1}{2}(4\pi r^2)$$

$$= \pi(3,5)(7,38) + 2\pi(3,5)^2)$$

$$= 158,12\,\text{cm}^2$$

8. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

9. Give learners an exercise to complete with a partner.

10. Walk around the classroom as learners do the exercise. Support learners where necessary.

11. Once the exercises have been completed and marked, learners should do a revision exercise. If there is not enough time available to do it in class time, then they should do it for homework.

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev Some Ch	288 289	5 Rev	278 280	w/sh	249	16.4 16.5	372 374	13.7	457

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=HIELx5sm5k0
(What is a cone)

https://www.youtube.com/watch?v=rd8tbD2eekM
(Surface area of a cone)

https://www.youtube.com/watch?v=v2CksDGfbPY
(Prisms and pyramids)

https://www.youtube.com/watch?v=XqWxpmd8x4w
(What is a pyramid)

https://www.asu.edu/courses/mat142ej/geometry/volume_and_surface_area/volume-and-surface-area-notes.pdf
(Summary notes and examples)