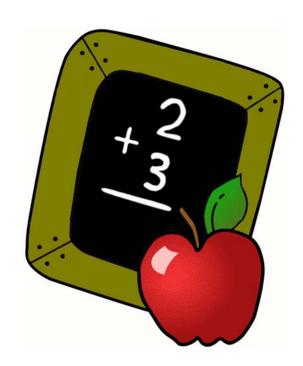
Minnesota Department of Education



Grade 11
Mathematics MCA-III
Item Sampler Teacher Guide

Grade 11 Mathematics MCA Item Sampler Parent/Teacher Guide

Minnesota Department of



The purpose of the Item Samplers is to familiarize students with the online MCA test format. The Item Samplers contain multiple choice items (MC) and technology enhanced items (TE).

This guide includes:

- A snapshot of each item
- Benchmark and examples from the Minnesota Academic Standards for Mathematics
- Item specifications (Content limits contained in the item specifications are intended for use by item developers. They should not be construed as instructional limits.)
- Vocabulary
- Depth of Knowledge (DOK) see more detail below
- Calculator designation (CL = calculator allowed; NC = no calculator)
- Correct answer
- Table of rationales (explanations for why a student might choose each incorrect answer option, e.g., mixed up addition and subtraction, used incorrect place value, etc.)
- Notes on grade expectations for some items

Cognitive Complexity/Depth of Knowledge (DOK)

Cognitive complexity refers to the cognitive demand associated with an item. The level of cognitive demand focuses on the type and level of thinking and reasoning required of the student on a particular item. Levels of cognitive complexity for MCA-III are based on Norman L. Webb's Depth of Knowledge¹ levels.

Level 1 (recall) items require the recall of information such as a fact, definition, term or simple procedure, as well as performing a simple algorithm or applying a formula. A well-defined and straight algorithmic procedure is considered to be at

¹ Webb, N. L. *Alignment of science and mathematics standards and assessments in four states* (Research Monograph No. 18). Madison: University of Wisconsin – Madison, National Institute for Science Education, 1999.

this level. A Level 1 item specifies the operation or method of solution and the student is required to carry it out.

Level 2 (skill/concept) items call for the engagement of some mental processing beyond a habitual response, with students required to make some decisions as to how to approach a problem or activity. Interpreting information from a simple graph and requiring reading information from the graph is a Level 2. An item that requires students to choose the operation or method of solution and then solve the problem is a Level 2. Level 2 items are often similar to examples used in textbooks.

Level 3 (strategic thinking) items require students to reason, plan or use evidence to solve the problem. In most instances, requiring students to explain their thinking is a Level 3. A Level 3 item may be solved using routine skills but the student is not cued or prompted as to which skills to use.

Level 4 (extended thinking) items require complex reasoning, planning, developing and thinking, most likely over an extended period of time. Level 4 items are best assessed in the classroom, where the constraints of standardized testing are not a factor.

Technology Enhanced Items

There are several types of technology enhanced items. To respond to these questions, students may be required to type a number into a blank, select their answer choice(s), or select and drag. When typing an answer into a blank, the test engine allows students to type in numbers, the division bar (/), decimal points, and negative signs (in certain grades only). The test engine does not allow students to type in other characters, symbols, or letters of the alphabet.

Grade 11 Mathematics MCA Item Sampler Answer Key

Item #	Correct Answer	Item Type	Benchmark
1	С	MC	9.2.4.8
2	D	MC	9.2.1.1
3	В	MC	9.2.3.7
4	Α	MC	9.2.1.2
5	С	MC	9.2.1.3
6	В	MC	9.2.1.8
7	С	MC	9.2.2.5
8	N/A	TE	9.2.2.1
9	С	MC	9.2.3.2
10	D	MC	9.2.3.5
11	N/A	TE	9.2.4.1
12	В	MC	9.2.4.2
13	N/A	TE	9.4.3.9
14	В	MC	9.2.4.7
15	С	MC	9.3.1.1
16	D	MC	9.3.4.7
17	С	MC	9.3.3.1
18	N/A	TE	9.3.3.3
19	С	MC	9.3.3.7
20	N/A	TE	9.3.3.8
21	В	MC	9.3.4.2
22	N/A	TE	9.3.2.4
23	В	MC	9.3.4.4
24	N/A	TE	9.4.3.1
25	D	MC	9.4.3.3
26	N/A	TE	9.4.3.5
27	С	MC	9.3.3.5
28	N/A	TE	9.3.1.3
29	N/A	TE	9.2.2.3
30	N/A	TE	9.2.4.4

The population of a type of bacteria doubles every 3.5 hours. Which expression is a reasonable approximation of how many bacteria there will be in 20 hours for an initial population, *P*, of bacteria?

- A. P
- B. 2P
- © C. 50P
- D. 64P

Benchmark: 9.2.4.8

Assess the reasonableness of a solution in its given context and compare the solution to appropriate graphical or numerical estimates; interpret a solution in the original context.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

DOK: 2 Answer: C

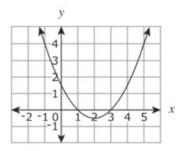
Α	Chose initial population from problem.
В	Did not include exponent to account for number of doubling periods.
С	Correct. $P(2)^{(20/3.5)} \approx 52.5P$
D	Approximated 20/3.5 as about 5, then found $P2(2)^5 = 64P$.

Notes on grade expectations: The number of 3.5-hour periods in 20 hours is about 5.714. So the initial amount, P, will double 5.714 times, which makes the multiplier for P about 50 because ($2^{5.714} \approx 52.5$).

Student may estimate $2^{5.714}$ by choosing the answer option with a coefficient between 2^5 and 2^6 (or a number between 32 and 64).

Or student may use doubling-time formula. Doubling-time formula: $A(t) = P(2)^{\frac{t}{d}}$, where A = amount after t units of time, P = initial amount, d = doubling time of a quantity, and t = number of years.

The graph of y = f(x) is shown.



What is a value of x for which f(x) = 4?

- O A. 1
- O B. 1.5
- O C. 4
- O D. 5

Benchmark: 9.2.1.1

Understand the definition of a function. Use functional notation and evaluate a function at a given point in its domain.

For example: If
$$f(x) = \frac{1}{x^2 - 3}$$
 find $f(-4)$.

Item Specifications

• Vocabulary allowed in items: relation, domain, range and vocabulary given at previous grades

DOK: 2 Answer: D

Α	Found that graph is one unit from <i>y</i> -axis at $f(x) = 4$; (-1, 4) is on graph.
В	Mixed up x and $f(x)$; (4, 1.5) is on graph.
С	Used 4 from question.
D	Correct. (5, 4) is on the graph.

How many solutions does $x-2=\sqrt{x}$ have?

O A. 0

D. 4

Benchmark: 9.2.3.7

Justify steps in generating equivalent expressions by identifying the properties used. Use substitution to check the equality of expressions for some particular values of the variables; recognize that checking with substitution does not guarantee equality of expressions for all values of the variables.

Item Specifications

· Vocabulary allowed in items: vocabulary given at previous grades

DOK: 2 Answer: B

Α	May have concluded that both solutions are extraneous. Or may have used guess-and-check and missed the solution of <i>x</i> =4.
В	Correct. Solving results in 2 solutions, $x = 1$ and $x = 4$, but $x = 1$ is extraneous because, when substituted into the equation, $1 - 2 = \sqrt{1}$ simplifies to $-1 = 1$ which is not a true statement.
С	Found two solutions: 1 and 4. Did not recognize that one of the solutions is extraneous.
D	Solved and included the opposites of the solutions: -4, -1, 1, 4 for a total of 4 solutions.

Notes on grade expectations: Student should know that extraneous means an apparent solution of a simplified version of an equation that does not satisfy the original equation. One approach is to solve directly, then check for extraneous solutions.

$$x-2 = \sqrt{x}$$

$$(x-2)^2 = (\sqrt{x})^2$$

$$x^2 - 4x + 4 = x$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0$$

$$x = 4, x = 1$$

Check for extraneous solutions by substituting into the original equation.

Try
$$x = 4$$
.

$$4 - 2 = \sqrt{4}$$

$$2 = 2$$

This is true, so x = 4 is a solution.

Try
$$x = 1$$
.

$$1 - 2 = \sqrt{1}$$

$$-1 = 1$$

This is not true, so x = 1 is extraneous. Therefore, there is only 1 solution to this equation.

Another approach is to guess and check to find the solutions. Students should find that x = 4 is the only solution that works, so there is exactly 1 solution to the equation.

For which equation is y a function of x?

$$A. y = \frac{5}{x-1}$$

B.
$$|y| = x - 8$$

C.
$$y^2 = x - 2$$

D.
$$y^2 = 4 - x^2$$

Benchmark: 9.2.1.2

Distinguish between functions and other relations defined symbolically, graphically or in tabular form.

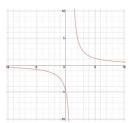
Item Specifications

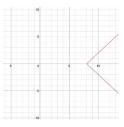
• Vocabulary allowed in items: relation, domain, range and vocabulary given at previous grades

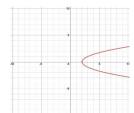
DOK: 1 Answer: A

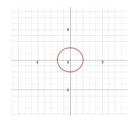
Α	Correct. The graph passes the vertical line test (each <i>x</i> -value has only 1 <i>y</i> -value).
В	The graph does not pass the vertical line test. For all <i>x</i> -values greater than 8, there is more than 1 <i>y</i> -value.
С	The graph does not pass the vertical line test. For all <i>x</i> -values greater than 2, there is more than 1 <i>y</i> -value.
D	The graph does not pass the vertical line test. For x -values between -2 and 2, there is more than 1 y -value.

Notes on grade expectations: Student should know that a function has exactly 1 *y*-value for every *x*-value. Graphs can be checked using the vertical line test. If a vertical line intersects more than 1 point on the graph, then the relation is not a function.









What is the domain of $f(x) = \frac{x}{2x^2 - 5x - 3}$?

- \bigcirc A. $\{x \mid -\infty < x < \infty\}$
- \bigcirc B. $\{x | x \neq 0\}$
- $^{\odot}$ C. $\left\{x\,|\,x
 eq-rac{1}{2}\,,\,x
 eq3
 ight\}$
- \bigcirc D. $\left\{x\,|\,x
 eq -3,\,x
 eq rac{1}{2}
 ight\}$

Benchmark: 9.2.1.3

Find the domain of a function defined symbolically, graphically or in a real-world context. For example: The formula $f(x) = \pi x^2$ can represent a function whose domain is all real numbers, but in the context of the area of a circle, the domain would be restricted to positive x.

Item Specifications

• Vocabulary allowed in items: relation, domain, range and vocabulary given at previous grades

DOK: 2 Answer: C

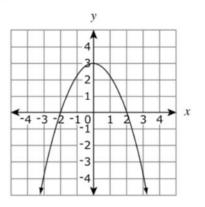
Α	Did not find values for which the relation is undefined (denominator of zero).
В	May have thought the value of the variable, <i>x</i> , in the denominator could not be 0.
С	Correct. $x \neq -1/2$, $x \neq 3$.
D	Mixed up positive/negative signs.

Notes on grade expectations: Factor the denominator to see what values of *x* make the relation undefined.

$$f(x) = \frac{x}{(2x+1)(x-3)}$$

Solving for 2x + 1 = 0 results in x = -1/2. Solving for x - 3 = 0 results in x = 3. If x = -1/2 or 3 the denominator will be 0, therefore x is undefined for x = -1/2 and 3.

The graph of a function is shown.



Which statement is true about the rate of change for this function when -3 < x < 0?

- A. The rate of change is constant.
- B. The rate of change is decreasing.
- C. The rate of change is increasing.
- D. The rate of change is negative.

Benchmark: 9.2.1.8

Make qualitative statements about the rate of change of a function, based on its graph or table of values.

For example: The function $f(x) = 3^x$ increases for all x, but it increases faster when x > 2 than it does when x < 2.

Item Specifications

• Vocabulary allowed in items: vocabulary given at previous grades

DOK: 2 Answer: B

Α	A constant rate of change is indicated by a horizontal line.
В	Correct. From -3 to 0, the slope of the line gets flatter (less steep).
С	Used the portion of the graph from 0 to 3; the slope of the line gets steeper.
D	Used the portion of the graph from 0 to 3; downward slope.

Notes on grade expectations: From previous grades, the student should know that the rate of change is the same as slope, or rise/run.

Emma takes a job with a starting salary of \$42,000. Her salary increases by 4% at the beginning of each year. What will be Emma's salary, to the nearest thousand dollars, at the beginning of year 10?

A. \$57,000

B. \$59,000

C. \$60,000

D. \$62,000

Benchmark: 9.2.2.5

Recognize and solve problems that can be modeled using finite geometric sequences and series, such as home mortgage and other compound interest examples. Know how to use spreadsheets and calculators to explore geometric sequences and series in various contexts.

Item Specifications

Vocabulary allowed in items: vocabulary given at previous grades

DOK: 2 Answer: C

Α	Used 8 instead of 9 for exponent; 42,000(1.04)8 = 57,480.
В	Calculated 59,779, then rounded down to 59,000 instead of rounding up to
Ь	nearest thousand.
С	Correct. $42,000(1.04)^9 = 59,779$; used 9 because it is 10 years after year 0.
D	Used 10 instead of 9 for exponent; 42,000(1.04) ¹⁰ = 62,170. This would be 10
U	years after year 1, but 11 years after year 0.

Notes on grade expectations: Student may use the exponential growth model, $A = P(1+r)^t$, where A = amount after t units of time, P = initial amount, r = rate of growth or decay, and t = number of units of time. This formula is not on the grade 11 formula sheet, but it is given in the problem when required on the MCA test. Students may also solve this problem by multiplying 42,000 by 1.04 nine times.

rag a word into each box.	
rag a word line each box.	
The total amount of money earned	Linear
for a job that pays \$20 per hour	
	Exponential
Model:	Quadratic
	The second second second
The height of a ball thrown	
into the air over 5 seconds	
The second second second second	
Model:	
Model:	
The volume of a room with	×
	x

Benchmark: 9.2.2.1

Represent and solve problems in various contexts using linear and quadratic functions. *For example*: Write a function that represents the area of a rectangular garden that can be surrounded with 32 feet of fencing, and use the function to determine the possible dimensions of such a garden if the area must be at least 50 square feet.

Item Specifications

• Vocabulary allowed in items: quadratic and vocabulary given at previous grades

DOK: 2 Answer:

This is a technology-enhanced item. The correct answer is shown. A student must correctly place the relationships into the boxes in order to receive 1 point.

Determine the best function model to represent Drag a word into each box.	nt each statement.
The total amount of money earned for a job that pays \$20 per hour Model: Linear	Exponential Quadratic
The height of a ball thrown into the air over 5 seconds Model: Quadratic	
The volume of a room with length $2x + 2$, width 3, and height x Model: Quadratic	

Notes on grade expectations:

- "The total amount of money earned for a job that pays \$20 per hour." The constant rate indicates a linear relationship. The equation would be y = 20x.
- "The height of a ball thrown into the air over 5 seconds." The heights of things that are falling or thrown can be found using: $h = (-1/2)g\frac{\ell^2}{2} + v_0t + h_0$. The squared variable indicates a quadratic relationship.
- "The volume of a room with length 2x + 2, width 3, and height x" can be modeled using (2x + 2)(3)(x) which simplifies to $6x^2 + 6x$. The squared variable indicates a quadratic relationship.

Divide.

$$(2x^3 + 9x^2 - 11x - 24) \div (x - 2)$$

What is the remainder?

- A. 0
- B. 2
- O C. 6
- O D. 18

Benchmark: 9.2.3.2

Add, subtract and multiply polynomials; divide a polynomial by a polynomial of equal or lower degree.

Item Specifications

• Vocabulary allowed in items: polynomial, degree of a polynomial and vocabulary given at previous grades

DOK: 2 Answer: C

Α	Divided incorrectly.
В	Divided incorrectly.
С	Correct.
D	Divided incorrectly.

Notes on grade expectations: Example of polynomial division:

$$\frac{2x^{2} + 13x + 15}{x - 2\sqrt{2x^{3} + 9x^{2} - 11x - 24}}$$

$$-(2x^{3} - 4x^{2})$$

$$\frac{13x^{2} - 11x}{-(13x^{2} - 26x)}$$

$$\frac{15x - 24}{-(15x - 30)}$$

$$6 = remainder$$

A root of function f(x) is -1-2i. Which could be an equation for this function?

A.
$$f(x) = x^2 - 2x - 3$$

B.
$$f(x) = x^2 - 2x + 5$$

$$\circ$$
 C. $f(x) = x^2 + 2x - 3$

① D.
$$f(x) = x^2 + 2x + 5$$

Benchmark: 9.2.3.5

Check whether a given complex number is a solution of a quadratic equation by substituting it for the variable and evaluating the expression, using arithmetic with complex numbers.

For example: The complex number $\frac{1+i}{2}$ is a solution of $2x^2 - 2x + 1 = 0$, since

$$2\left(\frac{1+i}{2}\right)^2 - 2\left(\frac{1+i}{2}\right) + 1 = i - (1+i) + 1 = 0.$$

Item Specifications

Vocabulary allowed in items: complex number and vocabulary given at previous grades

DOK: 2 Answer: D

Α	$(-1-2i)^2-2(-1-2i)-3=1+4i-4+2+4i-3=-4+8i$ Not equal to zero.
В	$(-1 - 2i)^2 - 2(-1 - 2i) + 5 = 1 + 4i - 4 + 2 + 4i + 5 = 4 + 8i$ Not equal to zero.
С	$(-1 - 2i)^2 + 2(-1 - 2i) - 3 = 1 + 4i - 4 - 2 - 4i - 3 = -8$ Not equal to zero.
D	Correct. $(-1 - 2i)^2 + 2(-1 - 2i) + 5 = 1 + 4i - 4 - 2 - 4i + 5 = 0$

Notes on grade expectations: Substitute the complex number into each of the answer options and see which one results in zero.

Another way to solve this problem is to use the complex conjugate of the given root, then multiply the factors $(x - \text{root}_1)(x - \text{root}_2)$.

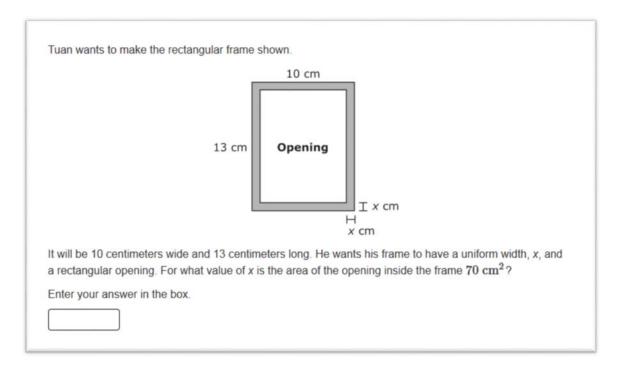
Complex conjugates are of the form (r + si), (r - si), so -1 - 2i and -1 + 2i are the roots of this function.

Multiply
$$(x - (-1 - 2i))$$
 by $(x - (-1 + 2i))$
 $(x + 1 + 2i)(x + 1 - 2i)$

$$x^2 + x - 2ix + x + 1 - 2i + 2ix + 2i + 4$$

$$x^2 + 2x + 5$$

Remember that $i^2 = -1$.



Benchmark: 9.2.4.1

Represent relationships in various contexts using quadratic equations and inequalities. Solve quadratic equations and inequalities by appropriate methods including factoring, completing the square, graphing and the quadratic formula. Find non-real complex roots when they exist. Recognize that a particular solution may not be applicable in the original context. Know how to use calculators, graphing utilities or other technology to solve quadratic equations and inequalities.

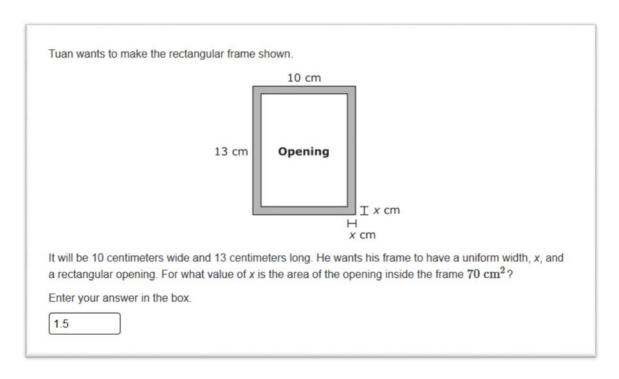
For example: A diver jumps from a 20-meter platform with an upward velocity of 3 meters per second. In finding the time at which the diver hits the surface of the water, the resulting quadratic equation has a positive and a negative solution. The negative solution should be discarded because of the context.

Item Specifications

- Items do not require the use of graphing technology
- Vocabulary allowed in items: quadratic, *n*th root and vocabulary given at previous grades

DOK: 2 Answer:

This is a technology-enhanced item. The correct answer is shown. A student must type the correct answer in the box in order to receive 1 point.



Notes on grade expectations: The area of the inside of the frame must be 70. The length and width of the frame opening are (13 - 2x) and (10 - 2x). Find the area by multiplying length and width, then set equal to 70 and solve for x.

$$(13 - 2x)(10 - 2x) = 70$$

 $(130 - 46x + 4x^2) = 70$
 $0 = 4x^2 - 46x + 60$
 $x = 1.5$ or 10

Use the quadratic formula or factor to solve the quadratic equation. The answer of 10 cannot be correct because the width of the whole frame is only 10 cm. The correct answer is 1.5 cm.

Note: The allowable characters that can be entered in the answer box are digits 0-9, fraction bar (/), decimal point (.), and negative sign (-). Students cannot enter a comma in numbers with more than 3 digits. Familiarity with calculators will help the students with this concept.

A scientist begins an experiment with 800 bacteria cells. After each 10-hour period, only $\frac{1}{2}$ of the bacteria cells remain. Let t be the number of hours since the beginning of the experiment. Which equation could be used to represent the situation when only 200 bacteria cells remain?

- \bigcirc A. $800(0.05)^t = 200$
- O B. $800(0.5)^{\frac{t}{10}} = 200$
- $^{\circ}$ C. $800(0.5)^{\frac{10}{t}} = 200$
- \bigcirc D. $800(0.5)^{10t} = 200$

Benchmark: 9.2.4.2

Represent relationships in various contexts using equations involving exponential functions; solve these equations graphically or numerically. Know how to use calculators, graphing utilities or other technology to solve these equations.

Item Specifications

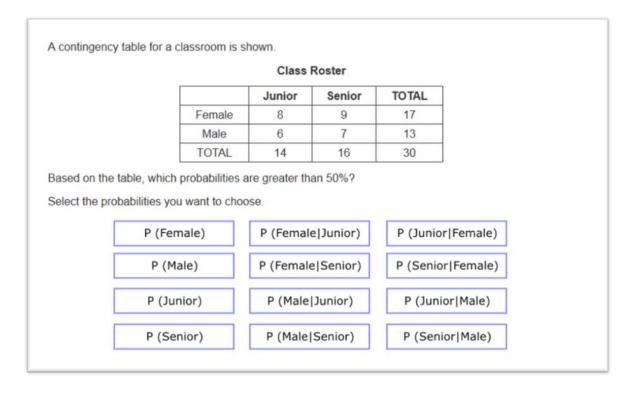
- Items do not require the use of graphing technology
- Vocabulary allowed in items: exponential and vocabulary given at previous grades

DOK: 2 Answer: B

Α	Used 0.05 instead of 0.5 for half-life; used <i>t</i> instead of <i>t</i> /10 for number of 10-hour time periods. May have divided base, 0.5, by 10 instead of dividing exponent, <i>t</i> , by 10.	
В	Correct.	
С	Used 10/t instead of t/10 for number of 10-hour time periods.	
D	Used 10 <i>t</i> instead of <i>t</i> /10 for number of 10-hour time periods.	

Notes on grade expectations: Student should be able to write and solve exponential decay problems, $y = ab^x$, or they may use the more specific half-life formula,

$$A(t) = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$$
, where A = amount after t units of time, P = initial amount, t = number of units of time, and h = the time it takes for the amount to be cut in half.



Benchmark: 9.4.3.9

Use the relationship between conditional probabilities and relative frequencies in contingency tables.

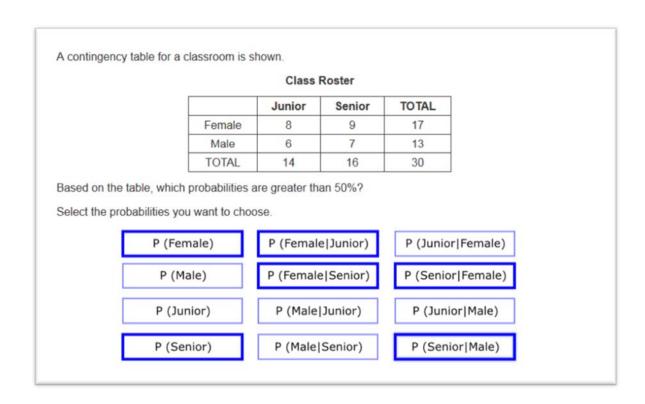
For example: A table that displays percentages relating gender (male or female) and handedness (right-handed or left-handed) can be used to determine the conditional probability of being left-handed, given that the gender is male.

Item Specifications

• Vocabulary allowed in items: conditional and vocabulary given at previous grades

DOK: 2 Answer:

This is a technology-enhanced item. The correct answer is shown. A student must select the correct probabilities in order to receive 1 point.



Notes on grade expectations: Student should be able to calculate simple and conditional probability. In this case, find all probabilities and select the ones that are greater than ½.

P(F) = 17/30	P(F Jr) = 8/14	P(Jr F) = 8/17
P(M) = 13/30	P(F Sr) = 9/16	P(Sr F) = 9/17
P(Jr) = 14/30	P(M Jr) = 6/14	P(Jr M) = 6/13
P(Sr) = 16/30	P(M Sr) = 7/16	P(Sr M) = 7/13

Comment on notation:

The vertical bar in the table above is read as "given". For example, in the last cell, P(Sr | M) is read as "the probability of being a Senior given that the student is Male". The numerator is the number of males that are seniors, and the denominator is the total number of males. The result is 7/13.

Which number is an extraneous solution of the equation $x=\sqrt{x}+6$?

- A. −6
- ◎ B. -2
- O C. 0
- O D. 3

Benchmark: 9.2.4.7

Solve equations that contain radical expressions. Recognize that extraneous solutions may arise when using symbolic methods.

For example: The equation $\sqrt{x-9}=9\sqrt{x}$ may be solved by squaring both sides to obtain x-9=81x, which has the solution $x=-\frac{9}{80}$. However, this is not a solution of the original equation, so it is an extraneous solution that should be discarded. The original equation has no solution in this case.

Another example: Solve $\sqrt[3]{-x+1} = -5$.

Item Specifications

Vocabulary allowed in items: extraneous and vocabulary given at previous grades

DOK: 2 Answer: B

Α	-6 is not a solution to the equation, so it cannot be an extraneous solution.
В	Correct. Substituting and simplifying, −2 ≠ 2, so this solution is extraneous.
С	May have interpreted "extraneous" as "not possible" and thought square root of 0 is not possible.
D	Ignored "extraneous" and found all apparent solutions.

Notes on grade expectations: When squaring both sides of an equation (or raising both sides to any power), an extraneous solution may be created. When using this method, students should check for extraneous solutions by checking the apparent solutions in the original equation.

$$x = \sqrt{x+6}$$

$$x^2 = (\sqrt{x+6})^2$$

$$x^2 = x+6$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3$$
; $x = ^{-}2$

Check apparent solutions in original equation:

Try
$$x = 3$$

 $3 = \sqrt{3+6}$
 $3 = \sqrt{9}$
 $3 = 3$

This is true, so x = 3 is a solution to the original equation.

Try
$$x = 2$$

 $-2 = \sqrt{-2+6}$
 $-2 = \sqrt{4}$
 $-2 = 2$

This is not true, so x = -2 is not a solution to the original equation. It is extraneous.

A quick analysis might be another approach to this problem. Since the left side of the equation, x, is equal to a positive square root, x must be a positive number. Therefore, any apparent solutions that have negative values must be extraneous.

The surface area of a baseball is 177 cm². What is the diameter of the baseball? (Use 3.14 for π.)

A. 3.75 cm

B. 7 cm

C. 7.5 cm

D. 14.1 cm

Benchmark: 9.3.1.1

Determine the surface area and volume of pyramids, cones and spheres. Use measuring devices or formulas as appropriate.

For example: Measure the height and radius of a cone and then use a formula to find its volume.

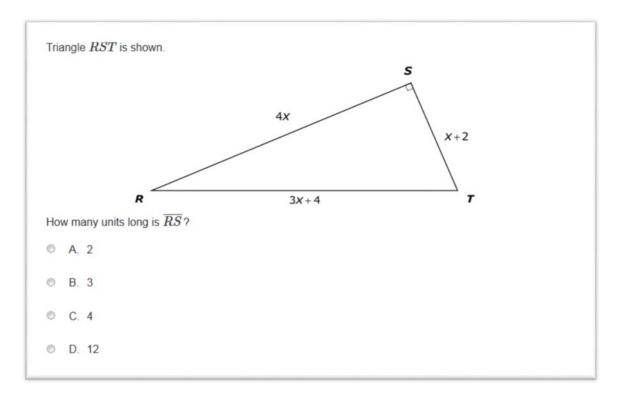
Item Specifications

• Vocabulary allowed in items: sphere and vocabulary given at previous grades

DOK: 2 Answer: C

Α	Mixed up radius with diameter; 3.75 is radius.
В	Divided by 2 instead of using the square root to find the radius. 177 = $4(3.14)r^2$; $r = 7.046$. Did not double radius to find diameter.
С	Correct. 177 = $4(3.14)r^2$; $r^2 \approx 14.09$; $r \approx 3.75$; $d = 2r = 2(3.75) = 7.5$
D	Solved for r^2 , but did not take square root of 14.1 when solving 177 = 4(3.14) r^2 for r .

Notes on grade expectations: Student should be able to choose the appropriate formula from the grade 11 formula sheet and calculate the surface area.



Benchmark: 9.3.4.7

Use algebra to solve geometric problems unrelated to coordinate geometry, such as solving for an unknown length in a figure involving similar triangles, or using the Pythagorean Theorem to obtain a quadratic equation for a length in a geometric figure.

Item Specifications

Vocabulary allowed in items: vocabulary given at previous grades

DOK: 2 Answer: D

Α	Solved $(2x+1) = 0$ incorrectly as $x = 1/2$ instead of $x = -1/2$; then used it to find RS: $4(1/2) = 2$.
В	Used value of <i>x</i> without multiplying by 4 to get length of <i>RS</i> .
С	Used 4 from 4x, the length of RS.
D	Correct. $x = 3$, and $RS = 4x$ or $4 \cdot 3$ which is 12.

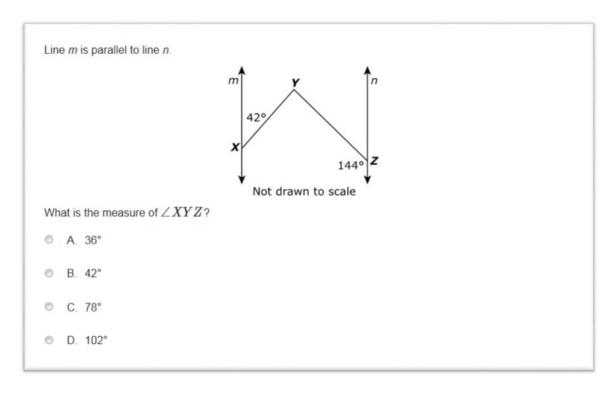
Notes on grade expectations: Student can use Pythagorean Theorem to write an equation to solve for x. $(4x)^2 + (x+2)^2 = (3x+4)^2$

$$8x^2 - 20x - 12 = 0$$

 $4(2x + 1)(x - 3) = 0$
 $x = -1/2, x = 3$

Another strategy is using guess and check by trying answer options A,B, C and D for RU to see if the results produce a right triangle.

12 = 4x; x = 3; 3 + 2 = 5; 3(3) + 4 = 13. Note that 5-12-13 is a Pythagorean Triple.



Benchmark: 9.3.3.1

Know and apply properties of parallel and perpendicular lines, including properties of angles formed by a transversal, to solve problems and logically justify results. *For example*: Prove that the perpendicular bisector of a line segment is the set of all points equidistant from the two endpoints, and use this fact to solve problems and justify other results.

Item Specifications

- Allowable notation: ⊥ (perpendicular), || (parallel)
- Vocabulary allowed in items: transversal, interior, exterior, corresponding, alternate and vocabulary given at previous grades

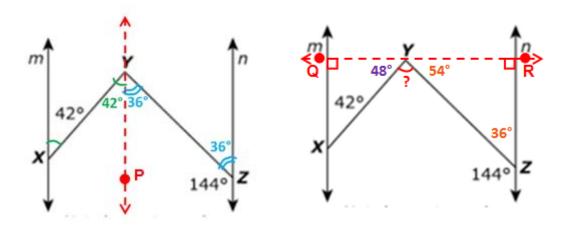
DOK: 3 Answer: C

Α	Used measure of angle adjacent to 144 instead of angle XYZ.
В	Used given angle or thought angle XYZ was congruent to given angle.
С	Correct.
D	Used measure of adjacent angle instead of angle XYZ.

Notes on grade expectations: By drawing a vertical line through angle XYZ, the rules for transversals may be used. Since alternate interior angles have equal measures, the measure of angle XYP must be 42° and the measure of angle ZYP must be 36°. The sum of XYP and ZYP gives the measure of angle XYZ, or 78°.

The second figure shows a different strategy for solving this item. By drawing a horizontal line through point Y, two right triangles are formed. Since the sum of the interior angles of a triangle is 180°, the measure of angle QYX is 48° and the measure of angle RYZ is 54°. A straight line has a measure of 180°, so the measure of angle XYZ can be found by subtracting: 180 - 48 - 54 to get 78°.

This is a DOK 3 item because the problem cannot be solved by simply making calculations with the numbers given, nor is there a formula that can be entered in the calculator that will calculate the angle. Prior knowledge that must be accessed includes standard 6.3.2. Prior experience with devising mathematically sound strategies is essential for solving DOK 3 items.



Roads connecting th	e towns of Oceanside, River City, and Lake View form a triangle. The distance from
Oceanside to River	City is 38 kilometers. The distance from River City to Lake View is 26 kilometers.
What is the smallest	possible whole number of kilometers between Lake View and Oceanside?
Enter your answer in	the box.
km	
km	

Benchmark: 9.3.3.3

Know and apply properties of equilateral, isosceles and scalene triangles to solve problems and logically justify results.

For example: Use the triangle inequality to prove that the perimeter of a quadrilateral is larger than the sum of the lengths of its diagonals.

Item Specifications

• Vocabulary allowed in items: equilateral, isosceles, scalene and vocabulary given at previous grades

DOK: 2 Answer:

This is a technology-enhanced item. The correct answer is shown. A student must type the correct answer in the box in order to receive 1 point.

100 S	
Roads	connecting the towns of Oceanside, River City, and Lake View form a triangle. The distance from
Oceans	ide to River City is 38 kilometers. The distance from River City to Lake View is 26 kilometers.
What is	the smallest possible whole number of kilometers between Lake View and Oceanside?
Enter yo	our answer in the box.
13	km

Notes on grade expectations: The Triangle Inequality Theorem states that the sum of two sides of a triangle must be greater than the third side. In this case, 26 + ? > 38. Since the missing side must be greater than 12, the smallest possible whole number is 13.

Note: The allowable characters that can be entered in the answer box are digits 0-9, fraction bar (/), decimal point (.), and negative sign (-). Students cannot enter a comma in numbers with more than 3 digits. Familiarity with calculators will help the students with this concept.

In figure RSTU, \overline{RT} and SU intersect at point M, so that $\overline{RM} \cong \overline{TM}$ and RS is parallel to \overline{TU} . Which additional information is needed to prove that figure RSTU is a rectangle?

- lacktriangledown A. $\overline{RU}{\cong}\overline{RS}$
- \odot B. $\overline{UM}{\cong}\overline{MS}$
- C. ∠RST≅∠STU
- \odot D. $\angle RUT \cong \angle TSR$

Benchmark: 9.3.3.7

Use properties of polygons—including quadrilaterals and regular polygons—to define them, classify them, solve problems and logically justify results.

For example: Recognize that a rectangle is a special case of a trapezoid.

Another example: Give a concise and clear definition of a kite.

Item Specifications

• Vocabulary allowed in items: regular polygon, isosceles and vocabulary given at previous grades

DOK: 2 Answer: C

Α	This would show that the figure is a rhombus, but angles must be 90 degrees for it to be a rectangle.
В	This would show that the figure is a rhombus, rectangle or square. Angles must be 90 degrees for it to be a rectangle.
С	Correct. The congruent consecutive angles differentiate this as a rectangle and not just a parallelogram.
D	This would show that the figure is a parallelogram, but angles must be 90 degrees for it to be a rectangle.

Maggie and Wei are measuring the distance across a circular fountain indirectly as shown in the diagram. S They find that the length of \overline{RS} is 15 meters and the length of \overline{ST} is 9 meters. \overline{RS} is tangent to circle F and point T is on \overline{FS} . To the nearest meter, what is the diameter of the fountain? Enter your answer in the box. $\boxed{ }$ meters

Benchmark: 9.3.3.8

Know and apply properties of a circle to solve problems and logically justify results. *For example*: Show that opposite angles of a quadrilateral inscribed in a circle are supplementary.

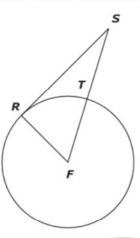
Item Specifications

• Vocabulary allowed in items: arc, central angle, inscribed, circumscribed, tangent, chord and vocabulary given at previous grades

DOK: 2 Answer:

This is a technology-enhanced item. The correct answer is shown. A student must type the correct answer in the box in order to receive 1 point.

Maggie and Wei are measuring the distance across a circular fountain indirectly as shown in the diagram.



They find that the length of \overline{RS} is 15 meters and the length of \overline{ST} is 9 meters. \overline{RS} is tangent to circle F and point T is on \overline{FS} . To the nearest meter, what is the diameter of the fountain?

Enter your answer in the box.

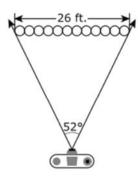
16 meters

Notes on grade expectations: Student should know that a line tangent to a circle is perpendicular to the radius at the point of tangency, so angle *SRF* is a right angle. Because this makes the triangle a right triangle, the Pythagorean Theorem may be used to solve for the missing length. Segments *RF* and *TF* are both radii of the circle, so they have equal length.

Use $15^2 + r^2 = (r + 9)^2$ to solve for the radius, r, then double the radius to find the diameter.

Note: The allowable characters that can be entered in the answer box are digits 0-9, fraction bar (/), decimal point (.), and negative sign (-). Students cannot enter a comma in numbers with more than 3 digits. Familiarity with calculators will help the students with this concept.

Twelve students are lined up to have their class picture taken.



The photographer's camera has a picture angle of 52°. The picture angle limits the width of the photo that can be taken. The line of students is approximately 26 feet long. About how far must the photographer be from the line of students in order to center all 12 students in the picture?

- A. 15 feet
- B. 27 feet
- C. 30 feet
- D. 53 feet

Benchmark: 9.3.4.2

Apply the trigonometric ratios sine, cosine and tangent to solve problems, such as determining lengths and areas in right triangles and in figures that can be decomposed into right triangles. Know how to use calculators, tables or other technology to evaluate trigonometric ratios.

For example: Find the area of a triangle, given the measure of one of its acute angles and the lengths of the two sides that form that angle.

Item Specifications

• Vocabulary allowed in items: trigonometric ratios, sine, cosine, tangent and vocabulary given at previous grades

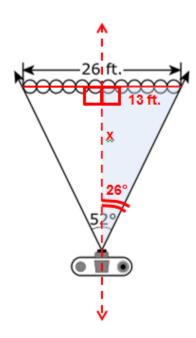
DOK: 3 Answer: B

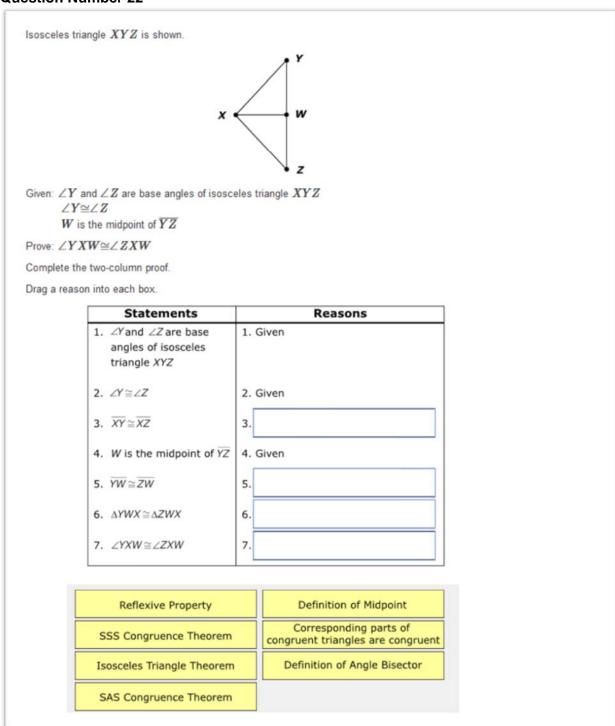
Α	Used cosine instead of tangent. Solved $\cos 26 = 13/x$.	
В	Correct. Split isosceles triangle into two right triangles, then solved tan 26 = 13/x.	
С	Used sine instead of tangent. Solved $\sin 26 = 13/x$.	
D	Split isosceles triangle into two right triangles, but used 26 for length instead of 13. Solved tan $26 = 26/x$.	

Notes on grade expectations: Draw a line through the vertex perpendicular to the base to divide the isoscles triangle into two right triangles. The length of the base of each right triangle is half the length of the total base, or 13 feet. The vertex gets cut in half, leaving an angle of 26° . See markings on figure below. The unknown distance is marked with x. Use tangent of 26° = 13/x to solve for x. (tan 26° =0.4877)

Note: Make sure your calculator is in degree mode.

This is a DOK 3 item because scaffolding is not done for the student. Prior experience with devising mathematically sound strategies is essential for solving DOK 3 items. Once the strategy for solving this item is chosen, then the steps to the solution are a straightforward application of the skills in the benchmark.





Benchmark: 9.3.2.4

Construct logical arguments and write proofs of theorems and other results in geometry, including proofs by contradiction. Express proofs in a form that clearly justifies the reasoning, such as two-column proofs, paragraph proofs, flow charts or illustrations.

For example: Prove that the sum of the interior angles of a pentagon is 540° using the fact that the sum of the interior angles of a triangle is 180°.

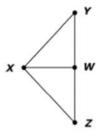
Item Specifications

- Allowable notation: CPCTC (Corresponding Parts of Congruent Triangles are Congruent), CPCFC (Corresponding Parts of Congruent Figures are Congruent), SAS (Side-Angle-Side), SSS (Side-Side-Side), ASA (Angle-Side-Angle)
- Vocabulary allowed in items: contradiction and vocabulary given at previous grades

DOK: 2 Answer:

This is a technology-enhanced item. The correct answer is shown. A student must place the correct justifications into the correct boxes in order to receive 1 point.

Isosceles triangle XYZ is shown.



Given: $\angle Y$ and $\angle Z$ are base angles of isosceles triangle XYZ

 $\angle Y \cong \angle Z$

W is the midpoint of \overline{YZ}

Prove: $\angle YXW \cong \angle ZXW$

Complete the two-column proof.

Drag a reason into each box.

Statements	Reasons
 ∠Yand ∠Z are base angles of isosceles triangle XYZ 	1. Given
2. ∠Y≅∠Z	2. Given
3. <i>XY</i> ≅ <i>XZ</i>	Isosceles Triangle Theorem
4. W is the midpoint of \overline{YZ}	4. Given
5. <i>YW</i> ≅ <i>ZW</i>	5. Definition of Midpoint
6. Δ <i>YWX</i> ≅ Δ <i>ZWX</i>	6. SAS Congruence Theorem
7. ∠ <i>YXW</i> ≅∠ <i>ZXW</i>	7. Corresponding parts of congruent triangles are congruent

Reflexive Property

SSS Congruence Theorem

Definition of Angle Bisector

An archaeologist used string to make a grid over an area where she was digging. She discovered a bowl buried at (1, 4) and a tool at (3, 10) on her grid. She suspected that an item was buried at the midpoint of the segment connecting the bowl and the tool. At which position on her grid should the archaeologist dig to look for the buried item?

- O A. (1, 3)
- ◎ B. (2, 7)
- \odot C. $\left(\frac{5}{2}, \frac{13}{2}\right)$
- O D. (4, 14)

Benchmark: 9.3.4.4

Use coordinate geometry to represent and analyze line segments and polygons, including determining lengths, midpoints and slopes of line segments.

Item Specifications

• Vocabulary allowed in items: midpoint and vocabulary given at previous grades

DOK: 2 Answer: B

Α	Used differences instead of sums to find averages for x-value and y-value.
В	Correct. Averaged x's to get x-value and averaged y's to get y-value of midpoint.
С	Averaged 1 and 4 to get x-value and averaged 3 and 10 to get y-value.
D	Added x-values and added y-values, but did not divide by 2 to find averages.

A group of he	ealth care providers consists of 4 doctors, 3 dentists, and 5 nurses. How many combinations
of 2 health ca	are providers of different types are possible?
Enter your ar	nswer in the box.

Benchmark: 9.4.3.1

Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.

For example: If one girl and one boy are picked at random from a class with 20 girls and 15 boys, there are $20 \times 15 = 300$ different possibilities, so the probability that a particular girl is chosen together with a particular boy is 1/300.

Item Specifications

Vocabulary allowed in items: vocabulary given at previous grades

DOK: 3 Answer:

This is a technology-enhanced item. The correct answer is shown. A student must type the correct answer in the box in order to receive 1 point.

A group of health care providers consists of 4 doctors, 3 dentists, and 5 nurses. How many combinations of 2 health care providers of different types are possible?	
Enter your answer in the box.	
47	

Notes on grade expectations: The correct answer of 47 can be found as follows. (1 doctor and 1 dentist) or (1 dentist and 1 nurse) or (1 nurse and 1 doctor) (4×3) + (3×5) + (5×4)

Students may be tempted to multiply $4 \times 3 \times 5$ to get 60 as the total number of possible combinations. Careful reading of the problem shows that the 2 health care providers must be "of different types," so this eliminates combinations such as doctor-doctor, nurse-nurse, dentist-dentist. The essential information is provided in this item but the scaffolding is not done for the student. The student must have a good grasp of the concept of sample space. Benchmark 6.4.1.1 is the basis for students' prior knowledge.

Note: The allowable characters that can be entered in the answer box are digits 0-9, fraction bar (/), decimal point (.), and negative sign (-). Students cannot enter a comma in numbers with more than 3 digits. Familiarity with calculators will help the students with this concept.

Isabella flipped a fair coin 100 times. Which statement about Isabella's outcomes is most likely true?

- A. The coin landed heads up 50 times and tails up 50 times.
- B. The number of times the coin landed heads up was less than 50.
- C. If Isabella continues to flip the coin, the experimental probability of the coin landing heads up will increase.
- © D. If Isabella continues to flip the coin, the experimental probability of the coin landing heads up will approach $\frac{1}{2}$.

Benchmark: 9.4.3.3

Understand that the Law of Large Numbers expresses a relationship between the probabilities in a probability model and the experimental probabilities found by performing simulations or experiments involving the model.

Item Specifications

• Vocabulary allowed in items: simulation and vocabulary given at previous grades

DOK: 2 Answer: D

Α	Used the Law of Large Numbers to predict outcomes of a small sample.
В	Guessed at the outcome.
С	Guessed at the outcome.
D	Correct. Per the Law of Large Numbers, as more trials are performed, the experimental probability will approach the theoretical probability.

Eli receives	shipment of 40 new books for his bookstore: 5 biographies, 12 mysteries, 10 romances, 11
technical boo	ks, and 2 cookbooks. Eli randomly picks 2 books from the shipment. What is the probability
that he picks	a biography first and then picks a technical book?
Enter your a	swer in the box.
	٦

Benchmark: 9.4.3.5

Apply probability concepts such as intersections, unions and complements of events, and conditional probability and independence, to calculate probabilities and solve problems.

For example: The probability of tossing at least one head when flipping a fair coin three times can be calculated by looking at the complement of this event (flipping three tails in a row).

Item Specifications

• Vocabulary allowed in items: intersections, unions, complements of events, conditional and vocabulary given at previous grades

DOK: 2 Answer:

This is a technology-enhanced item. The correct answer is shown. A student must type the correct answer in the box in order to receive 1 point.

technical books, and 2 cookbooks. Eli randomly picks 2 books from the shipment. What is the probability that he picks a biography first and then picks a technical book?
that he picks a biography first and then picks a technical book?
Enter your answer in the box.
55/1560

Notes on grade expectations: (Number of biographies ÷ total number of books) × (Number of technical books ÷ Number of remaining books) (5/40) × (11/39) = 55/1,560. Note that any equivalent form of the numerical answer such as 11/312 or 0.035 will receive 1 point.

Note: The allowable characters that can be entered in the answer box are digits 0-9, fraction bar (/), decimal point (.), and negative sign (-). Students cannot enter a comma in numbers with more than 3 digits. Familiarity with calculators will help the students with this concept.

Reginald is designing an outdoor art exhibit. He needs a metal equilateral triangle that measures 40 inches on each side. He wants to cut the triangle from a rectangular piece of metal that is 40 inches long. What is the minimum width of the rectangle Reginald needs to be able to cut out the triangle?

- \bigcirc A. $\frac{20\sqrt{3}}{3}$ inches
- \odot B. $20\sqrt{2}$ inches
- \odot C. $20\sqrt{3}$ inches
- D. 40 inches

Benchmark: 9.3.3.5

Know and apply properties of right triangles, including properties of 45-45-90 and 30-60-90 triangles, to solve problems and logically justify results.

For example: Use 30-60-90 triangles to analyze geometric figures involving equilateral triangles and hexagons.

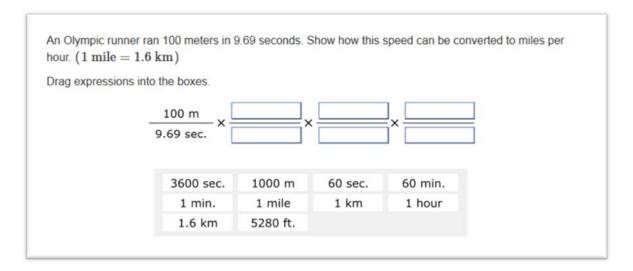
Another example: Determine exact values of the trigonometric ratios in these special triangles using relationships among the side lengths.

Item Specifications

Vocabulary allowed in items: vocabulary given at previous grades

DOK: 2 Answer: C

А	Divided 40 by 2 to get 20, then divided 20 by ($\sqrt{3}$) instead of multiplying by ($\sqrt{3}$).
В	Multiplied by square root of 2 instead of square root of 3.
С	Correct. Split equilateral triangle into two congruent 30-60-90 triangles, then
	found length of long side; $40(\sqrt{3})/2$.
D	Used length of sides of equilateral triangle instead of altitude.



Benchmark: 9.3.1.3

Understand that quantities associated with physical measurements must be assigned units; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert between measurement systems.

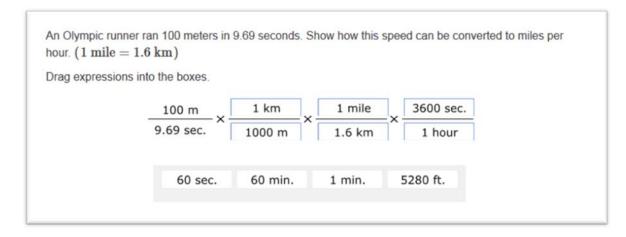
For example: 60 miles/hour = 60 miles/hour × 5280 feet/mile × 1 hour/3600 seconds = 88 feet/second.

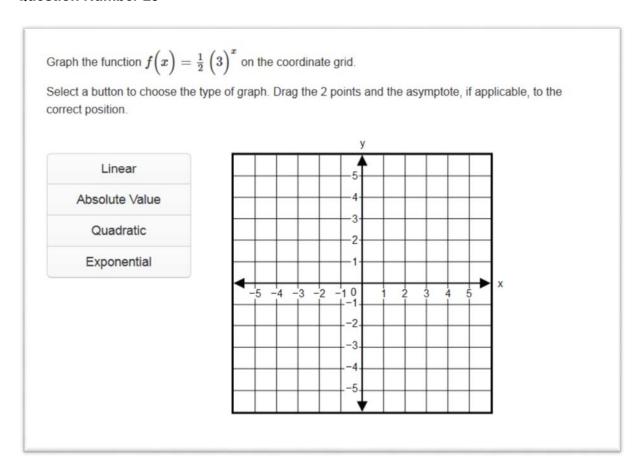
Item Specifications

Vocabulary allowed in items: vocabulary given at previous grades

DOK: 2 Answer:

This is a technology-enhanced item. A correct answer is shown. A student must place the correct labeled numbers into the correct boxes in order to receive 1 point. The rates may be in any order, i.e., "1 km/1,000 m" may be in the first pair of boxes, the second pair, or the third pair.





Benchmark: 9.2.2.3

Sketch graphs of linear, quadratic and exponential functions, and translate between graphs, tables and symbolic representations. Know how to use graphing technology to graph these functions.

Item Specifications

- Items do not require the use of graphing technology
- Vocabulary allowed in items: quadratic, exponential and vocabulary given at previous grades

DOK: 2 Answer:

This is a technology-enhanced item. A correct answer is shown. A student must select the correct type of graph, plot two correct points, and correctly place the asymptote in order to receive 1 point.

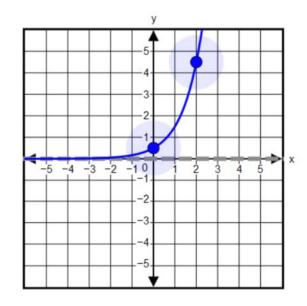
Graph the function $f\!\left(x
ight) = rac{1}{2} \left(3
ight)^x$ on the coordinate grid.

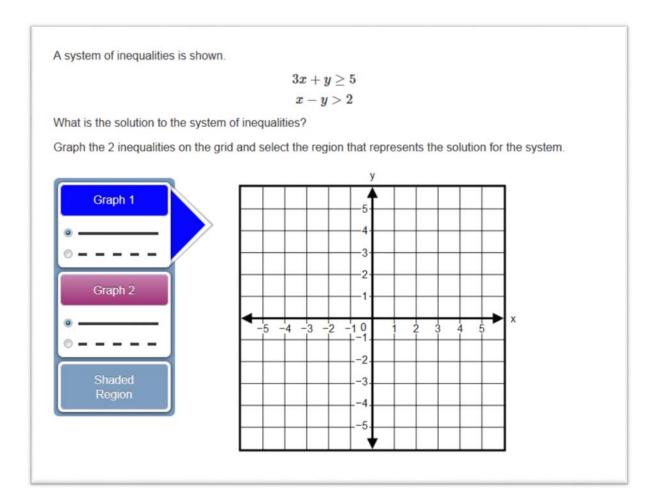
Select a button to choose the type of graph. Drag the 2 points and the asymptote, if applicable, to the correct position.

Absolute Value

Quadratic

Exponential





Benchmark: 9.2.4.4

Represent relationships in various contexts using systems of linear inequalities; solve them graphically. Indicate which parts of the boundry are included in and excluded from the solution set using solid and dotted lines.

Item Specifications

• Vocabulary allowed in items: boundry and vocabulary given at previous grades

DOK: 2 Answer:

This is a technology-enhanced item. A correct answer is shown. A student must correctly graph the system of inequalities in order to receive 1 point. The equations may be graphed in either order, i.e., $3x + y \ge 5$ may be represented by Graph 1 or Graph 2.

A system of inequalities is shown.

$$3x+y\geq 5$$

$$x-y>2$$

What is the solution to the system of inequalities?

Graph the 2 inequalities on the grid and select the region that represents the solution for the system.

