SENIOR SECONDARY IMPROVEMENT PROGRAMME 2013

education
Department: Education
GAUTENG PROVINCE

GRADE 12

MATHEMATICS

LEARNER HOMEWORK SOLUTIONS

The SSIP is supported by

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LEARNER HOMEWORK SOLUTIONS

SOLUTIONS TO HOMEWORK: SESSION

16.1 TOPIC: DATA HANDLING

QUESTION 1

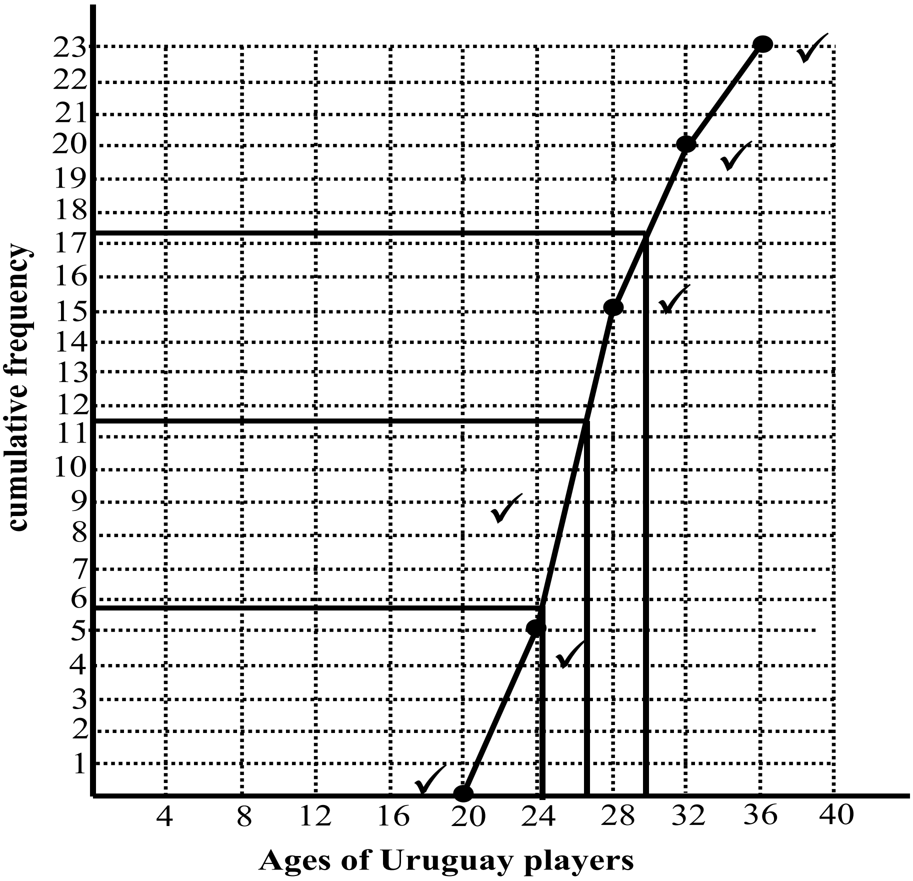
(a)

Class intervals (ages)	Frequency ✓	Cumulative frequency ✓
$16 \le x < 20$	0	0
$20 \le x < 24$	5	5
$24 \le x < 28$	10	15
$28 \le x < 32$	5	20
$32 \le x < 36$	3	23

(2)

(b)

Class intervals (ages)	Frequency	Cumulative frequency	Graph points
$16 \le x < 20$	0	0	$(20\,;0)$
$20 \le x < 24$	5	5	$(24\,;5)$
$24 \le x < 28$	10	15	$(28\,;15)$
$28 \le x < 32$	5	20	$(32\,;20)$
$32 \le x < 36$	3	23	$(36\,;23)$



(6)

(c)

Lower quartile $23 \times \frac{1}{4} = 5{,}75$ Therefore $Q_1 = 24$	✓
Median $23 \times \frac{1}{2} = 11.5$ Therefore Median = 26	✓
Upper quartile $23 \times \frac{3}{4} = 17.25$ Therefore $Q_3 = 30$	✓ (3)

[11]

QUESTION 2

(a)

Class intervals	Frequency (f)	Midpoint (m)	$f \times m$ ✓	$m - \overline{x}$ ✓	$(m-\overline{x})^2$ ✓	$f \times (m-\overline{x})^2$ ✓
$20 \leq x < 24$	5	22	110	-5	25	125
$24 \leq x < 28$	10	26	260	-1	1	10
$28 \leq x < 32$	5	30	150	3	9	45
$32 \leq x < 36$	3	34	102	7	49	147
			$\overline{x} = \frac{622}{23} = 27$ ✓			$\sum f \times (m-\overline{x})^2 = 327$

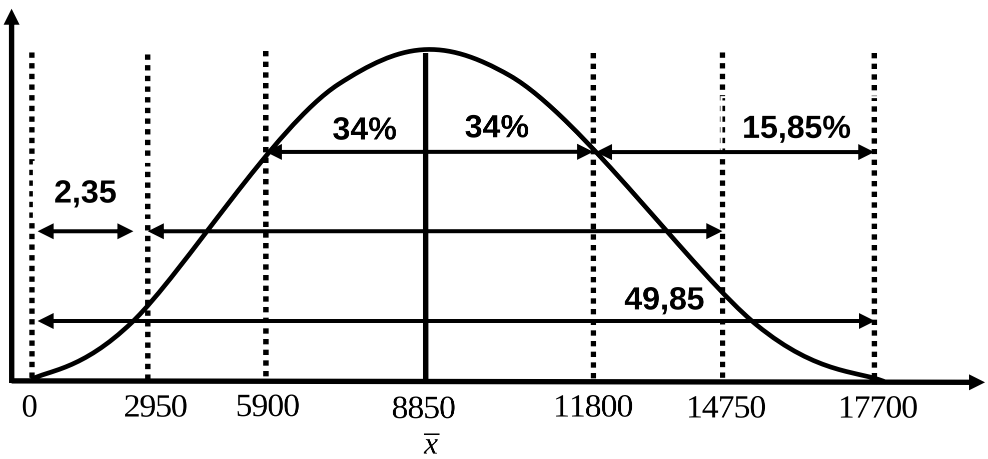
(5)

(b) $\text{SD} = \sqrt{\frac{\sum f.(x-\overline{x})^2}{23}} = \sqrt{\frac{327}{23}} = 3{,}8$	✓✓ (2)

(c) **CASIO fx-82ES PLUS:** MODE 2 : STAT 1 : 1 – VAR SHIFT SETUP 3: STAT 1: ON Enter the midpoints: 22= 26= 30= 34= Enter the frequencies: 5= 10= 5= 3= AC SHIFT 1 4: VAR 3 : $x\sigma n$ = The answer will read: 3,8 **SHARP DAL:** MODE 1= Enter data: 22 STO 3 M+ 26 STO 9 M+ 30 STO 8 M+ 34 STO 3 M+ RCL 6 to get 3,8	✓✓ (2)

[9]

QUESTION 3



One standard deviation interval:

$(\overline{x} - s\,;\,\overline{x} + s)$

$= (8850 - 2950 ; 8850 + 2950)$

$= (5900 ; 11800)$

Two standard deviation intervals:

$(\overline{x} - 2s\,;\,\overline{x} + 2s)$

$= (8850 - 2 \times 2950 ; 8850 + 2 \times 2950)$

$= (2950 ; 14750)$

Three standard deviation intervals:

$(\overline{x} - 3s \, ; \, \overline{x} + 3s)$

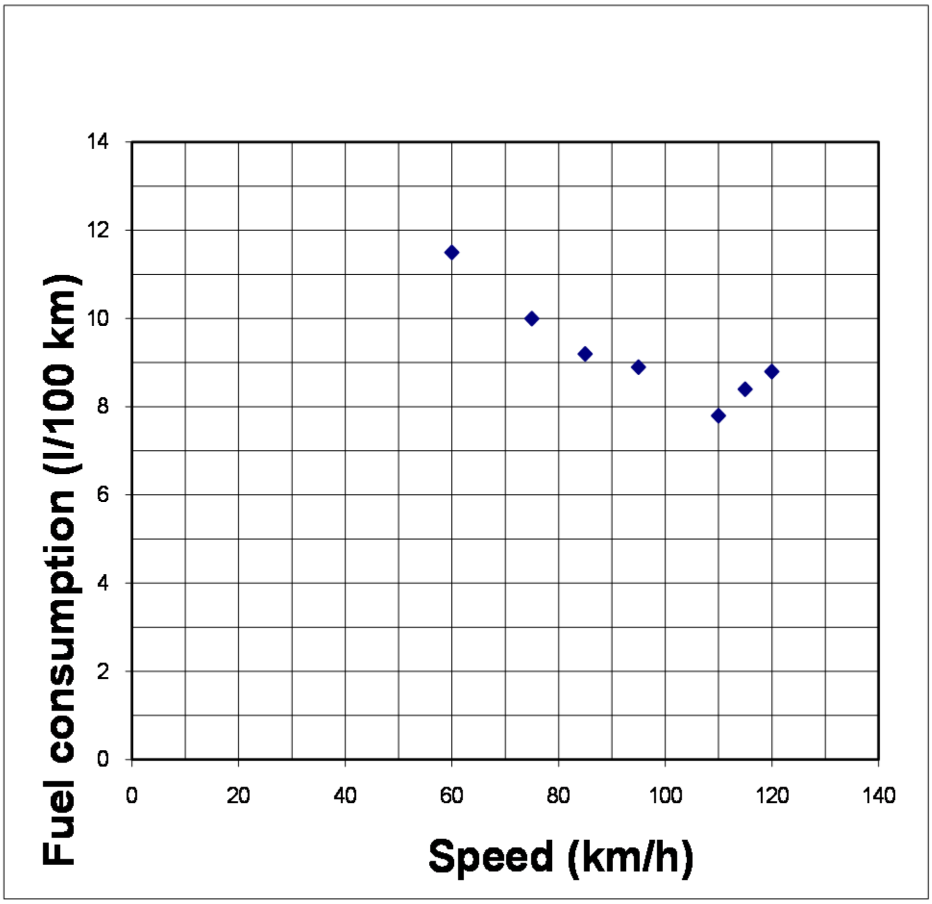
$= (8850 - 3 \times 2950 ; 8850 + 3 \times 2950)$

$= (0;17700)$

2%	✓✓	(2)
16%	✓	(1)
No, since there are some employees (less than 2%) earn below R3000,00. These employees will not live an acceptable lifestyle economically. **OR** Yes, there is a fair distribution of salaries since the majority of the employees,i.e. 68% earn a salary between R5 900 and R11 800 per month. Some employees will have more responsibilities or work longer hours and thus must be compensated accordingly. Less than 2% earn below R3000,00.	✓	(1) **[4]**

QUESTION 4

a.



✓
✓
✓

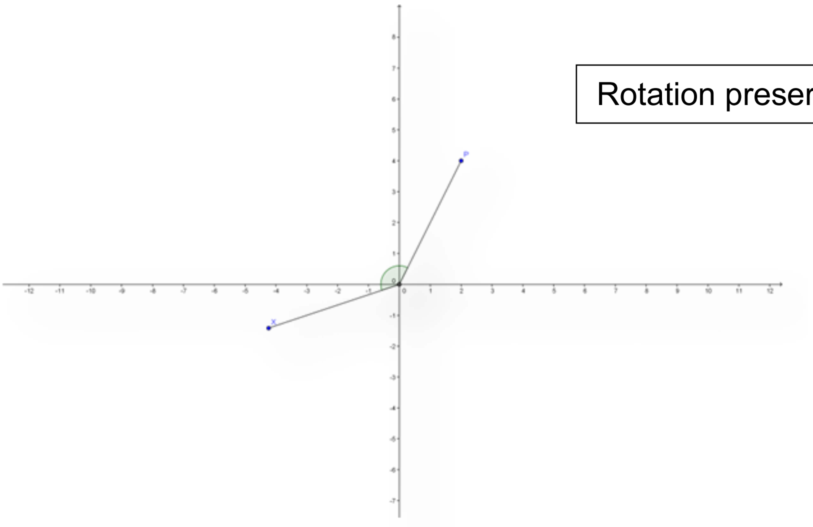
(3)

(b)	Quadratic	✓	(1)
(c)	Based on the quadratic trend the best fuel consumption occurs when the car is driven at 110 km/h. To keep its fuel bill to a minimum, drivers should drive at 110km/h	✓✓	(2)

[6]

SOLUTIONS TO HOMEWORK: SESSION 16.2 TOPIC : TRANSFORMATIONS

QUESTION 1



Rotation preserved

a) OP=OX

$(2-0)^2+(4-0)^2=\left(-3\sqrt{2}-0\right)^2+(y)^2$

20=18 $+y^2$

$\therefore y^2=2$

$\therefore$ $y=\pm\sqrt{2}$ but y<0 $\therefore$ $y=-\sqrt{2}$

X$(-3\sqrt{2};-\sqrt{2})$

b) x' = $x_A cos\theta - y_A sin\theta$ and y' = $y_A cos\theta + x_A sin\theta$

$-3\sqrt{2} = 2cos\theta - 4sin\theta$ ......(1) $-\sqrt{2} = 4cos\theta + 2sin\theta$ ....(2)

Multiply equation (1) by -2 and then add the equations

$6\sqrt{2} = -4cos\theta + 8sin\theta$

$\underline{-\sqrt{2} = 4cos\theta + 2sin\theta}$

$5\sqrt{2} = 10sin\theta$

$sin\theta = \frac{\sqrt{2}}{2}$

$\therefore$ θ = 45° but since θ is obtuse θ = 135°

QUESTION 2

2.1

$(4)^2+(3)^2=r^2$

$\therefore r^2=25$

$\therefore r=5$

2.2

$$4\cos\theta - 3\sin\theta = \frac{4\sqrt{3}-3}{2}......A$$

$$3\cos\theta + 4\sin\theta = \frac{3\sqrt{3}+4}{2}.......B$$

$$16\cos\theta - 12\sin\theta = 2(4\sqrt{3}-3)......A\times 4$$

$$9\cos\theta + 12\sin\theta = \frac{3(3\sqrt{3}+4)}{2}.......B\times 3$$

$$\therefore 25\cos\theta = 2(4\sqrt{3}-3) + \frac{3(3\sqrt{3}+4)}{2}$$

$$\therefore 25\cos\theta = \frac{25\sqrt{3}}{2}$$

$$\therefore \cos\theta = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = 30°$$

2.3

$$AB^2 = (5)^2 + (5)^2 - 2(5)(5)\cos 30°$$

$$\therefore AB^2 = 50 - 50\left(\frac{\sqrt{3}}{2}\right)$$

$$\therefore AB^2 = 50 - 25\sqrt{3}$$

$$\therefore AB^2 = 25(2-\sqrt{3})$$

$$\therefore AB = 5\sqrt{2-\sqrt{3}}$$

2.4

$$\text{Area } \Delta OAB = \frac{1}{2}(5)(5)\sin 30°$$

$$\therefore \text{Area } \Delta OAB = \frac{25}{4} \text{ units}^2$$

QUESTION 3

3.1 X(-6; 0) Y(3, 6) and Z(6; -6)

3.2 Here you will use Analytical geometry to help work out the angles of inclination

Mxy = ⅔ and	MYZ = -4	
Tanθ = ⅔	tanβ = -4	
Θ = 33.69….	β = 104.03…..	∴ α = 75.96…

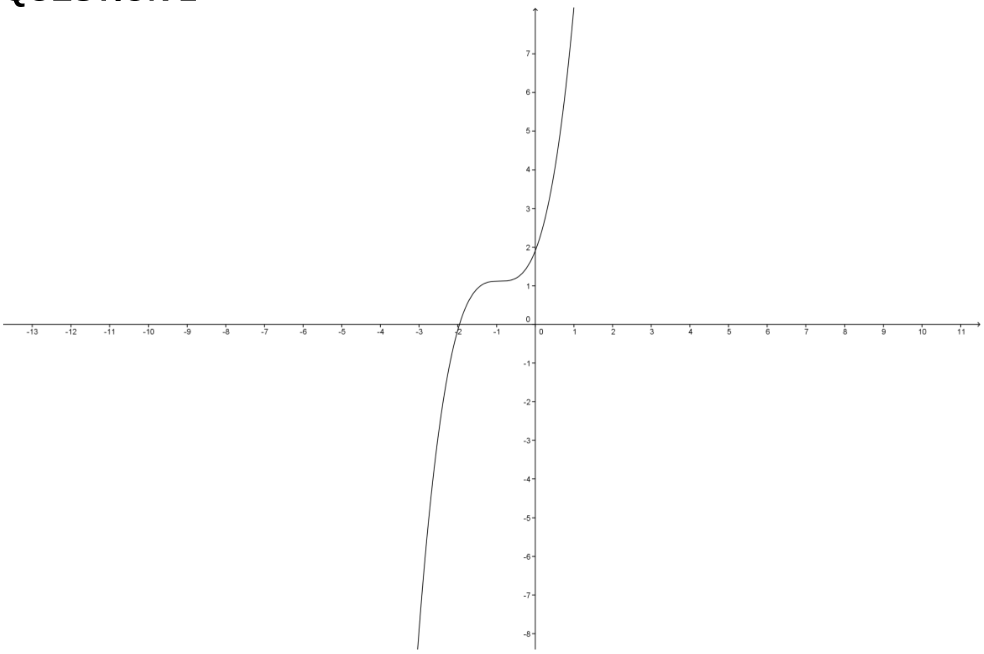
∴ $\hat{Y}$ = 180 – (75.96+33.69) = 70.4°

SOLUTIONS TO HOMEWORK:
SESSION 17.1 TOPIC : FUNCTIONS

QUESTION 1

$$f(x) = 2x$$
$$f\left(\frac{1}{x}\right) = 2\left(\frac{1}{x}\right)$$
$$\frac{1}{f(x)} = \frac{1}{2x}$$
$f^{-1}(x) = \frac{1}{2}x$ y=2x swop x and y to find inverse: x = 2y so y =½x
$$f(x) + f\left(\frac{1}{x}\right) + \frac{1}{f(x)} + f^{-1}(x) = 2x + \frac{2}{x} + \frac{1}{2x} + \frac{1}{2}x$$
$$= \frac{5x^2+5}{2x}$$
[6]

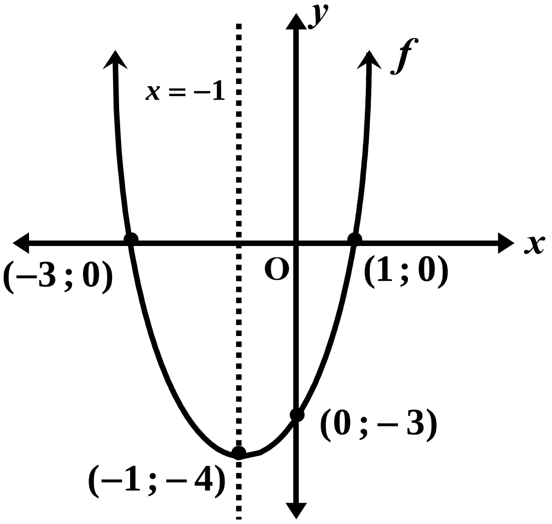
QUESTION 2



[5]

QUESTION 3

3.1



(6)

3.1.1 Range: $y \in [-4\,;\infty)$ (2)

[8]

QUESTION 4

4.1

$$y = a^x$$
$$\therefore \frac{1}{4} = a^2$$
$$\therefore a = \frac{1}{2}$$

(2)

4.2

$$y = \left(\frac{1}{2}\right)^x$$
$$\therefore x = \left(\frac{1}{2}\right)^y$$
$$\therefore y = \log_{\frac{1}{2}} x$$

(2)

4.3

$$y = \left(\frac{1}{2}\right)^x$$

(1)

4.4

$$y = 4x^2$$
$$\therefore x = 4y^2$$
$$\therefore \frac{x}{4} = y^2$$
$$\therefore y = \pm\sqrt{\frac{x}{4}}$$

(2)

4.5 $x > 0$ or $x < 0$ (2)

[9]

QUESTION 5

5.1 g(-½) = -1

$\log_a \frac{1}{2} = -1$

$\therefore a^{-1} = \frac{1}{2}$

$\therefore a = 2$ (2)

5.2 x> 0 and x≠ 1 (NB: The graph of g is only drawn for 0<x<1 but this is not the domain) (2)

5.3 $g^{-1}(x) = 2^x$

$x \in \mathbb{R}, x \neq 0$ (NB: From the log graph x≠ 1 so its' inverse will have y ≠ 1 the value that will make y=1 in $g^{-1}(x)$ is x=0 so it must be excluded from the domain.) (2)

[6]

SOLUTIONS TO HOMEWORK:
SESSION 17.2 TOPIC: CALCULUS

QUESTION 1

1.1.1	$P = 2h + 2r + \frac{1}{2} \times 2\pi r$ $\therefore P = 2h + 2r + \pi r$	✓ $2h + 2r$ ✓ πr (2)
1.1.2	$A = 2rh + \frac{1}{2}\pi r^2$	✓ $2rh$ ✓ $\frac{1}{2}\pi r^2$ (2)
1.2	$4 = 2rh + \frac{1}{2}\pi r^2$ $\therefore 8 = 4rh + \pi r^2$ $\therefore 8 - \pi r^2 = 4rh$ $\therefore \frac{8 - \pi r^2}{4r} = h$ $P = 2h + 2r + \pi r$ $\therefore P = 2\left(\frac{8 - \pi r^2}{4r}\right) + 2r + \pi r$ $\therefore P = \frac{8 - \pi r^2}{2r} + 2r + \pi r$ $\therefore P = \frac{4}{r} - \frac{\pi r}{2} + 2r + \pi r$ $\therefore P = \frac{4}{r} + \frac{\pi r}{2} + 2r$ $\therefore P = \frac{4}{r} + \left(\frac{\pi}{2} + 2\right)r$ $\therefore P = \left(\frac{\pi}{2} + 2\right)r + \frac{4}{r}$	✓ $4 = 2rh + \frac{1}{2}\pi r^2$ ✓ $\frac{8 - \pi r^2}{4r} = h$ ✓ $P = 2\left(\frac{8 - \pi r^2}{4r}\right) + 2r + \pi r$ ✓ $P = \left(\frac{\pi}{2} + 2\right)r + \frac{4}{r}$ (4)

1.3	$C=10\left(\frac{\pi}{2}+2\right)r+\frac{40}{r}$ $\therefore C=5\pi r+20r+40r^{-1}$ $\therefore C'(r)=5\pi+20-40r^{-2}$ $\therefore C'(r)=5\pi+20-\frac{40}{r^2}$ $\therefore 0=5\pi+20-\frac{40}{r^2}$ $\therefore \frac{40}{r^2}=5\pi+20$ $\therefore \frac{40}{5\pi+20}=r^2$ $\therefore \sqrt{\frac{40}{5\pi+20}}=r$ $\therefore r=1{,}06\text{m}$	✓ $C=10\left(\frac{\pi}{2}+2\right)r+\frac{40}{r}$ ✓ $C=5\pi r+20r+40r^{-1}$ ✓ $0=5\pi+20-\frac{40}{r^2}$ ✓ $r=1{,}06\text{m}$ (4) **[12]**

QUESTION 2

2.1.	$At\ A\ and\ B:\ f'(x)=0$ $f'(x)=12x^2+54x-30=0$ $2x^2+9x-5=0$ $(2x-1)(x+5)=0$ $x=\frac{1}{2}\ \ or\ \ x=-5$ $f\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)^3+27\left(\frac{1}{2}\right)^2-30\left(\frac{1}{2}\right)-1$ $=\frac{-35}{4}(-8{,}75)$ $f(-5)=4(-5)^3+27(-5)^2-30(-5)-1$ $=324$ $\therefore A(-5;324),\quad B\left(\frac{1}{2};\frac{-35}{4}\right)$	✓$=0$ ✓substitution of *x* values ✓✓$;324)$ ✓✓$\frac{-35}{4})$ (6)
2.2.	$Ave\ Grad=\frac{324-\left(\frac{-35}{4}\right)}{-5-\frac{1}{2}}$ $=\frac{-121}{2}\quad(-60{,}5)$	✓subs *x* and *y* values ✓$\frac{1}{-}$ $(-60{,}5)$ (2)
2.3.	$C(0;-1)$ $f'(0)=-30$ $Equ.of\ tangent: y=-30x-1$	✓$-1)$ ✓$=-30$ ✓ $-30x-1$ (3)

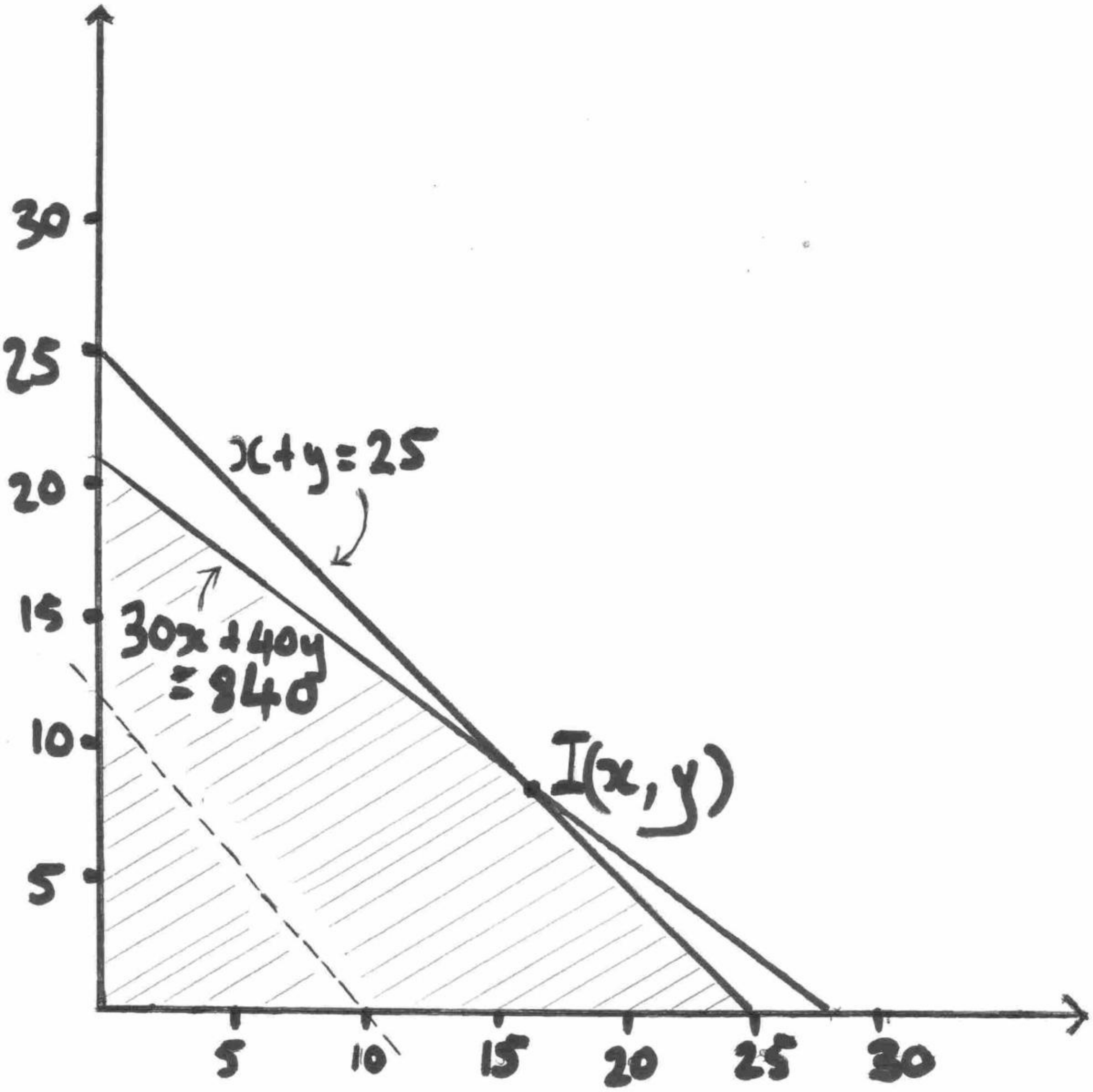
2.4.	$4x^3 + 27x^2 - 30x - 1 = -30x - 1$ $4x^3 + 27x^2 = 0$ $x^2(4x + 27) = 0$ $x = 0 \quad or \quad x = -\frac{27}{4}$ $\therefore x = \frac{-27}{4}$	✓cubic=tangent ✓$: + 27) = 0$ ✓$\frac{27}{4}$ (3)

[14]

SOLUTIONS TO HOMEWORK: SESSION 18
TOPIC : LINEAR PROGRAMMING

QUESTION 1

1.1	$x + y \leq 25$ $30x + 40y \leq 840$ $x \leq 0$ $y \leq 0$ $x, y \in \mathbb{N}$	✓ $x + y \leq 25$ ✓ $30x + 40y \leq 840$ ✓ $x \leq 0$, $y \leq 0$, $x, y \in \mathbb{N}$ (3)
1.2	see diagram on next page	✓ $x + y \leq 25$ ✓ $30x + 40y \leq 840$ ✓ $x \leq 0$ ✓ $y \leq 0$ ✓ $x, y \in \mathbb{N}$ (5)
1.3	$10x + 12y = P$ $\therefore y = -1.2x + \frac{P}{12}$ Intersection $I(x, y)$of $x + y = 25$ and $30x + 40y = 840$ $\therefore I(x, y) = (16, 9)$ Max at either $I(x, y)$or $(25, 0)$ Max at $I(x, y), P = 268$ $\therefore x = 16, y = 9$	✓ $10x + 12y = P$ ✓ Intersection $I(x, y)$of $x + y = 25$ and $30x + 40y = 840$ ✓ Check P at $I(x, y)$ and $(25, 0)$ ✓ Max at $I(x, y), P = 268$ (4)



[12]

SOLUTIONS TO HOMEWORK:
SESSION 18.2 TOPIC : TRIGONOMETRY

QUESTION 1

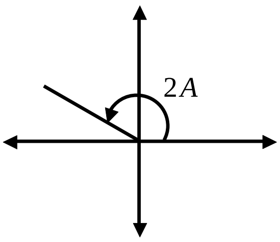
$$\frac{\sin(-145°).\cos(-215°)}{\sin 510°.\cos 340°}$$
$$= \frac{(-\sin 145°)(\cos 215°)}{(\sin 150°)(\cos 20°)}$$
$$= \frac{(-\sin 35°)(-\cos 35°)}{(\sin 30°)(\cos 20°)}$$

$$= \frac{\sin 35 \cos 35°°}{\left(\frac{1}{2}\right)(\cos 20°)}$$
$$= \frac{2\sin 35 \cos 35°°}{\cos 20°}$$
$$= \frac{\sin 70°}{\cos 20°}$$
$$= \frac{\cos 20°}{\cos 20°}$$
$$= 1$$

[8]

QUESTION 2

$$\sin 2A = \frac{\sqrt{5}}{3}$$
$$x^2 = r^2 - y^2$$
$$x^2 = 3^2 - \left(\sqrt{5}\right)^2$$
$$x^2 = 4$$
$$x = \pm 2$$
$$\therefore x = -2$$
$$\cos 2A = \frac{-2}{3}$$



[9]

QUESTION 3

$$\frac{\sin(90° + \theta) + \cos(180° + \theta)\sin(-\theta)}{\sin 180° - \tan 135°}$$
$$= \frac{\cos\theta + (-\cos\theta)(-\sin\theta)}{0+1}$$
$$= \cos\theta + \cos\theta.\sin\theta$$
$$= \cos\theta(1 + \sin\theta)$$

[5]

QUESTION 4

$$\frac{4\sin A\cos A\cos 2A.\sin 15°}{\sin 2A(1-2\sin^2 A)}$$
$$= \frac{4\sin A\cos A\cos 2A.\sin 15°}{2\sin A\cos A(1-2\sin^2 A)}$$
$$= \frac{2\cos 2A.\sin 15°}{\cos 2A}$$
$$= 2\sin 15°$$
$$= 2\sin(45° - 30°)$$
$$= 2[\sin 45°\cos 30° - \cos 45°\sin 30°]$$
$$= 2\left[\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}.\frac{1}{2}\right]$$
$$= 2\left[\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right]$$
$$= \frac{\sqrt{6}-\sqrt{2}}{2}$$

[6]

QUESTION 5

$$6\cos x - 5 = \frac{4}{\cos x}$$

$$6\cos^2 x - 5\cos x = 4$$

$$6\cos^2 x - 5\cos x - 4 = 0$$

$$(3\cos x - 4)(2\cos x + 1) = 0$$

$$\cos x = \frac{4}{3} \quad or \quad \cos x = \frac{-1}{2}$$

no solution or $x = 120° + k.360°\ , k \in Z$

or

$x = 240° + k.360°\ , k \in Z$

Alternative solution for $\cos x = \dfrac{-1}{2}$

$x = k.360° \pm 120°\ \ k \in Z$

[6]

> **Note:**
> If candidate puts $\pm k.360$ then $k \in N_0$

QUESTION 6

$$\cos^4 375° - \sin^4 345°$$

$$= \cos^4 15° - \sin^4 15°$$

$$= (\cos^2 15° + \sin^2 15°)(\cos^2 15° - \sin^2 15°)$$

$$= (1)(\cos 30°)$$

$$= \frac{\sqrt{3}}{2}$$

[6]

QUESTION 7

7.1

$$\sin 19° = \frac{t}{1}$$

$$x^2 + t^2 = 1^2$$

$$\therefore x^2 = 1 - t^2$$

$$\therefore x = \sqrt{1-t^2}$$

$$\sin 79°$$

$$= \sin(19° + 60°)$$

$$= \sin 19° \cos 60° + \cos 19° \sin 60°$$

$$= (t)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{1-t^2}}{1}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{t + \sqrt{3}\sqrt{1-t^2}}{2} = \frac{t + \sqrt{3-3t^2}}{2}$$

(7)

7.2

$$\tan 71°$$

$$= \frac{\sin 71°}{\cos 71°}$$

$$= \frac{\cos 19°}{\sin 19°}$$

$$= \frac{\frac{\sqrt{1-t^2}}{1}}{t}$$

$$= \frac{\sqrt{1-t^2}}{t}$$

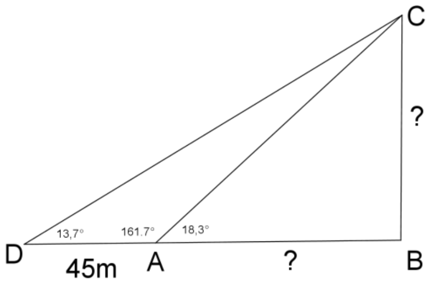
(3)

[10]

SOLUTIONS TO HOMEWORK:

SESSION 19.1 TOPIC: 2D TRIGONOMETRY

QUESTION 1



In ΔCDA $DAC = 180 - 18{,}3 = 161{,}7^{\circ}$
$DCA = 180 - (13{,}7 + 161{,}7) = 4{,}6^{\circ}$

$$\frac{AC}{sin13{,}7} = \frac{45}{sin4{,}6}$$

$$\therefore AC = \frac{45sin13{,}7}{sin4{,}6} = 133m$$

In ΔABC $sin18{,}3 = \frac{BC}{AC} = \frac{BC}{132{,}89}$

$\therefore BC = 132{,}89 \times sin18{,}3 = 42m$

Tree is 42m

Using Pythagoras: $AB = \sqrt{((132{,}89 \ldots)^2 - (41{,}7 \ldots))^2}$ = 126m = width of the river

QUESTION 2

2a)

$N\hat{D}B = 360° - 208° = 152°$

$\therefore M\hat{B}D = 28°$

$B\hat{D}A = 208° - 67° = 141°$

$$\frac{\sin D\hat{B}A}{97} = \frac{\sin 141°}{120}$$

$\therefore \sin D\hat{B}A = \frac{97 \sin 141°}{120}$

$\therefore \sin D\hat{B}A = 0{,}5087006494$

$\therefore D\hat{B}A = 30{,}58°$

$\therefore M\hat{B}A = 30{,}58° + 28°$

$\therefore M\hat{B}A = 58{,}58°$

SOLUTIONS TO HOMEWORK: SESSION 19.2 SELF

STUDY TOPIC: 3D TRIGONOMETRY

QUESTION 1

a) In ∆ABC

$AC^2 = AB^2 + BC^2 - 2AB.BCcos(90 - \alpha)$

$= d^2 + \left(\frac{1}{2}d\right)^2 - 2d(\frac{1}{2}d)sin\alpha$

$=\frac{5}{4}d^2 - d^2 sin\alpha = d^2\left(\frac{5}{4} - sin\alpha\right)$

∴ AC = $\frac{d\sqrt{(5-sin\alpha)}}{2}$

In ∆ACP

$tan\theta = \frac{PC}{AC}$

$PC = h = ACtan\theta = \frac{d\sqrt{(5-sin\alpha)}}{2}tan\theta$

b) $h = \frac{300(\sqrt{5-4sin32})}{2}tan63 = 500m$

QUESTION 2

a) ∠BAC =180 – (θ+β)

b) $\frac{AB}{sin\beta} = \frac{x}{\sin(180-(\theta+\beta))}$

$AB = \frac{xsin\beta}{\sin(\theta+\beta)}$

c i) IF AB = AC Then θ = β

$AB = \frac{xsin\theta}{sin2\theta} = \frac{xsin\theta}{2sin\theta cos\theta} = \frac{x}{2cos\theta}$

ii) In ∆BDA

$B = 90 - \theta$

$\frac{AB}{sin\theta} = \frac{AD}{\sin(90-\theta)}$ $\therefore AD = \frac{cos\theta(\frac{x}{2cos\theta})}{sin\theta} = \frac{x}{2sin\theta}$

QUESTION 3

a)

$$\frac{7}{PB} = \sin 18°$$
$$\therefore PB = \frac{7}{\sin 18°}$$
$$\therefore PB = 22{,}65247584..$$

b)

$$\frac{18}{PA} = \cos 23°$$
$$\therefore PA = \frac{18}{\cos 23°}$$
$$\therefore PA = 19{,}55448679....$$

c)

$$AB^2 = (22{,}65)^2 + (19{,}55)^2 - 2(22{,}65)(19{,}55).\cos 42°$$
$$\therefore AB^2 = 237{,}0847954...$$
$$\therefore AB = 15{,}40 \text{ m}$$

QUESTION 4

In ΔAEB:

$$EB^2 = 8^2 + 6^2$$
$$\therefore EB^2 = 100$$
$$\therefore EB = 10$$

In ΔGBC:

$$BC^2 = 15^2 + 8^2$$
$$\therefore BC^2 = 289$$
$$\therefore BC = 17$$

In ΔACB:

$$EG^2 = 15^2 + 6^2$$
$$\therefore EG^2 = 261$$
$$\therefore EG = \sqrt{261}$$

In ΔEGB:

$$\therefore \left(\sqrt{261}\right)^2 = 17^2 + 10^2 - \left(2(17)(10)\cos E\hat{B}G\right)$$
$$\therefore 261 = 389 - \left(340\cos E\hat{B}G\right)$$
$$\therefore -128 = -340\cos E\hat{B}G$$
$$\therefore \frac{32}{85} = \cos E\hat{B}G$$
$$\therefore E\hat{B}G = 67{,}88°$$