

Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

June 2016

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General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Advanced Learning in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" and/or "NR" only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Advanced Learning at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

Youyi Sun
Assessment Consultant
Grade 12 Pre-Calculus Mathematics
Telephone: 204-945-7590
Toll-Free: 1-800-282-8069, extension 7590
Email: youyi.sun@gov.mb.ca

Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a $\frac{1}{2}$ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

The total mark deduction for communication errors for any student response is not to exceed the marks given for that response. When multiple communication errors are made in a given response, any deductions are to be indicated in the order in which the errors occur in the response, without exceeding the given marks.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors ($\frac{1}{2}$ mark deduction), four E7 errors ($\frac{1}{2}$ mark deduction), and one E8 error ($\frac{1}{2}$ mark deduction). Although seven communication errors were committed in total, there is a deduction of only $1\frac{1}{2}$ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION									
Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error). Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).									
E1	<input checked="" type="radio"/>	E2	<input type="radio"/>	E3	<input type="radio"/>	E4	<input type="radio"/>	E5	<input type="radio"/>
E6	<input type="radio"/>	E7	<input checked="" type="radio"/>	E8	<input checked="" type="radio"/>	E9	<input type="radio"/>	E10	<input type="radio"/>

Example: Marks assigned to the student.

Marks Awarded	Booklet 1	Selected Response	Booklet 2	Communication Errors (Deduct)	Total
	25	7	40	$1\frac{1}{2}$	$70\frac{1}{2}$
Total Marks	36	9	45	maximum deduction of 5 marks	90

Scoring Guidelines



Booklet 1 Questions



A wheel has a diameter of 20 cm and rotates through a central angle of 252° .
Determine how far the wheel rolled.

Solution

$$\theta = (252^\circ) \left(\frac{\pi}{180^\circ} \right)$$

1 mark for conversion

$$= \frac{7\pi}{5}$$

$$s = \theta r$$

$$= \left(\frac{7\pi}{5} \right) \left(\frac{20}{2} \right)$$

1 mark for substitution

$$= 14\pi \text{ cm}$$

2 marks**or**

$$= 43.982 \text{ cm}$$

Exemplar 1

$$S = \theta R$$

$$S = (252)(10 \text{ cm})$$

$$S = 2520 \text{ radians}$$

1 out of 2

+ 1 mark for substitution

E5 (incorrect units of measure)

Exemplar 2

$$S = \theta r$$

$$\frac{252}{1} \frac{\pi}{180}$$

$$\theta = 4.398 \dots$$

$$S = ?$$

$$r = 20$$

$$S = (4.398) 20$$

$$S = 87.964 \text{ cm}$$

1½ out of 2

award full marks

– ½ mark for procedural error in line 2

Solve the following equation over the interval $[0, 2\pi]$:

$$3\sin^2 \theta - 10\sin \theta - 8 = 0$$

Solution

$$3\sin^2 \theta - 10\sin \theta - 8 = 0$$

$$(3\sin \theta + 2)(\sin \theta - 4) = 0$$

$$\sin \theta = -\frac{2}{3}$$

$$\sin \theta = 4$$

1 mark for solving for $\sin \theta$ (½ mark for each branch)

$$\theta_r = 0.729\ 728$$

No solution

2 marks for solving for θ (1 mark for indicating no solution, ½ mark for each value)

$$\theta = 3.871$$

$$\theta = 5.553$$

3 marks

Exemplar 1

$$(3\sin\theta - 1)(\sin\theta + 8) = 0$$

$$\sin\theta = \frac{1}{3}$$

$$\sin\theta = -8$$

$$\theta = \frac{1}{3}$$

$$\sin^{-1}\left(\frac{1}{3}\right) = 0.3398369$$

$$\theta = 0.3398369$$



$$2\pi - 0.3398369 = 5.94335$$

$$\theta = 0.340$$

$$\theta = 5.943$$

2 out of 3

+ 1 mark for solving for $\sin\theta$

+ 1 mark for indicating no solution

+ $\frac{1}{2}$ mark for one consistent value of θ

- $\frac{1}{2}$ mark for arithmetic error in line 1

E7 (notation error in line 3)

Exemplar 2

$$3\sin^2\theta - 10\sin\theta - 8 = 0$$

$$(3\sin\theta + 2)(\sin\theta - 4) = 0$$

$$\sin\theta = -\frac{2}{3}$$

$$\sin\theta = 4$$

$$\theta_r = -0.7568$$

$$\theta_r = 0.7297$$

$$Q_3 : 3.871$$

$$Q_4 : 5.553$$

2 out of 3

+ 1 mark for solving for $\sin\theta$

+ 1 mark for the values of θ

E7 (notation error in lines 5 and 6)

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Determine and simplify the fourth term in the expansion of $(2x^4 - 3y)^8$.

Solution

$$t_4 = {}_8C_3 (2x^4)^5 (-3y)^3$$

2 marks (1 mark for ${}_8C_3$, $\frac{1}{2}$ mark for each consistent factor)

$$= 56(32x^{20})(-27y^3)$$

$$= -48\,384x^{20}y^3$$

1 mark for simplification ($\frac{1}{2}$ mark for coefficient, $\frac{1}{2}$ mark for exponents)

3 marks

Exemplar 2

$$t_{k+1} = n \binom{n}{k} a^{n-k} b^k$$

$$t_4 = {}_8C_3 (2x^4)^5 - 3y^8$$

$$t_4 = 56(2x^{20}) - 3y^8$$

$$t_4 = -336x^{20}y^8$$

2 out of 3

+ 1 mark for ${}_8C_3$

+ ½ mark for a consistent factor

+ ½ mark for simplification of exponents

E4 (missing brackets but still implied in lines 2 and 3)

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Question 4

R10

Sheeva's bank is lending her \$50 000 at an annual interest rate of 6%, compounded monthly, to purchase a car.

Given that the last payment will be a partial payment, determine how many full monthly payments of \$800 Sheeva will have to make.

The formula below may be used.

$$PV = \frac{R \left[1 - (1 + i)^{-n} \right]}{i}$$

where $PV =$ the present value of the amount borrowed

$R =$ the amount of each periodic payment

$i = \frac{\text{annual interest rate (as a decimal)}}{\text{the number of compounding periods per year}}$

$n =$ the number of equal periodic payments

Express your answer as a whole number.

Solution

$$50\,000 = \frac{800 \left[1 - \left(1 + \frac{0.06}{12} \right)^{-n} \right]}{\frac{0.06}{12}}$$

½ mark for substitution

$$250 = 800 \left[1 - (1 + 0.005)^{-n} \right]$$

$$0.3125 = 1 - 1.005^{-n}$$

$$-0.6875 = -1.005^{-n}$$

$$0.6875 = 1.005^{-n}$$

½ mark for simplification

$$\log 0.6875 = -n \log 1.005$$

½ mark for applying logarithms

$$\frac{\log 0.6875}{-\log 1.005} = n$$

1 mark for power law

$$75.125\,880\,88 = n$$

½ mark for solving for n

∴ 75 full monthly payments are needed

3 marks

Exemplar 1

$$50000 = \frac{800 \left[1 - \left(1 + \frac{0.06}{12} \right)^{-n} \right]}{\frac{0.06}{12}}$$

$$500000 = \frac{800 \left[1 - (1.005)^{-n} \right]}{0.005}$$

$$250 = 800 \left[1 - (1.005)^{-n} \right]$$

$$1.3125 = (1.005)^{-n}$$

$$\log 1.3125 = \log 1.005^{-n}$$

$$\log 1.3125 = n \log 1.005$$

Sheeva will have to pay 66 months

2 out of 3

+ ½ mark for substitution

+ ½ mark for applying logarithms

+ 1 mark for power law

E7 (transcription error in line 6)

Exemplar 2

$$\frac{0,06}{12} (50\,000) = \left(\frac{800 \left[1 - \left(1 + \frac{0,06}{12} \right)^{-n} \right]}{\frac{0,06}{12}} \right) \frac{0,06}{12}$$

$$\frac{250}{800} = \frac{800 \left[1 - \left(1 + \frac{0,06}{12} \right)^{-n} \right]}{800}$$

$$0,3125 = 1 - \left(1 + \frac{0,06}{12} \right)^{-n}$$
$$-0,3125 + \left(1 + \frac{0,06}{12} \right)^{-n}$$

$$\left(1 + \frac{0,06}{12} \right)^{-n} = 1 - 0,3125$$

$$\left(\frac{1 + 0,06}{12} \right)^{-n} = 0,6875$$

$$\log \left(\frac{1 + 0,06}{12} \right)^{-n} = \log(0,6875)$$

$$\frac{-n \log \left(1 + \frac{0,06}{12} \right)}{\log \left(1 + \frac{0,06}{12} \right)} = \frac{\log 0,6875}{\log \left(1 + \frac{0,06}{12} \right)}$$

$$\rightarrow \frac{-n}{-1} = \frac{-75,125}{-1}$$

$$n = 75,125$$

$$n = 76 \text{ payments}$$

3 out of 3

award full marks

E6 (rounding error)

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An employee asked 10 people in an ice cream shop to wait in line.

Determine the number of different arrangements possible if two of the people, Jamie and John, refused to stand next to each other in the line.

Solution

$$10! - 9!2!$$

½ mark for $10!$

1 mark for product of $9!2!$ (½ mark for $9!$, ½ mark for $2!$)

½ mark for subtraction

$$2\,903\,040$$

2 marks

Exemplar 1

Case 1:
together

$$\underline{10} \quad \underline{9} \quad \underline{8} \quad \underline{7} \quad \underline{6} \quad \underline{5} \quad \underline{4} \quad \underline{3} \quad \underline{2} \quad \underline{1}$$

Case 2:
not together

$$\begin{array}{cccccccccc} \underline{10} & \underline{8} & \underline{8} & \underline{7} & \underline{6} & \underline{5} & \underline{4} & \underline{3} & \underline{2} & \underline{1} \\ \text{JAMIE} & & \text{John} & & \text{J} & & \text{J} & & \text{J} & \end{array}$$

$$\begin{array}{r} 3678800 \\ - 3225600 \\ \hline \end{array}$$

403200 possible
arrangements

1 out of 2

+ ½ mark for 10!

+ ½ mark for subtraction

Exemplar 2

$$\begin{aligned} \text{diff. arrangements} &= \frac{10!}{2!} \\ &= 1814400 \end{aligned}$$

½ out of 2

+ ½ mark for 10!

The point $(-2, 4)$ is on the graph of $f(x)$.

State the coordinates of the corresponding point when $f(x)$ is reflected over the y -axis.

Solution

$(2, 4)$

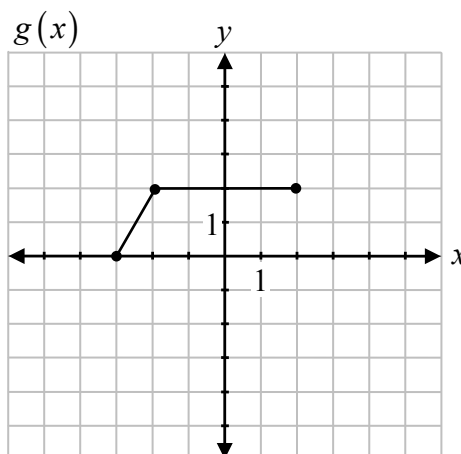
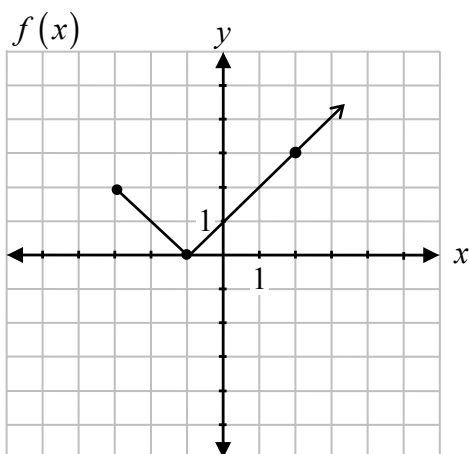
1 mark

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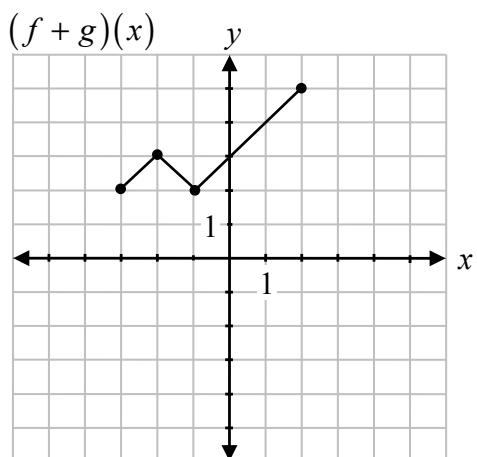
Question 7

R1

Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $(f + g)(x)$.



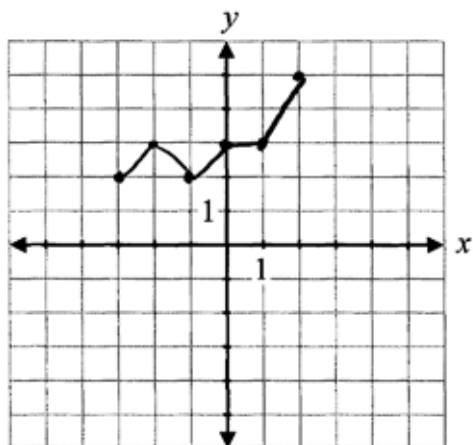
Solution



1 mark for operation of addition
1 mark for restricted domain

2 marks

Exemplar 1

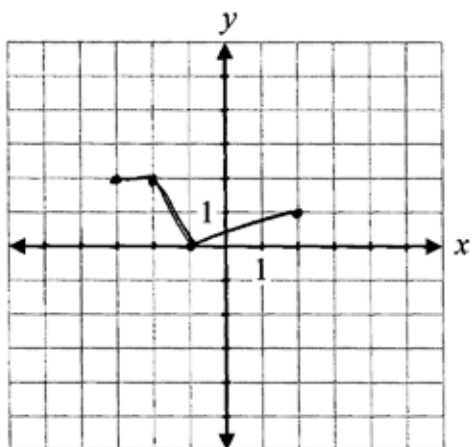


1½ out of 2

award full marks

– ½ mark for procedural error (one incorrect point)

Exemplar 2



1 out of 2

+ 1 mark for restricted domain

Using the laws of logarithms, fully expand the expression:

$$\log_2 \left(\frac{w^3 x}{y-1} \right)$$

Solution

$$3\log_2 w + \log_2 x - \log_2 (y-1)$$

1 mark for power law
1 mark for product law
1 mark for quotient law

3 marks

Exemplar 1

$$\log_2(w^3x) - \log_2(y-1)$$
$$\log_2 w^3 + \log_2 x - \frac{\log_2 y}{\log_2 1}$$

1 out of 3

- + 1 mark for product law
- + 1 mark for quotient law
- 1 mark for concept error in line 2

Exemplar 2

$$3\log_2 w + \log_2 x - \log_2 y - 1$$

3 out of 3

- award full marks
E7 (notation error of missing brackets)

Solve the following equation algebraically for θ , where $0 \leq \theta \leq 2\pi$:

$$2 \cos 2\theta = 1$$

Solution

Method 1

$$2(2 \cos^2 \theta - 1) = 1$$

1 mark for substitution of an appropriate identity

$$4 \cos^2 \theta - 2 = 1$$

$$\cos^2 \theta = \frac{3}{4}$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

1 mark for solving for $\cos \theta$ (½ mark for each value)

$$\theta = \frac{\pi}{6}, \frac{11\pi}{6}$$

$$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$$

2 marks (½ mark for each value of θ)

4 marks

Method 2

$$2(1 - 2 \sin^2 \theta) = 1$$

1 mark for substitution of an appropriate identity

$$2 - 4 \sin^2 \theta = 1$$

$$-4 \sin^2 \theta = -1$$

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \pm \frac{1}{2}$$

$$\sin \theta = \frac{1}{2}$$

$$\sin \theta = -\frac{1}{2}$$

1 mark for solving for $\sin \theta$ (½ mark for each value)

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

2 marks (½ mark for each value of θ)

4 marks

Note(s):

- Deduct a maximum of 1 mark if student omits second branch when taking the square root.

Exemplar 1

$$2 \cos 2\theta = 1$$

$$2(2\cos^2\theta - 1)$$

$$4\cos^2\theta - 2 = 1$$

$$4\cos^2\theta = 1$$

$$\cos^2\theta = \frac{1}{4}$$

$$\cos\theta = \pm\frac{1}{2}$$

$$\begin{array}{l} \theta_1 = \frac{\pi}{3} \\ \theta_2 = \frac{2\pi}{3} \\ \theta_3 = \frac{4\pi}{3} \\ \theta_4 = \frac{5\pi}{3} \end{array}$$

3½ out of 4

award full marks

– ½ mark for arithmetic error in line 4

E2 (changing an equation to an expression in line 2)

Exemplar 2

$$2\cos 2\theta = 1$$

$$2(\cos^2\theta - \sin^2\theta) = 1$$

$$2\cos^2\theta - 2\sin^2\theta = 1$$

$$2(1 - \sin^2\theta) - 2\sin^2\theta = 1$$

$$2 - 2\sin^2\theta - 2\sin^2\theta = 1$$

$$-4\sin^2\theta = -1$$

$$\sin^2\theta = \frac{1}{4}$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

3 out of 4

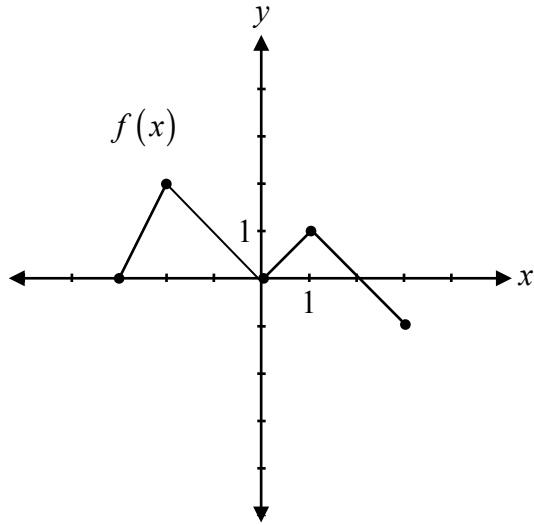
+ 1 mark for substitution of an appropriate identity

+ 1 mark for solving for $\sin\theta$

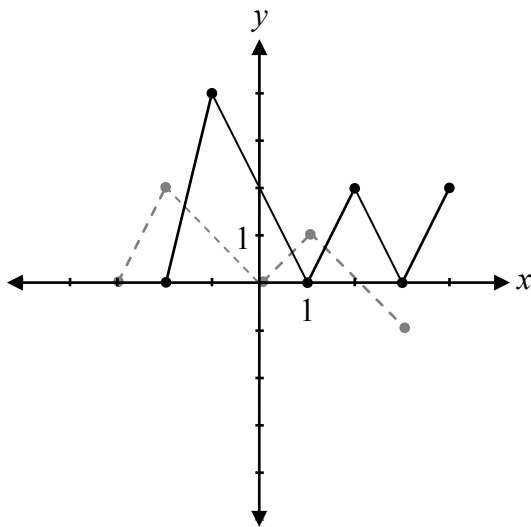
+ 1 mark for values of θ

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Given the graph of $y = f(x)$, sketch the graph of $y = 2|f(x-1)|$.



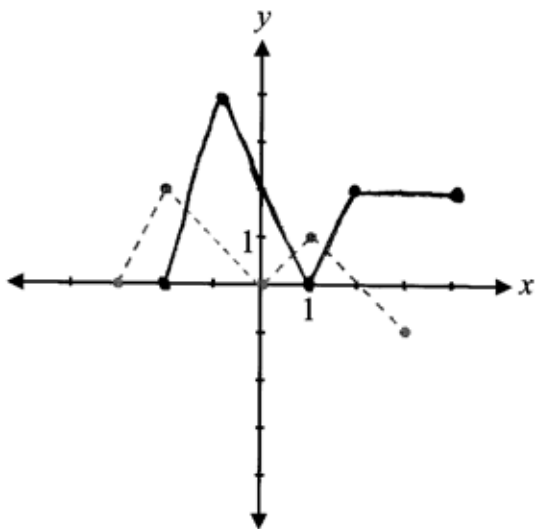
Solution



- 1 mark for vertical stretch
- 1 mark for horizontal translation
- 1 mark for absolute value

3 marks

Exemplar 1

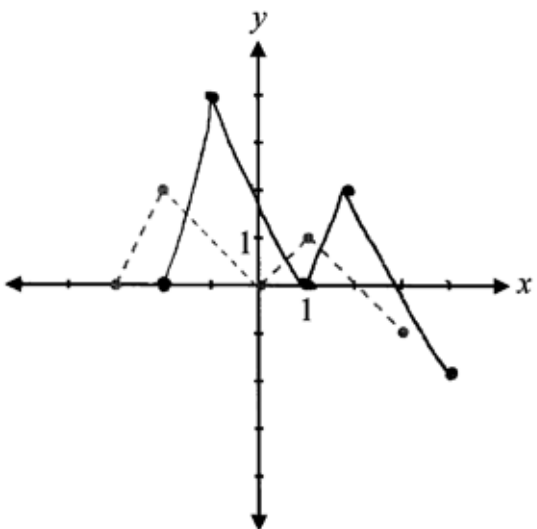


2 out of 3

+ 1 mark for vertical stretch

+ 1 mark for horizontal translation

Exemplar 2



2 out of 3

+ 1 mark for horizontal translation

+ 1 mark for vertical stretch

Prove the identity for all permissible values of θ :

$$\cos \theta + \tan \theta \sin \theta = \frac{\tan \theta \sin \theta}{1 - \cos^2 \theta}$$

Solution

Left-Hand Side	Right-Hand Side	
$\cos \theta + \tan \theta \sin \theta$	$\frac{\tan \theta \sin \theta}{1 - \cos^2 \theta}$	
$\cos \theta + \frac{\sin \theta}{\cos \theta} \sin \theta$	$\frac{\frac{\sin \theta}{\cos \theta} \cdot \sin \theta}{\sin^2 \theta}$	1 mark for correct substitution of identities
$\cos \theta + \frac{\sin^2 \theta}{\cos \theta}$	$\frac{\sin^2 \theta}{\cos \theta} \cdot \frac{1}{\sin^2 \theta}$	1 mark for algebraic strategies
$\frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta}$	$\frac{1}{\cos \theta}$	1 mark for logical process to prove the identity
$\frac{1}{\cos \theta}$		3 marks

Exemplar 1

Left-Hand Side	Right-Hand Side
$\cos\theta + \tan\theta \sin\theta$	$\frac{\sin\theta}{\cos\theta} \sin\theta$
$\cos\theta + \frac{\sin\theta}{\cos\theta} \sin\theta$	$\frac{\sin^2\theta}{\sin^2\theta}$
$\cos\theta + \frac{\sin^2\theta}{\cos\theta}$	
$\cos\theta + \frac{1 - \cos^2\theta}{\cos\theta}$	
$\cos\theta + 1 - \cos\theta$	
1	

1 out of 3

+ 1 mark for correct substitution of identities

Raoul has 8 shirts, 5 pairs of pants, and 3 hats. He adds the options together and determines that he has 16 different outfits to wear.

Raoul made an error in calculating the number of different outfits. Describe how to determine the correct number of outfits.

Solution

Raoul should have multiplied the number of clothing items to determine the total number of outfits.

1 mark

Exemplar 1

120 outfits

because five pants can be with one shirt and there is eight shirts along with 3 hats that he can or cannot wear

1 out of 1

Exemplar 2

Raul added the number of clothing items together. He must multiply $8! \cdot 5! \cdot 3!$ to figure out the correct number of variations for his outfits

0 out of 1

Given $f(x) = 2x - 1$ and $g(x) = x^2 + 1$:

- a) Determine $f(x) \cdot g(x)$.
b) Determine $g(g(x))$.

Solution

$$\begin{aligned} \text{a) } f(x) \cdot g(x) &= (2x - 1)(x^2 + 1) \\ &= 2x^3 + 2x - x^2 - 1 \\ &= 2x^3 - x^2 + 2x - 1 \end{aligned}$$

1 mark for product

1 mark

$$\begin{aligned} \text{b) } g(g(x)) &= (x^2 + 1)^2 + 1 \\ &= x^4 + 2x^2 + 1 + 1 \\ &= x^4 + 2x^2 + 2 \end{aligned}$$

1 mark for composition

1 mark

Exemplar 1

a)

$$(2x-1)(x^2+1)$$

$$2x^3+2x-x^2+1$$

$$2x^3-x^2+2x+1$$

½ out of 1

award full marks

– ½ mark for arithmetic error in line 2

b)

$$(x^2+1)(x^2+1)$$

$$x^4+x^2+x^2+1$$

$$x^4+2x^2+1$$

0 out of 1

Exemplar 2

a)

$$f(x) = (2x - 1)(x^2 + 1)$$

1 out of 1

award full marks

E7 (notation error in line 1)

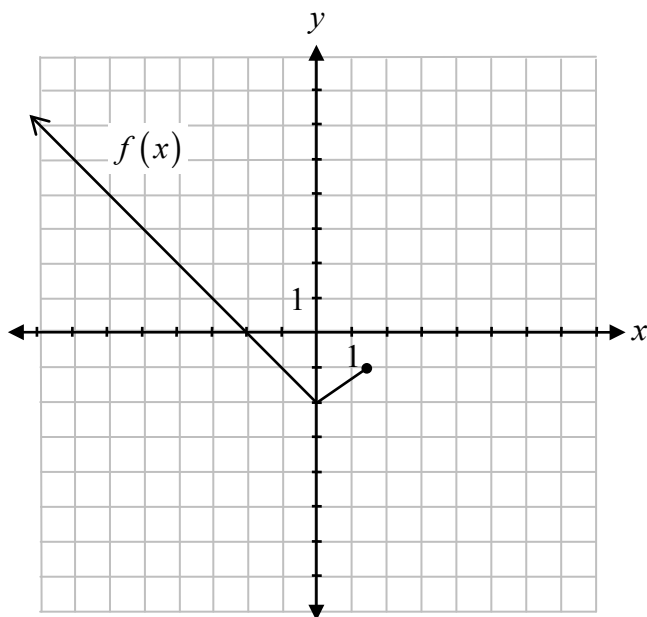
b)

$$\begin{aligned} f(x) &= (x^2 + 1) + 1 \\ &= x^2 + 2 \end{aligned}$$

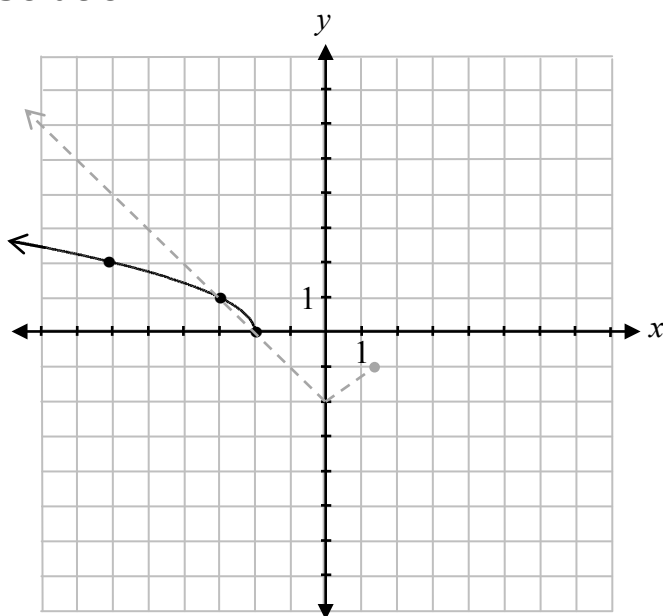
0 out of 1

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Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.



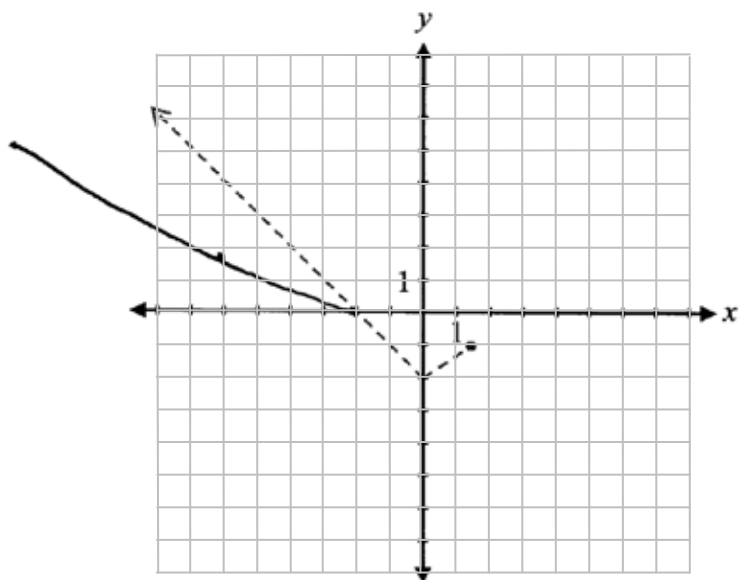
Solution



1 mark for restricting the domain
 ½ mark for shape between invariant points
 ½ mark for shape to the left of invariant points

2 marks

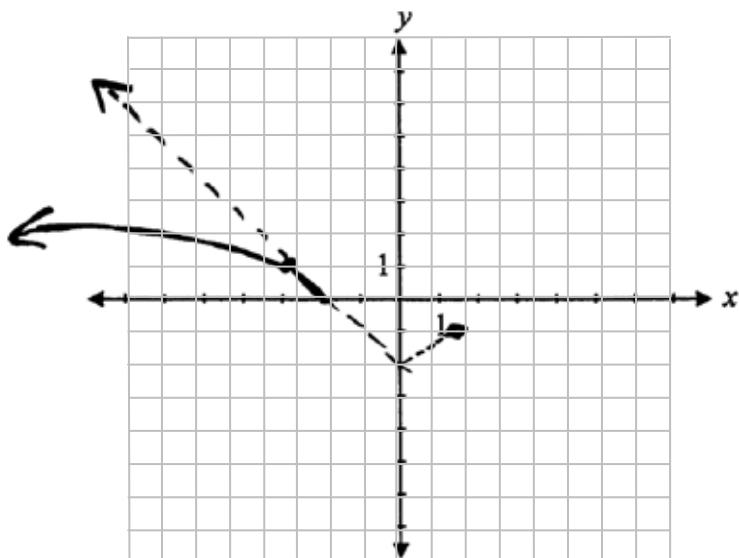
Exemplar 1



1 out of 2

+ 1 mark for restricting the domain
E9 (missing arrowhead)

Exemplar 2



1½ out of 2

+ 1 mark for restricting the domain
+ ½ mark for shape to the left of invariant points

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Booklet 2 Questions



Answer Key for Selected Response Questions

Question	Answer	Learning Outcome
15	C	R11
16	B	P3
17	D	R13
18	B	R7
19	A	T5
20	B	R13
21	D	R6
22	C	T1

Question 15

R11

Given the polynomial function $P(x) = x^4 - 5x^2 - 2x + 6$, if $P(1) = 0$, identify which statement is true.

- a) The y -intercept is 1.
- b) $P(x)$ has a factor of $(x + 1)$.
- c) The graph has a zero at 1
- d) The graph has a zero at -1 .

Question 16

P3

There are 6 different books that are being distributed evenly amongst three people.
Identify which expression represents the number of possible combinations.

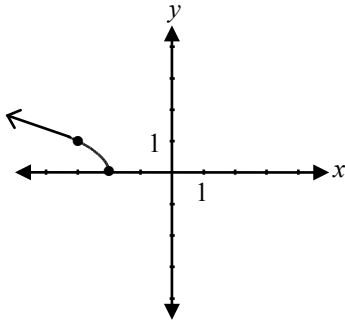
- a) ${}_6C_2 \cdot {}_6C_2 \cdot {}_6C_2$
- b) ${}_6C_2 \cdot {}_4C_2 \cdot {}_2C_2$
- c) ${}_2C_2 \cdot {}_2C_2 \cdot {}_2C_2$
- d) $3 \cdot {}_6C_2$

Question 17

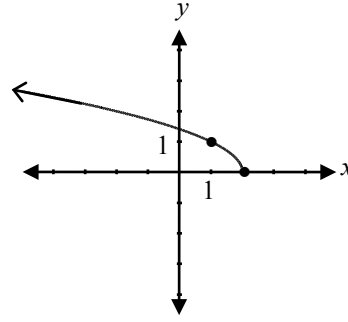
R13

Identify the graph that corresponds to the function $f(x) = -\sqrt{x-2}$.

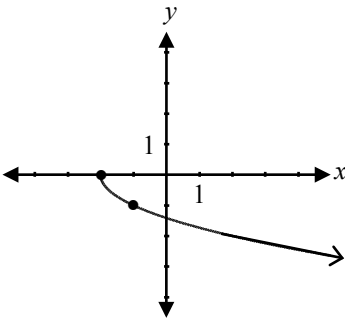
a)



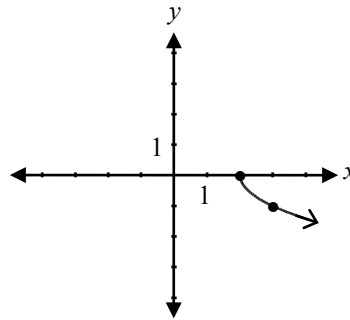
b)



c)



d)



Question 18

R7

Solve:

$$7^{\log_7 2} = x$$

a) $x = 1$

b) $x = 2$

c) $x = 7$

d) $x = 49$

Identify the equation that has a general solution of $\left. \begin{array}{l} \theta = \frac{\pi}{6} + 2\pi k \\ \theta = \frac{5\pi}{6} + 2\pi k \end{array} \right\}$ where $k \in \mathbb{Z}$.

a) $\sin \theta = \frac{1}{2}$

b) $\cos \theta = \frac{1}{2}$

c) $\sin \theta = \frac{\sqrt{3}}{2}$

d) $\cos \theta = \frac{\sqrt{3}}{2}$

Identify the function that has a domain of $x \leq -2$ and a range of $y \geq 3$.

a) $y = \sqrt{x+2} + 3$

b) $y = \sqrt{-(x+2)} + 3$

c) $y = -\sqrt{x-2} - 3$

d) $y = -\sqrt{-(x-2)} - 3$

Question 21

R6

Given $f(x) = 3x + 2$, identify $f^{-1}(x)$.

a) $f^{-1}(x) = -3x - 2$

b) $f^{-1}(x) = 2x + 3$

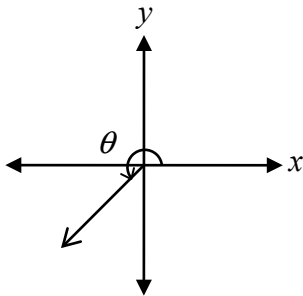
c) $f^{-1}(x) = \frac{x}{3} - 2$

d) $f^{-1}(x) = \frac{x - 2}{3}$

Question 22

T1

Identify a possible value for the angle θ sketched in standard position.



a) 2

b) 3

c) 4

d) 5

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Solve the following equation:

$$\log_3(x+3) + \log_3(x-5) = 2$$

Solution

$$\log_3[(x+3)(x-5)] = 2$$

1 mark for product law

$$(x+3)(x-5) = 3^2$$

1 mark for exponential form

$$x^2 - 2x - 15 = 9$$

$$x^2 - 2x - 24 = 0$$

$$(x-6)(x+4) = 0$$

$$x = 6 \quad \cancel{x = -4}$$

½ mark for solving for x

½ mark for rejecting extraneous root

3 marks

Exemplar 1

$$\log_3 \left(\frac{x+3}{x-5} \right) = 2$$

$$3^2 = \frac{x+3}{x-5}$$

$$9 = \frac{x+3}{x-5}$$

$$9(x-5) = x+3$$

$$9x - 45 = x + 3$$

$$8x = 48$$

$$\boxed{x = 6}$$

2 out of 3

award full marks

- 1 mark for concept error (using quotient law)

Exemplar 2

$$\log_3 (x+3)(x-5) = 2$$

$$\log_3 (x^2 - 2x - 15) = 2$$

$$\begin{array}{l} 36 - 12 \\ 24 - 15 \end{array}$$

$$3^2 = (x^2 - 2x - 15)$$

$$\underline{x = 6}$$

2½ out of 3

+ 1 mark for product law

+ 1 mark for exponential form

+ ½ mark for solving for x

Exemplar 3

$$\log_3 (x+3)(x-5) = 2$$

$$3^2 = (x+3)(x-5)$$

$$\cancel{09} = x^2 - 5x + 3x - 15 - 9$$

$$0 = x^2 - 2x - 24$$

$$(x+4)(x-6)$$

$$\underline{x = -4, 6}$$

2½ out of 3

+ 1 mark for product law

+ 1 mark for exponential form

+ ½ mark for solving for x

E2 (changing an equation to an expression in line 5)

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State a coterminal angle for $\theta = \frac{9\pi}{4}$.

Solution

$$\frac{9\pi}{4} + \frac{8\pi}{4} = \frac{17\pi}{4}$$

or

1 mark

$$405^\circ - 360^\circ = 45^\circ$$

Note(s):

- Other answers are possible.

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Sketch the graph of the function $f(x) = \frac{2x+2}{x^2-1}$.

Solution

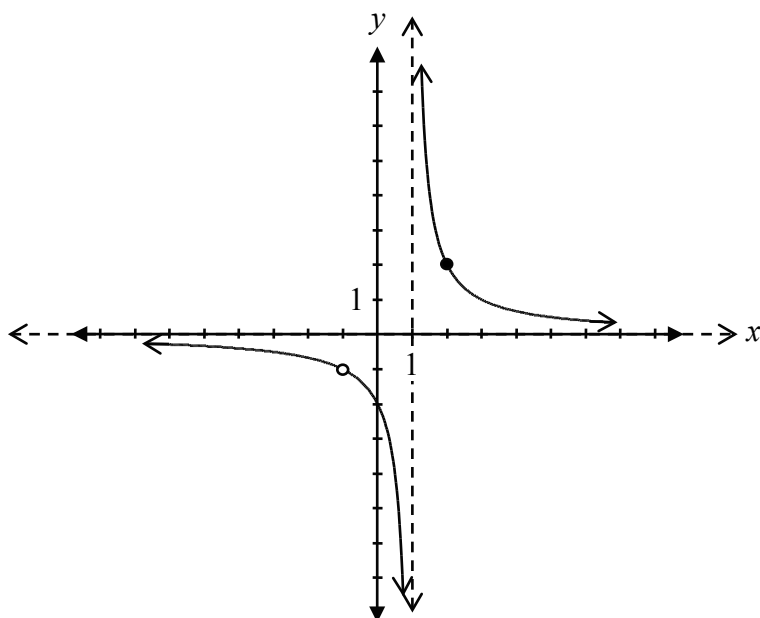
$$f(x) = \frac{2(\cancel{x+1})}{(x-1)(\cancel{x+1})}$$

$$= \frac{2}{x-1}$$

\therefore there is a point of discontinuity (hole) at $(-1, -1)$

vertical asymptote at $x = 1$

horizontal asymptote at $y = 0$



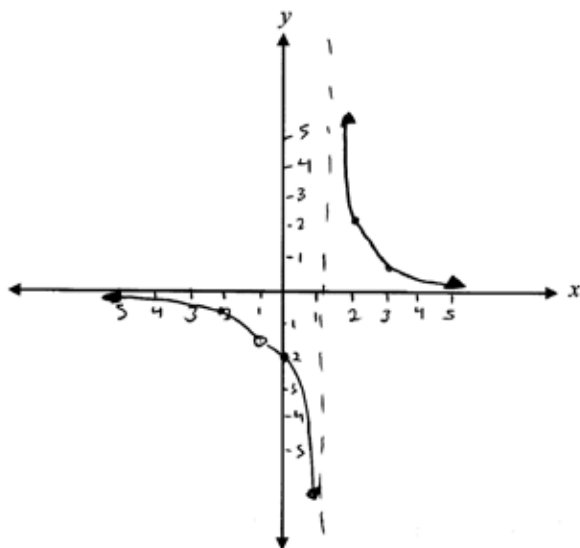
1 mark for asymptotic behaviour at $x = 1$
 1 mark for asymptotic behaviour at $y = 0$
 1 mark for point of discontinuity (hole) at $(-1, -1)$ ($\frac{1}{2}$ mark for $x = -1$, $\frac{1}{2}$ mark for $y = -1$)
 $\frac{1}{2}$ mark for graph left of $x = 1$
 $\frac{1}{2}$ mark for graph right of $x = 1$

4 marks

Exemplar 1

$$\frac{2(x+1)}{(x-1)(x+1)}$$

hole at $x = -1$
 $VA = 1$
 $HA = 0$



X	Y
-2	-2/3
0	-2
2	2
3	

$$\frac{2(-2+1)}{-2^2-1} = \frac{-2}{3}$$

$$\frac{2(0+1)}{0^2-1} = \frac{2}{-1}$$

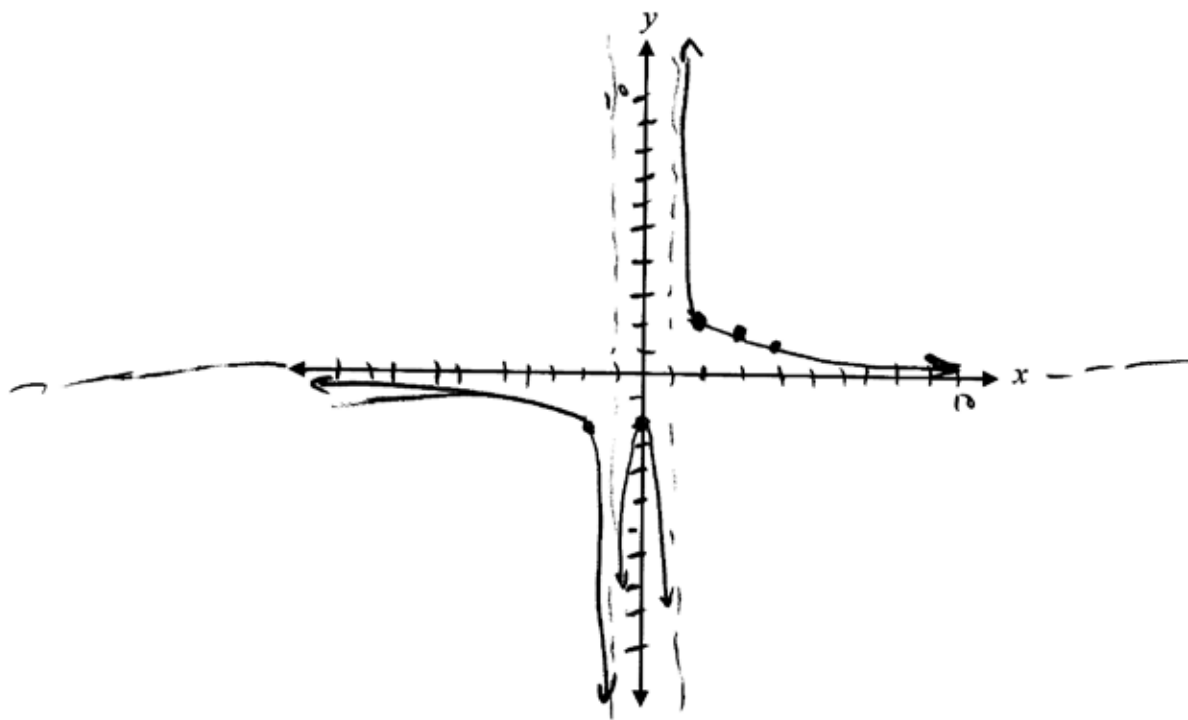
$$\frac{2(2+1)}{2^2-1} = \frac{6}{3} = 2$$

$$\frac{2(3+1)}{3^2-1} = \frac{8}{8}$$

3½ out of 4

- + 1 mark for asymptotic behaviour at $x = 1$
- + 1 mark for asymptotic behaviour at $y = 0$
- + ½ mark for $x = -1$ (point of discontinuity/hole)
- + ½ mark for graph left of $x = 1$
- + ½ mark for graph right of $x = 1$
- E10 (asymptote missing but still implied at $y = 0$)

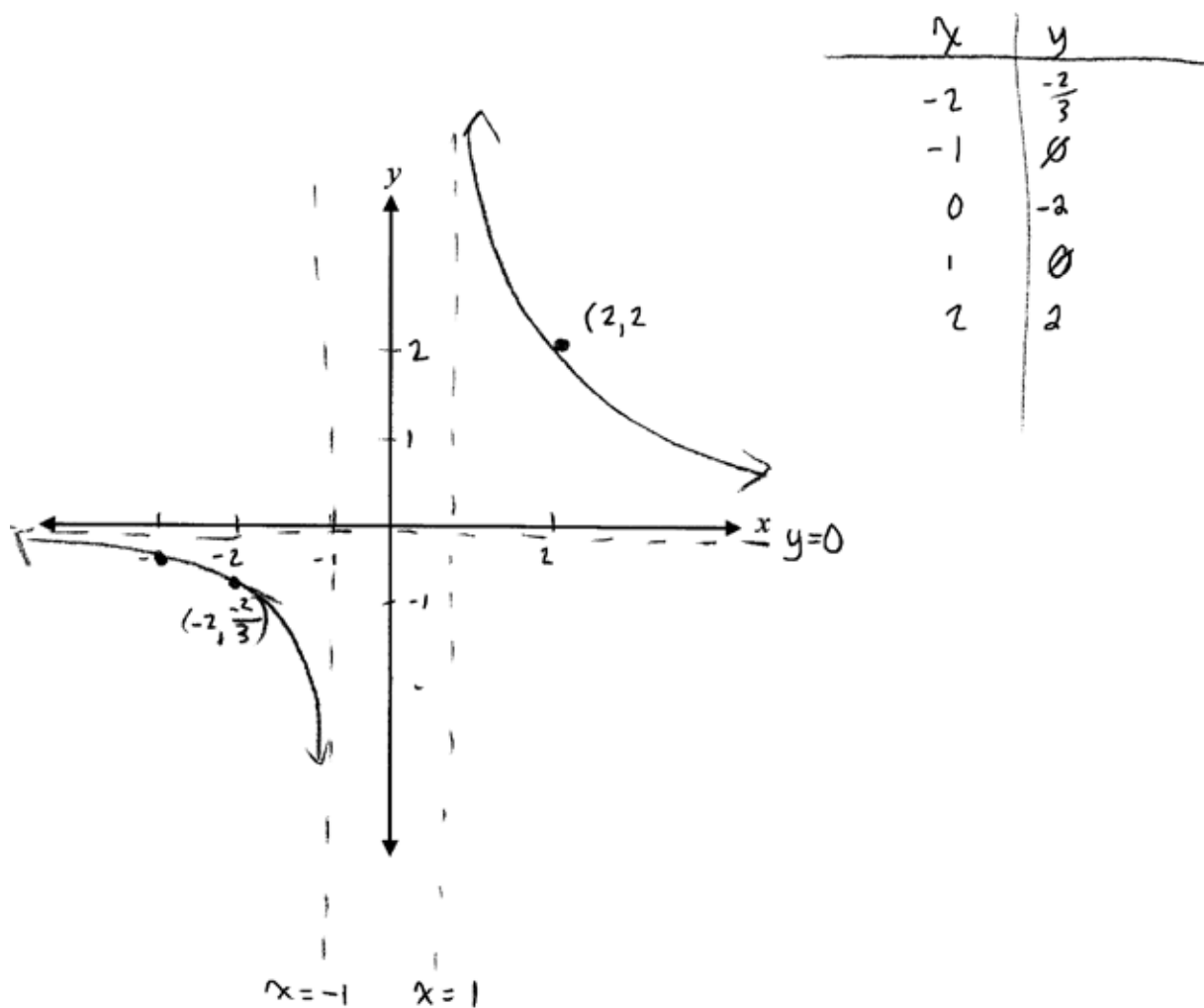
Exemplar 2



2½ out of 4

- + 1 mark for asymptotic behaviour at $x = 1$
- + 1 mark for asymptotic behaviour at $y = 0$
- + ½ mark for graph right of $x = 1$

Exemplar 3



3 out of 4

- + 1 mark for asymptotic behaviour at $x = 1$
- + 1 mark for asymptotic behaviour at $y = 0$
- + $\frac{1}{2}$ mark for graph right of $x = 1$
- + $\frac{1}{2}$ mark for graph left of $x = -1$ (consistent with error)

Justify why the binomial expansion of $(x + x^3)^7$ does not have a term containing x^{10} .

Solution

Method 1

$$(x)^7, (x)^6(x^3)^1, (x)^5(x^3)^2, \dots$$

$$x^7, x^9, x^{11}, \dots$$

The exponents increase by 2.

Therefore x^{10} is not in the pattern.

1 mark for determining the pattern

1 mark for justification

2 marks

Method 2

$$x^{7-r}(x^3)^r = x^{10}$$

$$x^{7+2r} = x^{10}$$

$$7 + 2r = 10$$

$$2r = 3$$

$$r = \frac{3}{2}$$

½ mark for substitution

½ mark for solving for r

The binomial expansion does not contain x^{10} because the value of r must be a whole number.

1 mark for justification

2 marks

Exemplar 1

$$\begin{array}{l} t_1 = (x)^7 (x^3)^0 \dots x^7 \\ t_2 = (x)^6 (x^3)^1 \dots x^9 \\ t_3 = (x)^5 (x^3)^2 \dots x^{11} \end{array} \left. \vphantom{\begin{array}{l} t_1 \\ t_2 \\ t_3 \end{array}} \right\} \text{ goes up by 2.}$$

2 out of 2

Exemplar 2

$$X^7 \cdot X^{3r} = X^{10}$$

$$X^{7+3r} = X^{10}$$

$$7+3r = 10$$

$$3r = 3$$

$$r = 1$$

yes it is part of it!

1½ out of 2

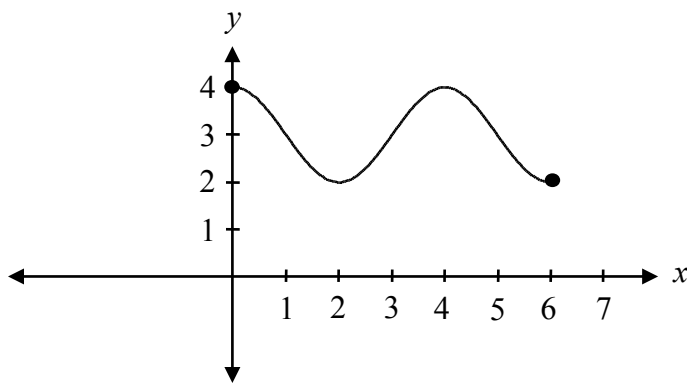
+ ½ mark for solving for r

+ 1 mark for justification

Sketch the graph of $y = -\sin\left(\frac{\pi}{2}(x-1)\right) + 3$ over the domain $[0, 6]$.

Solution

$$\text{period} = \frac{2\pi}{\frac{\pi}{2}} = 4$$



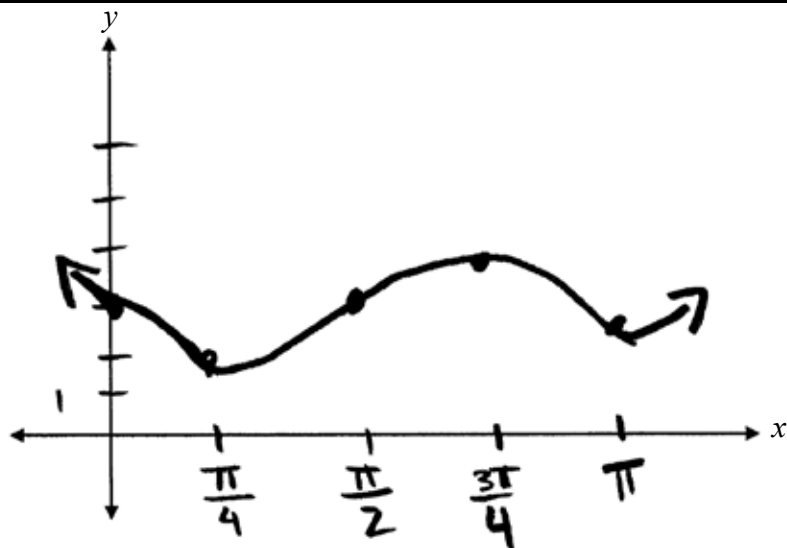
1 mark for vertical reflection
1 mark for horizontal translation
1 mark for period
1 mark for vertical translation

4 marks

Note(s):

- Deduct $\frac{1}{2}$ mark if the domain $[0, 6]$ is not completely sketched.

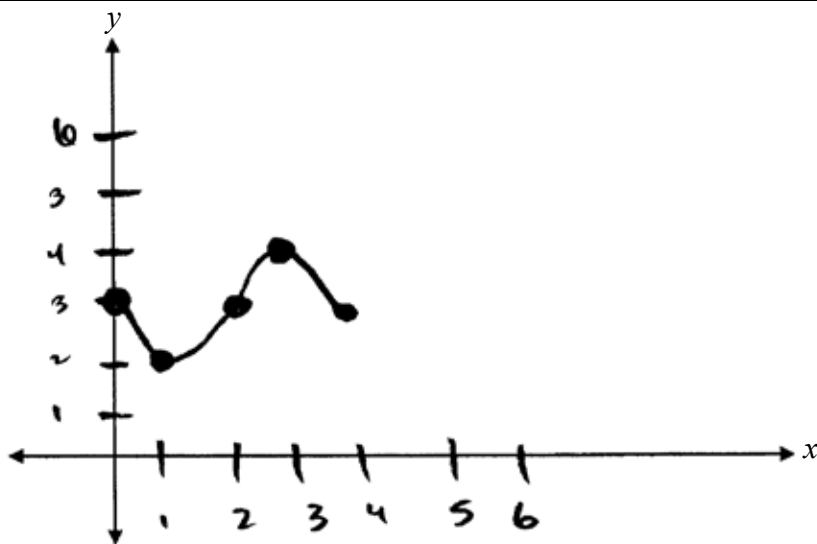
Exemplar 1



1½ out of 4

- + 1 mark for vertical reflection
- + 1 mark for vertical translation
- ½ mark for incorrect shape
- E8 (answer given outside the domain)

Exemplar 2



2½ out of 4

- + 1 mark for vertical reflection
- + 1 mark for vertical translation
- + 1 mark for period
- ½ mark for the domain $[0, 6]$ not completely sketched

When $P(x) = 3x^4 - kx^3 + 5x - 14$ is divided by $(x + 2)$, the remainder is -8 .

Determine the value of k .

Solution

Method 1

$$x = -2$$

$\frac{1}{2}$ mark for $x = -2$

$$-8 = 3(-2)^4 - k(-2)^3 + 5(-2) - 14$$

1 mark for remainder theorem

$$-8 = 48 + 8k - 10 - 14$$

$$-8 = 24 + 8k$$

$$-32 = 8k$$

$$k = -4$$

$\frac{1}{2}$ mark for solving for k

2 marks

Method 2

-2	3	$-k$	0	5	-14
	↓	-6	$2k + 12$	$-4k - 24$	$8k + 38$
	3	$-k - 6$	$2k + 12$	$-4k - 19$	$8k + 24$

$\frac{1}{2}$ mark for $x = -2$

1 mark for synthetic division
(or any other equivalent strategy)

$$-8 = 8k + 24$$

$$-32 = 8k$$

$$k = -4$$

$\frac{1}{2}$ mark for solving for k

2 marks

Exemplar 1

$$\begin{array}{r|rrrrr} -2 & 3 & -k & 0 & +5 & -14 \\ & \downarrow & & & & \\ & -6 & +4 & -8 & +6 & \\ \hline & 3 & -2 & +4 & -3 & -8 \end{array}$$

$$k = -4$$

$$\begin{array}{r|rrrrr} 2 & 3 & -4 & 0 & +5 & -14 \\ & \downarrow & & & & \\ & -6 & +4 & -8 & +6 & \\ \hline & 3 & -2 & +4 & -3 & -8 \end{array}$$

2 out of 2

Exemplar 2

$$\begin{aligned} 0 &= 3(2)^4 - k(2)^3 + 5(2) - 14 \\ 0 &= 48 - k(-8) - 10 - 14 \\ 0 &= 24 - k(-8) \\ -24 &= -k(-8) \\ 3 &= -k \\ -3 &= k \end{aligned}$$

1 out of 2

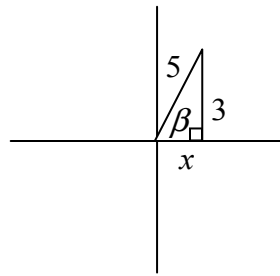
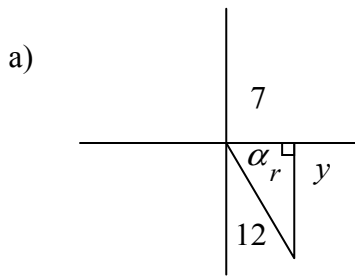
+ ½ mark for $x = -2$

+ ½ mark for solving for k

Given that $\cos \alpha = \frac{7}{12}$ where α is in quadrant IV, and $\sin \beta = \frac{3}{5}$ where β is in quadrant I, determine the exact value of:

- a) $\sin(\alpha - \beta)$
 b) $\csc(\alpha - \beta)$

Solution



$$12^2 - 7^2 = y^2 \quad 5^2 - 3^2 = x^2$$

$$95 = y^2 \quad 16 = x^2$$

$$\pm\sqrt{95} = y \quad \pm 4 = x$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$= \left(\frac{-\sqrt{95}}{12}\right)\left(\frac{4}{5}\right) - \left(\frac{7}{12}\right)\left(\frac{3}{5}\right)$$

$$= \frac{-4\sqrt{95}}{60} - \frac{21}{60}$$

$$= \frac{-4\sqrt{95} - 21}{60}$$

$$\text{b) } \csc(\alpha - \beta) = \frac{60}{-4\sqrt{95} - 21}$$

$\frac{1}{2}$ mark for value of x

$\frac{1}{2}$ mark for value of y

$\frac{1}{2}$ mark for $\sin \alpha$

$\frac{1}{2}$ mark for $\cos \beta$

1 mark for substitution into correct identity

3 marks

1 mark

Note(s):

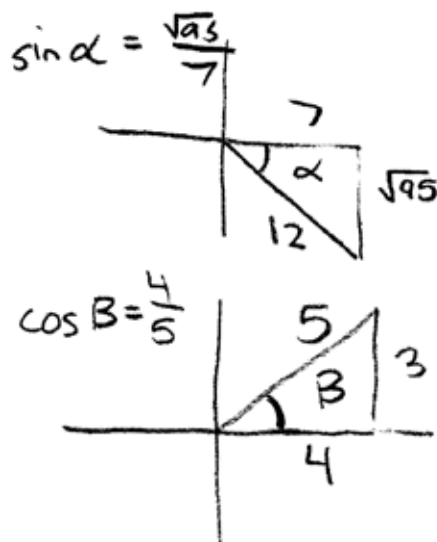
- accept any of the following values for x : $x = \pm 4$, or $x = 4$
- accept any of the following values for y : $y = \pm\sqrt{95}$, $y = \sqrt{95}$, or $y = -\sqrt{95}$

Exemplar 1

a)

$$\sin(\alpha + \beta) = \left(\frac{\sqrt{95}}{12} \cdot \frac{4}{5}\right) - \left(\frac{7}{12} \cdot \frac{3}{5}\right)$$
$$\sin(\alpha + \beta) = \frac{4\sqrt{95}}{60} - \frac{21}{60}$$

$$\sin(\alpha + \beta) = \frac{4\sqrt{95} - 21}{60}$$



2½ out of 3

+ ½ mark for value of x

+ ½ mark for value of y

+ ½ mark for $\cos \beta$

+ 1 mark for substitution into correct identity

E7 (transcription error in line 1)

b)

$$-\left(\frac{4\sqrt{95} - 21}{60}\right)$$

0 out of 1

Exemplar 2

a)

$$\begin{aligned}\sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ &= \sin \frac{\sqrt{95}}{12} \cos \frac{4}{5} - \cos \frac{7}{12} \sin \frac{3}{5}\end{aligned}$$

1½ out of 3

+ ½ mark for value of x

+ ½ mark for value of y

+ ½ mark for $\cos \beta$

+ 1 mark for substitution into correct identity

- 1 mark for concept error in line 2

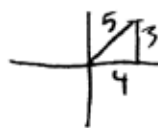
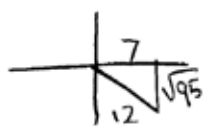
b)

$$\csc(\alpha - \beta) = \sin \frac{12}{7} \cos \frac{5}{3} - \cos \frac{12}{7} \sin \frac{5}{3}$$

0 out of 1

Exemplar 3

a)



$$\begin{aligned} &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ &= \left(\frac{\sqrt{95}}{12}\right)\left(\frac{3}{5}\right) - \left(\frac{7}{12}\right)\left(\frac{4}{5}\right) \\ &= \frac{3\sqrt{95}}{60} - \frac{28}{60} \end{aligned}$$

$$\sin(\alpha - \beta) = \frac{3\sqrt{95} - 28}{60}$$

2 out of 3

+ ½ mark for value of x

+ ½ mark for value of y

+ ½ mark for $\cos \beta$

+ 1 mark for substitution into correct identity

- ½ mark for procedural error (switching $\cos \beta$ and $\sin \beta$)

b)

$$\csc(\alpha - \beta) = \frac{60}{3\sqrt{95} - 28}$$

1 out of 1

work consistent with answer in a)

Describe the difference between the graph of $f(x) = \frac{7(x+2)}{x+2}$ and the graph of $g(x) = \frac{7(x-2)}{x+2}$ at $x = -2$.

Solution

The graph of $f(x) = \frac{7(x+2)}{x+2}$ has a point of discontinuity and the graph of $g(x) = \frac{7(x-2)}{x+2}$ has an asymptote.

1 mark

Exemplar 1

Because $f(x) = \frac{7(x+2)}{(x+2)}$ is a line and

$g(x) = \frac{7(x-2)}{(x+2)}$ has an asymptote

½ out of 1

award full marks

– ½ mark for lack of clarity in description

Exemplar 2

$f(x) = \frac{7(x+2)}{x+2}$ has a hole

$g(x) = \frac{7(x-2)}{x+2}$ has an asymptote.

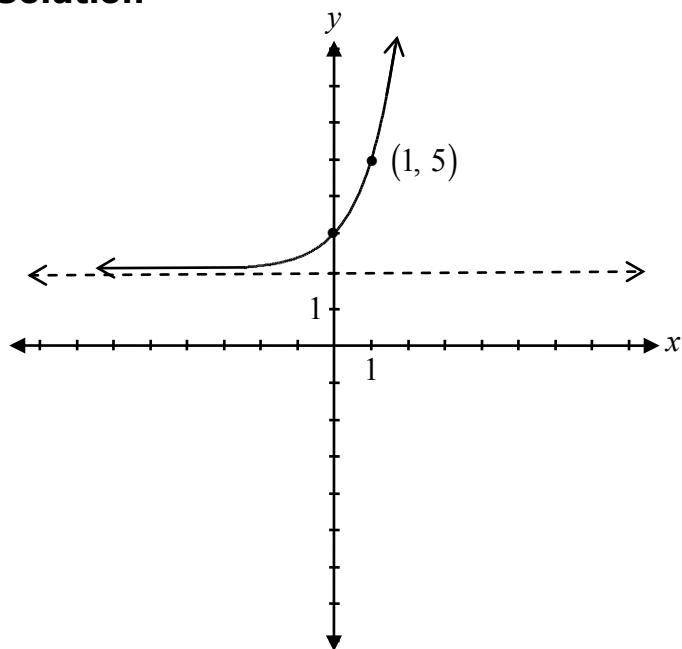
1 out of 1

Exemplar 3

One is positive and one
is negative

0 out of 1

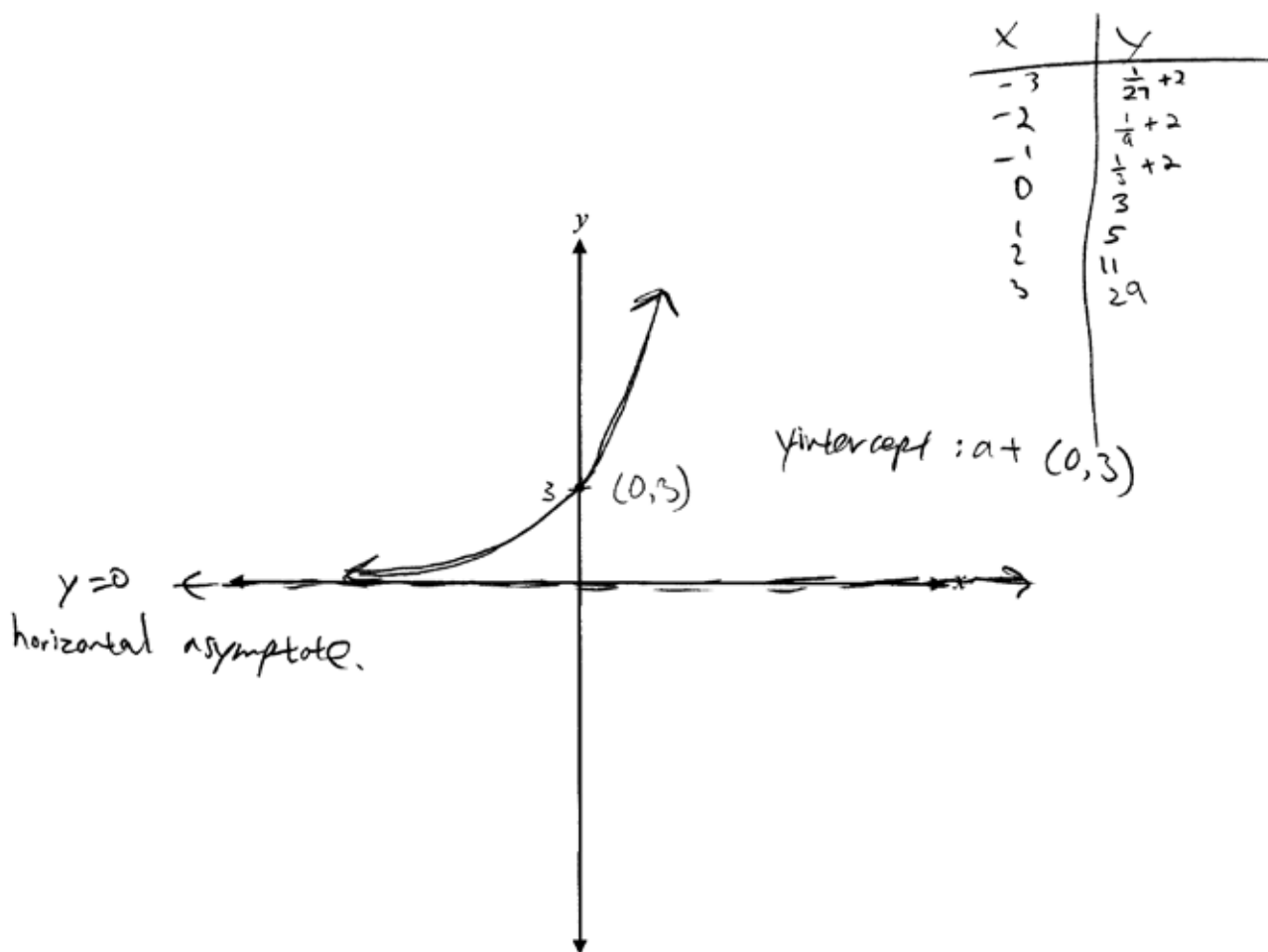
Sketch the graph of $f(x) = 3^x + 2$.

Solution

1 mark for increasing exponential function
1 mark for asymptotic behaviour at $y = 2$

2 marks

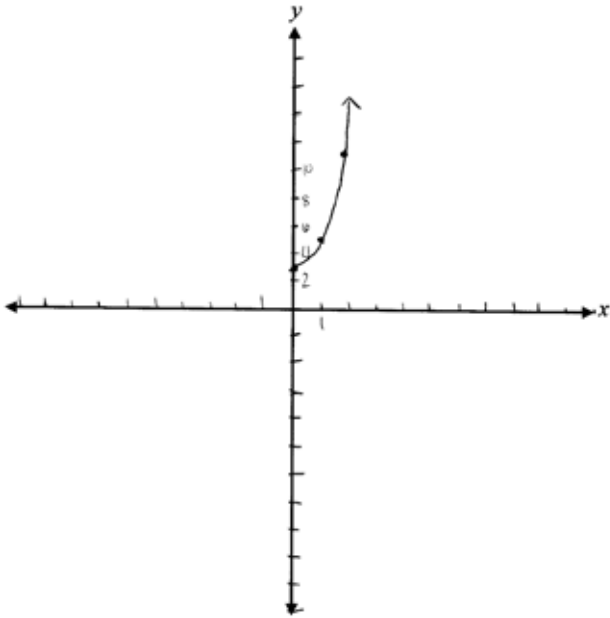
Exemplar 1



1 out of 2

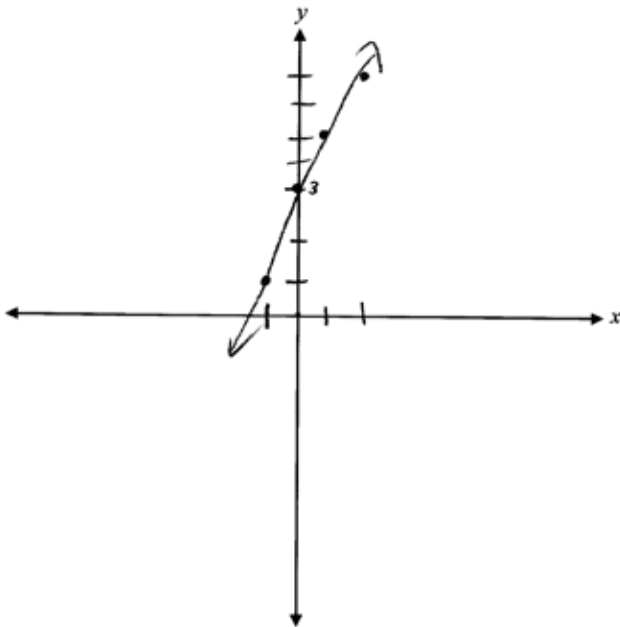
+ 1 mark for increasing exponential function

Exemplar 2



0 out of 2

Exemplar 3



0 out of 2

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Solve algebraically:

$${}_n C_3 = n - 2$$

Solution

$$\frac{n!}{(n-3)!3!} = n - 2$$

½ mark for substitution

$$\frac{n(n-1)\cancel{(n-2)}\cancel{(n-3)}!}{\cancel{(n-3)}!3!} = \cancel{n-2}$$

1 mark for factorial expansion

$$n(n-1) = 6$$

½ mark for simplification of factorials

$$n^2 - n - 6 = 0$$

$$(n-3)(n+2) = 0$$

$$n = 3 \quad \cancel{n = -2}$$

½ mark for rejecting extraneous root

½ mark for the value of n

3 marks

Exemplar 1

$$\frac{n!}{3!(n-3)!} = n-2$$

$$\frac{n(n-1)(n-2)(\cancel{n-3})!}{3!(\cancel{n-3})!} = n-2$$

$$\frac{n(n-1)(n-2)}{6} \rightarrow n-2$$

$$\frac{n(n-1)(\cancel{n-2})}{\cancel{n-2}} = 6$$

$$n(n-1) = 6$$

$$\boxed{n=3} \quad n \geq 3$$

3 out of 3

Exemplar 2

$$\frac{n!}{(n-3!)3!} = n-2$$

$$\frac{n(n-1)(\cancel{n-2})(\cancel{n-3!})}{(\cancel{n-3!})6} = \cancel{n-2}$$

$$n(n-1) = 6$$

$$n^2 - n - 6 = 0$$

$$(n-3)(n+2)$$

$$\boxed{n=3} \quad \cancel{n=2}$$

3 out of 3

award full marks

E7 (notation error in lines 1 and 2)

E2 (changing an equation to an expression in line 5)

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Describe the error that was made when solving the following equation:

$$\sin^2 \theta + \sin \theta - 2 = 1$$

$$\sin^2 \theta + \sin \theta = 3$$

$$\sin \theta (\sin \theta + 1) = 3$$

$$\sin \theta = 3 \quad \sin \theta + 1 = 3$$

\therefore No solution

$$\sin \theta = 2$$

\therefore No solution

Solution

The student did not apply the zero product principle before factoring.

1 mark

Exemplar 1

should have factored so the
equation equalled zero

½ out of 1

award full marks

– ½ mark for lack of clarity in explanation

Exemplar 2

It should = 0 to find the answer.

½ out of 1

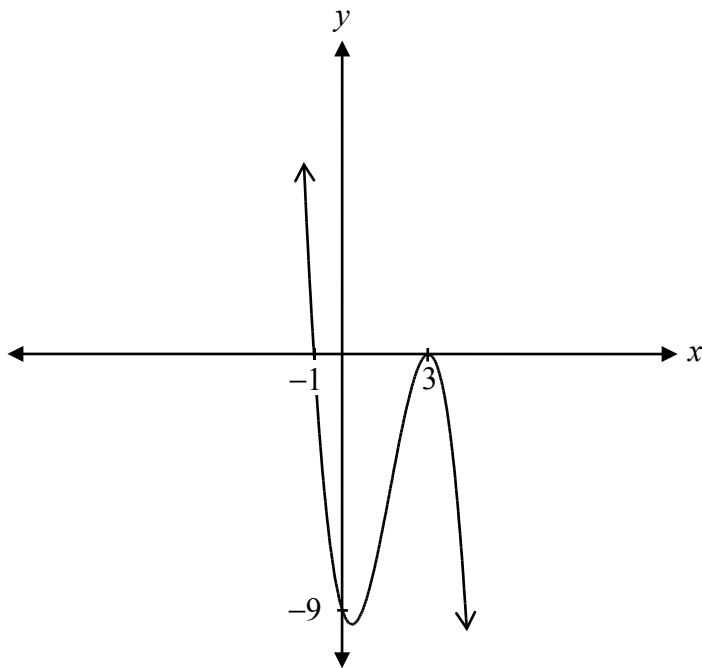
award full marks

– ½ mark for lack of clarity in explanation

Sketch the graph of the polynomial function with the following characteristics.

- a y -intercept of -9
- zeroes at -1 and 3
- the zero at -1 has a multiplicity of 1 and the zero at 3 has a multiplicity of 2

Solution



1 mark for x -intercepts

$\frac{1}{2}$ mark for y -intercept

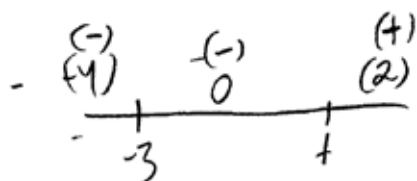
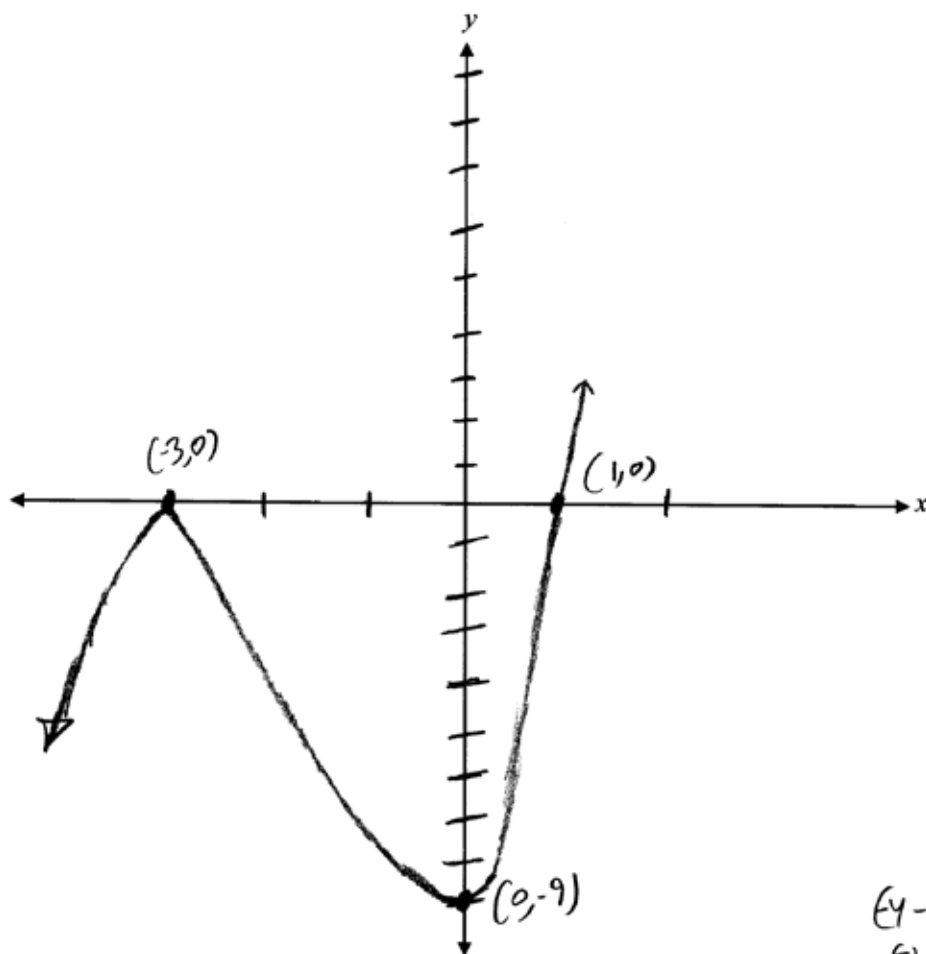
1 mark for multiplicity ($\frac{1}{2}$ mark for multiplicity at $x = 3$, $\frac{1}{2}$ mark for multiplicity at $x = -1$)

$\frac{1}{2}$ mark for shape of a cubic function

3 marks

Exemplar 1

$$(x-1)(x+3)^2$$



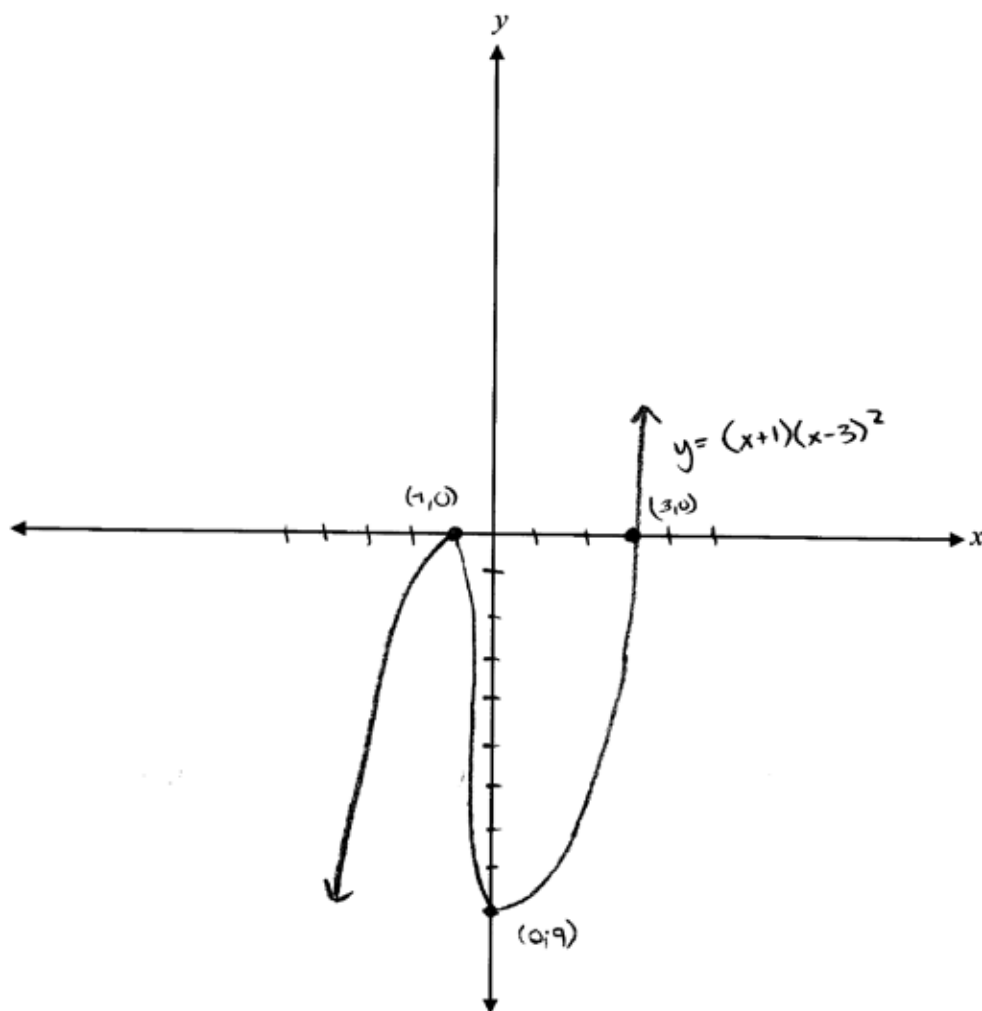
$$\begin{aligned} & (x-1)(x+3)^2 \\ & = (-) (+) \\ & \quad (-) \end{aligned}$$

$$\begin{aligned} & (x-1)(x+3)^2 \\ & = (+) (+) \\ & \quad (+) \end{aligned}$$

2 out of 3

- + ½ mark for y-intercept
- + ½ mark for shape of a cubic function
- + 1 mark for multiplicity

Exemplar 2



$$\begin{array}{ll} x = -1 & x = 3 \\ (x+1)^1 & (x-3)^2 \end{array}$$

$$f(x) = (x+1)(x-3)^2$$

$$y \text{ int @ } -9$$

$$\begin{array}{l} (x+1)(x-3)(x-3) \\ x^3 \end{array}$$

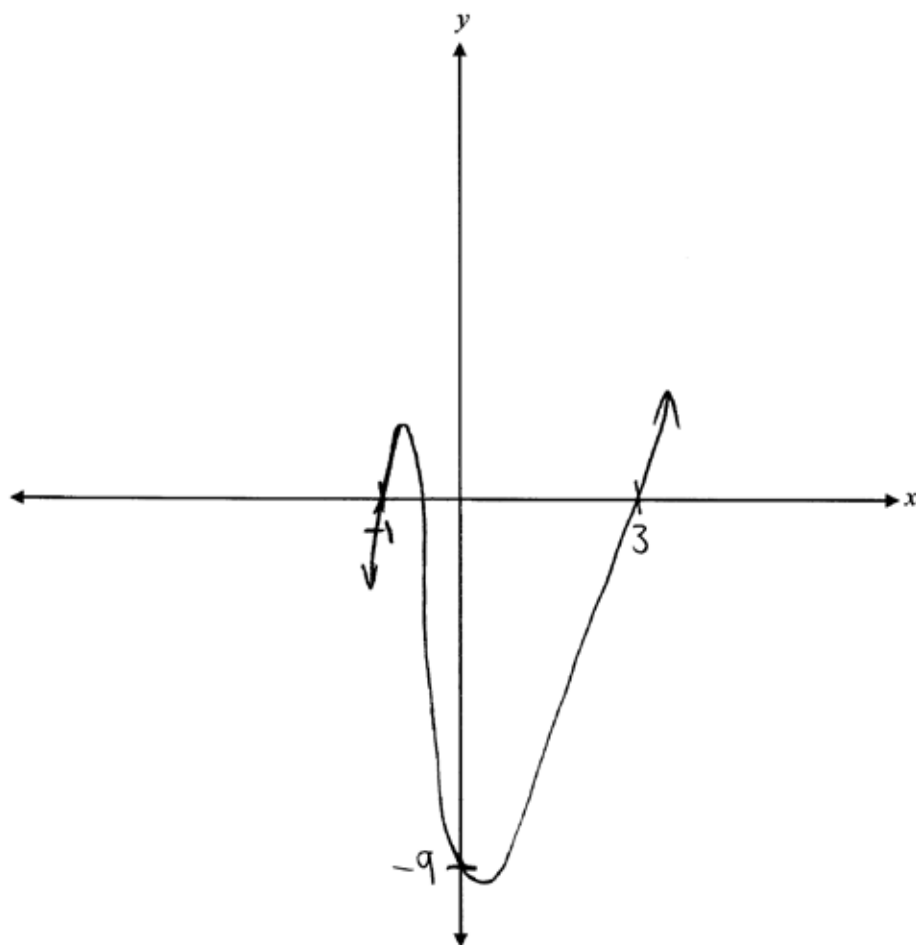
2 out of 3

+ 1 mark for x-intercept

+ ½ mark for y-intercept

+ ½ mark for shape of a cubic function

Exemplar 3



1½ out of 3

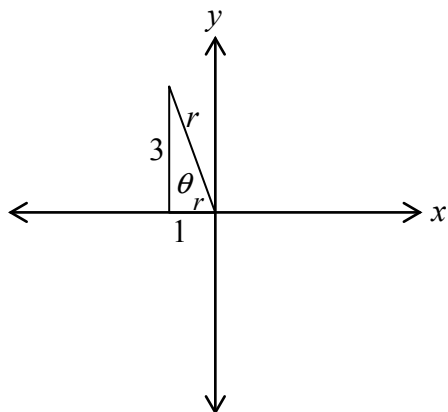
+ ½ mark for y-intercept

+ ½ mark for shape of a cubic function

+ ½ mark for multiplicity at $x = -1$

Given $\cot \theta = -\frac{1}{3}$, where θ is in quadrant II, determine the exact value of $\sin \theta$.

Solution



$$\cot \theta = \frac{x}{y}$$

$$r^2 = x^2 + y^2$$

$$r^2 = (-1)^2 + (3)^2$$

½ mark for substitution

$$r^2 = 10$$

$$r = \pm\sqrt{10}$$

½ mark for solving for r

$$\sin \theta = \frac{3}{\sqrt{10}}$$

1 mark for $\sin \theta$ (½ mark for the quadrant, ½ mark for the value)

2 marks

Note(s):

- accept any of the following values for r : $r = \pm\sqrt{10}$, $r = \sqrt{10}$

Exemplar 1

$$\cot \theta = \frac{x}{y}$$

$$\sin \theta = \frac{y}{r}$$

$$\begin{aligned} x &= 1 & x^2 + y^2 &= r^2 \\ y &= 3 & 1^2 + 3^2 &= r^2 \\ & & \sqrt{10} &= \sqrt{r^2} \\ & & r &= \pm\sqrt{10} \end{aligned}$$

Quad II
Sin = +

$$\sin \theta = \frac{3}{\sqrt{10}}$$

1½ out of 2

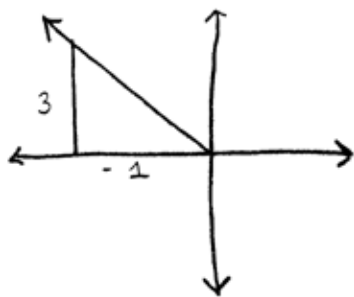
+ ½ mark for substitution

+ ½ mark for solving for r

+ ½ mark for the quadrant of $\sin \theta$

Exemplar 2

$$\tan \theta = -3$$



$$\begin{aligned} x &= -1 \\ y &= 3 \\ r^2 &= x^2 + y^2 \\ r^2 &= (-1)^2 + 3^2 \\ r^2 &= 1 + 9 \\ \sqrt{r^2} &= \sqrt{10} \\ r &= \pm\sqrt{10} \end{aligned}$$

$$\boxed{\sin \theta = \frac{-3}{\sqrt{10}}}$$

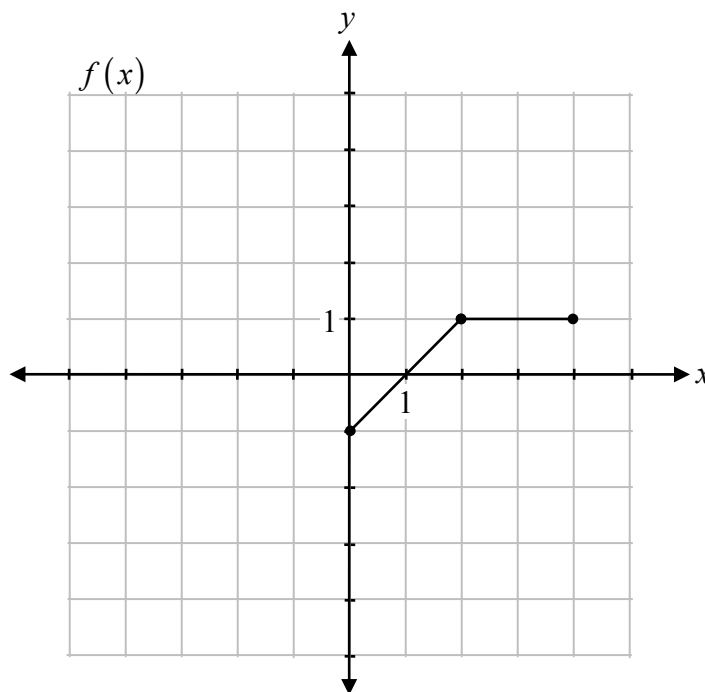
1 out of 2

+ ½ mark for substitution

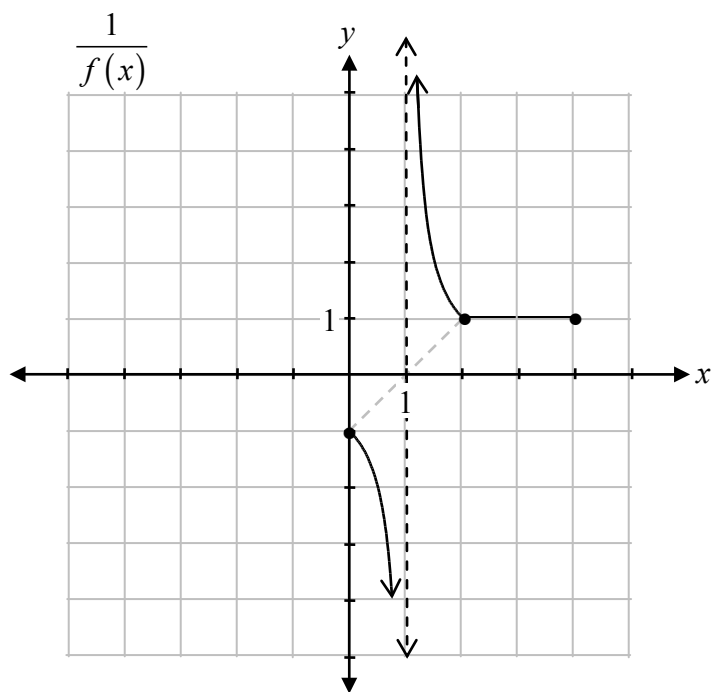
+ ½ mark for solving for r

E7 (notation error in line 4)

Given the function $f(x)$, sketch the graph of the reciprocal, $\frac{1}{f(x)}$.



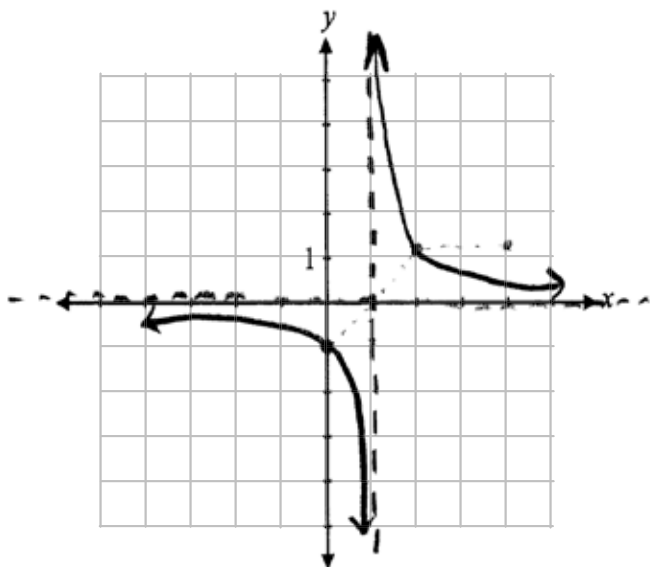
Solution



1 mark for asymptotic behaviour at $x = 1$
 ½ mark for graph left of vertical asymptote at $x = 1$
 ½ mark for graph right of vertical asymptote at $x = 1$

2 marks

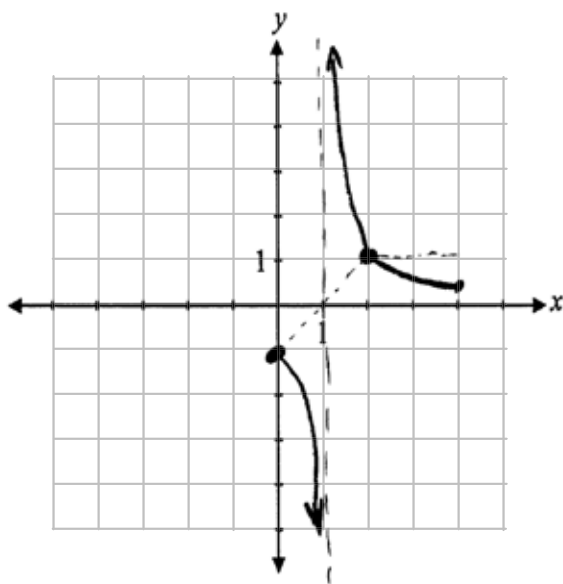
Exemplar 1



1 out of 2

+ 1 mark for asymptotic behaviour at $x = 1$

Exemplar 2

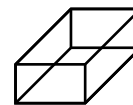


1½ out of 2

+ 1 mark for asymptotic behaviour at $x = 1$

+ ½ mark for graph left of vertical asymptote at $x = 1$

The volume of a planter, in the shape of a rectangular prism, can be modelled by the polynomial function $V(x) = x^3 + 3x^2 - 34x + 48$.



Determine the factors of the function, $V(x)$, which represent possible dimensions of this planter.

Solution

$$2^3 + 3(2)^2 - 34(2) + 48 = 0$$

$\therefore x - 2$ is a factor

2	1	3	-34	48
	↓	2	10	-48
	1	5	-24	0

1 mark for identifying one possible value of x

1 mark for synthetic division (or any other equivalent strategy)

$$V(x) = (x - 2)(x^2 + 5x - 24)$$

$$V(x) = \underline{(x - 2)(x + 8)(x - 3)}$$

1 mark for identifying all factors

3 marks

Exemplar 1

$$x = 2$$

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -34 & 48 \\ -x & \downarrow & -2 & -10 & +48 \\ \hline & 1 & 5 & -24 & 0 \end{array}$$

$$x^2 + 5x - 24$$

$$(x-3)(x+8)$$

$$x=3 \quad x=8$$

$$\begin{array}{l} x = 2 \\ x = 3 \\ x = -8 \end{array}$$

2 out of 3

+ 1 mark for identifying one possible value of x

+ 1 mark for synthetic division

Exemplar 2

$$\begin{aligned} v(1) &= 1^3 + 3(1)^2 - 34(1) + 48 \\ &= 1 + 3 - 34 + 48 \end{aligned}$$

$$\begin{aligned} v(2) &= 8 + 12 - 68 + 48 \\ &= -68 + 68 = 0 \end{aligned}$$

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -34 & 48 \\ & \downarrow & 2 & 10 & -48 \\ \hline & 1 & 5 & -24 & 0 \end{array}$$

$$(x-2)(x^2 + 5x - 24)$$

$$(x-2)(x-3)(x+8)$$

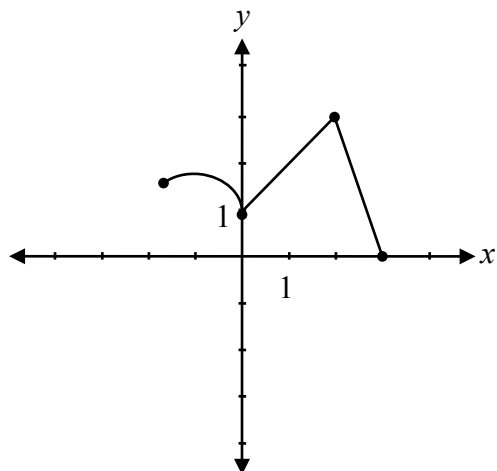
$x = 2 \quad x = 3 \quad x = -8$

2 out of 3

award full marks

- 1 mark for concept error (stating zeroes as final answer)

Describe how to determine the range of the inverse of the following graph.

**Solution**

The domain of the graph becomes the range of the inverse.

1 mark

Exemplar 1

Exchange the values of x and y $(x,y) \rightarrow (y,x)$

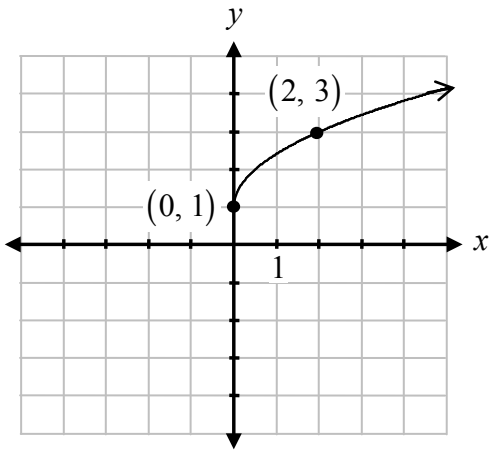
0 out of 1

Exemplar 2

$D: [0, 3]$
 $R: [-2, 3]$

0 out of 1

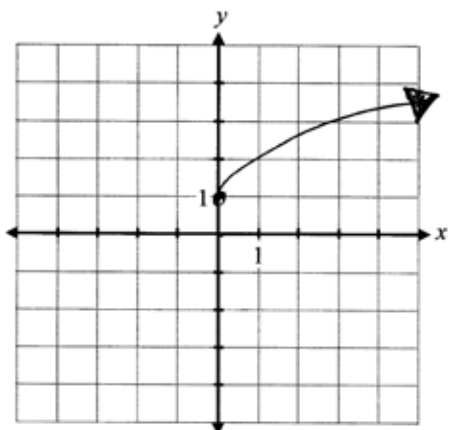
Sketch the graph of the function $y = \sqrt{2x} + 1$.

Solution

1 mark for shape of a radical function
1 mark for vertical translation
1 mark for horizontal compression

3 marks

Exemplar 1

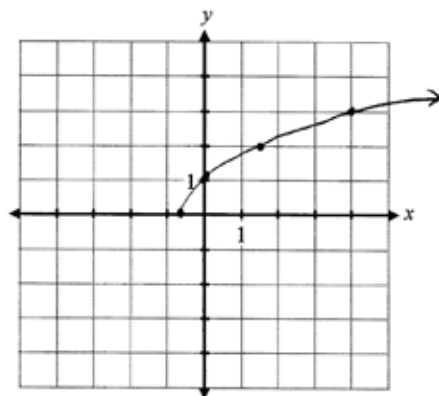


2 out of 3

+ 1 mark for shape of a radical function

+ 1 mark for vertical translation

Exemplar 2



$$y = \sqrt{2(x+0.5)}$$

$$2^{-0.5} \begin{array}{r|l} x & y \\ \hline 0 & 0 \\ 1 & 1 \end{array}$$

\Rightarrow

$$\begin{array}{r|l} x & y \\ \hline -0.5 & 0 \\ 0.5 & 1 \end{array}$$

$$\begin{array}{l} b \quad x \div 2 \\ \hline h \quad x - 0.5 \\ \hline \end{array}$$

2 out of 3

+ 1 mark for shape of a radical function

+ 1 mark for horizontal compression

Given the following characteristics of a sinusoidal function:

- an amplitude of 2
- a vertical translation down 3 units
- a period of $\frac{\pi}{4}$

- a) Determine an equation of this sinusoidal function in the form $y = a \sin b(x - c) + d$.
- b) Determine the range of this function.

Solution

a) $b = \frac{2\pi}{\frac{\pi}{4}}$

$$b = 8$$

$$y = 2 \sin(8x) - 3$$

1 mark for the value of b

$\frac{1}{2}$ mark for amplitude

$\frac{1}{2}$ mark for vertical translation

2 marks

b) Range: $\{y \mid y \in \mathbb{R}, -5 \leq y \leq -1\}$

or

1 mark

Range: $[-5, -1]$

Note(s):

- Other answers are possible for a).

Exemplar 1

a)

$$f(x) = 2\sin(8x) + 3$$

$$b = \frac{2\pi}{8}$$

$$b = \frac{\pi}{4}$$

1½ out of 2

+ ½ mark for amplitude

+ 1 mark for the value of b

b)

$$[1, 5]$$

1 out of 1

work consistent with answer in a)

Exemplar 2

a)

$$y = 2 \sin \left[\frac{\pi}{4} (x) \right] - 3$$

1 out of 2

+ ½ mark for amplitude

+ ½ mark for vertical translation

b)

$$[-1, -5]$$

1 out of 1

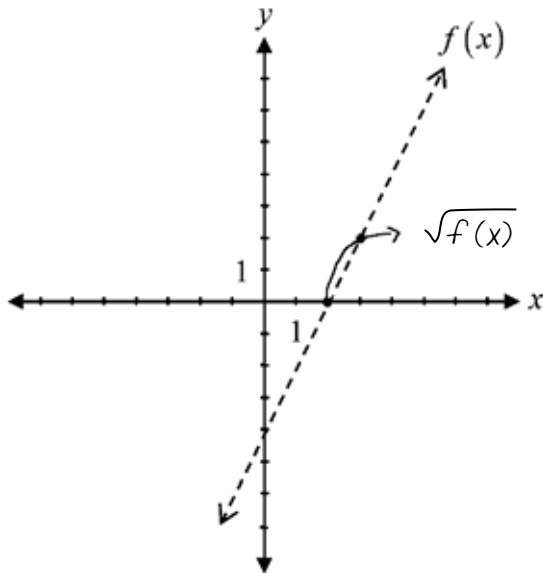
award full marks

E8 (range written in incorrect order)

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Suah was given the graph of $f(x)$ and asked to graph $y = \sqrt{f(x)}$.

Her solution is given on the graph below.



Describe the error Suah made when sketching the graph of $y = \sqrt{f(x)}$.

Solution

Suah's graph did not cross the invariant point at $y = 1$.

1 mark

Exemplar 1

Swah's graph crosses the line too high.

½ out of 1

award full marks

– ½ mark for lack of clarity in explanation

Solve:

$$9^{2x+1} = 27^x$$

Solution

$$3^{2(2x+1)} = 3^{3x}$$

1 mark for changing to a common base

$$3^{4x+2} = 3^{3x}$$

1 mark for exponent law (½ mark for each side)

$$4x + 2 = 3x$$

½ mark for equating exponents

$$x = -2$$

½ mark for solving for x

3 marks

Exemplar 1

$$3^{2(2x+1)} = 3^{3x}$$

$$3^{4x+1} = 3^{3x}$$

$$4x+1 = 3x$$

-3x -3x

$$x+1 = 0$$

$$x = -1$$

2 out of 3

award full marks

- ½ mark for arithmetic error in line 2
- ½ mark for procedural error in line 2

Exemplar 2

$$9^{2x+1} = 9^{3x}$$

$$2x+1 = 3x$$

$$1 = 3x - 2x$$

$$1 = x$$

2 out of 3

- + ½ mark for equating exponents
- + ½ mark for solving for x
- + 1 mark for exponent law

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Appendices



Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	<ul style="list-style-type: none">▪ answer given as a complex fraction▪ final answer not stated
E2 equation/expression	<ul style="list-style-type: none">▪ changing an equation to an expression▪ equating the two sides when proving an identity
E3 variables	<ul style="list-style-type: none">▪ variable omitted in an equation or identity▪ variables introduced without being defined
E4 brackets	<ul style="list-style-type: none">▪ "$\sin x^2$" written instead of "$\sin^2 x$"▪ missing brackets but still implied
E5 units	<ul style="list-style-type: none">▪ missing units of measure▪ incorrect units of measure▪ answer stated in degrees instead of radians or vice versa
E6 rounding	<ul style="list-style-type: none">▪ rounding error▪ rounding too early
E7 notation/transcription	<ul style="list-style-type: none">▪ notation error▪ transcription error
E8 domain/range	<ul style="list-style-type: none">▪ answer given outside the domain▪ bracket error made when stating domain or range▪ domain or range written in incorrect order
E9 graphing	<ul style="list-style-type: none">▪ incorrect or missing endpoints or arrowheads▪ scale values on axes not indicated▪ coordinate points labelled incorrectly
E10 asymptotes	<ul style="list-style-type: none">▪ asymptotes drawn as solid lines▪ asymptotes missing but still implied▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an Irregular Test Booklet Report should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all "NR") or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an Irregular Test Booklet Report.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an Irregular Test Booklet Report documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: _____

Date marked: _____

Booklet No.: _____

Problem(s) noted: _____

Question(s) affected: _____

Action taken or rationale for assigning marks: _____

Follow-up: _____

Decision: _____

Marker's Signature: _____

Principal's Signature: _____

For Department Use Only—After Marking Complete

Consultant: _____

Date: _____

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	Learning Outcome	Mark
6	R5	1
7	R1	2
10	R1, R2, R3	3
13 a)	R1	1
13 b)	R1	1
21	R6	1
36	R1	2
38	R6	1
Unit B: Trigonometric Functions		
Question	Learning Outcome	Mark
1	T1	2
22	T1	1
24	T1	1
27	T4	4
35	T2	2
40 a)	T4	2
40 b)	T4	1
Unit C: Binomial Theorem		
Question	Learning Outcome	Mark
3	P4	3
5	P1	2
12	P1	1
16	P3	1
26	P4	2
32	P3	3
Unit D: Polynomial Functions		
Question	Learning Outcome	Mark
15	R11	1
28	R11	2
34	R12	3
37	R12	3

Unit E: Trigonometric Equations and Identities

Question	Learning Outcome	Mark
2	T5	3
9	T5, T6	4
11	T6	3
19	T5	1
29 a)	T6	3
29 b)	T6	1
33	T5	1

Unit F: Exponents and Logarithms

Question	Learning Outcome	Mark
4	R10	3
8	R8	3
18	R7	1
23	R10	3
31	R9	2
42	R10	3

Unit G: Radicals and Rationals

Question	Learning Outcome	Mark
14	R13	2
17	R13	1
20	R13	1
25	R14	4
30	R14	1
39	R13	3
41	R13	1