Grade 12
Pre-Calculus Mathematics
[MPC40S]

Chapter 4

Trigonometry and the Unit Circle

Outcomes

T1, T2, T3, T5

12P.T.1. Demonstrate an understanding of angles in standard position expressed in degrees and radians.

12P.T.2. Develop and apply the equation of the unit circle.

12P.T.3 Solve Problems, using the six trigonometric ratios for angles expressed in radians and degrees.

12P.T.5. Solve, algebraically and graphically, first and second-degree trigonometric equations with the domain expressed in degrees and radians.

Chapter 4 – Homework

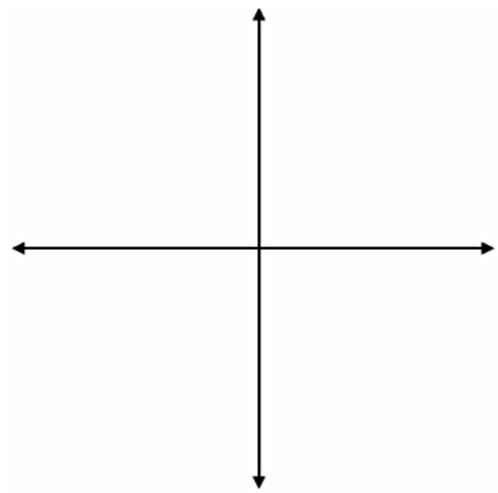
Section	Page	Questions

Chapter 4: TRIGONOMETRY AND THE UNIT CIRCLE

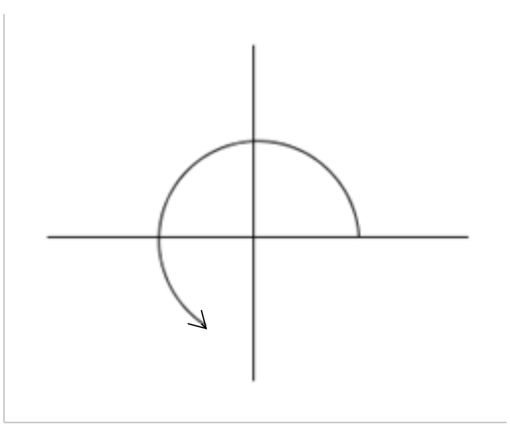
4.1 – Angles and Angle Measure

An ____ has its centre at the origin and its initial arm along the positive x-axis

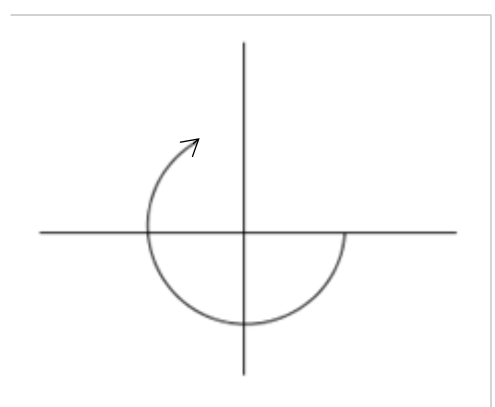
There are ____________________ and ___________________ angles.



Positive Angles
(Counter-clockwise)



Negative Angles
(Clockwise)



Example #1

In which **quadrant** is the terminal arm of each angle located?

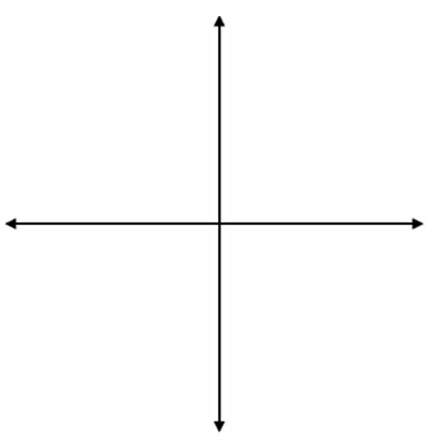
a) 400° _______ b) 700° _______

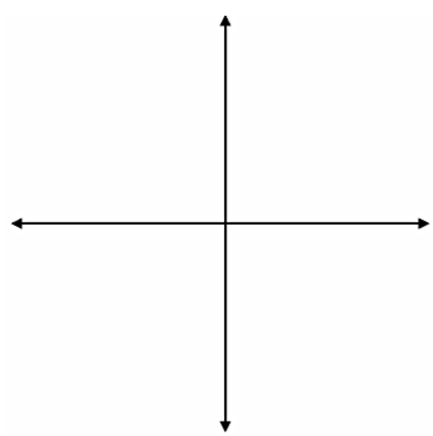
c) – 65° _______ d) – 150° _______

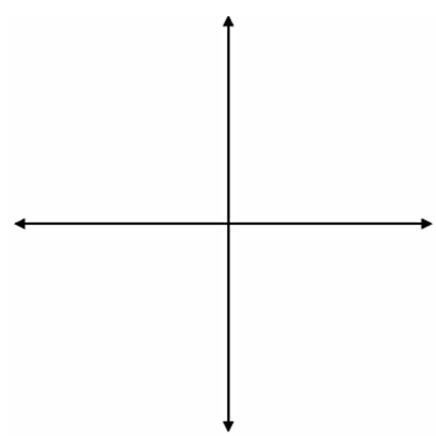
Example #2

Sketch each angle in **standard position**.

a) $286°$ b) $-190°$ c) $430°$





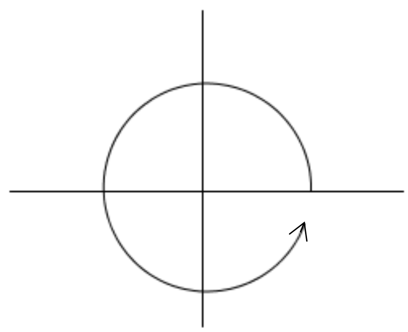


Radian Measure of an Angle

- The formula for the **circumference** of a circle is _______________
- The **unit circle** has a radius = __________
- Therefore, the circumference of the unit circle is ____________

$2\pi = 6.283185\ldots$

This means that the distance traveled from the initial arm all around the circle and back again is 6.283185…



Revolutions	Degrees	Radian Measure	
1 revolution		_____ radians	6.283185… radians
$\frac{1}{2}$ revolution		_____ radians	3.141592… radians
$\frac{1}{4}$ revolution		_____ radians	1.570796… radians
$\frac{3}{4}$ revolution		_____ radians	4.712388… radians
$\frac{1}{360}$ revolution		_____ radians	0.017453… radians

Note that 1 radian = $\left(\frac{180°}{\pi}\right) \approx 57.3°$

Converting Degrees to Radians: __________________________

Example #3

Express the following angle measures in **radians**.

a) $30°$	b) $225°$	c) $720°$

Converting Radians to Degrees: __________________________

Example #4

Express the following angle measures in **degrees**

a) $\frac{2\pi}{3}$	b) 1.6	c) $\frac{5\pi}{6}$

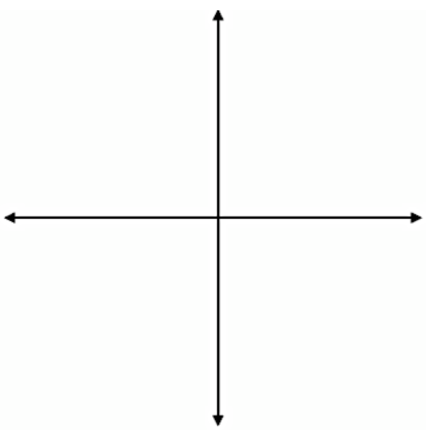
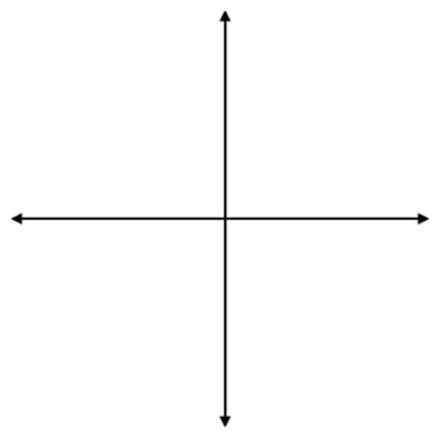
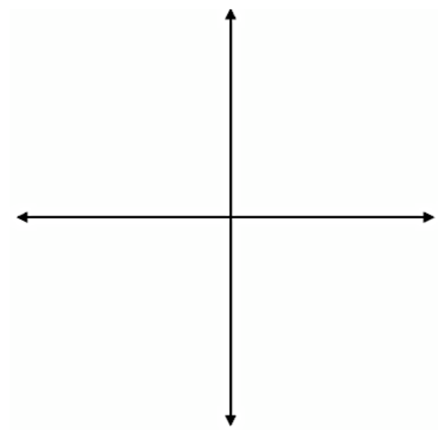
Coterminal Angles

Coterminal Angles are ____

____

Example #12

Sketch $\theta = 30°$ as an angle in standard position, and show that $\theta = 390°$ and $\theta = -330°$ are **coterminal angles**.

The coterminal angle can be found by **adding** or **subtracting** revolutions; either $\pm 360°$ when given degree measure or $\pm 2\pi$ when given radian measure. There are an infinite number of coterminal angles.

Example #5

Determine 3 **coterminal angles** for $40°$.

Example #6

Determine 3 **coterminal angles** for $\frac{\pi}{6}$

General Form of Coterminal Angles

Degrees: ________________________

Radians: ________________________

Example #7

Express the angles **coterminal** with $50°$ in general form.

Example #8

Express a general form for all **coterminal angles** of $\frac{5\pi}{3}$

Example #9

Determine a **coterminal angle** to $740°$ over the interval $-360° < \theta < 0°$

Example #10

Determine all **coterminal angles** to $\frac{5\pi}{3}$ over the interval $[-4\pi, 2\pi]$

Arc Length

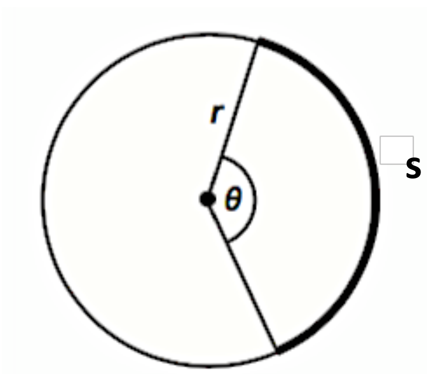
The central angle is the relationship between the length of the arc and the radius of the circle.

The equation that represents this relationship is:

$S = \theta r$ where: $S =$ ____________________

$r =$ ____________________

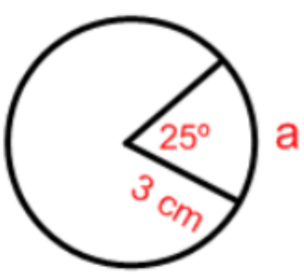
$\theta =$ ____________________



Note: If there is no unit attached to the angle measure (ex: $\theta = 2.5$) it is assumed to be in **radians**.

Example #11

Determine the **arc length**.



Example #12

A bicycle tire has a radius of 0.5 m and travels a distance of 1.5 m. Determine the rotated angle, in degrees.

Example #13

Given the following information determine the missing value.

a) $r = 8.7$ cm, $\theta = 75°$ determine arc length

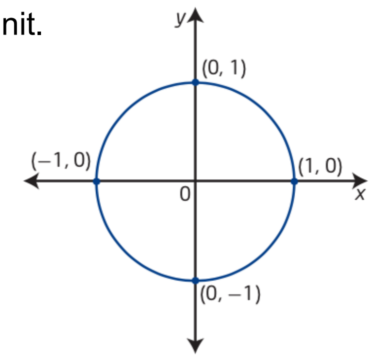
b) $\theta = 1.8$, $S = 4.7$ mm, determine the radius

c) $r = 5$ m, $S = 13$ m, determine the measure of the central angle

Chapter 4: TRIGONOMETRY AND THE UNIT CIRCLE

4.2 – The Unit Circle

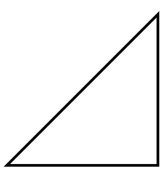
The unit circle is centered at the origin and has a radius of 1 unit.

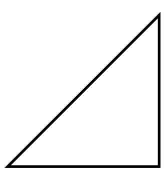


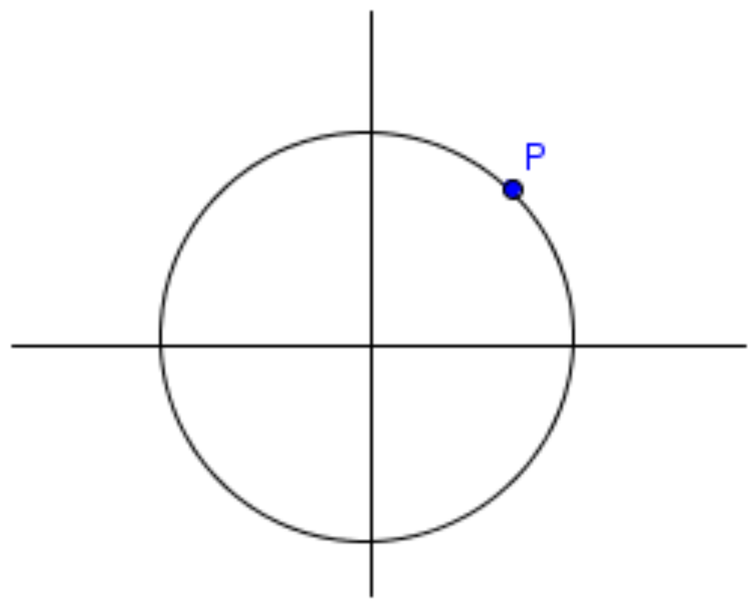
We use the notation $P(\theta)$ to indicate a point on the circle.

θ = arc length

$P(\theta)$ = defined by a point (x, y)







Since the radius is 1, then the equation of the unit circle is $x^2 + y^2 = 1$

Important ideas:

____

____

Example #1

Determine whether or not the point $\left(\frac{2}{5}, \frac{3}{5}\right)$ is on the unit circle. Justify your reasoning.

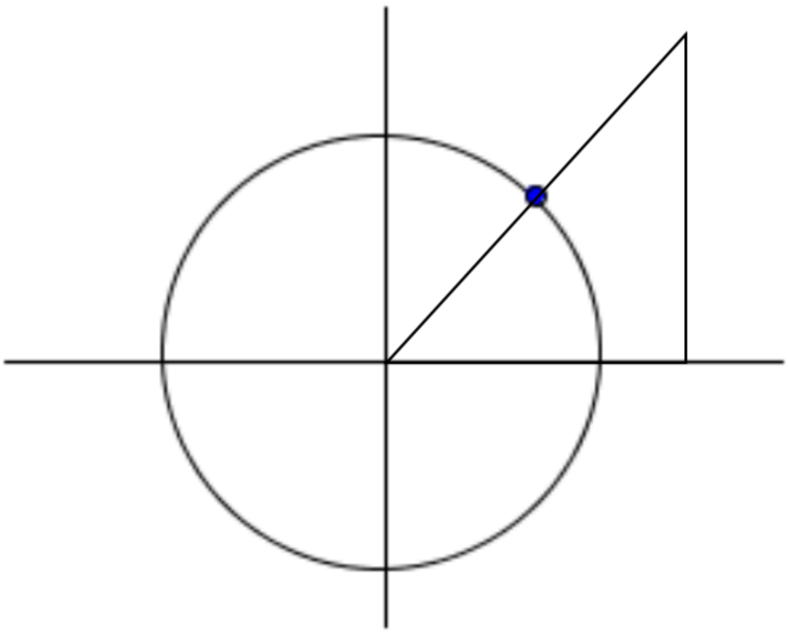
Example #2

A point $\left(\frac{2}{3}, y\right)$ is on the unit circle. Determine the value of y.

Example #3

The point $P(\theta)$ lies on the intersection of the unit circle and a line joining the origin to the point $(4, 3)$.

Determine the coordinates of $P(\theta)$.



Example #4

The point $P(\theta)$ lies on the intersection of the unit circle and a line joining the origin to the point (–3, 6).

Determine the coordinates of $P(\theta)$.

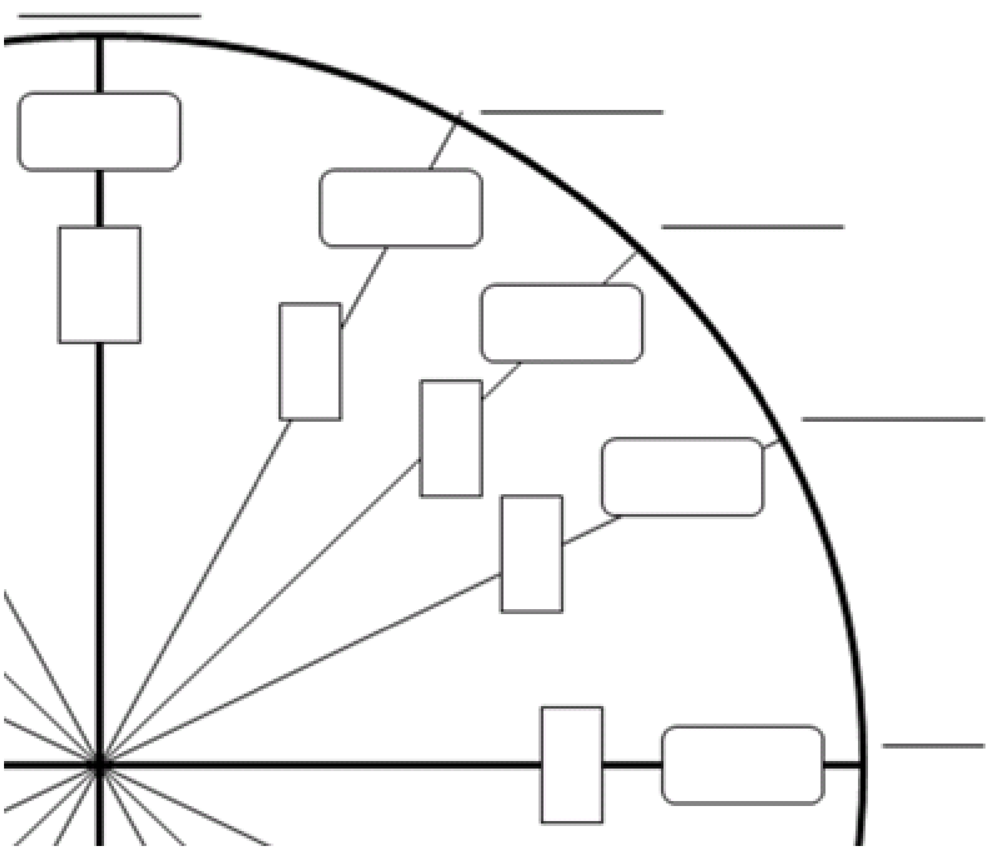
Example #5

Determine the values of $\cos\theta$ and $\tan\theta$ over the interval $\frac{3\pi}{2} \le \theta \le 2\pi$ when $\sin\theta = -\frac{3}{5}$.

Chapter 4: TRIGONOMETRY AND THE UNIT CIRCLE

The Unit Circle

QUADRANT 1



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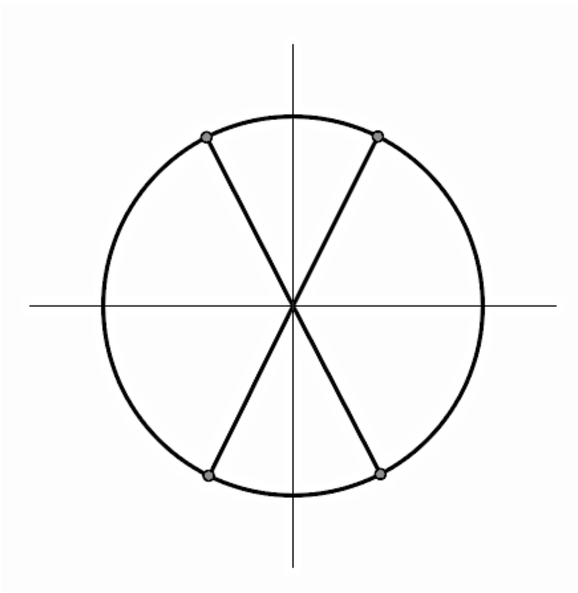
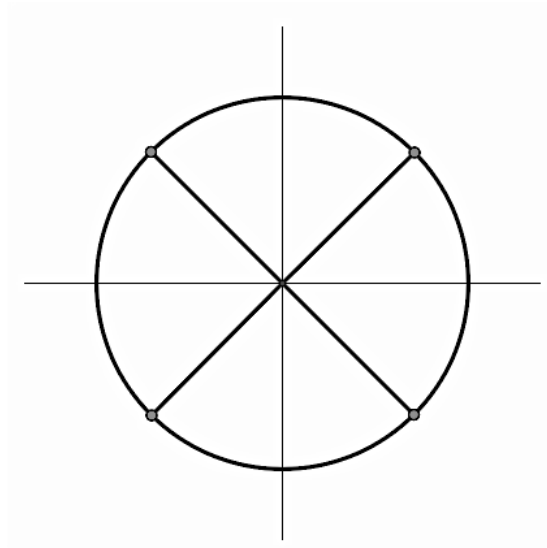
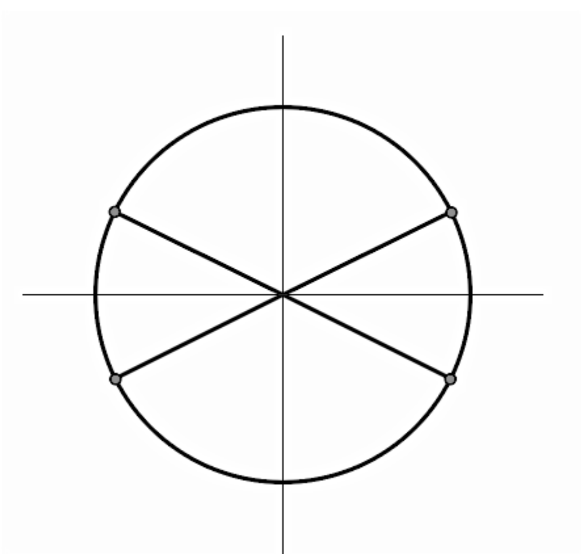
$\frac{\pi}{3}$ Family



$\frac{\pi}{4}$ Family

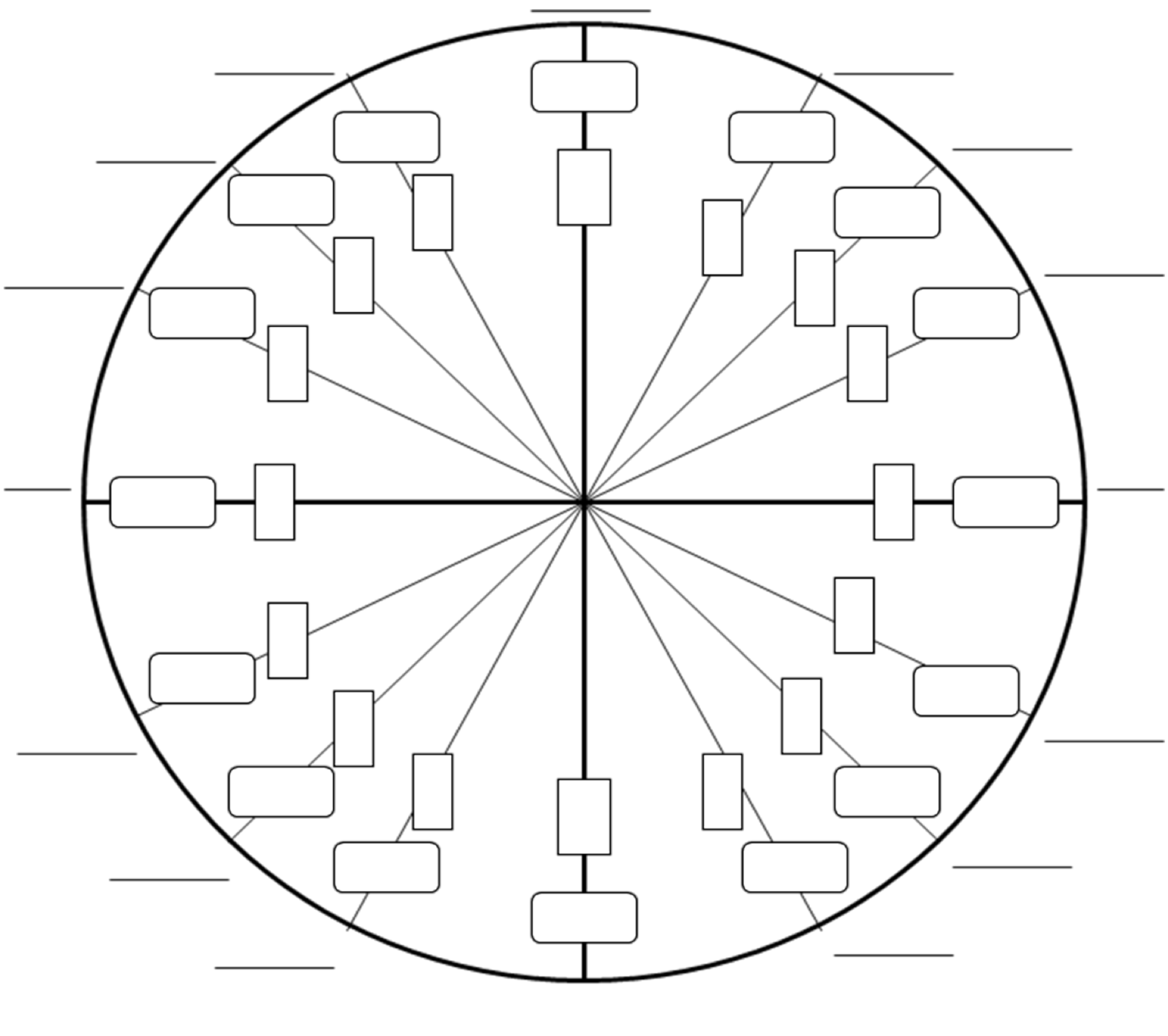


$\frac{\pi}{6}$ Family



 Date: ____________________

THE UNIT CIRCLE



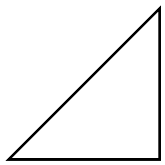
Chapter 4: TRIGONOMETRY AND THE UNIT CIRCLE

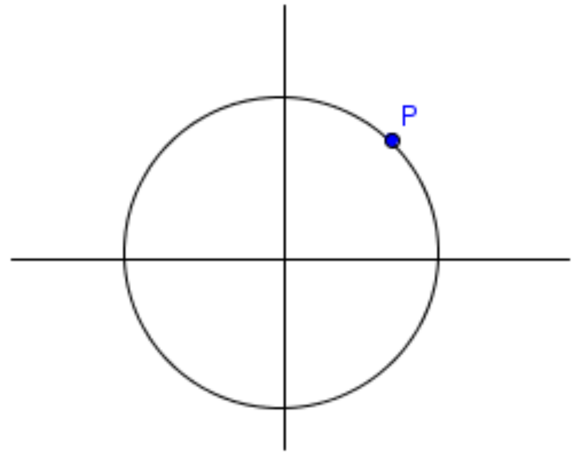
4.3 – Trigonometric Ratios

Recall:

θ = arc length

$P(\theta)$ = defined by a point (x, y)





If we use the trigonometric rations SOH CAH TOA, then

$\sin\theta = \frac{y}{1}$ → ____________

$\cos\theta = \frac{x}{1}$ → ____________

$\tan\theta = \frac{y}{x}$ → ____________

Thus, any point on the unit circle can be described as: $P(\theta) = (\cos\theta, \sin\theta)$

PRIMARY FUNCTIONS	RECIPROCAL FUNCTIONS $\left(\frac{1}{f(x)}\right)$
$\sin\theta = y$	cosecant ____________________
$\cos\theta = x$	secant ____________________
$\tan\theta = \frac{y}{x}$	cotangent ____________________

Example #1

The point $\left(\frac{5}{13}, -\frac{12}{13}\right)$ lies on the terminal arm of an angle θ in standard position.

a) Draw a diagram to represent this situation.

b) Find all 6 trigonometric ratios for θ.

Example #2

The point $\left(-\frac{3}{5}, \frac{4}{5}\right)$ lies on the terminal arm of an angle θ in standard position.

a) Draw a diagram to represent this situation.

b) Find all 6 trigonometric ratios for θ.

Determining Exact Values

Example #3

Determine the **exact** value of the following trigonometric ratios.

a) $\cos\frac{\pi}{3} =$	b) $\sec\frac{\pi}{3} =$
c) $\sin\left(-\frac{5\pi}{6}\right) =$	d) $\cos\frac{7\pi}{4} =$
e) $\cot(270°) =$	f) $\csc\left(\frac{2\pi}{3}\right) =$
g) $\tan\frac{17\pi}{4} =$	h) $\sec\frac{23\pi}{3} =$

Example #4

Determine the **exact** value of the following expressions.

a) $\cos(120°) - \tan(-135°)$

b) $\cot\left(-\frac{3\pi}{4}\right) + \csc\left(\frac{\pi}{2}\right)$

c) $\sin^2\left(\frac{7\pi}{6}\right) + \cos^2\left(\frac{7\pi}{6}\right)$

d) $\tan^2\left(\frac{-\pi}{3}\right)\sec\left(\frac{4\pi}{3}\right)$

Chapter 4: TRIGONOMETRY AND THE UNIT CIRCLE

4.4 – Trigonometric Equations

We can solve trigonometric equations just like we have been solving equations from previous units.

Note: If interval/domain is given in **radians**, your answer must be in **radians**.
If interval/domain is given in **degrees**, your answer must be in **degrees**.

Example #1

Solve the following trigonometric equation, over the given domain.

$\sin\theta = \frac{1}{2}, \quad 0 \le \theta \le 2\pi$

Example #2

Solve the following trigonometric equations, over the given domain.

a) $2\cos\theta + 3 = 1, \quad 0° \le \theta \le 540°$

b) $4\sec x + 8 = 0, \quad 0 \le x \le 2\pi$

Example #3

Solve the following trigonometric equations, over the given intervals.

a) $3\tan^2 x - 9 = 0, \quad 0° \leq x \leq 360°$

b) $2\cos^2\theta + \cos\theta = 1, \quad 0 \leq \theta \leq 2\pi$

Example #4

Solve the following trigonometric equations, over the given intervals.

a) $2\sin^2 x - 1 = \sin x, \quad 0 \leq x \leq 270°$

b) $\sin^2 x + \sin x - 12 = 0, \quad 0 \leq x \leq 2\pi$

Example #5

Solve the following trigonometric equations, over the given intervals.

a) $\csc^2 x + \csc x - 12 = 0,\ \ 0 \leq x \leq 2\pi$

b) $\tan^2\theta - 5\tan\theta + 4 = 0\,,\ -2\pi \leq x \leq 2\pi$

Example #6

Solve the following trigonometric equations, over the given interval.

$$2\cos^2\theta - 4\cos\theta - 5 = 0, \quad 0 \leq \theta \leq 2\pi$$

General Solution of Trigonometric Equations

If the domain is **real numbers**, there are an **infinite** number of rotations on the unit circle in both a positive and negative direction.

To determine a **general solution**, find the solutions in one positive rotation. Then use the concept of coterminal angles to write an expression that identifies all possible measures.

There are different was to request the **general solution** answers. They are:

- Domain is all real numbers
- $x \in R$ or $\theta \in R$
- General solution

Example #7

a) **Solve** $\cot\theta = \frac{1}{\sqrt{3}}$ over the interval $0 \leq \theta \leq 2\pi$

b) **Solve** the above equation if $\theta \in R$

Example #8

Solve each of the following trigonometric equations.

a) Solve $\tan\theta = -4$ if the domain is all real numbers, in radians.

b) Find the general solution of $\cos\beta = 0$, in degrees.

Example #9

Solve the following trigonometric equation, where $\theta \in R$. (In radians)

$$2\tan^2\theta - \tan\theta - 1 = 0$$