Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

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MATHEMATICS P2 MEMORANDUM

MARKS: 150

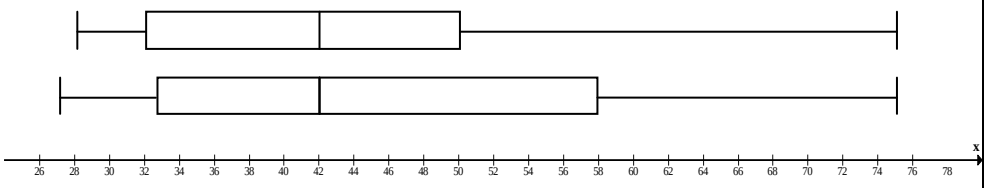
This memorandum consists of 14 pages.

QUESTION 1

<table>
<tr><td>1.1</td><td>55</td><td>✓ Answer</td><td>(1)</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>1.2</td><td>
<table>
<tr><th>AGES</th><th>FREQUENCY</th><th>CUMULATIVE FREQUENCY</th></tr>
<tr><td>$18 \leq x < 23$</td><td>4</td><td>4</td></tr>
<tr><td>$23 \leq x < 28$</td><td>8</td><td>12</td></tr>
<tr><td>$28 \leq x < 33$</td><td>13</td><td>25</td></tr>
<tr><td>$33 \leq x < 38$</td><td>15</td><td>40</td></tr>
<tr><td>$38 \leq x < 43$</td><td>10</td><td>50</td></tr>
<tr><td>$43 \leq x < 48$</td><td>4</td><td>54</td></tr>
<tr><td>$48 \leq x < 53$</td><td>1</td><td>55</td></tr>
</table>
</td><td>Cumulative Frequency
✓ First 4 correct values
✓ Remaining 3 correct

Frequency
✓ First 4 values correct
✓ Remaining 3 correct
(CA mark)</td><td>(4)</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>1.3</td><td>Median = 34 years</td><td>✓ Answer</td><td>(1)</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>1.4</td><td>Voters 35 years or older = 55 – 31
= 24</td><td>✓ Answer</td><td>(1)</td></tr>
<tr><td></td><td></td><td></td><td>[7]</td></tr>
</table>

QUESTION 2

2.1	27; 31; 31; 35; 39; 40; 44; 50; 54; 62; 65; 75 Min = 27, $Q_1=\frac{31+35}{2}$= 33 , $Q_2=\frac{40+44}{2}$= 42 , $Q_3=\frac{54+62}{2}$= 58, Max =75 See Bottom Box and Whisker Diagram  	✓ Min and Max ✓ Q_1 ✓ Q_2 ✓ Q_3	(4)
2.2	Min = 28, Q_1= 32, Q_2=40, Q_3= 32+18 = 50, Max = 75 See diagram in 2.1 (Top Box and Whisker Diagram)	✓ Min, Q_1,Q_2&max ✓ Q_3	(2)
2.3	Dutywa had more families with older people. • 25% 58 years and older. (i.e. Q_3 = 58) • 75% 33 years and older. • Any acceptable/reasonable reason which refers to figures. Accept: Both towns Reason: Have the same median 42.	✓ Dutywa ✓ Reason	(2)
			[8]

QUESTION 3

3.1

$$Mean = \frac{65,3 + 81,9 + 70 + 88,2 + 56,5 + 94,8 + 83 + 44,1 + 75 + 79,4}{10}$$

$$= \frac{738,2}{10}$$

= 73,82

✓ $\frac{738,2}{10}$
✓ 73,82

Answer only: 2/2 (2)

3.2

x	x-$\bar{x}$	$(x - \bar{x})^2$
65,3	65,3 – 73,82 = - 8,52	72,5904
81,9	81,9 – 73,82 = 8,08	65,2864
70	70 – 73,82 = - 3,82	14,5924
88,2	88,2 – 73,82 = 14,18	201,0724
56,5	56,5 – 73,82 = -17,32	299,9824
94,8	94,8 – 73,82 = 20,98	440,1604
83	83 – 73,82 = 9,18	84,2724
44,1	44,1 – 73,82 = - 29,72	883,2784
75	75 – 73,82 = 1,18	1.3924
79,4	79,4 – 73,82 = 5,58	31,1364
	Sum	2093,764

SD = $\sqrt{\frac{2093,764}{10}}$

SD = 14,49

✓ Sum
✓ $\sqrt{\frac{2093,764}{10}}$
✓ answer

Answer only:3/3 (3)

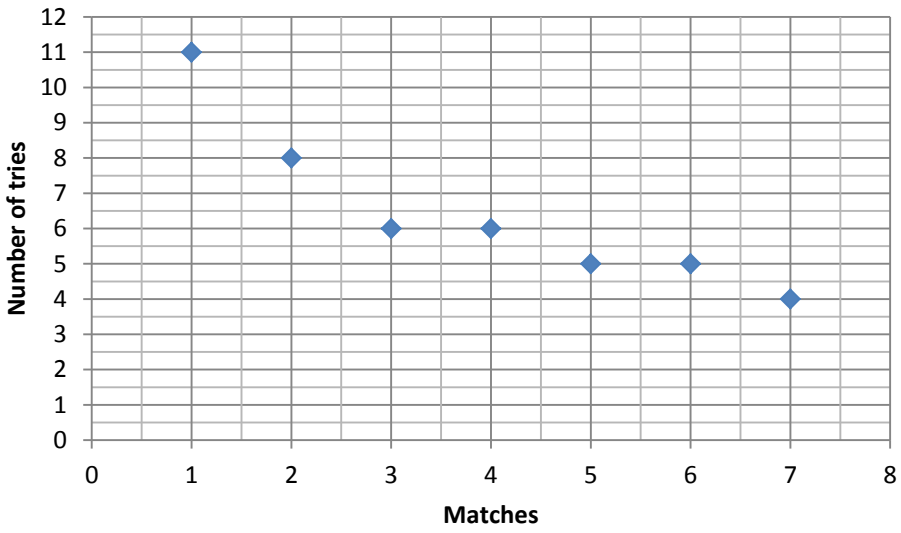
3.3

73,82 – 14,49 = 59,33

✓ method
✓ answer (2)

[7]

QUESTION 4

4.1		✓ First 4 points correct. ✓ Remaining 3 points correct	(2)
4.2	Exponential	✓ Answer	(1)
4.3	Less than 5 tries. As the number of matches increase, tries decrease.	✓ Answer with reason	(1)
			[4]

QUESTION 5

5.1	C(-4 ; 7) A(1;4) AC= $\sqrt{(-4-1)^2+(7-4)^2}$ AC= $\sqrt{34}$	✓ Substitution ✓ Answer	(2)
5.2	B(s ; -1) , M(-3 ; t) , C(-4 ; 7) $-3 = \frac{s-4}{2}$ - 6 = s – 4 s = -2 $t = \frac{-1+7}{2}$ t = 3	✓ Substitution ✓ s = -2 ✓ substitution ✓ t = 3	(4)
5.3	C(-4 ; 7) A(1 ; 4), B(- 2 ; -1) $m_{AB} = \frac{-1-4}{-2-1}$ $m_{AB} = \frac{5}{3}$ $m_{AC} = \frac{4-7}{1+4}$ $m_{AC} = -\frac{3}{5}$ $m_{AC} \times m_{AC} = -\frac{3}{5} \times \frac{5}{3} = -1$ ΔCAB is a right angled triangle at A.	✓ gradient of AB ✓ gradient of AC ✓ Product of gradients ✓ Conclusion	
	OR		
	$AB = \sqrt{(-4-1)^2+(7-4)^2}$ $AB = \sqrt{34}$ $AC = \sqrt{34}$ $BC = \sqrt{(-4+2)^2+(7+1)^2}$ $BC = \sqrt{68}$ $BC^2=AB^2+AC^2$ Hence ΔCAB is a right angled triangle at A.	✓ Length of AB ✓ Length of AC ✓ Length of BC ✓ Conclusion using Pythagoras theorem	(4)

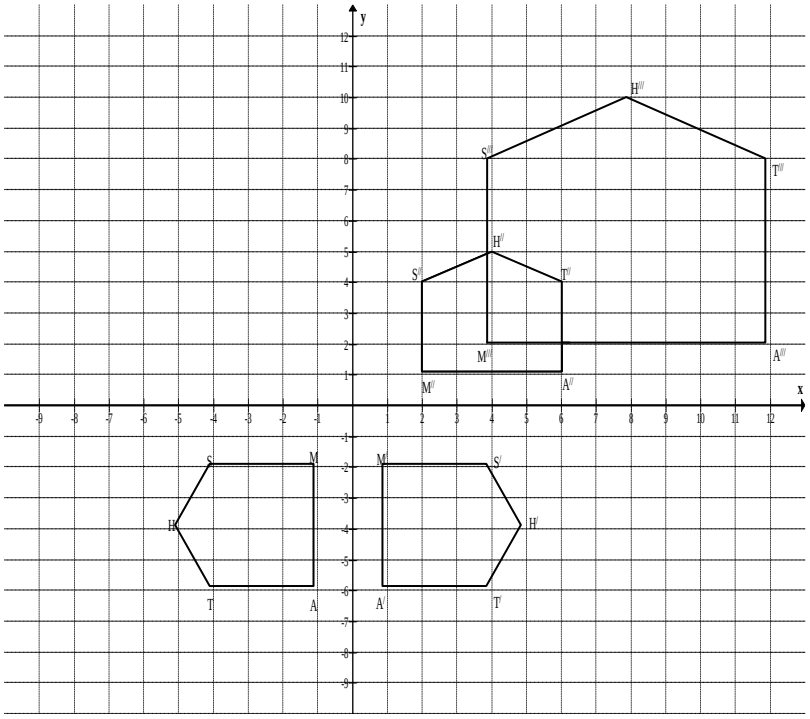
5.4	$m_{AB} = \frac{5}{3}$, C(- 4 ; 7) y – 7 = $\frac{5}{3}(x + 4)$ y = $\frac{5}{3}x + \frac{41}{3}$	✓ Gradient ✓ Substitution ✓ Answer	
	OR		
	$m_{AB} = \frac{5}{3}$, C(- 4 ; 7) y = mx + c 7 = $\frac{5}{3}$(- 4) + c c = $\frac{41}{3}$ y = $\frac{5}{3}x + \frac{41}{3}$	✓ Gradient ✓ Substitution ✓ Value of c/ equation	(3)
5.5	A(1 ; 4), B(-2 ; -1) $m_{AB} = \frac{-1-5}{-2-1} = \frac{5}{3}$ tan α = $\frac{5}{3}$ α = 59,04° $m_{AE} = -3$ tan $\theta = -3$ θ = 108,43° BAE = 108,43° − 59,04° = 49,39° CAE = 90° + 49,39° = 139,39°	✓ tan α = $\frac{5}{3}$ ✓ Size of α ✓ tan $\theta = -3$ ✓ θ = 108,43° ✓ Answer	(5)
5.6	C(-4 ; 7) A(1 ; 4), D(p ; 1) $m_{AC} = -\frac{3}{5}$ $m_{AD} = \frac{1-4}{p-1}$ $-\frac{3}{5} = \frac{-3}{p-1}$ $3p - 3 = 15$ $p = 6$	✓ Gradient of AD ✓ Equating gradients ✓ Answer	(3)
			[21]

QUESTION 6

6.1	6.1.1	E(2 ; -1), O(0 ; 0) Radius of the smaller circle = OE OE = $\sqrt{2^2 + (-1)^2}$ = $\sqrt{5}$	✓ Substitution ✓ Length of OE	(2)
	6.1.2	OE = $\sqrt{5}$ and E(2 ; -1), D(a ; -3) Therefore ED = 2 $\sqrt{5}$ $ED^2 = (a - 2)^2 + (-3+1)^2$ $20 = a^2 - 4a + 4 + 4$ $a^2 - 4a - 12 = 0$ $(a - 6)(a + 2) = 0$ $a = 6$ or $a = -2$ $a = 6$	✓ Length of ED ✓ Equation in standard form ✓ Factorisation ✓ $a = 6$	(4)
	6.1.3	D(6 ; -3) $r^2 = 20$ $(x - 6)^2 + (y + 3)^2 = 20$	✓ $r^2 = 20$ ✓ $(x - 6)^2$ ✓ $(y + 3)^2$	(3)
	6.1.4	E(2 ; -1), D(6 ; -3), O(0 ; 0) $m_{OE} = \frac{-1-0}{2-0} = -\frac{1}{2}$ $m_{Tangent} = 2$ $y + 1 = 2(x - 2)$ $y = 2x - 5$	✓ Gradient of radius ✓ Gradient of tangent ✓ Substitution ✓ Answer	(4)
6.2	6.2.1	$x^2 + y^2 - 4x + 5y + k = 0$ $x^2 - 4x + 4 + y^2 + 5y + \frac{25}{4} = -k + 4 + \frac{25}{4}$ $(x - 2)^2 + (y + \frac{5}{2})^2 = -k + \frac{41}{4}$ Centre $(2 ; -\frac{5}{2})$	✓ Completing the square ✓ Factor form ✓ x-value at centre ✓ y-value at centre	(4)
	6.2.2	diameter = 24, Therefore radius = 12 $-k + \frac{41}{4} = 144$ $-4k = 576 - 41$ $k = -\frac{535}{4} = -133{,}75$	✓ $r^2 = 144$ ✓ equating ✓ answer	(3)
				[20]

QUESTION 7

7.1	Reflection across the y-axes ($x = 0$)	✓ reflection ✓ y-axis	(2)
7.2	$(x ; y) \rightarrow (-y ; x)$ Accept: Rotation through 90° anticlockwise.	✓ $-y$ ✓ x	(2)
7.3	$(x ; y) \rightarrow (-x ; y) \rightarrow (-y ; -x)$ $(x ; y) \rightarrow (-y ; -x)$ Accept: Reflection in the line $y = -x$	✓ $-y$ ✓ $-x$	(2)

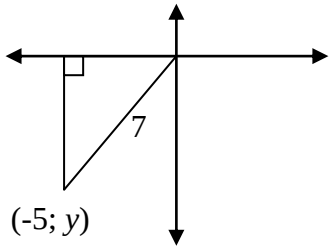
7.4	$M^{//}(2 ; 1)$, $A^{//}(6 ; 1)$, $T^{//}(6 ; 4)$, $H^{//}(4 ; 5)$, $S^{//}(2 ; 4)$
	$M^{///}(4 ; 2)$, $A^{///}(12 ; 2)$, $T^{///}(12 ; 8)$, $H^{///}(8 ; 10)$, $S^{///}(4 ; 8)$
	 
		✓ Two correct points
		✓ Remaining three correct points
		✓ Diagram
			(3)

7.5	$(x ; y) \rightarrow (-y ; -x) \rightarrow (-2y ; -2x)$ $(x ; y) \rightarrow (-2y ; -2x)$	✓ $-2y$ ✓ $-2x$	(2)
7.6	If area of MATHS = a, then area of $M^{///}A^{///}T^{///}H^{///}S^{///} = 2^2 \times a$ Hence area of MATHS : area of $M^{///}A^{///}T^{///}H^{///}S^{///} = 1 : 4$	✓ 1 ✓ 4	(2)
7.7	$(x ; y) \rightarrow (x - 4 ; y + 3)$ and M(-1 ; -2) L(-5 ; 1)	✓ -5 ✓ 1	(2)
			[15]

QUESTION 8

8.1	T′ $-\frac{5\sqrt{2}}{2};-\frac{\sqrt{2}}{2}$, T(-3;2) x' = x cosθ – y sinθ $-\frac{5\sqrt{2}}{2}$ = -3 cosθ – 2 sinθ …… ……. (1) y' = y cosθ – x sinθ $-\frac{\sqrt{2}}{2}$ = 2 cosθ – 3 sinθ .....(2) (1) × 2 and (2) × 3: $-\frac{10\sqrt{2}}{2}$ = -6 cosθ – 4 sinθ ……. (3) $-\frac{3\sqrt{2}}{2}$ = 6 cosθ – 9 sinθ .....(4) (3) + (4): –13sin$\theta = -\frac{13\sqrt{2}}{2}$ sin$\theta = \frac{\sqrt{2}}{2}$ θ = 45°	✓ Substitution into the formula for x' ✓ Substitution into the formula for y' ✓ (1) × 2 and (2) × 3 ✓ $-13\sin\theta = -\frac{13\sqrt{2}}{2}$ ✓ $\sin\theta = \frac{\sqrt{2}}{2}$ ✓ θ = 45°	(6)
	OR		
	Let $TOX = \beta$ $\tan\beta = \frac{2}{-3}$ ∴ β = 146,31° $\tan(\theta+\beta) = \frac{\frac{-\sqrt{2}}{2}}{\frac{-5\sqrt{2}}{2}}$ ∴ $\theta + \beta$ = 191,31° θ = 191,31° − 146,31° ∴ θ = 45°	✓ $\tan\beta = \frac{2}{-3}$ ✓ β = 146,31° ✓ $\tan(\theta+\beta)$ ✓ $\theta + \beta$ = 191,31° ✓ Method ✓ answer	(6)
8.2	T(-3 ; 2)→T′(3 ; -2) is rotation about the origin through 180°: $\theta + \alpha = 180°$ and θ = 45° α = 135°	✓ $\theta+\alpha = 180°$ ✓ α = 135°	(2)
			[8]

QUESTION 9

9.1	9.1.1	$7\cos\beta + 5 = 0$ and $\tan\beta > 0$ $\cos\beta = \frac{-5}{7}$  $y^2 + (-5)^2 = (7)^2$ $y = -\sqrt{24}$ $\tan\beta = \frac{\sqrt{24}}{5}$	✓ Diagram ✓ $y = -\sqrt{24}$ ✓ answer	(3)
	9.1.2	$\sin(450° + \beta) = \cos\beta$ $= \frac{-5}{7}$	✓ $\cos\beta$ ✓ answer	(2)
	9.1.3	$\sin 2\beta = 2\sin\beta\cos\beta$ $= 2 \times \frac{-\sqrt{24}}{7} \times \frac{-5}{7}$ $= \frac{10\sqrt{24}}{49}$	✓ $2\sin\beta\cos\beta$ ✓ answer	(2)
9.2		$\cos 2x - \frac{1}{3} = \frac{1}{3}\sin x$ $1 - 2\sin^2 x - \frac{1}{3} = \frac{1}{3}\sin x$ $6\sin^2 x + \sin x - 2 = 0$ $(3\sin x + 2)(2\sin x - 1) = 0$ $\sin x = -\frac{2}{3}$ or $\sin x = \frac{1}{2}$ $x = 221{,}81° + k.360°$ or $x = 318{,}19° + k.360°$ $(k \in \mathbb{Z})$ OR $x = 30° + k.360°$ or $x = 150° + k.360°$ $(k \in \mathbb{Z})$	✓ $1 - 2\sin^2 x$ ✓ Standard form ✓ Factors ✓ values of sin x ✓ $x = 221{,}81° + k.360°$ ✓ $x = 318{,}19° + k.360°$ ✓ $x = 30° + k.360°$ ✓ $150° + k.360°$ ✓ $(k \in \mathbb{Z})$	(9)
				[16]

QUESTION 10

10.1	$\frac{\tan\ 360° - x\ .\cos\ \ x - 90°\ \ + \cos(540° - x)}{\frac{\tan x}{-\tan x.\sin x - \cos x}}$ $\frac{\frac{\tan x}{-\frac{\sin x}{\cos x}.\sin x - \cos x}}{\frac{\frac{sinx}{cosx}}{\frac{-sin^2x - cos^2x}{\frac{cosx}{\frac{\sin x}{\cos x}}}}}$ $\frac{-1}{\cos x} \times \frac{\cos x}{\sin x}$ $-\frac{1}{\sin x}$	✓ $-\tan x$ ✓ sin x ✓ $-\cos x$ ✓ $\frac{sinx}{cosx}$ ✓ $-sin^2x - cos^2x$ ✓ -1 ✓ answer	(7)
10.2	10.2.1 LHS = $(\sin x + \cos x)^2$ $= sin^2 x + 2\sin x\cos x + cos^2x$ $= 2\sin x\cos x + 1$ = RHS	✓ squaring ✓ answer	(2)
	10.2.2 $3\sin 5\theta + 3\cos 5\theta$ $= 3(\sin 5\theta + \cos 5\theta)$ $= 3\ \sqrt{\sin 10\theta + 1}$ $= 3\ \sqrt{2}$	✓ common factor ✓ square root ✓ sin 10θ ✓ answer	(4)
10.3	LHS = $\frac{\sin 2x+1}{\cos 2x}$ $= \frac{\sin^2 x + 2\sin x\cos x + \cos^2 x}{\cos^2 x - \sin^2 x}$ $= \frac{\sin x+\cos x\ (\sin x+\cos x)}{\cos x-\sin x\ (\cos x+\sin x)}$ $= \frac{\sin x+\cos x}{\cos x-\sin x}$ = RHS	✓ $\sin^2 x + cos^2 x$ ✓ $2\sin x\cos x$ OR use 10.2.1 ✓ $\cos^2 x - \sin^2 x$ ✓ factorisation	(4)
			[17]

QUESTION 11

11.1	$x = -90°$ $x = 90°$	✓ $x = -90°$ ✓ $x = 90°$	(2)
11.2		$f(x) = \frac{1}{2}\tan x$ ✓ asymptotes ✓ x-intercepts ✓ shape $g(x) = \sin x + 1$ ✓ x- intercept ✓ y-intercept ✓ shape ✓ turning point	(7)
11.3	$x = 0°$ $x = 180°$	✓ $x = 0°$ ✓ $x = 180°$	(2)
11.4	$f(45°) - g(30°)$ $= 0,5 - 1,5$ $= -1$	✓ substitution ✓ answer	(2)
11.5	$m = -120°$ $m = 60°$	✓ $m = -120°$ ✓ $m = 60°$	(2)
11.6	90°	✓ answer	(1)
			[16]

QUESTION 12

12.1	$\text{UPQ} = 180° - (\theta + \alpha)$ $\sin \text{UPQ} = \sin\ 180° - \ \theta + \alpha$ $\therefore \sin \text{UPQ} = \sin\ (\theta + \alpha)$	✓ $\text{UPQ} = 180° - (\theta + \alpha)$ ✓ Answer	(2)
12.2	$\text{UPQ} = 180° - (\theta + \alpha)$ and PQ = 2t $\frac{\text{UQ}}{\sin\ \theta + \alpha} = \frac{2t}{\sin\alpha}$ $\text{UQ} = \frac{2t \sin\ \theta + \alpha}{\sin \alpha}$ $sin\theta = \frac{t}{QT}$ $\text{QT} = \frac{t}{\sin\theta}$ $\text{UT} = \frac{2t \sin\ \theta+\alpha}{\sin \alpha} + \frac{t}{\sin\theta}$	✓ Sine rule ✓ Substitution into sine rule ✓ $\text{UQ} = \frac{2t \sin\ \theta+\alpha}{\sin \alpha}$ ✓ $\text{QT} = \frac{t}{sin\theta}$ ✓ Answer	(5)
12.3	t = 3m, θ = 42° and α = 83° $\text{UQ} = \frac{2t \sin\ \theta + \alpha}{\sin\alpha}$ $\text{UQ} = \frac{2(3) \sin\ 83° + 42°}{\sin 83°}$ UQ = 4,95 m area of ΔUPQ = ½ × 4,95 × 6 × sin 42° = 9,94 m^2	✓ Substitution ✓ Answer ✓ Substitution ✓ Answer	(4)
			[11]
		TOTAL:	**150**