Grade 2 Elementary Mathematics Unit Plan

Japanese Elementary Mathematics Summer Institute
Hillcrest Elementary School, August 12-15, 2014

Teacher: Bill Jackson
Lesson Research Team: Bill Jackson and Makoto Yoshida

Title of Unit: Solving Problems Using Diagrams

Relationship of Unit to CCSS-m Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics

Relationship of Unit to CCSS-m Content Standards:

1. 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
2. 1. OA.4 Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.
3. 1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 + ? = 11$, $5 = \square - 3$, $6 + 6 = \square$.

Rationale for Teaching this Unit:

In grade K Units 5 and 6, students learned about addition and subtraction in the context of problem situations involving "putting together" (adding together), "adding to" (adding more), "taking from" (taking away), "taking a part" (part-whole), and "comparing" (comparison to find the difference). In these units, students are also learned to represent the situation of the problem using counting blocks and math sentences.

In grade 1 unit 8, students learned about addition and subtraction involving ordinal numbers and comparison involving two different kinds of quantities (units) with situations involving "more than" and "fewer than." In addition, students learned to represent the problem situation using counters and circles (drawing circles) and math sentences.

In this unit (Grade 1 Unit 14), students will learn addition and subtraction that involving reverse thought processes. For example, when the problem represents an addition situation but subtraction is used to solve for the unknown part. Another example is when the problem represents a subtraction situation but addition is used to solve for the unknown whole. Students represent the problem situations with tape diagrams and use the diagrams to think about what operation to use to solve problems. Through problem solving, students deepen their understanding of the meaning and relationship between addition and subtraction using a part-whole model.

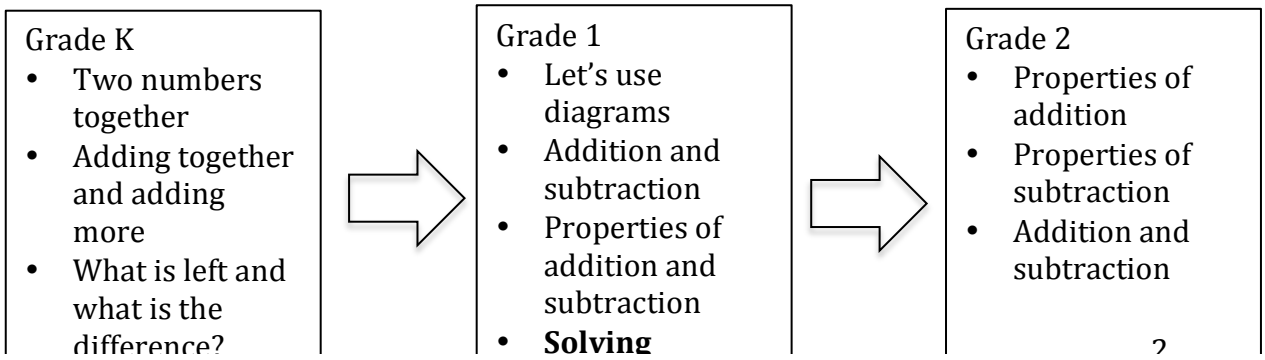
In grade 2, student further develop their understanding of the relationship between addition and subtraction and use tape diagrams to represent and solve problems. In grade 3, students will deepen their understanding of the relationship between addition and subtraction by representing unknowns using □ , and think about how to solve for unknowns.

Although the lessons in this unit involve addition and subtraction, they will become foundational for solving problems because students are representing the relationships among the quantities in a problem with tape diagrams and math sentences, skills that are further applied when students think, explain, and solve problems involving multiplication and division of fractions and decimal numbers in grades 4 through 6.

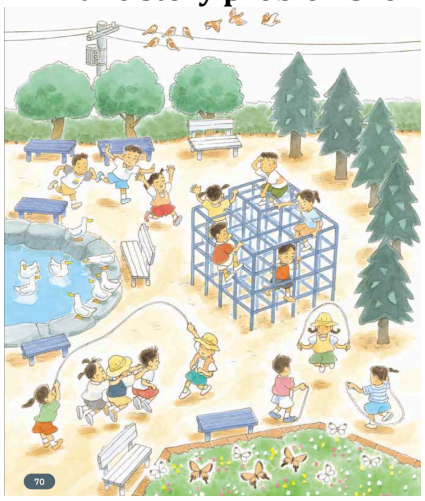
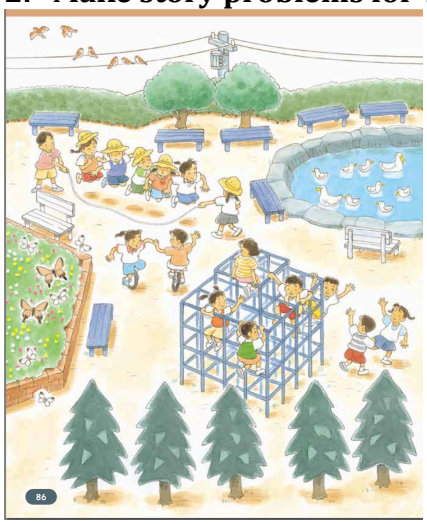
Focus of instruction: To improve students' skills to think, explain, and justify their thinking particularly using diagrams.

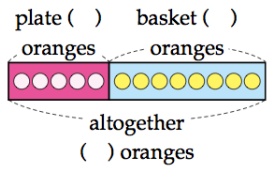
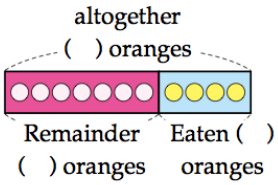
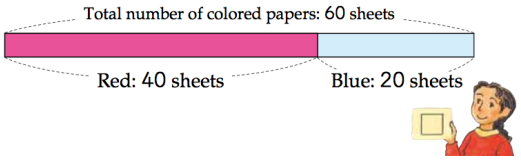
About diagrams:
Representing problem situations with tape diagrams is not easy task for young students. Drawing the diagram on their own is an even harder task for them. Thus, in this unit, we will focus on helping students to understand the structure of the problem situations and identifying the relationships between the quantities by using the tape diagrams, and not on requiring them to draw the diagrams. We will focus on helping students to use the tape diagram in addition and subtraction situations to determine the math sentences. Since problems will involve unknowns, it is important that students identify the location of the unknown on the tape diagram, and use the diagram to think about and explain how to solve the problem.

Relationship of Unit to IES (Japanese) Curriculum in Grades 1-3:



Plan of Unit:

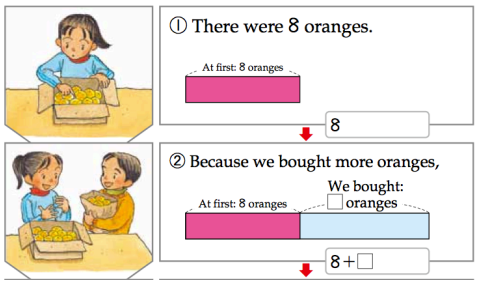
Date	Goals & Materials	Lesson Description
Mon. 8/11	• Recall addition and subtraction situations that students learned previously. • Represent addition and subtraction situations with countable objects.	**1. Make story problems for 5+3.**  • What do you see happening in the picture? • Use blocks to tell stories (adding to and putting together). **2. Make story problems for 7–2.**  • What do you see happening in the picture? • Use blocks to tell stories (taking away and taking apart).
Tues. 8/12	• Review the kinds of diagrams students have used to represent addition and subtraction situations. • Understand	**1. Think about addition situations we studied in first grade.** There are 5 oranges on the plate and 8 oranges in the basket. How many oranges are there altogether?

	relationship between addition and subtraction. Textbook p. 102	• Represent this situation with counters and strips of paper.  **2. Think about subtraction situations we studied in first grade.** There are 11 oranges altogether. If I eat 4 oranges, how many oranges will be left? • Represent this situation with counters and strips of paper.  • Think about how the diagrams are similar and different.
Wed. 8/13	• Use tape diagrams to discuss the relationship between addition and subtraction • Understand part-whole relationships in mathematical situations and equations Textbook pp. 103-104	**1. Grasp the problem situation.** There are some sheets of red and blue paper. There are 60 sheets of colored paper altogether. Of these, 40 sheets are red and 20 sheets are blue. • Let's show in a diagram what is happening in this picture.  2. **Try hiding one of the three one of the three numbers in the diagram with a □ .** • Think of a math sentence to hide the missing number. - Total unknown (40 + 20 = 60) - First part unknown (60 – 20 = 40) - Second part unknown (60 – 40 = 20)

<table>
<tr><td colspan="3">Optional lesson: If students need more practice on understanding the part-whole relationships in the model we will add lesson 2 (textbook p. 105).</td></tr>
<tr>
<td>Thurs.
8/14</td>
<td>• Solve unknown problem situations involving an unknown addend
• Understand the relationship between the unknown addend and subtraction

Textbook p. 106</td>
<td>1. Grasp the problem situation.

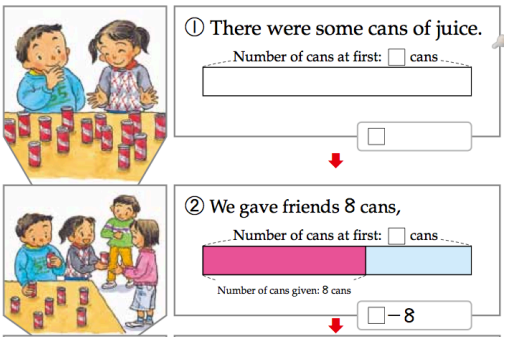
There were 8 oranges. We bought more oranges, and now there are 17 oranges. How many oranges did we buy?

• Represent with tape diagram and confirm that second quantity (part) is unknown.
 

2. Find out how many oranges we bought.
• Write a math sentence for the situation with an unknown initial quantity and solve.
• Clarify the relationship between the missing addend problem and subtraction.</td>
</tr>
<tr>
<td>Fri.
8/15</td>
<td>• Solve problems involving a subtraction situation with an unknown minuend (whole) by adding the parts.
• Develop a deeper understanding of the bidirectional relationship between addition and subtraction.

Textbook p. 107</td>
<td>1. Grasp the problem situation.

We have some cans of juice. We gave our friends 8 cans and we now have 6 cans left. How many cans did we have at first?

• Represent situation with tape diagram and confirm initial quantity (whole) is unknown.
 

2. Find out how many cans of juice we had at first.</td>
</tr>
</table>

		• Write a math sentence and find the answer. • Clarify the relationship between the minuend addend problem and addition.

Grade 2 Mathematics Lesson Plan

Hillcrest Elementary School August 11, 2014
Oakland, California

Teacher: Bill Jackson
Research Team: Bill Jackson and Makoto Yoshida

Name of Unit: Solving Problems Using Diagrams

Plan of Unit: (Total 5 Lessons)

1. Making stories for addition and subtraction situations (this lesson)
2. Exploring the relationship between addition and subtraction (2 lessons)
3. Addition situations with the initial quantity unknown (1 lesson)
4. Subtraction situations with an unknown whole (1 lesson)

Title of this Lesson: Making Math Stories

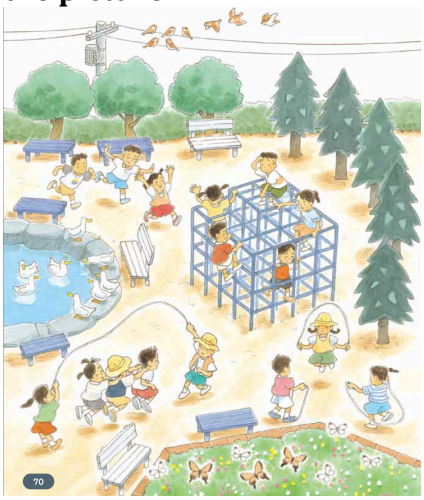
Goals of this Lesson:

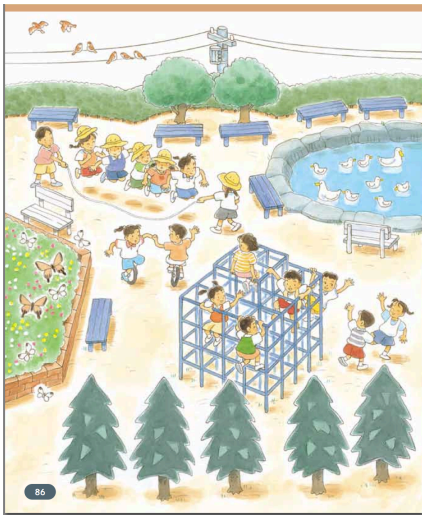
- Recall addition and subtraction situations that students learned before.
- Given an addition sentence, tell stories for situations involving adding to and putting together.
- Given a subtraction sentence, tell stories for situations involving taking from and taking apart.

Relationship of this lesson to the CCSS-m Content Standards:

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Learning Activities, Teachers Questions, and Anticipated Students' Reactions	**Teacher Support and Things to Remember**	**Method of Evaluation**
1. Introduction (10 min.) **Let's think about how to use a notebook to show our thinking.** • Date • Problem • My idea • Reflection	Give out notebooks and notebook page from textbook. Have students glue page of first page in in notebook. Go over important points of note taking. • Use one line to write sentences.	Do students have the date written in the right place?

	• When you make a mistake do not erase… Ask students to check each others' notebooks to make sure they put the date in the right place.	
2. Making Addition Stories (15 min.) **What do you see happening in the picture?**  **Make story problems for the math sentence 5+3.** <u>Example</u>: There are 5 white butterflies. There are 3 brown butterflies. If you put them together, how many are there? **Think about which addition story you want to tell and then tell your story to your friend.** <u>Anticipated Solutions</u>: **Add to (adding more) situations** • There are 5 children on the jungle gym and 2 children coming. How many children	Post picture on board and give to students. Ask students to share what they see and write important words on the board (e.g. jungle gym, ducks, children, trees, etc.). Give example of addition story with the butterflies. Use blocks to show putting together. Ask students to tell their addition stories to the class and move the blocks to show addition. Point out common mistakes such as… • Leaving out one or more quantities in the problem. • Not writing the question correctly. • Write a story that does not involve addition. Ask other students to retell the stories, making sure to include all the numbers. Write a couple of the stories on the board and ask students copy one of them in their notebooks.	Can students tell many different stories? Are students able to include a question in their stories? Can students retell and improve the stories of their friends? Are students able to see both types of situations as addition?

will be playing on the jungle gym? • There are 5 birds on the wire and 3 more birds are coming. How many birds will be on the wire? **Put together (adding together) situations** • There are 5 children jumping rope together and 3 children jumping rope separately. How many children will be jumping rope? • There are 5 blue benches and 3 white benches. How many benches are there altogether? • There are 5 ducks out of the water and 3 ducks in the water. How many ducks are there altogether? • There are 5 pine trees and 3 different (round) trees. How many trees are there altogether?	Help students see that even though the stories and situations are different they are all addition.	
3. Making Subtraction Stories (15 min.) **What do you see happening in the picture?** 	Post picture. Ask students to share what they see and write important words on the board (e.g. jungle gym, ducks, children, trees, etc.). Give example of subtraction story with the butterflies. Use blocks to show taking apart (or away). As students explain their stories have them move the blocks to show the subtraction. Point out common mistakes	Do the students' stories begin with the total (minuend)? Are students able to see both types of situations as subtraction?

Make story problems for the math sentence 7 – 2. <u>Example</u>: There are 7 butterflies. There are 2 brown butterflies. How many white ones are there? **Think about which subtraction story you want to tell and then tell your story to your friend.** <u>Anticipated Solutions</u>: **Take from (take away) situations** • There are 7 birds on the wire. 2 fly away. How many are left? • There are 7 children playing on the jungle gym. 2 children go home. How many are still playing? **Take apart (part-whole) situations** • There are 7 children playing jump rope. 2 are holding the rope. How many are jumping? • There are 7 ducks. 2 of them are parents. How many baby ducks are there? • There are 7 benches. 2 of them are white. How many are blue?	such as… • Leaving out one or more quantities in the problem. • Not writing the question correctly. • Not starting with the total (whole). • Write an addition story instead of subtraction. Ask other students to retell the stories, making sure to include all the numbers. Write a couple of the stories on the board and ask students copy one of them in their notebooks. Help students see that even though the stories and situations are different they are all subtraction.	
4. Summary and Reflection (10 min.) **It looks like there are many different stories for addition and subtraction.** **What did you learn today?** • Students share what they learned with whole class. **Please write your reflection in**	Ask several students share what they learned with the class before asking them to write a written reflection. Encourage students who are having trouble writing a reflection to think about what their friends' said.	Do students' oral and written reflections show that they understand there are different situations for addition and subtraction?

your notebook. • Prompt: “Today I learned…”		

Grade 2 Mathematics Lesson Plan

Hillcrest Elementary School August 12, 2014
Oakland, California

Teacher: Bill Jackson
Research Planning Team: Bill Jackson and Makoto Yoshida

Name of Unit: Solving Problems Using Diagrams

Plan of Unit: (Total 5 Lessons)

1. Making stories for addition and subtraction situations (1 lesson)
2. Exploring the relationship between addition and subtraction (this lesson)
3. Addition situations with the initial quantity unknown (1 lesson)
4. Subtraction situations with an unknown whole (1 lesson)
5. More problem situations (1 lesson)

Title of this Lesson: Using a Tape Model to Think About Addition and Subtraction

Goals of this Lesson:

- Use tape diagrams to discuss the relationship between addition and subtraction
- Understand part-whole relationships in mathematical situations and equations

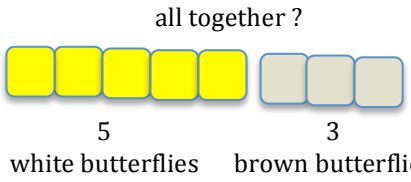
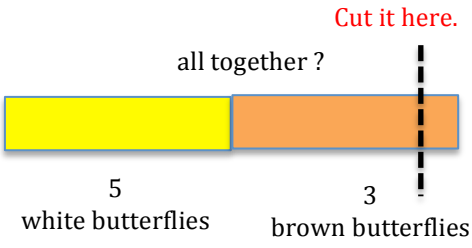
Relationship of this lesson to the CCSS-m Content Standards:

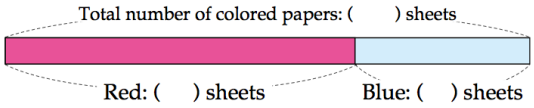
- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

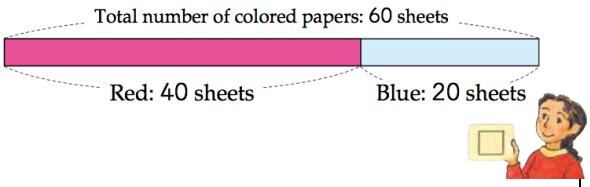
About this Lesson:
In yesterday's lesson, students represented different addition and subtraction situations with blocks and math sentences. These situations included addition by putting together and adding on, and subtraction by taking away, taking apart (part-whole), and comparison. In the introduction to today's lesson, I would like to help students make the connection between the concrete block representation, which is limited to small numbers, and the more abstract tape diagram, which can be used to represent the larger quantities they will encounter in today's problem and in the future.

Through today's problem, I would like to construct the tape diagram with the students and then focus on using the model to determine the operation needed to find a missing whole or part. Through discussion, students should see that the same model can represent both addition and subtraction (part-whole) situations, and the operation needed depends on the position of the unknown. This is very important

for the upcoming lessons where students encounter more difficult problem situations involving unknown augends, addends, minuends and subtrahends.

Learning Activities, Teachers Questions, and Anticipated Students' Reactions	Teacher Support and Things to Remember	Method of Evaluation
1. Introduction (5-10 min.) **Let's remember what we learned yesterday.** There are 5 white butterflies and 3 brown butterflies. If you put them all together, how many are there? **Let's label the blocks.**  **How can we represent the butterflies with the strips of paper?** • One brown butterfly strip needs to be shorter (3). • The longer strip is the bigger number (5 white butterflies). **Where should we cut the paper to show the correct size for the brown butterflies?**  	Write first problem from previous lesson on the board to make a link between concrete/semi-concrete and pictorial representations Clarify the sets by boxing in the blocks with lines. Show equal sized strips to students. After cutting the strip, label the diagram. Tell students that this is called a tape diagram.	Are students able to make connections from from the discrete model (counters) to the tape model? Are students able to see that the sizes of the quantities in the problem can be shown by using length (paper strips)?
2. Grasping the problem situation (10 min.). **What are the children doing in the picture?**	Pass out problem to students on paper to glue in notebooks.	

 (Problem) There are some sheets of red and blue paper. There are 60 sheets of colored paper altogether. Of these, 40 sheets are red and 20 sheets are blue. **What do you know from the problem?** • 60 sheets altogether • 40 sheets are red • 20 sheets are blue **Let's show this situation with these strips of paper.** **How should we cut the paper?** **Let's put important information from the problem in the tape diagram.** 	Discuss what the children are doing in the picture with the class and read the problem. Underline important information from the problem with red marker and list it on the board but do not fill in amounts on the diagram yet. Build the diagram as a class with the paper strips. Give the diagram to the students to glue in their notebooks and ask students to write the information in the tape model.	Can students see the relationship between the problem situation and the tape model? Are students able to associate the quantities from the problem with the parts and whole in the tape model?
3. Solving and Discussing (20 min.) **If you don't know one of the numbers how can you find it?**	Give students a square tile to cover up the quantities.	Are students able to see that the unknown parts or whole can



Let's write math sentences to show our thinking.

a) When you don't know the total number of colored papers.

Anticipated Solutions
- 40 + 20
- 20 + 40

b) When you don't know the number of red papers…

Anticipated Solutions
- 60 – 20
- ___ + 20 = 60
- 60 + 20

c) When you don't know the number of blue papers…

Anticipated Solutions
- 60 – 40
- 40 + ___ = 60
- 60 + 40

How are the three tape diagrams different?

Anticipated Responses
- Each one has a different unknown number.
- The square is in a different place.
- The part we need to find is different.

What part of the tape diagram do you find when you use addition?
What part do you find when you use subtraction?

Use a square paper to cover the amounts in the problem on the board.

Through discussion and comparison of the three diagrams help students to see…
- If you know both parts you can add to find the missing whole
- If you know the whole and one part you can subtract to find the missing part.

help determine the correct math operation?

Do students realize that the same model can represent both addition and subtraction situations, depending on what we are trying to find out (unknown)?

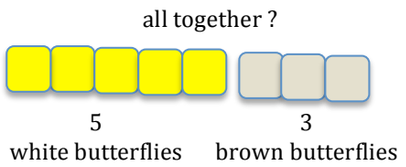
Can students see that when we are trying to find the whole we use addition?

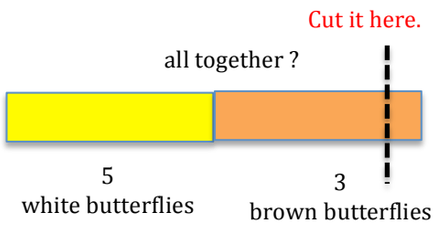
Can students see that when we are trying to find a part we use subtraction?

<u>Anticipated Responses</u> • We use addition when we want to find the whole (total). • We use subtraction when we are trying to find a missing part.		
4. Summarizing and Reflecting (5-10 min.) When using a tape diagram use addition to find the whole (total) and use subtraction to find a part. **Tell your friend what you learned today (reflection).**	Write summary on board and have students copy it into their notes. Ask students to reflect orally on what they learned.	Do students' oral reflections reflect the goals of the lesson?

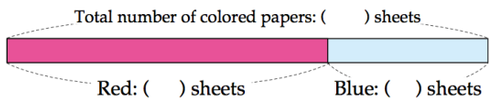
(Small Board)

There are 5 white butterflies.
There are 3 brown butterflies.
If you put them together, how many are there?





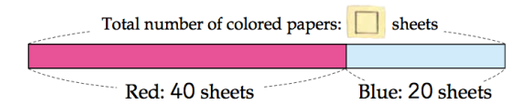
1 There are some sheets of red and blue paper.
There are 60 sheets of colored paper altogether.
Of these, 40 sheets are red and 20 sheets are blue.
Show this situation using a diagram.



When you don't know one of the Numbers in the diagram, how do You find it?

(Large Board)

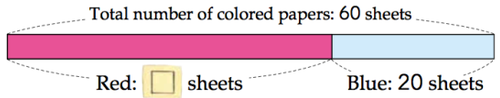
① When you don't know the total number of colored papers:



Math Sentence

Answer sheets

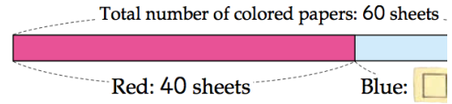
②When you don't know the number of red papers:



Math Sentence

Answer sheets

③When you don't know the number of blue pap



Math Sentence

Answer

Summary

When using a tape diagram, use addition to find the whc (total) and use subtraction t find a part.

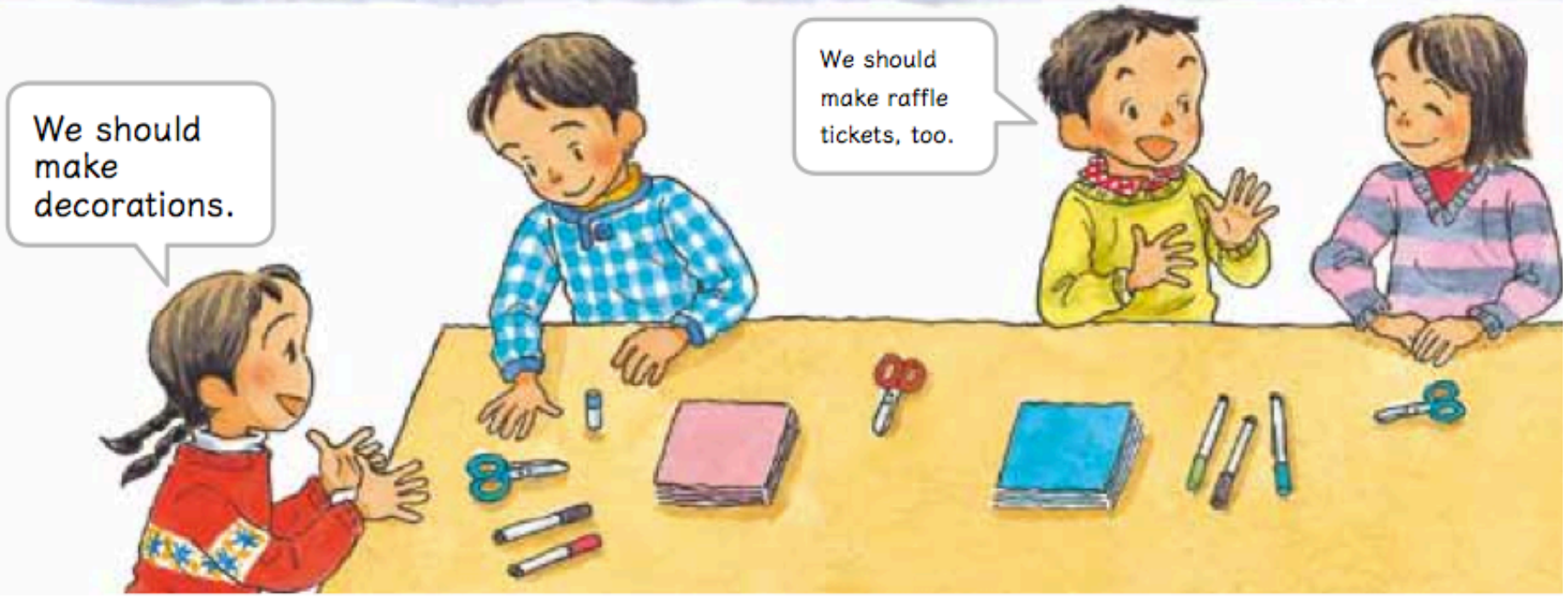
How are the three tape diagrams different?

What part of the tape diagram do you find when you use addition?
What part do you find when you use subtraction?

We are going to have a class party.

Need to prepare:

- Oranges
- Raffle tickets
- Juice
- Presents



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Of these, 40 sheets are red and 20 sheets are blue.
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Hillcrest Elementary School August 13, 2014
Oakland, California

Teacher: Bill Jackson
Research Planning Team: Bill Jackson and Makoto Yoshida

Name of Unit: Solving Problems Using Diagrams

Plan of Unit: (Total 5 Lessons)

1. Making stories for addition and subtraction situations (1 lesson)
2. Exploring the relationship between addition and subtraction (this is the second of 2 lessons)
3. Addition situations with the initial quantity unknown (1 lesson)
4. Subtraction situations with an unknown whole (1 lesson)

Title of this Lesson: Addition or Subtraction?

Goals of this Lesson:

- Understand that part-whole addition and subtraction situations share the same structure, and the operation needed to solve the problem depends on the position of the unknown
- Use tape diagrams, expression and equations to represent and discuss the relationship between addition and subtraction

Relationship of this lesson to the CCSS-m Content Standards:

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

About this Lesson:

In yesterday's lesson, some students were able to see the merits of the tape diagram. Other students, however, may not see the merits of the tape diagram yet so at the beginning of today's lesson we want to help students recall some of the things students' said about why the tape diagram is useful.

The focus of today's lesson is to use the tape diagram to help students see that addition and subtraction situations share the same structure but the operation is different depending on the position of the unknown. This understanding is important for students to be able to solve problems in upcoming lessons where they will see addition situations (missing augend or addend) that require subtraction to

solve and subtraction situations (missing minuend or subtrahend) that require addition to solve.

Learning Activities, Teachers Questions, and Anticipated Students' Reactions	Teacher Support and Things to Remember	Method of Evaluation
1. Introduction (5 min.) **Let's remember what we learned yesterday.** There are some sheets of red and blue paper. There are 60 sheets of colored paper altogether. Of these, 40 sheets are red and 20 sheets are blue. Show this situation using a tape diagram. **Let's review your ideas from yesterday.** • Tej (and others): drawing circles • Joe: drawing lines • Bryce: draw empty rectangles with numbers inside • Tape diagram **Why did we use a tape diagram?** • It is faster/easier than drawing blocks or circles. • We can show problems with larger numbers. • We can show the size of the parts with length.	Read problem from previous lesson and draw students' attention to the quantities in the problem and the tape model. If students cannot think of reasons why we use the tape diagram remind students of what Tej said in yesterday's lesson: "(With the tape diagram) we can do it faster and it is easier."	Can students articulate the advantages of the tape model?
2. Problem Solving (20 min.) How are the stories about the butterflies and the papers different? • They are about different things. • There is no question in the story about the papers? How can we make the story from yesterday into a problem? Can you be a teacher and make a problem?	Lead students to see that yesterday's story does not have a question by contracting it with the butterfly story. Give students a square tile to cover up the quantities. Use a square paper to cover the amounts in	Can students come up with questions to turn the story into a problem? Are students able to see that the unknown parts or whole can

(Problem) **Can you make a math problem from the story?** Students share their problems based on the story. • How many papers are there in all? • How many red papers are there? • How many blue papers are there? **Let's show each problem that we made with math sentences.** a) When you don't know the total number of colored papers. <u>Anticipated Solutions</u> • 40 + 20 = ___ • 20 + 40 = ___ b) When you don't know the number of red papers… <u>Anticipated Solutions</u> • 60 – 20 = ___ • ___ + 20 = 60 • 60 + 20 = ___ c) When you don't know the number of blue papers… <u>Anticipated Solutions</u> • 60 – 40 = ___ • 40 + ___ = 60 • 60 + 40 = ____	the problem on the board. Create tape diagrams on the board for each situation. Ask students to come to board to share their math sentences and ideas for each question. Help them to use a box to represent the unknown. Record students' ideas on the board with callouts.	help determine the correct operation? Can they realize that the math sentences change depending on what we are trying to find out (unknown)?
3. Discussion (15 min.) **How are the three tape diagrams the same and different?** <u>Anticipated Responses</u> • They look similar.	Give students time to discuss with partners before sharing as a class. Through discussion and comparison of the	Do students realize that the same model can represent both addition and

• Each one has a different unknown number. • The square is in a different place. • What we are trying to find out is different. • The part we need to find is different. **What part of the tape diagram do you find when you use addition?** **What part do you find when you use subtraction?** <u>Anticipated Responses</u> • We use addition when we want to find all the papers together (total/whole). • We use subtraction when we are trying to find only the red or blue papers (part).	three diagrams help students to see… • If you know both parts you can add to find the total (whole). • If you know the total (whole) and one part you can subtract to find the missing part.	subtraction situations? Can students see that when we are trying to find the whole we use addition? Can students see that when we are trying to find a part we use subtraction?
4. Reflection and Summary (10 min.) **Let's think about what we learned today.** **What do you think you learned? Please share.** Anticipated Summary: • We can make problems from a story. • We can use a box in the math sentence to show the problem even if we do not know something. • Tape diagram is helpful to write a math sentence.	Use the board to summarize main points of the lesson. If students have difficulty reflecting on their learning, ask them to look at the board.	Do students' reflections reflect the goals of the lesson?

There are some sheets of red and blue paper.
There are 60 sheets of colored paper altogether.
Of these, 40 sheets are red and 20 sheets are blue.

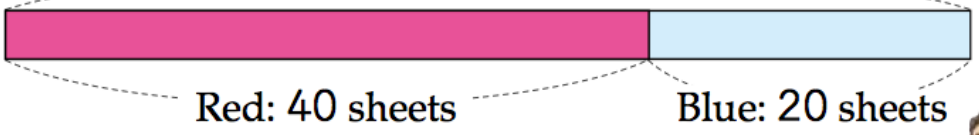
When you don't know one of the numbers in the diagram, how do you find it?

How are the three tape diagrams different?

What part of the tape diagram do you find when you use addition?

What part do you find when you use subtraction?

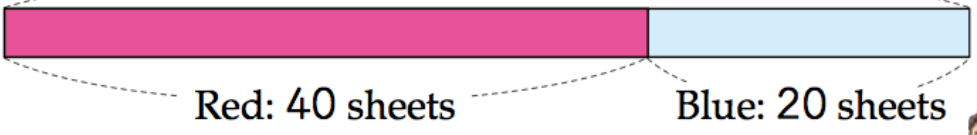
Total number of colored papers: 60 sheets



Red: 40 sheets

Blue: 20 sheets

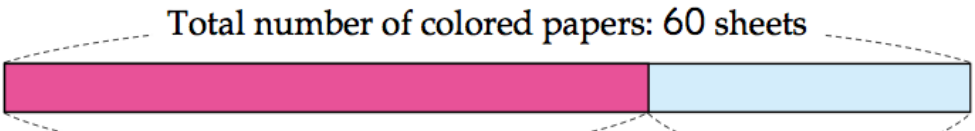
Total number of colored papers: 60 sheets



Red: 40 sheets

Blue: 20 sheets

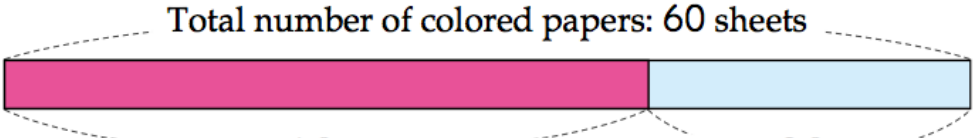
Total number of colored papers: 60 sheets



Red: 40 sheets

Blue: 20 sheets

Total number of colored papers: 60 sheets



Red: 40 sheets

Blue: 20 sheets

Grade 2 Mathematics Lesson Plan

Hillcrest Elementary School August 14, 2014
Oakland, California

Teacher: Bill Jackson
Research Planning Team: Bill Jackson and Makoto Yoshida

Name of Unit: Solving Problems Using Diagrams

Plan of Unit: (Total 5 Lessons)

1. Making stories for addition and subtraction situations (1 lesson)
2. Exploring the relationship between addition and subtraction (this is the third of 3 lessons)
3. Addition situations with the initial quantity unknown (1 lesson)

Title of this Lesson: Using Tape Models and Stories to See the Relationship between Addition and Subtraction

Goals of this Lesson:

- Understand that part-whole addition and subtraction situations share the same structure, and the operation needed to solve the problem depends on the position of the unknown
- Use tape diagrams, expression and equations to represent and discuss the relationship between addition and subtraction

Relationship of this lesson to the CCSS-m Content Standards:

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

About this Lesson:

Solving problems is the core of mathematics. The Japanese approach is to teach through problem solving as opposed to teaching for or about problem solving. To teach through problem solving there are certain dispositions that students need to have, such as being able to make sense of the structure of a problem, understand the components of a problem (what is known and unknown), and the operations needed to solve problems. Oftentimes students are taught to solve problems rotely, (e.g. “seven steps to problem solving,” etc.). Although this may help students to mimic and solve problems in the short run, it is not an effective way to develop problem solving dispositions in the long run. Furthermore, the tape (or bar) model is often used as a heuristic (problem solving strategy). This is a very limited view

and use of models. The question begs, why do we use models (discrete, tape, line, etc.)? There are several important reasons why models are useful, such as:

- Make sense of the problem situation.
- Reason about the size and relationship between the quantities in a problem.
- Translate a problem situation into a mathematical expression or equation (early grades – math sentence).
- Understand the relationship between the components of a problem (knowns and unknowns), representation of the problems (models and expressions/equations), and the method for solving problems.
- Understand the relationship between operations (e.g. addition and subtraction, multiplication and division).
- Develop algebraic thinking (e.g. how do I find an unknown?).

One difference between Japanese textbooks and most American textbooks is that Japanese textbooks focus on helping students understand the relationships in the problem to a much greater depth. Although the tendency for teachers might be to move forward even though students' understanding is incomplete, this may not be the best strategy. In other words, moving quickly may mean moving slowly as far as understanding is concerned, and vice versa.

The Japanese curriculum originally teaches the idea of using diagrams to solve problems with unknowns in all positions towards the end of second grade. The IES version attempts to bridge the gap between the original Japanese curriculum structured around the Japanese Course of Study, and the Common Core State Standards (CCSS). The CCSS spreads the idea of understanding the structure of problems across two grade levels, first and second grades, and then builds upon this in later grades.

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
- 2.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

This creates a bit of a quandary for U.S. teachers. According to the Japanese curriculum it is more developmentally appropriate to focus on the models towards the end of second grade, but the CCSS introduce these ideas in first grade and develop them in second. The following questions come to mind:

1. Is teaching the tape diagram (bar model) developmentally appropriate for first grade?

2. If so, what aspects of the multitude of skills that students need to understand to use a tape diagram/bar model correctly do students need to learn in first grade?

Besides the Japanese curriculum, we have also intensely studied the Singapore curriculum. The Singapore curriculum uses the bar (tape) model beginning in second grade. We can appreciate the power of the bar (tape) model as a heuristic (problem solving tool) but in our experience there are certain limitations to the Singapore bar model approach. These limitations are widely recognized in Singapore as well, and include:

1. Teaching the model as a heuristic (method for problem solving) as opposed to a way to derive mathematical equations and expressions may not be a good idea. One important consideration here is that if the model is a heuristic, the problems must be designed in a way that is conducive to solving by using a model, resulting in problems that are somewhat contrived. If this is the case, students may not see the advantage of using algebra later.
2. Teaching the model in a rote (direct teaching) way that students do not own or understand the merits of cold be counterproductive. You can teach "part + part = whole" and "whole – part = part" but if this is not coming from the students it basically becomes meaningless in the long run because problem situations are not that simple and pat.

During the time of this JEM conference the basic outline of the unit originally was:

1. Create story problems from pictures (this was Monday and most of the participants in the conference were not there for that lesson).
2. Understand how to represent a problem with a tape model the relationship between addition and subtraction.
3. Solve problems involving unknowns in different positions (initial quantity, etc.).

<u>Thinking About the Tape Model</u>

If the tape model is to be used as something beyond a heuristic, sufficient time must be devoted for students to make sense of the model. Through the discussion of the previous lessons we have gleaned some important insights:

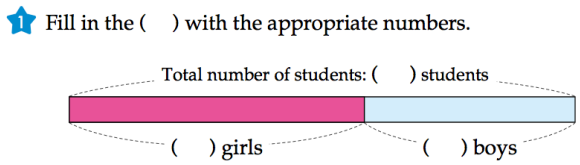
1. It is not enough for students to simply put numbers in the models. This is superficial understanding.
2. It is not enough to simply identify the unknown as "what you are trying to find out." The equation, including the unknown, shows the structure and constraints of the problem (what is known, what is unknown).
3. It is not enough to simply know that the sum of the parts makes the whole and the whole minus the part finds the other part. More importantly, students should represent a problem with a math sentence and the model should be, to the students, a helpful way to do this. All too often students are introduced to abstract representations such as tape (bar) models without

sufficient understanding of its merits and the model becomes rather painful to both teachers and students.

Today, we would like the students to make connections between the model, the equation, and the problem situation. Students should be able to see that a story situation can be represented with a model, a model can be used to derive an equation, and an equation can be used to solve a problem. This understanding becomes the crux for much of the mathematical and algebraic understanding students will need in the future.

It seems, based on our experience, that Hillcrest second grade students are able to make sense of a problem situation. However, this understanding is fragile at the moment so it is important to solidify it as we move to situations where students will encounter unknown augends, addends, minuends, and subtrahends. We would like students to be able to appreciate the model as a way to both understand the relationship between addition and subtraction and to derive an equation to solve the problem in a way that is developmentally appropriate for exiting first graders/entering second graders. Today's lesson is designed with that end in mind with the hopes that while students move forward they will be able to better grapple with complex problem situations.

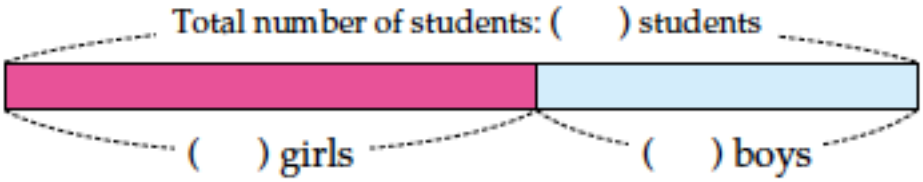
Learning Activities, Teachers Questions, and Anticipated Students' Reactions	**Teacher Support and Things to Remember**	**Method of Evaluation**
1. Introduction (10 min.) **Let's remember what we learned yesterday.** There are some sheets of red and blue paper. There are 60 sheets of colored paper altogether. Of these, 40 sheets are red and 20 sheets are blue. **Let's review some of your math sentences from yesterday.** • Brooke: 40 + 20 = _____ • Joe: 40 + ____ = 60 • Noah: 20 + _____ = 60 **Where do you see Brooke's math sentence in the tape diagram?** • 40 is red papers, 20 is blue papers, box is the total number of papers.	While students are sharing ideas, cover up the parts of both the tape model and the story that show the unknown quantity.	Can students relate the quantities and unknowns in the math sentences with the model?

Where do you see Joe's math sentence in the tape diagram? • 40 is red papers, box is blue papers, 60 is total. **Where do you see Noah's math sentence in the tape diagram?** • 20 is blue papers, box is re papers, 60 is total.		
2. Grasping the Problem (5 min.) **Let's think of a new story.** 12 students are playing in the schoolyard. There are 7 girls and 5 boys. **What do you notice about the story?** • There are no questions. • We know all the information. • We can cover one of the numbers to make a math sentence. **Let's fill in the tape diagram with the information from the story.** 	Give story and blank tape diagram. Allow students time to put the information in the diagram and then create tape diagram on the board as a class.	Do students realize that all the quantities are given in the problem? Can students complete the model with information from the story?
3. Problem Solving (15 min.) **Let's hide one of the numbers in the story and the tape diagram with a box. Think of a math sentence that will help us find the hidden number.** **(1) Write a math sentence that can be used to find the total number of girls and boys.** • 7 + 5 = _____	Pose and discuss problems one at a time. Give a new problem and tape diagram for each problem. Have them box the quantities they are trying to find out in each one.	Can students relate the quantities and unknowns in the story problem and the tape models with their math sentences?

• 5 + 7 = _____ **(2) Write a math sentence that can be used to find the number of girls.** • ____ + 5 = 12 • 12 – 5 = ____ **(3) Write a math sentence that can be used to find the number of boys.** • 7 + ____ = 12 • 12 – 7 = 5	Have students use the red pencil to box in the quantity they are trying to find. Relate math sentences to the diagram and the story by covering unknowns with the paper. Help students to see that the total number of students corresponds to the two parts in the tape diagram, and a part corresponds to the part of the diagram when the other part is removed.	
4. Discussion (10 min.) **How are the three tape diagrams the same and different?** <u>Anticipated Responses</u> • They look similar. • Each one has a different unknown number. • The square is in a different place. • What we are trying to find out is different. • The part we need to find is different. **What part of the tape diagram do you find when you use addition?** **What part do you find when you use subtraction?** <u>Anticipated Responses</u> • We use addition when we want to find all the papers together	Through discussion and comparison of the three diagrams help students to see… • If you know both parts you can add to find the total (whole). • If you know the total (whole) and one part you can subtract to find the missing part.	Do students realize that the same model can represent both addition and subtraction situations? Can students see that when we are trying to find the whole we use addition? Can students see that when we are trying to find a part we use subtraction?

(total/whole). • We use subtraction when we are trying to find only the red or blue papers (part).		
5. Reflection and Summary (10 min.) **Let's think about what we learned today.** **What do you think you learned? Please share.** Anticipated Summary: • We can add to find the total. • We can subtract to find a part. • We can subtract to solve a missing number in an addition problem.	Use the board to summarize main points of the lesson. If students have difficulty reflecting on their learning, ask them to look at the board.	Do students' reflections reflect the goals of the lesson?

12 students are playing in the schoolyard.
There are 7 girls and 5 boys.



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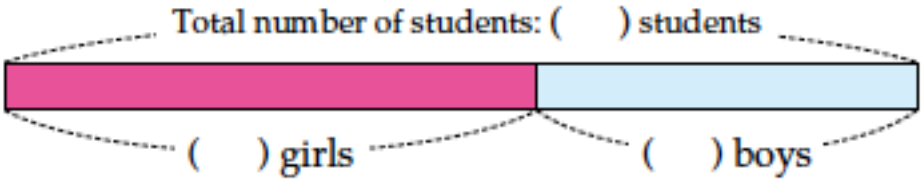

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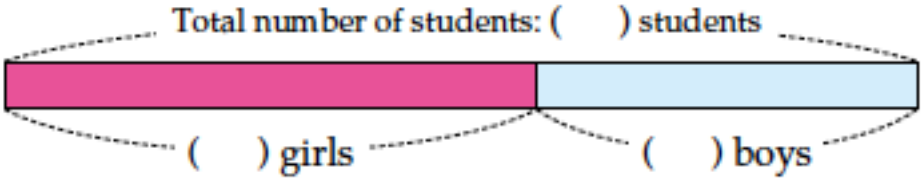

Grade 2 Mathematics Lesson Plan

Hillcrest Elementary School August 15, 2014
Oakland, California

Teacher: Bill Jackson
Research Planning Team: Bill Jackson and Makoto Yoshida

Name of Unit: Solving Problems Using Diagrams

Plan of Unit: (Total 5 Lessons)

1. Making stories for addition and subtraction situations (1 lesson)
2. Exploring the relationship between addition and subtraction (3 lessons)
3. Addition situations with the initial quantity unknown (This lesson)

Title of this Lesson: Let's Have Fun Solving a Problem

Goals of this Lesson:

- Solve unknown problem situations involving an unknown addend
- Understand the relationship between the unknown addend and subtraction
- Experience the joy and discovery of problem solving

Relationship of this lesson to the CCSS-m Content Standards:

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
- 1. OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

About this Lesson:

In this lesson we would like students to freely experience the joy of problem solving and come up with their own methods with as little prompting as possible. In this way, we can assess whether they can appreciate the merits of the methods they have been using this week such as:

- Identifying important information in the problem
- Representing the problem with a drawing or model (tape diagram)
- Writing a math sentence with a box for the unknown

Learning Activities, Teachers Questions, and Anticipated Students' Reactions	**Teacher Support and Things to Remember**	**Method of Evaluation**
1. Grasping and Solving the Problem (10 min.)	Give students the problem to glue into	Are students eager to solve

(Today's Problem) There were 8 oranges. We bought some more oranges. Now there are 17 oranges altogether. **Let's solve the problem and share our different ways.** <u>Anticipated Solutions</u> Addition a. 8 + 9 = 17 b. 8 + ___ = 17 c. 8 + 17 = 25 d. Draw 8 circles or objects and draw 9 more (add on) to get to 17. Subtraction e. 17 – 8 = 9 f. 17 – 8 = ___ g. Draw 17 circles and cross out 8 to get 9 left over.	their notebooks. Read the problem with the class. After posing the problem, and confirm that students understand the question. Give hint to students who cannot think of a way to solve encourage them to read the problem again. Have blank tape diagram and blocks prepared for students if they want to use them.	the problem? Do students understand what is happening in the problem? Do students understand what they are trying to find out? Can students come up with a math sentence that represents the missing addend situation?
h. Comparing and Discussing (20 min.) **Let's create a tape diagram to help us understand our math sentences.** <u>Anticipated Student Reactions</u> • 17 + 8 does not match the problem (or tape diagram). The total is 17 and this would be more. • 8 oranges + some oranges = total number of oranges • We can use 17 – 8 to find the oranges we don't know. • If we take away the oranges at first from the total, we get the remainder, 9.	Ask students to write their math sentences on the board. Create a tape diagram to clarify math sentences and help students explain their ideas (should come from students). Ask students to call on other students to explain their ideas.	Do students want to use a tape diagram to clarify their ideas? Can students understand and explain the ideas of their friends?
i. Summarizing and Reflecting (10	Use students' ideas on	Do students'

min.) **Let's think about and share what we learned today.** • We can use subtraction to find a missing number (part) in addition. • We can find may different ways to solve a problem. • We can use both addition and subtraction math sentences to solve the same problem.	the board to summarize the main points of the lesson.	oral reflections reflect the goals of the lesson?