Grade 5 Math Content [1]

Number and Operations: Whole Numbers

Multiplication and Division

In Grade 5, students consolidate their understanding of the computational strategies they use for multiplication. All students should be able to carry out strategies that involve breaking one or both factors apart, multiplying each part of one factor by each part of the other factor, then combining the partial products. They also practice notating their solutions clearly. They use representations and story contexts to connect these strategies, which are based on the distributive property of multiplication, to the meaning of multiplication. As part of their study of multiplication, students analyze and compare multiplication algorithms, including the U.S. algorithm for multiplication.

Examples of Multiplication Strategies

Breaking numbers apart by addition

148 x 42 =

40 x 100 = 4,000

40 x 40 = 1,600

40 x 8 = 320

2 x 100 = 200

2 x 40 = 80

2 x 8 = 16

4,000 + 1,600 + 320 + 200 + 80 + 16 = 6,216

148 x 42 =

100 x 42 = 4,200

48 x 40 = 1,920

48 x 2 = 96

4,200 + 1,920 + 96 = 6,216

[1] This document applies to the 2nd edition of *Investigations* (2008, 2012). See http://investigations.terc.edu/CCSS/ for changes when implementing *Investigations and the Common Core Standards*.

Changing one number to create an easier problem

148 x 42 =

150 x 42 = 6,300 (100 x 42 + 1/2 of 100 x 42)

2 x 42 = 84

6,300 – 84 = 6,216

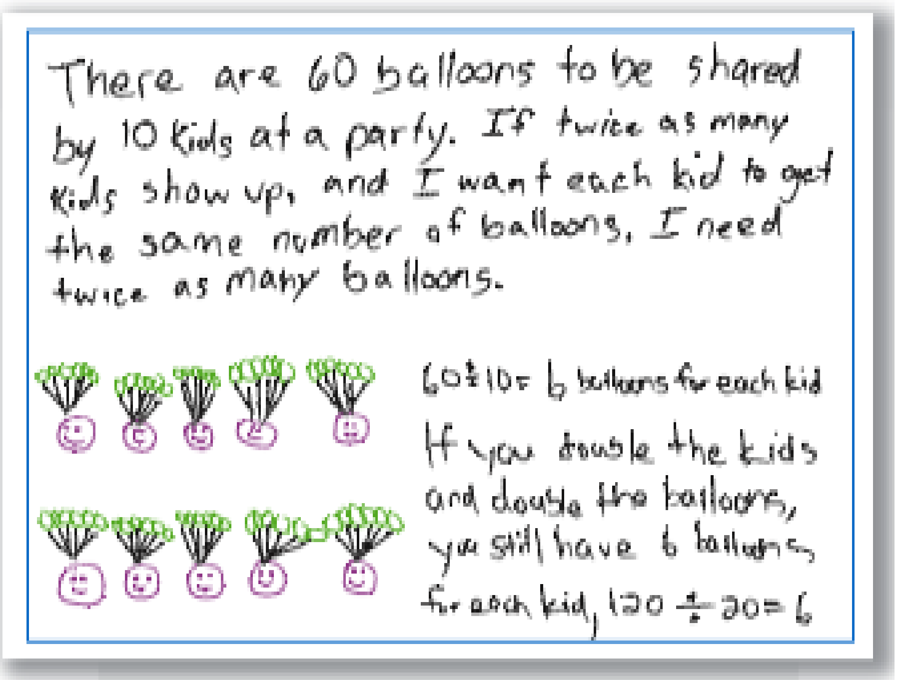
Students continue to learn ways to solve division problems fluently, focusing on the relationship between multiplication and division. They solve division problems by relating them to missing factor problems (e.g., 462 ÷ 21 = ____ and ____ x 21 = 462), by building up groups of the divisor, and by using multiples of 10 to solve problems more efficiently. As students refine their computation strategies for division, they find ways to use what they already know and understand well (familiar factor pairs, multiples of 10s, relationships between numbers, etc.) to break apart the harder problems into easier problems. They also work on notating their solutions clearly and concisely.

____ × 21 = 1,275 6 × 21 = 126 60 × 21 = 1,260 1,275 − 1,260 = 15 Answer: 60 R15	1,275 ÷ 21 = 630 ÷ 21 = 30 1,275 − 630 = 645 630 ÷ 21 = 30 645 − 630 = 15 1,260 ÷ 21 = 60 Answer: 60 R15
21)1,275 10 × 21 = 210 20 × 21 = 420 30 × 21 = 630 60 × 21 = 1,260 60 R15 21)1,275	21)1,275 − 420 20 × 21 = 420 855 − 630 30 × 21 = 630 225 − 210 10 × 21 = 210 15 20 + 30 + 10 = 60 Answer: 60 R15

Examples of clear and concise notation

Students also study underlying properties of numbers and operations and make and justify general claims based on these properties. They study the relationship between a number and its factors, which supports mental computation strategies for multiplication and division with whole numbers. For example, students consider multiplication expressions related by place value (e.g., 3 x 6 = 18; 3 x 60 = 3 x 6 x 10 = 180), and equivalent multiplication expressions (e.g., 24 x 18 = 12 x 36 or 24 x 18 = 72 x 6). This work includes finding longer and longer multiplication expressions for a number and considering the prime factorization of a number.

Students also investigate equivalent expressions in multiplication and division. For example, they investigate why doubling one factor and halving the other factor (or tripling and thirding, etc.) in a multiplication expression of the form *a x b* maintains the same product. They also examine how and why the ratio between dividend and divisor must be maintained to generate equivalent division expressions. In this work, students develop mathematical arguments based on representations of the operations.



Sample student work

The Algebra Connections pages in the two curriculum units that focus on multiplication and division show how students are applying the commutative and distributive properties of multiplication, as well as the inverse relationship between multiplication and division, as they solve problems. These pages also highlight particular generalizations about multiplication that students work on in Grade 5 as they create equivalent expressions for multiplication: If one factor in a multiplication expression is halved (or thirded) and another factor is doubled (or tripled), what is the effect on the product?

Emphases

Whole Number Operations

- Reasoning about numbers and their factors
- Understanding and using the relationship between multiplication and division to solve division problems
- Representing the meaning of multiplication and division
- Reasoning about equivalent expressions in multiplication and division

Computational Fluency

- Solving multiplication problems with 2-digit numbers
- Solving multiplication problems with 2- and 3-digit numbers
- Solving division problems with 2-digit divisors

Benchmarks

- Find the factors of a number
- Solve multiplication problems efficiently
- Solve division problems with 1-digit and 2-digit divisors
- Explain why doubling one factor in a multiplication expression (a x b) and dividing the other by 2 results in an equivalent expression
- Solve division problems efficiently

Addition, Subtraction, and the Number System

In Grade 5, students extend their knowledge of the base ten number system, working with numbers in the hundred thousands and beyond. In their place value work, students focus on adding and subtracting multiples of 100 and 1,000 to multi-digit numbers and explaining the results. This work helps them develop reasonable estimates for sums and differences when solving problems with large numbers. Students apply their understanding of addition to multi-step problems with large numbers. They develop increased fluency as they study a range of strategies and generalize the strategies they understand to solve problems with very large numbers.

90, 945 – 1,000 =

90,945 – 1,200 =

90,945 – 1,210 =

90,945 – 1,310 =

Students practice and refine their strategies for solving subtraction problems. They also classify and analyze the logic of different strategies; they learn more about the operation of subtraction by thinking about how these strategies work. Students consider which subtraction problems can be solved easily by changing one of the numbers and then adjusting the difference. As they discuss and analyze this approach, they visualize important properties of subtraction. By revisiting the steps and notation of the U.S. algorithm for subtraction and comparing it to other algorithms, students think through how regrouping enables subtracting by place, with results that are all in positive numbers.

Examples of Subtraction Strategies

Subtracting in parts

3,451 – 1,287 =

3,451 – 1,200 = 2,251

2,251 – 80 = 2,171

2,171 – 7 = 2,164

Adding up

3,451 – 1,287 =

1,287 + 13 = 1300

1,300 + 2,100 = 3,400

3,400 + 51 = 3,451

13 + 2,100 + 51 = 2,164

Subtracting back

3,451 – 1,287 =

3,451 – 51 = 3,400

3,400 – 2,100 = 1,300

1,300 – 13 = 1,287

51 + 2,100 + 13 = 2,164

Changing the numbers

3,451 – 1,287 =

3,451 – 1,300 = 2,151

2,151 + 13 = 2,164

3,451 – 1,287 =

(add 13 to both number to create an equivalent problem)

3,451 – 1,287 = 3,464 – 1300

= 2,164

The Algebra Connections page in the curriculum unit that focuses on addition and subtraction shows how students are applying the inverse relationship between addition and subtraction as they solve problems. It also highlights the algebraic ideas that underlie the generalizations students investigate and articulate when they create equivalent expressions in order to solve a problem (e.g., 892 – 567 = 895 – 570).

Emphases

The Base Ten Number System

- Extending knowledge of the base-ten number system to 100,000 and beyond

Computational Fluency

- Adding and subtracting accurately and efficiently

Whole Number Operations

- Examining and using strategies for subtracting whole numbers

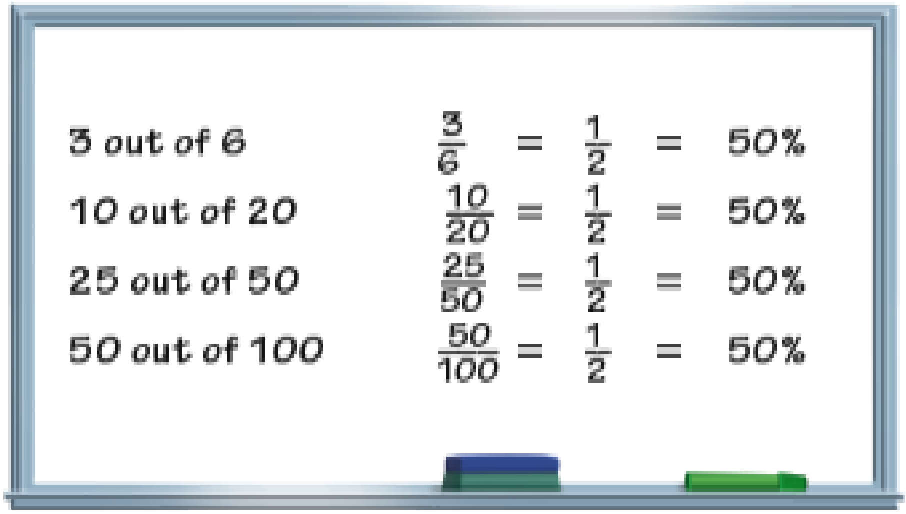
Benchmarks

- Read, write, and sequence numbers to 100,000
- Solve subtraction problems accurately and efficiently, choosing from a variety of strategies

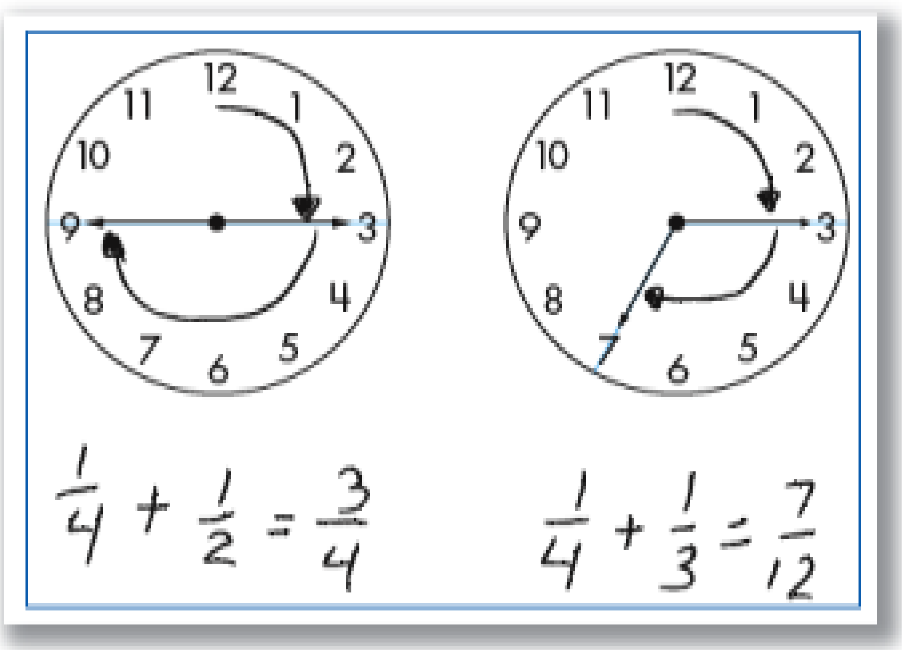
Number and Operations: Rational Numbers

The major focus of the work on rational numbers in grade 5 is on understanding relationships among fractions, decimals, and percents. Students make comparisons and identify equivalent fractions, decimals, and percents, and they develop strategies for adding and subtracting fractions and decimals.

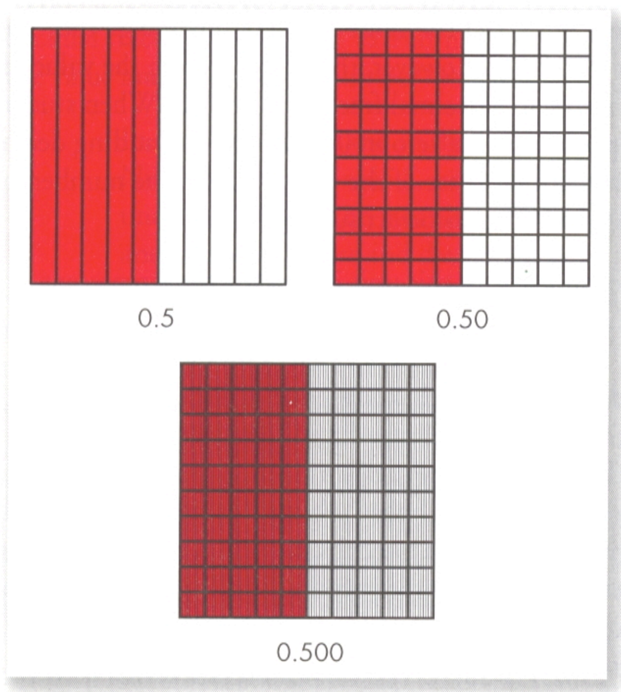
In a study of fractions and percents, students work with halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths. They develop strategies for finding percent equivalents for these fractions so that they are able to move back and forth easily between fractions and percents and choose what is most helpful in solving a particular problem, such as finding percentages or fractions of a group.



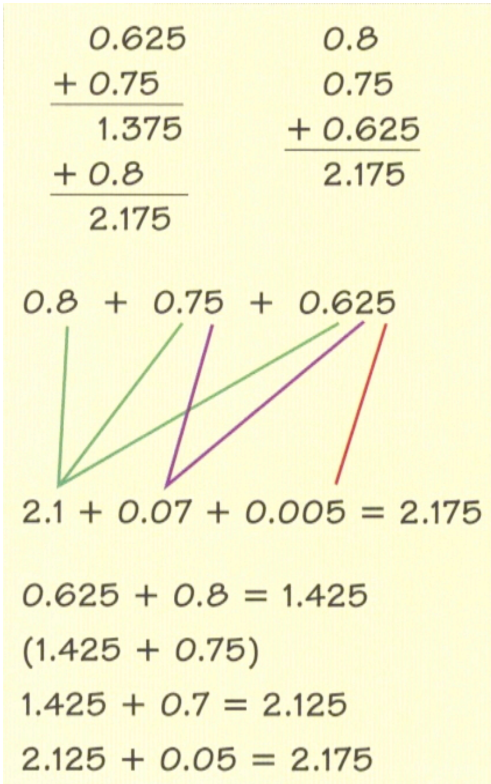
Students use their knowledge of fraction equivalents, fraction-percent equivalents, the relationship of fractions to landmarks such as ½, 1, and 2, and other relationships to decide which of two fractions is greater. They carry out addition and subtraction of fractional amounts in ways that make sense to them by using representations such as rectangles, rotation on a clock, and the number line to visualize and reason about fraction equivalents and relationships.



Students continue to develop their understanding of how decimal fractions represent quantities less than 1 and extend their work with decimals to thousandths. By representing tenths, hundredths, and thousandths on rectangular grids, students learn about the relationships among these numbers—for example, that one tenth is equivalent to ten hundredths and one hundredth is equivalent to ten thousandths—and how these numbers extend the place value structure of tens that they understand from their work with whole numbers.



Students extend their knowledge of fraction-decimal equivalents by studying how fractions represent division and carrying out that division to find an equivalent decimal. They compare, order, and add decimal fractions (tenths, hundredths, and thousandths) by carefully identifying the place value of the digits in each number and using representations to visualize the quantities represented by these numbers.



Emphases

Rational Numbers

- Understanding the meaning of fractions and percents
- Comparing fractions
- Understanding the meaning of decimal fractions
- Comparing decimal fractions

Computation with Rational Numbers

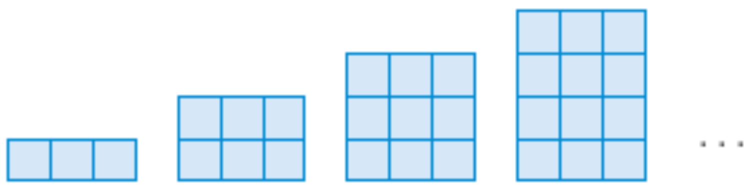
- Adding and subtracting fractions
- Adding decimals

Benchmarks

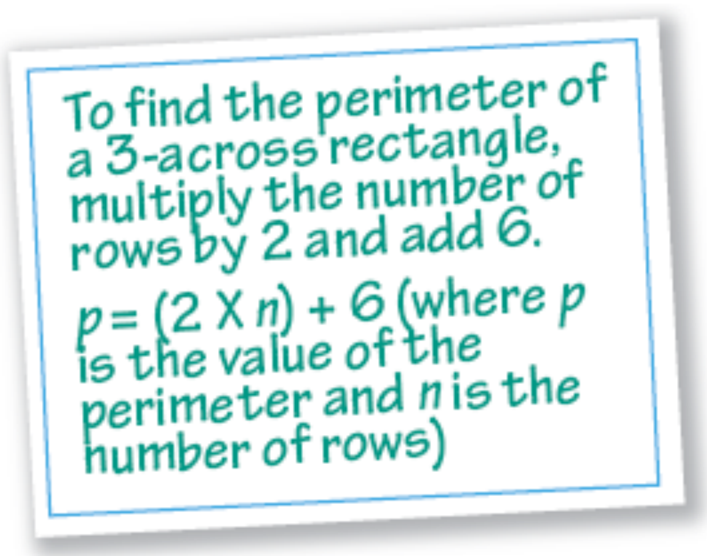
- Use fraction-percent equivalents to solve problems about the percentage of a quantity
- Order fractions with like and unlike denominators
- Add fractions through reasoning about fraction equivalents and relationships
- Read, write, and interpret decimal fractions to thousandths
- Order decimals to the thousandths
- Add decimal fractions through reasoning about place value, equivalents, and representations

Patterns, Functions, and Change

In Grade 5, students continue their work from Grades 3 and 4 by examining, representing, and describing situations in which the rate of change is constant. Students create tables and graphs to represent the relationship between two variables in a variety of contexts. They also articulate general rules for each situation. For example, consider the perimeters of the following set of rectangles made from rows of tiles with three tiles in each row:



If the value of one variable (the number of rows of three tiles) is known, the corresponding value of the other variable (the perimeter of the rectangle) can be calculated. Students express these rules in words and then in symbolic notation. For example:



For the first time in Grade 5, students create graphs for situations in which the rate of change is itself changing–for example, the change in the area of a square as a side increases by a constant increment–and consider why the shape of the graph is not a straight line as it is for situations with a constant rate of change.

Throughout their work, students move among tables, graphs, and equations and between those representations and the situation they represent. Their work with symbolic notation is closely related to the context in which they are working. By moving back and forth between the contexts, their own ways of describing general rules in words, and symbolic notation, students learn how this notation can carry mathematical meaning.

Emphases

Using Tables and Graphs

- Using graphs to represent change
- Using tables to represent change

Linear Change

- Describing and representing a constant rate of change

Nonlinear Change

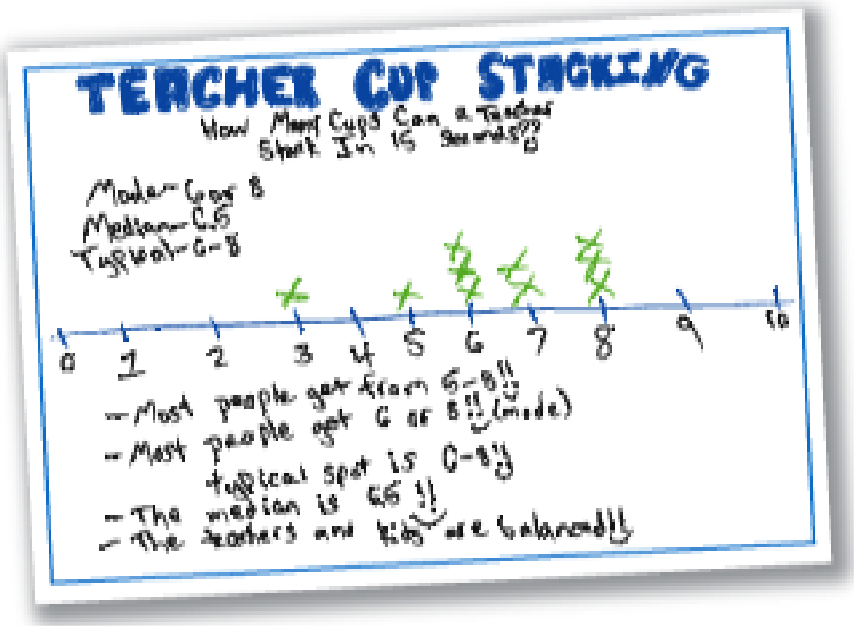
- Describing and representing situations in which the rate of change is not constant

Benchmarks

- Connect tables and graphs to represent the relationship between two variables
- Use tables and graphs to compare two situations with constant rates of change
- Use symbolic notation to represent the value of one variable in terms of another variable in situations with constant rates of change

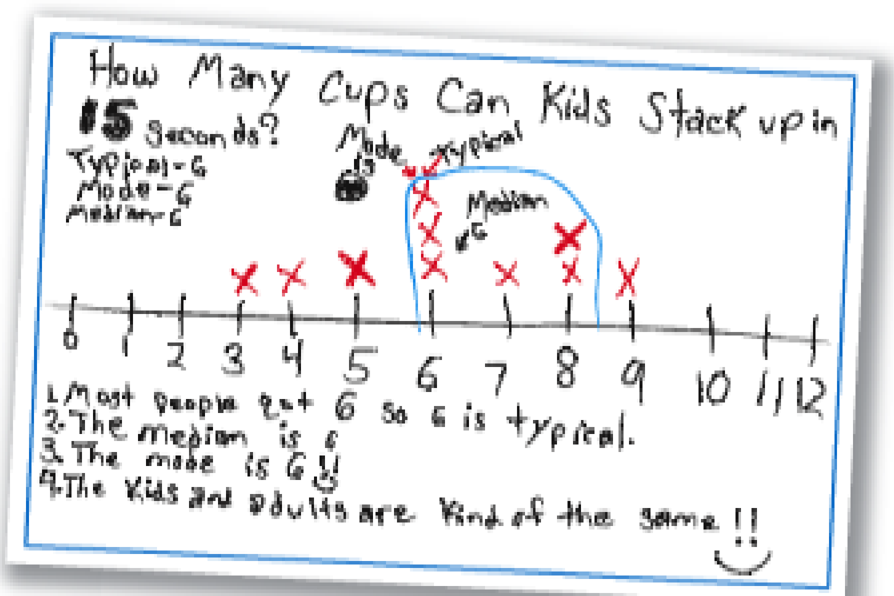
Data Analysis and Probability

Students continue to develop their understanding of data analysis in Grade 5 by collecting, representing, describing, and interpreting numerical data. Students' work in this unit focuses on comparing two sets of data collected from experiments. Students develop a question to compare two groups, objects, or conditions. (Sample questions: Which toy car goes farthest after rolling down the ramp? Which paper bridge holds more weight?). They consider how to ensure a consistent procedure for their experiment and discuss the importance of multiple trials. Using representations of data, including line plots and bar graphs, students describe the shape of the data—where the data are concentrated, how they are spread across the range. They summarize the data for each group or object or condition and use these summaries, including medians, to back up their conclusions and arguments. By carrying out a complete data investigation, from formulating a question through drawing conclusions from their data, students gain an understanding of data analysis as a tool for learning about the world.



Sample Student Work

In their work with probability, students describe and predict the likelihood of events and compare theoretical probabilities with actual outcomes of many trials. They use fractions to express the probabilities of the possible outcomes (e.g., landing on the green part of the spinner, landing on the white part of the spinner). Then they conduct experiments to see what actually occurs. The experiments lead to questions about theoretical and experimental probability, for example, if half the area of a spinner is colored green and half is colored white, why doesn't the spinner land on green exactly half the time?



Sample Student Work

Emphases

Data Analysis

- Representing data
- Describing, summarizing, and comparing data
- Analyzing and interpreting data
- Designing and carrying out a data investigation

Probability

- Describing the probability of an event
- Describe major features of a set of data represented in a line plot or bar graph, and quantify the description by using medians or fractional parts of the data

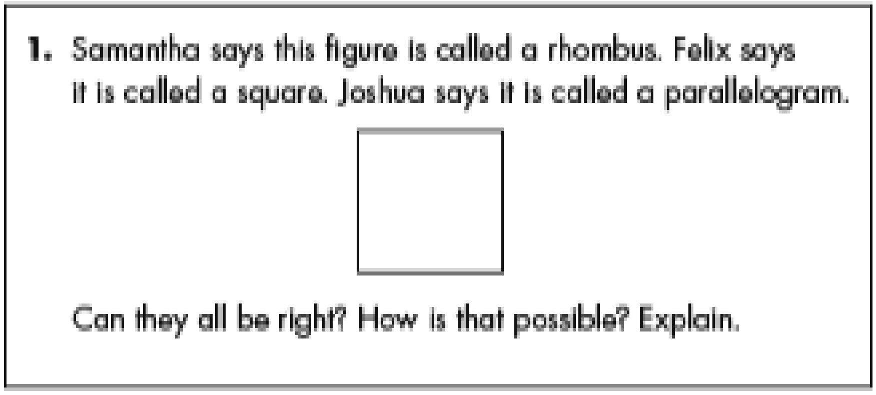
Benchmarks

- Draw conclusions about how 2 groups compare based on summarizing the data for each group
- Design and carry out an experiment in order to compare two groups
- Use a decimal, fraction, or percent to describe and compare the theoretical probabilities of events with a certain number of equally likely outcomes

Geometry and Measurement

In their work with geometry and measurement in grade 5, students further develop their understanding of the attributes of two-dimensional (2-D) shapes, find the measure of angles of polygons, determine the volume of three-dimensional (3-D) shapes, and work with area and perimeter. Students examine the characteristics of polygons, including a variety of triangles, quadrilaterals, and regular polygons. They consider questions about the classification of geometric figures, for example:

> Are all squares rectangles?
>
> Are all rectangles parallelograms?
>
> If all squares are rhombuses, then are all rhombuses squares?

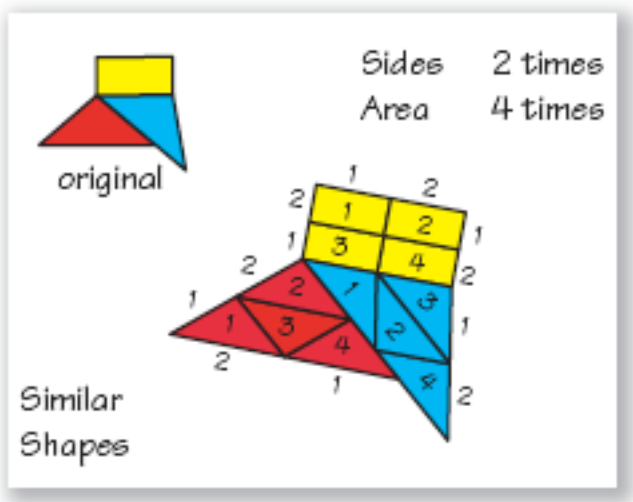


1. Samantha says this figure is called a rhombus. Felix says it is called a square. Joshua says it is called a parallelogram.

Can they all be right? How is that possible? Explain.

Question 1 from Resource Masters, M17 in Unit 5, Measuring Polygons

They investigate angle sizes in a set of polygons and measure angles of 30, 45, 60, 90, 120, and 150 degrees by comparing the angles of these shapes. Students also investigate perimeter and area. They consider how changes to the shape of a rectangle can affect one of the measures and not the other (e.g., two shapes that have the same area don't necessarily have the same perimeter), and examine the relationship between area and perimeter in similar figures.



Students continue to develop their visualization skills and their understanding of the relationship between 2-D pictures and the 3-D objects they represent. Students determine the volume of boxes (rectangular prisms) made from 2-D patterns and create patterns for boxes to hold a certain number of cubes. They develop strategies for determining the number of cubes in 3-D arrays by mentally organizing the cubes—for example as a stack of three rectangular layers, each three by four cubes. Students deepen their understanding of the relationship between volume and the linear dimensions of length, width, and height. Once students have developed viable strategies for finding the volume of rectangular prisms, they extend their understanding of volume to other solids such as pyramids, cylinders, and cones, measured in cubic units.

Emphases

Features of Shape
- Describing and classifying 2-D figures
- Describing and measuring angles
- Creating and describing similar shapes
- Translating between 2-D and 3-D shapes

Linear and Area Measurement

- Finding the perimeter and area of rectangles

Volume
- Structuring rectangular prisms and determining their volume
- Structuring prisms, pyramids, cylinders, and cones and determining their volume

Benchmarks

- Identify different quadrilaterals by attribute, and know that some quadrilaterals can be classified in more than one way
- Use known angle sizes to determine the sizes of other angles (30 degrees, 45 degrees, 60 degrees, 90 degrees, 120 degrees, and 150 degrees)
- Determine the perimeter and area of rectangles
- Identify mathematically similar polygons
- Find the volume of rectangular prisms
- Use standard units to measure volume
- Identify how the dimensions of a box change when the volume is changed
- Explain the relationship between the volumes of prisms and pyramids with the same base and height