Grade 6 Mathematics

Support Document for Teachers

Manitoba

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This resource is also available on the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/cur/math/supports.html>.

Available in alternate formats upon request.

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List of Blackline Masters (BLMs)

Grade 6 Mathematics Blackline Masters

Grades 5 to 8 Mathematics Blackline Masters

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Writer	Katarina Schilling	Retired Teacher
Grades 5 to 8 Mathematics Support Document Development Team (2006–2008)	Holly Forsyth	Fort La Bosse School Division
	Chris Harbeck	Winnipeg School Division
	Heidi Holst	Lord Selkirk School Division
	Linda Girling	Louis Riel School Division
	Steven Hunt	Independent School
	Jan Jebsen	Kelsey School Division
	Betty Johns	University of Manitoba
	Dianna Kicenko	Evergreen School Division
	Kelly Kuzyk	Mountain View School Division
	Judy Maryniuk	Lord Selkirk School Division
	Greg Sawatzky	Hanover School Division
	Darlene Willetts	Evergreen School Division
Manitoba Education and Advanced Learning School Programs Division Staff	Heather Anderson Consultant (until June 2007)	Development Unit Instruction, Curriculum and Assessment Branch
	Carole Bilyk Project Manager Acting Coordinator	Development Unit Instruction, Curriculum and Assessment Branch
	Louise Boissonneault Coordinator	Document Production Services Unit Educational Resources Branch
	Linda Girling Consultant (since January 2011)	Development Unit Instruction, Curriculum and Assessment Branch
	Lynn Harrison Desktop Publisher	Document Production Services Unit Educational Resources Branch
	Heather Knight Wells Project Leader (until December 2010)	Development Unit Instruction, Curriculum and Assessment Branch
	Grant Moore Publications Editor	Document Production Services Unit Educational Resources Branch

INTRODUCTION

Purpose of This Document

Grade 6 Mathematics: Support Document for Teachers provides various suggestions for instruction, assessment strategies, and learning resources that promote the meaningful engagement of mathematics learners in Grade 6. The document is intended to be used by teachers as they work with students in achieving the learning outcomes and achievement indicators identified in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)* (Manitoba Education).

Background

Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes is based on *The Common Curriculum Framework for K–9 Mathematics*, which resulted from ongoing collaboration with the Western and Northern Canadian Protocol (WNCP). In its work, WNCP emphasizes

- common educational goals
- the ability to collaborate and achieve common goals
- high standards in education
- planning an array of educational activities
- removing obstacles to accessibility for individual learners
- optimum use of limited educational resources

The growing effects of technology and the need for technology-related skills have become more apparent in the last half century. Mathematics and problem-solving skills are becoming more valued as we move from an industrial to an informational society. As a result of this trend, mathematics literacy has become increasingly important. Making connections between mathematical study and daily life, business, industry, government, and environmental thinking is imperative. The Kindergarten to Grade 12 mathematics curriculum is designed to support and promote the understanding that mathematics is

- a way of learning about our world
- part of our daily lives
- both quantitative and geometric in nature

Beliefs about Students and Mathematics Learning

The Kindergarten to Grade 8 mathematics curriculum is designed with the understanding that students have unique interests, abilities, and needs. As a result, it is imperative to make connections to all students' prior knowledge, experiences, and backgrounds.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with unique knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of manipulatives and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

Students need frequent opportunities to develop and reinforce their conceptual understanding, procedural thinking, and problem-solving abilities. By addressing these three interrelated components, students will strengthen their ability to apply mathematical learning to their daily lives.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Conceptual understanding: comprehending mathematical concepts, relations, and operations to build new knowledge. (Kilpatrick, Swafford, and Findell 5)

Procedural thinking: carrying out procedures flexibly, accurately, efficiently, and appropriately.

Problem solving: engaging in understanding and resolving problem situations where a method or solution is not immediately obvious. (OECD 12)

First Nations, Métis, and Inuit Perspectives

First Nations, Métis, and Inuit students in Manitoba come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings, including urban, rural, and isolated communities. Teachers need to recognize and understand the diversity of cultures within schools and the diverse experiences of students.

First Nations, Métis, and Inuit students often have a whole-world view of the environment; as a result, many of these students live and learn best in a holistic way. This means that students look for connections in learning, and learn mathematics best when it is contextualized and not taught as discrete content.

Many First Nations, Métis, and Inuit students come from cultural environments where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding.

A variety of teaching and assessment strategies are required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences, and learning styles of students. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks).

Affective Domain

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom learning activities, persist in challenging situations, and engage in reflective practices.

Teachers, students, and parents* need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward reaching these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessment of personal goals.

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* In this document, the term *parents* refers to both parents and guardians and is used with the recognition that in some cases only one parent may be involved in a child's education.

Middle Years Education

Middle Years education is defined as the education provided for young adolescents in Grades 5, 6, 7, and 8. Middle Years learners are in a period of rapid physical, emotional, social, moral, and cognitive development.

Socialization is very important to Middle Years students, and collaborative learning, positive role models, approval of significant adults in their lives, and a sense of community and belonging greatly enhance adolescents' engagement in learning and commitment to school. It is important to provide students with an engaging and social environment within which to explore mathematics and to construct meaning.

Adolescence is a time of rapid brain development when concrete thinking progresses to abstract thinking. Although higher-order thinking and problem-solving abilities develop during the Middle Years, concrete, exploratory, and experiential learning is most engaging to adolescents.

Middle Years students seek to establish their independence and are most engaged when their learning experiences provide them with a voice and choice. Personal goal setting, co-construction of assessment criteria, and participation in assessment, evaluation, and reporting help adolescents take ownership of their learning. Clear, descriptive, and timely feedback can provide important information to the mathematics student. Asking open-ended questions, accepting multiple solutions, and having students develop personal strategies will help students to develop their mathematical independence.

Adolescents who see the connections between themselves and their learning, and between the learning inside the classroom and life outside the classroom, are more motivated and engaged in their learning than those who do not observe these connections.

Adolescents thrive on challenges in their learning, but their sensitivity at this age makes them prone to discouragement if the challenges seem unattainable. Differentiated instruction allows teachers to tailor learning challenges to adolescents' individual needs, strengths, and interests. It is important to focus instruction on where students are and to see every contribution as valuable.

Mathematics Education Goals for Students

The main goals of mathematics education are to prepare students to

- communicate and reason mathematically
- use mathematics confidently, accurately, and efficiently to solve problems
- appreciate and value mathematics
- make connections between mathematical knowledge and skills and their applications
- commit themselves to lifelong learning
- become mathematically literate citizens, using mathematics to contribute to society and to think critically about the world

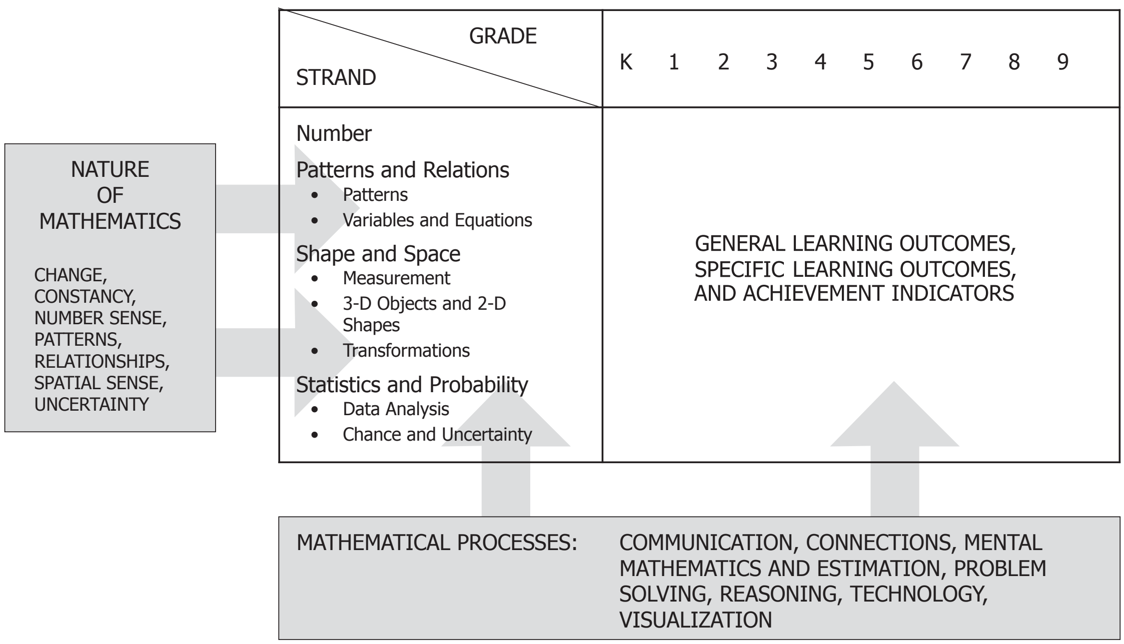
Mathematics education must prepare students to use mathematics to think critically about the world.

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

CONCEPTUAL FRAMEWORK FOR KINDERGARTEN TO GRADE 9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Mathematical Processes

There are critical components that students must encounter in mathematics to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The common curriculum framework incorporates these seven interrelated mathematical processes, which are intended to permeate teaching and learning:

- **Communication [C]:** Students communicate daily (orally, through diagrams and pictures, and by writing) about their mathematics learning. They need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. This enables them to reflect, to validate, and to clarify their thinking. Journals and learning logs can be used as a record of student interpretations of mathematical meanings and ideas.
- **Connections [CN]:** Mathematics should be viewed as an integrated whole, rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—concrete, pictorial, and symbolic (the symbolic mode consists of oral and written word symbols as well as mathematical symbols). The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas.
- **Mental Mathematics and Estimation [ME]:** The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills. Mental mathematics and estimation is a combination of cognitive strategies that enhances flexible thinking and number sense.
- **Problem Solving [PS]:** Students are exposed to a wide variety of problems in all areas of mathematics. They explore a variety of methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problems.
- **Reasoning [R]:** Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.
- **Technology [T]:** The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.
- **Visualization [V]:** Mental images help students to develop concepts and to understand procedures. Students clarify their understanding of mathematical ideas through images and explanations.

These processes are outlined in detail in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.

Strands

The learning outcomes in the Manitoba curriculum framework are organized into four strands across Kindergarten to Grade 9. Some strands are further subdivided into substrands. There is one general learning outcome per substrand across Kindergarten to Grade 9.

The strands and substrands, including the general learning outcome for each, follow.

Number

- Develop number sense.

Patterns and Relations

- Patterns
  - Use patterns to describe the world and solve problems.
- Variables and Equations
  - Represent algebraic expressions in multiple ways.

Shape and Space

- Measurement
  - Use direct and indirect measure to solve problems.
- 3-D Objects and 2-D Shapes
  - Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
- Transformations
  - Describe and analyze position and motion of objects and shapes.

Statistics and Probability

- Data Analysis
  - Collect, display, and analyze data to solve problems.
- Chance and Uncertainty
  - Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Learning Outcomes and Achievement Indicators

The Manitoba curriculum framework is stated in terms of general learning outcomes, specific learning outcomes, and achievement indicators:

- **General learning outcomes** are overarching statements about what students are expected to learn in each strand/substrand. The general learning outcome for each strand/substrand is the same throughout the grades from Kindergarten to Grade 9.
- **Specific learning outcomes** are statements that identify the specific skills, understanding, and knowledge students are required to attain by the end of a given grade.
- **Achievement indicators** are samples of how students may demonstrate their achievement of the goals of a specific learning outcome. The range of samples provided is meant to reflect the depth, breadth, and expectations of the specific learning outcome. While they provide some examples of student achievement, they are not meant to reflect the sole indicators of success.

In this document, the word *including* indicates that any ensuing items **must be addressed** to meet the learning outcome fully. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are **not requirements that must be addressed** to meet the learning outcome fully.

Summary

The conceptual framework for Kindergarten to Grade 9 mathematics describes the nature of mathematics, the mathematical processes, and the mathematical concepts to be addressed in Kindergarten to Grade 9 mathematics. The components are not meant to stand alone. Learning activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes, and lead students to an understanding of the nature of mathematics through specific knowledge, skills, and attitudes among and between strands. *Grade 6 Mathematics: Support Document for Teachers* is meant to support teachers to create meaningful learning activities that focus on formative assessment and student engagement.

ASSESSMENT

Authentic assessment and feedback are a driving force for the suggestions for assessment in this document. The purposes of the suggested assessment activities and strategies are to parallel those found in *Rethinking Classroom Assessment with Purpose in Mind: Assessment* for *Learning, Assessment* as *Learning, Assessment* of *Learning* (Manitoba Education, Citizenship and Youth). These include the following:

- assessing *for*, *as*, and *of* learning
- enhancing student learning
- assessing students effectively, efficiently, and fairly
- providing educators with a starting point for reflection, deliberation, discussion, and learning

Assessment *for* learning is designed to give teachers information to modify and differentiate teaching and learning activities. It acknowledges that individual students learn in idiosyncratic ways, but it also recognizes that there are predictable patterns and pathways that many students follow. It requires careful design on the part of teachers so that they use the resulting information to determine not only what students know, but also to gain insights into how, when, and whether students apply what they know. Teachers can also use this information to streamline and target instruction and resources, and to provide feedback to students to help them advance their learning.

Assessment *as* learning is a process of developing and supporting metacognition for students. It focuses on the role of the student as the critical connector between assessment and learning. When students are active, engaged, and critical assessors, they make sense of information, relate it to prior knowledge, and use it for new learning. This is the regulatory process in metacognition. It occurs when students monitor their own learning and use the feedback from this monitoring to make adjustments, adaptations, and even major changes in what they understand. It requires that teachers help students develop, practise, and become comfortable with reflection, and with a critical analysis of their own learning.

Assessment *of* learning is summative in nature and is used to confirm what students know and can do, to demonstrate whether they have achieved the curriculum learning outcomes, and, occasionally, to show how they are placed in relation to others. Teachers concentrate on ensuring that they have used assessment to provide accurate and sound statements of students' proficiency so that the recipients of the information can use the information to make reasonable and defensible decisions.

Overview of Planning Assessment			
	Assessment *for* Learning	**Assessment *as* Learning**	**Assessment *of* Learning**
Why Assess?	■ to enable teachers to determine next steps in advancing student learning	■ to guide and provide opportunities for each student to monitor and critically reflect on his or her learning and identify next steps	■ to certify or inform parents or others of the student's proficiency in relation to curriculum learning outcomes
Assess What?	■ each student's progress and learning needs in relation to the curriculum outcomes	■ each student's thinking about his or her learning, what strategies he or she uses to support or challenge that learning, and the mechanisms he or she uses to adjust and advance his or her learning	■ the extent to which each student can apply the key concepts, knowledge, skills, and attitudes related to the curriculum outcomes
What Methods?	■ a range of methods in different modes that make a student's skills and understanding visible	■ a range of methods in different modes that elicit the student's learning and metacognitive processes	■ a range of methods in different modes that assess both product and process
Ensuring Quality	■ accuracy and consistency of observations and interpretations of student learning ■ clear, detailed learning expectations ■ accurate, detailed notes for descriptive feedback to each student	■ accuracy and consistency of a student's self-reflection, self-monitoring, and self-adjustment ■ engagement of the student in considering and challenging his or her thinking ■ the student records his or her own learning	■ accuracy, consistency, and fairness of judgments based on high-quality information ■ clear, detailed learning expectations ■ fair and accurate summative reporting
Using the Information	■ provide each student with accurate descriptive feedback to further his or her learning ■ differentiate instruction by continually checking where each student is in relation to the curriculum outcomes ■ provide parents or guardians with descriptive feedback about student learning and ideas for support	■ provide each student with accurate, descriptive feedback that will help him or her develop independent learning habits ■ have each student focus on the task and his or her learning (not on getting the right answer) ■ provide each student with ideas for adjusting, rethinking, and articulating his or her learning ■ provide the conditions for the teacher and student to discuss alternatives ■ the student reports about his or her own learning	■ indicate each student's level of learning ■ provide the foundation for discussions on placement or promotion ■ report fair, accurate, and detailed information that can be used to decide the next steps in a student's learning

Source: Manitoba Education, Citizenship and Youth. *Rethinking Classroom Assessment with Purpose in Mind: Assessment* for *Learning, Assessment* as *Learning, Assessment* of *Learning*. Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2006, 85.

INSTRUCTIONAL FOCUS

The Manitoba mathematics curriculum framework is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of learning outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands.

Consider the following when planning for instruction:

- Routinely incorporating conceptual understanding, procedural thinking, and problem solving within instructional design will enable students to master the mathematical skills and concepts of the curriculum.
- Integration of the mathematical processes within each strand is expected.
- Problem solving, conceptual understanding, reasoning, making connections, and procedural thinking are vital to increasing mathematical fluency, and must be integrated throughout the program.
- Concepts should be introduced using manipulatives and gradually developed from the concrete to the pictorial to the symbolic.
- Students in Manitoba bring a diversity of learning styles and cultural backgrounds to the classroom and they may be at varying developmental stages. Methods of instruction should be based on the learning styles and abilities of the students.
- Use educational resources by adapting to the context, experiences, and interests of students.
- Collaborate with teachers at other grade levels to ensure the continuity of learning of all students.
- Familiarize yourself with exemplary practices supported by pedagogical research in continuous professional learning.
- Provide students with several opportunities to communicate mathematical concepts and to discuss them in their own words.

"Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways—individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. In addition, mathematics requires students to learn concepts and procedures, acquire skills, and learn and apply mathematical processes. These different areas of learning may involve different teaching and learning strategies. It is assumed, therefore, that the strategies teachers employ will vary according to both the object of the learning and the needs of the students" (Ontario 24).

Document Organization and Format

This document consists of the following sections:

- **Introduction:** The Introduction provides information on the purpose and development of this document, discusses characteristics of and goals for Middle Years learners, and addresses Aboriginal perspectives. It also gives an overview of the following:
  - **Conceptual Framework for Kindergarten to Grade 9 Mathematics:** This framework provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.
  - **Assessment:** This section provides an overview of planning for assessment in mathematics, including assessment *for*, *as*, and *of* learning.
  - **Instructional Focus:** This discussion focuses on the need to integrate mathematics learning outcomes and processes across the four strands to make learning experiences meaningful for students.
  - **Document Organization and Format:** This overview outlines the main sections of the document and explains the various components that comprise the various sections.
- **Number:** This section corresponds to and supports the Number strand for Grade 6 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Patterns and Relations:** This section corresponds to and supports the Patterns and Variables and Equations substrands of the Patterns and Relations strand for Grade 6 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Shape and Space:** This section corresponds to and supports the Measurement, 3-D Objects and 2-D Shapes, and Transformations substrands of the Shape and Space strand for Grade 6 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Statistics and Probability:** This section corresponds to and supports the Data Analysis and Chance and Uncertainty substrands of the Statistics and Probability strand for Grade 6 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Blackline Masters (BLMs):** Blackline masters are provided to support student learning. They are available in *Microsoft Word* format so that teachers can alter them to meet students' needs. They are also available in *Adobe* PDF format.
- **Bibliography:** The bibliography lists the sources consulted and cited in the development of this document.

Guide to Components and Icons

Each of the sections supporting the strands of the Grade 6 Mathematics curriculum includes the components and icons described below.

Enduring Understanding(s):

These statements summarize the core idea of the particular learning outcome(s). Each statement provides a conceptual foundation for the learning outcome. It can be used as a pivotal starting point in integrating other mathematics learning outcomes or other subject concepts. The integration of concepts, skills, and strands remains of utmost importance.

General Learning Outcome(s):

General learning outcomes (GLOs) are overarching statements about what students are expected to learn in each strand/substrand. The GLO for each strand/substrand is the same throughout Kindergarten to Grade 8.

Specific Learning Outcome(s):	**Achievement Indicators:**
Specific learning outcome (SLO) statements define what students are expected to achieve by the end of the grade. A code is used to identify each SLO by grade and strand, as shown in the following example: **6.N.1** The first number refers to the grade (Grade 6). The letter(s) refer to the strand (Number). The last number indicates the SLO number. [C, CN, ME, PS, R, T, V] Each SLO is followed by a list indicating the applicable mathematical processes.	Achievement indicators are examples of a representative list of the depth, breadth, and expectations for the learning outcome. The indicators may be used to determine whether students understand the particular learning outcome. These achievement indicators will be addressed through the learning activities that follow.

Prior Knowledge

Prior knowledge is identified to give teachers a reference to what students may have experienced previously. Teachers should assess students' prior knowledge before planning instruction.

Related Knowledge

Related knowledge is identified to indicate the connections among the Grade 6 mathematics learning outcomes.

Background Information

Background information is provided to give teachers knowledge about specific concepts and skills related to the particular learning outcome(s).

Mathematical Language

Lists of terms students will encounter while achieving particular learning outcomes are provided. These terms can be placed on mathematics word walls or used in a classroom mathematics dictionary. *Kindergarten to Grade 8 Mathematics Glossary: Support Document for Teachers* (Manitoba Education, Citizenship and Youth) provides teachers with an understanding of key terms found in Kindergarten to Grade 8 mathematics. The glossary is available on the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/cur/math/supports.html>.

Learning Experiences

Suggested instructional strategies and assessment ideas are provided for the specific learning outcomes and achievement indicators. In general, learning activities and teaching strategies related to specific learning outcomes are developed individually, except in cases where it seems more logical to develop two or more learning outcomes together. Suggestions for assessment include information that can be used to assess students' progress in their understanding of a particular learning outcome or learning experience.

Assessing Prior Knowledge

Suggestions are provided to assess students' prior knowledge and to help direct instruction.

Observation Checklist

Checklists are provided for observing students' responses during lessons.

- **Achievement indicators appropriate to particular learning experiences are listed.**

The instructional suggestions include the following:

- **Materials/Resources:** Outlines the resources required for a learning activity.
- **Organization:** Suggests groupings (individual, pairs, small group, and/or whole class).
- **Procedure:** Outlines detailed steps for implementing suggestions for instruction.

Some learning activities make use of BLMs, which are found in the Blackline Masters section in *Microsoft Word* and *Adobe* PDF formats.

Putting the Pieces Together

Putting the Pieces Together tasks, found at the end of the learning outcomes, consist of a variety of assessment strategies. They may assess one or more learning outcomes across one or more strands and may make cross-curricular connections.

Grade 6 Mathematics

Number

Grade 6: Number (6.N.1, 6.N.2)

Enduring Understanding(s):

The position of a digit in a number determines its value.

Each place value position is 10 times greater than the place value position to its right.

General Learning Outcome(s):

Develop number sense.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
6.N.1 Demonstrate an understanding of place value for numbers ▪ greater than one million ▪ less than one-thousandth. [C, CN, R, T]	➛ Explain how the pattern of the place value system (e.g., the repetition of ones, tens, and hundreds) makes it possible to read and write numerals for numbers of any magnitude. ➛ Provide examples of where large numbers and small decimals are used (e.g., media, science, medicine, technology).
6.N.2 Solve problems involving large numbers, using technology. [ME, PS, T]	➛ Identify which operation is necessary to solve a problem and solve it. ➛ Determine the reasonableness of an answer. ➛ Estimate the answer and solve a problem. ➛ Identify and correct errors in a solution to a problem that involves large numbers.

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Representing and describing whole numbers to 1 000 000
- Applying estimation strategies in problem-solving situations
- Describing and representing decimals (tenths, hundredths, thousandths)
- Comparing and ordering decimals (tenths, hundredths, thousandths)
- Comparing and ordering numbers to 10 000
- Demonstrating an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals)
- Describing and representing decimals (tenths and hundredths)

- Estimating quantities less than 1000
- Illustrating, concretely and pictorially, the meaning of place value of numerals to 1000

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of integers, concretely, pictorially, and symbolically

Background Information

Students extend their number pattern knowledge to numbers greater than a million and less than one-thousandth by knowing and understanding

- that each place represents ten times as much as the place to its right.
- that each place represents one-tenth as much as the place to its left.
- that places are organized into threes and that this pattern goes beyond millions into billions, trillions, and also into ten-thousandths, hundred-thousandths, etc.
- that when we write numbers, spaces, not commas, are used to show the place value of numbers, with the exception of 4-digit numbers (e.g., 5432).
- that when we read numbers greater than a million, we read 4 567 890 123 as four billion, five hundred sixty-seven million, eight hundred ninety thousand, one hundred twenty-three. The word "and" is used for decimal numbers (e.g., 654.78 is read as six hundred fifty-four **and** seventy-eight hundredths (not six hundred fifty four **point** seventy eight).
- that 0 is an important place holder. The number 30 indicates that there are 3 tens and 0 is in the ones place. In the number 285 027, the 0 indicates that there are no hundreds (or there are 50 hundreds) and in decimal numbers such as 1.09, the zero indicates that there are no tenths.

Teachers should have students

- explore and explain how the numbers system pattern works.
- investigate and be able to explain where and when large numbers are used and where or when we need to use the precision of small decimals.
- read numbers using the appropriate vocabulary and place value.

With this knowledge, students will develop flexibility with reading, identifying, and representing numbers, which will support the development of mental math and estimation skills.

Glossary terms can be found at <www.edu.gov.mb.ca/k12/curr/math/glossary_k-8/index.html>.

Base-Ten Number System: The system of numbers we use is called the **base-ten number system**. It is a place-value number system in which 10 digits, 0 through 9, are used to represent a number, and the value of each place is 10 times the value of the place to its right. The value of any digit in the number is the product of that digit and its place value.

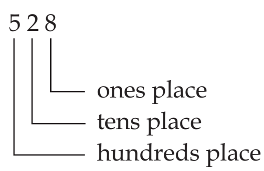
10 ones = 1 ten
10 tens = 1 hundred
10 hundreds = 1 thousand
10 thousands = 1 ten thousand
10 ten thousands = 1 hundred thousand
10 hundred thousands = 1 million

10 tenths = 1 one
10 hundredths = 1 tenth
10 thousandths = 1 hundredth
10 ten-thousandths = 1 thousandth
10 hundred-thousandths = 1 ten-thousandth
10 millionths = 1 hundred-thousandth

Place value is the value of a digit in a number based on its position.

Example

In the number 528, the 5 has a value of 5 hundreds (or 500), the 2 has a value of 2 tens (or 20), and the 8 has a value of 8 ones (or 8).



Numeral is the written symbol that represents a number.

Number is the concept of an amount, quantity, or how many items there are in a collection. Numbers play an important part in our lives. We use numbers in school, at the workplace, and in our daily living.

Decimal is a fractional number written in base-ten form; a mixed decimal number has a whole number part as well (e.g., 0.32 is a decimal number and 3.5 is a mixed decimal number). A period or dot (such as in the previous example) separating the ones place from the tenths place in decimal numbers or dollars from cents in money is called a **decimal point**. When numbers are spoken, the decimal point is read as "and" (e.g., 3.2 is read as three and two-tenths).

Suggested Activities

- Explore number patterns using models such as base-ten blocks, Cuisinaire rods, number lines, thousands grids, and decimal squares.
- Create an "A-B-C" book of real-work numbers (e.g., A is for an ant's antenna measured in small decimal points, C is for the population of Canada—32 000 000).
- Place the place-value game using playing cards with the face cards removed (tens can be zeros). Have students draw nine lines on a piece of paper in place-value format (___ ___ ___   ___ ___ ___   ___ ___ ___). Select one card at a time and ask the students to make the largest number that they can, using the numbers as they are drawn. Once the number is placed, it cannot be moved. **Variation:** Place a decimal in the visual and have students make the smallest number that could be made using the cards.
- Compare the populations of large and small cities and draw conclusions or create problems using these numbers (e.g., world population is 6 767 805 208, the population of China is 1 338 612 968, and the population of Canada is 32 440 970).

Mathematical Language

decimal number

decimal point

place value

Learning Experiences

Assessing Prior Knowledge

Organization: Whole class

Materials: BLM 5–8:17: Number Fan

Procedure:

1. Tell students you want to check what they remember about the decimal system.
2. Have a class discussion. Ask questions such as:
   a) "What are the 10 digits we use for representing all numbers?"
   b) "What makes it possible for us to represent large numbers, using only the 10 digits?"
   c) "What makes it possible for us to represent small numbers that are less than one, using only the 10 digits?"
3. Discuss place value and decimal place.

4. Provide students with a number fan and have them show you
   a) twenty-eight hundredths
   b) nineteen tenths
   c) one thousand nine hundred eight
   d) a number larger than one thousand twenty-three
   e) a number smaller than one-hundredth

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Demonstrate an understanding of place value related to whole numbers.
- ❒ Demonstrate an understanding of place value related to decimal numbers.

Suggestions for Instruction

- **Explain how the pattern of the place value system (e.g., the repetition of ones, tens, and hundreds) makes it possible to read and write numerals for numbers of any magnitude.**
- **Provide examples of where large numbers and small decimals are used (e.g., media, science, medicine, technology).**

Materials:

- pencil and paper

Organization: Whole class/small groups

Procedure:

1. Write a few three-digit numerals on the board, such as the following:
   a) 385
   b) 761
   c) 208
2. Ask a student volunteer to read each numeral.
3. Write a few six-digit numerals on the board, such as the following:
   a) 378 321
   b) 618 026
   c) 320 180

4. Ask a student volunteer to read each numeral.
5. Discuss what students noticed about this second set.
6. Write a few numerals on the board that are greater than one million, such as the following:
   a) 38 851 406
   b) 76 211 318
   c) 208 182 281
7. For each numeral, read the numeral to the students first, and then ask the students to read each numeral.
8. Tell students to discuss the following questions with their group members:
   a) How did you know how to read each numeral?
   b) Did you detect a pattern? Explain.
   c) Where would you find large numbers that are greater than one million?
9. Ask one student volunteer to explain orally the replies of his or her group.
10. Tell each group to
    a) think of a few large numbers and where those numbers could be found or used
    b) write the numerals on one side of their paper, and the examples of their use on the other side
11. Have members of one group go in front of the class and present their numbers.
12. Have the rest of the class read them out loud and provide examples of their use.
13. Have one of the group members write the examples on the board.
14. Repeat with the other groups.
15. At the end, check if the examples on the board covered all the examples provided on the backs of the papers.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❐ Read numerals greater than one million.
- ❐ Write numerals greater than one million.
- ❐ Provide examples of where large numbers are used.

Suggestions for Instruction

- **Explain how the pattern of the place value system (e.g., the repetition of ones, tens, and hundreds) makes it possible to read and write numerals for numbers of any magnitude.**
- **Provide examples of where large numbers and small decimals are used (e.g., media, science, medicine, technology).**

Materials:

- pencil and paper
- BLM 6.N.1.1: Small Decimals

Organization: Whole class/pairs

Procedure:

1. Place a transparent copy of BLM 6.N.1.1 on the overhead projector or display it electronically.
2 Read each numeral and have the students repeat each one right after you.
3 Discuss what students noticed about this set of numerals (e.g., decimal place, place value).
4 Discuss where such small decimals are to be used.
5 Tell each student to
   a) think of a few small decimals and where those numbers could be found or used
   b) write the numerals on one side of the paper, and the examples of their use on the other side
6 Have each student read his or her partner's numbers and provide examples of their use.
7 Check whether the examples the student provided cover all the examples provided on the backs of the papers.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Read numerals smaller than one-thousandth.
- ❒ Write numerals smaller than one-thousandth.
- ❒ Provide examples of where small decimal numbers are used.

Suggestions for Instruction

- **Identify which operation is necessary to solve a problem and solve it.**

Materials:

- pencil and paper
- BLM 6.N.2.1: Nedy's Bike Ride

Organization: Whole class/individual

Procedure:

1. Place a transparent copy of BLM 6.N.2.1 on the overhead projector or display it electronically.
2. Have the student
   a) read the problem
   b) choose between "Today" and "Yesterday"
   c) identify which operation is necessary to solve the problem
   d) solve the problem
3. Discuss what students noticed about the problem (e.g., two units of measure are used: metres and kilometres).
4. Have each student design a problem.
5. Let students exchange their work with their neighbour.
6. Tell them to
   a) read the problem
   b) identify which operation is necessary to solve the problem
   c) solve the problem
7. Have a class discussion on these student-made problems.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Read the problem.
- ❒ Identify which operation is necessary to solve the problem.
- ❒ Solve the problem.
- ❒ Notice that two units of measure are used: metres and kilometres.
- ❒ Convert units.

- **Determine the reasonableness of an answer.**

Materials:

- pencil
- BLM 6.N.2.2: Am I Reasonable?

Organization: Whole class/individual

Procedure:

1. Using large numbers, write on the board one question for each operation (addition, subtraction, multiplication, and division). For example:
   a) 29 000 000 + 63 000 000 =
   b) 48 000 000 – 29 000 000 =
   c) 3 000 000 × 18 =
   d) 86 000 000 ÷ 21 =
2. Discuss what is a reasonable answer for each of the examples. What strategies did students use to determine reasonable answers?
3. Distribute to each student a copy of BLM 6.N.2.2 (or display it for the whole class on an overhead/screen/interactive whiteboard).
4. Tell students to
   a) read each problem carefully
   b) determine if the answer is reasonable
5. Have a class discussion regarding the reasonableness of each problem. What makes an answer reasonable? Do you always need an exact answer? When is it okay to make an estimate?

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Read large numerals.
- ❒ Add large numbers.
- ❒ Subtract large numbers.
- ❒ Multiply large numbers.
- ❒ Divide large numbers.
- ❒ Determine the reasonableness of an answer given operations with large numbers.

Suggestions for Instruction

- **Estimate the answer and solve a problem.**

Materials:

- pencil
- calculator
- BLM 6.N.2.3: Estimate and Solve

Organization: Whole class/individual

Procedure:

1. Discuss estimation with the class.
2. Using large numbers, write on the board one question for each operation (addition, subtraction, multiplication, and division). For example:
   a) 38 000 000 + 23 000 500 =
   b) 42 006 000 – 17 895 000 =
   c) 4 000 000 × 19 =
   d) 64 030 000 ÷ 21 =
3. For each of the examples,
   a) have the class estimate the answer
   b) discuss how they estimated the answer
   c) have students use their calculator to solve the problem

4. Distribute to each student a copy of BLM 6.N.2.3 (or display for class).
5. Tell students to
   a) estimate the answer
   b) use their calculator to solve the problem
6. Have a class discussion regarding the closeness of their estimation to the solution of each problem.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Estimate large numerals.
- ❒ Use a calculator to add, subtract, multiply, and divide large numbers.

Suggestions for Instruction

- **Identify and correct errors in a solution to a problem that involves large numbers.**

Materials:

- pencil
- calculator
- BLM 6.N.2.4: Identify and Correct

Organization: Whole class/individual

Procedure:

1. For each operation (addition, subtraction, multiplication, and division), ask a different student to
   a) use large numbers and write one question on the board
   b) write an equation choosing either a correct or an incorrect solution

   Examples:

   a) $47\ 000\ 000 + 45\ 000\ 000 \stackrel{?}{=} 92\ 000\ 000$

   b) $128\ 000\ 000 - 29\ 000\ 000 \stackrel{?}{=} 101\ 000\ 000$

   c) $3\ 000\ 000 \times 18 \stackrel{?}{=} 54\ 000\ 000$

   d) $96\ 000\ 000 \div 32 \stackrel{?}{=} 30\ 000\ 000$

2. Discuss the correctness of each reply ((a) and (c) are correct, (b) and (d) are not correct).
3. Have the class identify the errors (incorrect subtraction for (b), and incorrect place value for (d)).
4. Correct errors.

   For the above example (b), the correct statement should read:

   128 000 000 – 29 000 000 = 99 000 000

   For the above example (d), the correct statement should read:

   96 000 000 ÷ 32 = 3 000 000
5. Distribute to each student a copy of BLM 6.N.2.4 (or display for class).
6. Tell students to
   a) read each problem carefully
   b) identify the errors
   c) correct the errors
7. Have a class discussion regarding the errors and corrections to each problem.
8. Discuss common computational errors with students.
9. Have students create their own questions with errors and switch with a partner.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Identify errors in a solution to a problem that involves large numbers.
- ❐ Correct errors in a solution to a problem that involves large numbers.

Grade 6: Number (6.N.3)

Enduring Understanding(s):

Some numbers have only two factors: the number itself, and one.

Some numbers have many factors; they can also be said to be composed of multiples of numbers other than one and itself.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	**Achievement Indicators:**
6.N.3 Demonstrate an understanding of factors and multiples by ▪ determining multiples and factors of numbers less than 100 ▪ identifying prime and composite numbers ▪ solving problems involving factors or multiples [PS, R, V]	➛ Identify multiples for a number and explain the strategy used to identify them. ➛ Determine all the whole-number factors of a number using arrays. ➛ Identify the factors for a number and explain the strategy used (e.g., concrete or visual representations, repeated division by prime numbers or factor trees). ➛ Identify common factors and common multiples for 2 or 3 numbers. ➛ Provide an example of a prime number and explain why it is a prime number. ➛ Provide an example of a composite number and explain why it is a composite number. ➛ Sort a set of numbers as prime and composite. ➛ Solve a problem involving factors, multiples, the largest common factor, or the lowest common multiple. ➛ Explain why 0 and 1 are neither prime nor composite.

Prior Knowledge

Students may have had experience with the following:

- Representing and describing whole numbers to 1 000 000
- Determining multiplication facts (to 81) and related division facts
- Applying mental mathematics strategies for multiplication

- Demonstrating an understanding of multiplication to solve problems
- Demonstrating an understanding of division with and without concrete materials, and interpreting remainders to solve problems

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of integers, concretely, pictorially, and symbolically
- Demonstrating an understanding of multiplication and division of decimals

Background Information

Number is the concept of an amount, quantity, or how many items there are in a collection. Numbers play an important part in our lives. We use numbers in school, at the workplace, and in our daily living. The written symbol that represents a number is called a **numeral**.

Numbers can be prime or composite. A **prime number** is a number greater than 1 that has exactly two different factors: 1 and itself (e.g., 3 is a prime number, as its only factors are 1 and 3).

Prime Numbers		Non-Prime Numbers	
Number	**Factors**	**Number**	**Factors**
2	1, 2	6	1, 2, 3, 6
7	1, 7	8	1, 2, 4, 8
11	1, 11	15	1, 3, 5, 15
17	1, 17	25	1, 5, 25

A **composite number** is a whole number that has more than two factors (e.g., 4 is a composite number because it has three factors: 1, 2, and 4).

Each number has multiples and factors.

A **multiple** is the product of a given whole number and any other whole number.

Examples

18 is a multiple of 6 (since $6 \times 3 = 18$).

18 is a multiple of 18 (since $18 \times 1 = 18$).

18 is NOT a multiple of 8.

To find the first few multiples of 3, multiply 3 by 1, 2, 3 to get 3, 6, 9. Multiples can also be found using skip counting.

A whole number that is a multiple of two or more given numbers is called a **common multiple** (e.g., common multiples of 2, 3, and 4 are 12, 24, 36, 48, . . .).

Factors are numbers that are multiplied to get a product (3 and 4 are factors of 12). Factors can be used in the following ways:

a) A number or expression that is multiplied by another to yield a product (e.g., a factor of 24 is 8 because $8 \times 3 = 24$, and a factor of $3n$ is n because $3 \cdot n = 3n$).

b) To express as a product of two or more factors (e.g., if the question is "factor 36," the answer could be 2×18 or $2 \times 3 \times 6$). A whole number that is a factor of two or more given numbers or expressions is called a **common factor** (e.g., 7 is a common factor of 14, $14x$, $14w$, . . . 21, $21x$, $21w$, . . . 28, $28x$, $28w$, . . .).

Factors of a number are never greater than the number. The greatest factor is always the number itself and the least factor is always 1. The second factor is always half of the number or less (unless the number is prime).

A **prime number** has only two factors: 1 and itself (e.g., 29 has only 2 factors, 1 and 29). The concept of prime numbers applies only to whole numbers.

Composite numbers have more than two factors and include all non-prime numbers other than 1 and 0 (e.g., 9 has the factors 1, 3, and 9). The numbers 0 and 1 are neither prime nor composite because 1 has only one factor (itself) and 0 has an infinite number of divisors and cannot be uniquely written as a product of 2 factors.

A **whole number** that is a factor of two or more given numbers or expressions is called a **common factor** (e.g., a common factor for 18 and 21 is 3, because $3 \times 6 = 18$ and $3 \times 7 = 21$).

Mathematical Language

composite number

factor

multiple

prime number

Assessing Prior Knowledge

Materials:

- demonstration board/small white boards for each student

Organization: Whole class

Procedure:

1. Tell students they will be learning about composites and primes, but first you want to check what they know about factors and multiples.
2. Tell them they will need to write their answers on the demonstration board and show their responses to the teacher. They can use numbers between 1 and 50 for their responses.
3. Ask the following questions:
   a) Which numbers are multiples of 6?
   b) Which numbers have a factor of 10?
   c) What are the factors of 12?
   d) Which numbers are multiples of 9?
   e) What are the factors of 30?
   f) Which numbers have a factor of 7?
4. Discuss what they know about factors and multiples.
5. Tell them that this time they need to examine a few special numbers, and then write on the board:

   2, 7, 13, 19, 31
6. Ask what students observe about these numbers.
7. Ask them to provide more examples of these numbers without factors.
8. How are they different from numbers such as: 8, 12, 20, 24, or 30?
9. Discuss prime and composite numbers and brainstorm other examples.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Name multiples of a given number between 1 and 50.
- ❒ Name factors of a given number.
- ❒ Relate prior knowledge of operations on whole numbers to reasoning about factors, multiples, primes, and composites.

Suggestions for Instruction

- **Identify multiples for a number and explain the strategy used to identify them.**
- **Provide an example of a prime number and explain why it is a prime number.**
- **Provide an example of a composite number and explain why it is a composite number.**
- **Sort a set of numbers as prime and composite.**
- **Explain why 0 and 1 are neither prime nor composite.**

Materials:

- coloured pencils (9 distinct colours)
- BLM 6.N.3.1: Multiples and Factors, Primes and Composites, or a hundred board

Organization: Pairs or individuals

Procedure:

1. Student directions:
   a) Place a yellow checkmark in the squares that are multiples of 2. Look at the pattern and describe it. (alternating squares, checkerboard, even numbers)
   b) Place a red checkmark in the squares that are multiples of 3. Look at the pattern and describe it. (Some numbers have two checkmarks or factors, some odd and some even numbers)
   c) Place a dark blue checkmark in the squares that are multiples of 4. Look at the pattern and describe it. (Even, some numbers have three factors)
   d) Continue filling in the hundreds chart using
      - purple for multiples of 5
      - green for multiples of 6
      - orange for multiples of 7
      - black for multiples of 8
      - pink for multiples of 9
      - light blue for multiples of 10
2. Have a discussion about multiples and factors. Have students identify what numbers are composite and what numbers are prime. Ask students to explain why 0 and 1 are neither prime nor composite.
3. Students should keep the chart and include a key so that they can check on factors in future activities.

Variation:

As an alternative activity, use BLM 6.N.3.1. Repeat the student directions above, but this time use cube links instead of pencil crayons and build factor towers over each number on the hundred board.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Name multiples of a given number.
- ❒ Identify the factors of a given number.
- ❒ Identify the prime numbers to 100.
- ❒ Explain the strategy used to identify the multiples of a given number.
- ❒ Explain what makes a number prime.
- ❒ Identify prime and composite numbers to 100.
- ❒ Explain why 0 and 1 are neither prime nor composite.

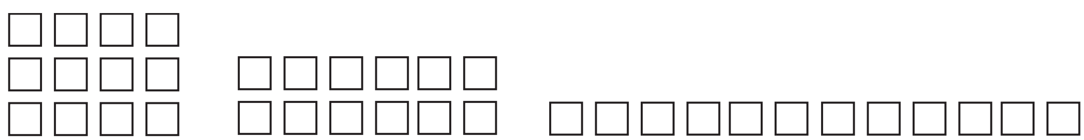
Suggestions for Instruction

- **Determine all the whole-number factors of a number using arrays.**

Materials:
- paper and pencil

Organization: Whole class/small group

Procedure:
1. Discuss factors with the class.
2. Ask students what are the whole-number factors of a few numbers of your choice. Choose numbers up to 20 first, and then you can increase numbers to 50.
3. Ask students what they know about arrays.
4. Ask a few students to come to the board and give an example of an array.
5. Tell students to
   a) work in small groups.
   b) use arrays to determine all the whole-number factors of 12 (see example).



3 x 4 2 x 6 1 x 12

6. Repeat this process using 64.

Extension:
1. Work as a class to build a display of all the possible arrays for every number from 1 to 100. They can use BLM 5–8.9: Centimetre Grid Paper to cut out arrays. Some arrays may have to be taped together (for example, a 21 × 1 array would need a whole column of 20 squares plus one more square taped to it). The display can stay on the classroom wall, as it will be useful when investigating area and perimeter.
2. For each array, label the perimeter and area of the rectangle.
3. For each number, label it as prime or composite.
4. Once the display is complete, have students write a reflection in their journal describing any patterns they notice or describing interesting observations they made about the display.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Explain what factors are.
- ❒ Name factors of a given number.
- ❒ Explain what an array is.
- ❒ Use arrays to determine all the whole-number factors of a number.

Suggestions for Instruction

- **Identify the factors for a number and explain the strategy used (e.g., concrete or visual representations, repeated division by prime numbers or factor trees).**

Materials:

- paper
- pencil
- overhead projector/smart board or projector

Organization: Pairs

Procedure:

1. Discuss the different strategies for identifying factors for a number.
2. Discuss what they like and what they dislike about each strategy.
3. Use a smart board, a projector, or an overhead projector to write "36." Show students how to identify factors using arrays, repeated division, number facts, or factor trees.
4. Tell students to
   a) identify the factors of various numbers
   b) explain to a partner the strategy they used to identify the factors of the number
5. Circulate to check if they are identifying the correct factors for the number.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Name different strategies for identifying factors for a number.
- ❒ Identify factors of a given number.
- ❒ Explain the strategy used to identify the factors of a given number.

Suggestions for Instruction

- **Identify common factors and common multiples for 2 or 3 numbers.**

Materials:

- paper
- pencil
- BLM 6.N.3.2: What's Common?

Organization: Small groups

Procedure:

1. Write two numerals on the board (e.g., 12 and 15). Discuss how to find the common factors of these numbers, and demonstrate various strategies.
2. Ask students to identify a common factor for 24 and 32. Have students share the strategy they used.
3. Discuss what students know about common factors for two numbers.
4. Repeat with three numbers (e.g., 64, 24, 32).
5. Place two other numerals on the board (e.g., 8 and 12).
6. Ask students to identify common multiples for the two numbers.
7. Discuss what students know about common multiples for the two numbers.
8. Repeat with three numbers (5, 15, 2).
9. Have students complete BLM 6.N.3.2.
10. Circulate to check if they are identifying the correct common factors and correct common multiples for each set of numbers.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify common factors of two or three numbers.
- ❒ Identify common multiples of two or three numbers.

Suggestions for Instruction

- **Provide an example of a prime number and explain why it is a prime number.**
- **Provide an example of a composite number and explain why it is a composite number.**
- **Sort a set of numbers as prime and composite.**
- **Solve a problem involving factors, multiples, the largest common factor, or the lowest common multiple.**
- **Explain why 0 and 1 are neither prime nor composite.**

Materials:

- paper
- pencil
- BLM 6.N.3.3: Dilly's Dilemma

Organization: Pairs

Procedure:

1. Tell students they will work in pairs to solve a problem.
2. Ask students to tell you what they remember about the largest common factor.
3. Have students provide an example of the largest common factor of two numbers (e.g., 14 and 21).
4. Ask students the following:
   a) Are the factors always prime numbers?
   b) What is a prime number?
   c) Are 0 and 1 prime or composite? Why?
5. Ask students to tell you what they remember about the lowest common multiple.
6. Have students provide an example of the lowest common multiple of two numbers (e.g., 14 and 21).

7. Distribute to each student a copy of BLM 6.N.3.3, or display it on an interactive whiteboard.
8. Ask students to complete the following:
   a) Find the largest common factor for the three numbers.
   b) Find the lowest common multiple for the three numbers.
   c) Should Dilly give Bobby the largest common factor for the three numbers? Why?
   d) Should Dilly give Johnny the lowest common multiple for the three numbers? Why?
   e) How much money would Dilly have if she gave both twins what they asked for?
9. Have students share their answers and justify their solutions.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Explain what is a prime number.
- ❐ Explain what is the largest common factor.
- ❐ Provide an example of the largest common factor.
- ❐ Explain what is the lowest common multiple.
- ❐ Provide an example of the lowest common multiple.
- ❐ Explain why 0 and 1 are neither prime nor composite.
- ❐ Solve a problem involving the largest common factor.
- ❐ Solve a problem involving the lowest common multiple.

Putting the Pieces Together

Factors and Multiples

Purpose: The purpose of this activity is for students to apply their knowledge of factors and multiples. Students will need some prior knowledge, such as how to multiply and divide numbers. The processes that are demonstrated by this task are communication, connections, problem solving, and reasoning.

Curricular Links:

Materials/Resources:

- paper and pencil
- poster-sized paper
- BLM 6.N.3.4: The Ten of Us

Organization:

Small groups

Inquiry:

Scenario

Students will be working in small groups. They will examine some numbers of their choice in order to find five pairs of numbers that fit the described criteria.

Procedure

Hand out to each group a copy of the following:

1. Instruction sheet
2. BLM 6.N.3.4: The Ten of Us

1. Students read BLM 6.N.3.4 carefully.
2. Students discuss with their group members how they will go about finding the five pairs of numbers as described in BLM 6.N.3.4, where
   - the second number is two times the first number
   - the second number has twice as many factors as the first one
3. Students find five pairs of numbers as described in BLM 6.N.3.4, where
   - the second number is two times the first number
   - the second number has twice as many factors as the first one
4. Students state which numbers are prime and which numbers are composite.
5. In their journals, students describe
   - how they found the five pairs of numbers
   - how they figured out which numbers are prime and which numbers are composite

6. On a poster-sized paper, students record the five pairs of numbers and their factors by making a chart similar to the one below:

First Number	A =	B =	C =	D =	E =
Factors of First Number					
Second Number	2 times A =	2 times B =	2 times C =	2 times D =	2 times E =
Factors of Second Number					

7. Then, students show on the (same or different) poster-sized paper the mathematical procedures they used to find the factors.

   Web link: www.ixl.com/math/grade/six/

Assessment:

Use the following observation checklist to assess student learning:

The student can do the following:	Yes	No	Comment
Solve a problem involving factors and multiples.			
Identify a prime number.			
Identify a composite number.			
Multiply numbers.			
Divide numbers.			
Factor numbers.			
Effectively communicate with peers.			
Cooperate with peers.			

Extension:

Taking it further

Students use their calculators this time to repeat the task using very large numbers.

Extension Activities:

Have students

- express 36 as the product of two factors in as many ways as possible
- find the number less than 50 (or 100) that has the most factors
- show all the factors of 48 by drawing or colouring arrays on square grid paper
- solve problems involving factors and multiples (e.g., Ms. Sherry has 34 students in her class. How many different-sized groups of students can she make so that all groups are the same size?—1, 2, 3, 4, 6, 8, 12, 24)
- use a computer or a calculator to help students determine the prime numbers up to 100
- list all the factors of 8 and the first ten multiples of 8

Ask students

- to explain, without dividing, why 2 cannot be a factor of 47
- to identify a number with five factors
- to find three pairs of prime numbers that differ by two (e.g., 5 and 7)
- to explain why it is easy to know that certain large numbers are not prime, even without factoring them (e.g., 4 283 495)
- to explain why 2 and 3 are consecutive prime numbers but why can there be no other examples of consecutive prime numbers

Grade 6: Number (6.N.4)

Enduring Understanding(s):

Improper fractions and mixed numbers are two ways of writing fractions greater than 1.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
6.N.4 Relate improper fractions to mixed numbers. [CN, ME, R, V]	➛ Demonstrate using models that an improper fraction represents a number greater than 1. ➛ Express improper fractions as mixed numbers. ➛ Express mixed numbers as improper fractions. ➛ Place a set of fractions, including mixed numbers and improper fractions, on a horizontal or vertical number line, and explain strategies used to determine position.

Prior Knowledge

Students may have had experience with the following:

- Determining multiplication facts (to 81) and related division facts
- Demonstrating an understanding of division (3-digit numerals by 1-digit numerals) with and without concrete materials, and interpreting remainders to solve problems
- Demonstrating an understanding of fractions

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of common factors and multiples
- Demonstrating and explaining the meaning of preservation of equality

BACKGROUND INFORMATION

This section of the Grade 6 Number Strand deals with improper fractions and mixed numbers. Before students explore improper fractions and mixed numbers, they need to review the concept of fractions.

A **fraction** is a number that represents part of a whole, part of a set, or a quotient in the form $\left(\frac{a}{b}\right)$, which can be read as *a* divided by *b*.

A fraction whose numerator is smaller than its denominator is referred to as a proper fraction (e.g., $\frac{3}{5}$).

An **improper fraction** is a fraction whose numerator is greater than its denominator (i.e., a fraction with a value greater than 1).

A fraction (proper and improper) can be represented as

- a fraction of a region or whole
- a measurement, such as a name for a point on a number line
- parts of a group or set

Students need to recognize fractions as less than 1 (proper) and as greater than 1 (improper). This recognition is of greater importance than the terminology.

Improper fractions can be expressed as mixed numbers.

A **mixed number** is a number larger than 1 composed of a whole number and a proper fraction (e.g., $1\frac{1}{2}$).

MATHEMATICAL LANGUAGE

denominator

improper fraction

mixed number

numerator

Assessing Prior Knowledge

Materials:

- paper and pencil

Organization: Whole class/individual

Procedure:

1. Tell students during the next few classes they will be learning about improper fractions and mixed numbers, but first you want to check what they remember about equivalent fractions.
2. Write down the following fractions:

$$\frac{3}{5} \quad \frac{12}{20} \quad \frac{18}{25} \quad \frac{27}{45}$$

   a) State whether the fractions are equivalent.
   b) Give your reasons why you believe that they are or they are not equivalent fractions.
3. Discuss the student responses.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Recognize equivalent fractions.
- ❒ Explain what makes equivalent fractions.

- **Place a set of fractions, including mixed numbers and improper fractions, on a horizontal or vertical number line, and explain the strategies to determine position.**

Materials:

- Fraction Cards $\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{3}{8}, \frac{5}{8}, \frac{3}{16}, \frac{7}{16}, \frac{15}{16}, \frac{9}{16}, \frac{15}{24}, \frac{12}{32}, \frac{1}{1}, \text{ and } \frac{2}{1}\right)$
  BLM 6.N.4.1: Fractions (Fraction cards can be printed on card stock paper. Set D [Fractions] and Set I [Mixed Fractions] can be downloaded from the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/cur/math/my_games/index.html>.)
- Clothespins

Organization: Whole class/individual

Procedure:

1. Draw on the board a large horizontal number line from 0 to 2 or hang a clothesline across the front of the class.
2. Have students identify a few benchmarks by taping the cards on the board at $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \text{ and } 2,$ or have students clothespin the numbers on the clothesline at these points.
3. Tell students to draw in their notebooks a number line just like the one on the board, making sure they also identify the benchmarks.
4. Then, hand out a few fraction cards and have students place them on the board or clothesline in order, such as :
   $\frac{7}{8} \quad \frac{3}{8} \quad \frac{5}{8} \quad \frac{3}{16} \quad \frac{7}{16} \quad \frac{15}{16} \quad \frac{9}{16} \quad \frac{15}{24} \quad \frac{12}{32}$
   a) Have students rationalize their choice of position and check for accuracy.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use the number line to order fractions.
- ❒ Order fractions with like and unlike denominators.
- ❒ Explain the strategy used to determine the order.
- ❒ Relate improper fractions to mixed numbers.
- ❒ Place fractions, mixed numbers, and improper fractions on a number line.

- **Demonstrate using models that an improper fraction represents a number greater than 1.**

Materials:

- scissors
- envelopes
- small cards
- BLM 6.N.4.2 Fraction Circles or disk sectors (downloadable from the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/cur/math/my_games/110_disc_sectors.pdf.>

Organization: Five groups

Procedure:

1. Before the students enter, arrange the desks so you have five groups.
2. Place one copy of a different **Fraction Circles** sheet for each group, an envelope identifying the fraction, and a card inside the envelope stating five improper fractions appropriate for the particular fraction circles. For example:
   - Group A will have a copy of Fraction Circles $\left(\frac{1}{2}\text{s}\right)$, an envelope with the word "halves" written on it, and a card inside the envelope stating the following fractions:

     $\frac{3}{2}$ $\frac{5}{2}$ $\frac{7}{2}$ $\frac{9}{2}$ $\frac{11}{2}$

   - Group B will have a copy of Fraction Circles $\left(\frac{1}{3}\text{s}\right)$, an envelope with the word "thirds" written on it, and a card inside the envelope stating the following fractions:

     $\frac{4}{3}$ $\frac{5}{3}$ $\frac{10}{3}$ $\frac{11}{3}$ $\frac{14}{3}$

   - Group C will have a copy of Fraction Circles $\left(\frac{1}{4}\text{s}\right)$, an envelope with the word "quarters" written on it, and a card inside the envelope stating the following fractions:

     $\frac{7}{4}$ $\frac{9}{4}$ $\frac{11}{4}$ $\frac{15}{4}$ $\frac{19}{4}$

   - Group D will have a copy of Fraction Circles $\left(\frac{1}{5}\text{s}\right)$, an envelope with the word "fifths" written on it, and a card inside the envelope stating the following fractions:

     $\frac{6}{5}$ $\frac{9}{5}$ $\frac{12}{5}$ $\frac{19}{5}$ $\frac{21}{5}$

- Group E will have a copy of Fraction Circles $\left(\frac{1}{6}\text{s}\right)$, an envelope with the word "sixths" written on it, and a card inside the envelope stating the following fractions:

  $\frac{8}{6} \quad \frac{13}{6} \quad \frac{17}{6} \quad \frac{20}{6} \quad \frac{25}{6}$

3. Give each group a BLM sheet with fraction circles and an envelope with a card inside.
4. Take the sheet with the fraction circles and cut out each circle carefully. Then cut each circle into its marked fractional parts. Place the fractional parts into your marked envelope.
5. Distribute to each group a copy of the blank circles from BLM 6.N.4.2.
6. Tell each group to do the following:
   a) Take the card from the envelope.
   b) Use the fractional parts and the sheet with the circles to produce the stated improper fractions.
   c) Take turns making each improper fraction marked on the envelope.
   d) When you have completed all the stated improper fractions, exchange with another group your envelope of fractional parts and card.
   e) Repeat the process with the new set of fractional parts and card.
   f) Each group should reproduce the improper fractions stated on all five cards.
7. Check that each group
   - understands the task
   - uses all five envelopes
8. Discuss what students observed about improper fractions.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Demonstrate using models that an improper fraction represents a number greater than 1.

Suggestions for Instruction

- **Express improper fractions as mixed numbers.**

Materials:

- pencil
- scissors
- BLM 6.N.4.2: Fraction Circles
- BLM 6.N.4.3: Improper Fractions and Mixed Numbers

Organization: Pairs/individual

Procedure:

1. Distribute to each student a copy of the blank circles and the $\frac{1}{5}$s from BLM 6.N.4.2 and BLM 6.N.4.3.
2 Have students do the following:
   a) Take the sheet with the fifths
   b) Cut out each circle carefully.
   c) Cut each circle into its marked fractional parts.
3 When all the cutting is completed, have students do the following:
   a) Complete the worksheet in BLM 6.N.4.3.
   b) Use the fractional parts and the sheet containing blank circles from BLM 6.N.4.2 to help you complete each task as marked.
   c) Discuss with your partner your work on how to record mixed numbers.
   d) In the last four blank rows, have students share two improper fractions with a partner using fifths.
   e) Write both partners' improper fractions on the worksheet.
   f) Again, use the fractional parts and the sheet containing blank circles from BLM 6.N.4.2 to help you make a pictorial representation, and write an appropriate mixed number.
4 Circulate to check that students have the correct mixed number for each improper fraction.
5 Have students record in their journals what they learned about improper fractions and mixed numbers.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Use concrete objects to represent improper fractions.
- ❐ Make pictorial representations of improper fractions.
- ❐ Provide a correct mixed number for an improper fraction.

Suggestions for Instruction

- **Express mixed numbers as improper fractions.**

Materials:

- pencil
- BLM 6.N.4.4: State My Fraction

Organization: Small groups

Procedure:

1. Distribute to each student a copy of BLM 6.N.4.4.
2. Student directions:
   a) One member of your group will state a mixed number.
   b) Everyone will record it.
   c) Discuss with your group members what kind of pictorial representation you can have.
   d) Draw the pictorial representation.
   e) State the improper fraction.
   f) Have another member of your group state a mixed number.
   g) Repeat the process.
   h) Take turns until the entire sheet is filled.
3. Circulate to check that students understand the task and are stating the correct improper fraction for each mixed number.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ State a mixed number.
- ❒ Give a pictorial representation of a mixed number.
- ❒ Express mixed numbers as improper fractions.

Suggestions for Instruction

- **Express mixed numbers as improper fractions.**

Materials:

- pencil
- BLM 6.N.4.6: Vertical Number Line

Organization: Groups of three

Procedure:

1. Distribute to each student a copy of BLM 6.N.4.6.
2. Tell students they will play a game called "Proper or Improper," and state the rules of the game:
   a) Name some proper and some improper fractions.
   b) Make sure all the fractions fall between 0 and 8.
   c) Make sure all the fractions are thirds or sixths.
   d) Use a sheet of paper for keeping a record of correct scores.
   e) Roll a die to determine who starts.
   f) The one who rolls the smallest number will be the first caller and will start the game.
   g) The caller will state a fraction and mark a dot on his/her vertical number line, making sure the group partners do not see where the dot is placed.
   h) The student on the left of the caller will
      - place a dot on the appropriate spot on his or her vertical number line to mark the fraction
      - say whether the fraction is proper or improper
      - explain to the group the strategy he or she used to determine the position of the fraction

i) The caller and the student on the left of the caller will compare the position of their dots.

j) The student on the left of the caller will receive one point if the dots match or no points if the dots do not match.

k) Repeat the process.

l) The student on the left of the first caller will be the new caller. No one loses a turn.

m) Play 12 rounds.

n) The student with the highest score is the winner.

Variation: Use BLM 6.N.4.5 and play the game again but use only quarters and halves.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ State a fraction.
- ❐ Recognize whether a fraction is proper or improper.
- ❐ Place a set of fractions, including mixed numbers and improper fractions, on a vertical number line, and explain strategies used to determine position.

Grade 6: Number (6.N.5)

Enduring Understanding(s):

Ratios are not numbers; rather, they are comparisons of numbers or like items.

Percents, fractions, decimals, and ratios are different representations of the same quantity.

General Learning Outcome(s):

Develop number sense.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
6.N.5 Demonstrate an understanding of ratio, concretely, pictorially, and symbolically. [C, CN, PS, R, V]	➛ Provide a concrete or pictorial representation for a ratio. ➛ Write a ratio from a concrete or pictorial representation. ➛ Express a ratio in multiple forms, such as 3:5, $\frac{3}{5}$, or 3 to 5. ➛ Identify and describe ratios from real-life contexts and record them symbolically. ➛ Explain the part/whole and part/part ratios of a set (e.g., for a group of 3 girls and 5 boys, explain the ratios 3:5, 3:8, and 5:8). ➛ Solve a problem involving ratio.

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Demonstrating an understanding of division with and without concrete materials, and interpreting remainders to solve problems
- Demonstrating an understanding of fractions
- Demonstrating an understanding of decimals
- Relating decimals to fractions
- Comparing and ordering decimals

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of common factors and multiples
- Demonstrating an understanding of multiplication and division of decimals
- Demonstrating and explaining the meaning of preservation of equality

Background Information

This section of the Grade 6 Number Strand deals with ratios. Prior to teaching ratios, it would help students if they had a quick review of fractions. Although a ratio is not a fraction, it may be written in the form of a fraction.

Students may find ratios difficult to understand; therefore, it is imperative that students understand the similarities and differences between fractions and ratios.

A **fraction** is a number that represents part of a whole, part of a set, or a quotient in the form $\frac{a}{b}$, which can be read as a divided by b.

Ratio is a comparison of two numbers or two like quantities by division (e.g., the ratio of girls to boys is three to five $\left(\frac{3}{5}\right)$, 3 to 5, or 3:5).

Some ratios are comparisons of one part of a whole to another part of a whole. This is sometimes called a part-to-part ratio. For example:

- To make this recipe, you need 2 kg of white flour to every 3 kg of whole wheat flour.

In this example, the amount of white flour is compared to the whole wheat flour, and the ratio is 2:3.

- You buy 12 doughnuts: 5 chocolate and 7 glazed.

The ratio of chocolate to glazed is 5:7. But you could also compare the number of chocolate doughnuts to the total number of doughnuts. This ratio is 5:12, and is sometimes called a part-to-whole ratio.

When connecting fractions and ratios, it is important to maintain the meaning of the numerator and denominator. Students are likely more familiar with the fraction, meaning where the numerator is part of a whole (or a set). For example, $\frac{5}{12}$ of the doughnuts are chocolate. Thinking of the numerator as part of a part may be more difficult and confusing for students. For example, there are $\frac{7}{5}$ as many glazed doughnuts as chocolate.

Mathematical Language

part/part
part/whole
percent
ratio

Learning Experiences

Assessing Prior Knowledge

Materials:

- BLM 5–8.12: Fraction Bars (found in the complete package of *Grades 5 to 8 Blackline Masters*)

Organization: Small groups

Procedure:

1. Distribute to each student a copy of BLM 5-8.12, and tell them to
   a) discuss with their group members which fraction needs to be written in each space, and fill in each bar appropriately
   b) select one fraction and use it in a sentence that describes an everyday occurrence
2. Circulate to check that they are
   a) understanding the work
   b) recording the fractions correctly
3. Discuss fractions with the class.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understand the meaning of fractions.
- ❒ Record fractions correctly.
- ❒ Use a fraction to describe a real-world situation.

Suggestions for Instruction

- **Provide a concrete or pictorial representation for a ratio.**
- **Solve a problem involving ratio.**

Materials:

- pencil and paper
- candies or small blocks

Organization: Pairs/whole class

Procedure:

1. Present students with the following scenario:

   It is Halloween, and your little brother was too sick to go out trick or treating. You decided that you would collect candies, and for every four that you kept, you would give three to your little brother.

2. Ask students
   a) to model this scenario concretely
   b) to predict the number of candies that you and your little brother will each receive
   c) who will receive more candies, and how you know this
3. As a class, discuss students' answers to the questions and derive a meaning for ratio.

Suggestions for Instruction

- **Provide a concrete or pictorial representation for a ratio.**

Materials:

- Cube A Links

Organization: Whole class/pairs

Procedure:

1. Prepare 10 cubes linked together in this formation: G (green) Y (yellow)

   G G G G Y G G G G Y

2. Ask students what colours are described by the following ratios:
   a) 2:8
   b) 4:1
   c) 8:10

3. Hand out a 10-by-10 grid. Colour the grid using the pattern above. How will the ratios used above change?
4. Have students create their own 10-cube link using two colours. Using the model above, create three ratios reflected in your pattern, and ask a partner to identify the colours.
5. What will the same ratios be in a 10-by-10 grid using the new pattern?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Produce a concrete representation by demonstrate a ratio.
- ❒ Produce a pictorial representation for a ratio.

Suggestions for Instruction

- **Identify and describe ratios from real-life contexts and record them symbolically.**
- **Write a ratio from a concrete or pictorial representation for a ratio.**

Materials:
- pencil and paper
- 10 blocks or cubes (two different colours)
- brown paper bag

Organization: Pairs

Procedure:

Using the coloured blocks, each partner will have a chance to guess the ratio of coloured cubes in a paper bag.

Student Directions:

1. Have partner "A" come up to the front and select 10 blocks in any combination of two colours (e.g., four pink and six black, two white, and eight blue). Place the cubes secretively in the paper bag.
2. Partner "B" will draw one cube at a time from the paper bag, and will record the colour of the cube drawn out of the bag, replacing the cube each time. Partner "B" will do this 10 times. After 10 pulls, partner "B" will guess the ratio of coloured cubes by writing down the ratio of cubes he or she believes is in the bag.

3. See how close your ratio is to the real ratio of coloured cubes. Reverse roles and repeat.

Note: This activity can also be used for Outcome 6.SP.4: Demonstrate an understanding of probability.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Represent a ratio using concrete material.

Suggestions for Instruction

- **Express a ratio in multiple forms, such as 3:5, $\frac{3}{5}$, or 3 to 5.**

Materials:

- pencil
- BLM 6.N.5.1: Uncle Farley's Farm Animals
- BLM 6.N.5.2: Ratio Map for Uncle Farley's Farm Animals

Organization: Pairs

Procedure:

1. Distribute to each student BLM 6.N.5.1 and BLM 6.N.5.2.
2. Tell students to use the information in BLM 6.N.5.1 to make ratios using BLM 6.N.5.2.
3. Tell students to work in pairs to fill in BLM 6.N.5.2.
4. Circulate to check that students record the correct ratios.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Compare two items or numbers.
- ❒ Provide three different forms of representing a ratio.
- ❒ Understand the various forms that represent ratios.

- **Identify and describe ratios from real-life contexts and record them symbolically.**

Materials:

- pencil and paper
- Internet
- magazines

Organization: Small groups (3 to 4 students)

Procedure:

1. Have students seated in groups of three or four.
2. Tell students that you want each group to
   a) identify 10 ratios from real-life contexts
   b) describe the ratios they identified from real-life contexts
   c) record the ratios symbolically
3. Circulate to check that students are identifying real-life ratios, describing them, and recording them correctly.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify ratios from real-life contexts.
- ❒ Describe the ratios they identified from real-life contexts.
- ❒ Record the ratios symbolically.

- **Explain the part/whole and part/part ratios of a set (e.g., for a group of 3 girls and 5 boys, explain the ratios 3:5, 3:8, and 5:8).**

Materials:

- pencil

Organization: Whole class/small groups

Procedure:

1. Discuss part/whole and part/part ratios of a set. Ask students the following questions:
   a) How many girls are in this class?
   b) How many boys are in this class?
   c) How many students are in this class?
   d) What is the ratio of girls to boys?
   e) What is the ratio of boys to girls?
   f) What is the ratio of girls to the total number of students?
   g) What is the ratio of boys to the total number of students?
   h) What do you notice about the first two ratios?
   i) What do you notice about the last two ratios?
2. Distribute to each student a copy of BLM 6.N.5.1.
3. Tell students to write in their journals a few part/whole and part/part ratios of a set.
4. Circulate to check that students are providing correct examples of part/whole and part/part ratio.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Provide examples of part/whole and part/part ratios of a set.
- ❒ Explain the part/whole and part/part ratios of a set.

- **Solve a problem involving ratio.**

Materials:

- pencil
- BLM 6.N.5.3: Ratio Problems

Organization: Individual/small groups

Procedure:

1. Distribute to each student a copy of BLM 6.N.5.3.
2. Tell students the following:
   a) Work individually to complete each problem.
   b) Compare your responses to the responses of the other group members.
   c) Discuss your work, explaining what you did to solve each problem.
   d) Write in your journal what you learned about solving problems involving ratio.

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:
- ❐ Solve problems involving ratio.
- ❐ Use the correct ratio to solve a problem.
- ❐ Explain their solution.

- **Solve a problem involving ratio.**

Materials:

- pencil and paper
- BLM 6.N.5.4: Uncle Bert's Ratio Riddle

Organization: Individual

Procedure:

1. Place on an overhead projector a copy of BLM 6.N.5.4.
2. Say the following to the students:
   a) Danny wants to find out how far is the nearest store. Can you help him?
   b) Explain how you are going to solve the problem.
   c) Write down your solution.
   d) Explain why you chose to solve it that way.
3. Circulate to check that students are using the correct ratios.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Explain how they are going to solve a problem.
- ❒ Use the correct ratio to solve a problem.
- ❒ Write down the correct solution.
- ❒ Explain why they chose a certain way to solve a problem involving ratio.

Grade 6: Number (6.N.6)

Enduring Understanding(s):

Percents can be thought of as a ratio comparing to 100 or as a fraction out of 100.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
6.N.6 Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially, and symbolically. [C, CN, PS, R, V]	➛ Explain that "percent" means "out of 100." ➛ Explain that percent is the ratio of a certain number of units to 100 units. ➛ Use concrete materials and pictorial representations to illustrate a percent. ➛ Record the percent displayed in a concrete or pictorial representation. ➛ Express a percent as a fraction and a decimal. ➛ Identify and describe percents from real-life contexts and record them symbolically. ➛ Solve a problem involving percents.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of division with and without concrete materials, and interpreting remainders to solve problems
- Demonstrating an understanding of fractions
- Demonstrating an understanding of decimals
- Relating decimals to fractions
- Demonstrating an understanding of division

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of ratio

Background Information

Percent (%) is a number expressed in relation to 100, represented by the symbol % (e.g., 40 parts out of 100 is 40%).

Percent can be expressed as a ratio, fraction, or decimal. The above example, 40%, when expressed as a ratio, is 40:100; when expressed as a fraction, it is $\frac{40}{100}$, and when expressed as a decimal, it is 0.40 (or 0.4).

The tasks are designed to: (a) develop students' understanding of percents, and (b) have students relate percents to real-life contexts.

Mathematical Language

decimal

fraction

percent

ratio

Assessing Prior Knowledge

Materials:

- BLM 6.N.6.1: What Is My Equal?

Organization: Pairs

Procedure:

1. Tell students you want to check what they remember about fractions and decimals.
2. Have students complete BLM 6.N.6.1.
3. Have students discuss their results with their partner.
4. Circulate among the students to check that they know how to convert decimals to fractions and fractions to decimals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Convert decimals (hundredths) to fractions.
- ❒ Convert fractions $\left(\frac{x}{100}\right)$ to decimals.

Suggestions for Instruction

- **Explain that "percent" means "out of 100."**

Materials:

- pencil
- notebook
- BLM 6.N.6.2: Gizzy Saw These Birds

Organization: Small groups

Procedure:

1. Display BLM 6.N.6.2 on an overhead projector or interactive whiteboard.
2. Have students read the information on the BLM and discuss with their group members what Gizzy means when she says that
   a) 25% of the birds she saw were yellow
   b) 42% were black
   c) 10% were blue
   d) 15% were white
   e) 8% were red
3. Circulate among the students to check that they understand the meaning of percent.
4. Tell students to explain in their notebooks that "percent" means "out of 100."

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Relate examples stated in % form and numbers out of 100.
- ❐ Explain orally and in writing that "percent" means "out of 100."

Suggestions for Instruction

- **Explain that percent is the ratio of a certain number of units to 100 units.**

Organization: Whole class

Procedure:

1. Discuss ratios and percents.
2. Have students do the following:
   a) Provide examples of percents in real-life contexts.
   b) Write on the board the examples as provided.
   c) Write the example as a ratio of a certain number to 100 units.
3. Discuss what students noticed about the percent written as a ratio of a certain number to 100 units.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Provide an example of ratio.
- ❒ Provide an example of percent.
- ❒ Explain that percent is the ratio of a certain number of units to 100 units.

Suggestions for Instruction

- **Explain that percent is the ratio of a certain number of units to 100 units.**

Materials:

- scissors
- BLM 6.N.6.3: My Ratio Is... Who Has?

Organization: Two groups

Procedure:

1. Distribute to each group a copy of the two sheets of BLM 6.N.6.3.
   a) Students cut on the lines to create a set of 24 cards.
   b) Students then shuffle the cards and place them upside down on the desk.
   c) The shortest person in the group will be the dealer.
   d) The dealer distributes all the cards, one at a time, to each group member. Depending on the size of the group, some group members may have more cards than others.
   e) The person who was dealt the first card will start the game by reading his or her card (one of the cards) to the group.
   f) The person who has the match will read his or her match to the first question, and then read the next question stated on his or her card.
   g) The person who has the match to the second question will state that match and continue the game.
   h) The game is finished when all the cards are read. If done correctly, each card will be read once and the answer to the last card will be the statement on the first card.

2. Circulate to check that students are calling out the correct replies.
3. Discuss what students observed about percents and the ratio of a certain number of units to 100 units.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Match correctly a ratio and a percent.
- ❒ Explain that percent is the ratio of a certain number of units to 100 units.

Suggestions for Instruction

- **Use concrete materials and pictorial representations to illustrate a percent.**
- **Explain that percent is a ratio of a certain number of units to 100 units.**

Materials:

- pencil crayons
- BLM 6.N.6.4: 100-Square Grid Paper (or use a geoboard with different coloured elastics)

Organization: Individual

Procedure:

1. Distribute to each student a copy of BLM 6.N.6.4.
2. Tell students to colour
   a) 45% of the grid yellow
   b) 30% of the grid red
   c) 10% of the grid blue
   d) 15% of the grid black
3. Ask students what the ratio is for each coloured section, and include this in a legend or code linking colours of the grid to % and ratios.
4. Ask students this extension question: Shannon colours a shape on a 10-by-10 grid and says the ratio of her coloured part to the whole square is 3:5. On a blank 10-by-10 grid paper square, record three shapes that could represent this ratio. What percent is this shape of the entire grid?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use grid paper to represent percent.
- ❒ Illustrate percent correctly.
- ❒ Link ratios to percent.

Suggestions for Instruction

- **Use concrete materials and pictorial representations to illustrate a percent.**
- **Explain that percent is a ratio of a certain number of units to 100 units.**

Materials:

- rice (white and brown)

Organization: Pairs

Procedure:

1. Tell students:
   a) Bring 100 grains of rice to school. Make sure 30% of the grains are white and 70% are brown.
   b) If each grain of rice corresponds to one day, how many days are represented by the 100 grains of rice?
   c) If each white grain represents snowy days, how many snowy days are represented?
   d) If each brown grain of rice represents slushy days, how many slushy days are represented?
   e) If 50% of the days were windy, use your grains of rice to show one possibility.
   f) Describe your solution in your notebook.
   g) Are there other possibilities? Explain to your partner.
2. Circulate among the students to check that they understand the process.
3. Discuss what students observed about a percent. Extension: Read *The King's Chessboard* by David Birch.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use grains of rice to represent percent.
- ❒ Illustrate percent correctly.

Suggestions for Instruction

- **Record the percent displayed in a concrete or pictorial representation.**

Materials:

- pencil
- pencil crayons
- BLM 6.N.6.5: Percent Grids (Full Sheet)

Organization: Pairs

Procedure:

1. Distribute to each student a copy of BLM 6.N.6.5.
2. Tell students the following:
   a) They will be working in pairs.
   b) Each student will shade in an area of each grid to represent a different percent (e.g., the first grid may have 24% of the squares shaded, the second grid may have 90% of the squares shaded, and the third grid may have 51% of the squares shaded).
   c) Each student will shade the grids in the first row representing any percent of squares shaded that the student chooses. The three grids in a row should represent different percents.
   d) They will exchange papers with their partners.
   e) They will record the percent displayed above each grid.
   f) Each will then choose a different partner.
   g) They will repeat the process with their new partner by shading the grids in the second row. There should be no repeats of percents in any grid.
   h) Upon completing the process, they will choose a third partner.
   i) They will repeat the process with their third partner by shading the grids in the third row.

j) After completing the process, they will choose a fourth partner.

k) They will repeat the process with their fourth partner and by shading the grids in the fourth row.

3. Circulate to check that students are doing their task.
4. Have students record in their journals what they learned about displaying and recording a percent.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Create a visual representation of a percent.
- ❐ Record the percent displayed in a pictorial representation.

Suggestions for Instruction

- **Express a percent as a fraction and a decimal.**

Materials:

- pencil
- BLM 6.N.6.6: Percent, Fraction, and Decimal Sheet

Organization: Small groups

Procedure:

1. Distribute to each student BLM 6.N.6.6. Give students the following instructions:
   a) Have one student in your group call out a "percent."
   b) Have every student in the group write it down, including the person who called it.
   c) Have every student write down the fraction equivalent and the decimal equivalent.
   d) Repeat the process by having another student call out a "percent."
   e) Take turns calling out a "percent" until you have all 20 lines filled.
   f) Discuss your results.
   g) Correct your errors.
2. Circulate to check that students understand the intent of the task.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Express a percent as a fraction.
- ❒ Express a percent as a decimal.

Suggestions for Instruction

- **Express a percent as a fraction and a decimal.**

Materials:

- scissors
- numbered cubes
- BLM 6.N.6.7: Say My Equal Fraction, Say My Equal Decimal (both sheets)

Organization: Groups of six

Procedure:

1. Tell students the following instructions:
   a) Each group will receive two sheets of cards.
   b) Students cut the sheets on the lines to create a set of 24 cards.
   c) Students then shuffle the cards and place the set upside down in the centre of the desk.
   d) Each student will roll a numbered cube.
   e) The one with the smallest number starts.
   f) This student picks up the top card and reads it to the group.
   g) This student gives the card to the student on his or her left, who says the reply.
   h) If the reply is correct, the student puts it at the bottom of the pile and picks up a new card. Then the process repeats with the new person reading the card.
   i) If the reply is incorrect, the student who said the reply keeps the card and loses his or her turn to read a question. The student to his or her left picks up a card and reads it. Then the process repeats.
2. Circulate to check that the students understand the process.
3. Discuss what they observed about percents, fractions, and decimals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Express a percent as a fraction.
- ❒ Express a percent as a decimal.

Suggestions for Instruction

- **Identify and describe percents from real-life contexts and record them symbolically.**

Materials:

- newspapers
- paper and pencil

Organization: Small groups

Procedure:

1. Distribute a newspaper section to each group.
2. Provide students with the following instructions:
   a) Look through your newspaper.
   b) Identify percents that were taken from real-life contexts.
   c) Record the percents symbolically.
   d) Take turns identifying and describing other percents from real-life contexts not found in your newspaper. Share with the class.
3. Tell students to describe in their journals the percents from real-life contexts that the group members identified, and record them symbolically.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify percents from real-life contexts.
- ❒ Describe percents from real-life contexts.
- ❒ Record symbolically the percents they identified and described from real-life contexts.

Suggestions for Instruction

- **Solve a problem involving percents.**

Materials:

- pencil
- pencil crayons
- BLM 6.N.6.8: Percent Grids
- BLM 6.N.6.9: Grandpa's Berry Bushes

Organization: Individual

Procedure:

1. Display BLM 6.N.6.9.
2. Distribute to each student a copy of BLM 6.N.6.8.
3. Tell students to do the following:
   a) Work individually on this task.
   b) Read the information Grandpa provided.
   c) Discuss what each of those percents mean.
   d) Use your pencil crayons to create three different designs for Grandpa's berry bushes, using the percents Grandpa provided for you.
   e) Figure out, if Grandpa planted 50 bushes this year, how many would be
      i. raspberry?
      ii. gooseberry?
      iii. blackberry?
      iv. red current?

4. Circulate among the students to check that they are colouring the correct percentage for each type of bushes and performing the correct calculations

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ State the meaning of a given percent.
- ❒ Solve a problem involving percents.
- ❒ Create different designs involving percents.

Putting the Pieces Together

Plan a School Party

Purpose: The purpose of this activity is to have students connect and apply their skills learned in the mathematics classroom to a real-life event. Students will need to apply their knowledge of fractions (proper and improper), ratios, and percents (N.4-6). They will also need to collect data, design a questionnaire, select a type of graph appropriate for displaying the data, and graph the data (SP.2&3). To complete this task, students will have to rely on some prior knowledge, such as comparison of numbers and knowledge of multiplication facts. The processes that are demonstrated by this task are communication, connection, mental mathematics and estimation, and problem solving. To make this task enjoyable, students will have to cooperate with their group members and with the students they survey.

Curricular Links: This task can be linked to health and social studies.

Materials/Resources:

- paper and pencil
- poster-sized paper

Students will need to find out what kind of food and drinks most students would select in order to create an afternoon of fun or a good school party

Organization:

- Put desks together for group work.
- Split class into two groups.

Inquiry:

Scenario

Students will be working in two groups in their classroom as they design a survey questionnaire. Each of the four groups will be responsible for designing a specific section of the questionnaire (food and drinks). 100 copies of the survey questionnaire will be made, and the completed questionnaires will be analyzed and the results will be displayed.

Procedure

This task will require a few days.

Teacher:

i) Divide the class into two groups. One group will work on the food items and the other group will work on the drink items.

ii) Hand out a copy of the instruction sheet to each group.

Students' Directions:

Day 1: Let's design a survey questionnaire!

- Each group (food and drinks) working separately:
  a) Brainstorm and list items for your section of the survey questionnaire.
  b) Discuss which items your peers would most likely be interested to choose from.
  c) Choose seven items your peers would most likely be interested to choose from.
  d) Design a questionnaire similar to the one provided here.

Items	My Favourite One	I Also Like
1		
2		
3		
4		
5		
6		
7		

  e) List the seven chosen items under the "Items" column.
  f) Type up the questionnaire on the computer.
  g) Print 100 copies.

Day 2: Let's design a food and drinks chart!

- Make a specific chart for your group, and then do the following:
  a) List each of the seven items on the survey questionnaire.
  b) Put next to each item the quantity in which the item can be purchased (e.g., sets of 2, sets of 3, a dozen).
  c) Have a column for stating the number of students who chose the item as their favourite.
  d) Have a column for stating the fraction (the number of students who chose the item as their favourite/the quantity in which the item can be purchased).
  e) Have a column for stating the mixed number (the number of sets of 2s or dozens or whatever quantity in which the item can be purchased).

Sample Chart:

Item	**Set in Which Item Can Be Purchased**	**Number of Favourites**	**Fraction**	**Mixed Number**
1				
2				
3				
4				
5				
6				
7				

Day 3: Let's do the survey!

- Work in pairs (one member of the food group and one member of the drink group).
- Each pair will
  a) visit a designated class
  b) explain the purpose of the task
  c) ask for volunteers to fill out the survey questionnaires
  d) hand out the questionnaires to the volunteers (Each student volunteer will fill out a food and a drink questionnaire.)
  e) collect all survey questionnaires
  f) take all survey questionnaires back to the classroom

Day 4: Let's record the data!

- Separate the food survey questionnaires from the drink survey questionnaires.
- Work with members of your own group:
  a) Check the replies for each questionnaire.
  b) Record how many times each item was chosen as
    i) favourite
    ii) also liked
  c) Make a chart similar to the one below (like the questionnaire) on a poster-sized paper and fill in the data.

Items	Total Number For ″My Favourite One″	Total Number For "I Also Like"
1		
2		
3		
4		
5		
6		
7		

d) Place the completed poster on the board.

e) Use the data on the poster to calculate the following for each item:

i) The % of students who chose it as "My favourite."

ii) The % of students who chose it as "I also like it."

iii) The ratio of "My favourite" to "I also like it."

Day 5: Let's display the results!

- Display each group (food and drinks) separately:

a) Use the data to fill in the five-column chart made on Day 2.

b) Estimate, using the last column of the five-column chart, how many sets of each item you would need to purchase if you had a party and wanted to serve the most favourite foods and drinks (e.g., How many sets of 2s of an item, or how many dozens of another item).

c) Discuss what is the best graph to display the "My favourite" items.

d) Create a graph to display the "My favourite" items.

Literature Link

Read: *The Grizzly Gazette* by Stuart J. Murphy and illustrated by Steve Bjorkman. This is a story about a summer camp where children have a club, a newsletter, a parade, and where everyone can vote for a mascot. It includes an interesting investigation into percentage as the vote goes on.

Assessment:

Use the following observation checklist to assess students' learning.

The student can do the following:	Yes	No	Comment
Design a questionnaire.			
Use fractions (proper and improper) correctly.			
Use percents.			
Use mixed numbers.			
Use ratios.			
Select an appropriate graph to display data.			
Design a graph.			
Correctly estimate quantities.			
Effectively communicate with peers.			
Cooperate with their peers.			

Extension:

Taking it further

Based on the ideas in this task or the ideas in *The Grizzly Gazette*, create your own story using percent, ratio, improper fractions, or mixed numbers.

Proportions and Giants

Purpose: The purpose of this activity is to have students connect and apply the skills they learned in the mathematics classroom to a real-life event. Students will need to apply their knowledge of fractions (proper and improper), ratios, and percents (N.4–6). They will also need to collect data, design a questionnaire, select a type of graph appropriate for displaying the data, and graph the data (SP.2&3). To complete this task, students will have to rely on some prior knowledge, such as comparing numbers and knowledge of multiplication facts. The processes that are demonstrated by this task are communication, connection, mental mathematics and estimation, and problem solving. To make this task enjoyable, students will have to cooperate with their group members and with the students they survey.

Curricular Links: This task can be linked to health, social studies, and English language arts.

Materials/Resources:

- *Jim and the Beanstalk* by Raymond Briggs
- Measurement tools, such as tape measures, rulers, metre sticks
- Calculators

Organization:

- Put desks together for group work.
- Split class into two groups.

Inquiry Procedure:

- Read *Jim and the Beanstalk*.
- Students will be working in groups to determine ratios and measurements. Question: At one point in the story, the giant says that his favourite food is "Three fried boys on toast." Using this fact, determine the size of the giant.
- Have students determine the steps they would need to take to accomplish this task.

Suggested Procedure

1. Have three boys lie on paper or on the ground. Draw a large piece of toast around their bodies or use tape to outline the size of the toast.
2. Have students measure this large toast and then a regular-sized piece of toast to get the correct ratio.
3. Using this ratio, measure an average child in class and determine how big the giant would be.

Extensions

Have students outline the giant in the classroom to visualize his size.

Have students suggest other literature that includes giants or little people to determine sizes and proportions. Compare them to Jim's giant. Make a classroom chart or make outlines of each character for the wall. Use visuals and descriptors of people and objects in each story for the ratios.

Examples

Haggar in *Harry Potter* by J.K. Rowling

The Hobbits in *The Lord of the Rings* by J.R.R. Tolkien

The Lilliputians in *Gulliver's Travels* by Jonathan Swift

The BFG by Roald Dahl

Spiderwick: A Giant Problem by Tony DiTerlizzi

The children in *Honey, I Shrunk the Kids* by Nancy Krulik

The Borrowers by Mary Norton

Also:

Jim spends a lot of time measuring the giant. Measure the same body parts on your own body. What is the ratio of your head circumference to the giant's? Is this same ratio consistent throughout? What is the ratio of your arm to your leg? Your head circumference to your wrist? Are these ratios the same for the students in your class?

Grade 6: Number (6.N.7)

Enduring Understanding(s):

Numbers can be positive or negative. Positive numbers are greater than zero. Negative numbers are less than zero.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
6.N.7 Demonstrate an understanding of integers, concretely, pictorially, and symbolically. [C, CN, R, V]	➛ Extend a horizontal or vertical number line by adding numbers less than zero and explain the pattern on each side of zero. ➛ Place a set of integers on a horizontal or vertical number line and explain how integers are ordered. ➛ Describe contexts in which integers are used (e.g., on a thermometer). ➛ Compare two integers, represent their relationship using the symbols <, >, and =, and verify using a horizontal or vertical number line. ➛ Order a set of integers in ascending or descending order.

Prior Knowledge

Students may have had experience with the following:

- Representing and describing whole numbers to 1 000 000
- Comparing and ordering numbers to 10 000
- Demonstrating an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals)

Background Information

Numbers, whether large or small, can be positive or negative.

Positive number is any number greater than 0, located to the right of 0 on a horizontal number line or above 0 on a vertical number line.

Negative number is a number that is less than 0, located to the left of 0 on a horizontal number line or below 0 on a vertical number line.

Not all numbers are large or small, positive or negative. The number that indicates no quantity, size, or magnitude is called zero. Zero is neither negative nor positive; zero is the additive identity. The zero is called the additive identity because, when any number and zero are added, the sum will be the same number (e.g., 8 + 0 = 8, and [–5] + 0 = [–5]).

Numbers can be grouped in sets. A **set** is any collection of things, without regard to their order. The members (or elements) of a set could be numbers, names, shapes, and so on.

The set of numbers consisting of the whole numbers (e.g., 1, 2, 3, 4, . . .), their opposites (e.g., –1, –2, –3, –4, . . .), and 0 is called integers.

The concept of integers is often difficult for students to understand, so we often use a number line when we introduce integers.

A **number line** is a line (vertical or horizontal) on which each point represents a number (for an example, see BLM 6.N.4.4).

It is a good idea to stress the use of correct terminology (i.e., when using integers, it is important that students say "negative" five or "negative" two rather than "minus" five or two).

It might be helpful to students if the teacher pointed out that

a) zero is neither positive nor negative and is always in the middle of the number line (horizontal or vertical)
b) on a horizontal number line, the positive numbers are always on the right-hand side and the negative numbers are always on the left-hand side of zero
c) on a vertical number line, the positive numbers are always above and the negative numbers are always below zero
d) the numbers immediately next to zero are 1 and –1
e) the larger the number is (positive or negative), the further it is located from zero.

Students also need to know, and be able to use correctly, the relational symbols: =, >, and <.

- = means both “equal to” and “balanced with” and is used to show that the left and the right side of a mathematical statement are equal, such as: 6 = 6, (–7) = (–7), (–9) + 9 = 0, 18 + 0 = 18, 23 – 9 = 16 – 2, 56 = 8 × 7, and 54 ÷ 6 = 9.
- > means “greater than” and is used to show that the left side of a mathematical statement is greater than the right side, such as: 296 > 184, 2 > 0, 0 > (–8), and (–2) > (–95) (conversely, the right-hand side is less than the left-hand side).

  296 + 100 > 184

  175 – 171 > –4

  16 + 18 > 15 + 18

  95 > 9 × 10
- < means “less than” and is used to show that the left side of a mathematical statement is less than the right side, such as: 28 < 31, 0 < 7, (–8) < 0, and (–836) < (–5) (conversely, the right-hand side is greater than the left-hand side).

  8 x 7 < 60

  21 – 18 < 18 + 21

  4 + 7 < 5 × 3

Mathematical Language

integer

negative number

number line

positive number

set

zero

Assessing Prior Knowledge

Organization: Whole class

Procedure:

1. Tell students they will be learning about integers, but first you want to check what they remember about the number line.
2. Draw a number line in the positive direction only.
3. Mark 0 and 10 on the number line. Mark every whole number increment between 0 and 10, but do not write the numerals under the markings.
4. Ask one student at a time to come to the board and do the following (keeping in mind that the number line is not limited to the numbers or increments marked on it):
   a) Place 5 on your number line.
   b) Place a number on the number line that is less than 5.
   c) Place 8 on your number line.
   d) Place 12 on your number line.
   e) Place a number on the number line that is less than 10.
   f) Place a number on the number line that is greater than 2.
   g) Place 7 on your number line.
   h) Place 14 on your number line.
5. Discuss with students how they decided where to place each numeral. Since 12 and 14 do not fall between 0 and 10, where would you place them?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❐ Place a positive number on a number line
- ❐ Determine if a number is less than or greater than another number.

Suggestions for Instruction

- **Extend a horizontal or vertical number line by adding numbers less than zero and explain the pattern on each side of zero.**

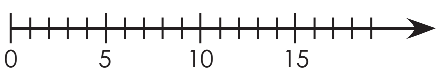
Materials:

- pencil and paper

Organization: Whole class/individual

Procedure:

1. Draw a horizontal number line on the board.
2. Place 0 in the middle of the number line.
3. Show notches for each increment to the right of zero (i.e., in the positive direction only). (Make all increments be of equal size.)
4. Mark every fifth increment (i.e., 5, 10, 15), like in the example below.



5. Write the following numerals on the board:
   a) 2
   b) 6
   c) 9
   d) 11
   e) 13
6. Ask a different student volunteer to come to the board each time (for a, b, c, d, and e) to place a dot in the correct spot on the number line, and write the numeral next to the dot.
7. Ask students what they noticed about the pattern on the right side of zero.
8. Show notches for each increment to the left of zero.
9. Mark every fifth increment, but leave out the negative sign for the sake of discussion (i.e., mark them incorrectly as 5, 10, and 15).
10. Have a discussion on this "new" number line that extends in two directions (i.e., to both sides of zero). (How would they know whether to place a dot on the right side or left side of zero?)
11. After the discussion, tell students: "Mathematicians solved this problem by putting a negative sign in front of each numeral on the left-hand side of zero (like this: –5, –10, –15)." Place a negative sign in front of the increments on the left-hand side of zero incorrectly marked as 5, 10, 15.

12. Write the following numerals on the board:
    a) –1
    b) 3
    c) –4
    d) –7
    e) 8
    f) –12
    g) –14
13. Have a discussion on which side of the zero they would place these numerals, and why.
14. Ask students to explain in their journal the pattern on the
    a) right-hand side of zero
    b) left-hand side of zero

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Explain the pattern on the right-hand side of zero.
- ❒ Explain the pattern on the left-hand side of zero.

Suggestions for Instruction

- **Place a set of integers on a horizontal or vertical number line and explain how integers are ordered.**

Materials:

- pencil
- BLM 5–8.19: Number Line

Organization: Individual/pairs

Procedure:

1. Distribute to each student a copy of BLM 5-8.19.
2 Write on the board: 2, –3, –6, 7, –11, 14.
3 Tell students to
   a) work individually
   b) label their number line (either vertically or horizontally)
   c) mark a dot for each numeral in the correct spot on the number line
   d) place the numeral next to each dot
4 Circulate to check that students are placing the dots in the correct spots on the number line.
5 Tell students to do the next part with a partner.
6 Provide students with the following instructions:
   a) One student will mark one dot on each side of zero.
   b) Tell the numeral to your partner.
   c) Compare your results.
   d) Repeat the game with the other student marking one dot on each side of zero.
   e) Do four rounds.
7 Discuss with your partner how integers are ordered, and write it in your journals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Place integers correctly on a vertical number line.
- ❒ Explain how integers are ordered.

Suggestions for Instruction

- **Place a set of integers on a horizontal or vertical number line and explain how integers are ordered.**

Materials:

- pencil
- 6-sided die
- BLM 5–8.19: Number Line

Organization: Groups of three/whole class

Procedure:

1. Distribute to each student a copy of BLM 5-8.19.
2 Distribute to each group a 6-sided die.
3 Provide students with the following instructions:
   a) Each member of the group roles the die once.
   b) The one with the lowest number starts the game and will be the caller.
   c) The caller will place a dot on his/her number line and say the numeral.
   d) The other two group members will place a dot in the correct spot on their number line and mark the numeral next to it.
   e) The two will compare their work with the caller.
   f) Repeat. The student to the left of the caller will be the next caller.
   g) Repeat until everyone has had four turns.
4 Circulate to check that students are placing the dots in the correct spots on the number line.
5 Discuss with the whole class how integers are ordered.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Place integers correctly on a vertical number line.
- ❒ Explain how integers are ordered.

- **Describe contexts in which integers are used (e.g., on a thermometer).**

Materials:

- pencil
- BLM 5–8.19: Number Line

Organization: Whole class

Procedure:

1. Place BLM 5-8.19 on the overhead projector or an interactive whiteboard, and label the appropriate increments.
2 Ask for individual volunteers to place on the number line dots representing –16, –8, –4, –1, 2, 6, and 11.
3 Ask students:
   a) What does the vertical number line remind you of? Why?
   b) How is a thermometer similar to a vertical number line?
   c) Where do you think integers are used?
   d) Describe contexts in which integers are used.
4 Have students use their journals to describe contexts in which integers are used.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Place a dot correctly on a vertical number line.
- ❒ Explain how integers are used.
- ❒ Describe contexts in which integers are used.

Suggestions for Instruction

- **Compare two integers, represent their relationship using the symbols <, >, and =, and verify using a horizontal or vertical number line.**

Materials:

- pencil
- BLM 6.N.7.1: Integers

Organization: Small groups

Procedure:

1. Place on the overhead projector a transparency copy of BLM 6.N.7.1 or display it electronically.
2 Provide students with the following instructions:
   a) Compare each set of integers.
   b) Represent the relationship of each set of integers using the symbols <, >, and =.
   c) Draw a number line.
   d) Verify the relationship of each set of integers using a number line.
3 Discuss your results with your group members.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Compare two integers.
- ❒ Represent the relationship of two integers using the symbols <, >, and =.
- ❒ Draw a number line.
- ❒ Verify the relationship of two integers using a number line.

Suggestions for Instruction

- **Compare two integers, represent their relationship using the symbols <, >, and =, and verify using a horizontal or vertical number line.**

Materials:

- pencil
- BLM 5–8.19: Number Line
- BLM 6.N.7.2: Compare Integers

Organization: Groups of four

Procedure:

1. Distribute to each group a 6-sided die, a copy of BLM 5-8.19, and a copy of BLM 6.N.7.2.
2 Provide students with the following instructions:
   a) The object of the game is to tell if integer A <, >, or = integer B.
   b) Let every member of the group roll a die to see who will start the game.
   c) The person with the smallest number starts the game. He or she will choose integer A, record it on the Compare Integers sheet, and state integer A to the group.
   d) The person next to the game starter will choose integer B, record it on the Compare Integers sheet, and state integer B to the group.
   e) The next left person will compare the two integers, on the Compare Integers sheet represent the relationship using the symbols <, >, or =; and state to the group whether integer A is <, >, or = integer B.
   f) The fourth person will plot a dot for each integer on the number line to verify the relationship.
   g) For the next round, let the person who called integer B start the game.
   h) Repeat the game until your Compare Integers sheet is filled, making sure that the starter of the game for each new round is the person who called integer B.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Compare two integers.
- ❒ Represent the relationship of two integers using the symbols <, >, and =.
- ❒ Use a number line.
- ❒ Verify the relationship of two integers using a number line.

Suggestions for Instruction

- **Order a set of integers in ascending or descending order.**

Materials:

- pencil and paper

Organization: Whole class/small groups

Procedure:

1. Provide each group with the following instructions:
   a) Choose 12 integers.
   b) Write each one down as you are choosing it.
   c) Decide how you want to order them (ascending or descending order).
   d) Order your integers
2. Circulate to make sure students choose negative numbers as well as positive.
3. Have one member of each group write the 12 chosen numerals on the board in the order the group chose.
4. Discuss with the class the different sets of integers and their order.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Choose integers.
- ❒ Write numerals for each integer.
- ❒ Order a set of integers in ascending or descending order.

Putting the Pieces Together

How Cold Is It?

Purpose: The purpose of this activity is for students to connect integers to real life. Students will need to apply their knowledge of integers and data collection and analysis. Students will need some prior knowledge such as comparing numbers. The processes that are demonstrated by this task are communication and connection. Since the purpose of this activity is to work with integers, have students select a Canadian city that has positive and negative temperatures. Doing this activity between November and March might ensure more variability in temperature.

Curricular Links: This task can be linked to science and social studies.

Materials/Resources:

- paper and pencil

Students will need to compare temperature in different cities.

Organization:

- Small groups

Inquiry:

Scenario

Students will be working in small groups. Each group will choose a city and follow the temperature changes during a five-day period. They will record the temperature each morning and afternoon at as close to the same time of day as possible (for example, 8:00 a.m. and 3:00 p.m. or midnight and noon each day).

Procedure

Teacher:

1. Divide the students into small groups.
2. Distribute the directions to each group.
3. Tell students to follow the directions.

Students:

1. Each group needs to choose a Canadian city and monitor its daily temperature.
2. Watch the weather station on a television set to monitor the temperature or go to the StatsCanada website at <http://climate.weather.gc.ca/>.

3. Make a chart like the one provided here:

Day	Morning Temperature	Afternoon Temperature
Monday		
Tuesday		
Wednesday		
Thursday		
Friday		

4. Record the temperature of your chosen city on the chart twice a day for five days in a row: in the morning and in the afternoon.
5. Make three number lines:
   - morning temperature
   - afternoon temperature
   - all temperatures
6. Compare the five morning temperatures and write up your observations.
7. Compare the five afternoon temperatures and write up your observations.
8. Get together with another group.
9. Compare your three number lines with the three number lines of the other group.
10. Write up your observations. What were the differences between the morning and afternoon temperatures each day?

Assessment:

Use the following observation checklist to assess students' learning.

The student can do the following:	Yes	No	Comment
Design a number line.			
Use a number line correctly.			
Use integers.			
Compare integers.			
Record data.			
Analyze data.			

Extension:

Record the daily high and low temperature for several days. Have the whole class do it together. You can use the daily paper for your information. Then:

- Ask students to predict the daily high and low temperature for the next two days based on their data.
- Ask them to compare their predictions with the actual temperatures.

Literature link:

The daily newspaper is an excellent source for data collection. The *Winnipeg Free Press* has a weather report where students can check for information, such as the daily temperature, the UV index, sunrise, sunset, moonrise, and moonset.

Grade 6: Number (6.N.8)

Enduring Understanding(s):

The position of a digit in a number determines its value.

Each place value position is 10 times greater than the place value position to its right.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
6.N.8 Demonstrate an understanding of multiplication and division of decimals (involving 1-digit whole-number multipliers, 1-digit natural number divisors, and multipliers and divisors that are multiples of 10), concretely, pictorially, and symbolically, by ▪ using personal strategies ▪ using the standard algorithms ▪ using estimation ▪ solving problems [C, CN, ME, PS, R, V]	➛ Estimate a product using front-end estimation (e.g., for 15.205 m × 4, think 15 m × 4, so the product is greater than 60 m), and place the decimal in the appropriate place. ➛ Estimate a quotient using front-end estimation (e.g., for $26.83 ÷ 4, think 24 ÷ 4, so the quotient is greater than $6), and place the decimal in the appropriate place. ➛ Predict products and quotients of decimals using estimation strategies. ➛ Identify and correct errors of decimal point placement in a product or quotient by estimating. ➛ Solve a problem that involves multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9. ➛ Use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10 (e.g., $2.47 \times 10 = 24.7$; $31.9 \div 100 = 0.319$). ➛ Model and explain the relationship that exists among an algorithm, place value, and number properties. *continued*

Specific Learning Outcome(s):	Achievement Indicators:
	➛ Determine products and quotients using the standard algorithms of vertical multiplication (numbers arranged vertically and multiplied using single digits which are added to form a final product) and long division (the multiples of the divisor are subtracted from the dividend). ➛ Solve multiplication and division problems in context using personal strategies, and record the process. ➛ Refine personal strategies, such as mental math, to increase their efficiency when appropriate (e.g., 4.46 ÷ 2 think 446 ÷ 2 = 223, and then use front-end estimation to determine the placement of the decimal 2.23).

Prior Knowledge

Students may have had experience with the following:

- Applying estimation strategies in problem-solving situations
- Determining multiplication facts (to 81) and related division facts
- Applying mental mathematics strategies for multiplication
- Demonstrating an understanding of multiplication to solve problems
- Demonstrating an understanding of division with and without concrete materials, and interpreting remainders to solve problems
- Describing and representing decimals
- Comparing and ordering decimals

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of place value

Background Information

In previous years, students learned to describe and represent decimals. In Grade 6, students will learn to multiply and divide decimals, and will be required to demonstrate an understanding of these two operations with decimals.

The concept of decimals may present a difficulty for some students. Teachers need to place special emphasis on the decimal point (i.e., its meaning and placement—see definition below).

One of the difficulties a teacher might encounter occurs while teaching multiplication and division of decimals involving 1-digit whole number multipliers and 1-digit natural number divisors. A student, for example, may not understand whether he or she is supposed to keep track of the number of digits in front or behind the decimal point. When multiplying 8.23 × 3, a student may incorrectly write the answer as 2.469, saying that since there is only one numeral in front of the decimal point in the question, then there should only be one numeral before the decimal point in the reply. Or when dividing 397.26 ÷ 6, a student may incorrectly write 662.1 for the same reason. With these types of examples, the teacher needs to get the student used to looking at the whole number part of the numeral. For the multiplication example, ask the student "What is the answer to 8 × 3?" After the student replies that it is 24, continue saying "If the answer to 8 × 3 is 24, then the answer to 8.23 × 3 is greater than 24 (in this example, 24.69)." For the division example, ask the student "What is the answer to 397 ÷ 6?" After the student replies that it is 66 and the remainder is 1, continue saying "If the answer to 397 ÷ 6 is 66 and the remainder is 1, then the answer to 397.26 ÷ 6 is greater than 66."

When teaching multiplication and division of decimals involving multipliers and divisors that are multiples of 10, there may be a difficulty when assigning questions such as: 37.296 × 100, 42.36 × 10, 58.79 ÷ 10, and 394.32 ÷ 100. The student may express confusion about which way to move the decimal point. With these types of examples, try to get the student used to thinking: 37 × 100 is 3700, so 37.296 × 100 is greater than 3700; 42 × 10 is 420, so 42.36 × 10 is greater than 420; 58 ÷ 10 is 5.8, so 58.79 ÷ 10 is greater than 5.8; and 394 ÷ 100 is 3.94, so 394.32 ÷ 100 is greater than 3.94.

Definitions may also be beneficial to students. If they are exposed to the definitions of the required terminology in each lesson, they may understand them better. Terminology and definitions needed for 6.N.8 are listed below:

Decimal is a fractional number written in base-10 form; a mixed decimal number has a whole number part as well (e.g., 0.32 is a decimal number and 3.5 is a mixed decimal number).

Decimal point is a period or dot separating the ones place from the tenths place in decimal numbers, or dollars from cents in money. When numbers are spoken, the decimal point is read as "and" (e.g., 3.2 is read as three and two-tenths).

Divisor is the number by which the dividend is divided (e.g., in 12 ÷ 3 = 4, 3 is the divisor).

Estimate is an answer that is an approximation.

Front-end estimation (also called front-end rounding) is a method for estimating an answer to a calculation problem by focusing on the front-end or left-most digits of a number.

Example:

> ***Question***
>
> You buy a hamburger for $4.59, a drink for $1.96, and an ice cream cone for $0.95. Will a five-dollar bill cover the cost?
>
> ***Front-end Strategy Solution Process***
>
> Total the front-end (dollar) amounts: $4 + $1 + $0 = $5.
>
> ***Solution***
>
> A five-dollar bill will not cover the cost because the front-end estimate, which is always an underestimate, is $5.

Mental mathematics is a mathematical process by which computation is done "in the head," either in whole or in part.

Multiplier is the number by which the multiplicand is multiplied in a multiplication problem (e.g., in $1.2 \times 3 = 3.6$, 3 is the multiplier).

An **algorithm** is a system of finite procedures for solving a particular class of problems. The best known algorithms are the traditional paper-and-pencil procedures for adding, subtracting, multiplying, and dividing. Along with these standard algorithms, mathematics instruction should include an emphasis on understanding through mental mathematics, estimation, the use of technology, the development of invented procedures, and the use of alternative algorithms, such as area model multiplication, and adding up to solve subtraction problems.

By encouraging students to develop their own computation strategies and allowing them to use alternative algorithms, the emphasis in mathematics instruction is shifted to reasoning, problem solving, and conceptual understanding. Providing students with opportunities to invent their own strategies and use alternative algorithms enhances their number and operation sense. Students become more flexible in their thinking, more aware of the different ways to solve a problem, and more adept at selecting the most appropriate procedure for solving a problem. Discussion of the algorithms or strategies and their relationship to place value and number properties can also help students develop better reasoning and communication skills.

The standard algorithm for multiplication is where numbers are arranged vertically and multiplied using single digits, which are added to form a final product. Students are expected to use the standard algorithm as one of the tools for computation.

When teaching the traditional algorithm for multiplication, it is important to follow the concrete, pictorial, and symbolic sequence of teaching. The important idea is to allow students to construct meaning, not memorize procedures without understanding. Students' misconceptions (or their fuzzy understandings) can be reinforced by a poorly understood algorithm. Students should be able to explain the relationship that exists among an algorithm, place value, and number properties.

Teachers can facilitate students' understanding and use of a variety of computational strategies by

- providing a supporting and accepting environment
- allowing time for exploration and experimentation
- embedding computational tasks in real-life situations
- allowing students to discuss, analyze, and compare their solution strategies
- encouraging discussion that focuses on place value and number properties when defending the choice of a particular algorithm or strategy
- understanding that a child needs to be efficient at computation and that this looks different for each student

Models for Dividing Decimals

The focus is on dividing 2 numbers by building understanding through the use of models, non-traditional algorithms, and traditional algorithms.

Note: The term *traditional algorithm* is used to indicate the symbolic algorithm traditionally taught in North America. Throughout the world, many other algorithms are traditionally used.

Base-10 Blocks

Base-10 blocks can be used to represent the operations of addition, subtraction, multiplication, and division of decimals.

Students must have a fluent understanding of the numeric values for the model. If they lack this understanding, their attention will be focused on trying to make sense of the model instead of learning to compute with whole numbers. Have students physically separate the blocks, or cut paper grids, to help them attach meaningful values to the representations. Spend time naming various combinations of blocks and creating representations of various whole numbers to develop fluency.

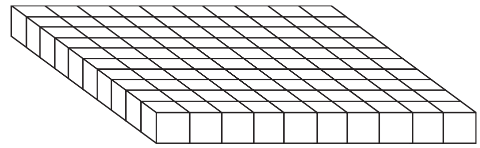
Base-10 grid paper serves as a two-dimensional representation of the base-10 blocks (see BLM 5-8.10: Base-Ten Grid Paper).

Note: It is important that students work flexibly with various representations to develop their understanding of operations with whole numbers, rather than memorizing the steps without understanding their meaning.

Representations:

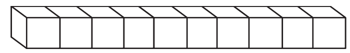
If the flat represents one whole,

- its value is 1.0



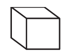
then the rod represents a tenth,

- its value is 0.1



and the cube represents a hundredth.

- its value is 0.01



Note: Stress to students that although they may have used these materials before using different names, the materials can represent whatever they want them to as long as the relationship among the materials is mathematically correct.

Example:

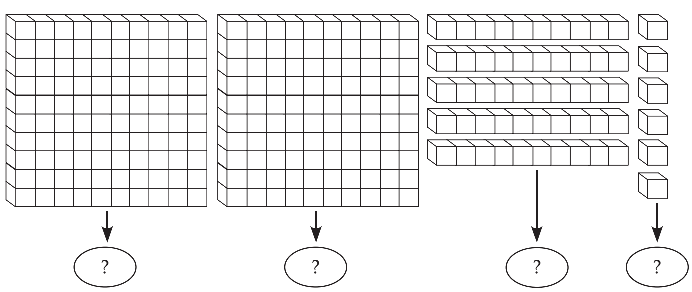
2.56 ÷ 4

Estimate:

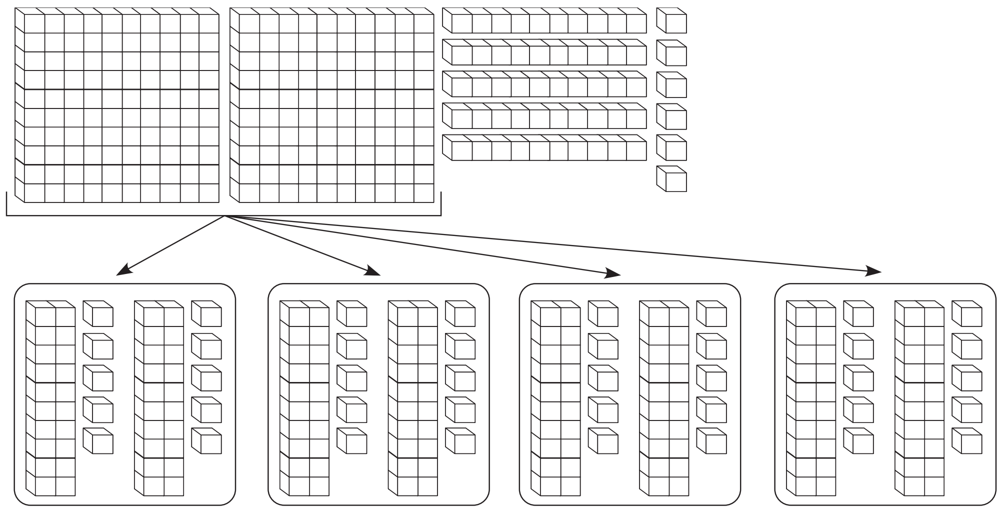
Think: 2 ÷ 4 = 0.5 (or 2 is half of 4). The answer will be less than 1 but greater than one-half.

Build 2.56 with the materials.

Use models to help find an exact number. What is 2.56 divided into 4 groups?

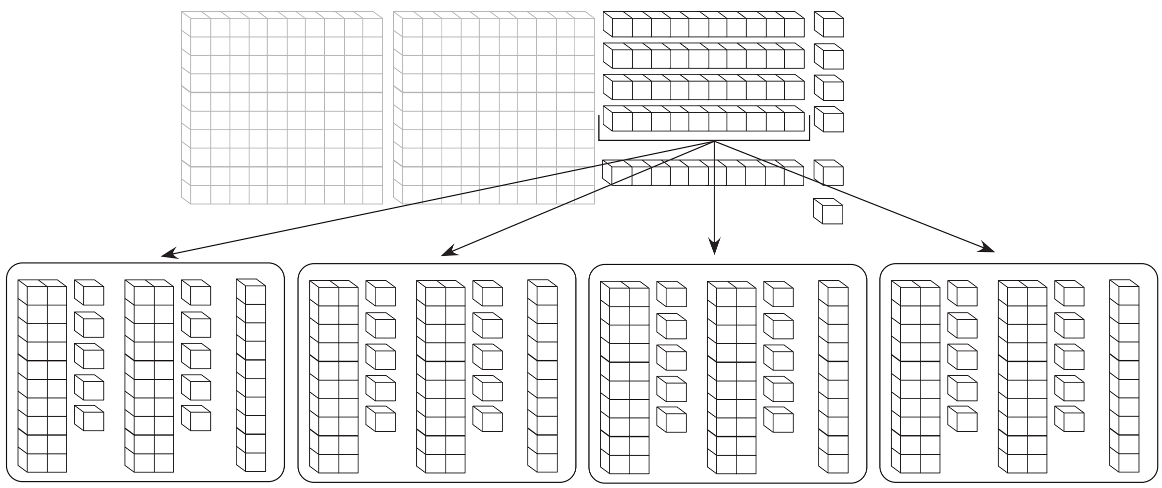


Think: “I can break each whole into 4 groups. There are 2 tenths and 5 hundredths in each group for every hundred I divide.”

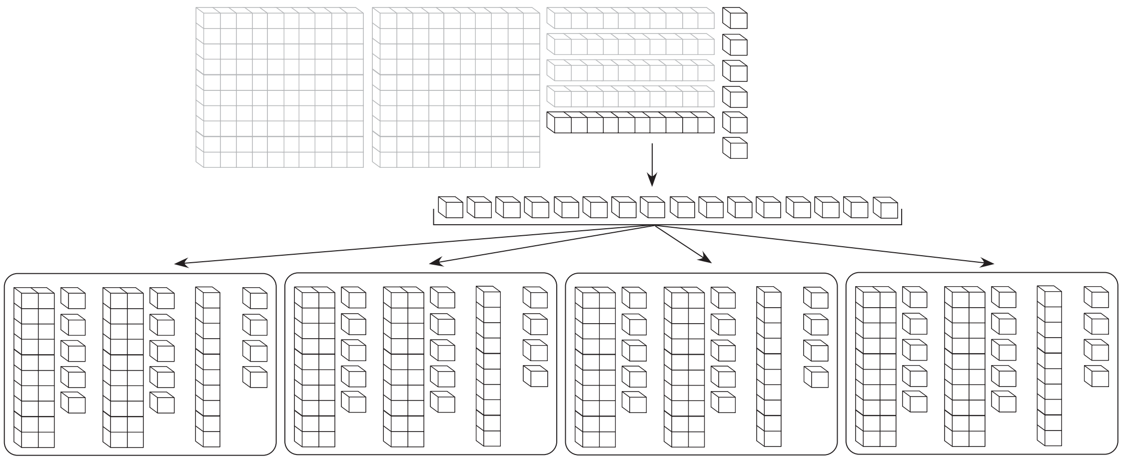


Think: “After I divide the 2.0 into 4 groups, there is nothing left over.”

Think: “I can move one tenth to each group.”



Think: "That leaves one tenth (or an additional 10 hundredths). There are 16 hundredths left, so I can move 4 more to each group."



Think: In each group of 4, there are 6 tenths and 4 hundredths or 0.64. So, 2.56 ÷ 4 is 0.64.

Long Division

Long division is a more compact algorithm used to show the division of multi-digit numbers. Once students have an understanding of the above models and methods, and they are confident with their understanding of division and place value, the "traditional algorithm" for division can be a quick and precise method of dividing. Algorithm 1 shows one way to divide decimals while the traditional algorithm shows the same process in a different way.

Students follow the same steps as they do when dividing whole numbers but should be aware of the correct placement of the decimal. Although 2.56 ÷ 4 and 25.6 ÷ 4 produce a quotient with the digits 64, students should estimate that 2.56 ÷ 4 will be less than 1.0 and 25.6 ÷ 4 will be close to 6.0.

Example:

Algorithm 1		Traditional Algorithm
$4\overline{)256}$	6 groups of 4	0.64 $4\overline{)2.56}$
$\underline{-24}$		$\underline{-2.4}$
16	4 groups of 4	0.16
$\underline{16}$		$\underline{-0.16}$
0	64	0

Models for Multiplying Decimals

The focus is on multiplying numbers by building understanding through the use of models, non-traditional algorithms, and traditional algorithms.

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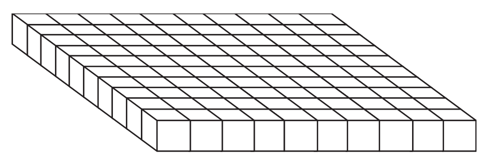
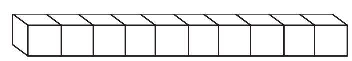


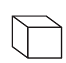
then the rod represents a tenth,

- its value is 0.1



and the cube represents a hundredth.

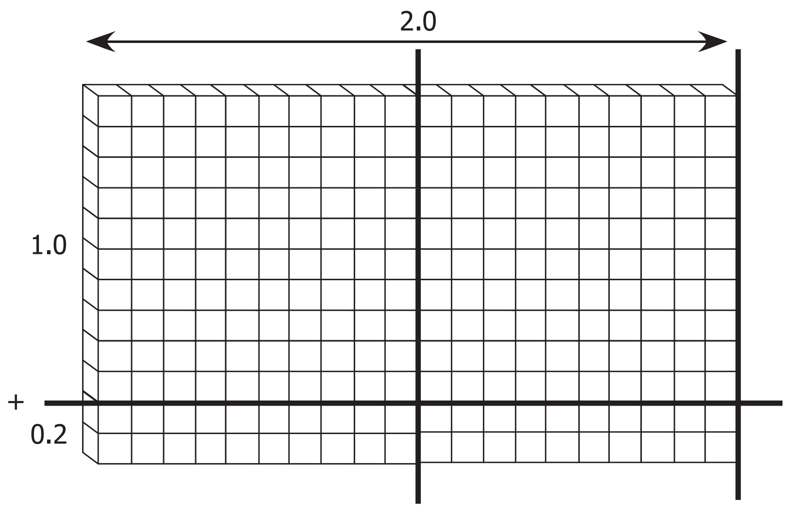
- its value is 0.01



An area model can be used to multiply decimals. As with whole number multiplication, the algorithm can be modelled using base-10 blocks or pictorially.

Example:

2.0×1.2



To determine the partial products, think of numbers that are easier to multiply. For example, represent numbers according to place value and think:

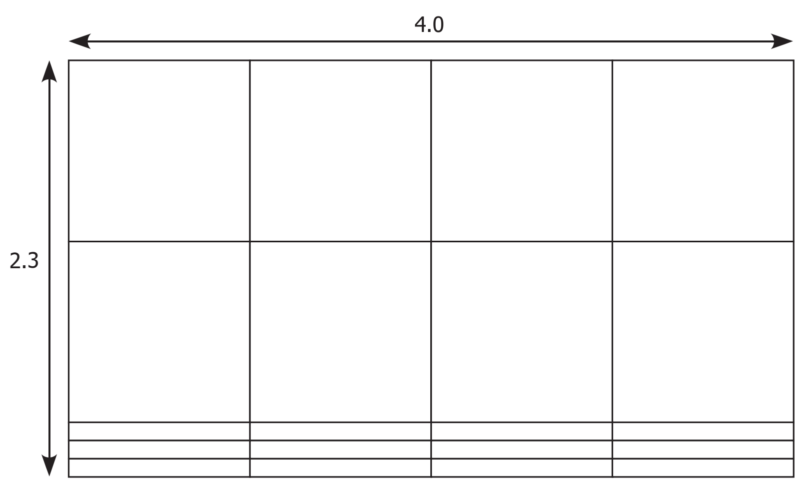
- "What is 2.0×1.0?" "2.0"
- "What is 2.0×0.2?" "0.4"

"So then, 2×1.2 must be $2.0 + 0.4$ or 2.4."

Example:

4.0×2.3

$$\begin{array}{r} 4.0 \\ \times\ 2.3 \\ \hline 1.2 \\ +\ 8.0 \\ \hline 9.2 \end{array}$$



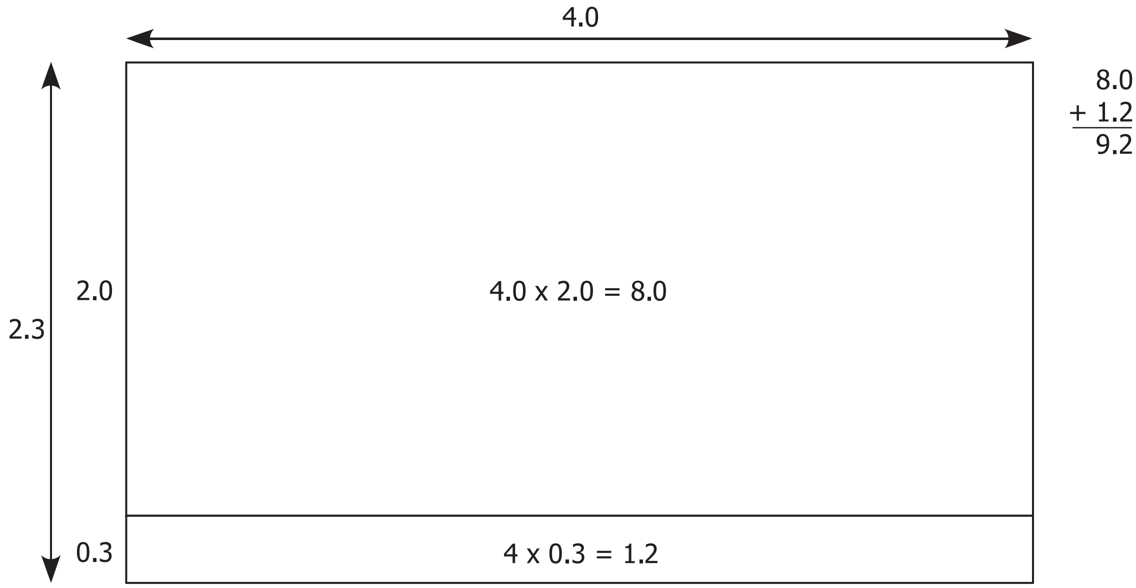
Think:

- $4.0 \times 2.0 = 8.0$
- $4.0 \times 0.3 = 1.2$

Then, $8.0 + 1.2 = 9.2$.

Note: Because of the commutative property of multiplication, area models can be drawn in either orientation.

4.0×2.3 or 2.3×4.0



The partial products are determined as

$4.0 \times 2.0 = 8.0$

$4.0 \times 0.3 = 1.2$

So, 8.0 + 1.2 = 9.2.

It is important to establish a convention of keeping the blocks organized, as it will help with developing future representations.

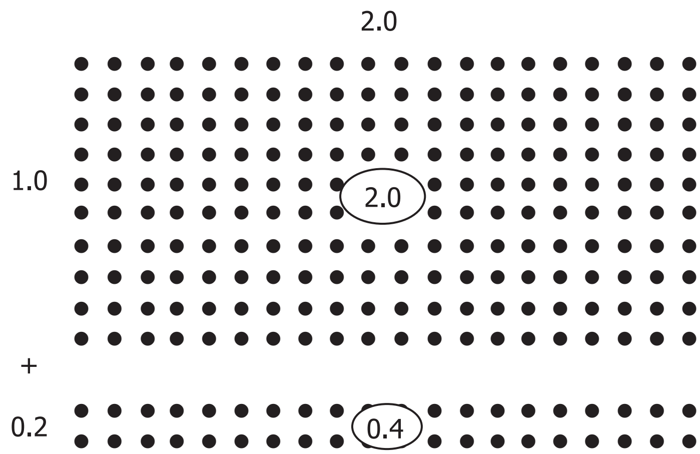
Base-10 blocks can be used to represent the active understanding of multiplication as a specific number of groups of a specific size, or the non-active array representation of a quantity.

When the blocks are arranged as a rectangle, the rectangle may be rotated and the quantity does not change. This is a verification of the commutative property of multiplication. The orientation of the array has no effect on the result, but in some places a convention has been established of representing the first number horizontally and the second vertically.

The array also serves as a model for area. It represents the area covered by a rectangle with a length of the multiplicand and a width of the multiplier, or vice versa.

Example:

2.0×1.2



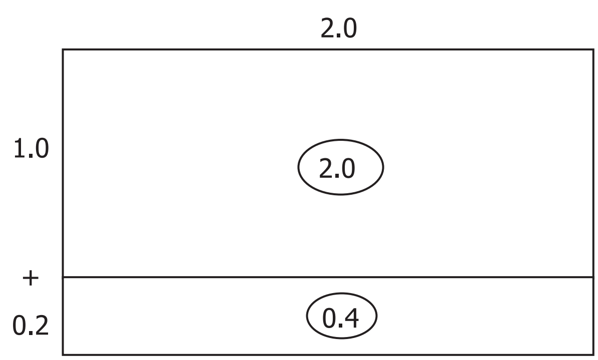
The partial products are determined in the same way as they are in the previous example.

The Area Model

Once students have an understanding of multiplication using base-10 blocks and/or base-10 paper, and they have a thorough understanding of the numerical value they represent, they can simply move to an area model. In this model, the length represents the multiplicand and the width represents the multiplier, or vice versa; however, the lengths and widths do not have to represent an exact measurement.

Example:

2.0×1.2



The partial products are determined in the same way as they are in the previous example.

The Traditional Algorithm

Once students have an understanding of the above models and methods and are confident with multiplication, the traditional algorithm can be quick and useful. Students will multiply as if the decimal is not there. This works because the digits are the same. It is important, however, to estimate to make sure the decimal is placed correctly.

Example:

$$\begin{array}{r} 2.33 \\ \times\ 4 \\ \hline \end{array}$$

Think:

$$\begin{array}{lr} 4 \times 0.03 = & 0.12 \\ 4 \times 0.3 = & 1.2 \\ 4 \times 2.0 = & 8.0 \\ \hline & 9.32 \end{array}$$

Traditional Algorithm:

$$\begin{array}{r} {}^{1\ 1} \\ 2.33 \\ \times\ 4 \\ \hline 9.32 \end{array}$$

Even though students have learned the standard algorithm for multiplication, it is important to understand the rationale for place value placement of the decimal.

How are 2.33 × 4 and 23.3 × 4 both similar and different? They have the same digits and, when multiplied, will produce the same digits, 932. Discuss how 2.33 × 4 produces a product near to 8 while 23.3 × 4 produces a product near to 80.

Note: For further information regarding using models and methods of division to build understanding, see resources such as the following:

- "Big Ideas for Teaching Mathematics Grades 4–8" by Marian Small (found in Chapter 2 of *Varied Approaches for Multiplication and Division*)
- "Making Math Meaningful to Canadian Students K–8" by Marian Small (found in Chapter 10 of *Computational Strategies: Operations with Whole Numbers*)
- *Teaching Student Centered Mathematics* by John A. Van de Walle

Mathematical Language

decimal
decimal point
divisor
estimate
mixed decimal number
mental mathematics
multiples

Learning Experiences

Assessing Prior Knowledge

Organization: Whole class

Procedure:

1. Tell students that they will learn how to multiply and divide decimals, but first you want to check what they remember about decimals.
2. Write a few decimals on the board and ask students to read them.

   Examples:

   0.15

   0.398

   0.0317

   0.207

   0.004
3. Have a class discussion. Ask questions such as the following:
   a) Where are decimals used?
   b) What does the dot mean?
   c) If I want to write four-hundredths,
      - does it matter if I place zeros after the 4? Why?
      - does it matter how many zeros I place before the four? Why?
4. Discuss place value and decimal place.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understand place value.
- ❒ Read decimals correctly.
- ❒ Use the decimal point correctly.

Suggestions for Instruction

- **Estimate a product using front-end estimation (e.g., for 15.205 m x 4, think 15 m x 4, so the product is greater than 60 m), and place the decimal in the appropriate place.**

Materials:

- pencil and paper

Organization: Whole class

Procedure:

1. Write a few decimals on the board, such as the following:
   a) 3.85
   b) 7.69
   c) 2.08
   d) 8.321
   e) 6.18 026
   f) 3.20 18
   g) 9.341
2. Ask a few student volunteers to do the following:
   a) Read each decimal.
   b) Say what each number will be after using front-end rounding.
3. Discuss with the class
   a) what the estimated product of each decimal × 3 will be after using front-end rounding
   b) where the decimal point will be placed in each product

4. Have students explain the following:
   a) 8.216×3 is greater than 24
   b) 21.08×4 is greater than 84
5. Have students place the decimal point in each product using front-end estimation:
   a) $25.024 \times 3 = 75072$
   b) $19.128 \times 5 = 9564$
   c) $84.301 \times 2 = 168602$
   d) $7.932 \times 10 = 7932$
   e) $3\,869.54 \times 100 = 386954$
6. Discuss any other strategies students may have used to determine where to place the decimal in the product.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Read decimal numbers
- ❒ Apply front-end rounding.
- ❒ Place the decimal point correctly.

Suggestions for Instruction

- **Estimate a quotient using front-end estimation (e.g., for $26.83 ÷ 4, think $24 ÷ 4, so the quotient is greater than $6), and place the decimal in the appropriate place.**

Materials:
- pencil and paper
- BLM 6.N.8.1: Izabella's Teacher

Organization: Whole class/pairs

Procedure:
1. Display a copy of BLM 6.N.8.1.
2. Tell students to read the question carefully.

3. Discuss with the class
   a) what Izabella's teacher means by using front-end estimation to solve the decimal questions
   b) what Izabella's teacher means by wanting the class to carefully consider where they place the decimal point in the quotient
   c) how you solve the decimal questions presented on the BLM
4. Provide students with the following instructions:
   a) Place the decimal in each quotient.
   b) Discuss your work with your partner.
5. Circulate to make sure students understand how to divide decimals using front-end estimation and that they place the decimal point in the correct position.
6. Discuss any other strategies students may have used to determine where to place the decimal in the quotient.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Apply front-end rounding.
- ❒ Use the decimal point correctly.

Suggestions for Instruction

- **Predict products and quotients of decimals using estimation strategies.**

Materials:

- pencil and paper
- BLM 6.N.8.2: Decimal Products and Quotients

Organization: Small groups

Procedure:

1. Hand out to each student a copy of BLM 6.N.8.2.
2. Provide students with the following instructions:
   a) Read the questions carefully.
   b) Do not calculate.

c) Predict the products of decimal using estimation strategies.

d) Write down your prediction for each product.

e) Discuss your predictions with your group members.

f) Next, predict the quotients of decimal using estimation strategies.

g) Write down your prediction for each quotient.

h) Discuss your predictions with your group members.

3. Circulate to see that the students understand the work and are doing it correctly.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Multiply and divide by single digits.
- ❒ Multiply and divide by multiples of 10.
- ❒ Use estimation strategies.
- ❒ Predict the products of decimal using estimation strategies.
- ❒ Predict the quotients of decimal using estimation strategies.

Suggestions for Instruction

- **Identify and correct errors of decimal point placement in a product or quotient by estimating.**

Materials:

- pencil and paper
- BLM 6.N.8.3: Marie's Cell Phone Bill

Organization: Individual

Procedure:

1. Display BLM 6.N.8.3.
2. Provide students with the following instructions:

a) Use the information on Marie's cell phone bill to calculate her payment requirement.

b) Does your calculation of Marie's payment requirement match with the payment requirement on Marie's cell phone bill?

c) Use estimation and check how your estimation results of the payment requirement match your calculations and Marie's bill.

d) In your journal, describe your calculations and why your payment requirement does or does not match with the payment requirement on Marie's cell phone bill.

3. Circulate to see that students understand the problem and are solving it correctly.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve a problem involving decimals.
- ❒ Use estimation strategies to identify errors of decimal point placement in a product and a quotient.

Suggestions for Instruction

- **Identify and correct errors of decimal point placement in a product or quotient by estimating.**

Materials:

- pencil and paper
- BLM 6.N.8.4: Errors of Decimal Point Placement

Organization: Small groups

Procedure:

1. Distribute to each student a copy of BLM 6.N.8.4.
2. Provide students with the following instructions:
   a) Read the questions carefully.
   b) Use estimation strategies to
      - identify errors of decimal point placement
      - correct errors of decimal point placement

      Note: Some products and quotients may be correct.
   c) Write down your correction for each product and quotient.
   d) Discuss your errors and corrections with your group members.
3. Circulate to see that students understand the placement of the decimal point and are doing their work correctly.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use estimation strategies to identify errors of decimal point placement in a product and quotient.
- ❒ Use estimation strategies to correct errors of decimal point placement in a product and quotient.

Suggestions for Instruction

- **Solve a problem that involves multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9.**

Materials:

- pencil and paper
- BLM 6.N.8.5: Multiplication and Division Problems involving Decimals

Organization: Pairs/whole class

Procedure:

1. Distribute to each student a copy of BLM 6.N.8.5.
2. Provide students with the following instructions:
   a) Read each problem carefully.
   b) Decide with your partner which problem set you will do (A or B).
   c) Discuss with your partner how you will solve each problem.
   d) Solve each problem on your own.
   e) Use estimation strategies to verify your placement of the decimal point in each reply.
   f) Compare your final answer and discuss your method of solution with your partner.
3. Discuss with the whole class the method used for solving the problems and the errors and difficulties with each problem.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve problems that involve multiplication and division of decimals.
- ❒ Use estimation strategies to verify placement of the decimal point.

Suggestions for Instruction

- **Use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10 (e.g., 2.47 X 10 = 24.7; 31.9 ÷ 100 = 0.319).**

Materials:

- calculator
- BLM 6.N.8.6: Complete the Charts

Organization: Individual/partner/whole class

Procedure:

1. Distribute a copy of BLM 6.N.8.6.
2. Provide students with the following instructions:
   a) Use your calculators to complete individually the multiplication charts on the BLM 6.N.8.6 sheet.
   b) Discuss with your partner the patterns you see for multiplication.
   c) Then, use your calculators to complete individually the division charts on the BLM 6.N.8.6 sheet.
   d) Discuss with your partner the patterns you see for division.
3. Discuss with the whole class the patterns students see.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Discover a pattern.

- **Use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10 (e.g., 2.47 X 10 = 24.7; 31.9 ÷ 100 = 0.319).**

Materials:

- BLM 6.N.8.7: Use Mental Math

Organization: Whole class/individual

Procedure:

1. Display BLM 6.N.8.7.
2. Have students look at the questions.
3. Discuss how to determine the product or the quotient using mental math.
4. Ask different student volunteers to state the product or the quotient using mental math.
5. Tell students to explain in their journals how to use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understanding how to use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10.
- ❒ Use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10.

Suggestions for Instruction

- **Model and explain the relationship that exists between an algorithm place value, and number properties.**
- **Determine products and quotients using the standard algorithms of vertical multiplication (numbers arranged vertically and multiplied using single digits, which are added to form a final product) and long division (the multiples of the divisor are subtracted from the dividend).**

Materials:

- Base-10 blocks

Organization: Small groups of 3 or 4

Procedure:

1. Explain that the base-10 blocks will represent decimals.

   1 flat = 1 whole 1.0

   1 rod = 1 tenth 0.1

   1 cube = 1 hundredth 0.01

2. Students will be given 3 multiplication of decimal questions and 3 division of decimal questions. For each multiplication question, students will

   a) estimate the answer

   b) solve using an area model

   c) solve using repeated addition

   d) solve using the traditional algorithm

   For each division question, students will

   a) estimate the answer

   b) solve using the base-10 model

   c) solve using algorithm 1 (see Background Information)

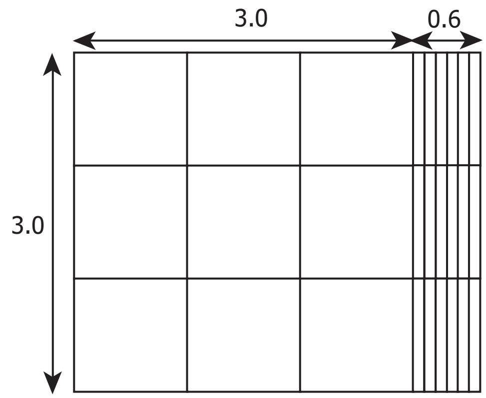
   d) solve using the traditional algorithm

Examples: Multiplication

1. 3.6×3.0

   a) Group estimate: $3 \times 3 = 9.0$

   b) Person #1: Draws the area model

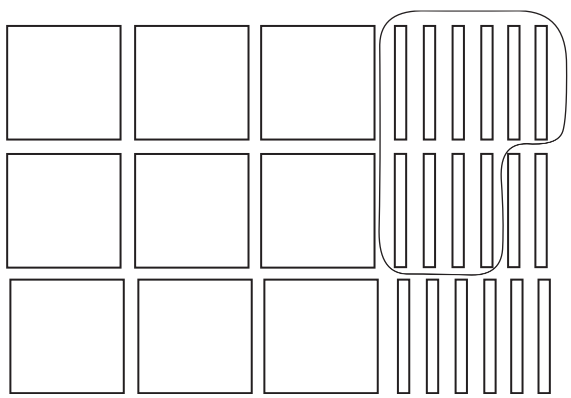


$3.0 \times 3.0 = 9.0$

$3.0 \times 0.6 = 1.8$

Total = 10.8

   c) Person #2: Models repeated addition



Regrouping into 10 and 8 tenths

10.8

   d) Person #3: Uses the traditional algorithm

$$\begin{array}{r} 3.6 \\ \times\ 3 \\ \hline 1.8 \\ 9.0 \\ \hline 10.8 \end{array}$$

Think 3×0.6

Think 3×3.0

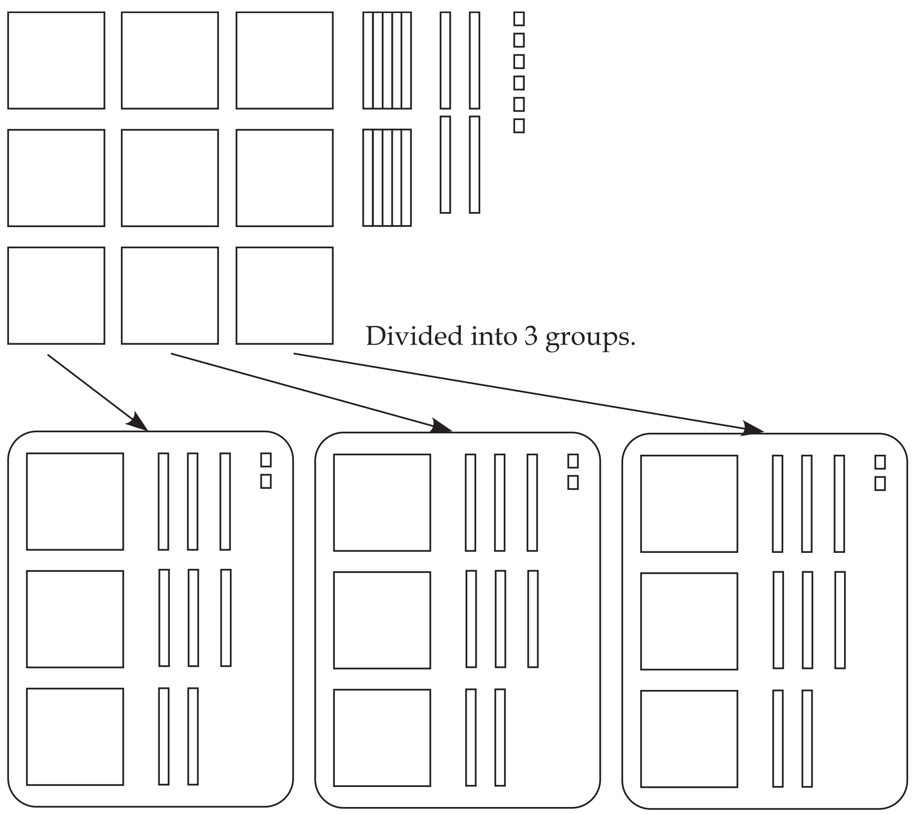
$$\begin{array}{r} {}^{1} \\ 3.6 \\ \times\ 3 \\ \hline 10.8 \end{array}$$

2. 6.2 × 5
   a) Estimate the answer
   b) Person #1: Shows repeated addition
   c) Person #2: Shows the traditional algorithm
   d) Person #3: Shows the area model

3. 9.1 × 2
   a) Estimate the answer
   b) Person #1: Shows the traditional algorithm
   c) Person #2: Shows the area model
   d) Person #3: Shows repeated addition

Examples: Division

1. 11.46 ÷ 3
   a) Group estimate: 11 ÷ 3 = answer between 3 and 4
   b) Person #1: Models using base-10 blocks



Each group has: 3 wholes
8 tenths = 3.82
3 hundredths

c) Person #2: Uses algorithm 1

$$
\begin{array}{rl}
3\overline{)11.46} & \\
\underline{-9.0} & \quad 3.0 \text{ groups of } 3 \\
2.46 & \quad 0.8 \text{ groups of } 3 \\
\underline{-2.4} & \\
0.06 & \quad 0.02 \text{ groups of } 3 \\
\underline{0.06} & \\
0 & \quad \overline{3.82}
\end{array}
$$

d) Person #3: Uses the traditional algorithm

$$
\begin{array}{r}
3.82 \\
3\overline{)11.46} \\
\underline{-9}\downarrow \\
2.4 \\
\underline{-2.4}\downarrow \\
0.06 \\
\underline{-0.06} \\
0
\end{array}
$$

2. 16.8 ÷ 4
   a) Estimate the answer (16.8 ÷ 4 = 4)
   b) Person #1: Uses algorithm 1
   c) Person #2: Uses traditional algorithm
   d) Person #3: Uses base-10 blocks

3. 9.12 ÷ 2
   a) Estimate the answer (9.12 ÷ 2 will be between 4 and 5)
   b) Person #1: Uses traditional algorithm
   c) Person #2: Uses the base-10 blocks
   d) Person #3: Uses algorithm 1

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine products and quotients using the standard algorithms.
- ❒ Explain and model the relationship between algorithms and place value.

Suggestions for Instruction

- **Solve multiplication and division problems in context using personal strategies, and record the process.**
- **Refine personal strategies, such as mental math, to increase their efficiency when appropriate (e.g., 4.46 ÷ 2 think 446 ÷ 2 = 223, and then use front-end estimation to determine the placement of the decimal 2.23).**

Materials:

- BLM 6.N.8.8: Question Sheet

Organization: Small groups

Procedure:

1. Circulate BLM 6.N.8.8 to students in small groups and have them solve the questions.
2. Have students communicate their group answers and strategies by participating in a whole-class group share. Groups can take turns sharing an answer to a question and other students can compare their group's strategies, ask questions, and share solutions.

Observation Checklist

☑ Observe students' responses to determine if they can do the following:

- ❒ Solve problems using personal strategies.
- ❒ Record the problem-solving process.
- ❒ Refine personal strategies to increase efficiency.

Calculate the Distance

Purpose: The purpose of this activity is for students to connect decimals to real life. Students will need to apply their knowledge of decimals, measurements, and recording and analysis of data. Students will need some prior knowledge, such as multiplying and dividing by single-digit whole numbers and multiples of 10. The following processes are demonstrated by this task: communication, connections, mental mathematics and estimation, problem solving, and reasoning.

Curricular Links: This task can be linked to science.

Materials/Resources:

- paper and pencil
- metre stick
- poster-sized paper

Students will need to measure lengths in different areas of the school.

Organization:

- Four groups

Inquiry:

Scenario

Students will be working in four separate groups. Each group will measure the distance to one of these rooms: library, gym, music room, and lunch room.

Procedure

Teacher:

1. Divide the students into four groups.
2. Assign a room to each group (library, gym, music room, and lunch room).
3. Distribute the directions to each group.
4. Tell students to follow the directions.

Students:

1. Each group needs to measure the exact distance in metres and the decimal parts of a metre (extra centimetres) from the classroom door to the door of the designated room.
2. Record the exact distance in metres and the decimal parts of a metre from the classroom door to the door of the designated room.
3. Record how many times per week you go from the classroom to the designated room.

4. Make a chart like the one provided below:

Room	Number of Metres (Use Decimals to Record the Exact Number of Centimetres)	Number of Times per Week

5. Calculate how many metres you walk each week from the classroom door to the door of the designated room and back.
6. How many metres do you walk from the classroom door to the door of the designated room in four weeks?
7. Get together with another group.
8. Record the information from your chart and the other group's chart on the poster-sized paper.
9. Compare their distance in metres in a week to yours using estimation strategies and mental math.
10. Compare your distance in metres in four weeks to the distance of the other group, using estimation strategies and mental math.
11. Write up your observations.

Assessment:

Use the following observation checklist to assess students' learning.

The student can do the following:	Yes	No	Comment
Use the metre stick to to take measurements.			
Record the measurements accurately.			
Use decimals.			
Multiply decimals.			
Record data.			
Analyze data.			
Effectively communicate with peers.			
Cooperate with peers.			

Extension:

Taking it further

Based on the ideas in this task, create your own activity that involves multiplying or dividing decimals.

NOTES

Grade 8: Number (6.N.9)

Enduring Understanding(s):

When a mathematics question is composed of more than one operation, the solution depends on the order in which the operations are done.

It is important to follow a standardized order of operations so that everyone solving the identical problem will obtain the same answer.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	**Achievement Indicators:**
6.N.9 Explain and apply the order of operations, excluding exponents (limited to whole numbers). [CN, ME, PS, T]	➛ Demonstrate and explain with examples why there is a need to have a standardized order of operations. ➛ Apply the order of operations to solve multi-step problems with or without technology.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of multiplication (1- and 2-digit multipliers and up to 4-digit multiplicands) to solve problems
- Demonstrating an understanding of division (1- and 2-digit divisors and up to 4-digit dividends) with and without concrete materials, and interpreting remainders to solve problems
- Demonstrating an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals)

Related Knowledge

Students should be introduced to the following:

- Demonstrating and explaining the meaning of and preservation of equality

Background Information

In previous years, students have learned about the importance that numbers play in our daily lives. They also learned to perform the four basic mathematical operations: addition, subtraction, multiplication, and division.

In Grade 6, students will learn to recognize that the order in which the four mathematical operations are performed is very important. Mathematicians realized that a standardized order of operations is required if everyone is to produce the same answer to a multi-operation question.

Without a standardized rule, a multi-operation question can be solved in a different order, yielding a variety of replies.

Order of Operations

Order of operations is a specified sequence in which mathematical operations are expected to be performed. An arithmetic expression is evaluated by following these ordered steps:

1. Simplify within grouping symbols, such as parentheses or brackets, starting with the innermost.
2. Apply exponents—powers and roots.
3. Perform all multiplications and divisions in order from left to right.
4. Perform all additions and subtractions in order from left to right.

A common way to remember this is to use the acronym BEDMAS: Brackets, Exponents, Division, Multiplication, Addition, Subtraction.

Note: In Kindergarten to Grade 7, exponents are not used. Students are first exposed to squares and square roots in Grade 8, and in Grade 9 students will revisit the order of operations and work with exponents.

Mathematical Language

order of operations

Assessing Prior Knowledge

Organization: Whole class

Procedure:

1. Tell students you want to check what they remember about the four mathematical operations.
   a) Ask students to solve questions, such as the following:

      $23 + 15 =$

      $68 - 29 =$

      $15 \times 8 =$

      $63 \div 7 =$

2. Have a class discussion. Ask questions such as the following:
   a) "How did you know which mathematical operation you had to use?" Discuss the importance of the symbols representing the four operations (i.e., addition, subtraction, multiplication, and division).
   b) "Was there an order, or a rule, you had to follow to solve the questions?"
   c) "Would you get the same answer if you reversed the order?"

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve simple questions using any one of the four operations.
- ❒ Understand the four operations.
- ❒ Understand the need for specific symbols.
- ❒ Understand the need for specific order.

- **Demonstrate and explain with examples why there is a need to have a standardized order of operations.**

Materials:

- pencil and paper
- BLM 6.N.9.1: One Solution, Two Solutions?

Organization: Whole class

Procedure:

1. Place on the overhead a transparency of BLM 6.N.9.1. Make sure the two solutions are covered, showing the question part only, as shown below.

> **BLM 6.N.9.1: One Solution, Two Solutions?**
>
> Bonny and Jenny loved to compare their work. Yesterday, their math teacher assigned the following question for homework:
>
> $3 + 5 \times 7 - 2 + 9 \div 3 =$
>
> Both girls decided that the question needs to be split into many parts.

2. Provide students with the following instructions:
   a) Read the question carefully.
   b) Discuss with your group members how you would solve the problem.
   c) Do not solve the problem.
3. Have a class discussion on what each group decided as to how to solve the problem.
4. Then say "Let's see how Bonny and Jenny solved the problem."
5. Now show the solution part of BLM 6.N.9.1, as shown below.

Bonny solved the problem like this:	Jenny solved the problem like this:
$3 + 5 = 8$	$5 \times 7 = 35$
$8 \times 7 = 56$	$9 \div 3 = 3$
$56 - 2 = 54$	$3 + 35 = 38$
$54 + 9 = 63$	$38 - 2 = 36$
$63 \div 3 = 21$	$36 + 3 = 39$
Then, Bonny stated:	Then, Jenny stated:
$3 + 5 \times 7 - 2 + 9 \div 3 = 21$	$3 + 5 \times 7 - 2 + 9 \div 3 = 39$

6. Tell students to look at each solution carefully, and then discuss the following with their group members:
   a) What was different in the two solutions?
   b) Did the solution the group decided on match either girl's solution?
   c) What was the same? What was different?
   d) What would you suggest to Bonny and Jenny to do so they could both get the same solution?
7. Discuss the following with the whole class:
   a) the students' observations about this question and the solutions
   b) what is needed in order to assure that everyone solving the same question will arrive at the same answer
   c) why there is a need to have a standardized order of operations
8. Have students give examples that show there is a need to have a standardized order of operations.
9. Explain why a standardized order of operations would ensure sameness of replies.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❐ Understand a multi-step problem.
- ❐ Take apart a multi-step problem.
- ❐ Realize that a rule is needed for solving a multi-step problem.
- ❐ Demonstrate and explain with examples why there is a need to have a standardized order of operations.

Suggestions for Instruction

- **Demonstrate and explain with examples why there is a need to have a standardized order of operations.**

Materials:

- pencil and paper

Organization: Whole class/pairs

Procedure:

1. Write on the board the following question:

   $8 + 3 \times 4 - 12 \div 2 =$

2. Provide students with the following instructions:
   a) Copy the question.
   b) Discuss with your partner the different ways you could solve this problem if a standardized order of operations did not exist.
   c) Show the different ways you could solve this problem if a standardized order of operations did not exist.

3. Have a class discussion about why it is important to have a standardized order of operations.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Recognize that without a standardized order of operations, a multi-step problem could be solved in a variety of ways.
- ❐ Realize that a rule is needed for solving a multi-step problem.
- ❐ Demonstrate and explain with examples why there is a need to have a standardized order of operations.

- **Demonstrate and explain with examples why there is a need to have a standardized order of operations.**

Materials:

- pencil and paper

Organization: Individual

Procedure:

1. Provide students with the following instructions:
   a) Work individually on this task.
   b) Construct an example of a math question that includes the four operations: addition, subtraction, multiplication, and division.
   c) Show the variety of ways you could solve the problem if there was no standardized order of operations.
2. Have students write in their journals why it is important to have a standardized order of operations.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Provide an example of a multi-step problem that shows a need for a standardized order of operations.
- ❒ Demonstrate and explain with examples why there is a need to have a standardized order of operations.

- **Apply the order of operations to solve multi-step problems with or without technology.**

Materials:

- pencil
- BLM 6.N.9.2: Use Your Pencil: Set A

Organization: Individual/whole class

Procedure:

1. Distribute to each student a copy of BLM 6.N.9.2.
2. Provide students with the following instructions:
   a) Work individually using a pencil only.
   b) Apply the order of operations in solving each question.
3. When the set is completed, discuss with the class how the order of operations helped solve the multi-step problems.

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Work individually on a set of problems.
- ❒ Solve problems using a pencil only.
- ❒ Apply the order of operations to solve multi-step problems without technology.

- **Apply the order of operations to solve multi-step problems with or without technology.**

Materials:

- pencil
- BLM 6.N.9.3: Use Your Pencil: Set B

Organization: Individual/pairs

Procedure:

1. Distribute to each student a copy of BLM 6.N.9.3.
2. Provide students with the following instructions:
   a) Work individually using a pencil only.
   b) Apply the order of operations in solving each question.
   c) When the set is completed, compare your work with your partner.
3. When the set is completed, discuss with your partner whether the order of operations helped you both get the same solution to the multi-step problems.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Work individually on a set of problems.
- ❐ Cooperate with a partner.
- ❐ Apply the order of operations to solve multi-step problems without technology.

Suggestions for Instruction

- **Apply the order of operations to solve multi-step problems with or without technology.**

Materials:

- calculator
- BLM 6.N.9.4: Use Your Calculator: Set A
- BLM 6.N.9.5: Use Your Calculator: Set B

Organization: Individual/small groups

Procedure:

1. Use two overhead projectors.
   a) Place a transparency of BLM 6.N.9.4 on one overhead projector
   b) Place a transparency of BLM 6.N.9.5 on the second overhead projector
2. Provide students with the following instructions:
   a) Choose either set A or set B to solve.
   b) Work individually using a calculator.
   c) Apply the order of operations in solving each question.
   d) When the set is completed, compare your work with the other group members who chose the same set as you.
   e) Discuss how the order of operations helped you all get the same solution to the multi-step problems.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Work individually on a set of problems.
- ❐ Cooperate with group members.
- ❐ Use a calculator to solve a problem.
- ❐ Apply the order of operations to solve multi-step problems with technology.

- **Apply the order of operations to solve multi-step problems with or without technology.**

Materials:
- BLM 5–8.6: Hundred Chart
- three 10-sided dice
- paper

Organization: Whole group

Procedure:
1. Roll three numbers and record them on the board.
2. Allow students one minute to use the three numbers and the order of operations to create as many different answers as they can. They will record their number sentences on paper. For example, if the numbers rolled were 4, 9, and 2, students could write the following:

   $4 + 9 + 2 = 15$

   $9 - 4 \times 2 = 1$

   $(9 - 4) \times 2 = 10$

   Etc.
3. Have students mark the numbers they find onto BLM 5-8.6.
4. The first student to create a 3-by-3 grid (or whatever you choose) wins.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Work individually on a set of problems.
- ❒ Cooperate with group members.
- ❒ Use a calculator to solve a problem.
- ❒ Apply the order of operations to solve multi-step problems with technology.

Putting the Pieces Together

Tina's Ten Turkeys

Purpose: The purpose of this activity is for students to connect the order of operations to real life. Students will need to apply their knowledge of the four basic mathematical operations (addition, subtraction, multiplication, and division), as well as the use of brackets. They will also need to be able to draw simple geometric shapes, such as a triangle, square, and rectangle. The processes that are demonstrated by this task are problem solving, reasoning, communication, and connection.

Curricular Links: This task can be linked to reading and science.

Materials/Resources:

- BLM 6.N.9.6: Tina's Ten Turkeys
- Scrap paper
- Recipe cards
- Poster paper
- Markers

Organization:

- Small groups

Procedure:

Teacher's instructions:

Arrange the classroom so that it is conducive to small-group work.

Hand out the following to each group:

- Several pieces of scrap paper
- Three recipe cards
- A poster-sized paper
- A marker
- A copy of BLM 6.N.9.6
- A copy of the instruction sheet

Students' instructions:

1. Work together.
2. Have one group member carefully read "Tina's Ten Turkeys" to the group.
3. Decide amongst yourselves who will do each task, and use your geometry set to do the following:
   a) Draw a large triangle on one recipe card.
   b) Draw a large square on the second recipe card.
   c) Draw a large rectangle on the third recipe card.

4. Above the triangle, write “omelet.”
5. Above the square, write “waffle.”
6. Above the rectangle, write “cake.”
7. Inside the triangle, write the numeral representing the number of eggs it takes to make an omelet.
8. Inside the square, write the numeral representing the number of eggs it takes to make a waffle.
9. Inside the rectangle, write the numeral representing the number of eggs it takes to make a cake.
10. Discuss each of the following before you do them on your scrap paper:
    a) Show how using the order of operations helps you find out how many turkey eggs Tina would have used up during week 1.
    b) Show how using the order of operations helps you find out how many turkey eggs Tina would have leftover at the end of week 1.
    c) Show how using the order of operations helps you find out how many turkey eggs Tina would have used up during week 2.
    d) Show how using the order of operations helps you find out how many turkey eggs Tina would have used up during the two-week period.
    e) Show how using the order of operations helps you find out how many turkey eggs Tina would have leftover at the end of week 2.
11. Discuss and check your work.
12. Make corrections if needed.
13. Use the marker to write the corrected version of your work on the poster-sized paper.
14. Place the poster on the board.

Assessment:

Use the following observation checklist to assess students’ learning.

The student can do the following:	Yes	No	Comment
Use numerals and mathematical operations to solve a word problem.			
Use the order of operations correctly.			
Explain the order of operations.			
Analyze their work.			
Effectively communicate with peers.			
Cooperate with peers.			

Extension:

Taking it further

Based on the ideas in this task, create your own activity using the order of operations.

Grade 6 Mathematics

Patterns and Relations

Grade 6: Patterns and Relations (Patterns) (6.PR.1, 6.PR.2)

Enduring Understanding(s):

Words, tables, graphs, and expressions are different representations of the same pattern.

General Learning Outcome(s):

Use patterns to describe the world and solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
6.PR.1 Demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, PS, R]	➢ Generate values in one column of a table of values, values in the other column, and a pattern rule. ➢ State, using mathematical language, the relationship in a table of values. ➢ Create a concrete or pictorial representation of the relationship shown in a table of values. ➢ Predict the value of an unknown term using the relationship in a table of values and verify the prediction. ➢ Formulate a rule to describe the relationship between two columns of numbers in a table of values. ➢ Identify missing elements in a table of values. ➢ Identify and correct errors in a table of values. ➢ Describe the pattern within each column of a table of values. ➢ Create a table of values to record and reveal a pattern to solve a problem.
6.PR.2 Represent and describe patterns and relationships using graphs and tables. [C, CN, ME, PS, R, V]	➢ Translate a pattern to a table of values and graph the table of values (limit to linear graphs with discrete elements). ➢ Create a table of values from a pattern or a graph. ➢ Describe, using everyday language, orally or in writing, the relationship shown on a graph.

Prior Knowledge

Students may have had experience with the following:

- Adding and subtracting whole numbers, each numeral not greater than four digits
- Multiplying whole numbers, using 2- or 3-digit numerals by 1-digit numerals
- Dividing whole numbers, using a 1-digit divisor and up to a 2-digit dividend
- Using concrete materials to reproduce a pattern shown in a table or chart
- Making predictions about subsequent elements based on the pattern rule
- Solving one-step single-variable equations, using whole numbers only

Related Knowledge

Students should be introduced to the following:

- Using concrete objects, pictures, and symbols to demonstrate an understanding of integers
- Drawing conclusions after they create, label, and interpret line graphs
- Solving problems by graphing collected data and analyzing the graph

Background Information

In earlier grades, students were developing their understanding of repeating patterns, increasing patterns, and decreasing patterns by using manipulatives, sounds, actions, diagrams, and numbers. Students later used tables and charts to identify, describe, and reproduce patterns, identify and explain mathematical relationships, and determine a pattern rule. Look at the following pattern:

□□ □□□□ □□□□□□ □□□□□□□□ □□□□□□□□□□

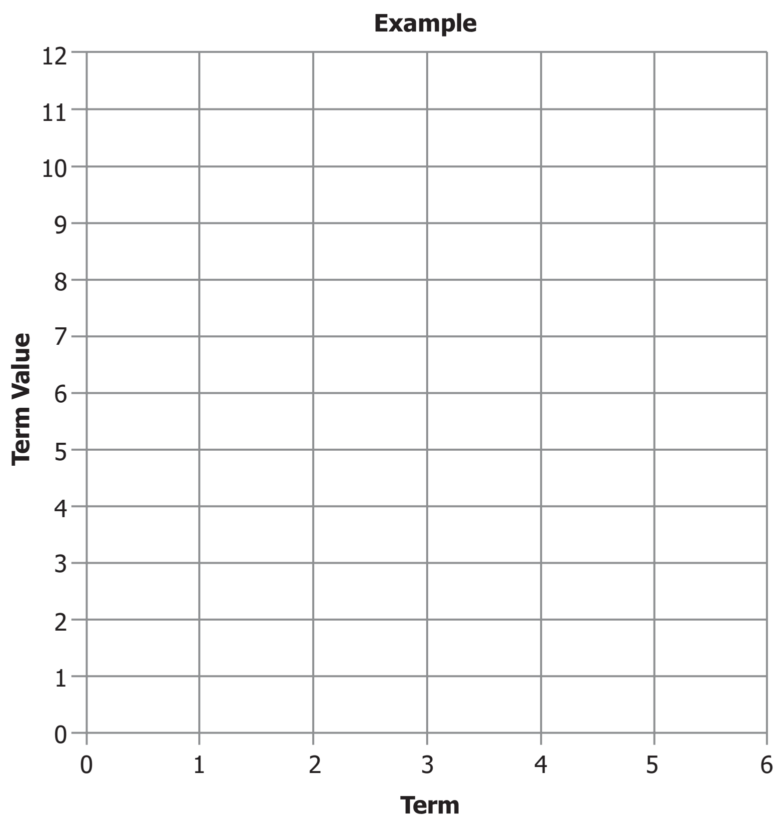
Mathematical patterns, such as the one above, can be used to give elaborate visual representations of changes. They show a step-by-step change in quantities. The step-by-step change in quantities can also be represented in a table, as shown here.

Term	1	2	3	4	5
Term value	2	4	6	8	10

This table shows that the value of each term is two more than the value of the previous term. This type of generalization (rule), by which you can find the value of a term based on the term value of the previous terms, is called a r**ecursive generalization**. Along with using pictorial representations, tables, and words, you can also use algebraic expressions to represent patterns or describe rules. The recursive generalization that describes the above pattern can be expressed by the algebraic expression $n + 2$. Using a recursive generalization works well if you have a small number of terms, such as in the above example. Finding the value of the 86th term, though, would take a lengthy process, since you would need to find the value of each of the 85 terms preceding it. It is also possible to look at this table in a slightly different way.

To find the value of a particular term in the above shown table, we simply take the term number and double it. For example, we can see two squares in the first term and we say that the first term has a term value of 2; similarly, we can say that the third term has a term value of 6. The rule for the above shown table of values is $2n$. This means that if we extend the above shown pattern, term 6 would have a term value of 12, term 10 would have a term value of two times ten, which is twenty ($2 \times 10 = 20$), and term 86 would have a term value of two times eighty-six, which is one hundred seventy-two ($2 \times 86 = 172$). This type of generalization (rule), which expresses the relationship between the term number and its value, is called an **explicit generalization**. Students might also benefit from a discussion concerning the two types of generalizations as stated above (a recursive generalization and an explicit generalization).

Transferring information from tables to graphs will be easier for students if they are reminded to label the horizontal axis as the "term" and the vertical axis as the "term value."



It might be helpful for students if they are reminded that a table of values and a graph are two ways of representing the same information. Ask students to describe how the two representations are similar.

Mathematical Language

data
element
explicit generalization
graph
pattern
pattern rule
recursive generalization
table of values
term

Learning Experiences

Assessing Prior Knowledge

Materials:

- overhead projector
- transparencies
- paper and pencil

Organization: Groups of four (seated)

Procedure:

1. Tell students that the following few lessons will be spent on expanding their knowledge of patterns.
2. Tell them to put on their "thinking caps" and see what they can remember from Grade 5 or earlier grades.
3. Place the word "Patterns" on an overhead transparency and ask students: "What do you know about patterns? Discuss in your groups your ideas about patterns."
4. Listen to their discussions. Are these discussions revealing their knowledge as to the following questions: What is a pattern? What is the next term? How do I find the next term? What is the pattern rule?
5. Ask students to give an example of a pattern, and write down what makes their example a pattern.

Suggestions for Instruction

- **Generate values in one column of a table of values, values in the other column, and a pattern rule.**
- **Formulate a rule to describe the relationship between two columns of numbers in a table of values.**
- **Create a table of values to record and reveal a pattern to solve a problem.**

Materials:

- BLM 6.PR.1: Pattern Introduction

Procedure:

1. Using an overhead projector, show the class the following pattern, as found in BLM 6.PR.1.

2. Ask students to discuss the following questions:
   a) How many terms are there in this pattern?
   b) How many circles do you see in the first term?
   c) How many circles do you see in the second term?
3. Let them discuss with their group members what a table of values would look like for this pattern, and then, using a pencil and paper, let each of them draw the table of values.
4. Ask students to discuss what pattern rule they see, and write it down.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Generate values in one column of a table of values, values in the other column, and a pattern rule.
- ❒ Formulate a rule to describe the relationship between two columns of numbers in a table of values, as in the above example $n + 1$.
- ❒ Create a table of values to record and reveal a pattern to solve a problem.

Suggestions for Instruction

- **State, using mathematical language, the relationship in a table of values.**
- **Create a concrete or pictorial representation of the relationship shown in a table of values.**
- **Predict the value of an unknown term using the relationship in a table of values and verify the prediction.**
- **Formulate a rule to describe the relationship between two columns of numbers in a table of values.**

Materials:

- BLM 6.PR.2: Horizontal Table #1

Procedure:

1. Display the following table on a poster-size paper, or distribute to students copies of BLM 6.PR.2.

Term	1	2	3	4	5
Number of Tiles	3	5	7	9	11

2. In groups of two, have students discuss the pattern.
3. Have each pair of students create the pattern with building blocks.
4. Ask each student to draw the pattern on paper.
5. Ask students to predict how many tiles term 6 would have if the pattern were extended.
6. Ask students to predict how many tiles term 32 would have if the pattern were extended.
7. Ask students to formulate a rule using words, symbols, and diagrams (emphasize multiple representations).
8. Ask students to verify their predictions.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ State, using mathematical language, the relationship in a table of values.
- ❒ Create a concrete or pictorial representation of the relationship shown in a table of values.
- ❒ Predict the value of an unknown term using the relationship in a table of values, and verify the prediction, as 13 is the value of term 6.
- ❒ Formulate a rule to describe the relationship between two columns of numbers in a table of values (the pattern above could be described as 2 more than the number before, one more than twice the term number, or $2t + 1$).

Note: It is important that students are able to see both the pattern in the number of tiles (recursive generalization) and the relationship between the term number and the number of tiles (explicit generalization). Both of these are valuable skills.

Suggestions for Instruction

- **Formulate a rule to describe the relationship between two columns of numbers in a table of values.**
- **Identify missing elements in a table of values.**
- **Describe the pattern within each column of a table of values.**
- **Identify and correct errors in a table of values.**

Materials:

- BLM 6.PR.3: Horizontal Table #2

Procedure:

1. Have students sit in groups of four. Let each of them have a copy of the pattern.
2. Using an overhead projector, show the following pattern from BLM 6.PR.3 to the class:

Term	1	2	3	4			7	8	9
Term Value	9	19	29		49	59		87	91

3. Ask students to discuss and fill in the missing elements.
4. Ask students to discuss what pattern rule they see and to write it down.
5. Ask them how the term values are related to each other.
6. Ask them if they can identify the two errors in the table of values and correct them.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Formulate a rule to describe the relationship between two rows of numbers in a table of values, such as the term value is one less than 10 times the term number or $10n - 1$ for the above table.
- ❒ Identify missing elements in a table of values.
- ❒ Describe the pattern within each row of a table of values. (In the second row, each number is 10 more than the number before.)
- ❒ Identify and correct errors in a table of values. (Errors: 87 and 91 corrections: 79 and 89).

Suggestions for Instruction

- **Formulate a rule to describe the relationship between two rows of numbers in a table of values.**
- **Identify missing elements in a table of values.**
- **Create a table of values to record and reveal a pattern to solve a problem.**

Procedure:

1. Hand out the following question to each student.

   The music teacher was happy to see a daily increase in students in the choir. On the first day of classes, eight students came to choir. On the second day, 11 students came to choir. The following day, three new students came to choir. How many students will show up to choir on day 10 if this pattern continues?

Day	1	2	3	4	5
Number of Students					

2. Ask students to enter all known information into the table of values, working individually.

3. Ask them to find the pattern rule (half of the class can use the recursive generalization and the other half of the class can use the explicit generalization), and complete the missing values.
4. Find the answer to the question.
5. How many students would show up to choir on day 25 if this pattern continued?
6. Have a class discussion on the different methods of completing the problem (e.g., ask students to list the positive aspects of each method).

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Create a table of values to record and reveal a pattern to solve a problem.
- ❒ Formulate a rule to describe the relationship between two rows of numbers in a table of values, such as the above rule $(n - 1)(3) + 8$.
- ❒ Identify missing elements in a table of values.

Suggestions for Instruction

- **Formulate a rule to describe the relationship between two columns of numbers in a table of values.**
- **Identify missing elements in a table of values.**
- **Describe the pattern within each column of a table of values.**

Materials:

- BLM 6.PR.4: Vertical Table #1

Procedure:

1. Tell students that today you want them to be little detectives.
2. Show on the overhead projector BLM 6.PR.4, and read the text part to them.
3. Ask students to look at the table carefully and write in their journals what these numbers could mean.
4. Ask students to discuss what pattern rule they see and write it down.
5. Ask them to fill in the missing numbers.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Formulate a rule to describe the relationship between two columns of numbers in a table of values, such as the term value is two less than three times the term number, or $3n - 2$ for the vertical table #1.
- ❒ Identify missing elements in a table of values.
- ❒ Describe the pattern within each column of a table of values, such as in the term value column each number is three more than the number before it.

Suggestions for Instruction

- **Create a table of values from a pattern or a graph.**
- **Describe, using everyday language, orally or in writing, the relationship shown on a graph.**

Materials:

- BLM 6.PR.4: Vertical Table
- BLM 6.PR.5: King Klonig's Graph

Procedure:

1. Distribute copies of BLM 6.PR.5 on coloured paper, and have students sit in groups with other students who received the same coloured paper (about four or five groups).
2. Ask students to discuss the following questions with their group members:
   a) What could King Klonig's graph represent?
   b) How many terms are shown on King Klonig's graph?
   c) What is the term value of the first term?
   d) What is the term value of the last term shown on this graph?
   e) How simple or difficult would it be to predict the term value of term 38?
3. Let each group choose one person to present the main ideas of their discussion to the class.
4. Ask "What would a table of values look like for this pattern?" Then, using a pencil and paper, let each of them draw the table of values.

5. Ask students to discuss what pattern rule they see and, using everyday language, describe it in their journals.
6. Read out some answers orally.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Create a table of values from a pattern or a graph.
- ❒ Describe, using everyday language, orally or in writing, the relationship shown on a graph, as the term value is five more than two times the term number for this example.

Suggestions for Instruction

- **Translate a pattern to a table of values and graph the table of values (limit to linear graphs with discrete elements).**
- **Describe, using everyday language, orally or in writing, the relationship shown on a graph.**

Materials:

- BLM 6.PR.6: Lily's Pattern

Procedure:

1. Distribute copies of BLM PR.6.
2. Ask for four volunteers, each representing a term, to clap according to his or her term value.
3. Ask students to create a table of values for this pattern in their notebooks, and then extend the table to term 6.
4. Then ask "What would the value of term 10 be if the pattern were extended?" Let them give their answers orally.
5. Tell students to create a graph for their table of values.
6. Have a class discussion on what pattern rule they see, and then ask them to describe it in their journals using everyday language.

Observation Checklist

- ☑ Check students' replies to determine whether they can do the following:
  - ❐ Translate a pattern to a table of values and graph the table of values (limit to linear graphs with discrete elements).
  - ❐ Describe, using everyday language, orally or in writing, the relationship shown on a graph, as the term value is one less than two times the term number for the above-stated example.

Putting the Pieces Together

How Many Sticks to Each Bottle Cap?

Purpose: The purpose of this activity is to have students refresh and use their knowledge of number concepts, patterns, and statistics. Specifically, this activity was designed for students to do the following:

- Count objects and record the data
- Compare and order numbers
- Arrange numbers in ascending order
- Add, subtract, multiply, and divide whole numbers
- Determine the rule for the pattern
- Predict larger and smaller values

As well, the following mathematical processes are demonstrated by this activity:

- Communication
- Connections
- Mental mathematics
- Problem solving
- Reasoning

Curricular Links: Science

Materials/Resources:

- several boxes
- a lot of bottle caps and sticks
- paper and pencil

Organization: Small Groups

Inquiry:

Scenario

Arrange desks in such a way that students can work in small groups. Choose your pattern rule. Based on your pattern rule, prepare a box for each group containing some bottle caps and some sticks. Let's say you choose the rule: the number of sticks = twice the number of bottle caps +3, and that you have six groups. Then, one box might contain eight bottle caps and 19 sticks; another box might contain 17 bottle caps and 37 sticks; still another box might contain 24 bottle caps and 51 sticks. You can choose larger or smaller amounts for the other three boxes as long as the pattern rule for each box is the same.

Procedure:

- Have a box with a predetermined number of bottle caps and sticks on a desk for each group.
- Tell students that each group has some treasures in their boxes and they need to figure out the similarities or differences.
- Ask them to discuss which mathematical processes they need to use, and write them down.
- Have each group post their findings on the board at the front of the classroom.
- Tell students to discuss in their groups what they need to do in order to find a pattern rule.
- Tell students to record their processes and rules.
- Have each group post their new work on the board under their previous work.

Assessment:

Check students' replies to determine whether they can do the following:

- ❑ Compare and order numbers
- ❑ Arrange numbers in ascending order
- ❑ Add, subtract, multiply, and divide whole numbers
- ❑ Determine the rule for the pattern
- ❑ Predict larger and smaller values and verify their predictions.

Grade 6: Patterns and Relations (Variables and Equations) (6.PR.3, 6.PR.4)

Enduring Understanding(s):

Preservation of equality is used to solve equations.

Number patterns and relationships can be represented using variables.

General Learning Outcome(s):

Represent algebraic expressions in multiple ways.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
6.PR.3 Represent generalizations arising from number relationships using equations with letter variables. [C, CN, PS, R, V]	➛ Write and explain the formula for finding the perimeter of any rectangle. ➛ Write and explain the formula for finding the area of any rectangle. ➛ Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication (e.g., $a + b = b + a$ or $a \times b = b \times a$). ➛ Describe the relationship in a table using a mathematical expression. ➛ Represent a pattern rule using a simple mathematical expression, such as $4d$ or $2n + 1$.

(continued)

Specific Learning Outcome(s):	**Achievement Indicators:**
6.PR.4 Demonstrate and explain the meaning of preservation of equality, concretely, pictorially, and symbolically. [C, CN, PS, R, V]	➢ Model the preservation of equality for addition using concrete materials, such as a balance or using pictorial representations, and orally explain the process. ➢ Model the preservation of equality for subtraction using concrete materials, such as a balance or using pictorial representations, and orally explain the process. ➢ Model the preservation of equality for multiplication using concrete materials, such as a balance or using pictorial representations, and orally explain the process. ➢ Model the preservation of equality for division using concrete materials, such as a balance or using pictorial representations, and orally explain the process. ➢ Write equivalent forms of an equation by applying the preservation of equality, and verify using concrete materials [e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$].

Prior Knowledge

Students may have had experience with the following:

- Using manipulatives and diagrams (0–100) to demonstrate and explain the meaning of equality and inequality
- Solving problems by identifying and explaining mathematical relationships using charts and diagrams
- Solving one-step equations that involve a symbol that represents an unknown number
- Making predictions about subsequent elements based on the pattern rule
- Solving one-step single-variable equations and problems involving these equations using whole numbers only

Related Knowledge

Students should be introduced to the following:

- Using whole numbers to explain and apply the order of operations, excluding exponents
- Solving problems by demonstrating an understanding of the relationships within tables of values

Background Information

In prior grades, students were exposed to translating word problems into equations and then solving them. Those equations consisted of whole number operations, such as $2 + 5 = 7$, $3 = 27 \div 9$, or $7 - 2 = 8 - 3$. Students were solving those equations by using a variety of methods, such as concrete objects, pictorial representations, and an empty square representing the unknown value.

Later, students were introduced to using letters as representations of unknown quantities. This change from using simple number operations to using letter operations is a big leap for students. Instead of having to solve something familiar, such as $28 + \square = 35$, they are looking at $28 + x = 35$. Students need to get comfortable using letters for representing unknown quantities. They need to be reminded that the letter representation means the same thing as the empty box; both can be used to represent an unknown quantity.

Using letters is simply one way to write a mathematical rule.

In a rectangle, the two opposite sides are equal in measure.

If we want to find a general rule that describes the perimeter of each of the above-shown rectangles, we could use words, such as the following:

1. the perimeter of each of these rectangles is equal to its length plus its width plus its length plus its width ($l + w + l + w$)
2. stated more concisely, the perimeter of each of these rectangles is equal to the sum of twice its length plus twice its width ($2l + 2w$)
3. the perimeter or each of these rectangles is equal to twice the sum of its length plus its width $2(l + w)$

We can also describe the perimeter of the above-shown rectangles by a mathematical equation using letters for variables, such as $p = 2l + 2w$, $p = 2w + 2l$, $2l + 2w = p$, $2(l + w) = p$, $p = 2(w + l)$, or $p = 2(l + w)$.

(Note: All of these equations are equivalent forms of the same equation.) The general formula tells us that **the perimeter of a specific rectangle is always equal to twice the sum of the length plus the width of that specific rectangle, no matter how small or how large that rectangle might be. If the dimensions of the rectangle increase, then the perimeter also increases; if the dimensions decrease, then the perimeter decreases as well. The equality is always preserved.**

Note: It might be beneficial to students if you review the commutative property of addition while discussing the perimeter of a rectangle (i.e., $l + w = w + l$).

In a rectangle, each of the four interior angles is a right angle.

If we want to find a general rule that describes the area of each of the above-shown rectangles, we could use words, such as: the area of each of these rectangles is equal to its length times its width or its width times its length. We can also describe the area of the above-shown rectangles by a mathematical equation using letters for variables, such as $a = l \cdot w$, $a = w \cdot l$, $w \cdot l = a$, or $l \cdot w = a$.

Note: All of the equations are equivalent forms of the same equation. The general formula tells us that the area of a specific rectangle is always equal to its length times its width regardless of the size of the rectangle. The equality is always preserved.

Note: It might be beneficial to students if you reviewed the commutative property of multiplication while discussing the area of a rectangle (i.e., $l \cdot w = w \cdot l$).

Mathematical Language

equal

equation

mathematical expression

unknown

variable

Assessing Prior Knowledge

Materials:

- BLM 5–8.2: Concept Description Sheet #1
- pencil
- overhead projector

Organization: Small groups

Procedure:

1. Tell students that the following few lessons will be spent on expanding their knowledge of equations.
2. Ask them to try to remember what they learned in Grade 5 or earlier grades about equations.
3. Place the word “Equation” in the oval middle section on Concept Description Sheet #1 overhead transparency and ask them: “What do you know about equations?”
4. Tell students to write all they can about equations on their concept description sheet.
5. Walk around and observe student responses for each section of the concept description sheet. Are these responses revealing their knowledge as to the following questions: What is an equation? What is not an equation? What are the characteristics of an equation? What is a good picture or a diagram of an equation?
6. After students complete the concept sheet on equations, write on the board an arithmetic equation, such as $18 + 7 = 25$, and an algebraic equation, such as $6y + 4 = 22$.
7. Ask students what is different about these two equations, and have a discussion.
8. Tell students that variables are very important in mathematics. Then ask them what they remember about variables.
9. Write the word “Variable” on the board and tell students to do a concept sheet on variables using the same procedure as they did for the equation.
10. Again, walk around and observe student responses for each section of the concept description sheet. Consider whether these responses reveal their knowledge as to the following questions: What is a variable? What is not a variable? What are the characteristics of a variable? What is a good picture or a diagram of a variable?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Define what is an equation.
- ❒ Identify examples and non-examples of an equation.
- ❒ Define what is a variable.
- ❒ Identify examples and non-examples of a variable.
- ❒ Tell the difference between a simple (arithmetic) equation and an equation with a variable (algebraic equation).

Suggestions for Instruction

- **Write and explain the formula for finding the perimeter of any rectangle.**

Materials:

- large brown envelopes
- white envelopes
- rectangular sheets of paper
- recipe cards
- blank business cards
- rulers

Organization: Individual

Procedure:

1. Tell students that you will hand out five items, and that each item will have a rectangular shape.
2. Hand out a large brown envelope containing a rectangular sheet of paper, a recipe card, a blank business card, and a white envelope. Make sure none of the items are identical in size to any other item.
3. Tell students that each item will need a ruler, and then provide the following instructions:
   a) One item at a time, measure the perimeter and record your measurements on the item.
   b) When all five items are measured and recorded, write your observations in your journal.

c) Write a formula for finding the perimeter of any rectangle.

d) Explain your formula.

4. Have a class discussion on what the students noticed about all these objects. Ask them what was common to all these perimeters, and then have a few (e.g., three or four) students write their formula on the chalkboard and explain their formula to the class.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Tell that the opposite sides of a rectangle are equal in measure.
- ❒ Write the formula for finding the perimeter of any rectangle.
- ❒ Explain the formula for finding the perimeter of any rectangle.

Suggestions for Instruction

- **Write and explain the formula for finding the area of any rectangle.**

Materials:

- BLM 6.PR.7: Mrs. Dean's Carpet

Organization: Groups of two

Procedure:

1. Have students work in groups of two.
2. Distribute to each student a copy of BLM 6.PR.7.
3. Ask students to read the question and discuss it with their partner.
4. Ask students to create a table in their notebooks. (Observe that they write the correct numerals for the words.)
5. Have students discuss with their partner how Mrs. Dean figured out how much carpet she needed of each width.
6. Ask students to write a formula for finding the area of any rectangle, and to explain in their journals how they figured out their formula.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❐ Multiply a double-digit by a single-digit whole number.
- ❐ Multiply a fraction by a single-digit whole number.
- ❐ Write the formula for finding the area of any rectangle.
- ❐ Explain the formula for finding the area of any rectangle.

Suggestions for Instruction

- **Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication (e.g., $a + b = b + a$ or $a \times b = b \times a$).**

Materials:

- BLM 6.PR.8: Poff and Gloff's Math Homework

Organization: Individual

Procedure:

1. Distribute a copy of BLM 6.PR.8.
2. Tell students to read the question and do the work individually.
3. Check that students are on the right track.
4. Ask a student to write his or her equation on the chalkboard and explain it.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❐ Write a simple mathematical expression.
- ❐ Develop equations using letter variables that illustrate the commutative property of addition (e.g., $a + b = b + a$).
- ❐ Justify equations using letter variables that illustrate the commutative property of addition (e.g., $a + b = b + a$).

Suggestions for Instruction

- **Develop and justify equations using letter variables that illustrate the commutative property of multiplication (e.g., *a* x *b* = *b* x *a*).**

Materials:

- BLM 6.PR.9: Equation Pairs

Organization: Individual/whole class

Procedure:

1. Distribute a copy of BLM 6.PR.9 to each student.
2. Ask students to write on their sheets the two appropriate equations for each pair of measurements.
3. Ask a student to write on the chalkboard a pair of numbers of his or her choice and the two appropriate multiplication equations.
4. Have a class discussion based on their work. Discuss the commutative property of multiplication.
5. Tell students to write an equation using letter variables, and to explain in their journals why their equation works.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Multiply simple mathematical expressions.
- ❒ Develop equations using letter variables that illustrate the commutative property of multiplication (e.g., $a \times b = b \times a$).
- ❒ Justify equations using letter variables that illustrate the commutative property of multiplication (e.g., $a \times b = b \times a$).

- **Describe the relationship in a table using a mathematical expression.**

Materials:

- BLM 6.PR.10: Baskets and Oranges
- a large sheet of chart paper

Organization: Whole class

Procedure:

1. Copy the table from BLM 6.PR.10 on a large sheet of chart paper and post it on the board.
2. Ask students to analyze the table and write their observations in their math notebooks.
3. Ask a student to provide orally his or her observations to the class.
4. Ask the class to use a mathematical expression to describe the relationship in the table "Baskets and Oranges."
5. As a class, discuss the observations students made and the mathematical expressions they used to describe the relationship.
6. Have students analyze another pattern, such as $3n$, $5n$, or $2n + 1$. (Fit the level of difficulty according to your students' needs).

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❐ Multiply two single-digit numbers.
- ❐ Write a simple mathematical expression.
- ❐ Describe the relationship in a table using a mathematical expression.

Suggestions for Instruction

- **Represent a pattern rule using a simple mathematical expression, such as 4*d* or 2*n* + 1.**

Organization: Whole class

Procedure:

Tell students that today's math class will be a little different. Then say the following:

1. "Look at my hands when I wave them and tell me how many fingers you see." (Wave both of your hands in the same direction and slowly, showing all 10 fingers.)
2. "How many fingers do you see waving at you?"
3. "We need one person to come to the front of the class and wave his or her hands with me." (Stand side by side and both of you wave slowly showing all your fingers.)
4. "How many fingers do you see waving now?"
5. "We need one more person to come to the front of the class and wave his or her hands with us." (Stand side-by-side and all three of you wave slowly showing all your fingers.)
6. "How many fingers do you see waving now?"
7. "If we had one more person with us at the front, how many fingers would you see then?"
8. "How many fingers would you see if nine of us were standing and waving?"
9. "Thanks to all of you for participating. And now, write in your journals a simple mathematical expression to describe the pattern rule for our little game, and describe how it works."

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Multiply by 10 orally.
- ❒ Write a simple mathematical expression.
- ❒ Represent a pattern rule using a simple mathematical expression.

Suggestions for Instruction

- **Model the preservation of equality for addition using concrete materials, such as a balance or using pictorial representations, and orally explain the process.**
- **Model the preservation of equality for subtraction using concrete materials, such as a balance or using pictorial representations, and orally explain the process.**

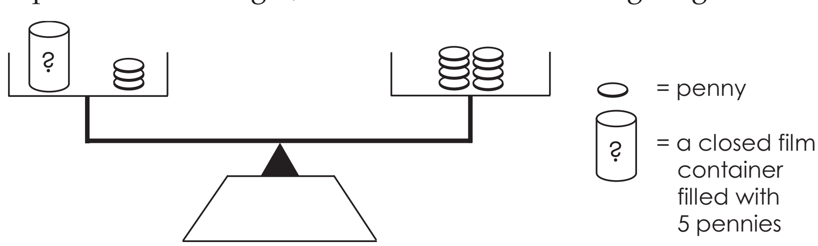
Materials:

- film containers
- pennies
- balance scales

Organization: Small groups

Procedure:

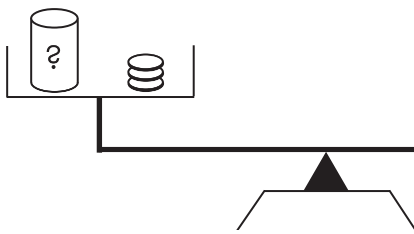
1. Have students sit in small groups (three to five students per group).
2. Distribute the following to each group:
   a) a closed film container containing five pennies
   b) a balance scale
   c) an empty film container identical to the filled container
   d) 11 pennies
3. Tell students to
   a) balance the scale with the empty film container on the left pan, and then remove the container
   b) put the filled film container and three pennies on the left pan of the scale and eight pennies on the right, as shown in the following diagram



4. Tell students to answer the following:
   a) "How many pennies are in the container?"
   b) "If there are eight pennies on one side, how many pennies are on the other side?"
   c) "What will happen if you remove three pennies from each side?" Remove the pennies. Empty the container to verify the number of hidden pennies.

5. Ask students to discuss in their groups what would happen if they added five more pennies to each side, and to make a pictorial representation in their journals.
6. Have students create their own equations using the containers and pennies, and have their partners solve them.

Note: Another way to account for the mass of the container is to omit the empty container in step 3 and to add the open empty container and lid to the right side of the balance.



Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Balance an equation (addition and subtraction).
- ❒ Model the preservation of equality for addition.
- ❒ Model the preservation of equality for subtraction.

Suggestions for Instruction

- **Model the preservation of equality for multiplication using concrete materials, such as a balance or using pictorial representations, and orally explain the process.**
- **Model the preservation of equality for division using concrete materials, such as a balance or using pictorial representations, and orally explain the process.**

Organization: Whole class

Procedure:

1. Draw two large circles on the chalkboard and put an equals sign between them.
2. Draw six little triangles in each large circle.
3. Have students discuss as one group the meaning of your diagram.
4. Draw two new circles underneath the first two circles and also place an equals sign between the new circles.

5. Ask the students what would happen if you doubled each side. Discuss.
6. Ask one student to place the correct amount of little triangles into each new circle.
7. Tell students to draw two circles in their journals and show what would happen if they divided each side by three.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Multiply simple numbers orally.
- ❒ Model the preservation of equality for multiplication
- ❒ Model the preservation of equality for division.

Suggestions for Instruction

- **Write equivalent forms of an equation by applying the preservation of equality, and verify using concrete materials [e.g., 3*b* = 12 is the same as 3*b* + 5 = 12 + 5 or 2*r* = 7 is the same as 3(2*r*) = 3(7)].**

Materials:

- BLM 6.PR.11: Equivalent Forms of an Equation

Organization: Small groups

Procedure:

1. Distribute to each student a copy of BLM 6.PR.11.
2. Have students work in small groups (three to five students).
3. Distribute to each group a small container containing 40 buttons.
4. Tell students to discuss the work with their group members, and then write the equations and verify their work.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Add, subtract, multiply, and divide small numbers (under 100).
- ❒ Write equivalent forms of an equation by applying the preservation of equality.
- ❒ Verify equivalent forms of an equation using concrete materials.

Suggestions for Instruction

- **Write equivalent forms of an equation by applying the preservation of equality, and verify using concrete materials [e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$].**

Materials:

- BLM 6.PR.12: I Have, Who Has…?

Organization: Whole class

Procedure:

1. Tell students that they will be playing an "equivalent forms of an equation" version of the game "I have, who has…?". Cut BLM 6.PR.12 into card-size pieces. Explain that each student will get one card (some students may get two cards if there are fewer than 36 students in the class).
2. Have one student start the game by reading his or her card. (Help them play the game once.)
3. Have the student who has the answer to the question read his or her card.
4. Continue the game in this fashion until it gets back to the person who started the game.
5. Play the game several times, each time asking a different student to start it.
6. Divide the class into four groups.
7. Have each student make his or her equivalent forms of an equation version of the game "I have, who has…?" and play it with the other members of the group.

Extension:

When a student answers, have the student explain what operation was done to both sides to make it equivalent.

For example: Question: Who has $6t = 0$?

Answer: I have $6t + 3 = 3$ because 3 was added to both sides of the equation.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Add, subtract, multiply, and divide small numbers (under 100).
- ❒ Recognize equivalent forms of an equation.
- ❒ Write equivalent forms of an equation by applying the preservation of equality.

- **Write equivalent forms of an equation by applying the preservation of equality, and verify using concrete materials [e.g., 3*b* = 12 is the same as 3*b* + 5 = 12 + 5 or 2*r* = 7 is the same as 3(2*r*) = 3(7)].**

Materials:

- BLM 6.PR.13: Same As Cards
- BLM 6.PR.14: Same As Reply Sheet A
- BLM 6.PR.15: Same As Reply Sheet B

Organization: Groups of four

Procedure:

1. Have students work in groups of four.
2. Distribute to each group one set of BLM 6.PR.13: Same As Cards (64 cards) and two copies of Same As Reply Sheets A and B.
3. Tell students the following:
   a) Students sitting opposite each other should have the same reply sheets.
   b) The dealer will deal three cards to each student and place the rest of the cards face down in the centre.
   c) The person on the right side of the dealer will start by picking up one card from the top of the pile. If the new card has an equation equivalent to any of the four equations on his or her reply sheet, then he or she will read the two equivalent equations out loud (e.g., $3x + 7 = 3 + 7$ is the same as $3x = 3$) and place them on the reply sheet. If the new card does not match any of the equations on his or her reply sheet, then the student keeps the card with the first three cards.
   d) The next person (opposite the dealer) repeats the same procedure. This process continues until all the cards are picked up.
   e) When there are no more cards in the centre, the next student to pick a card will ask the person to the right to give him or her a card, and the process continues until someone has all eight equivalent forms of one of the equations on his or her reply sheet.
   f) The winner (the student with all eight equivalent forms of one of the equations on his or her reply sheet) will display and read all eight forms of the equation to the group.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❐ Add, subtract, multiply, and divide small numbers (under 100).
- ❐ Recognize equivalent forms of an equation.

Suggestions for Instruction

- **Write equivalent forms of an equation by applying the preservation of equality, and verify using concrete materials [e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$].**

Materials:

- BLM 6.PR.13: Same As Cards
- BLM 6.PR.16: Same As Record Sheet

Organization: Small group

Procedure:

1. Have students work in small groups (three to five students).
2. Distribute a set of Same As Cards to each group and a copy of BLM 6.PR.16: Same As Record Sheet to each student.
3. Have one student start by picking a card and reading the equation to the group.
4. Have everyone in the group write on the record sheet next to the simple form an equivalent equation to the one read and using the same operation (e.g., card reads: $8x + 13 = 25$; under the addition column, next to $8x = 12$, the students write an equation such as $8x + 5 = 12 + 5$, $8x + 5 = 17$, $8x + 2 = 12 + 2$, $8x + 2 = 14$, or $8x + 100 = 12 + 100$).
5. Have the next student pick a card, read the equation, and follow in this fashion until the record sheet is filled.
6. Have a class discussion on the preservation of equality.
7. Tell students to write in their journals what they learned from using the Same As Cards.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❐ Add, subtract, multiply, and divide small numbers.
- ❐ Recognize equivalent forms of an equation.
- ❐ Write equivalent forms of an equation by applying the preservation of equality.

GRADE 6 MATHEMATICS

Shape and Space

Grade 6: Shape and Space (Measurement) (6.SS.1, 6.SS.2)

Enduring Understanding(s):

All measurements are comparisons.

The unit of measure must be of the same nature as the property of the object being measured.

Many geometric properties and attributes of shapes are related to measurement.

General Learning Outcome(s):

Use direct or indirect measurement to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
6.SS.1 Demonstrate an understanding of angles by ■ identifying examples of angles in the environment ■ classifying angles according to their measure ■ estimating the measure of angles using 45°, and 90°, and 180° as reference angles ■ determining angle measures in degrees ■ drawing and labelling angles when the measure is specified [C, CN, ME, V]	➛ Provide examples of angles found in the environment. ➛ Classify a set of angles according to their measure (e.g., acute, right, obtuse, straight, reflex). ➛ Sketch 45°, 90°, and 180° angles without the use of a protractor, and describe the relationship among them. ➛ Estimate the measure of an angle using 45°, 90°, 180° as reference angles. ➛ Measure, using a protractor, angles in various orientations. ➛ Draw and label an angle in various orientations using a protractor. ➛ Describe the measure of an angle as the measure of rotation of one of its sides. ➛ Describe the measure of an angle as the measure of an interior angle of a polygon.
6.SS.2 Demonstrate that the sum of interior angles is ■ 180° in a triangle ■ 360° in a quadrilateral [C, R]	➛ Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles. ➛ Explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.

Prior Knowledge

Students may have had experience with the following:

- Dividing 3-digit numerals by 1-digit numerals
- Solving one-step single-variable equations, and problems involving these equations, using whole numbers only
- Adding and subtracting 1-, 2-, and 3-digit numerals with answers to 1000

Related Knowledge

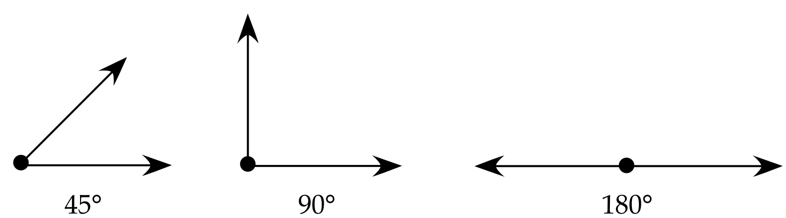
Students should be introduced to the following:

- Demonstrating and explaining the meaning of preservation of equality, concretely, pictorially, and symbolically

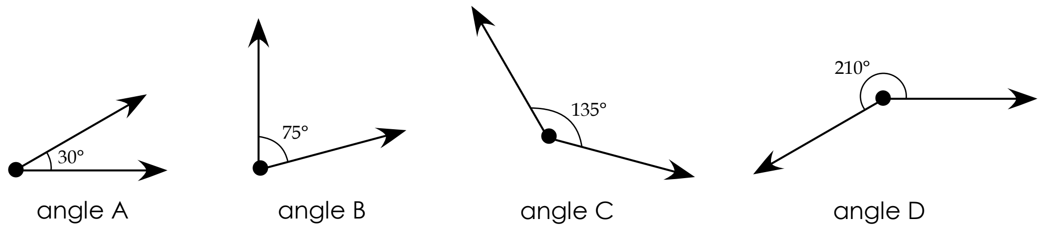
Background Information

An angle is the space between two rays or line segments that are joined at a common point. There are many different sizes of angles, some are small and some are large. Angles can be seen by observing different geometric shapes as well as the environment around us. Angles are measured in degrees using a protractor.

Grade 6 is the first year students are formally learning about angles and angle measures. This year, students will learn to measure angles using a protractor, recognize reference angles (45º, 90º, and 180º), and estimate measures of angles using the reference angles. Look at reference angles shown below:



Based on these angles, students will be able to recognize whether an angle is smaller than 45° (see angle A below), between 45° and 90° (see angle B below), between 90° and 180° (see angle C below), or bigger than 180° (see angle D below).

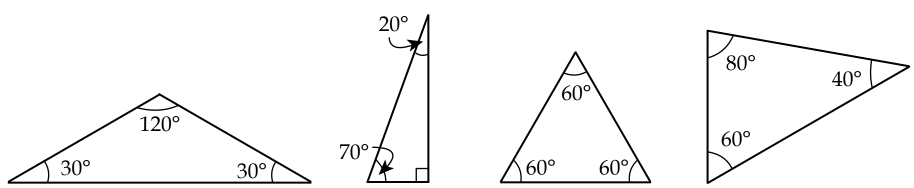


Based on the angle measure, students will learn to name and classify angles as acute (between 0° and 90°, such as angles A, B, and reference angle 45° above), right (90°, such as reference angle 90° above), obtuse (between 90° and 180°, such as angle C above), straight (180°, such as reference 180° above), and reflex (between 180° and 360°, such as angle D above).

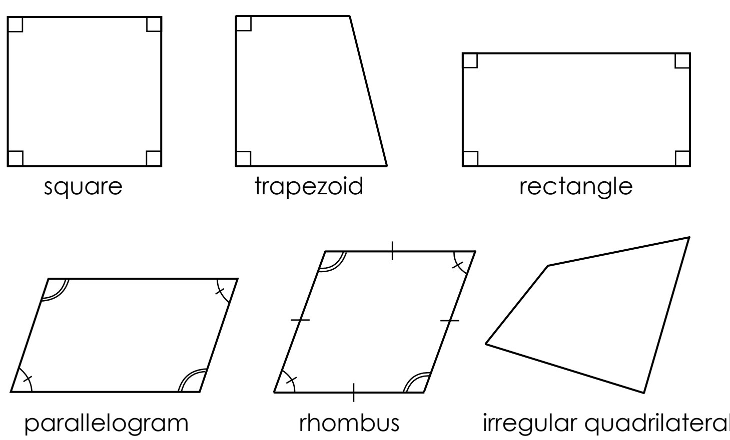
Students will also learn to identify examples of angles found in the environment, such as the space between the ceiling and the wall, the space between the ground and a lamp post, or the space between two branches on a tree.

Measuring interior angles of polygons will reveal some interesting geometric rules. Students will learn that

- the sum of interior angles is 180° in a triangle



- the sum of interior angles is 360° in a quadrilateral



Mathematical Language

angle
acute angle
degree
interior angle
polygon
quadrilateral
reflex angle
straight angle
triangle

Learning Experiences

Assessing Prior Knowledge

Materials:

- BLM 5–8.9: Centimetre Grid Paper

Organization: Individual/whole class

Procedure:

1. Distribute to each student the centimetre grid paper from BLM 5-8.9.
2. Ask students to use the centimetre grid to draw the following:
   a) Four kinds of polygons with 2 cm sides each
   b) Four kinds of polygons with 3 cm sides each
   c) Four kinds of polygons with 4 cm sides each
3. Ask them to write the name of the polygon inside each shape.
4. Ask students to write inside each polygon the number of sides it has.
5. Have one student draw one of his or her polygons on the board.
6. With a piece of paper, cover most of the polygon, leaving only one angle visible, and say to the students: "This is an angle. Who could come to the board and show another angle?"
7. Have students point out different angles inside the polygon.
8. Draw each angle separately, next to the polygon.

9. Discuss angles. Ask students questions such as the following:
   a) Are all of these angles the same?
   b) What is the same?
   c) What is different?
10. Tell students to count the number of angles they find in each polygon, and write the number inside each polygon.
11. Discuss the relationship between the number of sides and the number of angles

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Construct different kinds of polygons (i.e., triangle, quadrilateral, pentagon, and hexagon).
- ❒ Construct polygons given measure of sides.
- ❒ Name polygons according to the sides that they have.
- ❒ Recognize angles.
- ❒ Recognize the relationship between sides and angles of a polygon.

Suggestions for Instruction

- **Provide examples of angles found in the environment.**

Materials:
- BLM 5-8.2: Concept Description Sheet #1

Organization: Whole class/small group/individual

Procedure:
1. Have a class discussion on the concept of angles. Have students answer questions, such as the following:
   a) What are angles?
   b) Where can you find angles?
   c) Are all angles the same?

2 Distribute to each student a copy of BLM 5-8.2: Concept Description Sheet #1.

3 Tell students to write the word “angle” inside the oval, and complete the sheet, doing the following:

a) Describe the characteristics of an angle.

b) Provide examples of angles.

c) Provide examples of non-angles.

d) Draw pictures of angles.

4 Tell students to discuss with their group members the angles they can find

a) in the classroom

b) in their homes

c) on the playground

5 Have students record their examples of angles in their journals.

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Describe the characteristics of an angle.
- ❒ Provide examples of angles.
- ❒ Provide examples of non-angles.
- ❒ Draw pictures of angles.
- ❒ Provide examples of angles found in the classroom.
- ❒ Provide examples of angles found in their homes.
- ❒ Provide examples of angles found on the playground.

Suggestions for Instruction

- **Classify a set of angles according to their measure (e.g., acute, right, obtuse, straight, reflex).**
- **Provide examples of angles found in the environment.**

Materials:

- BLM 6.SS.1.1: Angles

Organization: Whole class/individual

Procedure:

1. Draw a right angle on the board and write "right angle" under it.
2. Tell students to look around the classroom to spot some right angles.
3. Discuss the examples of right angles they spotted in the classroom.
4. Draw a straight angle on the board and write "straight angle" under it.
5. Tell students to look around the classroom to spot some straight angles.
6. Discuss the examples of straight angles they spotted in the classroom.
7. Discuss other angles and name them.
8. Ask students to draw examples of acute, obtuse, and reflex angles on the board.
9. Distribute a copy of BLM 6.SS.1.1, and ask students to write the name of each angle inside it.
10. Discuss their classifications.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Recognize a right angle.
- ❒ Recognize a straight angle.
- ❒ Recognize an acute angle.
- ❒ Recognize an obtuse angle.
- ❒ Recognize a reflex angle.
- ❒ Provide examples of angles found in the environment.

Suggestions for Instruction

- **Sketch 45°, 90°, and 180° angles without the use of a protractor, and describe the relationship among them.**
- **Provide examples of angles found in the environment.**

Materials:

- BLM 6.SS.1.2: Reference Angles

Organization: Whole class/individual

Procedure:

1. Place on the overhead projector a transparency of BLM 6.SS.1.2.
2. Draw two vertical lines on the chalkboard in order to create three sections.
3. Write on the top part of the first section "45°", second section "90°", and third section "180°". Then say to the students "Look around the classroom and find possible examples of each angle."
4. Have a class discussion about the possible examples for each angle.
5. Ask for three volunteers (one for each section) to record possible examples of each angle, and sketch the angle (without a protractor).
6. Discuss the relationship among the three angles and their importance (reference angles).
7. Tell students to sketch a "45°", "90°", and "180°" angle without a protractor, and describe the relationship among these three angles in their journals.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Provide an example of a 45° angle.
- ❒ Provide an example of a 90° angle.
- ❒ Provide an example of a 180° angle.
- ❒ Sketch a 45° angle without a protractor.
- ❒ Sketch a 90° angle without a protractor.
- ❒ Sketch a 180° angle without a protractor.
- ❒ Describe the relationship among these three (45°, 90°, 180°) angles.

Suggestions for Instruction

- **Estimate the measure of an angle using 45º, 90º, 180º as reference angles.**
- **Measure, using a protractor, angles in various orientations.**

Materials:

- BLM 6.SS.1.1: Angles
- BLM 6.SS.1.2: Reference Angles

Organization: Individual

Procedure:

1. Distribute to each student a copy of BLM 6.SS.1.1.
2. Place a transparency of the BLM 6.SS.1.2 on the overhead projector.
3. Tell students to use the reference angles to estimate the measure of each angle on their paper, and write the estimated measure next to the angle.
4. Ask a few students to read out loud their estimated measures of each angle.
5. Say "Take out your protractors, measure each angle, and record the angle measurement next to the angle."
6. When they complete their work, ask students to record the difference between the measured angle and its estimate.
7. Let students know that, with practice, their estimates can get quite close to the actual angle measurement.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Use the reference angles to estimate the measure of an angle.
- ❒ Estimate the measure of an acute angle, using 45º and 90º angles as reference angles.
- ❒ Estimate the measure of an obtuse angle, using 90º and 180º angles as reference angles.
- ❒ Estimate the measure of a reflex angle, using a 90º angle as a reference angle.
- ❒ Use a protractor to measure angles in standard position.
- ❒ Use a protractor to measure angles in various orientations.

- **Draw and label an angle in various orientations using a protractor.**

Materials:

- poster-sized paper
- protractor
- pencil

Organization: Groups of four

Procedure:

1. Seat students in groups of four.
2. Distribute a large poster-sized paper to each group.
3. Say to the class:
   a) I want each of you to choose an angle measure. Make sure that the number of degrees you choose is not the same as the angle chosen by any of the other three members in your group.
   b) Use your protractor to draw your angle on the poster-sized paper in front of you.
   c) Then, using the same angle measure, draw the angle in three other orientations.
   d) Label each angle (write the angle measure inside the angle).
4. Have one member of each group place the paper (completed work) on the board.
5. Ask students to have a class discussion on what they have observed about angles in various orientations.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Use a protractor to measure an angle.
- ❒ Draw an angle.
- ❒ Replicate an angle.
- ❒ Draw an angle in various orientations.

Suggestions for Instruction

- **Describe the measure of an angle as the measure of rotation of one of its sides.**

Organization: Whole class

Procedure:

1. Say and demonstrate the following to the class:
   a) Today's activity will be a live demonstration of an angle measure. It will involve some physical activity on your part.
   b) I want everyone to stand up facing me. Stand with your feet together and pretend your feet are the two sides of an angle.
   c) Now, keeping your heels together, rotate your left foot so that you make an angle. (Demonstrate).
   d) How big of an angle can you make without moving your right foot and without falling?
   e) Now, make a zero degree angle by rotating your left foot back to a "feet together" position. (Demonstrate).
   f) Show the person next to you a variety of angles you can make by keeping your heels together and rotating your right foot.
   g) Thanks for the great participation. Now, go back to your seats.
2. Discuss the number of angles and the kinds of angles that students made with their feet.
3. Tell students to describe in their journals what happens to the measure of an angle when you rotate one of its sides.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Follow directions.
- ❒ Make an angle using their feet.
- ❒ Make a variety of angles using their feet.
- ❒ Associate the change in measure of an angle with the rotation of its side.

Suggestions for Instruction

- **Describe the measure of an angle as the measure of rotation of one of its sides.**

Materials:

- a variety of craft materials
- scissors

Organization: Pairs

Procedure:

1. Let students know that they will be making their own protractors using what they know about angles.
2. Have them work in pairs to formulate a plan for how they are going to make their protractor.
3. Ask them to document the steps they take, making specific reference to reference angles and rotation.
4. Allow students to construct a protractor.
5. Have the students all measure the same angles from the classroom using their protractors (i.e., the edge of a desk, the binding and edge of a three-ring binder).
6. Facilitate a class discussion about the students' protractors.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Reason mathematically.
- ❒ Apply their knowledge about reference angles.
- ❒ Apply their knowledge about rotation.
- ❒ Measure angles using a protractor.

Suggestions for Instruction

- **Describe the measure of an angle as the measure of an interior angle of a polygon.**

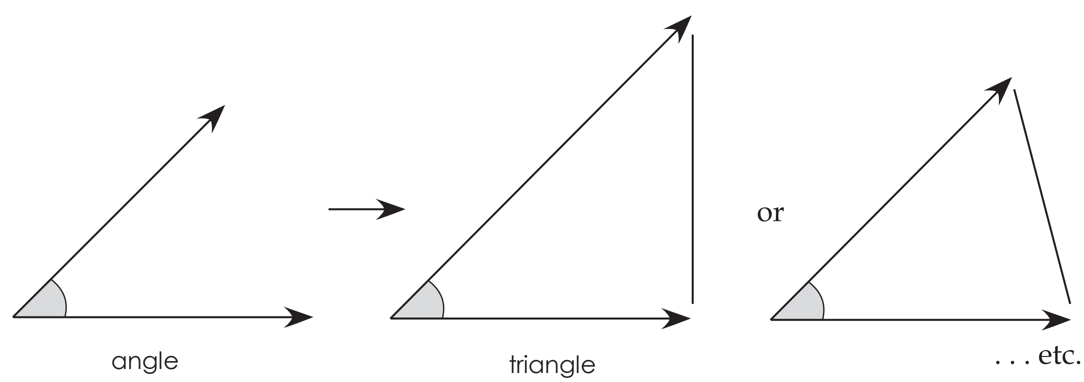
Materials:

- protractor
- ruler
- pencil

Organization: Whole class/individual

Procedure:

1. Draw four distinct angles on the board (e.g., 30º, 45º, 90º, and 120º).
2. Measure each angle using a protractor.
3. Write the measure of each angle inside it.
4. Have a discussion on angles and angle measures.
5. Draw a triangle (i.e., choose one of the angles you drew on the board and connect its two adjacent sides by a third line segment, creating a side opposite to the angle). See example below.



6. Ask students the following:
   a) What happened to the angle?
   b) Did the angle measure change?
7. Ask a student to measure the angle that was made into an interior angle of a triangle.
8. Discuss interior angles of a polygon.
9. Ask one of the students to come up to the board and draw a polygon, but not a triangle, using another one of the angles.
10. Ask students to predict the measure of the angle, which is now an interior angle.
11. Ask a different student to come up to the board and measure that particular interior angle.

12. Ask students what would happen to the other two angles if they were made into interior angles.
13. Discuss the measure of angles as the measure of an interior angle of a polygon.
14. Tell students to do the following:
    a) Draw an angle in their journals.
    b) Describe the measure of the angle as the measure of an interior angle of a polygon of their choice.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:
- ❒ Sketch an angle (approximate size).
- ❒ Measure an angle with a protractor.
- ❒ Know what an interior angle of a polygon is.
- ❒ Describe the measure of an angle as the measure of an interior angle of a polygon.

Suggestions for Instruction

- **Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles.**

Materials:
- BLM 6.SS.1.3: Sum of Interior Angles of a Triangle
- straws
- pipe cleaners
- scissors
- protractor
- pencil

Organization: Whole class/four groups

Procedure:

1. Pre-cut straws to the following lengths: 6 cm, 9 cm, 12 cm, and 15 cm. It would be helpful if you could have a different colour for each length, such as red, green purple, and white.
2. For each group, you will need the following:
   a) two straws of each: 6 cm (red)
15 cm (white)
   b) four straws of each: 9 cm (green)
12 cm (purple)
3. Distribute to each **group** the 12 pre-cut straws and 12 pipe cleaners.
4. Distribute to each **student** a copy of BLM 6.SS.1.3.
5. Tell students to do the following:
   a) Separate the straws into four piles according to size (colour).
   b) Slightly bend each pipe cleaner.
   c) Take one red straw, one purple straw, and one white straw
   d) Use the pipe cleaners to join two straws together.
   e) You will need three bent pipe cleaners to join the three straws and form a triangle.
   f) Using three different lengths of straws will make a *scalene* triangle.
   g) Measure each angle of the *scalene* triangle.
   h) On BLM 6.SS.1.3, mark "*scalene*" under "Triangle Name."
   i) Write the measure of each interior angle under "Interior Angle Measures."
   j) Add up the measure of the three interior angles and write the sum under "Sum of Interior Angles of Triangle."
6. Place on the overhead projector a transparency copy of BLM 6.SS.1.3.
7. Ask students for information to record under each heading.
8. Discuss the results they got by measuring the interior angles of the scalene triangle.
9. Tell students to use one green straw, one purple straw, and one white straw to construct a *right* triangle.
10. Tell students to record the name, each angle measure, and sum of interior angles of the right triangle on the BLM sheet.
11. Tell students to use three green straws to make an equilateral triangle, and the two purples and one red straw to make an isosceles triangle.
12. Have them record the results of each triangle.

13. Discuss the results they got by measuring the interior angles of the other three triangles. For example, compare the
    a) interior measures of each triangle to the interior measures of the scalene triangle
    b) sum of the interior measures of each triangle to the sum of the interior measures of the scalene triangle
    c) interior measures of each triangle to the interior measures of the other triangles
    d) sum of the interior measures of each triangle to the sum of the interior measures of the other triangles
14. Tell students to write in their journals what they observed about how different measures of the interior angles of a triangle influence the sum of the interior angles of a triangle.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:
- ❒ Follow instructions.
- ❒ Construct a model.
- ❒ Measure interior angles of a triangle accurately.
- ❒ Explain, using models, that the sum of the interior angles of a triangle is 180º.
- ❒ Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles.

Suggestions for Instruction

- **Explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.**

Materials:

- BLM 6.SS.1.4: Sum of Interior Angles of a Quadrilateral
- BLM 6.SS.2.3: Sides for Flexible Quadrilaterals
- protractor
- scissors
- pencil
- small binder rings (safety pins or paper clips will also work)

Organization: Whole class/small groups

Procedure:

1. Have students seated in small groups.
2. Distribute to each student a copy of BLM 6.SS.2.2 and BLM 6.SS.2.3.
3. Tell students to do the following:
   a) Take BLM 6.SS.2.3.
   b) Cut the strips of paper along the lines. You should have 16 strips of paper.
      1. 5 large
      2. 1 medium
      3. 10 small
   c) Use a hole-puncher to punch holes on each end of the strips of paper.
   d) Separate the strips of paper into four piles according to the following classifications: parallelogram P1, parallelogram P2, trapezoid T1, and trapezoid T2.
   e) Join two strips of paper by lining up the holes and putting a ring through the holes. Then, using the same method, join to them the third and the fourth strip of paper to form the four sides of each quadrilateral.
   f) Measure each angle of the "parallelogram P1" without moving the sides of the model.
   g) On your sheet BLM 6.SS.2.2, mark "parallelogram P1" under the "Name of Quadrilateral."
   h) Write the measure of each interior angle under "Interior Angle Measures."
   i) Add up the measure of the four interior angles and write the sum under "Sum of Interior Angles of Quadrilaterals."
4. Write on the board "*parallelogram P1*."
5. Tell students to discuss with their group members their angle measures and the sum of their interior measures.
6. Ask a few students to record on the board their angle measures and the sum of their interior measures.
7. Discuss with the class the results they got by measuring the interior angles of "*parallelogram P1*" and how some results may be similar or different from other students' results.
8. Tell students to do the following:
   a) Measure the interior angles of the other three quadrilaterals.
   b) Record the measure of each interior angle.
   c) Record the sum of the measures of each quadrilateral.

9. Discuss the results they got by measuring the interior angles of the other three quadrilaterals. For example: compare the following:
    a) Interior measures of each quadrilateral to the interior measures of the *"parallelogram P1"*
    b) Sum of the interior measures of each quadrilateral to the sum of the interior measures of the *"parallelogram P1"*
    c) Interior measures of each quadrilateral to the interior measures of the other quadrilaterals
    d) Sum of the interior measures of each quadrilateral to the sum of the interior measures of the other quadrilaterals
10. Tell students to write in their journals what they observed about how different measures of the interior angles of a quadrilateral influence the sum of the interior angles of a quadrilateral.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❐ Follow instructions.
- ❐ Construct a model.
- ❐ Measure interior angles of a quadrilateral accurately.
- ❐ Explain, using models, that the sum of the interior angles of a quadrilateral is 360°.
- ❐ Explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.

Grade 6: Shape and Space (Measurement) (6.SS.3)

Enduring Understanding(s):

All measurements are comparisons.

There is no direct relationship between perimeter and area.

Perimeter, area, and volume are measurable properties of objects.

The units of measure must be of the same nature as the property being measured.

General Learning Outcome(s):

Use direct or indirect measurement to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
6.SS.3 Develop and apply a formula for determining the ▪ perimeter of polygons ▪ area of rectangles ▪ volume of right rectangular prisms [C, CN, PS, R, V]	➛ Explain, using models, how the perimeter of any polygon can be determined. ➛ Generalize a rule for determining the perimeter of polygons. ➛ Explain, using models, how the area of any rectangle can be determined. ➛ Generalize a rule for determining the area of rectangles. ➛ Explain, using models, how the volume of any right rectangular prism can be determined. ➛ Generalize a rule for determining the volume of right rectangular prisms. ➛ Solve a problem involving the perimeter of polygons, the area of rectangles, or the volume of right rectangular prisms.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of multiplying 2-digit numerals by 2-digit numerals to solve problems
- Demonstrating an understanding of division of 2-digit numerals by 3-digit numerals
- Using perimeter or area or both (whole numbers) to design and construct different rectangles and draw conclusions
- Demonstrating an understanding of measuring length

- Demonstrating an understanding of volume
- Identifying and sorting quadrilaterals
- Describing orally and in writing the rule for pattern
- Demonstrating an understanding of area of regular and irregular 2-D shapes
- Describing and constructing rectangular and triangular prisms
- Demonstrating an understanding of perimeter of regular and irregular shapes
- Adding and subtracting 1-, 2-, and 3-digit numerals with answers to 1000

Related Knowledge

Students should be introduced to the following:

- Explaining and applying the order of operations, excluding exponents
- Representing generalizations arising from number relationships

Background Information

Perimeter, **area**, and **volume** are not of the same nature; therefore, they do not use the same units of measurement.

Perimeter is the distance around a shape, and it is measured in linear units such as kilometres (km), metres (m), centimetres (cm), and millimetres (mm).

Area is the amount of surface a shape covers, and it is measured in square units such as square kilometres (km^2), square metres (m^2), square centimetres (cm^2), and square millimetres (mm^2).

Volume is the amount of space an object occupies or, if the object is hollow, the amount of space inside the object (capacity). Volume is measured in cubic units such as cubic metres (m^3), cubic centimetres (cm^3), and cubic millimetres (mm^3).

When given either perimeter, or area, or both, students in Grade 5 learned to: (a) design and construct different rectangles; (b) draw conclusions. They also demonstrated an understanding of volume by: (a) selecting and justifying referents for cm^3 and m^3 units; (b) estimating volume by using referents for cm^3 and m^3; (c) measuring and recording volume (cm^3 and m^3); and (d) constructing rectangular prisms for a given volume.

In Grade 6, students will learn through their activities to develop and apply a formula for determining the (a) perimeter of polygons; (b) area of rectangles; and (c) the volume of right rectangular prisms.

Mathematical Language

centimetre
millimetre
kilometre
height
length
width
perimeter
area
volume
polygon
rectangle
right rectangular prism

Learning Experiences

Assessing Prior Knowledge

Materials:

- BLM 5-8.9: Centimetre Grid Paper
- ruler
- pencil
- scissors
- tape

Organization: Individual

Procedure:

1. Tell students that you wish to assess their understanding of perimeter and area of rectangles and also their understanding of volume of rectangular prisms.
2. Distribute to each student a copy of BLM 5-8.9.
3. Tell students to draw two rectangles:
   a) One with a 14 cm perimeter and mark P = 14 cm inside it.
   b) One with a 24 cm² area and mark A = 24 cm² inside it.

4. Circulate to check that students chose a correct rectangle for each scenario.
5. Construct a rectangular prism that has a 60 cm^3 volume.
6. Circulate to check that students chose a correct rectangular prism.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Use a given perimeter to find the dimensions of a rectangle.
- ❒ Use a given area to find the dimensions of a rectangle.
- ❒ Divide two-digit numerals by one-digit numerals.
- ❒ Draw a rectangle.
- ❒ Use a given volume to find the dimensions of a rectangular prism.
- ❒ Construct a rectangular prism.

Suggestions for Instruction

- **Explain, using models, how the perimeter of any polygon can be determined.**

Materials:
- geoboard
- elastic band
- pegs

Organization: Whole class/small group/individual

Procedure:
1. Distribute to each student:
   a) 1 geoboard
   b) 6 pegs
   c) 1 elastic band
2. Say the following to the students:
   a) Use the geoboard, elastic band, and any number of pegs you want to make a polygon.
   b) Analyze your polygon to determine its perimeter.

c) Write in your notebook how you determined the perimeter of your polygon.

d) Use your geoboard, elastic band, and pegs to make a different polygon.

e) Analyze your new polygon to determine its perimeter.

f) Write in your notebook how you determined the perimeter of your new polygon.

g) Use your geoboard, elastic band, and pegs to make two more types of polygons, and go through the same process.

h) Analyze the notes you wrote about how to determine the perimeter of each polygon.

i) Write in your notebook how you would determine the perimeter of any polygon.

j) Repeat the process for regular polygons.

3. Discuss together what students found out about determining the perimeter of any polygon.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Construct a polygon using a geoboard, elastic band, and pegs.
- ❒ Determine the perimeter of a polygon.
- ❒ Add lengths together.
- ❒ Explain how they determined the perimeter of a polygon.

- **Generalize a rule for determining the perimeter of polygons.**

Materials:

- collection of polygons (both regular and irregular)
- BLM 6.SS.3.1: Polygon Collection: Set 1–5, with the polygons cut out and placed in envelopes

Organization: Four or five groups

Procedure:

1. Distribute a different collection of polygons to each group.
2. Tell students to do the following:
   a) Sort the polygons by their number of sides.
   b) Estimate and then measure the perimeter of each polygon.
   c) Can you find a shortcut or rule for finding the perimeter of polygons?
   d) Exchange your polygon collection with another group.
   e) Verify that the shortcut or rule you developed works for other polygons.
   f) Report to the class how your group calculated the perimeters, including the shortcut (or rule) you found.
   g) Discuss similarities or differences among your findings.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Sort polygons by the number of their sides.
- ❒ Estimate the perimeter of a polygon.
- ❒ Measure the perimeter of a polygon.
- ❒ Explain the shortcut or rule for finding the perimeter of a polygon.

Suggestions for Instruction

- **Explain, using models, how the area of any rectangle can be determined.**

Materials:

- geoboard
- elastic band
- pegs

Organization: Whole class/individual

Procedure:

1. Distribute the following to each student:
   a) 1 geoboard
   b) 1 elastic band
   c) 4 pegs
2. Say the following to the students:
   a) Use the geoboard, elastic band, and the four pegs to make a rectangle.
   b) Analyze your rectangle to determine its area.
   c) Write in your notebook how you determined the area of your rectangle.
   d) Use your geoboard, elastic band, and pegs to make a different rectangle.
   e) Analyze your new rectangle to determine its area.
   f) Write in your notebook how you determined the area of your new rectangle.
   g) Use your geoboard, elastic band, and pegs to make two more types of rectangles, and go through the same process.
   h) Analyze the notes you wrote about how to determine the area of each rectangle.
   i) Write in your notebook how you would determine the area of any rectangle.
3. Discuss together what students found out about determining the area of any rectangle.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Construct a rectangle using a geoboard, elastic band, and pegs.
- ❒ Determine the area of a rectangle.
- ❒ Explain how they determine the area of a rectangle.

- **Generalize a rule for determining the area of rectangles.**

Materials:

- square tiles or a geoboard

Organization: Groups

Procedure:

1. Distribute square tiles or a geoboard to each group.
2. Tell students to do the following:
   a) Make a collection of rectangles.
   b) Look for shortcuts that can be used in determining area.
   c) Determine the area of each rectangle.
   d) Draw each rectangle on grid paper.
   e) Record each rectangle's dimensions on grid paper.
   f) Record the area of each rectangle inside it.
   g) Exchange collections with another group.
   h) Verify that the shortcut or rule you developed works for other rectangles.
   i) Report to the class how your group calculated the area, including any shortcut (or rule) you may have found.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❐ Construct different rectangles.
- ❐ Determine the area of different rectangles.
- ❐ Measure the sides of a rectangle.
- ❐ Use multiplication facts.
- ❐ Find a shortcut or rule for finding the area of a rectangle.
- ❐ Explain the shortcut or rule for finding the area of a rectangle.

Suggestions for Instruction

- **Explain, using models, how the volume of any right rectangular prism can be determined.**
- **Generalize a rule for determining the volume of right rectangular prisms.**

Materials:

- centicubes

Organization: Small groups

Procedure:

1. Tell students to:
   a) Make right rectangular prisms whose volumes are 48 cm^3, 36 cm^3, 28 cm^3, 21 cm^3, and 12 cm^3.
   b) Make a table, such as the one below.
   c) Record each prism's
      - dimensions
      - area of its base
      - volume

Width in cm	Length in cm	Area of Base in cm^2	Height in cm	Volume in cm^3

   d) Explain in your journals how the volume of each of these right rectangular prisms was found.
   e) Discuss with your group members how a prism's dimensions are related to its area and volume.
   f) Explain in your journals how the volume of any right rectangular prisms can be found.
   g) Write a shortcut method or rule to be used to calculate volume of any right rectangular prism.
   h) Verify your shortcut rules by predicting the volume of a right rectangular prism that is 4-cm wide by 3-cm long by 10-cm high.
   i) Build the prism using centicubes and count the number of cubes. The count should match the volume calculated by your rule.
   j) Share your conclusions with the class.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Construct a right rectangular prism using centicubes, given a specific volume.
- ❒ Calculate dimensions of a right rectangular prism, given a specific volume.
- ❒ Find a shortcut or rule for finding the volume of a right rectangular prism.
- ❒ Explain the shortcut or rule for finding the volume of a right rectangular prism.

Suggestions for Instruction

- **Explain, using models, how the volume of any right rectangular prism can be determined.**
- **Generalize a rule for determining the volume of right rectangular prisms.**

Materials:

- centicubes

Organization: Small groups

Procedure:

1. Have a class discussion about volume and right rectangular prisms. Be sure everyone understands before you move on.
2. Present students with the following problem:

   A grocery store chain is opening up a new store in Stonewall, MB. They are given very specific instructions as to how to place each shelf so that all of the items will fit. The page about juice boxes has a small tear in the bottom and some information seems to be missing.

The volume of juice boxes is 36 cm^3 and 48 cm^3 and they are to be displayed on the same shelf.

The space to the next shelf must be

The manager does not want the owners to doubt his abilities, so he has the stocking clerks try to figure out the height that the shelf should be. They can record their information in the table below:

Width in cm	**Length in cm**	**Area of Base in cm^2**	**Height in cm**	**Volume in cm^3**

3. Have students explain in their journals how the volume relates to the width, length, area of base, and height of their right rectangular prisms. Ask them to use this relationship to determine the volume of a juice box that is 5 cm long by 3 cm wide by 6 cm tall.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Construct a right rectangular prism using centicubes, given a specific volume.
- ❒ Find a shortcut or rule for finding the volume of a right rectangular prism.
- ❒ Explain the shortcut or rule for finding the volume of a right rectangular prism.

- **Solve a problem involving the perimeter of polygons, the area of rectangles, or the volume of right rectangular prisms.**

Materials:

- BLM 6.SS.3.2: Dolly Made a Garden

Organization: Individual

Procedure:

1. Distribute to each student a copy of BLM 6.SS.3.2.
2. Tell students to do the following:
   a) Read the problem carefully.
   b) Name the polygon.
   c) Explain how they will find the perimeter of Dolly's garden.
   d) Solve the perimeter problem of Dolly's garden.
3. Discuss with the class the polygon created by Dolly.
4. Ask a student to write the name of the polygon on the board.
5. Have a student tell the class how he or she found the perimeter of Dolly's garden.
6. Invite comments by other students.
7. Ask a student to write his or her solution on the board.
8. Discuss the results with the class.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Recognize and name a polygon.
- ❒ Explain how they will find the perimeter of a polygon.
- ❒ Solve the perimeter of a polygon.

Suggestions for Instruction

- **Solve a problem involving the perimeter of polygons, the area of rectangles, or the volume of right rectangular prisms.**

Materials:

- BLM 6.SS.3.3: David's Playroom
- A poster-sized paper

Organization: Small group

Procedure:

1. Distribute to each student a copy of BLM 6.SS.3.3 and a poster-sized paper.
2. Tell students to do the following:
   a) Read the problem carefully.
   b) Name the quadrilateral.
   c) Explain how they will find the area of David's playroom.
   d) Solve the area problem of David's playroom.
3. Discuss with the class the quadrilateral created by David.
4. Ask each group to use the poster-sized paper to do the following:
   a) Write the name of the quadrilateral on top of the poster paper.
   b) Draw the shape of David's playroom.
   c) Write down how they found the area of David's playroom.
   d) Write their solution to the problem involving the area of David's playroom.
5. Place each poster on the board.
6. Discuss the results with the class.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:
- ❐ Recognize and name a quadrilateral.
- ❐ Explain how they will find the area of a quadrilateral.
- ❐ Solve the area of a quadrilateral.

- **Solve a problem involving the perimeter of polygons, the area of rectangles, or the volume of right rectangular prisms.**

Materials:

- BLM 6.SS.3.4: Peter's Toy Box

Organization: Individual

Procedure:

1. Distribute to each student a copy of BLM 6.SS.3.4.
2. Tell students to do the following:
   a) Read the problem carefully.
   b) Create a chart based on the problem.
   c) Explain how you will find the volume of Peter's toy box.
   d) Solve the volume problem of Peter's toy box.
3. Discuss with the class the dimensions of Peter's toy box.
4. Write the dimensions on an overhead transparency.
5. Have a student tell the class how he or she found the volume of Peter's toy box.
6. Ask if anyone has a different explanation.
7. Ask a student to write his or her solution on the board.
8. Discuss the results with the class.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Create a chart based on a problem.
- ❒ Explain how they will find the volume of a right rectangular prism.
- ❒ Solve a problem involving the volume of a right rectangular prism.

Grade 6: Shape and Space (3-D Objects and 2-D Shapes) (6.SS.4, 6.SS.5)

Enduring Understanding(s):

Shapes are distinguished by their properties

General Learning Outcome(s):

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
6.SS.4 Construct and compare triangles in different orientations, including ■ scalene ■ isosceles ■ equilateral ■ right ■ obtuse ■ acute [C, PS, R, V]	➛ Sort a set of triangles according to the length of the sides. ➛ Sort a set of triangles according to the measures of the interior angles. ➛ Identify the characteristics of a set of triangles according to their sides or their interior angles. ➛ Sort a set of triangles and explain the sorting rule. ➛ Draw a triangle (e.g., scalene). ➛ Replicate a triangle in a different orientation and show that the two are congruent.
6.SS.5 Describe and compare the sides and angles of regular and irregular polygons. [C, PS, R, V]	➛ Sort a set of 2-D shapes into polygons and non-polygons, and explain the sorting rule. ➛ Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing. ➛ Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by measuring. ➛ Demonstrate that the sides of a regular polygon are of the same length and that the angles of a regular polygon are of the same measure. ➛ Sort a set of polygons as regular or irregular and justify the sorting. ➛ Identify and describe regular and irregular polygons in the environment.

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Designing and constructing different rectangles
- Identifying and sorting quadrilaterals according to their attributes
- Demonstrating an understanding of area of regular and irregular 2-D shapes
- Describing and constructing rectangular and triangular prisms
- Demonstrating an understanding of line symmetry
- Sorting regular and irregular polygons according to the number of sides

RELATED KNOWLEDGE

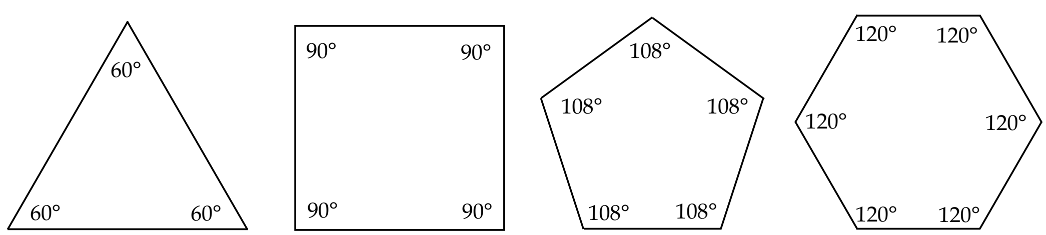
Students should be introduced to the following:

- Demonstrating an understanding of angles
- Performing a combination of transformations on a single 2-D shape, and drawing and describing the image

BACKGROUND INFORMATION

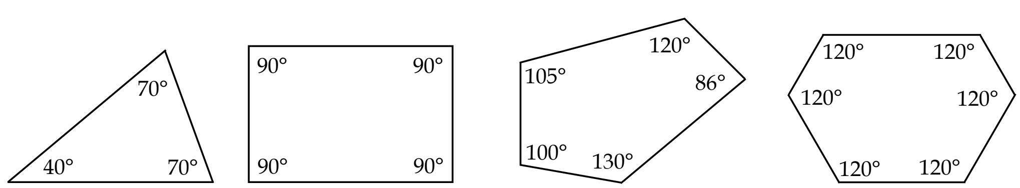
A **polygon** is a closed plane figure formed by three or more line segments. The simplest polygon is a triangle. There are regular polygons and irregular polygons.

A **regular polygon** is a polygon in which all sides and all angles are congruent. See examples below.



triangle quadrilateral pentagon hexagon

An **irregular polygon** is a polygon whose sides and angles are not all congruent. See examples below.

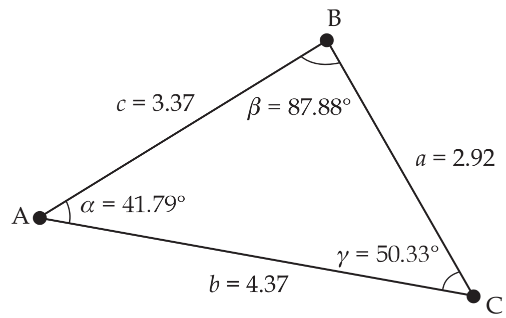


triangle quadrilateral pentagon hexagon

A **triangle** is a polygon with three sides and three angles. Triangles are sorted according to their sides and angles.

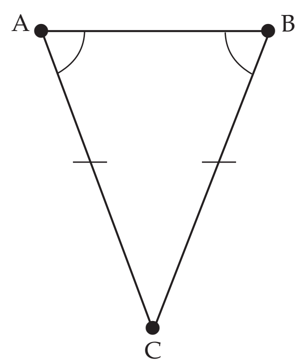
- A **scalene triangle** is a triangle with no congruent sides and no congruent angles.

  Example:



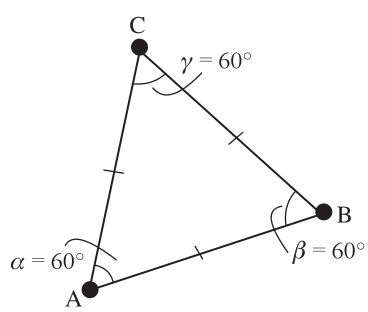
- An **isosceles triangle** is a triangle with at least two congruent sides and two congruent angles.

  Example:



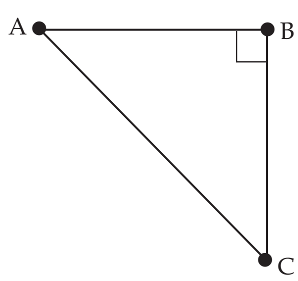
- An **equilateral triangle** is a triangle with three congruent sides and three congruent angles.

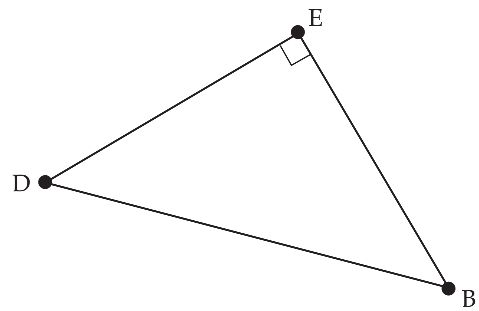
  Example:



- A **right triangle** is a triangle with one right angle.

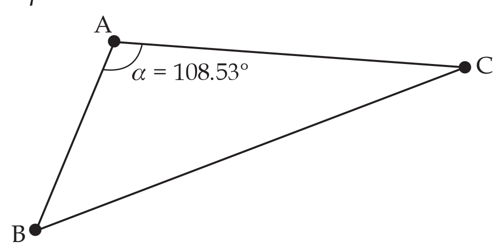
  Examples:





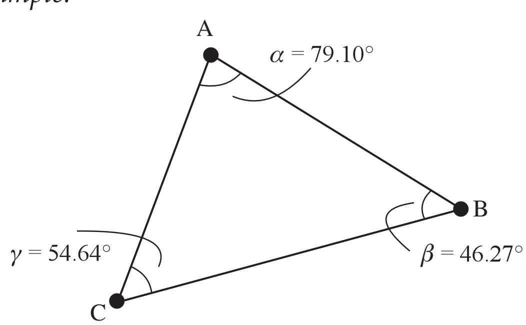
- An **obtuse triangle** is a triangle containing one obtuse angle (greater than 90º and less than 180º).

  Example:



- An **acute triangle** is a triangle in which all three angles are acute (greater than 0º and less than 90º).

  Example:



When two figures have the same shape and size, they are **congruent**.

Examples:

- Two sides (line segments) are congruent if they are the same length.
- Two angles are congruent if they have the same measure.

Mathematical Language

acute
congruent
equilateral
hexagon
isosceles
obtuse
pentagon
polygon
right
scalene
square
triangle

Learning Experiences

Assessing Prior Knowledge

Materials:

- BLM 5-8.9: Centimetre Grid Paper
- ruler
- protractor
- pencil

Organization: Individual

Procedure:

1. Distribute to each student a copy of BLM 5-8.9.
2. Ask students to use the centimetre grid to draw the following:
   a) One triangle using a ruler and protractor.
   b) One quadrilateral using a ruler and protractor.
3. Tell students to do the following:
   a) Mark each interior angle of each shape with a different letter of the alphabet.
   b) Name each interior angle of each polygon according to its measure.

4. Circulate to check that students do the following:
   a) Draw the correct shape.
   b) Write the correct name for each angle.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Draw a triangle using a ruler and protractor.
- ❒ Draw a quadrilateral (e.g., square, rectangle, parallelogram, trapezoid, or other irregular quadrilateral) using a ruler and protractor.
- ❒ Recognize interior angles in a polygon.
- ❒ Mark interior angles in a polygon.
- ❒ Name interior angles of a polygon according to their measure (e.g., acute, right, obtuse, straight, reflex).

Suggestions for Instruction

- **Sort a set of triangles according to the length of the sides.**

Materials:
- scissors
- ruler
- BLM 6.SS.4.1: Cards of Triangles #1
- BLM 6.SS.4.2: Sorting of Triangles According to the Length of the Sides

Organization: Whole class/pairs

Procedure:
1. Distribute to each pair a copy of BLM 6.SS.4.1, and to each student a copy of BLM 6.SS.4.2.
2. Have the students cut the cards of triangles in BLM 6.SS.4.1 so they get eight cards (that is, four cards per student).

3. Tell students to do the following:
   a) Measure all three sides of each triangle.
   b) On your copy of BLM 6.SS.4.2, mark down
      - the name of the triangle
      - length of each side
      - how many sides have the same length—that is,
        - 3 sides have the same length
        - 2 sides have the same length
        - No sides are the same length
   c) Discuss the results with your partner.
   d) Copy your partner's results onto your sheet.
   e) Sort the triangles according to the number of sides that have the same length (3, 2, none).
4. Have a class discussion on sorting triangles according to the length of the sides.
5. Tell students to write in their journals their observations on sorting triangles according to the length of the sides.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Measure the sides of a triangle.
- ❒ Compare triangles according to the length of the sides.
- ❒ Sort triangles according to the length of the sides.

Suggestions for Instruction

- **Sort a set of triangles according to the measures of the interior angles.**

Materials:

- scissors
- protractor
- BLM 6.SS.4.3: Cards of Triangles #2
- BLM 6.SS.4.4: Sorting of Triangles According to the Measure of the Interior Angles

Organization: Whole class/pairs

Procedure:

1. Distribute to each pair a copy of BLM 6.SS.4.3, and to each student a copy of BLM 6.SS.4.4.
2. Have students cut the cards of triangles in BLM 6.SS.4.3 so they get eight cards (that is, four cards per student).
3. Tell students the following:
   a) Measure all three interior angles of each triangle.
   b) On your copy of BLM 6.SS.4.4, mark down
      - the name of the triangle
      - the measure of each interior angle
3. Ask students to use the measure of the interior angles to sort their triangles into groups. Have them explain their sorting rules on the table provided.
4. Discuss the various sorting rules as a class. Through the discussion, encourage students to sort the triangles by
   a) the number of angles that are equal (3 – equilateral, 2 – isosceles, none – scalene)
   b) the type of angles it contains (one obtuse – obtuse, one right – right, all acute – acute)
5. Encourage correct use of mathematical terminology
6. Ask students to show their knowledge of different types of triangles through their choice of
   a) a journal entry
   b) a flip book
   c) an interview
   d) a poem
   e) etc.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Measure interior angles of a triangle.
- ❒ Compare triangles according to the measure of the interior angles.
- ❒ Sort triangles according to the measure of the interior angles.

Suggestions for Instruction

- **Identify the characteristics of a set of triangles according to their sides or their interior angles.**

Materials:

- ruler
- protractor
- BLM 6.SS.4.5: Triangle Identification

Organization: Whole class/individual

Procedure:

1. Distribute to each student a copy of BLM 6.SS.4.5.
2. Tell students to analyze the set of triangles.
3. Say to the students the following:
   a) You need to measure the sides or interior angles of the triangles.
   b) Record their measurements.
   c) Identify the characteristics of the set of triangles based on your measurements.
4. Have a class discussion.
5. Tell students to write their observations regarding the identification of the set of triangles in their journals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Measure the sides of triangles.
- ❒ Measure the interior angles of triangles.
- ❒ Compare triangles according to the length of their sides.
- ❒ Compare triangles according to their interior angles.
- ❒ Identify the characteristics of a set of triangles according to their sides or their interior angles.

Suggestions for Instruction

- **Sort a set of triangles according to the length of the sides.**
- **Sort a set of triangles according to the measures of the interior angles.**
- **Identify the characteristics of a set of triangles according to their sides or their interior angles.**

Materials:

- paper
- pencil
- ruler
- protractor
- grid paper

Organization: Small groups

Procedure:

1. Seat students in small groups.
2. Hand out a grid paper to each student.
3. Tell students the following:
   a) Each group needs to make a set of triangles (that is, a group of triangles with similar characteristics).
   b) Discuss with group members what kind of triangles each person will make.
   c) Discuss with group members how many triangles each person will make.
   d) Make your triangles.

4. When your set of triangles is complete, change sets with another group.
5. Identify the characteristics of the other group's set of triangles according to their sides or their interior angles.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Work well as a member of a group.
- ❒ Construct a variety of triangles.
- ❒ Identify the characteristics of the other group's set of triangles according to their sides or their interior angles.

Suggestions for Instruction

- **Sort a set of triangles, and explain the sorting rule.**

Materials:

- protractor
- ruler
- BLM 6.SS.4.6: Triangle Page

Organization: Whole class/small groups

Procedure:

1. Distribute to each student a copy of BLM 6.SS.4.6.
2. Tell students to do the following:
   a) Analyze the triangles carefully.
   b) Sort the triangles and record them in your notebooks.
   c) Explain your sorting rule in your notebooks.
   d) Discuss the results with your group members.
3. Have a few students orally present their work.
4. Use the presentations as the basis for a class discussion on the sorting of triangles.
5. Tell students to write their observations on sorting triangles in their journals (e.g., Did everyone use the same sorting rule? Why? What are the sorting methods some students used?).

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Measure the sides of triangles.
- ❒ Measure the interior angles of triangles.
- ❒ Sort triangles according to their sides or interior angles.
- ❒ Explain the sorting of triangles.

Suggestions for Instruction

- **Draw a triangle (e.g., scalene).**
- **Replicate a triangle in a different orientation and show that the two are congruent.**

Materials:

- protractor
- ruler
- a blank sheet of paper
- pencil

Organization: Individual

Procedure:

1. Have a class discussion on triangles (e.g., Can students name some triangles and describe them according to their sides or interior angles?).
2. Distribute to each student a blank sheet of paper (computer paper is fine).
3. Tell students to do the following:
   a) Draw a triangle using a ruler and a protractor.
   b) Replicate the triangle in a different orientation.
   c) Show that the two triangles are congruent.
4. Ask a few students to give an oral presentation to the class.

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Draw a triangle using a ruler and protractor.
- ❒ Measure sides of a triangle accurately.
- ❒ Measure inside angles of a triangle accurately.
- ❒ Replicate triangles using a ruler and protractor.

Suggestions for Instruction

- **Sort a set of 2-D shapes into polygons and non-polygons, and explain the sorting rule.**

Materials:

- BLM 6.SS.5.1: Polygons or Non-polygons?

Organization: Whole class/pairs

Procedure:

1. Distribute to each pair a copy of BLM 6.SS.5.1.
2. On the board, create two columns. Write the word “polygons” in one column and the word “non-polygons” in the other column.
3. Tell students to do the following:
   a) Analyze the 2-D shapes in BLM 6.SS.5.1.
   b) Discuss the 2-D shapes with your partners (e.g., Which column does each 2-D shape belong to? Why?).
   c) Copy the chart off the board into your notebooks.
   d) Place each 2-D shape in the appropriate column.
4. Ask a few students to help you complete the chart on the board.
5. Have a class discussion on the results of the sorting of the 2-D shapes.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Recognize polygons.
- ❐ Recognize non-polygons
- ❐ Explain what makes a polygon.
- ❐ Explain what makes a non-polygon.

Suggestions for Instruction

- **Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing.**

Materials:

- BLM 6.SS.5.2: Equilateral Triangle

Organization: Whole class/small groups

Procedure:

1. Distribute to each student a copy of BLM 6.SS.5.2.
2. Discuss with the class the characteristics of an equilateral triangle.
3. Tell students to do the following:
   a) Fold your equilateral triangle, matching up two sides and two angles.
   b) Rotate the paper, and match up two other sides and angles until all the sides and angles are checked for congruence.
   c) Discuss the congruence by folding and superimposing with your group members.
4. Have a student give a class presentation on what he or she learned about congruence by folding and superimposing.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Superimpose two sides and two interior angles of a triangle by folding.
- ❒ Recognize that all three sides of an equilateral triangle are congruent.
- ❒ Recognize that all three interior angles of an equilateral triangle are congruent.

Suggestions for Instruction

- **Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by measuring.**

Materials:

- BLM 6.SS.5.2: Equilateral Triangle
- ruler
- protractor
- pencil

Organization: Whole class/pairs

Procedure:

1. Distribute to each pair a copy BLM 6.SS.5.2.
2. On the board, create two columns. Write the word "Sides of an Equilateral Triangle" in one column and the word "Angles of an Equilateral Triangle" in the other column.
3. Tell students to do the following:
   a) Mark each side of the triangle with a lower-case letter.
   b) Mark each interior angle of the triangle with an upper-case letter.
   c) Measure each side of the triangle with your ruler.
   d) Record the length of each side of the triangle.
   e) Measure each interior angle of the triangle with your protractor.
   f) Record the measure of each interior angle of the triangle.
4. Ask a few students to help record the results on the board.
5. Have a class discussion on demonstrating congruence in a regular triangle by measuring.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Measure sides of a triangle with a ruler.
- ❒ Recognize congruence of sides by measuring.
- ❒ Measure inside angles of triangle with a protractor.
- ❒ Recognize congruence of interior angles by measuring.
- ❒ Explain congruence by measuring sides and angles.

Suggestions for Instruction

- **Demonstrate that the sides of a regular polygon are of the same length and that the angles of a regular polygon are of the same measure.**

Materials:

- BLM 6.SS.5.3: Regular Pentagon
- ruler
- protractor
- pencil

Organization: Whole class/small group

Procedure:

1. Distribute to each pair a copy BLM 6.SS.5.3.
2. On the board, create two columns. Write the word "Sides of regular pentagon" in one column and the word "Angles of a regular pentagon" in the other column.
3. Tell students to do the following:
   a) Mark each side of the pentagon with a lower-case letter.
   b) Mark each interior angle of the pentagon with an upper-case letter (with the same letter as the side opposite the angle).
   c) Choose a method to demonstrate that the sides are the same length.
   d) Choose a method to demonstrate that the interior angles are of the same measure.
4. Have a student demonstrate to the class that the sides are the same length.
5. Have a student demonstrate to the class that the interior angles are of the same measure.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Demonstrate that the sides of a regular polygon are the same length.
- ❒ Demonstrate that the interior angles of a regular polygon are of the same measure.

Suggestions for Instruction

■ **Sort a set of polygons as regular or irregular, and justify the sorting.**

Materials:

- BLM 6.SS.5.4: Polygons: Regular and Irregular
- BLM 6.SS.5.5: Am I a Regular Polygon?

Organization: Individual/whole class

Procedure:

1. Distribute to each student a copy of BLM 6.SS.5.4 and BLM 6.SS.5.5.
2. Tell students to do the following:
   a) Analyze the polygons on BLM 6.SS.5.4.
   b) Complete the chart in BLM 6.SS.5.5 by classifying each polygon and providing a justification.
3. Place on the overhead projector a transparency copy of the chart in BLM 6.SS.5.5.
4. Ask a few students to help you complete the chart.
5. Have a class discussion on the results of sorting the polygons.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Recognize regular polygons.
- ❒ Recognize irregular polygons
- ❒ Explain what makes a polygon regular.
- ❒ Explain what makes a polygon irregular.

Suggestions for Instruction

- **Identify and describe regular and irregular polygons in the environment.**

Materials:

- ruler
- protractor
- pencil and paper

Organization: Whole class/small group

Procedure:

1. Discuss with the class the characteristics of regular and irregular polygons.
2. Tell students to do the following:
   a) Find polygons in the classroom, hallway, gym floor, or on the playground (e.g., square tiles in the hallway, large square drawn on the gym floor, large, square cement sidewalk tiles).
   b) Identify the polygons as regular or irregular.
   c) Discuss with your group members the characteristics of each polygon.
3. Have a class discussion regarding the polygons students found in the environment.
4. Tell students to do the following:
   a) Describe in your journals the polygons you found
   b) Describe what makes each example a regular or irregular polygon.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Recognize regular polygons in the environment.
- ❒ Recognize irregular polygons in the environment.
- ❒ Explain what makes a polygon regular.
- ❒ Explain what makes a polygon irregular.

Putting the Pieces Together

What's There?

Purpose: The purpose of this activity is to assess students' knowledge in measuring angles and distance, calculating perimeter, distinguishing between regular and irregular polygons, and justifying choices.

The mathematical processes demonstrated by this task are communication, connections, problem solving, reasoning, and visualization.

In order for students to be able to perform this task, they need to have some prior knowledge related to the task.

Prior Knowledge: Students should be able to do the following:

- Use perimeter or area or both (whole numbers) to design and construct different rectangles and draw conclusions.
- Demonstrate an understanding of measuring length.
- Identify and sort quadrilaterals.
- Describe orally and in writing the rule for pattern.
- Demonstrate an understanding of area of regular and irregular 2-D shapes.
- Sort regular and irregular polygons according to the number of sides.
- Demonstrate an understanding of perimeter of regular and irregular shapes.
- Add and subtract 1-, 2-, and 3-digit numerals with answers to 1000.

Related Knowledge: Students should be able to do the following:

- Demonstrate an understanding of angles.
- Perform a combination of transformations on a single 2-D shape, and draw and describe the image.
- Demonstrate and explain the meaning of preservation of equality, concretely, pictorially, and symbolically.
- Represent generalizations arising from number relationships.

Curricular Links: This task links to other curriculum such as social studies and English language arts.

Materials/Resources:

- BLM 6.SS.5.4: Polygons: Regular and Irregular
- map of Manitoba
- scissors
- ruler
- protractor

Note: During this task, students will use angles and polygons to get familiar with the map of Manitoba. (You may use a different map to fit your social studies lesson.)

Organization:

1. Prior to the students entering the room, place the following on each board:
   a) A map of Manitoba
   b) Images of two angles and two regular polygons (different ones on each board)
   c) A copy of BLM 6.SS.1.2: Reference Angles
2. Have students work in small groups.

Inquiry:

Scenario:

Each group of students will be responsible for naming places on a map that lie:

a) a certain distance and direction from a given point
b) inside or outside of the perimeter of a regular polygon

Procedure:

Before the students enter the room,

a) arrange the desks so they are suitable for working in small groups
b) provide the following for each group:
   - Map
   - Transparent copy of BLM 6.SS.5.4
c) provide each student with one of the following:
   - A pair of scissors
   - Ruler
   - Protractor
   - Looseleaf paper
   - A copy of the directions
   - A copy of BLM 5-8.9: Centimetre Grid Paper
   - A copy of BLM 5-8.4: How I Worked in My Group

Student Directions:

1. Look at the transparency of BLM 6.SS.5.4.
2. Each student needs to choose a polygon. Make sure that each group member chooses a different polygon.

3. Tell students the following:

   On your looseleaf,

   a) identify your polygon as regular or irregular

   b) specify the polygon according to the number of sides and angles, such as triangle, quadrilateral, pentagon, and hexagon

   c) justify your classification by describing the characteristics of your polygon

4. Carefully cut out your polygon, making sure that all sides of your polygon stay intact and none of the other polygons on the transparent BLM copy get damaged. Your group members will need the other polygons.

5. On your centimetre grid paper,

   a) replicate your polygon in two different orientations **and**

   b) inside each polygon, write its name, such as polygon #1 and polygon #2

6. On your looseleaf, describe why polygon #1 and polygon #2 are congruent to each other and to the transparent polygon.

7. Calculate the perimeter of your polygon and record your calculations on your looseleaf.

8. Place your transparent polygon on the map in front of you.

9. On your looseleaf, record the specific places or landmarks (city, river, lake, road) that lie inside the perimeter of your polygon.

10. Choose a city or town that lies inside your polygon (e.g., Winnipeg).

11. Choose a city or town that lies on the left side your polygon (e.g., Brandon).

12. Choose a city or town that lies above your polygon (e.g., Churchill).

13. Look at the reference angles placed on the board.

14. Based on the reference angles, identify the angle you would get if you imagined that a line going from (for example) Brandon to Winnipeg would be one side of an angle and Winnipeg to Churchill would be the other side of the angle.

15. On your looseleaf, record your observations regarding the angle.

16. On your centimetre grid paper, replicate your angle in two different orientations and write the name inside each angle, such as “angle A” and “angle B.”

17. Discuss your observations with your group members.

Literature Link:

After students complete this activity, read to the class the book entitled *Hamster Champs* by Stuart J. Murphy and illustrated by Pedro Martin. This is a well written and neatly illustrated book about three hamsters using a protractor and a straight edge to perform stunts for a cat that has nothing to do except to chase them.

Assessment:

Use the following observation checklist to assess student learning.

The student can do the following:	Yes	No	Comment
Identify a polygon as regular or irregular.			
Identify the characteristics of a given polygon.			
Identify a polygon according to the number of its sides and angles (e.g., triangle, quadrilateral, pentagon, hexagon).			
Replicate a polygon in two different orientations.			
Justify that two replicated polygons are congruent to each other and the original polygon.			
Identify an angle based on the reference angles.			
Replicate an angle in two different orientations.			
Communicate observations orally or in writing.			

Extension:

Taking it further

Have students describe in detail how they would design a map of their classroom. Students should be reminded to use correct mathematical terminology.

Grade 6: Shape and Space (Transformations) (6.SS.6, 6.SS.7, 6.SS.8, 6.SS.9)

Enduring Understanding(s):

The position of shapes can be changed by translating, rotating, or reflecting them.

General Learning Outcome(s):

Describe and analyze position and motion of objects and shapes.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
6.SS.6 Perform a combination of transformations (translations, rotations, or reflections) on a single 2-D shape, and draw and describe the image. [C, CN, PS, T, V]	➛ Demonstrate that a 2-D shape and its transformation image are congruent. ➛ Model a set of successive translations, successive rotations, or successive reflections of a 2-D shape. ➛ Model a combination of two different types of transformations of a 2-D shape. ➛ Draw and describe a 2-D shape and its image, given a combination of transformations. ➛ Describe the transformations performed on a 2-D shape to produce a given image. ➛ Model a set of successive transformations (translation, rotation, or reflection) of a 2-D shape. ➛ Perform and record one or more transformations of a 2-D shape that will result in a given image.
6.SS.7 Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations. [C, CN, T, V]	➛ Analyze a design created by transforming one or more 2-D shapes, and identify the original shape and the transformations used to create the design. ➛ Create a design using one or more 2-D shapes and describe the transformations used.

(continued)

Specific Learning Outcome(s):	**Achievement Indicators:**
6.SS.8 Identify and plot points in the first quadrant of a Cartesian plane using whole-number ordered pairs. [C, CN, V]	➢ Label the axes of the first quadrant of a Cartesian plane and identify the origin. ➢ Plot a point in the first quadrant of a Cartesian plane given its ordered pair. ➢ Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair. ➢ Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole-number ordered pairs. ➢ Draw shapes or designs, given ordered pairs in the first quadrant of a Cartesian plane. ➢ Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane. ➢ Draw shapes or designs in the first quadrant of a Cartesian plane and identify the points used to produce them.
6.SS.9 Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole-number vertices). [C, CN, PS, T, V]	➢ Identify the coordinates of the vertices of a 2-D shape (limited to the first quadrant of a Cartesian plane). ➢ Perform a transformation on a given 2-D shape and identify the coordinates of the vertices of the image (limited to the first quadrant). ➢ Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to first quadrant).

Prior Knowledge

Students may have had experience with the following:

- Performing a single transformation of a 2-D shape and drawing and describing the image
- Identifying a single transformation of 2-D shapes
- Identifying triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides
- Identifying triangles, squares, rectangles, and circles

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of integers, concretely, pictorially, and symbolically
- Demonstrating and explaining the meaning of preservation of equality, concretely, pictorially, and symbolically
- Constructing and comparing triangles in different orientations
- Creating, labelling, and interpreting line graphs to draw conclusions
- Graphing collected data and analyzing the graph to solve problems

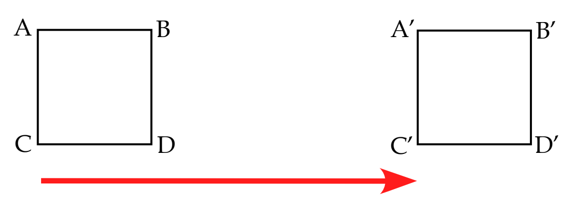
Background Information

Transformations play an important role in the mathematics curriculum. In the Middle Years, the study of transformation can support students' work in patterning, algebra, problem solving, geometry, and statistics. In high school and beyond, the study of transformations helps students recognize the connections between algebra and geometry, and enhances their understanding of other topics such as matrices, scaling, and complex numbers.

A **transformation** can be thought of as a change in the position, size, or shape of a figure.

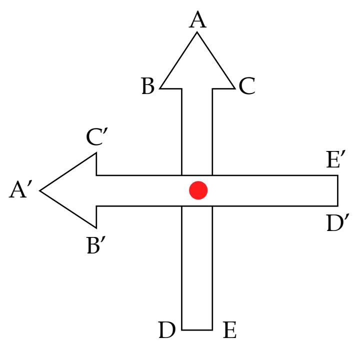
In the learning activities that follow, students are asked to perform three transformations that change the position of a figure. Informally, these transformations are referred to as slides, flips, and turns. Formally, they are known as translations, reflections, and rotations.

A **translation** "slides" a figure a fixed distance in a given direction. The figure and its translation are congruent (same size and shape) and face in the same direction. In the diagram shown below, square ABCD has been translated to a new position represented by square A'B'C'D'.

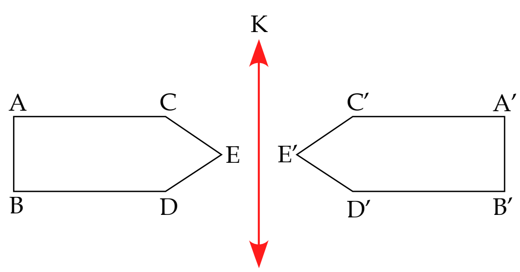


Note that Square A'B'C'D', which is called the image of Square ABCD, is congruent to Square ABCD and faces in the same direction. The arrow indicates the distance and the direction of the translation.

A rotation "turns" a figure any number of degrees around a fixed point called the centre (or point) of rotation. The centre of rotation may be any point within or outside the figure. The figure and its image (the result of the transformation) are congruent, but they may have different orientations (e.g., in the diagram below, the arrow ABCDE has been rotated 90° counter-clockwise about its midpoint). The image arrow A'B'C'D'E' is congruent to Arrow ABCDE, but faces in a different direction.

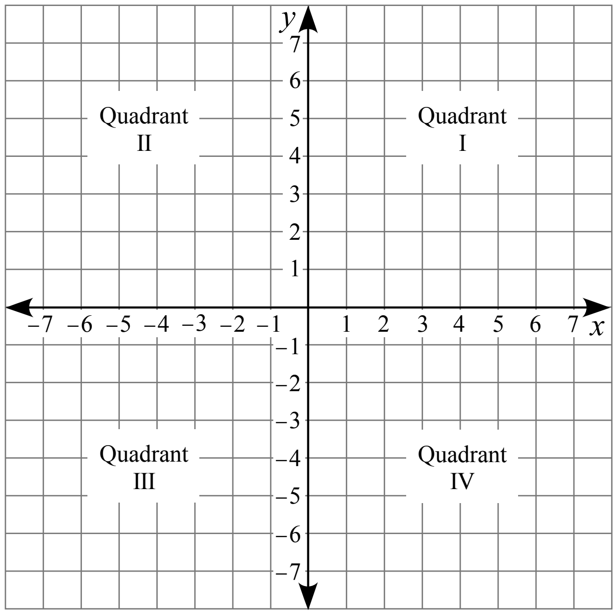


A reflection "flips" the figure over a line, creating a mirror image. The figure and its image are congruent but have different orientations. The line the figure is flipped over is called the line of reflection, and it is the same distance from the figure as its image (e.g., in the diagram below, pentagon ABCDE has been flipped over line K).



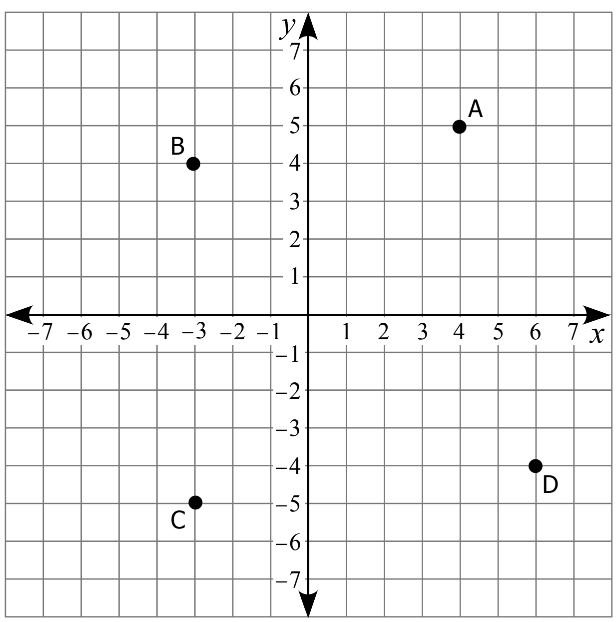
Note that Pentagon A'B'C'D'E' is congruent to Pentagon ABCDE but faces in the opposite direction. Line *k*, the line of reflection, is equidistant from the two pentagons.

In Grade 5, students were introduced to all three above-named transformations using both the formal and informal terminology. In Grade 6, students continue their study of transformations, which gets expanded with the introduction of the Cartesian plane (also called the coordinate plane). The Cartesian plane is formed by a horizontal axis and a vertical axis, often labelled the *x*-axis and *y*-axis respectively. It contains quadrants 1 to 4 (the quadrants are often labelled using Roman numerals I to IV).



Each point on the Cartesian plane is identified by a unique ordered pair (a set of two numbers named in a specific order), represented by (x, y). The first number, x, represents the x-coordinate and the second number, y, represents the y-coordinate. (See points A, B, C, and D below.)

Image of points A, B, C, and D on a Cartesian plane.



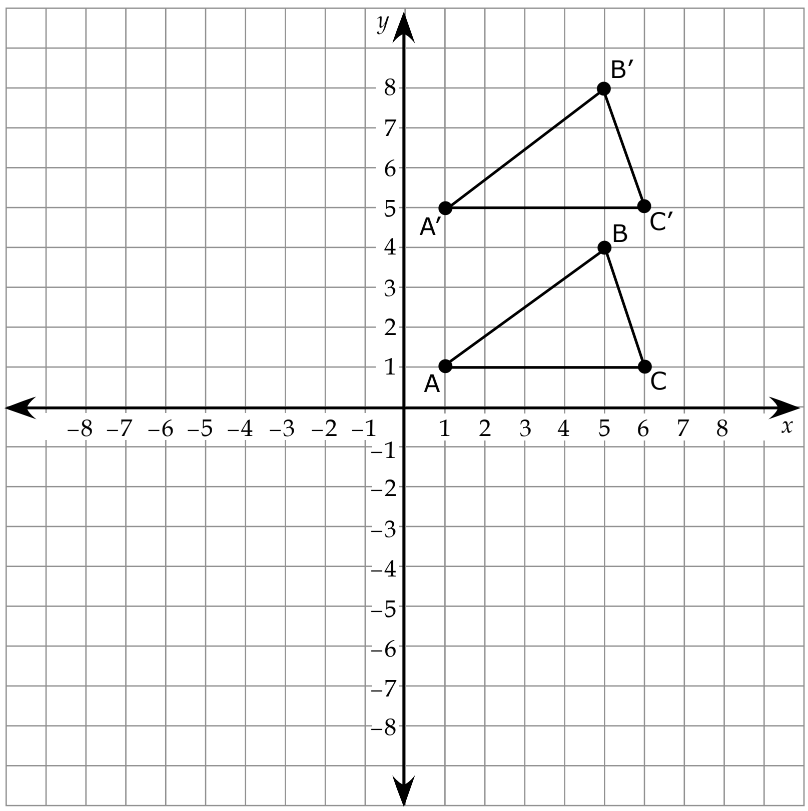
On the Cartesian plane above, point A has coordinates (4, 5), point B has coordinates (–3, 4), point C has coordinates (–3, –5), and point D has coordinates (6, –4).

The only point in the above example with both coordinates being positive is point A, which lies in the first quadrant.

In Grade 6, students will explore the first quadrant of the Cartesian plane by

a) identifying and plotting points (such as point A, on a Cartesian plane above)
b) performing and describing single transformations of a 2-D shape (such as shown below)

2-D shape and transformation image on a Cartesian plane.



The triangle shown above has the following vertices: A (1, 1), B (5, 4), and C (6, 1); its image has the following vertices: A′ (1, 5), B′ (5, 8), and C′ (6, 5). This means that the triangle has been translated four units in the vertical upward direction.

Mathematical Language

axes
Cartesian plane
horizontal
image
line of reflection
ordered pairs
origin
point
quadrant
reflection
rotation
transformation
translation
vertical
vertices

Learning Experiences

Assessing Prior Knowledge

Materials:

- carpeted area or floor mats

Organization: Whole class

Procedure:

1. Tell students you want to check what they remember about transformations.
2. Have a class discussion and demonstration about each type of transformation.
3. Have students lie down on a carpet or mat. Ask them to slide a short distance in one direction. Have them repeat the movement several times by asking them to slide up, down, and sideways.

4. After each slide, ask students the following questions:
   a) What changed?
   b) What remained the same?
5. Emphasize that when a slide is made, the direction in which an object is pointing does not change.
6. Ask the students what the mathematical terminology is for this type of transformation.
7. Have students demonstrate flips. At the end of a flip, students should have changed from stomach to back or back to stomach. Discuss the different ways flips can be completed. For example, students may roll to the left or to the right, or over the feet or head. Have students try these different ways.
8. After each flip, ask students the following questions:
   a) What changed?
   b) What remained the same?

   Emphasize that when an object is flipped, its orientation changes.
9. Ask students how this is different from looking at their reflection in the mirror. Emphasize that a true reflection of oneself would have exactly the same image, just in a different orientation.
10. Ask the students what the mathematical terminology is for this type of transformation.
11. Have students demonstrate turns. To perform a turn, students must keep either their feet or their heads (or belly button) at the same location for the duration of the turn. If the feet are the point (centre) of rotation, then the arms and head are used to move the body. If the head is the point (centre) of rotation, the feet are used to make the move.
12. Have students turn all the way around or partway around. Have them turn in either a clockwise or counter-clockwise direction.
13. After each turn ask students the following questions:
    a) What changed?
    b) What remained the same?
14. Discuss the fact that, after a turn, the direction in which the head points is different, except when a complete turn is made.
15. Ask the students what the mathematical terminology is for this type of transformation.
16. Inform students that in the next few lessons they will be learning more about translations, reflections, and rotations.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Perform a slide.
- ❒ Know the mathematical terminology of a slide.
- ❒ Perform a flip.
- ❒ Know the mathematical terminology of a flip.
- ❒ Perform a turn.
- ❒ Know the mathematical terminology of a turn.

Suggestions for Instruction

- **Demonstrate that a 2-D shape and its transformation image are congruent.**

Materials:

- ruler
- protractor
- BLM 6.SS.6.1: Shape and Image #1

Organization: Whole class/small groups

Procedure:

1. Have a class discussion on transformations (translations, rotations, and reflections).
2. Place on the overhead projector a transparency copy of BLM 6.SS.6.1.
3. Ask students to identify which transformation is presented on the overhead projector, and why they believe that.
4. Distribute to each student a copy of BLM 6.SS.6.1.
5. Tell students to
   a) analyze and discuss with their partners the shape and its image
   b) demonstrate that the 2-D shape and its transformation image are congruent
6. Have a student volunteer come up to the overhead projector and using the transparency copy of BLM 6.SS.6.1; demonstrate that the 2-D shape and its transformation image are congruent.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify the transformation.
- ❒ Demonstrate that the 2-D shape and its transformation image are congruent.

Suggestions for Instruction

- **Model a set of successive translations, successive rotations, or successive reflections of a 2-D shape.**

Materials:

- scissors
- grid paper
- BLM 6.SS.6.1: Shape and Image #1

Organization: Groups of three

Procedure:

1. Distribute to each student a sheet of grid paper, a pair of scissors, and a copy of BLM 6.SS.6.1.
2. Have students cut out the 2-D shape.
3. To model a set of successive translations, have student A
   a) draw a set of successive translations in the horizontal or vertical direction
4. Have the rest of the students in the group figure out the translations that took place.
5. After each slide, student A will ask other students in the group the following questions:
   a) In which direction did the 2-D shape move?
   b) What changed?
   c) What remained the same?
6. To model a set of successive rotations, have student B do a few rotations in the clockwise direction and a few rotations in the counter-clockwise direction, always making sure that the corner of the 2-D shape that touches the dot stays touching the dot.

7. After each rotation, student B will ask other students in the group the following questions:
   a) In which direction did the 2-D shape move?
   b) What changed?
   c) What remained the same?
8. To model a set of successive reflections, have student C do a few reflections downward or upward and a few to the left or right, always making sure that the side of the 2-D shape touching the dot remains touching the dot and on the same line as it was before it got flipped.
9. After each reflection, student C will ask other students in the group the following questions:
   a) In which direction did the 2-D shape move?
   b) What changed?
   c) What remained the same?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Model a successive translation.
- ❒ Model a successive rotation.
- ❒ Model a successive reflection.
- ❒ Describe the transformation performed on a 2-D object.

Suggestions for Instruction

- **Model a combination of two different types of transformations of a 2-D shape.**

Materials:
- scissors
- grid paper
- BLM 6.SS.6.2: Shape and Image #2

Organization: Whole class/small class

Procedure:

1. Distribute to each student a sheet of grid paper, a pair of scissors, and a copy of BLM 6.SS.6.2.
2. To model a combination of two different types of transformations of a 2-D shape, have students do the following:
   a) Draw a vertical line on the grid paper.
   b) Place a dot somewhere in the middle of the line.
   c) Cut out the 2-D shape.
   d) Place the 2-D shape on the left-hand side of the line so that one corner is touching the dot.
   e) Turn the 2-D shape clockwise, making sure that the corner of the 2-D shape that touches the dot stays touching the dot.
   f) Slide the 2-D shape up along the line, making sure they do not change the direction of the 2-D shape.
3. After the two transformations have been completed, ask students the following questions:
   a) What changed?
   b) What remained the same?
4. Have students work in small groups.
5. Let them repeat the activity by doing the following:
   a) Each group member designs an activity using two transformations of his or her own choice.
   b) Let the other members describe the combination of transformations.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Model a combination of two different types of transformations of a 2-D shape.
- ❒ Describe a combination of two different types of transformations of a 2-D shape.

Suggestions for Instruction

- **Draw and describe a 2-D shape and its image, given a combination of transformations.**

Materials:

- grid paper
- ruler
- protractor
- pencil

Organization: Individual

Procedure:

1. Distribute to each student a grid paper
2. Ask students to draw a 2-D shape on the grid paper.
3. Tell students to describe in their journals the 2-D shape that they drew on the grid paper.
4. Tell students to do the following:
   a) Rotate the 2-D shape 90° in the clockwise direction, then make a vertical reflection (flip it over) to the right.
   b) Draw on the grid paper the image that results from the combination of the two transformations.
   c) Describe in their journals the image of the 2-D shape that they drew on the grid paper.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Draw a 2-D shape.
- ❒ Describe a 2-D shape.
- ❒ Draw an image of a 2-D shape given a combination of transformations.
- ❒ Describe an image of a 2-D shape given a combination of transformations.

- **Describe the transformations performed on a 2-D shape to produce a given image.**

Materials:

- BLM 6.SS.6.2: Shape and Image #2

Organization: Individual

Procedure:

1. Review the different types of transformations (translations, rotations, and reflections).
2. Distribute to each student a copy of BLM 6.SS.6.2.
3. Tell students to analyze the 2-D shape and its image in order to determine the different types of transformations that were performed to produce the image.
4. Have students describe in their journals the transformations performed on the 2-D shape shown on the BLM to produce the image shown.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Analyze the transformations performed on the 2-D shape to produce a given image.
- ❒ Describe the transformations performed on the 2-D shape to produce a given image.

Suggestions for Instruction

- **Model a set of successive transformations (translation, rotation, or reflection) of a 2-D shape.**

Materials:

- scissors
- grid paper
- BLM 6.SS.6.2: Shape and Image #2

Organization: Whole class

Procedure:

1. Distribute to each student a sheet of grid paper, a pair of scissors, and a copy of BLM 6.SS.6.2.
2. To model a set of successive transformations of a 2-D shape, tell students to do the following:
   a) Cut out the 2-D shape.
   b) Place the 2-D shape on the grid paper near the top so that one side of the 2-D shape lies along a perpendicular line.
   c) Move the 2-D shape down along the line, stopping after every third square until you reach the bottom of the grid paper.
3. Ask students the following questions:
   a) What was the set of successive transformations of a 2-D shape you performed?
   b) What other sets of successive transformations of a 2-D shape can you perform?
4. Ask a student volunteer to come to the front of the class and demonstrate another set of successive transformations of a 2-D shape.
5. Repeat the activity, making sure that students use all three types of transformations (translation, rotation, and reflection).

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Model a set of successive transformations (translation, rotation, and reflection) of the 2-D shape.

- **Model a set of successive transformations (translation, rotation, or reflection) of a 2-D shape.**

Materials:

- grid paper
- small plastic triangle

Organization: Four groups/whole class

Procedure:

1. Divide the class into four groups.
2. Each group needs to create an activity in order to model a set of successive transformations (translation, rotation, or reflection) of a 2-D shape.
3. Each group will choose a presenter and three recorders.
4. The presenter will use the grid paper and small plastic triangle to demonstrate to the other three groups the original (starting) position of the triangle and the end result of each successive transformation.
5. Each recorder will be responsible to record the correct questions for one designated group (e.g., recorder #1 will record the correct questions asked by members of group B, recorder #2 will record the correct questions asked by members of group C, etc.).
6. To play the game "Guess My Transformation":
   a) The presenter presents (e.g., from group A).
   b) One member from the other three groups will ask a question (e.g., one student from group B asks one question). Members of each group can only ask one question at a time (followed by one student from group D, then back to group B…).
   c) Recorder #1 will record the question for group B only if it is correct.
   d) One member from another group will ask a question (e.g., one student from group C).
   e) Recorder #2 will record the question for group C only if it is correct.
   f) One member from the third group will ask a question (e.g., one student from group D).
   g) Recorder #3 will record the question for group D only if it is correct.
   h) Repeat this form of questioning and recording until all the conditions are satisfied for creating the resultant image. The game is over when all the necessary conditions are filled.
   i) Each question needs to be simple and can request one piece of information only (see sample questions below).
   j) The group with the greatest number of correct questions wins.

7. The following are sample questions:
   a) Is it a translation?
   b) Is it to the right?
   c) Is it three units?
   d) Is it a rotation?
   e) Is it clockwise?
   f) Is it 45º?
   g) Is it a reflection?
   h) Is it vertical?
8. Repeat the game with another group presenting and recording.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Demonstrate understanding of a set of successive transformations (translation, rotation, and reflection) of the 2-D shape.
- ❒ Model a set of successive transformations (translation, rotation, and reflection) of the 2-D shape.
- ❒ Recognize a set of successive transformations (translation, rotation, and reflection) of the 2-D shape.

Suggestions for Instruction

- **Perform and record one or more transformations of a 2-D shape that will result in a given image.**

Materials:
- scissors
- BLM 6.SS.6.3: Envelope Shape

Organization: Individual

Procedure:

1. Distribute to each student a sheet of grid paper, a pair of scissors, and a copy of BLM 6.SS.6.3.
2. Tell students to do the following:
   a) Cut out the shape so that the image and the rest of the grid paper stay as they are.
   b) Place the 2-D shape back on the grid paper in its original position.
   c) Move the 2-D shape on the grid paper, using necessary transformations to obtain the given image.
3. Discuss with the class what transformations of a 2-D shape they performed in order to obtain the given image.
4. Tell students to record in their notebooks the transformations they used to obtain the given image.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Perform the necessary transformations of a 2-D shape to obtain the given image.
- ❒ Record the transformations of a 2-D shape they used to obtain the given image.

Suggestions for Instruction

- **Analyze a design created by transforming one or more 2-D shapes, and identify the original shape and the transformations used to create the design**
- **Create a design using one or more 2-D shapes and describe the transformations used.**

Materials:

- grid paper
- pencil
- ruler
- transparency of BLM 6.SS.7.1: Design

Organization: Whole class/individual

Procedure:

1. Place on the overhead projector a transparency of BLM 6.SS.7.1.
2. Discuss the design.
   a) What do they see?
   b) What type of transformations created the design?
   c) What is the original position of the 2-D shape?
3. Tell students to create and draw on the grid paper their own design using one or more 2-D shapes.
4. Tell students to describe in their notebooks the transformations they used to create the design.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify the original 2-D shape used to create a design.
- ❒ Identify the transformations used to create a design.
- ❒ Create a design using one or more 2-D shapes.
- ❒ Describe the transformations they used to create a design.

Suggestions for Instruction

- **Label the axes of the first quadrant of a Cartesian plane and identify the origin.**
- **Plot a point in the first quadrant of a Cartesian plane given its ordered pair.**

Materials:

- grid paper
- pencil
- ruler
- overhead projector

Organization: Whole class/individual

Procedure:

1. Show a transparency copy of a grid paper on the overhead projector.
2. Have a class discussion regarding a plane and points on a plane.
3. Draw a horizontal line through the middle of your grid paper.
4. Discuss what happens to the plane when you draw a horizontal line through the middle.
5. Draw a vertical line on the grid paper through the middle of the horizontal line.
6. Discuss what happens to the plane when you draw a vertical line through the middle of the horizontal line.
7. Mark the top right quadrant with a Roman numeral, such as *I*.
8. Tell students that this year they are going to be exploring the first quadrant.
9. Label the horizontal line x and the vertical line y.
10. Discuss horizontal and vertical axes.
11. Label the origin.
12. Tell students to do the following:
    a) Draw on their grid paper the two axes and label them.
    b) Label the origin.
    c) Mark quadrant I.
13. Discuss the significance of the origin.
14. Plot a point such as (2, 1), and explain that the first numeral always shows the x-value (horizontal direction) and the second numeral always shows the y-value (vertical direction).
15. Plot another point, explaining the x- and y-coordinates.
16. Ask a student volunteer to come up to the overhead projector and plot a point in the first quadrant. For example: (3, 5).
17. Have several student volunteers come up to the overhead projector and plot a point in the first quadrant (a different point by each student).
18. Provide an ordered pair for several points in Quadrant I, and tell students to plot them.

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Label the axes of the first quadrant of a Cartesian plane.
- ❒ Identify the origin.
- ❒ Plot a point in the first quadrant of a Cartesian plane given its ordered pair.

Suggestions for Instruction

- **Label the axes of the first quadrant of a Cartesian plane and identify the origin.**
- **Plot a point in the first quadrant of a Cartesian plane given its ordered pair.**
- **Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair.**

Materials:

- pencil
- BLM 6.SS.8.1: Matching Game

Organization: Whole class/individual

Procedure:

1. Before you play the Matching Game, discuss the Cartesian plane.
   a) Have a student volunteer draw a Cartesian plane on the chalkboard, and label the axes and the origin.
   b) Provide an ordered pair, and have one student plot the required point in the first quadrant of the Cartesian plane.
   c) Provide an ordered pair, and have another student plot the designated point in the first quadrant of the Cartesian plane.
   d) Have students place a few points on the chalkboard, and then tell students that now they can match dots by playing the Matching Game.
2. Distribute to each student a copy of BLM 6.SS.8.1.
3. Tell students to do the following:
   a) Match each point in the first quadrant of a Cartesian plane with its corresponding ordered pair.
   b) Write next to each dot the corresponding letter.
4. Discuss the results of the Matching Game after each student has completed the game.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Label the axes of the first quadrant of a Cartesian plane.
- ❐ Identify the origin.
- ❐ Plot a point in the first quadrant of a Cartesian plane, given its ordered pair.
- ❐ Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair.

Suggestions for Instruction

- **Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole-number ordered pairs.**

Materials:

- BLM 6.SS.8.2: Cartesian Plane #1
- BLM 6.SS.8.3: Cartesian Plane #2

Organization: Whole class/individual

Procedure:

1. Discuss the Cartesian plane.
2. Place on the overhead projector a transparency of BLM 6.SS.8.2.
3. Choose several whole-number ordered pairs, and plot and label the points. Make sure you choose numbers that are not marked on the interval. For example: (5, 3).
4. Have some students come up and plot a point in the first quadrant of the Cartesian plane after you provide its ordered pair.
5. Distribute to each student a copy of BLM 6.SS.8.2.
6. Write several ordered pairs on the board, and have students plot the points.
7. Repeat the activity with BLM 6.SS.8.3.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Plot points in the first quadrant of a Cartesian plane with intervals of 2 on its axes, given whole-number ordered pairs.
- ❐ Plot points in the first quadrant of a Cartesian plane with intervals of 5 on its axes, given whole-number ordered pairs.

Suggestions for Instruction

- **Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole-number ordered pairs.**
- **Draw shapes or designs, given ordered pairs in the first quadrant of a Cartesian plane.**

Materials:

- pencil crayons
- ruler
- BLM 6.SS.8.2: Cartesian Plane #1

Organization: Pairs

Procedure:

1. Discuss using ordered pairs to draw shapes or designs in the first quadrant of a Cartesian plane.
2. Distribute to each student a copy of BLM 6.SS.8.2.
3. Tell students that they will be playing with a partner a game called "Guess My Shape."
4. Explain the rules of the game:
   a) Student A will plot points to make a 2-D shape or designs in the first quadrant of a Cartesian plane.
   b) Student A will call out the coordinates of the points he or she used in producing the shape of design.
   c) Student B plots the points using the ordered pairs that student A tells him or her.
   d) When student A has all the points called out, he or she will say "Guess my shape!"

e) Student B will guess the shape.

f) Student A and student B will compare their shapes to make sure they are the same.

5. Repeat the game with student B making the shape and student A guessing.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Plot points in the first quadrant of a Cartesian plane with intervals of two on its axes, given whole-number ordered pairs.
- ❒ Draw shapes or designs, given ordered pairs in the first quadrant of a Cartesian plane.

Suggestions for Instruction

- **Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole-number ordered pairs.**
- **Draw shapes or designs, given ordered pairs in the first quadrant of a Cartesian plane.**
- **Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane.**

Materials:

- pencil crayons
- ruler
- BLM 6.SS.8.2: Cartesian Plane #1

Organization: Small groups/individual/class

Procedure:

1. Distribute to each student a copy of BLM 6.SS.8.2.
2. Tell students that they will be drawing four lines.

   a) The first line will be blue. It will go through points A, B, and C, whose ordered pairs are: A (3, 15), B (8, 15), and C (15, 15)

   b) The second line will be orange. It will go through points D, E, and F, whose ordered pairs are: D (3, 7), E (4, 7), and F (13, 7)

   c) The third line will be black. It will go through points G, H, and I, whose ordered pairs are: G (4, 4), H (8, 4), and I (15, 4)

d) The fourth line will be yellow. It will go through points J, K, and L, whose ordered pairs are: J (3, 0), K (8, 0), and L (13, 0)

3. Discuss the four lines. (What is similar? What is different?)
4. Tell students to determine the distance between points:
   a) A and B
   b) B and C
   c) A and C
5. Students discuss with their group members.
6. Discuss with the whole class the three distances on the first horizontal line.
7. Repeat the process for the three distances for the other three horizontal lines.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Plot points in the first quadrant of a Cartesian plane with intervals of 2 on its axes, given whole-number ordered pairs.
- ❒ Draw horizontal lines given ordered pairs in the first quadrant of a Cartesian plane.
- ❒ Determine the distance between points along horizontal lines in the first quadrant of a Cartesian plane.

Suggestions for Instruction

- **Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole-number ordered pairs.**
- **Draw shapes or designs, given ordered pairs in the first quadrant of a Cartesian plane.**
- **Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane.**

Materials:

- pencil crayons
- ruler
- BLM 6.SS.8.3: Cartesian Plane #2

Organization: Small groups

Procedure:

1. Distribute to each student a copy of BLM 6.SS.8.3.
2. Tell students that they will be playing a game called "How many units did they move?" using dots on vertical lines.
3. Explain the rules of the game:
   a) Student #1 will call out the ordered pairs for three points not on the same vertical line. For example: (2, 3); (5, 1); and (6, 7) for points A, B, and C.
   b) The other students will plot the points and draw the shape.
   c) Student #1 will do the following:
      - Draw a vertical line through each point and move the three points either up or down the same number of units.
      - Say to the rest of the group that the three points each found a new location.
      - Call out the three new ordered pairs for the three new points. For example: (2, 9); (5, 7); and (6, 13).
   d) The other students will do the following:
      - Plot the new points.
      - Draw the new shape.
   e) Student #1 will ask the following:
      - How many units did point A move, and in which direction?
      - How many units did point B move, and in which direction?
      - How many units did point C move, and in which direction?
   f) The rest of the group members will provide the required reply. (In this example, each point moved 6 units up.)
4. Repeat the game with Student #2, and so on.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Plot points in the first quadrant of a Cartesian plane with intervals of 2 on its axes, given whole-number ordered pairs.
- ❒ Draw vertical lines given ordered pairs in the first quadrant of a Cartesian plane.
- ❒ Determine the distance between points along vertical lines in the first quadrant of a Cartesian plane.

Suggestions for Instruction

- **Draw shapes or designs in the first quadrant of a Cartesian plane and identify the points used to produce them.**

Materials:

- grid paper
- pencil
- ruler
- overhead projector

Organization: Whole class/individual

Procedure:

1. Show a transparency copy of a grid paper on the overhead projector.
2. Have a class discussion regarding points and ordered pairs on a plane.
3. Draw and label the axes of the first quadrant of a Cartesian plane, and identify the origin.
4. Draw a capital **N** shape on grid paper without plotting the dots first. For example: draw line segments from (1, 1) to (1, 7), from (1, 7) to (4, 1), and from (4, 1) to (4, 7)
5. Ask students to identify the points used to draw the **N** shape.
6. Draw a square on the grid paper without plotting the dots first. For example, draw line segments from (6, 2) to (6, 7), from (6, 7) to (11, 7), from (11, 7) to (11, 2) and from (11, 2) to (6, 2).
7. Ask students to identify the points used to draw the square.
8. Distribute to each student a grid paper.
9. Tell students to draw and label the axes of the first quadrant of a Cartesian plane, and identify the origin.
10. Tell students to do the following:
    a) Draw one of each:
        - Triangle
        - Rectangle
        - Pentagon
        - Hexagon

    b) Identify the points used to draw each shape.
11. Circulate to make sure they are: (a) drawing the correct shapes, and (b) identifying the correct points.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Label the axes of the first quadrant of a Cartesian plane.
- ❒ Identify the origin.
- ❒ Draw shapes or designs in the first quadrant of a Cartesian plane.
- ❒ Identify the points used to produce the shapes or designs.

Suggestions for Instruction

- **Identify the coordinates of the vertices of a 2-D shape (limited to the first quadrant of a Cartesian plane).**

Materials:

- pencil
- BLM 6.SS.8.2: Cartesian Plane #1
- BLM 6.SS.9.1: Identification Game

Organization: Whole class/individual

Procedure:

1. Discuss drawing shapes in the first quadrant of a Cartesian plane, and identifying the points used to produce the designs.
2. Place on the overhead projector a transparency of BLM 6.SS.8.2.
3. Draw a triangle in the first quadrant. For example, use the following ordered pairs (0, 4), (6, 0), and (5, 8) as the vertices of the triangle.
4. Ask students to identify the coordinates of the vertices of your triangle.
5. Discuss what they had to do to identify the coordinates of the vertices of your triangle.
6. Distribute to each student a copy of BLM 6.SS.9.1.
7. Tell students to do the following:
   a) Identify the coordinates of the vertices of the 2-D shape.
   b) Write the coordinates of each vertex next to it.
8. Circulate to make sure that students correctly identify each vertex.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify the coordinates of the vertices of a 2-D shape (limited to the first quadrant of a Cartesian plane).

Suggestions for Instruction

- **Perform a transformation on a given 2-D shape and identify the coordinates of the vertices of the image (limited to the first quadrant).**

Materials:

- pencil
- BLM 6.SS.8.2: Cartesian Plane #1

Organization: Whole class/individual

Procedure:

1. Discuss transformations (translations, rotations, and reflections).
2. Place on the overhead projector a transparency copy of BLM 6.SS.8.2.
3. Draw a triangle in the first quadrant. For example, use the following ordered pairs (8, 2), (12, 1), and (11, 5) as the vertices of the triangle.
4. Tell students to perform the following transformation:
   a) Horizontal translation 5 units to the left.
5. Ask students to identify the coordinates of the vertices of the image.
6. Repeat the activity with different transformations.
7. Circulate to make sure that students correctly identify each vertex.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Perform a transformation on a given 2-D shape (limited to the first quadrant).
- ❒ Identify the coordinates of the vertices of the image (limited to the first quadrant).

- **Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to first quadrant).**

Materials:

- pencil
- BLM 6.SS.9.2: Dizzy Pentagon

Organization: Whole class/individual

Procedure:

1. Discuss transformations (translations, rotations, and reflections).
2. Distribute to each student a copy of BLM 6.SS.9.2.
3. Tell students to do the following:
   a) Analyze the Dizzy Pentagon and its image.
   b) Analyze the vertices of the Dizzy Pentagon and its image.
   c) Describe in their journals how the vertices of the Dizzy Pentagon changed to the corresponding vertices of its image.
4. Circulate to make sure that students correctly describe the positional change in each vertex.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to first quadrant).

Ten Flags

Introduction: Ten Flags (Based on NCTM article from *Mathematics Teaching in the Middle Schools* 16.2, Sept. 2010, pp. 72-75.)

Purpose: The purpose of this activity is for students to put into practice skills that they acquired inside the mathematics classroom. Students will need to apply their knowledge of the first quadrant of the Cartesian plane. They will also have to rely on some prior knowledge, such as the following:

a) the metric system—in particular, measuring distance

b) angle measures

The processes that are demonstrated by this task are communication, connections, problem solving, reasoning, and visualization. To make the task fun, students will have to collaborate as well as cooperate with each other.

Curricular Links: This task can be linked to the social studies curriculum. The flags can be those of different provinces within Canada, or they can be of different countries of the world.

Materials/Resources:

- 10 flags
- one stand-alone post
- 10 high-stools (with square seats)
- 10 protractors
- 10 boxes

For each student:

- one instruction sheet
- measuring tape
- pencil
- notepad
- assessment sheet
- BLM 5-8.4: How I Worked in My Group

Note: During this task, students will use angles and polygons to get familiar with the map of Manitoba. (You may use a different map to fit your social studies lesson.)

Organization:

- Use the gym for this task, and set it up as a large-scale Cartesian plane.
- Small groups

Inquiry:

Scenario:

Students will be located along two walls: (1) the wall by the entrance of the gym, and (2) the wall that is perpendicular to and to the left of the entrance. The flags will be standing up seemingly scattered but actually strategically placed by the teacher. Teacher will ensure that each flag is clearly visible from all ten marked spots (high stools).

Procedure:

Day 1: Let's Prepare!

Students will be divided into small groups. They will be working in the classroom.

Students will do the following:

1. Number the 10 high-stools as 1, 2, 3, 4, 5, A, B, C, D, E.
2. Make an open box (seat cover) to be placed upside-down on each box so that the box covers the seat of the high stool and the sides of the box hang over the sides of the seat, thus preventing itself (the box) from moving.
3. Tape a large copy of a protractor onto each box, making sure that the straight edge of the protractor is even with a straight edge of the box.
4. Practise measuring angles. Use the taped-on protractor and a straight edge to measure various angles in the classroom.

The teacher will do the following:

1. Take as many pieces of equal-sized paper as there are groups.
2. Mark each paper with a different number set, using two numbers of the 10 high stools such as (1, D), (3, B), (4, A), (5, C), (2, E), and so on.
3. Fold each piece of paper so that the number is not displayed.
4. Place all 10 pieces of marked and folded paper into a paper bag.

Day 2: Let's Get Moving!

Everyone will be in the gym.

Before students enter the gym, the teacher strategically places the flags, ensuring that each is clearly visible from all 10 marked spots (high stools).

1. Students will mark a line parallel to and four metres away from the wall along the entrance, as well as a line parallel to and four metres away from the wall that is perpendicular to, and to the left of, the entrance. The two lines will be the two axes of the "life-size" Cartesian plane. Where the two perpendicular lines meet, students will place a stand-alone post, which will mark the origin.

2. Along each line (each axis), starting at the intersection of the two lines (the origin), students will measure and mark two-metre intervals by placing a high stool on each two-metre mark. Along each line, therefore, there will be five high stools: at the two-metre, four-metre, six-metre, eight-metre, and 10-metre marks. To achieve good results, accuracy in measuring is important.
3. Each group of students will do the following:
   a) Draw a number set out of a paper bag.
   b) Stand by the first specified high stool.
   c) Measure the angles made by all 10 flags from the first location.
   d) Record the angles made by all 10 flags measured from the first location.
   e) Stand by the second specified high stool.
   f) Measure the angles made by all 10 flags from the second location.
   g) Record the angles made by all 10 flags measured from the second location.

Day 3: Let's Get the Record Straight!

The teacher will have a transparency of the correct locations of each flag.

1. Each group of students will do the following:
   a) Design a Cartesian plane.
   b) Use a dot to record the correct spot for each flag.
   c) Describe the location of each flag in terms of the two axes.
   d) Check for accuracy of location for each flag.
   e) Do self-assessment.

Assessment:

Use the following observation checklist to assess student learning.

The student can do the following:	Yes	No	Comment
Design a Cartesian plane.			
Correctly identify the origin.			
Correctly identify each axis.			
Plot points correctly.			
Demonstrate an understanding of an angle.			
Measure angles accurately.			
Replicate an angle.			

Extension:

Taking it further

Have students describe how they would design a task involving the Cartesian plane.

NOTES

Grade 6 Mathematics

Statistics and Probability

Grade 6: Statistics and Probability (Data Analysis) (6.SP.1, 6.SP.2, 6.SP.3)

Enduring Understanding(s):

Graphs are a way of organizing, representing, and communicating information. Information for the purpose of data can be collected by a variety of methods, including questionnaires, experiments, databases, and electronic media.

General Learning Outcome(s):

Collect, display, and analyze data to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
6.SP.1 Create, label, and interpret line graphs to draw conclusions. [C, CN, PS, R, V]	➛ Determine the common attributes (title, axes, and intervals) of line graphs by comparing a set of line graphs. ➛ Determine whether a set of data can be represented by a line graph (continuous data) or a series of points (discrete data), and explain why. ➛ Create a line graph from a table of values or a set of data. ➛ Interpret a line graph to draw conclusions.
6.SP.2 Select, justify, and use appropriate methods of collecting data, including ▪ questionnaires ▪ experiments ▪ databases ▪ electronic media [C, PS, T]	➛ Select a method for collecting data to answer a question, and justify the choice. ➛ Design and administer a questionnaire for collecting data to answer a question and record the results. ➛ Answer a question by performing an experiment, recording the results, and drawing a conclusion. ➛ Explain when it is appropriate to use a database as a source of data. ➛ Gather data for a question by using electronic media, including selecting data from databases.
6.SP.3 Graph collected data and analyze the graph to solve problems. [C, CN, PS]	➛ Select a type of graph for displaying a set of collected data, and justify the choice of graph. ➛ Solve a problem by graphing data and interpreting the resulting graph.

Prior Knowledge

Students may have had experience with the following:

- Collecting, organizing, and using first-hand data (tally marks, line plots, charts, and lists) to answer questions
- Drawing conclusions by constructing and interpreting pictographs and bar graphs. involving many-to-one correspondence
- Drawing conclusions by constructing and interpreting double bar graphs
- Representing and describing decimals to hundredths
- Representing and describing whole numbers to 1 000 000

Related Knowledge

Students should be introduced to the following:

- Using graphs and tables to represent and describe patterns and relationships
- Using technology to solve problems involving large numbers

Background Information

In Grade 5, students learned to differentiate between first-hand data (information they obtained directly by asking questions, measuring, observing, or doing experiments) and second-hand data (readily available and obtainable from sources such as newspapers, magazines, journals, and electronic media). They also learned to construct and interpret double bar graphs (two sets of comparable data) to draw conclusions.

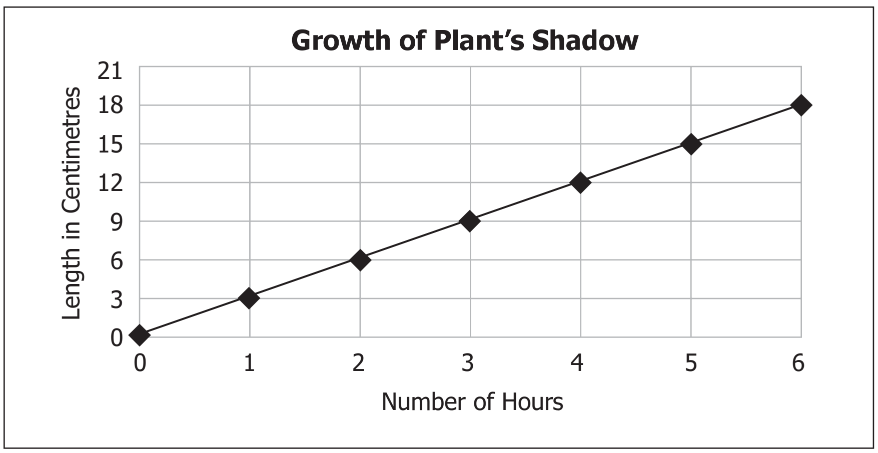
In Grade 6, students will learn to draw conclusions by creating, labelling, and interpreting line graphs. They will also solve problems by selecting, justifying, and using appropriate methods of collecting data, and by graphing collected data and analyzing the graph.

Line graphs are used when continuous data need to be presented, such as distance between a student and the school as he or she is nearing or leaving the school, shadow of a tree from sunrise to sunset, age, height, and temperature.

The method of data collection (including questionnaires, experiments, databases, and electronic media) depends on the question the student wants answered. Collected data is easier analyzed when it is presented by a graph. The title of the graph is placed above the graph, as shown below. The axes are labelled. By looking at the graphs below, we see that, for Graph 1, the x-axis represents the number of hours over which the measurements were taken, and the y-axis represents the length of the plant's shadow in centimetres. For Graph 2, the x-axis represents the time in months over which the measurements were taken, and the y-axis represents the weight of the puppy in kilograms. The increments should be of equal size, numbered, and start at 0. If part of the scale has been omitted, then the graph will have a squiggle in its scale in order to indicate that an amount has been skipped. The independent values are placed along the horizontal axis and the dependent values are placed along the vertical axis. The two line graphs below will clarify this further.

Imagine that you want to measure the shadow of a certain flower between noon and evening, and use your data to create a graph. If this flower had no shadow at noon, had a 3 cm shadow at 1 p.m., and 3 more cm each hour, your graph should look much like Graph 1 below. Since your measurements were taken on the hour, each hour, the dots are placed above the numbers denoting the time. The shadow of the little flower did not grow in leaps of 3 cm when you were doing the measuring, but it continuously grew throughout the entire time (during and in between the time of the measurements). Since the growth is continuous, we represent the continuous growth by connecting the dots and forming a solid line.

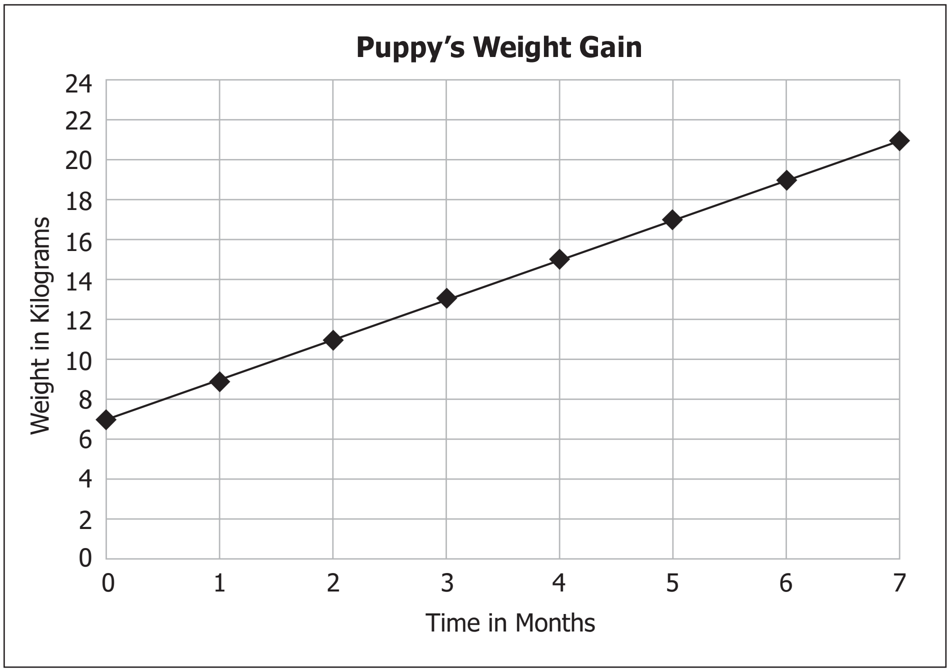
Graph 1



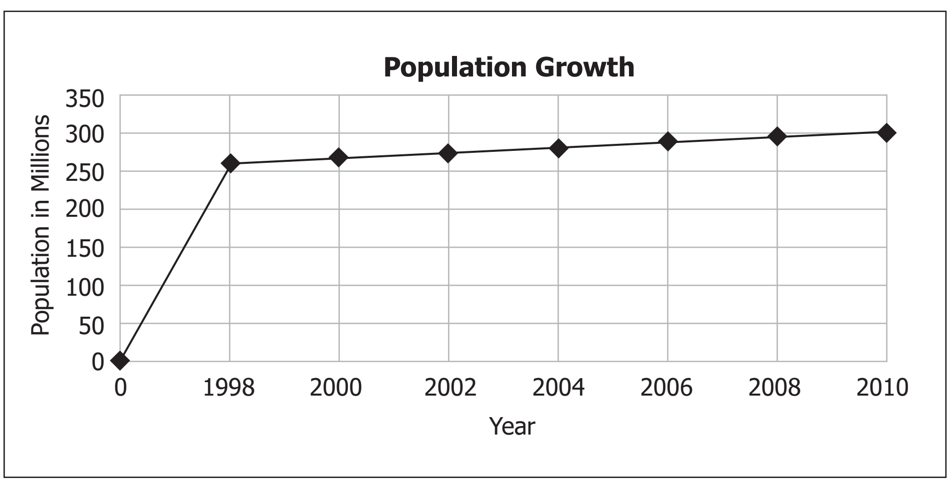
Also, imagine that a little boy named Leo got a puppy for his birthday. If the puppy weighed 7 kg when Leo got it, 9 kg one month later, and gained 2 more kg each month, the graph representing the growth of the puppy should look much like Graph 2 below. Since the weight measurements were taken each month, the dots were placed above the numbers denoting the number of months passed since Leo got the puppy. The weight of the puppy did not increase in leaps of 2 kg on the day when the measurements were taken, but there was a continuous fluctuation of weight during each month between the measurements.

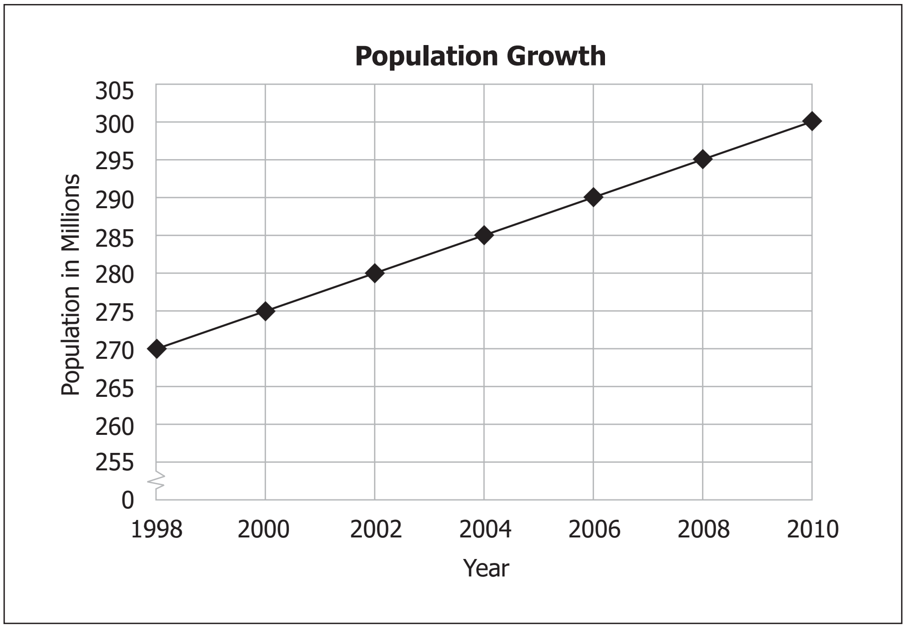
Although the numbering along the two axes starts at zero, the graph itself may or may not start at zero. Graph 1 starts at 0 because there was no shadow when the original measurement started, but Graph 2 starts at $y = 7$ because the puppy weighed 7 kilograms when Leo got it.

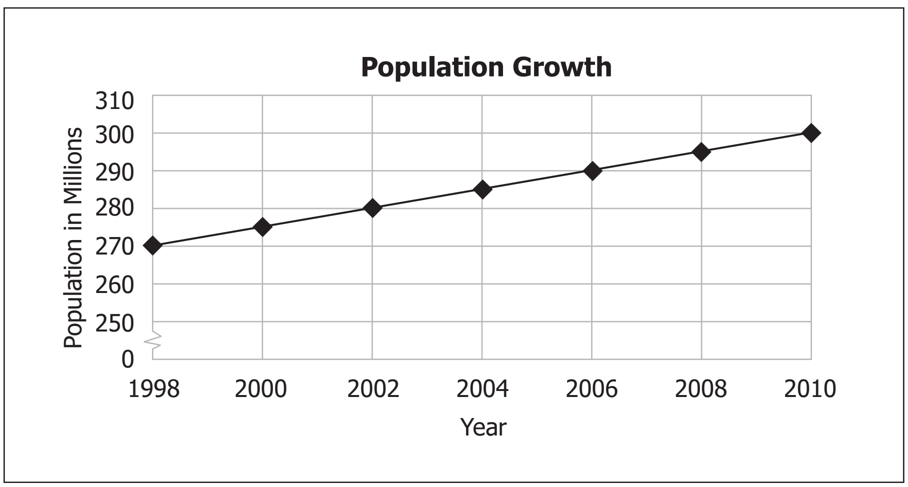
Graph 2



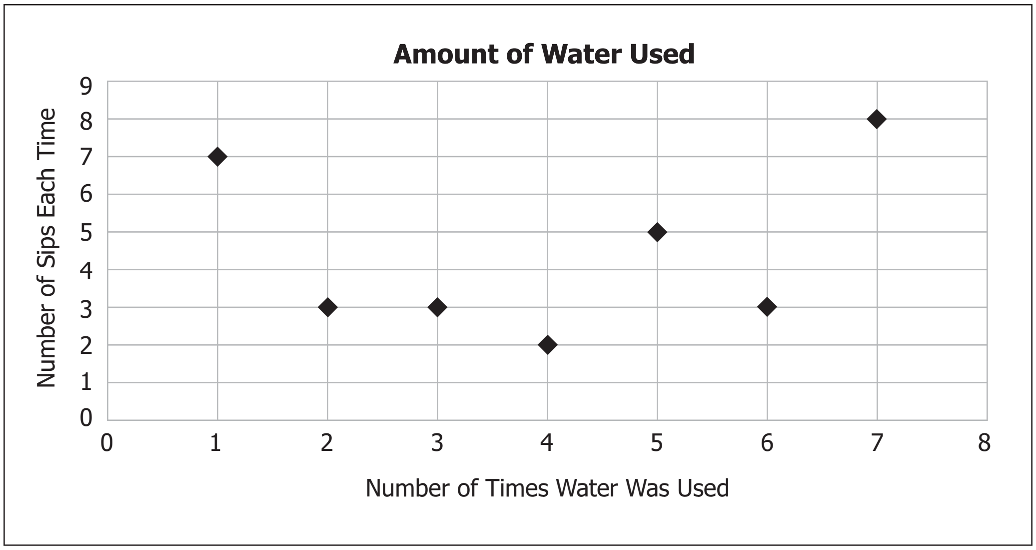
Now, imagine that you read in the daily newspaper that a certain country had a population of 270 million in the year 1998, 275 million in 2000, 280 million in 2002, 285 million in 2004, 290 million in 2006, 295 million in 2008, and 300 million in 2010. If you were to graph this data, you would need to place a squiggle between the zero and 1996 on the *x*-axis, and you would also need to place a squiggle between zero and 270 million to indicate that part of the scale has been omitted.







It is important that students understand that they can connect the dots to form a line only if the graph represents continuous data. If the data is discrete, the dots are not connected, as in the example below.



Imagine that during your math class you wanted to record the number of sips you had each time you drank some water. The first time you reached for your water bottle, you were very thirsty so you had seven sips of water; the second and third time, you were less thirsty so you had three sips each time; the fourth time, you only took two sips; the fifth time, you got a little thirstier so you took five sips; the sixth time around, you took three sips; and by the end of the math class, you were quite thirsty again so you drank eight sips of water. You would not connect these dots because these seven dots represent seven distinct actions. You were drinking only at those specified seven times you recorded and not in between—that is, you were not drinking continuously starting from the first time all the way through to the seventh time.

Mathematical Language

data
graph
line graph
title
axes
intervals
continuous data
discrete data

Assessing Prior Knowledge

Materials:

- graph paper
- BLM 6.SP.1.3: Prior Knowledge
- pencil
- straight edge

Organization: Individual/whole class

Procedure:

1. Let students know that they will be doing an activity and that the purpose of this activity is to find out what they know about line graphs.
2. Distribute to each student a copy of BLM 6.SP.1.3: Prior Knowledge and tell them to complete the activity.
3. Have students share their graphs and conclusions with the rest of the class.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Make a line graph.
- ❒ Explain when a line graph is used.
- ❒ Include a title with the graph.
- ❒ Label the axes.
- ❒ Use appropriate scale with equal increments that start at zero.
- ❒ List more than one way of collecting data.

Suggestions for Instruction

- **Determine the common attributes (title, axes, and intervals) of line graphs by comparing a set of line graphs.**

Materials:

- BLM 6.SP.1.4: Common Attributes of Line Graphs

Organization: Pairs/whole class

Procedure:

1. Have students work in groups of two.
2. Distribute to each student a copy of BLM 6.SP.1.4.
3. Ask students to analyze the line graphs and discuss with their partner what they observed.
4. Have a class discussion on what is similar and/or different.
5. Write the common attributes (title, axes, and intervals) on the board, and discuss the importance of each.
6. Ask students to write in their journals the common attributes of a line graph, and explain why each is important.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Tell that each line graph has a title and that the title of the graph is placed above the graph.
- ❒ Tell that each line graph has two axes (horizontal and vertical) and that the axes are labelled.
- ❒ Tell that each axis is marked with increments of equal size, which are numbered, and start at zero.

Suggestions for Instruction

- **Determine whether a set of data can be represented by a line graph (continuous data) or a series of points (discrete data), and explain why.**

Materials:

- BLM 6.SP.1.5: Timmy's Mom Had a New Baby

Organization: Pairs/whole class

Procedure:

1. Have students do this activity individually.
2. Distribute to each student a copy of BLM 6.SP.1.5.
3. Ask students to analyze the set of data and answer the following questions:
   a) How tall was the baby at the time of the first check-up?
   b) How much did the baby grow between check-ups?
   c) Would the baby grow more if Timmy sang to the baby? Why?
   d) How much do you think the baby would grow between check-ups if Grandma came for a visit? Why?
   e) Would you use a line graph or a series of points to represent this data? Why?
4. Have a class discussion.
5. Ask students to write in their journals if they would use the same type of graph to record the number of candy bars they ate each day, and explain their reason for their decision.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Tell that the increase in height was the same each month.
- ❒ Tell that the data was continuous.
- ❒ Tell that they need to use a line graph to represent continuous data.

- **Create a line graph from a table of values or a set of data.**

Materials:

- BLM 6.SP.1.6: Grandma's Lilac Bush

Organization: Individual

Procedure:

1. Have students do this activity individually.
2. Distribute to each student a copy of BLM 6.SP.1.6.
3. Tell students to answer the following questions:
   a) What was the height of Grandma's lilac bush after three months?
   b) What do you think it means that the lilac bush was 40 cm at 0 months? Explain.
   c) Do you think the lilac bush would grow faster if Grandma would plant another lilac bush next to it? Explain.
4. Tell students to create a line graph from the table of values.
5. Circulate to see if they record the title of the graph, mark the axes, and use equally spaced intervals.
6. Have a class discussion.
7. Ask students to write in their journals what they learned about creating a line graph from a table of values.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Draw a line graph.
- ❒ Place the title above the graph.
- ❒ Label each axis.
- ❒ Mark each axis with increments of equal size, number increments, and start at 0.

Suggestions for Instruction

- **Interpret a line graph to draw conclusions.**

Materials:

- BLM 6.SP.1.7: Mom's Distance from Home

Organization: Small groups (3 or 4 students)/whole class

Procedure:

1. Have students do this activity in groups of 3 or 4.
2. Place on the overhead projector a transparency of BLM 6.SP.1.7.
3. Tell students to study the graph, and discuss with their group members what they observed.
4. Circulate to listen for their observation. (Did the students observe the title, axes, and intervals?) Following are some questions you might want to ask your students:
   a) What does the horizontal portion of the graph represent?
   b) What does the vertical portion of the graph represent?
   c) Do you think mom was walking or driving? Why?
   d) What do you think the graph would look like if she started running?
   e) Do you think the graph would look different if you accompanied mom? Why?
   f) What do you think the graph would look like if dad took the same route? Why?
5. Have a class discussion.
6. Ask students to write in their journals what they learned.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Recognize a line graph.
- ❒ Observe the title above the graph.
- ❒ Observe that each axis is labelled.
- ❒ Observe that each axis is marked with increments of equal size, that increments are numbered, and that they start at 0.

Suggestions for Instruction

- **Select a method for collecting data to answer a question, and justify the choice.**
- **Explain when it is appropriate to use a database as a source of data.**

Materials:

- BLM 6.SP.1.8: Questions for Data Collection #1

Organization: Whole class

Procedure:

1. Place on the overhead projector a transparency of BLM 6.SP.1.8.
2. Read the first question.
3. Let students choose the method of collecting data they think is appropriate for the question.
4. Discuss their choice.
5. Read the next question and repeat the process until all six questions have been discussed.
6. Discuss the database as a source of data. If students chose the database as a source of data for any of the questions, discuss why they did so. If students did not choose the database as a source of data for any of the questions, discuss why they did not.
7. Ask students to write in their journals what they learned about data sources—i.e., give an example of when they might use
   a) questionnaires as a method of data collection
   b) experiments as a method of data collection
   c) databases as a method of data collection
   d) electronic media as a method of data collection

Observation Checklist

☑ Check students' replies to determine whether they can do the following:
- ❒ Recognize the appropriate method of collecting data.
- ❒ Recognize when it is appropriate to use a database as a method of collecting data.
- ❒ Provide a good justification for choosing a particular method of collecting data.

Suggestions for Instruction

- **Select a method for collecting data to answer a question, and justify the choice.**
- **Explain when it is appropriate to use a database as a source of data.**

Materials:

- Posters marked "questionnaire," "experiment," "database," "electronic media" (each posted in a different corner of the room)

Organization: Whole class

Procedure:

1. Let students know that you will be playing a game called "Four Corners."
2. Provide students with a data collection scenario.
3. Ask them to think individually about which would be the best way to collect the data (questionnaire, experiment, database, electronic media).
4. Let students know that each corner of the room represents a different type of data collection.
5. Ask them to go to the corner that represents the method of data collection that they think is best suited to the scenario.
6. When they are there, chat with the others who also selected that method, and prepare to share the reasoning with the whole class.
7. One by one, the groups will share their reasoning, trying to convince other members of the class to come to their corner.

Suggestions for Instruction

- **Select a method for collecting data to answer a question, and justify the choice.**
- **Explain when it is appropriate to use a database as a source of data.**

Materials:

- BLM 6.SP.1.9: Questions for Data Collection #2

Organization: Individual/whole class

Procedure:

1. Have students work individually.
2. Distribute to each student a copy of BLM 6.SP.1.9.
3. Say the following to students
   a) Read each question carefully.
   b) For each question, write down the method you would choose to collect your data.
   c) Explain why you would choose that method of data collection.
4. Circulate to see if they choose the appropriate method of data collection for each question.
5. Have a class discussion.
6. Ask students to write in their journals what they learned.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Recognize the appropriate method of collecting data.
- ❒ Recognize when it is appropriate to use a database as a method of collecting data.
- ❒ Provide a good justification for choosing a particular method of collecting data.

- **Design and administer a questionnaire for collecting data to answer a question and record the results.**

Organization: Small groups (three or four students)/whole class

Procedure:

1. Have students sit in small groups.
2. Discuss with the whole class possible questions that lend themselves to a questionnaire type of data collecting. Choose types of questions that can be done in the classroom.
3. Write on the board the questions appropriate for the questionnaire type of data collecting.
4. Choose one of the questions and, together, design a questionnaire for data collecting.
5. Discuss what makes a good questionnaire.
6. Tell each group to choose one question from the board and design a questionnaire.
7. Circulate to see if the questionnaires are asking the appropriate information to collect good data for each particular question.
8. Have a class discussion.
9. Make copies of student questionnaires.
10. Ask students to administer the questionnaire to other groups, and then record the results.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Recognize the appropriate method of collecting data.
- ❒ Recognize when it is appropriate to use a questionnaire as the method of collecting data.
- ❒ Design and administer a questionnaire for collecting data.
- ❒ Record results of a questionnaire.

Suggestions for Instruction

- **Answer a question by performing an experiment, recording the results, and drawing a conclusion.**

Materials:

- computer
- respiration rate belt
- relative pressure sensor

Organization: Whole class

Procedure:

1. Use Internet Explorer (or any other web browser) to find *PASCO:AP Biology Probeware Labs*.
2. Click on the following links:
   a) "educator resources" (in the blue title area at the top)
   b) Scroll down to "Experiments" and click on "Online Experiments"
   c) "Take My Breath Away" (under Featured Experiments)
   d) Scroll down to "Downloads," and click on "Take My Breath Away - Science Workshop"
3. Read the experiment "Take My Breath Away - Science Workshop for Systems" and follow the instructions.
4. Under "Data Analysis," omit question 2. (Mean is not part of the Grade 6 curriculum.)
5. Omit "Conclusions and Extensions."
6. Have a class discussion and ask students to record their answers to the following questions:
   a) What was your breathing rate at the start of the experiment?
   b) What was your breathing rate during your exercise?
   c) What was your breathing rate at the end of your exercise?
   d) What do you think your breathing rate would be if you were sleeping? Why?
   e) What do you think your breathing rate would be if your friend would read you a story? Why?
   f) How do you think the graph would look if your breathing rate was measured while you played soccer? Why?

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Use technology as an aid to conduct an experiment.
- ❒ Use technology as the method of collecting data.
- ❒ Record the results of an experiment.
- ❒ Draw a conclusion from a computer-designed graph.

Suggestions for Instruction

- **Answer a question by performing an experiment, recording the results, and drawing a conclusion.**

Materials:

- BLM 6.SP.1.8: Questions for Data Collection #1
- BLM 6.SP.1.9: Questions for Data Collection #2

Organization: Whole class

Procedure:

1. Have students sit in small groups.
2. Place a transparency copy of BLM 6.SP.1.8 and BLM 6.SP.1.9.
3. Tell students to read the questions and choose the ones that lend themselves to collecting data by performing an experiment.
4. Have a class discussion.
5. Discuss with the whole class other possible questions that lend themselves to data collecting by performing an experiment. Try to select questions that lend themselves to performing experiments in class.
6. Discuss what makes a good experiment.
7. Tell each group to choose one question from those they discussed, perform an experiment in class, record the results, and draw a conclusion.
8. Circulate to see if the students are collecting the appropriate data, recording the results, and drawing a conclusion.
9. Have a class discussion.
10. Tell students to record in their journals what they learned about answering a question by performing an experiment, recording the results, and drawing a conclusion.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Recognize when it is appropriate to use an experiment as the method of collecting data.
- ❒ Perform an experiment for collecting data.
- ❒ Record the results of an experiment.
- ❒ Draw a conclusion from an experiment.

Suggestions for Instruction

- **Gather data for a question by using electronic media, including selecting data from databases.**

Materials:

- BLM 6.SP.1.8: Questions for Data Collection #1
- BLM 6.SP.1.9: Questions for Data Collection #2

Organization: Individually/whole class

Procedure:

1. Have students work individually.
2. Distribute to each student a copy of BLM 6.SP.1.8 and BLM 6.SP.1.9.
3. Tell students to read the questions and choose the ones that lend themselves to collecting data by using electronic media, including selecting data from databases.
4. Have a class discussion.
5. Tell students that each of them needs to choose a question from those discussed earlier and gather data for his or her question by using the computer.
6. Have each student work alone at a computer.
7. Circulate to see if the students are finding the appropriate data.
8. Have a class discussion.
9. Tell students to think of who might use this data and for what purpose. Have them record their ideas in their journals.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Recognize when it is appropriate to gather data by using electronic media.
- ❒ Use electronic media for collecting data.

Suggestions for Instruction

- **Select a type of graph for displaying a set of collected data, and justify the choice of graph.**
- **Solve a problem by graphing data and interpreting the resulting graph.**

Materials:

- BLM 6.SP.1.10: Bobby Planted Peas
- BLM 6.SP.1.11: Data or Not?

Organization: Two groups

Procedure:

1. Tell students that before they do the activity, there will be a small review.
2. Discuss with the class the different types of graphs they learned about in the previous grades and in Grade 6.
3. Ask individual students to draw examples of graphs on the board.
4. Discuss what the best use would be for each graph.
5. Tell students to draw a sample of each graph, such as a pictograph, single bar graph, double bar graph (from previous grades), and a line graph (Grade 6) and give an example of when each would be used, as stated in the class discussion.
6. Circulate and check that students draw a sample of each graph (i.e., a pictograph, single bar graph, double bar graph, and a line graph), and that they give an example of when each would be used, as stated in the class discussion.
7. Then divide class into two groups (group A and B).
8. Distribute to each student in group A a copy of BLM 6.SP.1.10. Distribute to each student in group B a copy of BLM 6.SP.1.11.

9. Say the following to the students:
   a) Read the question carefully.
   b) Select a type of graph for displaying the data collected from your question.
   c) Justify your choice of graph.
10. Have a class discussion.
11. Tell each group to do the following :
   a) Graph the data on a poster-sized sheet.
   b) Interpret the resulting graph.
   c) Post the graph on the board.
12. Ask for three volunteers from group A to come up to the front of the class and explain their work.
13. Let students from group B ask questions for clarification.
14. Ask for three volunteers from group B to come up to the front of the class and explain their work.
15. Let students from group A ask questions for clarification.
16. Tell students to write in their journals what they learned about selecting graphs and interpreting the resulting graph.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:
- ❒ Select the appropriate type of graph for displaying a set of collected data.
- ❒ Justify their choice of graph.
- ❒ Graph data.
- ❒ Interpret the resulting graph.

Grade 6: Statistics and Probability (Chance and Uncertainty) (6.SP.4)

Enduring Understanding(s):

Chance is an element of many aspects of our lives. The chance that an event will occur varies from impossible to certain.

Experimental results of a small-scale experiment may be quite different from the theoretical probability, while experimental results of a very large experiment should be approaching the theoretical probability.

General Learning Outcome(s):

Use experimental and theoretical probabilities to represent and solve problems involving uncertainty.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
6.SP.4 Demonstrate an understanding of probability by ▪ identifying all possible outcomes of a probability experiment ▪ differentiating between experimental and theoretical probability ▪ determining the theoretical probability of outcomes in a probability experiment ▪ determining the experimental probability of outcomes in a probability experiment ▪ comparing experimental results with the theoretical probability for an experiment [C, ME, PS, T]	➛ List the possible outcomes of a probability experiment, such as ▪ tossing a coin ▪ rolling a die with any number of sides ▪ spinning a spinner with any number of sectors ➛ Determine the theoretical probability of an outcome occurring for a probability experiment. ➛ Predict the probability of an outcome occurring for a probability experiment by using theoretical probability. ➛ Conduct a probability experiment, with or without technology, and compare the experimental results to the theoretical probability. ➛ Explain that as the number of trials in a probability experiment increases, the experimental probability approaches theoretical probability of a particular outcome. ➛ Distinguish between theoretical probability and experimental probability, and explain the differences.

Prior Knowledge

Students may have had experience with the following:

- Using words, such as *impossible*, *possible*, and *certain*, to describe the likelihood of a single outcome occurring
- Using words, such as *less likely*, *equally likely*, and *more likely*, to compare the likelihood of two possible outcomes occurring
- Having an understanding of fractions

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of ratio
- Demonstrating and explaining the meaning of *preservation of equality*, *concretely*, *pictorially*, and *symbolically*

Background Information

In Grade 5, students worked on developing their knowledge of the terms associated with probability, understanding the role chance plays in their lives, and awareness that some events are more probable than others. In Grade 6, students learn to distinguish between theoretical and experimental probability. They will predict possible outcomes of a probability experiment based on theoretical probability, conduct probability experiments, and compare their experimental results with the theoretical probability for an experiment.

When students perform the coin toss experiment, they will learn that no matter how many times they toss a coin,

a) there are always two **possible outcomes** for each toss (i.e., heads and tails)

b) the probability of getting heads is 1 out of 2

c) the probability of getting tails is 1 out of 2 (The **theoretical probability** of getting either outcome in any coin toss is always 1 out of 2, since there are only two sides to a coin and therefore two possible outcomes.)

When students roll, for example, a six-sided die they will learn that no matter how many times they roll a six-sided die of equal size,

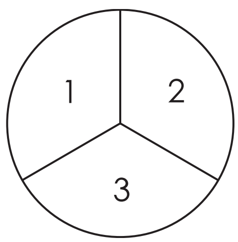
a) there are always six **possible outcomes** (i.e., side 1, side 2, side 3, side 4, side 5, and side 6)

b) the probability of rolling any one of the sides is 1 out of 6 (The **theoretical probability** of rolling any one of the six sides on any six-sided die of equal size is always 1 out of 6, since there are six sides, an equal opportunity to land on each side, and therefore six possible outcomes.)

When students spin, for example, a spinner with **three equal-sized sectors**, like "Spinner #1" below, they will learn that no matter how many times they spin that spinner,

a) there are always three **possible outcomes** (i.e., sector 1, sector 2, and sector 3)

b) the probability of landing on any one of the three sectors is 1 out of 3 (The **theoretical probability** of landing on any one of the three sectors on any spinner with three equal-sized sectors is always 1 out of 3, since there are three equal sectors and three possible outcomes.)



When students spin a spinner with three sectors where the sectors are not all of equal size but for example like "Spinner #2" above (i.e., if sector 1 is a quarter of a circle, sector 2 is also a quarter of a circle, and sector 3 is half of a circle, they will learn that no matter how many times they spin that spinner),

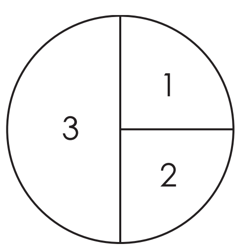
a) there are always three **possible outcomes** (i.e., sector 1, sector 2, and sector 3)

b) the probability of landing on sector 1 is 1 out of 4

c) the probability of landing on sector 2 is 1 out of 4

d) the probability of landing on sector 3 is 2 out of 4

(On this type of a spinner, the **theoretical probability** of landing on sector 1 and sector 2 is 1 out of 4 each, since sector 1 and sector 2 each occupy a quarter of a circle, but the **theoretical probability** of landing on sector 3 is 2 out of 4, since sector 3 occupies one-half of a circle, which can also be viewed as two-quarters of a circle.)



When students try to compare their results of a **probability experiment** (any probability experiment they perform), they will notice that the **experimental probability** may not necessarily be the same as they may have predicted based on the outcomes of **theoretical probability**. In fact, “as the number of trials increases, the experimental probability approaches theoretical probability of a particular outcome.” (Manitoba Education and Training 124)

Mathematical Language

probability

experimental probability

theoretical probability

possible outcomes

experimental results

accurate

Learning Experiences

Assessing Prior Knowledge

Organization: Whole class

Procedure:

1. Tell students that during the following few lessons, the subject of their discussion and activities will be the topic of probability.
2. Ask them to try to remember what they learned in Grade 5 about chance and uncertainty.
3. Place the terms *Impossible*, *Possible*, *Certain*, *Less likely*, *More likely*, and *Equally likely* on the board.
4. Discuss each term separately.
5. Provide examples of each term orally and write them on the board.
6. Tell students to copy the terms and examples (off the board) into their journals.
7. Then tell them to provide their own examples and notes to further clarify their understanding of each term.

8. Walk around and observe student notes and examples for each term. Check whether the notes reveal their knowledge as to the following questions:
   a) What is the meaning of each term (*Impossible*, *Possible*, *Certain*, *Less likely*, *More likely*, and *Equally likely*)?
   b) What is an example of each term (*Impossible*, *Possible*, *Certain*, *Less likely*, *More likely*, and *Equally likely*)?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
   - ❒ Define the probability terms learned in Grade 5 (*Impossible*, *Possible*, *Certain*, *Less likely*, *More likely*, and *Equally likely*).
   - ❒ Provide examples for the probability terms learned in Grade 5 (*Impossible*, *Possible*, *Certain*, *Less likely*, *More likely*, and *Equally likely*).

Suggestions for Instruction

- **List the possible outcomes of a probability experiment, such as**
  - **tossing a coin**
  - **rolling a die with any number of sides**
  - **spinning a spinner with any number of sectors**

Materials:

- pennies

Organization: Whole class

Procedure:

1. Write on the board "penny."
2. Discuss with the class the possible outcomes of tossing a penny one time.
3. Write it on the board.
4. Ask students to write in their notebooks all possible outcomes for tossing a penny a second time, a third time, a fourth time, and a fifth time.
5. Discuss the possible outcomes for each scenario. How does each subsequent toss compare to the first toss?
6. Write on the board the word *quarter*.

7. Have students write in their journals the possible outcomes for tossing a quarter once, twice, and three times. How do these results compare to the results of tossing a penny?

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ List the two possible outcomes of a coin (heads, tails).
- ❒ Tell that each coin has the same two possible outcomes (heads, tails).

Suggestions for Instruction

- **List the possible outcomes of a probability experiment, such as**
  - **tossing a coin**
  - **rolling a die with any number of sides**
  - **spinning a spinner with any number of sectors**

Materials:

- six-sided die

Organization: Whole class/fours

Procedure:

1. Show to the class a large six-sided die.
2. Have a class discussion on what the possible outcomes would be for rolling a six-sided fair die once.
3. Tell students to discuss with their group members the possible outcomes for rolling a six-sided fair die a second time.
4. Ask one student to write on the board the replies of his or her group.
5. Ask students to discuss their replies compared to the one on the board.
6. Ask students why they think they got those particular replies.
7. Have students write in their journals the possible outcomes for rolling a six-sided fair die a third time, and state how these results compare to the results of the first and second roll. Why?

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❐ List the six possible outcomes of rolling a six-sided fair die (side 1, side 2, side 3, side 4, side 5, and side 6).
- ❐ Tell that each roll has the same six possible outcomes (side 1, side 2, side 3, side 4, side 5, and side 6).

Suggestions for Instruction

- **List the possible outcomes of a probability experiment, such as**
  - **tossing a coin**
  - **rolling a die with any number of sides**
  - **spinning a spinner with any number of sectors**

Materials:

- BLM 6.SP.4.1: Spinner A
- BLM 6.SP.4.2: Spinner B
- BLM 6.SP.4.3: Spinner C

Organization: Whole class/small groups

Procedure:

1. Show on the overhead projector *Spinner A*.
2. Discuss with the class the possible outcomes of spinning *Spinner A* once.
3. Ask a student to write the outcomes on the board.
4. Show on the overhead projector *Spinner B*.
5. Tell students to write in their notebooks all possible outcomes of spinning *Spinner B* once.
6. Discuss the possible outcomes. How do these outcomes compare to the outcomes of *Spinner A*?
7. Show on the overhead projector *Spinner C*.
8. Tell students to observe carefully *Spinner C* and discuss in their group the possible outcomes of this spinner.
9. Tell students to write in their notebooks all possible outcomes of spinning once a spinner like *Spinner C*, and state how these results compare to the results of the other two spinners. Why?

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ List the possible outcomes of spinning a spinner with three sectors (sector 1, sector 2, and sector 3).
- ❒ List the possible outcomes of spinning a spinner with four sectors (sector 1, sector 2, sector 3, and sector 4).
- ❒ Tell that for each spinner the number of sectors determines the number of possible outcomes.

Suggestions for Instruction

- **Determine the theoretical probability of an outcome occurring for a probability experiment.**

Materials:

- dime

Organization: Whole class/individual

Procedure:

1. Tell students the following:
   a) "We have already discussed the possible outcomes of tossing a penny one time, a second time, a third time, a fourth time, and a fifth time, and we concluded that there are two possible outcomes—that is, heads or tails."
   b) "You have also recorded in your journals the possible outcomes of tossing a quarter one time, a second time, a third time, and how those results compared to the results of tossing a penny. Do you remember what we concluded?"
   c) Have students recall the conclusion. If need be, remind them that tossing a quarter also yielded two possible outcomes—that is, heads or tails.
2. Then say: "Today, we are going to discuss the theoretical probability of each of those outcomes for a probability experiment."
3. Discuss with the class the theoretical probability of obtaining "heads" when you toss a dime once.
4. Discuss with the class the theoretical probability of obtaining a "tails" when you toss a dime once.
5. Ask a student to come up to the board and write up the results as discussed.

6. Discuss standard probability notation with students:

$$P(\text{favourable outcomes}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

For example,

$$P(\text{heads}) = \frac{1}{2}$$

7. Ask students to write in their notebooks what they believe is the theoretical probability of
   a) obtaining "heads" when tossing a dime a second time, and a third time
   b) obtaining "tails" when tossing a dime a second time, and a third time

   Tell them to explain why.
8. Circulate and check that they have recorded the theoretical probability of obtaining each outcome as "one out of two" for each scenario.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:
- ❒ Determine the theoretical probability of obtaining "heads" for one coin toss.
- ❒ Determine the theoretical probability of obtaining "tails" for one coin toss.
- ❒ Determine the theoretical probability of obtaining "heads" for subsequent coin tosses.
- ❒ Determine the theoretical probability of obtaining "tails" for subsequent coin tosses.

Suggestions for Instruction

- **Determine the theoretical probability of an outcome occurring for a probability experiment.**

Materials:
- six-sided die

Organization: Whole class/small groups

Procedure:

1. Have a class review what the possible outcomes would be for rolling a six-sided fair die once, and a second time.
2. Then say: "Today, we are going to discuss the theoretical probability of each of those outcomes for a probability experiment."
3. Discuss with the class the theoretical probability of rolling "side 1" when rolling a six-sided fair die once.
4. Tell students to discuss with their group members the theoretical probability of rolling "side 1" when rolling a six-sided fair die a second time.
5. Ask one student from each group to give an oral report of the replies of his or her group.
6. Tell students to discuss with their group members the theoretical probability of rolling "side 2" when rolling a six-sided fair die
   a) once
   b) a second time
7. Ask one student from each group to give an oral report of the replies of his or her group and why they chose those particular replies.
8. Have students write in their journals what they believe is the theoretical probability of rolling "side 3," "side 4," "side 5," and "side 6" when rolling a six-sided fair die
   a) once
   b) a second time
9. Have them state why they chose those particular replies.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Determine the theoretical probability of rolling "side 1" for one single roll when rolling a six-sided fair die once.
- ❒ Determine the theoretical probability of rolling "side 1" for one single roll when rolling a six-sided fair die a second time.
- ❒ Determine the theoretical probability of rolling any one side for one single roll when rolling a six-sided fair die once.
- ❒ Determine the theoretical probability of rolling any one side for one single roll when rolling a six-sided fair die a second time.

Suggestions for Instruction

> ■ **Determine the theoretical probability of an outcome occurring for a probability experiment.**

Materials:

- BLM 6.SP.4.1: Spinner A
- BLM 6.SP.4.2: Spinner B
- BLM 6.SP.4.4: Spinner D

Organization: Whole class/pairs

Procedure:

1. Review possible outcomes of spinning once a spinner using *Spinner A*, *Spinner B*, and *Spinner D* as examples.
2. Then say: "Today, we are going to discuss the theoretical probability of each of those outcomes for a probability experiment."
3. Have students express the theoretical probabilities of each of the sectors in *Spinner B*.
4. Look carefully at *Spinner A*. What can you observe about the size of the sectors?
5. Discuss their observations about the sectors.
6. Have a class review of fractions using *Spinner A*.
7. Then discuss as a class the theoretical probability of a spinner like *Spinner A* stopping on
   a) yellow
   b) red
   c) blue
8. Have students work in pairs in order to write in their notebooks the theoretical probability of a spinner like *Spinner D* stopping on
   a) Sector 1
   b) Sector 2
   c) Sector 3
   d) Sector 4
   e) Sector 5

   based on their class discussion.
9. Circulate and check that students used the correct fraction for each sector.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Determine the theoretical probability of a spinner like "Spinner D" stopping on

  Sector 1

  Sector 2

  Sector 3

  Sector 4

  Sector 5
- ❒ Understand the notion of equivalent fractions, such as one-half equals two-quarters, one-quarter equals two-eighths, and one-third equals two-sixths.
- ❒ Reason mathematically in order to determine the theoretical probabilities of sectors of a spinner with uneven sector sizes.

Suggestions for Instruction

- **List the possible outcomes of a probability experiment, such as**
  - **tossing a coin**
  - **rolling a die with any number of sides**
  - **spinning a spinner with any number of sectors**
- **Determine the theoretical probability of an outcome occurring for a probability experiment.**
- **Predict the probability of an outcome occurring for a probability experiment by using theoretical probability.**

Materials:

- BLM 6.SP.4.5: Probability Bingo
- BLM 6.SP.4.1: Spinner A
- bingo chips
- coin

Organization: Whole class

Procedure:

1. Let students know that they will be playing a game called *Probability Bingo,* and explain the rules:
   a) They will be given a blank *Probability Bingo* sheet in which they will need to fill in the possible outcomes from tossing a coin, rolling a regular die, and spinning *Spinner A*. Outcomes may repeat as many times as they wish.
   b) Alternate between tossing a coin, rolling the die, and spinning the spinner.
   c) Each time, call out the result and students will mark one space on their board (e.g., if they wrote "heads" in three times, they would have to select which one to cover).
   d) The first student to get a Bingo (line, X, blackout, etc.) wins.
   e) Students will need to use their knowledge about probability to fill in the chart to give them the best chance of winning.
2. Play the game.

Suggestions for Instruction

- **Predict the probability of an outcome occurring for a probability experiment by using theoretical probability.**
- **Conduct a probability experiment, with or without technology, and compare the experimental results to the theoretical probability.**
- **Explain that as the number of trials in a probability experiment increases, the experimental probability approaches the theoretical probability of a particular outcome.**

Materials:

- pennies
- BLM 6.SP.4.6: Record Sheet #1

Organization: Pairs

Procedure:

1. Tell students that they will be doing a probability experiment, but first they need to line up in a single row.
2. Then have the student closest to you call out "one," the next student call out "two," the third student call out "one," the fourth student call out "two," and so on, alternating between "one" and "two" until you run out of students.
3. Tell the "ones" to turn around, facing the "twos." They then shake hands and remember that they are partners.
4. Tell the "ones" to pick up a Record Sheet from your desk and go back to their seats.
5. Tell the "twos" to come and get a penny from you and then join their partners.

6. Then ask students to do the following:
   a) Using theoretical probability, students predict the probability of a coin toss resulting in "heads" and write it on *Recording Sheet #1* under a heading marked "prediction."
   b) In their partner groups, student 1 tosses the coin 10 times and student 2 does the recording for each coin toss.
   c) Next, student 2 tosses the coin 10 times and student 1 does the recording for each coin toss.
   d) Students copy the results of all 20 coin tosses into their notebooks.
7. Discuss as a group and then have students record in their notebooks their replies to the following questions:
   a) Using theoretical probability what were your predictions for obtaining "heads" for your 10 coin tosses?
   b) Using theoretical probability what were your partner's predictions for obtaining "heads" for his or her 10 coin tosses?
   c) Would you have changed your predictions if you were allowed to guess after each toss?
   d) What is the theoretical probability of a coin toss resulting in "heads?"
   e) What were the experimental results of your 10 coin tosses?
   f) What were the experimental results of your partner's 10 coin tosses?
   g) How did your experimental results match with your predictions?
   h) How did the experimental probability compare to the theoretical probability?
   i) How did your partner's experimental results match with his or her predictions?
   j) If you were to repeat the experiment, do you think you would get the same results? Why?
   k) When can the experimental probability approach the theoretical probability of obtaining "heads" when tossing a coin? Explain.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:
- ❒ Predict the probability of obtaining "heads" for a coin-toss probability experiment by using theoretical probability.
- ❒ Conduct a coin-toss probability experiment.
- ❒ Compare the experimental results to the theoretical probability.
- ❒ Explain that as the number of coin-tosses increase, the experimental probability approaches the theoretical probability of obtaining "heads."

Suggestions for Instruction

- **Distinguish between theoretical probability and experimental probability, and explain the differences.**

Materials:

- one coin per student (the same type for the whole class—e.g., a penny)

Organization: Class

Procedure:

1. Have students determine the theoretical probability for heads and for tails in a toss.
2. Have each student
   a) toss a coin 10 times
   b) record results for each toss (heads, tails)
   c) record the experimental probability for the tosses
3. Compare students' results by collecting data from all students and charting results by name.
4. Combine all data, chart the results, and record the experimental probability for the class.
5. Ask students to compare their results with the combined class results and write the theoretical probability.
6. Explain the difference between experimental probability and theoretical probability.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Determine the theoretical probability (in a toss) for:
  - Heads
  - Tails
- ❒ Determine the experimental probability based on their experimental results.
- ❒ Compare experimental probability to theoretical probability.
- ❒ Explain the difference between experimental probability and theoretical probability.

- **Distinguish between theoretical probability and experimental probability, and explain the differences.**

Materials:

- dice (numbered 1 to 6)
- a variety of number polyhedral (multisided) dice
- paper and pencil

Organization: Pairs

Procedure:

Say to the students:

1. "With your partner, determine the theoretical probability for rolling each number cube. Record."
2. "Conduct an experiment to determine the experimental probability for rolling the cube 50 times. Record."
3. "When you have determined the experimental probability, compare it with the theoretical probability."
4. "Repeat the investigation with other number polyhedrals."
   a) "What happens to the theoretical probability of rolling a particular number when the number of sides increases?"
   b) "What happens to the experimental probability?"
5. "Explain the difference between experimental probability and theoretical probability."

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Determine the theoretical probability for rolling each number cube.
- ❒ Determine the experimental probability based on their experimental results.
- ❒ Compare experimental probability to theoretical probability.
- ❒ Explain the difference between experimental probability and theoretical probability.

Suggestions for Instruction

- **Predict the probability of an outcome occurring for a probability experiment by using theoretical probability.**
- **Conduct a probability experiment, with or without technology, and compare the experimental results to the theoretical probability.**
- **Explain that as the number of trials in a probability experiment increases, the experimental probability approaches theoretical probability of a particular outcome.**

Materials:

- six-sided dice
- BLM 5–8.5: Number Cards
- BLM 6.SP.4.7: Record Sheet #2

Organization: Six groups

Procedure:

1. Make several copies of *Number Cards* (use the cards numbered from 1 through 6, inclusive).
2. Divide the class into six groups by handing out one numbered card to each student.
3. Then say: "Students holding cards numbered 'one' will sit in front of the board (pick any spot you like). Students holding cards numbered 'two' will sit by the window" (and so on, designating a specific spot for each of the six groups).
4. Tell students :
   a) "Each group will represent one side of a six-sided die."
   b) "Look at your cards to tell which side you represent." (For example, the group of students holding cards numbered "one" represent "side one" of the six-sided die; the group of students holding cards numbered "two" represent "side two" of the six-sided die, and so on.)
   c) "I want each group to predict the probability of the outcome they represent. No two groups will predict the probability of the same outcome."
   d) "More specifically, using theoretical probability, predict the probability of rolling a 'number representing your side of the six-sided die' for each of the rolls."
5. Then point at each group, one at a time, and ask: "Using theoretical probability, your group is predicting the probability of occurring of what outcome?" (Each group will need to state their particular outcome based on the numbered cards you handed to them.)
6. Hand a copy of *Record Sheet* #2 to each student.
7. Say to the class: "Write your predictions on your sheet under a heading marked 'theoretical probability.'"

8. Circulate to check that students write the correct prediction of an outcome occurring.
9. Then say: "Now you are going to do a probability experiment using a six-sided die." Each group will toss the die 10 times. One person will do the recording.
10. Give each group a six-sided die.
11. Ask one student from each group to copy the results from each Record Sheet onto one large sheet of paper and post it on the board.
12. Ask another student to give an oral report of the replies of his or her group, comparing their group's results to the class's results.
13. Discuss as a group first and then have students record in their notebooks the replies to the following questions:
    a) What were the experimental results of your 10 rolls for each possible outcome?
    b) What were the experimental results of the class's 10 rolls for each possible outcome?
    c) How did your experimental results match with your predictions of the theoretical outcomes?
    d) How did the class's experimental results match with your predictions of the theoretical outcomes?
    e) If you were to repeat the experiment, do you think you would get the same results? Why?
    f) When can the experimental probability approach the theoretical probability of obtaining a "your particular outcome" when rolling a six-sided fair die? Explain.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ By using theoretical probability, predict the probability of rolling a "particular side" for one single roll when you roll a six-sided fair die once.
- ❒ For a probability experiment, predict the probability of rolling a "particular side" for each single roll when you roll a six-sided fair die 10 times.
- ❒ Conduct a "six-sided fair die" roll probability experiment.
- ❒ Compare the experimental results to the theoretical probability.
- ❒ Explain that as the number of rolls increase, the experimental probability approaches the theoretical probability of obtaining a particular outcome.

Suggestions for Instruction

- **Predict the probability of an outcome occurring for a probability experiment by using theoretical probability.**
- **Conduct a probability experiment, with or without technology, and compare the experimental results to the theoretical probability.**
- **Explain that as the number of trials in a probability experiment increases, the experimental probability approaches theoretical probability of a particular outcome.**

Materials:

- BLM 6.SP.4.8: Spinners
- BLM 6.SP.4.7: Record Sheet #2

Organization: Five groups

Procedure:

1. Divide the class into up to five groups.
2. Hand out a copy of *Record Sheet #2* to each student and one spinner to each group. No two groups should have the same spinner.
3. Then say:
   a) "Today, each of you will predict an outcome occurring and will also conduct an experiment using a particular spinner."
   b) "Look at your spinner carefully, noting the number of sectors and the size of the sectors."
   c) "Now make a comparison of 'sector 1' to the other sectors. Discuss with your group members the size of sector 1 in relationship to the rest of the sectors, and in relationship to the whole spinner."
4. Circulate to check that students are discussing the correct proportions or fractions.
5. Then say: "I want each group to record the theoretical probabilities for all possible outcomes on the spinner your group has been assigned."
6. Circulate to check that students write the correct prediction of their specific outcome occurring.
7. Then say: "Now take turns spinning the spinner in your group until you have spun 100 times. Record the results after each spin."
8. Discuss the theoretical and the experimental probabilities of each group.
9. Tell students to record in their journals their replies to the following questions:
   a) "What were the experimental results for each possible outcome?"
   b) "How did your experimental results match with the theoretical results?"
   c) "If there were differences, why do you think that might be the case?"

d) Compare your theoretical and experimental probabilities for sector 1 with those of the class.

e) If you were to repeat the experiment, do you think you would get the same results? Why?

f) When can the experimental probability approach the theoretical probability of obtaining "your particular outcome" when spinning your particular spinner? Explain.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ By using theoretical probability, predict the probability of landing on a "particular sector" for one single spin.
- ❒ For a probability experiment, predict the probability of landing on a "particular sector" for each single spin when they spin a particular spinner.
- ❒ Conduct a probability experiment using a spinner with four sectors.
- ❒ Compare the experimental results to the theoretical probability.
- ❒ Explain that as the number of spins increase, the experimental probability approaches the theoretical probability of obtaining a particular outcome.

How Predictable Are They?

Purpose: The purpose of this activity is for students to do the following:

- Demonstrate their understanding of the difference between experimental and theoretical probability by completing and analyzing a statistical experiment.
- Show that they can collect data by observation.
- Show that they can use data to determine the experimental probability of an event happening in the future.

This activity is designed to also show that students can do the following:

- Demonstrate an understanding of fractions.
- Demonstrate an understanding of ratio.

Curricular Links: English language arts

Materials/Resources:

- paper and pencil
- large poster paper

Organization:

Groups of three to five students

Inquiry:

Scenario

Have students work in small groups discussing who or what person or animal they will observe, and what they will observe about that person or animal. Each student may choose a person or a pet. They may not choose to observe a student who is a member of their own group.

Procedure

1. Tell students that each of them needs to choose one person or a pet to observe for five days.
2. The person can be another student or a family member whom they see every day. The person cannot be a member of their group.
3. Tell students to discuss what they might be observing for their data collection.

   Examples:

   a) When does a person arrive at school?
   b) When does a person arrive at home?
   c) When does a person go for a walk?
   d) When does a person smile?

e) When does a cat meow or a dog bark?

f) At what time does your pet rub against you?

4. Let students choose the five behaviours that they think provide the best data for their observations.
5. Write their observations daily in their journals.
6. State the theoretical probability for the observations.
7. At the end of the five days, record all data on a large sheet of paper.
8. Analyze the data and record the experimental probability.
9. Make predictions based on the experimental probability.
10. Choose and create a graph that best fits your data.
11. Make a presentation to the other group members.

Literature Link

For collecting data and predictions on temperature, wind, and pressure, students can check *The Weather Network* at <www.theweathernetwork.com> (they can choose the city for which they collect data).

Assessment:

Use the following checklist to assess students' replies to determine whether they can do the following:

- ❑ Use fractions.
- ❑ Make observations.
- ❑ Make predictions based on the experimental probability.
- ❑ Choose which graph best fits the data.
- ❑ Compare experimental probability to theoretical probability.

Use the following observation checklist to assess student learning:

The student can do the following:	Yes	No	Comment
Make good choices for observations.			
Record daily observations.			
Record observations in detail.			
Express theoretical probability as a fraction.			
Express experimental probability as a fraction.			
Make predictions based on experimental probability.			
Choose a graph that best fits data.			
Create a graph that best fits data.			
Compare experimental probability to theoretical probability.			

Extension:

Taking it further

Read to the class the book *Same Old Horse* by Stuart J. Murphy and illustrated by Steve Bjorkman.

Ask students the following:

- "Jazz and Majesty said that Hankie was predictable. Can you predict when Hankie will sneeze next? Why?"
- "What other predictions can you make about Hankie?"
- "What is the theoretical probability that Hankie will wear a red saddle on week 11?"
- "What is the experimental probability that Hankie will wear a red saddle on week 11?"
- "Write your own story using your knowledge of statistics and probability."

NOTES

Grade 6 Mathematics

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