Grade 6: Unit 2: Ratios, Rates and Proportions

Approximate Time Frame: 6-7 weeks

Connections to Previous Learning:

The study of ratios and proportional relationships extends students' work in measurement and in multiplication and division from the elementary grades. It is expected that students will have prior knowledge and experience related to concepts and skills such as multiples, factors, and divisibility rules. This background knowledge about relationships and rules for multiplication and division of whole numbers connects to the understanding of how to complete tables to help support the development of ratio and rate reasoning.

Focus of this Unit:

Students learn that a ratio expresses the comparison between two quantities. Special types of ratios are rates, unit rates, measurement conversions, and percentages and are concepts that are applied to a variety of real world and mathematical situations. Students gain a deeper understanding of proportional reasoning through instruction and practice. They develop and use multiplicative thinking to develop a sense of proportional reasoning as they describe ratio relationships between two quantities.

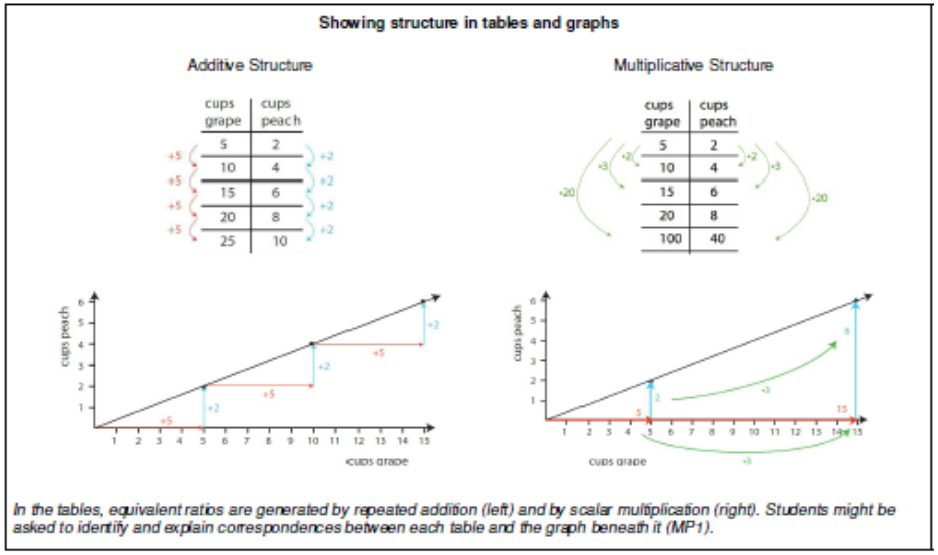
Connections to Subsequent Learning:

Ratios and proportional relationships are foundational for further study in mathematics and science and useful in everyday life. Students use ratios in geometry and in algebra when they study similar figures and slopes of lines, and later when they study sine, cosine, tangent, and other trigonometric ratios in high school. Students use ratios when they work with situations involving constant rates of change, and later in calculus when they work with average and instantaneous rates of change of functions. An understanding of ratio is essential in the sciences to make sense of quantities that involve derived attributes such as speed, acceleration, density, surface tension, electric or magnetic field strength, and to understand percentages and ratios used in describing chemical solutions. Ratios and percentages are also useful in many situations in daily life, such as in cooking and in calculating tips, miles per gallon, taxes, and discounts. They are also involved in a variety of descriptive statistics, including demographic, economic, medical, meteorological, and agricultural statistics (e.g., birth rate, per capita income, body mass index, rain fall, and crop yield) and underlie a variety of measures, for example, in finance (exchange rate), medicine (dose for a given body weight), and technology (kilobits per second).

From the 6-7, Ratios and Proportional Relationships Progression Document, pp. 5-7:

Representing and reasoning about ratios and collections of equivalent ratios: Because the multiplication table is familiar to sixth graders, situations that give rise to columns or rows of a multiplication table can provide good initial contexts when ratios and proportional relationships are introduced. Pairs of quantities in equivalent ratios arising from whole number measurements such as "3 lemons for every $1" or "for every 5 cups of grape juice, mix in 2 cups of peach juice" lend themselves to being recorded in a table. Initially, when students make tables of quantities in equivalent ratios, they may focus only on iterating the related quantities by repeated addition to generate equivalent ratios.

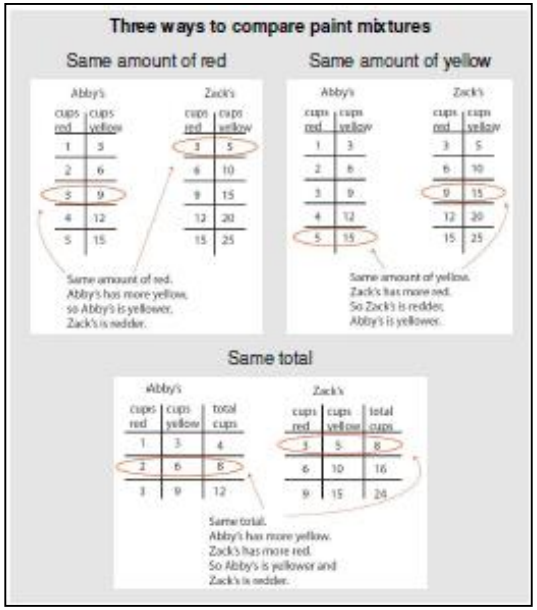
As students work with tables of quantities in equivalent ratios (also called ratio tables), they should practice using and understanding ratio and rate language. It is important for students to focus on the meaning of the terms "for every", "for each", "for each 1", and "per" because these equivalent ways of stating ratios and rates are at the heart of understanding the structure in these tables, providing a foundation for learning about proportional relationships in Grade 7.



In the tables, equivalent ratios are generated by repeated addition (left) and by scalar multiplication (right). Students might be asked to identify and explain correspondences between each table and the graph beneath it (MP1).

Students graph the pairs of values displayed in ratio tables on coordinate axes. The graph of such a collection of equivalent ratios lies on a line through the origin, and the pattern of increases in the table can be seen in the graph as coordinated horizontal and vertical increases.

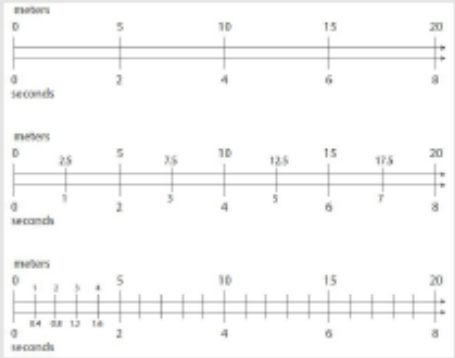
By reasoning about ratio tables to compare ratios, students can deepen their understanding of what a ratio describes in a context and what quantities in equivalent ratios have in common. For example, suppose Abby's orange paint is made by mixing 1 cup of red paint for every 3 cups of yellow paint and Zack's orange paint is made by mixing 3 cups of red for every 5 cups of yellow. Students could discuss that all the mixtures within a single ratio table for one of the paint mixtures are the same shade of orange because they are multiple batches of the same mixture. For example, 2 cups red and 6 cups yellow is two batches of 1 cup red and 3 cups yellow; each batch is the same color, so when the two batches are combined, the shade of orange doesn't change. Therefore, to compare the colors of the two paint mixtures, any entry within a ratio table for one mixture can be compared with any entry from the ratio table for the other mixture.



It is important for students to focus on the rows (or columns) of a ratio table as multiples of each other. If this is not done, a common error when working with ratios is to make additive comparisons. For example, students may think incorrectly that the ratios 1:3 and 3:5 of red to yellow in Abby's and Zack's paints are equivalent because the difference between the number of cups of red and yellow in both paints is the same, or because Zack's paint could be made from Abby's by adding 2 cups red and 2 cups yellow. The margin shows several ways students could reason correctly to compare the paint mixtures.

Strategies for solving problems: Although it is traditional to move students quickly to solving proportions by setting up an equation, the Standards do not require this method in Grade 6. There are a number of strategies for solving problems that involve ratios. As students become familiar with relationships among equivalent ratios, their strategies become increasingly abbreviated and efficient. For example, suppose grape juice and peach juice are mixed in a ratio of 5 to 2 and we want to know how many cups of grape juice to mix with 12 cups of peach juice so that the mixture will still be in the same ratio. Students could make a ratio table as shown in the margin, and they could use the table to find the grape juice entry that pairs with 12 cups of peach juice in the table. This perspective allows students to begin to reason about proportions by starting with their knowledge about multiplication tables and by building on this knowledge.

Double number line diagrams used for situations with different units



Double number lines indicate coordinated multiplying and dividing of quantities. This can also be indicated in tables.

As students generate equivalent ratios and record them in tables, their attention should be drawn to the important role of multiplication and division in how entries are related to each other. Initially, students may fill ratio tables with columns or rows of the multiplication table by skip counting, using only whole number entries, and placing these entries in numerical order. Gradually, students should consider entries in ratio tables beyond those they find by skip counting, including larger entries and fraction or decimal entries. Finding these other entries will require the explicit use of multiplication and division, not just repeated addition or skip counting. For example, if Seth runs 5 meters every 2 seconds, then Seth will run 2.5 meters in 1 second because in half the time he will go half as far. In other words, when the elapsed time is divided by 2, the distance traveled should also be divided by 2. More generally, if the elapsed time is multiplied (or divided) by *N*, the distance traveled should also be multiplied (or divided) by *N*. Double number lines can be useful in representing ratios that involve fractions and decimals.

As students become comfortable with fractional and decimal entries in tables of quantities in equivalent ratios, they should learn to appreciate that unit rates are especially useful for finding entries. A unit rate gives the number of units of one quantity per 1 unit of the other quantity. The amount for *N* units of the other quantity is then found by multiplying by *N*. Once students feel comfortable doing so, they may wish to work with abbreviated tables instead of working with long tables that have many values. The most abbreviated tables= consist of only two columns or two rows; solving a proportion is a matter of finding one unknown entry in the table.

A progression of strategies for solving a proportion

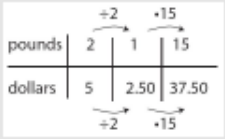
If 2 pounds of beans cost $5, how much will 15 pounds of beans cost?

Method 1

pounds	2	4	6	8	10	12	14	1	15
dollars	5	10	15	20	25	30	35	2.50	37.50

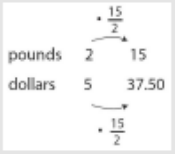
"I found 14 pounds costs $35 and then 1 more pound is another $2.50, so that makes $37.50 in all."

Method 2



"I found 1 pound first because if I know how much it costs for each pound then I can find any number of pounds by multiplying."

Method 3



The previous method, done in one step.

With this perspective, the second column is seen as the first column times a number. To solve the proportion one first finds this number.

Measurement conversion provides other opportunities for students to use relationships given by unit rates. For example, recognizing "12 inches in a foot", "1000 grams in a kilogram", or "one kilometer is $\frac{5}{8}$ of a mile" as rates, can help to connect concepts and methods developed for other contexts with measurement conversion.

Representing a problem with a tape diagram

Slimy Gloopy mixture is made by mixing glue and liquid laundry starch in a ratio of 3 to 2. How much glue and how much starch is needed to make 85 cups of Slimy Gloopy mixture?

5 parts $\longrightarrow$ 85 cups
1 part $\longrightarrow$ $85 \div 5 = 17$ cups
3 parts $\longrightarrow$ $3 \cdot 17 = 51$ cups
2 parts $\longrightarrow$ $2 \cdot 17 = 34$ cups

51 cups glue and 34 cups starch are needed.

Tape diagrams can be useful aids for solving problems.

Representing a multi-step problem with two pairs of tape diagrams

Yellow and blue paint were mixed in a ratio of 5 to 3 to make green paint. After 14 liters of blue paint were added, the amount of yellow and blue paint in the mixture was equal. How much green paint was in the mixture at first?

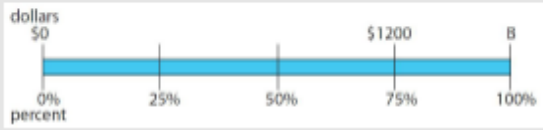
2 parts $\longrightarrow$ 14 liters
1 part $\longrightarrow$ $14 \div 2 = 7$ liters
(original total) 8 parts $\longrightarrow$ $8 \cdot 7 = 56$ liters

There was 56 liters of green paint to start with.

This problem can be very challenging for sixth or seventh graders.

Solving a percent problem

If 75% of the budget is \$1200, what is the full budget?



"I said 75% is 3 parts and is \$1200
25% is 1 part and is $\$1200 \div 3 = \$400$
100% is 4 parts and is $4 \cdot \$400 = \$1600$"

portion	75	3	1200
whole	100	4	1600

75% is $\frac{1200}{B}$ 75% of B is 1200

$\frac{75}{100} = \frac{1200}{B}$ $\frac{75}{100} \cdot B = 1200$

$B = 1600$

In reasoning about and solving percent problems, students can use a variety of strategies. Representations such as this, which is a blend between a tape diagram and a double number line diagram, can support sense-making and reasoning about percent.

Desired Outcomes

Standard(s):

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.2 Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with b ≠0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a) Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b) Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
c) Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
d) Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph order pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Transfer: *Students will apply…*

Students will apply ratio and rate concepts and procedures and equations with dependent and independent variables to represent and solve real-world and mathematical problems (rate and unit rate problems, scaling, unit pricing, statistical analysis, etc).

Understandings: *Students will understand that …*

- A ratio expresses the comparison between two quantities. Special types of ratios are rates, unit rates, measurement conversions, and percents.
- A ratio or a rate expresses the relationship between two quantities. Ratio and rate language is used to describe a relationship between two quantities (including unit rates.)
- A rate is a type of ratio that represents a measure, quantity, or frequency, typically one measured against a different type of measure, quantity, or frequency.
- Ratio and rate reasoning can be applied to many different types of mathematical and real-life problems (rate and unit rate problems, scaling, unit pricing, statistical analysis, etc.).

Essential Questions:

- When is it useful to be able to relate one quantity to another?
- How are ratios and rates similar and different?
- What is the connection between a ratio and a fraction?

Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)

* **1. Make sense of problems and persevere in solving them**. Students understand the problem context in order to translate them into ratios/rates. Students interpret real world situations by making visual representations or equations.

2. Reason abstractly and quantitatively. Students understand the relationship between two quantities in order to express them mathematically. They use ratio and rate notation as well as visual models and contexts to demonstrate reasoning.

3. Construct viable arguments and critique the reasoning of others. Students construct and critique arguments regarding appropriateness of representations given ratio and rate contexts. For example, does a tape diagram adequately represent a given ratio scenario.

* **4. Model with mathematics.** Students can model problem situations symbolically (tables, expressions or equations), visually (graphs or diagrams) and contextually to form real-world connections.

5. Use appropriate tools strategically. Students choose appropriate models for a given situation, including tables, expressions or equations, tape diagrams, number line models, etc.

* **6. Attend to precision**. Students use and interpret mathematical language to make sense of ratios and rates.

* **7. Look for and make use of structure.** The structure of a ratio is unique and can be used across a wide variety of problem-solving situations. For instance, students recognize patterns that exist in ratio tables, including both the additive and multiplicative properties. In addition, students use their knowledge of the structures of word problems to make sense of real-world problems.

* **8. Look for and express regularity in repeated reasoning.** Students utilize repeated reasoning by applying their knowledge of ratio, rate and problem solving structures to new contexts. Students can generalize the relationship between representations, understanding that all formats represent the same ratio or rate.

Prerequisite Skills/Concepts:

Students should already understand:

- Multiples and Factors
- Divisibility Rules
- Relationships and rules for multiplication and division of whole numbers as they apply to decimal fractions
- Understanding of common fractions

Advanced Skills/Concepts:

Some students may be ready to:

Students will use ratios, rates, unit rates and percent skills:

- in grade 7 when working with proportional relationships and probability
- in geometry and in algebra when studying similar figures and slopes of lines

Knowledge: *Students will know…*

- A **ratio** compares two related quantities.
- Ratios can be represented in a variety of formats including *each, to, per, for each, %, 1/5*, etc.
- A **percent** is a type of ratio that compares a quantity to 100.
- A **unit rate** is the ratio of two measurements in which the second term is 1.
- When it is appropriate to use ratios/rates to solve mathematical or real life problems.
- Mathematical strategies for solving problems involving ratios and rates, including tables, tape diagrams, double line diagrams, equations, equivalent fractions, graphs, etc.

Skills: *Students will be able to…*

- Use ratio language to describe a ratio relationship between two quantities. (6.RP.1)
- Represent a ratio relationship between two quantities using manipulatives and/or pictures, symbols and real-life situations. (*a to b,* a:b, or a/b) (6.RP.1)
- Represent unit rate associated with ratios using visuals, charts, symbols, real-life situations and rate language. (6.RP.2)
- Use ratio and rate reasoning to solve real-world and mathematical problems. (6.RP.3)
- Make and interpret tables of equivalent ratios. (6.RP.3)
- Plot pairs of values of the quantities being compared on the coordinate plane. (6.RP.3)
- Use multiple representations such as tape diagrams, double number line diagrams, or equations to solve rate and ratio problems. (6.RP.3)
- Solve unit rate problems (including unit pricing and constant speed). (6.RP.3)
- Solve percent problems, including finding a percent of a quantity as a rate per 100 and finding the whole, given the part and the percent. (6.RP.3)
- Use variables to represent two quantities in a real-world problem that change in relationship to one another. (6.EE.9)
- Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. (6.EE.9)
- Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. (6.EE.9)

WIDA Standard: (English Language Learners)

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
English Language Learners benefit from:

- practice with manipulatives (such as fraction-decimal-percent equivalence towers, fraction squares for multiplication and division, etc.) and visuals (such as tape diagrams).
- explicit vocabulary instruction to connect the content to language.

Academic Vocabulary:

Critical Terms:

Percent
Proportion
Rate
Ratio
Rational number
Unit Ratio
Quantity

Supplemental Terms:

Tape diagram
Double number line
Numerator
Denominator
Equivalent

Assessment

Pre-Assessments	Formative Assessments	Summative Assessments	Self-Assessments
• Equivalence & Scaling • Lesson 1 Pre and Post-Assessment	• Matching Ratios • Sample Exit Slips • Representing Percent • Swimming Laps – Representing Ratios • Fastest Texter in the World-Interpreting a Table • Growing Panda • How Tall is an Ice Cream Cone • Multiple Representations	• Using Models to Solve Ratio Problems • Unit 2 Pre- and Post-Test • Colored Sand 6RP1, 6RP3c • Math at the Zoo-Unit 2 Summative • The Zoo Mosaic	• Ratio Reflection

Sample Lesson Sequence

1. **Ratio and Rate 6.RP.1, 6.RP.2 & RP.3 (Model Lesson)**
2. Ratio Tables and Measurement 6.RP.3a&d
3. Percent 6.RP.3c
4. Independent and Dependent Variables 6.EE.9
5. Solving Ratio and Rate Problems 6.RP.3b and 6.EE.9