UNIVERSITY OF WATERLOO

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Centre for Education in
Mathematics and Computing

Grade 7 & 8 Math Circles

February 2-3, 2016

Logic Puzzles

Introduction

Math is not always numbers, equations and procedures. There are many math problems which don't even contain a single number, variable or equation, solely relying on logic, problem solving and finding patterns and connections. A logic puzzle is any problem, game or question which requires us to use critical thinking to solve it. I often have to reread logic puzzles once or twice before I get an idea of how to approach the problem at hand - but the first step is usually the hardest part, so don't worry if it takes you a minute or two before you have anything to write.

Some Strategies

At first glance, you may be tempted to start right away using the "guess and check" method, but there is often a much easier way to solve the problem if you just spend a few moments before you start writing anything. Here are some tips that you may find helpful when approaching these problems:

- Reread through the problem a few times before writing anything down, until you understand the exact goal of the problem.
- Make sure you understand and take note of any conditions on the solution.
- Using a table or diagram to organize the information given is often very helpful.
- Don't be overwhelmed if the problem seems huge - tackle it one step at a time and you'll find that it often solves itself once the sub-problems are worked out.
- Write down everything no matter how small or presumable it seems.
- Try setting up equations or dividing the possible outcomes into cases.
- "Process of elimination" can be quite helpful - but don't ever guess or assume anything, always use logical reasoning to eliminate any 'impossible' cases.

- Make sure to check your solution and see if you've 'violated' any of the conditions.
- Lastly, don't forget to enjoy this - that is what it is there for anyways!

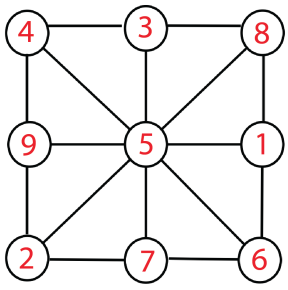
Examples

1. **Magic Boxes**
   Fill in the missing numbers so that the sum of every column, row and diagonal adds up to the same number. No number is repeated within each square.
   Hints:
   - *What is our first goal? Before we start filling in the blanks, what do we need to know?*
   - *Don't forget to check all that all the initial conditions are still satisfied once you believe you're done.*

4	9	2
3	5	7
8	1	6

4	9	8
11	7	3
6	5	10

2. Put the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ exactly once in the bubbles so that each column, row and diagonal adds up to the same number.
   Note: This is similar to Example 1, but with a little twist!
   Hints:
   - *What is the sum of all the numbers 1 through 9?*
   - *So what is the sum of each column, row and diagonal?*
   - *What are possible cominations of the numbers 1 through 9 which produce this sum?*



3. Santa's elves Theodore, Judy, and Charlie have just competed in the North Pole's 162^{nd} annual Toboggan Race. Theodore, Judy, and Charlie each finished in first, second, or third in the toboggan race (there were no ties). Each elf also works in a different department in Santas workshop. One works in the Toy Building department, one works in the Wrapping & Bows department, and one works in the Reindeer Care department.

   Using the following clues, determine who placed first, second and third, and in which department each elf works.

   1. Judy was faster than Charlie.
   2. Charlie cannot care for the reindeer since he is allergic to them and he did not finish before Theodore.
   3. The elf who builds toys was faster than the elf who works in wrapping.
   4. Judy does not build toys and Theodore does not make bows or wrap gifts.
   5. The elf who came in first does not work with the reindeer.

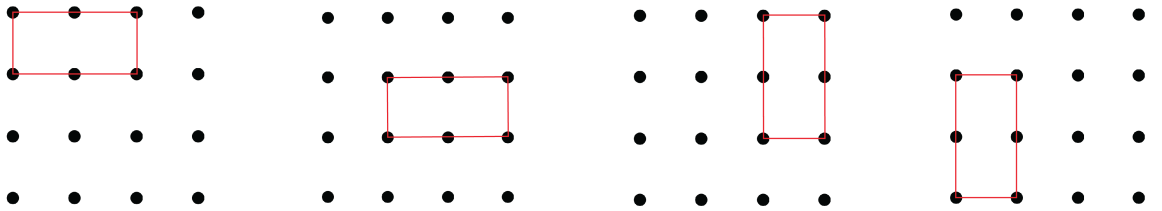
   You may find this table a helpful way to organize your solution to this problem, by placing ✓ (or ✗), one a time, wherever a combination is for sure correct (or impossible).

   For example: The first statement says that "Judy was faster than Charlie." This implies that Judy cannot be in last place and Charlie cannot be in first place. So place an ✗ in the boxes corresponding to those combinations, as shown below.

	Building	Wrapping	Caring	First	Second	Third
Theodore	✓	X	X	✓	X	X
Judy	X	X	✓	X	✓	X
Charlie	X	✓	X	X	X	✓
First	✓	X	X			
Second	X	X	✓			
Third	X	✓	X			

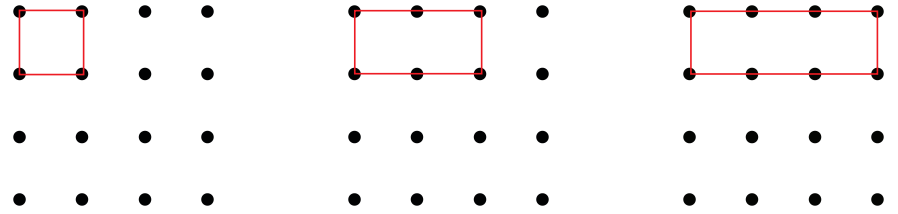
4. How many noncongruent squares and rectangles can be drawn on a 4×4 grid by connecting the dots?

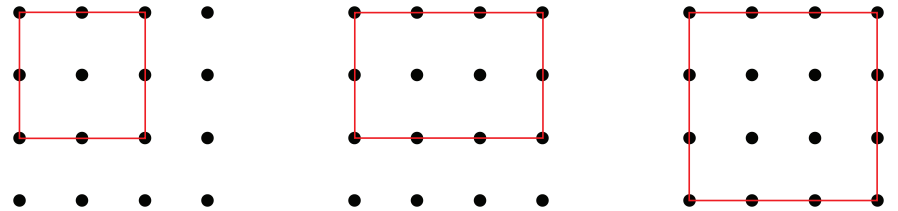
Note: Two polygons are congruent if a series of transformations (rotation, shift or reflection) to one polygon produces the other. For example, all these rectangles are congruent (so this is counted as 1):



Hints:

- *What are the possible side lengths of squares and rectangles in a 4×4 grid?*
- *What are the possible combinations for each of these side lengths?*





5. The digits $1, 2, 3, 4, 5$ are each used exactly once to create a five digit number $abcde$ which satisfies the following two conditions:
   (i) the two digit number ab is divisible by 4, and
   (ii) the two digit number cd is divisible by 3.
   Find all five digit numbers that satisfy both conditions.

   Hints:

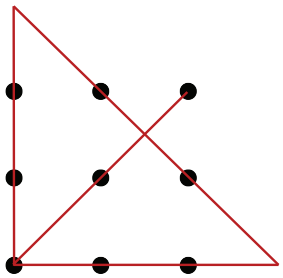
   - *List all the possible 2-digit numbers divisible by 4, and those divisible by 3.*
   - *What are all the possible combinations?*
   - *Are there any that you have to eliminate?*

   There are only eight 5-digit numbers which satisfy these 2 conditions, which are:
   12453, 12543, 24153, 24513, 32154, 32514, 32451, 32541

Problem Set

1. Without lifting your pencil, connect all the dots below with 4 straight lines.

   Hint: Think outside the box!



2. Use each of the numbers $1, 2, 3, 4, 5, 6$ exactly once to fill in the slots below:

   Hints:

   - *Which box should we try filling first?*
   - *Which two single digit numbers produce a different number between 1 and 6 when multiplied together? Where do these have to go?*

$$\begin{array}{r} \boxed{5}\boxed{4} \\ \times \boxed{3} \\ \hline \boxed{1}\boxed{6}\boxed{2} \end{array}$$

3. **Magic Box:** Fill in the missing numbers so that the sum of each column, row and diagonal adds up to the same number. No number is repeated within each square.

The sum is ____.

6	1	8
7	5	3
2	9	4

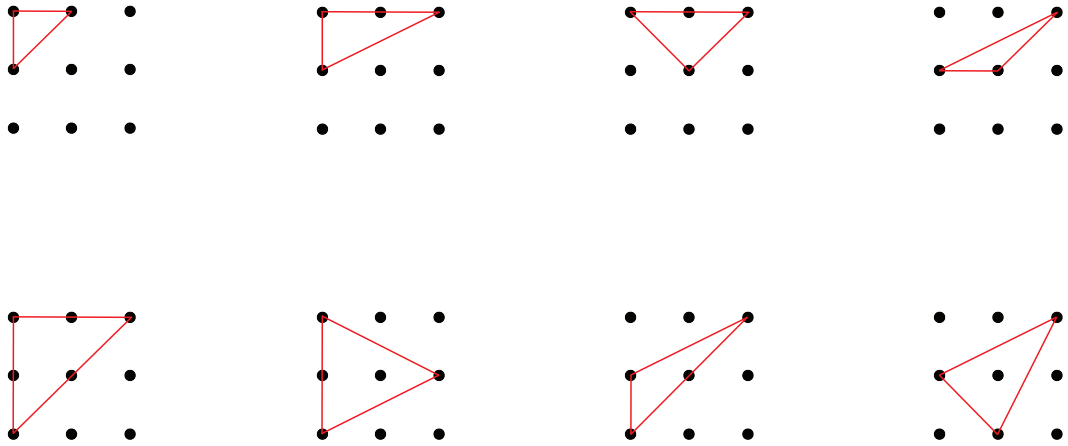
The sum is 111.

31	73	7
13	37	61
67	1	43

The sum is 34.

13	8	12	1
2	11	7	14
3	10	6	15
16	5	9	4

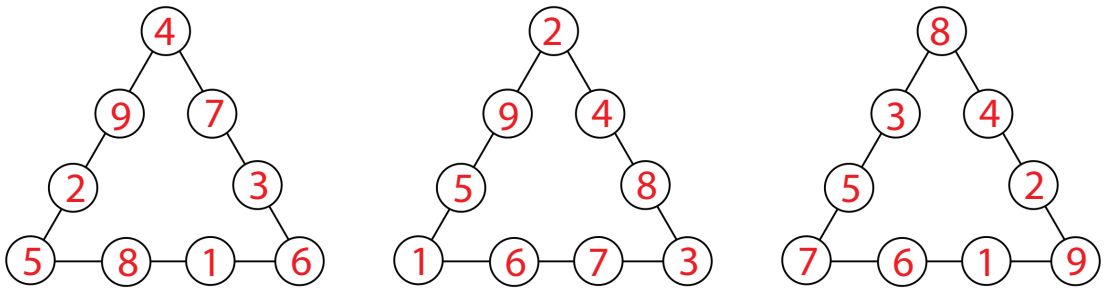
4. How many noncongruent triangles can be drawn on this 3 by 3 grid by connecting the dots?
   Hint: Look back at what we did for Example 3. How is this different?



5. Find a 10-digit number where the first digit is number of zeros in the number, the second digit is the number of 1s in the number etc. until the tenth digit which is the number of 9s in the number.
   Hints:
   - *Which box should we start with?*
   - *Don't forget to change any digits you've previously written every time you add a number.*

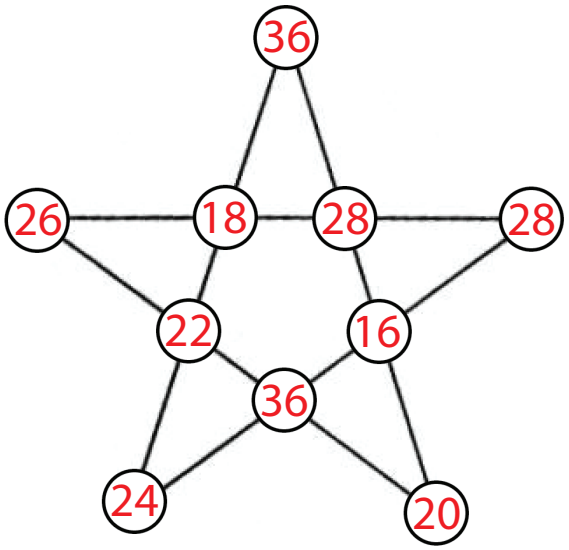
6	2	1	0	0	0	1	0	0	0

6. Put the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 exactly once in the bubbles so that each edge adds up to the same number.
   Hint: Look back at what we did for Example 2. How is this different?



7. Put the numbers 16, 18, 20, 22, 24, 26, 28, 28, 32, 36 exactly once in the bubbles so that each of the 5 straight edges add up to 100.
   Hints:

   - *Look back at what we did for Example 2 and Problem 6.*
   - *Which combinations of the numbers given add up to 100?*



8. In the recent Math Circles annual marathon, there were 10 participating students: Alan, Brian, Frank, Graham, Jimmy, Kevin, Matthew, Peter, Tom and Zach. Now the professors are comparing notes to determine who finished when. Can you help them to reconstruct the final result? *You may use the first letter of their name in your solution and final answer.*

   - Matthew beat Tom and Jimmy.
   - Peter beat Jimmy, Tom and Alan.
   - Graham beat Tom.
   - Zach beat Frank, but not Peter.
   - Graham lost to Frank and Peter.
   - Tom beat Brian.
   - Alan beat Zach, Kevin and Graham.
   - Kevin lost to Graham and Matthew.
   - Brian beat Kevin.
   - Matthew lost to Alan and Zach.
   - Frank beat Tom, Matthew and Brian.
   - Tom lost to Jimmy and Alan.
   - Jimmy beat Graham and Brian.

P	A	Z	F	M	J	G	T	B	K

First place *Last place*

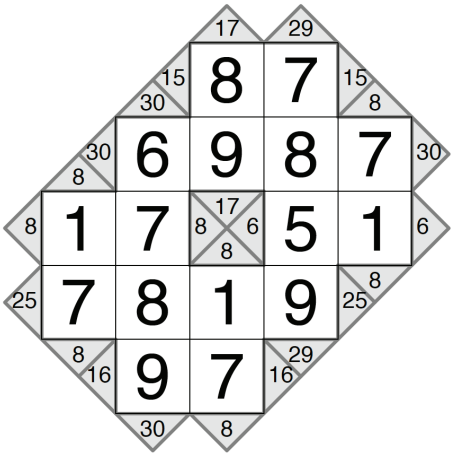
9. Daisy, Delilah, Donny, and Duke have each decided to build a snowman. In decorating their snowman, each person has chosen a different colour of scarf from purple, red, green, or blue. In addition they have chosen one accessory from a top hat, earmuffs, flower, or carrot nose. Daisy, Delilah, Donny, and Duke have all chosen to give their snowmen coal buttons. However, each person has decided to give their snowman a different number of buttons. One snowman has 2 buttons, one has 3 buttons, one has 4 buttons and one has 5 buttons.
Using the following clues, determine the combination of accessories and buttons that each person used to create their snowman.
   - The snowman built by Daisy, who is wearing the top hat, has one fewer button than the snowman wearing the red scarf, but one more button than the snowman with the carrot nose.
   - The snowman wearing the blue scarf, who is also wearing earmuffs, has two fewer buttons than the snowman wearing the green scarf.
   - The snowman wearing the purple scarf has one more button than the snowman built by Delilah.
   - The snowman built by Duke is wearing the flower.

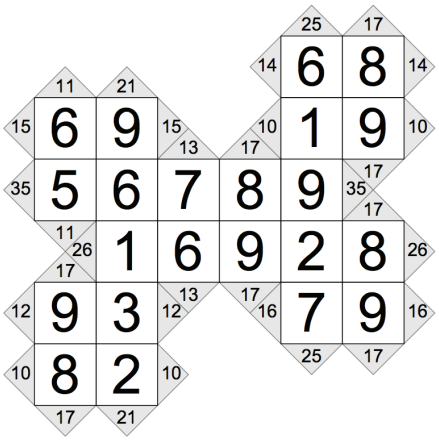
	Purple	Red	Green	Blue	2 Buttons	3 Buttons	4 Buttons	5 Buttons	Top Hat	Earmuffs	Flower	Carrot Nose
Daisy	X	X	✓	X	X	X	✓	X	✓	X	X	X
Delilah	X	X	X	✓	✓	X	X	X	X	✓	X	X
Donny	✓	X	X	X	X	✓	X	X	X	X	X	✓
Duke	X	✓	X	X	X	X	X	✓	X	X	✓	X
Top Hat	X	X	✓	X	X	X	✓	X				
Earmuffs	X	X	X	✓	✓	X	X	X				
Flower	X	✓	X	X	X	X	X	✓				
Carrot Nose	✓	X	X	X	X	✓	X	X				
2 Buttons	X	X	X	✓								
3 Buttons	✓	X	X	X								
4 Buttons	X	X	✓	X								
5 Buttons	X	✓	X	X								

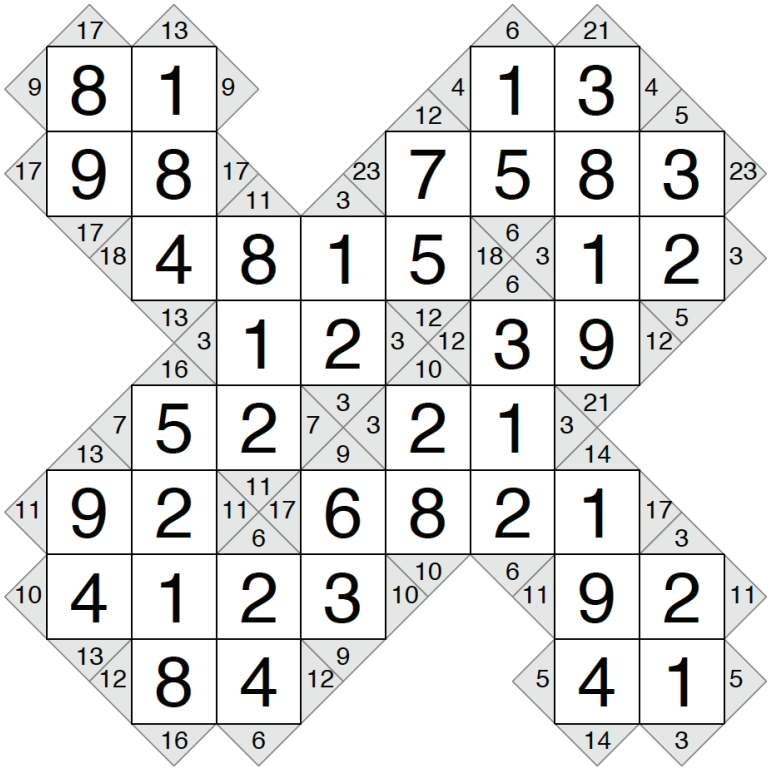
The snowman built by Daisy is wearing the green scarf, has 4 buttons, and is wearing the top hat.
The snowman built by Delilah is wearing the blue scarf, has 2 buttons, and is wearing the earmufs.
The snowman built by Donny is wearing the purple scarf, has 3 buttons, and has the carrot nose.
The snowman built by Duke is wearing the red scarf, has 5 buttons, and is wearing the flower.

10. **Kakuro**

Kakuro puzzles are like a cross between a crossword and a Sudoku puzzle. Instead of letters, each block contains the digits 1 through 9. The same digit will never repeat within a word. If you add the digits in a word, the sum will be the number shown in the clue. Clues are shown on the left and right sides of across words, and on the top and bottom sides of down words.







11. **Inkies**

Fill in the blank squares so that each row and each column contain all of the digits 1 through 5 (for the 5 by 5) or 6 (for the 6 by 6) etc. The heavy lines indicate areas (called cages) that contain groups of numbers that can be combined (in any order) to produce the result shown in the cage, with the indicated math operation. For example, 12× means you can multiply the values together to produce 12. Numbers in cages may repeat, as long as they are not in the same row or column.

5	4	2	3	1
2	3	1	5	4
1	5	4	2	3
4	2	3	1	5
3	1	5	4	2

3	4	5	2	1
4	2	1	3	5
1	5	2	4	3
5	3	4	1	2
2	1	3	5	4

12. **Siguru**

The heavy lines indicate areas, called cages, from one to five squares in size. Fill each cage with unique digits, counting up from 1. So for example a 2-square cage contains the numbers 1 and 2; and a 5-square cage contains the numbers from 1 to 5. Adjacent (touching) squares, even ones that touch diagonally, may never contain the same number.

1	3	1	2	5	4
5	2	4	3	1	2
4	3	5	2	5	3
1	2	1	3	4	2
3	5	4	2	5	3
1	2	3	1	4	1

3	1	5	2	4	2
2	4	3	1	5	1
5	1	5	2	3	4
4	3	4	1	5	1
1	2	5	2	4	2
4	3	1	3	1	3

13. **Bridges**

Connect these islands with bridges until each island can be reached from any other island, and each island has as many outgoing bridges as its number. You may only connect islands vertically or horizontally and bridges may not cross. There may be one or two bridges connecting pairs of islands, but no more than two. Each puzzle has a unique solution that can be found without making guesses.