GRADE 7 MATH LEARNING GUIDE

Lesson 10: Principal Roots and Irrational Numbers   **Time: 2 hours**
Prerequisite Concepts: Set of rational numbers
About the Lesson:
        This is an introductory lesson on irrational numbers, which may be daunting to students at this level. The key is to introduce them by citing useful examples.

Objectives:
In this lesson, you are expected to:
1. describe and define irrational numbers;
2. describe principal roots and tell whether they are rational or irrational;
3. determine between what two integers the square root of a number is;
4. estimate the square root of a number to the nearest tenth;
5. illustrate and graph irrational numbers (square roots) on a number line with and without appropriate technology.

Lesson Proper:
I. Activities
A. Take a look at the unusual wristwatch and answer the questions below.

1. Can you tell the time?

2. What time is shown in the wristwatch?

3. What do you get when you take the $\sqrt{1}$? $\sqrt{4}$? $\sqrt{9}$? $\sqrt{16}$?

4. How will you describe the result?

5. Can you take the exact value of $\sqrt{130}$?

6. What value could you get?

http://walyou.com/math-wristwatch-
is-a-square-root-geek-gadget/

Taking the square root of a number is like doing the reverse operation of squaring a number. For example, both 7 and -7 are square roots of 49 since $7^2 = 49$ and $(-7)^2 = 49$. Integers such as 1, 4, 9, 16, 25 and 36 are called perfect squares. Rational numbers such as 0.16, $\frac{4}{100}$ and 4.84 are also, perfect squares. Perfect squares are numbers that have rational numbers as square roots. The square roots of perfect squares are rational numbers while the square roots of numbers that are not perfect squares are *irrational numbers*.

Any number that cannot be expressed as a quotient of two integers is an irrational number. The numbers $\sqrt{2}$, π, and the special number *e* are all irrational numbers. Decimal numbers that are non-repeating and non-terminating are irrational numbers.

B. Activity
Use the $\sqrt[n]{}$ button of a scientific calculator to find the following values:

  1. $\sqrt[6]{64}$ 2. $\sqrt[4]{-16}$ 3. $\sqrt[3]{90}$ 4. $\sqrt[5]{-3125}$ 5. $\sqrt{24}$

II. Questions to Ponder (Post-Activity Discussions)

Let us answer the questions in the opening activity.

1. Can you tell the time? *Yes*
2. What time is it in the wristwatch? *10:07*
3. What do you get when you take the $\sqrt{1}$? $\sqrt{4}$? $\sqrt{9}$? $\sqrt{16}$? *1, 2, 3, 4*
4. How will you describe the result? *They are all positive integers.*
5. Can you take the exact value of $\sqrt{130}$? *No.*
6. What value could you get? *Since the number is not a perfect square you could estimate the value to be between $\sqrt{121}$ and $\sqrt{144}$, which is about 11.4.*

Let us give the values asked for in Activity B. Using a scientific calculator, you probably obtained the following:
1. $\sqrt[6]{64}$ = *2*
2. $\sqrt[4]{-16}$ *Math Error, which means not defined*
3. $\sqrt[3]{90}$ = *4.481404747, which could mean non-terminating and non-repeating since the calculator screen has a limited size*
4. $\sqrt[5]{-3125}$ = *-5*
5. $\sqrt{24}$ = *4.898979486, which could mean non-terminating and non-repeating since the calculator screen has a limited size*

On Principal n^{th} Roots
Any number, say a, whose n^{th} power (n, a positive integer), is b is called the nth root of b. Consider the following: $(-7)^2 = 49$, $2^4 = 16$ and $(-10)^3 = -1000$. This means that -7 is a 2nd or square root of 49, 2 is a 4th root of 16 and -10 is a 3rd or cube root of -1000.

However, we are not simply interested in any n^{th} root of a number; we are more concerned about the principal n^{th} root of a number. **The principal n^{th} root of a positive number is the positive n^{th} root. The principal n^{th} root of a negative number is the negative n^{th} root if n is odd. If n is even and the number is negative, the principal n^{th} root is not defined. The notation for the principal n^{th} root of a number b is $\sqrt[n]{b}$. In this expression, n is the index and b is the radicand. The nth roots are also called radicals.**

Classifying Principal n^{th} Roots as Rational or Irrational Numbers
To determine whether a principal root is a rational or irrational number, determine if the radicand is a perfect n^{th} power or not. If it is, then the root is rational. Otherwise, it is irrational.

Problem 1. Tell whether the principal root of each number is rational or irrational.

$$\text{(a) } \sqrt[3]{225} \qquad \text{(b) } \sqrt{0.04} \qquad \text{(c) } \sqrt[5]{-111} \qquad \text{(d) } \sqrt{10000} \qquad \text{(e) } \sqrt[4]{625}$$

Answers:

a) $\sqrt[3]{225}$ *is irrational*

(b) $\sqrt{0.04} = 0.2$ *is rational*

(c) $\sqrt[5]{-111}$ *is irrational*

(d) $\sqrt{10000} = 100$ *is rational*

(e) $\sqrt[4]{625} = 5$ *is rational*

If a principal root is irrational, the best you can do for now is to give an estimate of its value. Estimating is very important for all principal roots that are not roots of perfect n^{th} powers.

Problem 2. The principal roots below are between two integers. Find the two closest such integers.

$$\text{(a) } \sqrt{19} \qquad\qquad \text{(b) } \sqrt[3]{101} \qquad\qquad \text{(c) } \sqrt{300}$$

Solution:

(a) $\sqrt{19}$

  16 is a perfect integer square and 4 is its principal square root. 25 is the next perfect integer square and 5 is its principal square root. Therefore, $\sqrt{19}$ is between 4 and 5.

(b) $\sqrt[3]{101}$

  64 is a perfect integer cube and 4 is its principal cube root. 125 is the next perfect integer cube and 5 is its principal cube root. Therefore, $\sqrt[3]{101}$ is between 4 and 5.

(c) $\sqrt{300}$

  289 is a perfect integer square and 17 is its principal square root. 324 is the next perfect integer square and 18 is its principal square root. Therefore, $\sqrt{300}$ is between 17 and 18.

Problem 3. Estimate each square root to the nearest *tenth*.

$$\text{(a) } \sqrt{40} \qquad\qquad \text{(b) } \sqrt{12} \qquad\qquad \text{(c) } \sqrt{175}$$

Solution:

(a) $\sqrt{40}$

  The principal root $\sqrt{40}$ is between 6 and 7, principal roots of the two perfect squares 36 and 49, respectively. Now, take the square of 6.5, midway between 6 and 7. Computing, $(6.5)^2 = 42.25$. Since $42.25 > 40$ then $\sqrt{40}$ is closer to 6 than to 7. Now, compute for the squares of numbers between 6 and 6.5: $(6.1)^2 = 37.21$, $(6.2)^2 = 38.44$, $(6.3)^2 = 39.69$, and $(6.4)^2 = 40.96$. Since 40 is close to 39.69 than to 40.96, $\sqrt{40}$ is approximately 6.3.

(b) $\sqrt{12}$

      The principal root $\sqrt{12}$ is between 3 and 4, principal roots of the two perfect squares 9 and 16, respectively. Now take the square of 3.5, midway between 3 and 4. Computing $(3.5)^2 = 12.25$. Since $12.25 > 12$ then $\sqrt{12}$ is closer to 3 than to 4. Compute for the squares of numbers between 3 and 3.5: $(3.1)^2 = 9.61$, $(3.2)^2 = 10.24$, $(3.3)^2 = 10.89$, and $(3.4)^2 = 11.56$. Since 12 is closer to 12.25 than to 11.56, $\sqrt{12}$ is approximately 3.5.

(c) $\sqrt{175}$

      The principal root $\sqrt{175}$ is between 13 and 14, principal roots of the two perfect squares 169 and 196. The square of 13.5 is 182.25, which is greater than 175. Therefore, $\sqrt{175}$ is closer to 13 than to 14. Now: $(13.1)^2 = 171.61$, $(13.2)^2 = 174.24$, $(13.3)^2 = 176.89$. Since 175 is closer to 174.24 than to 176.89 then, $\sqrt{175}$ is approximately 13.2.

Problem 4. Locate and plot each square root on a number line.
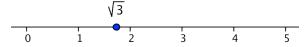
      (a) $\sqrt{3}$ (b) $\sqrt{21}$ (c) $\sqrt{87}$

Solution: You may use a program like Geogebra to plot the square roots on a number line.
(a) $\sqrt{3}$

      This number is between 1 and 2, principal roots of 1 and 4. Since 3 is closer to 4 than to 1, $\sqrt{3}$ is closer to 2. Plot $\sqrt{3}$ closer to 2.

(b) $\sqrt{21}$

      This number is between 4 and 5, principal roots of 16 and 25. Since 21 is closer to 25 than to 16, $\sqrt{21}$ is closer to 5 than to 4. Plot $\sqrt{21}$ closer to 5.

(c) $\sqrt{87}$

      This number is between 9 and 10, principal roots of 81 and 100. Since 87 is closer to 81, then $\sqrt{87}$ is closer to 9 than to 10. Plot $\sqrt{87}$ closer to 9.

III. Exercises

A. Tell whether the principal roots of each number is rational or irrational.

1. $\sqrt{400}$
2. $\sqrt{64}$
3. $\sqrt{0.01}$
4. $\sqrt{26}$
5. $\sqrt{\dfrac{1}{49}}$

6. $\sqrt{13,689}$
7. $\sqrt{1000}$
8. $\sqrt{2.25}$
9. $\sqrt{39}$
10. $\sqrt{12.1}$

B. Between which two consecutive integers does the square root lie?

1. $\sqrt{77}$
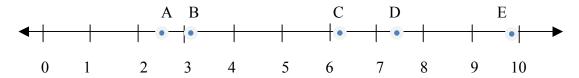2. $\sqrt{700}$
3. $\sqrt{243}$
4. $\sqrt{444}$
5. $\sqrt{48}$

6. $\sqrt{90}$
7. $\sqrt{2045}$
8. $\sqrt{903}$
9. $\sqrt{1899}$
10. $\sqrt{100000}$

C. Estimate each square root to the nearest *tenth* and plot on a number line.

1. $\sqrt{50}$
2. $\sqrt{72}$
3. $\sqrt{15}$
4. $\sqrt{54}$
5. $\sqrt{136}$

6. $\sqrt{250}$
7. $\sqrt{5}$
8. $\sqrt{85}$
9. $\sqrt{38}$
10. $\sqrt{101}$

D. Which point on the number line below corresponds to which square root?

1. $\sqrt{57}$ _____
2. $\sqrt{6}$ _____
3. $\sqrt{99}$ _____
4. $\sqrt{38}$ _____
5. $\sqrt{11}$ _____

Summary

In this lesson, you learned about irrational numbers and principal n^{th} roots, particularly square roots of numbers. You learned to find two consecutive integers between which an irrational square root lies. You also learned how to estimate the square roots of numbers to the nearest tenth and how to plot the estimated square roots on a number line.