GRADE 7 MATH TEACHING GUIDE

Lesson I: SETS: AN INTRODUCTION
Pre-requisite Concepts: Whole numbers

Objectives:

In this lesson, you are expected to:

- 1. describe and illustrate
 - a. well-defined sets;
 - b. subsets:
 - c. universal set; and
 - d. the null set.
- 2. use Venn Diagrams to represent sets and subsets.

NOTE TO THE TEACHER:

This lesson looks easy to teach but don't be deceived. The introductory concepts are always crucial. What differentiates a set from any group is that a set is well defined. Emphasize this to the students.

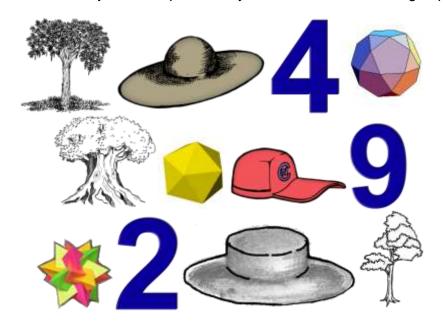
You may vary the activity by giving the students a different set of objects to group. You may make this into a class activity by showing a poster of objects in front of the class or even make it into a game. The idea is for them to create their own well-defined groups according to what they see as common characteristics of elements in a group.

Lesson Proper:

Α.

I. Activity

Below are some objects. Group them as you see fit and label each group.



Answer the following questions:

- a. How many groups are there?
- b. Does each object belong to a group?
- c. Is there an object that belongs to more than one group? Which one?

NOTE TO THE TEACHER:

You need to follow up on the opening activity hence, the problem below is important. Ultimately, you want students to apply the concepts of sets to the set of real numbers.

The groups are called <u>sets</u> for as long as the objects in the group share a characteristic and are thus, <u>well defined</u>.

Problem: Consider the set consisting of whole numbers from 1 to 200. Let this be set U. Form smaller sets consisting of elements of U that share a different characteristic. For example, let E be the set of all even numbers from 1 to 200.

Can you form three more such sets? How many elements are there in each of these sets? Do any of these sets have any elements in common?

Did you think of a set with no element?

NOTE TO THE TEACHER:

Below are important terms, notations and symbols that students must remember. From here on, be consistent in your notations as well so as not to confuse your students. Give plenty of examples and non-examples.

Important Terms to Remember

The following are terms that you must remember from this point on.

- 1. A **set** is a <u>well-defined</u> group of objects, called **elements** that share a common characteristic. For example, 3 of the objects above belong to the set of head covering or simply hats (ladies hat, baseball cap, hard hat).
- 2. Set *F* is a **subset** of set *A* if all elements of *F* are also elements of *A*. For example, the even numbers 2, 4 and 12 all belong to the set of whole numbers. Therefore, the even numbers 2, 4, and 12 form a subset of the set of whole numbers. *F* is a **proper subset** of *A* if *F* does not contain all elements of *A*.
- 3. The **universal set** *U* is the set that contains all objects under consideration.
- 4. The **null set**∅ is an empty set. The null set is a subset of any set.
- 5. The **cardinality of set** *A* is the number of elements contained in *A*.

Notations and Symbols

In this section, you will learn some of the notations and symbols pertaining to sets.

1. Uppercase letters will be used to name sets, and lowercase letters will be used to refer to any element of a set. For example, let H be the set of all objects on page 1 that cover or protect the head. We write

 $H = \{ \text{ladies hat, baseball cap, hard hat} \}$

This is the listing or roster method of naming the elements of a set.

Another way of writing the elements of a set is with the use of a descriptor. This is the rule method. For example, $H = \{x \mid x \text{ covers and protects the head}\}$. This is read as "the set H contains the element x such that x covers and protects the head."

- 2. The symbol \varnothing or $\{\ \}$ will be used to refer to an empty set.
- 3. If F is a subset of A, then we write $F \subseteq A$. We also say that A contains the set F and write it as $A \supseteq F$. If F is a proper subset of A, then we write $F \subseteq A$.
- 4. The cardinality of a set A is written as n(A).
 - **II. Questions to Ponder (Post-Activity Discussion)**

NOTE TO THE TEACHER:

It is important for you to go over the answers of your students to the questions posed in the opening activity in order to process what they have learned for themselves. Encourage discussions and exchanges in the class. Do not leave questions unanswered.

Let us answer the questions posed in the opening activity.

1. How many sets are there?

There is the set of head covers (hats), the set of trees, the set of even numbers, and the set of polyhedra. But, there is also a set of round objects and a set of pointy objects. There are 6 well-defined sets.

- 2. Does each object belong to a set? Yes.
- 3. Is there an object that belongs to more than one set? Which ones are these? All the hats belong to the set of round objects. The pine trees and two of the polyhedra belong to the set of pointy objects.

III. Exercises

Do the following exercises. Write your answers on the spaces provided:

1. Give 3 examples of well-defined sets.

Possible answers: The set of all factors of 24, The set of all first year students in this school, The set of all girls in this class.

2. Name two subsets of the set of whole numbers using both the listing or roster method and the rule method.

Example:

Listing or Roster Method:

Rule Method:

 $E = \{2x \mid x \text{ is a whole number}\}\$ $O = \{2x+1 \mid x \text{ is a whole number}\}\$ 3. Let $B = \{1, 3, 5, 7, 9\}$. List all the possible subsets of B.

 $\{\ \}, \{1\}, \{3\}, \{5\}, \{7\}, \{9\}, \{1, 3\}, \{1, 5\}, \{1, 7\}, \{1, 9\}, \{3, 5\}, \{3, 7\}, \{3, 9\}, \{5, 7\}, \{5, 9\}, \{7, 9\}, \{1, 3, 5\}, \{1, 3, 7\}, \{1, 3, 9\}, \{3, 5, 7\}, \{3, 5, 9\}, \{5, 7, 9\}, \{1, 5, 7\}, \{1, 5, 9\}, \{1, 7, 9\}, \{3, 7, 9\}, \{1, 3, 5, 7\}, \{1, 3, 5, 9\}, \{1, 5, 7, 9\}, \{3, 5, 7, 9\}, \{1, 3, 7, 9\}, \{1, 3, 7, 9\}, \{1, 3, 7, 9\}$

4. Answer this question: How many subsets does a set of n elements have? There are 2ⁿ subsets in all.

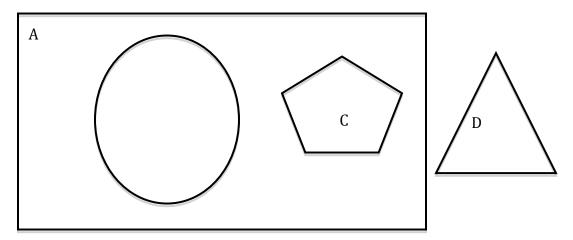
B. Venn Diagrams

NOTE TO THE TEACHER:

A lesson on sets will not be complete without using Venn Diagrams. Note that in this lesson, you are merely introducing the use of these diagrams to show sets and subsets. The extensive use of the Venn Diagrams will be introduced in the next lesson, which is on set operations. The key is for students to be able to verbalize what they see depicted in the Venn Diagrams.

Sets and subsets may be represented using <u>Venn Diagrams</u>. These are diagrams that make use of geometric shapes to show relationships between sets.

Consider the Venn diagram below. Let the universal set U be all the elements in sets A, B, C and D.



Each shape represents a set. Note that although there are no elements shown inside each shape, we can surmise or guess how the sets are related to each other. Notice that set B is inside set A. This indicates that all elements in B are contained in A. The same with set C. Set D, however, is separate from A, B, C. What does it mean?

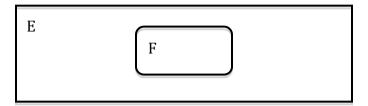
Exercise

Draw a Venn diagram to show the relationships between the following pairs or groups of sets:

1.
$$E = \{2, 4, 8, 16, 32\}$$

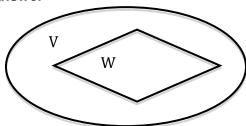
 $F = \{2, 32\}$

Sample Answer



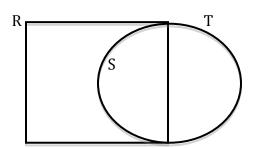
2. V is the set of all odd numbers W = {5, 15, 25, 35, 45, 55,...}

Sample Answer



3. R = {x| x is a factor of 24} S = {} T = {7, 9, 11}

Sample Answer:



NOTE TO THE TEACHER: End the lesson with a good summary.

Summary

In this lesson, you learned about sets, subsets, the universal set, the null set, and the cardinality of the set. You also learned to use the Venn diagram to show relationships between sets.

Lesson 2.1: Union and Intersection of Sets

Pre-requisite Concepts: Whole Numbers, definition of sets, Venn diagrams

Objectives:

In this lesson, you are expected to:

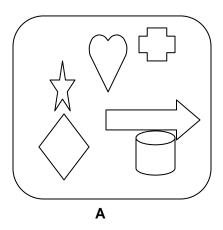
- 1. describe and define
 - a. union of sets;
 - b. intersection of sets.
- 2. perform the set operations
 - a. union of sets;
 - b. intersection of sets.
- 3. use Venn diagrams to represent the union and intersection of sets.

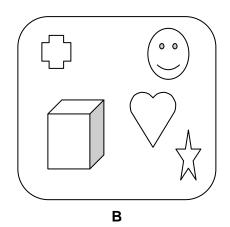
Note to the Teacher:

Below are the opening activities for students. Emphasize that just like with the whole number, operations are also used on sets. You may combine two sets or form subsets. Emphasize to students that in counting the elements of a union of two sets, elements that are common to both sets are counted only once.

Lesson Proper:

I. Activities





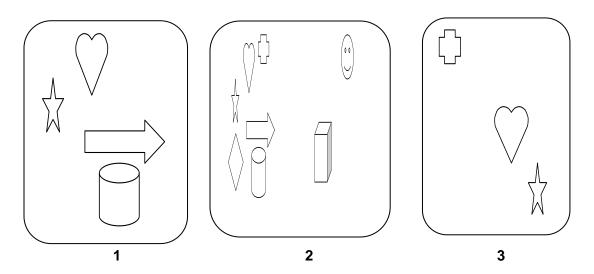
Time: 1.5 hours

Answer the following questions:

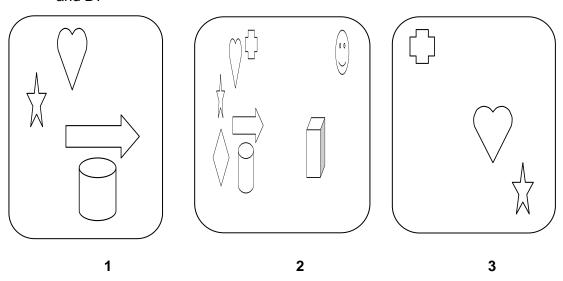
1. Which of the following shows the union of set A and set B? How many elements are in the union of A and B?







2. Which of the following shows the intersection of set A and set B? How many elements are there in the intersection of A and B?



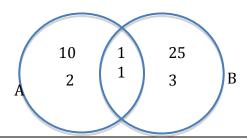
Here's another activity: Let

$$V = \{ \ 2x \mid x \in \ I, \ 1 \le x \le 4 \}$$

$$W = \{ x^2 \mid x \in \ I, \ -2 \le x \le 2 \}$$

What elements are found in the intersection of V and W? How many are there? What elements are found in the union of V and W? How many are there?

Do you remember how to use Venn Diagrams? Based on the diagram below, (1) determine the elements that belong to both A and B; (2) determine the elements that belong to A or B or both. How many are there in each set?



NOTE TO THE TEACHER:

Below are important terms, notations and symbols that students must remember. From here on, be consistent in your notations as well so as not to confuse your students. Give plenty of examples and non-examples.

Important Terms/Symbols to Remember

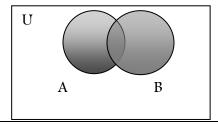
The following are terms that you must remember from this point on.

1. Let A and B be sets. The *union* of sets A and B, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

An element x belongs to the union of the sets A and B if and only if x belongs to A or x belongs to B. This tells us that

 $A \cup B = \{x \mid x \text{ is in } A \text{ or } x \text{ is in } B\}$

Venn diagram:



Note to the Teacher:

Explain to the students that in general, the <u>inclusive OR</u> is used in mathematics. Thus, when we say, "elements belonging to A or B," includes the possibility that the elements belong to both. In some instances, "belonging to both" is explicitly stated when referring to the intersection of two sets. Advise students that from here onwards, OR is used inclusively.

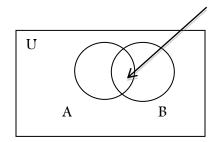
2. Let A and B be sets. The *intersection* of sets A and B, denoted by $A \cap B$, is the set containing those elements in both A and B.

An element x belongs to the intersection of sets A and B if and only if x belongs to A and x belongs to B. This tells us that

 $A \cap B = \{x \mid x \text{ is in } A \text{ and } x \text{ is in } B\}$

 $A \cap B$

Venn diagram:



Sets whose intersection is an empty set are called *disjoint sets*.

3. The cardinality of the union of two sets is given by the following equation:

$$n (A \cup B) = n (A) + n (B) - n (A \cap B).$$

II. Questions to Ponder (Post-Activity Discussion)

NOTE TO THE TEACHER

It is important for you to go over the answers of your students posed in the opening activities in order to process what they have learned for themselves. Encourage discussions and exchanges in the class. Do not leave questions unanswered. Below are the correct answers to the questions posed in the activities.

Let us answer the questions posed in the opening activity.

- 1. Which of the following shows the union of set A and set B? Why? Set 2. This is because it contains all the elements that belong to A or B or both. There are 8 elements.
- 2. Which of the following shows the intersection of set A and set B? Why? Set 3. This is because it contains all elements that are in both A and B. There are 3 elements.

In the second activity:

$$V = \{ 2, 4, 6, 8 \}$$

 $W = \{ 0, 1, 4 \}$

Therefore, $V \cap W = \{4\}$ has 1 element and $V \cup W = \{0, 1, 2, 4, 6, 8\}$ has 6 elements. Note that the element $\{4\}$ is counted only once.

On the Venn Diagram: (1) The set that contains elements that belong to both A and B consists of two elements {1, 12}; (2) The set that contains elements that belong to A or B or both consists of 6 elements {1, 10, 12, 20, 25, 36}.

NOTE TO THE TEACHER:

Always ask for the cardinality of the sets if it is possible to obtain such number, if only to emphasize that

$$n (A \cup B) \neq n (A) + n (B)$$

because of the possible intersection of the two sets. In the exercises below, use every opportunity to emphasize this. Discuss the answers and make sure students understand the "why" of each answer.

III. Exercises

1. Given sets A and B,

Set A Students who play the guitar	Set B Students who play the piano	
Ethan Molina	Mayumi Torres	
Chris Clemente	Janis Reyes	
Angela Dominguez	Chris Clemente	
Mayumi Torres	Ethan Molina	
Joanna Cruz	Nathan Santos	

determine which of the following shows (a) union of sets A and B; and (b) intersection of sets A and B?

Set 1	Set 2	Set 3	Set 4
Ethan Molina	Mayumi Torres	Mayumi Torres	Ethan Molina
Chris Clemente	Ethan Molina	Janis Reyes	Chris Clemente
Angela	Chris Clemente	Chris Clemente	Angela
Dominguez		Ethan Molina	Dominguez
Mayumi Torres		Nathan Santos	Mayumi Torres
Joanna Cruz			Joanna Cruz
			Janis Reyes
			Nathan Santos

Answers: (a) Set 4. There are 7 elements in this set. (b) Set 2. There are 3 elements in this set.

2. Do the following exercises. Write your answers on the spaces provided:

A =
$$\{0, 1, 2, 3, 4\}$$
 B = $\{0, 2, 4, 6, 8\}$ C = $\{1, 3, 5, 7, 9\}$ Answers:

Given the sets above, determine the elements and cardinality of:

a.
$$A \cup B = \{0, 1, 2, 3, 4, 6, 8\}; n (A \cup B) = 7$$

b.
$$A \cup C = \{0, 1, 2, 3, 4, 5, 7, 9\}; n(A \cup C) = 8$$

c.
$$A \cup B \cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}; n (A \cup B \cup C) = 10$$

d.
$$A \cap B = \{0, 2, 4\}; n (A \cap B) = 3$$

e.
$$B \cap C = \emptyset$$
; $n(B \cap C) = 0$

f.
$$A \cap B \cap C = \emptyset$$
; $n(A \cap B \cap C) = 0$

g.
$$(A \cap B) \cup C = \{0, 1, 2, 3, 4, 5, 7, 9\}; n((A \cap B) \cup C) = 8$$

NOTE TO THE TEACHER:

In Exercise 2, you may introduce the formula for finding the cardinality of the union of 3 sets. But, it is also instructive to give students the chance to discover this on their own. The formula for finding the cardinality of the union of 3 sets is:

$$n (A \cup B \cup C) = n (A) + n (B) + n (C) - n (A \cap B) - n (A \cap C) - n (B \cap C) + n (A \cap B \cap C).$$

3. Let
$$W = \{ x \mid 0 < x < 3 \}, Y = \{ x \mid x > 2 \}, \text{ and } Z = \{ x \mid 0 \le x \le 4 \}.$$

Determine (a) $(W \cup Y) \cap Z$; (b) $W \cap Y \cap Z$. Answers:

Since at this point students are more familiar with whole numbers and fractions greater than or equal to 0, use a partial real numberline to show the elements of these sets.

(a)
$$(W \cup Y) \cap Z = \{x \mid 0 < x \le 4\}$$

(b)
$$W \cap Y \cap Z = \{x \mid 2 < x < 3\}$$

NOTE TO THE TEACHER:

End with a good summary. Provide more exercises on finding the union and intersection of sets of numbers.

Summary

In this lesson, you learned about the definition of union and intersection of sets. You learned also how to use Venn diagrams to represent the unions and the intersection of sets.

Pre-requisite Concepts: sets, universal set, empty set, union and intersection of sets, cardinality of sets, Venn diagrams

About the Lesson:

The complement of a set is an important concept. There will be times when one needs to consider the elements not found in a particular set A. You must know that this is when you need the complement of a set.

Objectives:

In this lesson, you are expected to:

- 1. describe and define the complement of a set;
- 2. find the complement of a given set;
- 3. use Venn diagrams to represent the complement of a set.

NOTE TO THE TEACHER

Review the concept of universal set before introducing this lesson. Emphasize to the students that there are situations when it is more helpful to consider the elements found in the universal set that are not part of set A.

Lesson Proper:

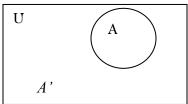
I. Problem

In a population of 8 000 students, 2 100 are Freshmen, 2 000 are Sophomores, 2 050 are Juniors, and the remaining 1 850 are either in their fourth or fifth year in university. A student is selected from the 8 000 students and he/she is not a Sophomore, how many possible choices are there?

Discussion

Definition: The complement of set A, written as A', is the set of all elements found in the universal set, U, that are not found in set A. The cardinality n (A') is given by

$$n(A') = n(U) - n(A)$$
.



Time: 1.5 hours

Examples:

1. Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and $A = \{0, 2, 4, 6, 8\}$.

Then the elements of *A'* are the elements from U that are not found in A.

Therefore, $A' = \{1, 3, 5, 7, 9\}$

2. Let U = {1, 2, 3, 4, 5}, A = {2, 4} and B = {1, 5}. Then,

$$A' = \{1, 3, 5\}$$

 $B' = \{2, 3, 4\}$
 $A' \cup B' = \{1, 2, 3, 4, 5\} = U$

3. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 7, 8\}$. Then,

$$A' = \{5, 6, 7, 8\}$$

 $B' = \{1, 2, 5, 6\}$
 $A' \cap B' = \{5, 6\}$

4. Let U =
$$\{1, 3, 5, 7, 9\}$$
, A = $\{5, 7, 9\}$ and B = $\{1, 5, 7, 9\}$. Then,
A \cap B = $\{5, 7, 9\}$
 $(A \cap B)' = \{1, 3\}$

5. Let U be the set of whole numbers. If $A = \{x \mid x \text{ is a whole number and } x > 10\}$, then

 $A' = \{x \mid x \text{ is a whole number and } 0 \le x \le 10\}.$

The opening problem asks for how many possible choices there are for a student that was selected and known to be a non-Sophomore. Let U be the set of all students and n (U) = 8 000. Let A be the set of all Sophomores then n (A) = 2 000. Set A' consists of all students in U that are not Sophomores and n (A') = n (U) – n (A) = 6 000. Therefore, there are 6000 possible choices for that selected student.

NOTE TO THE TEACHER:

Pay attention to how students identify the elements of the complement of a set. Teach them that a way to check is to take the union of a set and its complement. The union is the universal set U. That is, $A \cup A' = U$. Have them recall as well that $n(A \cup A') = n(A) + n(A') - n(A \cap A') = n(A) + n(A') = n(U)$ since $A \cap A' = \emptyset$ and therefore, $n(A \cap A') = 0$.

In the activity below, use Venn diagrams to show how the different sets relate to each other so that it is easier to identify unions and intersections of sets and complements of sets or complements or unions and intersections of sets. Watch as well the language that you use. In particular, $(A \cup B)'$ is read as "the complement of the union of A and B"

whereas $A' \cup B'$ is read as the union of the complement of A and the complement of B."

II. Activity

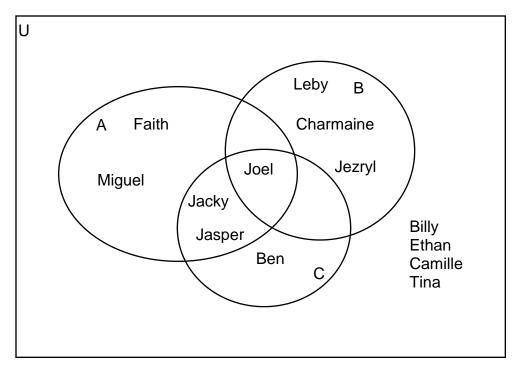
Shown in the table are names of students of a high school class by sets according to the definition of each set.

Α	В	С	D
Like Singing	Like Dancing	Like Acting	Don't Like Any
Jasper	Charmaine	Jacky	Billy
Faith	Leby	Jasper	Ethan
Jacky	Joel	Ben	Camille
Miguel	Jezryl	Joel	Tina
Joel			

After the survey has been completed, find the following sets:

- a. U =
- b. A ∪ B' =
- c. A' ∪ C =
- d. (B ∪ D)' =
- e. A' ∩ B =
- f. $A' \cap D' =$
- g. $(B \cap C)' =$

The easier way to find the elements of the indicated sets is to use a Venn diagram showing the relationships of U, sets A, B, C, and D. Set D does not share any members with A, B, and C. However, these three sets share some members. The Venn diagram below is the correct picture:



Now, it is easier to identify the elements of the required sets.

- a. U = {Ben, Billy, Camille, Charmaine, Ethan, Faith, Jacky, Jasper, Jezryl, Joel, Leby, Miguel, Tina}
- b. $A \cup B' = \{Faith, Miguel, Joel, Jacky, Jasper, Ben, Billy, Ethan, Camille, Tina\}$
- c. A' ∪ C = {Jasper, Jacky, Joel, Ben, Leby, Charmaine, Jezryl, Billy, Ethan, Camille, Tina}
- d. $(B \cup D)' = \{\text{Faith, Miguel, Jacky, Jasper, Ben}\}$
- e. $A' \cap B = \{Leby, Charmaine, Jezryl\}$
- f. $A' \cap D' = \{Leby, Charmaine, Jezryl, Ben\}$
- g. $(B \cap C)' = \{\text{Ben, Billy, Camille, Charmaine, Ethan, Faith, Jacky, Jasper, Jezryl, Leby, Miguel, Tina}\}$

NOTE TO THE TEACHER

Below are the answers to the exercises. Encourage discussions among students. Take note of the language they use. It is important that students say the words or phrases correctly. Whenever appropriate, use Venn diagrams.

III. Exercises

1. True or False. If your answer is false, give the correct answer.

Let U = the set of the months of the year

X = {March, May, June, July, October}

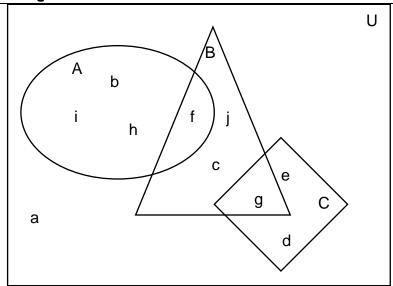
Y = {January, June, July}

Z = {September, October, November, December}

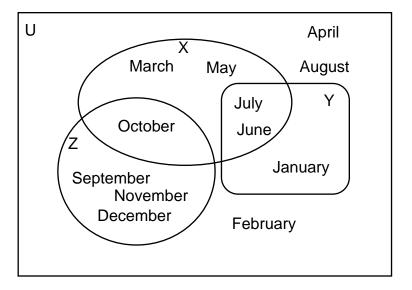
- a. Z' = {January, February, March, April, May, June, July, August}True
- b. X' ∩ Y' = {June, July} False. X' ∩ Y' = {February,
 April, August, September, November, December}
- c. $X' \cup Z' = \{January, February, March, April, May, June, July, August, September, November, December\}$ **True**
- d. $(Y \cup Z)' = \{February, March, April, May\}$ False. $(Y \cup Z)' = \{February, March, April, May, August\}$.

NOTE TO THE TEACHER

The next exercise is a great opportunity for you to develop students' reasoning skills. If the complement of A, the complement of B and the complement of C all contain the element \underline{a} then \underline{a} is outside all three sets but within U. If B' and C' both contain b but A' does not, then A contains b. This kind of reasoning must be clear to students.



2. Place the elements in their respective sets in the diagram below based on the following elements assigned to each set:



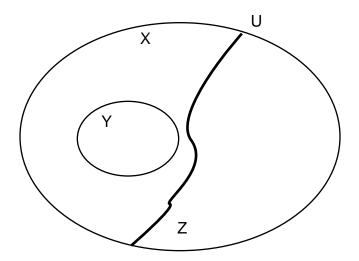
$$U = \{a, b, c, d, e, f, g, h, i, j\}$$

 $A' = \{a, c, d, e, g, j\}$
 $B' = \{a, b, d, e, h, i\}$
 $C' = \{a, b, c, f, h, i, j\}$

NOTE TO THE TEACHER:

In Exercise 3, there are many possible answers. Ask students to show all their work. This is a good opportunity for them to argue and justify their answers. Engage them in meaningful discussions. Encourage them to explain their work. Help them decide which diagrams are correct.

- 3. Draw a Venn diagram to show the relationships between sets U, X, Y, and Z, given the following information.
 - U, the universal set contains set X, set Y, and set Z.
 - $X \cup Y \cup Z = U$
 - Z is the complement of X.
 - Y' includes some elements of X and the set Z



NOTE TO THE TEACHER End with a good summary.

Summary

In this lesson, you learned about the complement of a given set. You learned how to describe and define the complement of a set, and how it relates to the universal set, U, and the given set.

Lesson 3: Problems Involving Sets

Prerequisite Concepts: Operations on Sets and Venn Diagrams

Objectives:

In this lesson, you are expected to:

1. solve word problems involving sets with the use of Venn diagrams

Time: 1 hour

2. apply set operations to solve a variety of word problems.

NOTE TO THE TEACHER

This is an important lesson. Do not skip it. This lesson reinforces what students learned about sets, set operations and the Venn diagram in solving problems.

Lesson Proper:

I. Activity

Try solving the following problem:

In a class of 40 students, 17 have ridden an airplane, 28 have ridden a boat, 10 have ridden a train, 12 have ridden both an airplane and a boat, 3 have ridden a train only, and 4 have ridden an airplane only. Some students in the class have not ridden any of the three modes of transportation and an equal number have taken all three.

- a. How many students have used all three modes of transportation?
- b. How many students have taken only the boat?

NOTE TO THE TEACHER

Allow students to write their own solutions. Allow them to discuss and argue. In the end, you have to know how to steer them to the correct solution.

II. Questions/Points to Ponder (Post-Activity Discussion)

Venn diagrams can be used to solve word problems involving union and intersection of sets. Here are some worked out examples:

- A group of 25 high school students was asked whether they use either Facebook or Twitter or both. Fifteen of these students use Facebook, and twelve use Twitter.
 - a. How many use Facebook only?
 - b. How many use Twitter only?
 - c. How many use both social networking sites?

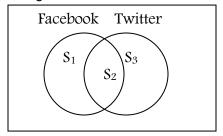
Solution:

Let S_1 = set of students who use Facebook only

 S_2 = set of students who use both social networking sites

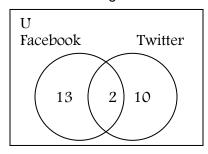
 S_3 = set of students who use Twitter only

The Venn diagram is shown below



Finding the elements in each region:

The number of elements in each region is shown below



- 2. A group of 50 students went on a tour to Palawan. Out of the 50 students, 24 joined the trip to Coron; 18 went to Tubbataha Reef; 20 visited El Nido; 12 made a trip to Coron and Tubbataha Reef; 15 saw Tubbataha Reef and El Nido; 11 made a trip to Coron and El Nido, and 10 saw the three tourist spots.
 - a. How many of the students went to Coron only?
 - b. How many of the students went to Tubbataha Reef only?
 - c. How many joined the El Nido trip only?
 - d. How many did not go to any of the tourist spots?

Solution:

To solve this problem, let

 P_1 = students who saw the three tourist spots

 P_2 = those who visited Coron only

 P_3 = those who saw Tubbataha Reef only

 P_4 = those who joined the El Nido trip only

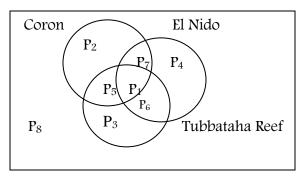
P₅ = those who visited Coron and Tubbataha Reef only

P₆ = those who joined the Tubbataha Reef and El Nido trip only

 P_7 = those who saw Coron and El Nido only

P₈ = those who did not see any of the three tourist spots

Draw the Venn diagram as shown below and identify the region where the students went.



Determine the elements in each region starting from P₁.

 P_1 consists of students who went to all three tourist spots. Thus, $n(P_1) = 10$.

- $P_1 \cup P_5$ consists of students who visited Coron and Tubbataha Reef but this set includes those who also went to El Nido. Therefore, $n(P_5) = 12 10 = 2$ students visited Coron and Tubbatha Reef only.
- $P_1 \cup P_6$ consists of students who went to El Nido and Tubbataha Reef but this set includes those who also went to Coron. Therefore, $n(P_6) = 15 10 = 5$ students visited El Nido and Tubbataha Reef only.
- $P_1 \cup P_7$ consists of students who went to Coron and El Nido but this set includes those who also went to Tubbataha Reef. Therefore, $n(P_7) = 11 10 = 1$ student visited Coron and El Nido only.

From here, it follows that

$$n(P_2) = 24 - n(P_1) - n(P_5) - n(P_7) = 24 - 10 - 2 - 1 = 11$$
 students visited Coron only.

$$n(P_3) = 18 - n(P_1) - n(P_5) - n(P_6) = 18 - 10 - 2 - 5 = 1$$
 student visited
Tubbataha Reef only

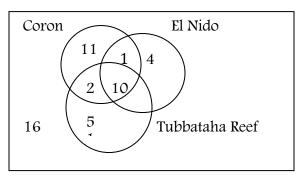
$$n(P_4) = 20 - n(P_1) - n(P_6) - n(P_7) = 20 - 10 - 5 - 1 = 4$$
 students visited Coron and El Nido only.

Therefore

$$n(P_8) = 50 - n(P_1) - n(P_2) - n(P_3) - n(P_4) - n(P_5) - n(P_6) - n(P_7) = 16$$

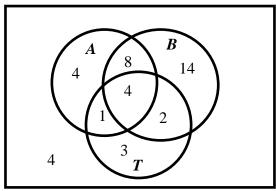
students did not visit any of the three spots.

The number of elements is shown below.



Now, what about the opening problem? Solution to the Opening Problem (Activity):

Can you explain the numbers?



NOTE TO THE TEACHER

Discuss the solution thoroughly and clarify all questions your students might have. Emphasize the notation for the cardinality of a set.

III. Exercises

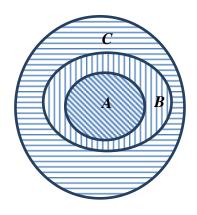
Do the following exercises. Represent the sets and draw a Venn diagram when needed.

- 1. If **A** is a set, give two subsets of **A**. **Answer:** Ø **and A**
- 2. **(a)** If A and B are finite sets and $A \subset B$, what can you say about the cardinalities of the two sets?
 - **(b)** If the cardinality of A is less than the cardinality of B, does it follow that $A \subset B$?

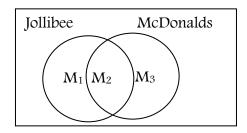
Answer: (a) n(A) < n(B); (b) **No. Example:** $A = \{1, 2\}, B = \{2, 4, 6\}$

- 3. If A and B have the same cardinality, does it follow that A = B? Explain.

 Answer: Not necessarily. Example, A = {1, 2, 3} and B = {4, 8, 9}.
- 4. If $A \subset B$ and $B \subset C$. Does it follow that $A \subset C$? Illustrate your reasoning using a Venn diagram. **Answer: Yes.**



5. Among the 70 kids in Barangay Magana, 53 like eating in Jollibee, while 42 like eating in McDonalds. How many like eating both in Jollibee and McDonalds? In Jollibee only? In McDonalds only?



Solution:

Let $n(M_1)$ = kids who like Jollibee only $n(M_2)$ = kids who like both Jollibee and McDonalds $n(M_3)$ = kids who like McDonalds only

Draw the Venn diagram

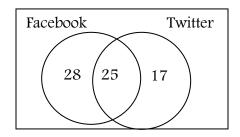
Find the elements in each region

$$n(M_1) + n(M_2) + n(M_3) = 70
n(M_1) + n(M_2) = 53$$

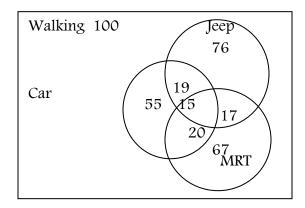
$$n(M_1) + n(M_2) + n(M_3) = 70
n(M_2) + n(M_3) = 42$$

$$n(M_1) + n(M_2) + n(M_3) = 42$$

Check using Venn diagram



6. The following diagram shows how all the First Year students of Maningning High School go to school.



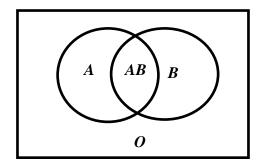
- a. How many students ride in a car, jeep and the MRT in going to school? **15**
- b. How many students ride both in a car and a jeep? 34
- c. How many students ride both in a car and the MRT? 35
- d. How many students ride both in a jeep and the MRT? 32
- e. How many students go to school

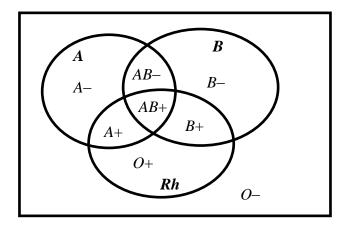
in a car only? **55** in a jeep only? **76** in the MRT only? **67** walking? **100**

- f. How many First Year students of Maningning High School are there in all? **269**
- 7. The blood-typing system is based on the presence of proteins called antigens in the blood. A person with antigen A has blood type A. A person with antigen B has blood type B, and a person with both antigens A and B has blood type AB. If no antigen is present, the blood type is O. Draw a Venn diagram representing the ABO System of blood typing.

A protein that coats the red blood cells of some persons was discovered in 1940. A person with this protein is classified as Rh positive (Rh+), and a person whose blood cells lack this protein is Rh negative (Rh-). Draw a

Venn diagram illustrating all the blood types in the ABO System with the corresponding Rh classifications.





NOTE TO THE TEACHER

The second problem is quite complex. Adding the 3rd set Rh captures the system without altering the original diagram in the first problem.

Summary

In this lesson, you were able to apply what you have learned about sets, the use of a Venn diagram, and set operations in solving word problems.

Lesson 4.1: Fundamental Operations on Integers: Addition of Integers

Time: 1 hour

Pre-requisite Concepts: Whole numbers, Exponents, Concept of Integers

Objectives:

In this lesson, you are expected to:

- 1. add integers using different approaches;
- 2. solve word problems involving addition of integers.

NOTE TO THE TEACHER

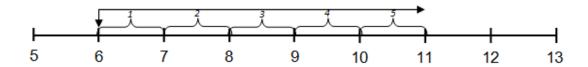
This lesson is a review and deepening of the concept of addition of integers. Keep in mind that the definitions for the operations on integers must retain the properties of the same operations on whole numbers or fractions. In this sense, the operations are merely extended to cover a bigger set of numbers. We present here two models for addition that are used to represent addition of whole numbers.

Lesson Proper:

I. Activity

Study the following examples:

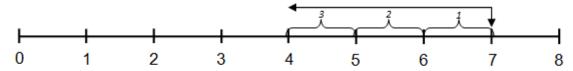
- A. Addition Using Number Line
- 1. Use the number line to find the sum of 6 & 5.



On the number line, start with point 6 and count 5 units to the right. At what point on the number line does it stop?

It stops at point 11; hence, 6 + 5 = 11.

2. Find the sum of 7 and (-3).



On the number line, start from 7 and count 3 units going to the left since the sign of 3 is negative.

At which point does it stop?

It stops at point 4; hence, (-3) + (7) = 4.

After the 2 examples, can you now try the next two problems?

- a. (-5) + (-4)
- b. (-8) + (5)

NOTE TO THE TEACHER

More examples may be given to emphasize an interpretation of the negative sign as a direction to the left of the number line.

We now have the following generalization:

Adding a positive integer n to m means moving along the real line a distance of n units to the right from m. Adding a negative integer – n to m means moving along the real line a distance of n units to the left from m.

NOTE TO THE TEACHER

Other objects might be used in this next activity. Signed tiles could be algebra tiles or counters with different colors on each side. Bottle caps are easily obtained and will be very good visual and hands-on materials.

B. Addition Using Signed Tiles

This is another device that can be used to represent integers. The tile represents integer 1, the tile represents -1, and the flexible represents 0.

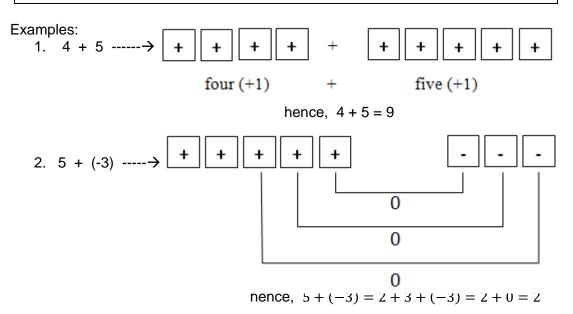
Recall that a number and its negative cancel each other under the operation of addition. This means

$$4 + (-4) = 0$$

 $15 + (-15) = 0$
 $(-29) + 29 = 0$
In general, $n + (-n) = (-n) + n = 0$.

NOTE TO THE TEACHER

Get the students to model the above equations using signed tiles or colored counters.



3.
$$(-7) + (-6)$$

hence
$$(-7) + (-6) = -13$$

Now, try these:

- 1. (-5) + (-11)
- 2. (6) + (-9)

Solution:

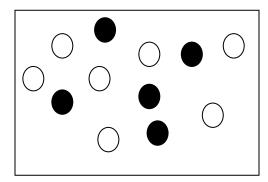
hence,
$$(-5) + (-11) = -16$$
.

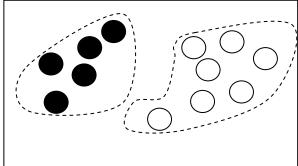
hence,
$$(6) + (-9) = -3$$
.

If colored counters (disks) or bottle caps are used, one side of the counter denotes "positive," while the other side denotes "negative." For example, with counters having black and red sides, black denotes "positive," while red denotes "negative." For this module, we will use white instead of red to denote negative.

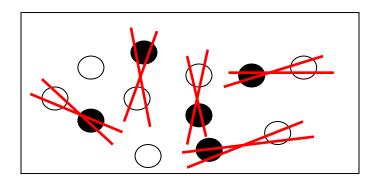
Examples:

1. The configurations below represent 5 + (-7)



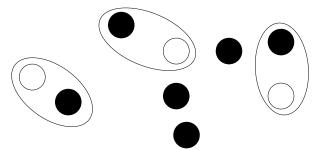


Keeping in mind that a black disk and a white disk cancel each other, take out pairs consisting of a black and a white disk until there are no more pairs left.



This tells us that 5 + (-7) = -2

2. Give a colored-counter representation of (-3) + 6



Therefore, (-3) + 6 = 3

The signed tiles model gives us a very useful procedure for adding large integers having different signs.

Examples:

1.
$$-63 + 25$$

Since 63 is bigger than 25, break up 63 into 25 and 38. Hence-63 + 25 = -38 + (-25) + 25 = -38 + 0 = -38

2.
$$724 + (-302) = 422 + 302 + (-302) = 422 + 0 = 422$$

II. Questions/ Points to Ponder

Using the above model, we summarize the procedure for adding integers as follows:

1. If the integers have the same sign, just add the positive equivalents of the integers and attach the common sign to the result.

a.
$$27 + 30 = +(/27/ +/30/)$$

= $+(/57/)$
= $+57$

b.
$$(-20) + (-15) = -(/20/ + /15/)$$

= - $(20 + 15)$
= - (35)
= - 35

2. If the integers have different signs, get the difference of the positive equivalents of the integers and attach the sign of the larger number to the result.

a.
$$(38) + (-20)$$

Get the difference between 38 and 20: 18
Since 38 is greater than 20, the sign of the sum is positive.
Hence $38 + (-20) = 18$

b. (-42) + 16

Get the difference between 42 and 16: 26 Since 42 is greater than 16, the sum will have a negative sign. Hence (-42) + 16 = -26

NOTE TO THE TEACHER

Provide more examples as needed.

If there are more than two addends in the problem, the first step to do is to combine addends with the same signs and then get the difference of their sums. Examples:

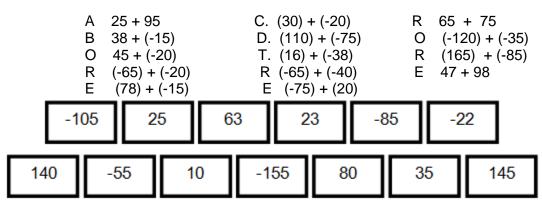
1.
$$(-14) + (22) + (8) + (-16) = -(14 + 16) + (22 + 8)$$

 $= -30 + 30 = 0$
2. $31 + 70 + 9 + (-155) = (31 + 70 + 9) + (-155)$
 $= 110 + (-155) = -45$

III. Exercises

A. Who was the first English mathematician who first used the modern symbol of equality in 1557?

(To get the answer, compute the sums of the given exercises below. Write the letter of the problem corresponding to the answer found in each box at the bottom).



Answer: ROBERT RECORDE

B. Addthe following:

1.
$$(18) + (-11) + (3)$$

$$2. (-9) + (-19) + (-6)$$

3.
$$(-4) + (25) + (-15)$$

4.
$$(50) + (-13) + (-12)$$

5.
$$(-100) + (48) + (49)$$

Answers:

1. 10 2. -34 3. 6 4. 25 5. -3

C. Solve the following problems:

1. Mrs. Reyes charged P3,752.00 worth of groceries on her credit card. Find her balance after she made a payment of P2,530.00.

Answer: PhP1,222.00

- In a game, Team Azcals lost 5 yards in one play but gained 7 yards in the next play. What was the actual yardage gain of the team? Answer: (-5) + 7 = 2 yards
- A vendor gained P50.00 on the first day; lost P28.00 on the second day, and gained P49.00 on the third day. How much profit did the vendor gain in 3 days? Answer: 50 + (-28) + 49 = 71. Profit is PhP71.00
- 4. Ronnie had PhP2280 in his checking account at the beginning of the month. He wrote checks for PhP450, P1200, and PhP900. He then made a deposit of PhP1000. If at any time during the month the account is overdrawn, a PhP300 service charge is deducted. What was Ronnie's balance at the end of the month?

Answer: 2 280 + (-450) + (-1 200) + (-900) = -270 (-270) + (-300) + 1 000 = 430 Balance is PhP430.00

NOTE TO THE TEACHER

Summarize the two models used in this lesson. It is always good to keep these models in mind, but make sure that students learn to let go of these models and should be able to add integers eventually even without these models.

Summary

In this lesson, you learned how to add integers using two different methods. The number line model is practical for small integers. For larger integers, the signed tiles model is a more useful tool.

Lesson 4.2: Fundamental Operation on Integers: Subtraction of Integers

Time: 1 hour

Prerequisite Concepts: Whole numbers, Exponents, Concept of Integers, Addition of Integers

About the Lesson: This lesson focuses on the subtraction of integers using different approaches. It is a review of what the students learned in Grade 6.

Objectives:

In this lesson, you are expected to:

- 1. subtract integers using
 - a. number line
 - b. signed tiles
- 2. solve problems involving subtraction of integers.

NOTE TO THE TEACHER

This lesson is a continuation of lesson 4.1 in a sense that mastery of the law of signs in addition of integers makes subtraction easy for the learners. Emphasis must be given on how the law of signs in addition is connected to that of subtraction.

Lesson Proper:

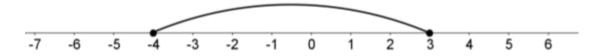
I. Activity

Study the material below.

1. Subtraction as the reverse operation of addition.

Recall how subtraction is defined. We have previously defined subtraction as the reverse operation of addition. This means that when we ask "what is 5 minus 2?", we are also asking "what number do we add to 2 in order to get 5?" Using this definition of subtraction, we can deduce how subtraction is done using the number line.

a. Suppose you want to compute (-4) - 3. You ask "What number must be added to 3 to get -4?"

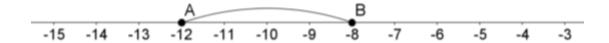


To get from 3 to -4, you need to move 7 units to the left. This is equivalent to adding -7 to 3. Hence in order to get -4, -7 must be added to 3. Therefore,

$$(-4) - 3 = -7$$

b. Compute (-8) - (-12)

What number must be added to -12 to get -8?



To go from -12 to -8, move 4 units to the right, or equivalently, add 4. Therefore,

$$(-8) - (-12) = 4$$

2. Subtraction as the addition of the negative

Subtraction is also defined as the addition of the negative of the number. For example, 5-3=5+(-3). Keeping in mind that n and -n are negatives of each other, we can also have 5-(-3)=5+3. Hence the examples above can be solved as follows:

$$(-4) - 3 = (-4) + (-3) = -7$$

$$(-8) - (-12) = (-8) + 12 = 4$$

This definition of subtraction allows the conversion of a subtraction problem to an addition problem.

NOTE TO THE TEACHER

You need to follow up on the opening activity, hence the problem below is important to reinforce what was discussed.

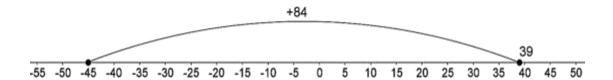
Problem:

Subtract (-45) from 39 using the two definitions of subtraction.

Can you draw your number line?Where do you start numbering it to make the line shorter?

Solution:

1.
$$39 - (-45)$$



What number must be added to -45 in order to obtain 39?

$$39 - (-45) = 84$$

2.
$$39 - (-45) = 39 + 45 = 84$$

II. Questions/Points to Ponder

Rule in Subtracting Integers

In subtracting integers, add the negative of the subtrahend to the minuend,

$$a - b = a + (-b)$$

$$a - (-b) = a + b$$

NOTE TO THE TEACHER

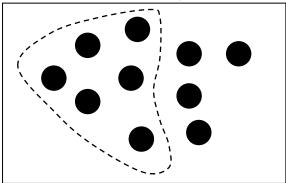
Give more examples as needed. The next section relies on the use of colored counters or signed tiles. Study the material so that you will be able to guide your students in understanding the use of these tiles correctly.

Using signed tiles or colored counters

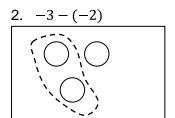
Signed tiles or colored counters can also be used to model subtraction of integers. In this model, the concept of subtraction as "taking away" is utilized.

Examples:

1. 10 - 6 means take away 6 from 10. Hence

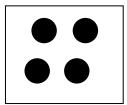


$$10 - 6 = 4$$

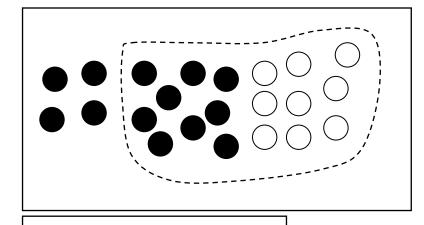


$$-3 - (-2) = -1$$

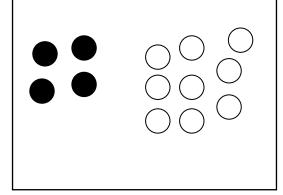
3. 4-9



Since there are not enough counters from which to take away 9, we add 9 black counters and 9 white counters. Remember that these added counters are equivalent to zero.

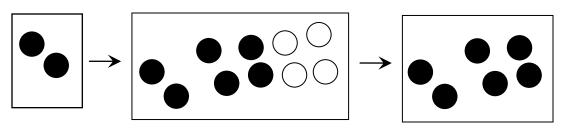


We now take away 9 black counters.



Notice that this configuration is the same configuration for 4 + (-9). We proceed with the addition and obtain the answer -5

4. 2 - (-4)



Hence 2 - (-4) = 6

The last two examples above illustrate the definition of subtraction as the addition of the negative.

$$m-n=m-n+[n+(-n)]=[m-n+n]+(-n)=m+(-n)$$

III. Exercices

A. What is the name of the 4th highest mountain in the world?

(Decode the answer by finding the difference of the following subtraction problems. Write the letter to the answer corresponding to the item in the box provided below:

- O Subtract (-33) from 99
- L Subtract (-30) from 49
- H 18 less than (-77)
- E Subtract (-99) from 0
 - T How much is 0 decreased by (-11)?
 - S(-42) (-34) (-9) 18

79 -95 132	11	-17	99
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Answer: LHOTSE

B. Mental Math

Give the difference:

1. 53 -25

6.25 - 43

2. (-6) - 123

7. (-30) - (-20)

3. (-4) - (-9)

8. (-19) - 2

4. 6 - 15

9. 30 –(-9)

5. 16 - (-20)

10. (-19) - (-15)

Answers:

- 1. 28
- 2. -129
- 3. 5 4. –9

- 6, –18
- 7. –10
- 8. -21 9. 39
- 5. 36 10. -4

- C. Solve the following problems:
 - 1. Maan deposited P53,400.00 in her account and withdrew P19,650.00 after a week. How much of her money was left in the bank?

Answer: PhP33.750.00

2. Two trains start at the same station at the same time. Train A travels 92km/h, while train B travels 82km/h. If the two trains travel in opposite directions, how far apart will they be after an hour? If the two trains travel in the same direction, how far apart will they be in two hours?

Answer: 92 - (-82) = 174 km apart $2 \times 92 - 2 \times 82 = 20 \text{ km apart}$

3. During the Christmas season, the student gov't association was able to solicit 2 356 grocery items and distribute 2 198 grocery items to one barangay. If this group decides to distribute 1 201 grocery items to the next barangay, how many more grocery items do they need to solicit?

Answer: 2 356 – 2 198 = 158 left after the first barangay 1 201 – 158 = 1 043 needed for the second barangay

NOTE TO THE TEACHER

To end, emphasize the new ideas that this lesson discussed, particularly the new concepts of subtraction and how these concepts allow the conversion of subtraction problems to addition problems.

Summary

In this lesson, you learned how to subtract integers by reversing the process of addition, and by converting subtraction to addition using the negative of the subtrahend.

Lesson 4.3: Fundamental Operations on Integers: Multiplication of Integers Time: 1 hour

Prerequisite Concepts: Operations on whole numbers, addition and subtraction of integers

About the Lesson: This is the third lesson on operations on integers. The intent of the lesson is to deepen what students have learned in Grade 6, by expounding on the meaning of multiplication of integers.

Objective:

In this lesson; you are expected to:

- 1. multiply integers
- 2. apply multiplication of integers in solving problems.

NOTE TO THE TEACHER

The repeated addition model for multiplication can be extended to multiplication of two integers in which one of the factors is positive. However, for products in which both factors are negative, repeated addition does not have any meaning. Hence multiplication of integers will be discussed in two parts: the first part looks into products with at least one positive factor, while the second studies the product of two negative integers.

Lesson Proper:

I. Activity

Answer the following question.

How do we define multiplication?

We learned that with whole numbers, *multiplication is repeated addition*. For example, 4×3 means three groups of 4. Or, putting it into a real context, 3 cars with 4 passengers each, how many passenger in all? Thus

$$4 \times 3 = 4 + 4 + 4 = 12$$
.

But, if there are 4 cars with 3 passengers each, in counting the total number of passengers, the equation is $3 \times 4 = 3 + 3 + 3 + 3 = 12$. We can say then that $4 \times 3 = 3 \times 4$ and

$$4 \times 3 = 3 \times 4 = 3 + 3 + 3 + 3 = 12$$
.

We extend this definition to multiplication of a negative integer by a positive integer. Consider the situation when a boy loses P6 for 3 consecutive days. His total loss for three days is

$$(-6) \times 3$$
. Hence, we could have

$$(-6) \times 3 = (-6) + (-6) + (-6) = -18.$$

II. Questions/Points to Ponder

The following examples illustrate further how integers are multiplied.

Example 1. Multiply: $5 \times (-2)$

However,

$$5 \times (-2) = (-2) \times (5)$$

Therefore:

$$(-2) \times (5) = (-2) + (-2) + (-2) + (-2) + (-2) = -10$$

The result shows that the product of a negative multiplier and a positive multiplicand is a negative integer.

Generalization: Multiplying unlike signs

We know that adding negative numbers means adding their positive equivalents and attaching the negative sign to the result, then

$$a \times (-b) = (-b) \times a = \underbrace{(-b) + (-b) + \dots + (-b)}_{aaddends} = -\underbrace{(b + b + \dots + b)}_{aaddends} = -ab$$

for any positive integers a and b.

We know that any whole number multiplied by 0 gives 0. Is this true for any integer as well? The answer is YES. In fact, any number multiplied by 0 gives 0. This is known as the *Zero Property*.

FOR THE TEACHER: PROOF OF THE ZERO PROPERTY

Since 1 is the identity for multiplication, for any integer a, $a \times 1 = a$.

The identity for addition is 0, so $a \times 1 = a \times (1+0) = a$.

By the distributive law, $a \times (1+0) = a \times 1 + a \times 0 = a$.

Hence a+a×0=a.

Now 0 is the only number which does not change a on addition. Therefore $a \times 0=0$.

What do we get when we multiply two negative integers?

Example 2. Multiply: $(-8) \times (-3)$

We know that $(-8) \times 3 = -24$.

Therefore,

$$-24 + (-8) \times (-3) = (-8) \times 3 + (-8) \times (-3)$$

$$= (-8) \times [3 + (-3)] (Distributive Law)$$

$$= (-8) \times 0 \quad (3 \text{ and } -3 \text{ are additive inverses})$$

$$= 0 \quad (Zero Property)$$

The only number which when added to -24 gives 0 is the additive inverse of -24. Therefore, $(-8) \times (-3)$ is the additive inverse of 24, or

$$(-8) \times (-3) = 24$$

The result shows that the product of two negative integers is a positive integer.

NOTE TO THE TEACHER

The above argument can be generalized to obtain the product (-a)×(-b). The proof may be presented to more advanced students.It is important to note that the definition of the product of two negative integers is not based on the same model as the product of whole numbers (i.e., repeated addition). The basis for the definition of the product of two negative numbers is the preservation of the properties or axioms of whole number operations (distributive law, identity and inverse property).

Generalization: Multiplying Two Negative Integers
If a and b are positive integers, then $(-a) \times (-b) = ab$.

Rules in Multiplying Integers:

6. (9)(-8)(-9) 7. (-9)(-4)(-6)

In multiplying integers, find the product of their positive equivalents.

- 1. If the integers have the same signs, their product is positive.
- 2. If the integers have different signs, their product is negative.

III. Exercises

A. Find the product of the following:

Column I

1.	(5)(12)	
2.	(-8)(4)	Angword
3.	(-5)(3)(2)	Answers: 1. 60 232 330 4. 56
4.	(-7)(4)(-2)	548 6. 648 7216
5.	(3)(8)(-2)	340 0. 040 7210

MATH DILEMMA

B. How can a person fairly divide 10 apples among 8 children so that each child has the same share?

To solve the dilemma, match the letter in Column II with the number that corresponds to the numbers in Column I.

Column II

Column I	Colur	nn II
1. (6)(-12)	С	270
2. (-13)(-13)	Р	-72
3. (19)(-17)	E	300
4. (-15)(29)	K	-323
5. (165)(0)	Α	-435
6. (-18)(-15)	M	0
7. (-15)(-20)	L	16
8. (-5)(-5)(-5)	J	-125
9. (-2)(-2)(-2)	U	169
10. (4)(6)(8)	I	192

	5	4		3	7	
4		1	1	9	- <u>-</u>	7
8		2	10	6		7

Answer: MAKE APPLE JUICE

C. Problem Solving

1. Jof has twenty P5 coins in her coin purse. If her niece took 5 of the coins, how much has been taken away?

Answer: PhP25 $(5 \times 5 = 25)$

2. Mark can type 45 words per minute, how many words can Mark type in 30 minutes?

Answer: 1 350 words $(45 \times 30 = 1350)$

- 3. Give an arithmetic equation which will solve the following
 - a. The messenger came and delivered 6 checks worth PhP50 each. Are you richer or poorer? By how much?
 - b. The messenger came and took away 3 checks worth PhP120 each. Are you richer or poorer? By how much?
 - c. The messenger came and delivered 12 bills for PhP86 each. Are you richer or poorer? By how much?
 - d. The messenger came and took away 15 bills for PhP72 each. Are you richer or poorer? By how much?

Answers:

a. $6 \times 50 = 300$	Richer by PhP300
b. $-3 \times 120 = -360$	Poorer by PhP360
c. $12 \times (-86) = -1032$	Poorer by PhP1,032
d. $(-15) \times (-72) = 1080$	Richer by PhP1,080

NOTE TO THE TEACHER

Give additional problems and drills, if only to reinforce the rules for multiplying integers. Summarize by emphasizing as well the different types of problems given in this lesson.

Summary

This lesson emphasized the meaning of multiplication to set the rules for multiplying integers. To multiply integers, first find the product of their positive equivalents. If the integers have the same signs, their product is positive. If the integers have different signs, their product is negative.

Lesson 4.4: Fundamental Operations on Integers: Division of Integers

Time: 1 hour

Prerequisite Concepts: Addition and subtraction of Integers, Multiplication of Integers

Objective:

In this lesson you are expected to:

- 1. find the quotient of two integers
- 2. solve problems involving division of integers.

NOTE TO THE TEACHER

This is a short lesson because the sign rules for division of integers are the same as with the multiplication of integers. Division is to be understood as the reverse operation of multiplication, hence making the rules the same with respect to the sign of the quotient.

Lesson Proper:

I. Activity

Answer the following questions:

What is $(-51) \div (-3)$?

What is $(-51) \div 3$?

What is $51 \div (-3)$?

What are the rules in dividing integers?

NOTE TO THE TEACHER

This exercise emphasizes the need to remember the sign rules for dividing integers.

II. Questions/Points to Ponder

We have learned that Subtraction is the inverse operation of Addition, In the same manner, Division is the inverse operation of Multiplication.

Example 1. Find the quotient of (-51) and (-3)

Solution:

Since division is the inverse of multiplication, determine what number multiplied by (-3) produces (-51).

If we ignore the signs for the meantime, we know that

$$3 \times 17 = 51$$

We also know that in order to get a negative product, the factors must have different signs. Hence $(-3) \times 17 = -51$

Therefore

$$(-51) \div (-3) = 17$$

Example 2. What is $(-57) \div 19$?

Solution: $19 \times 3 = 57$

Hence

$$19 \times (-3) = -57$$

Therefore

$$(-57) \div 19 = -3$$

Example 3.Show why $273 \div (-21) = -13$.

Solution: $(-13) \times (-21) = 57$ Therefore, $273 \div (-21) = -13$

NOTE TO THE TEACHER

It is important to give more examples to students. Always, ask students to explain or justify their answers.

Generalization

The quotient of two integers with the same signs is a positive integer, and the quotient of two integers having unlike signs is a negative integer. However, division by zero is not possible.

NOTE TO THE TEACHER

Since we introduced division as the reverse operation of multiplication, it is now easy to show why division by 0 is not possible.

What is $(-10) \div 0$? Because division is the reverse of multiplication, we must find a number such that when multiplied by 0 gives -10. But, there is no such number. In fact, no number can be divided by 0 for the same reason.

When several operations have to be performed, the GEMDAS rule applies.

Example 4. Perform the indicated operations

- 1. $2-3\times(-4)$
- 2. $4 \times 5 + 72 \div (-6)$
- 3. $9+6-(-3)\times 12\div (-9)$

Solution:

- 1. $2-3\times(-4)=2-(-12)=14$
- 2. $4 \times 5 + 72 \div (-6) = 20 + (-12) = 8$
- 3. $9+6-(-3)\times 12\div (-9)=9+6-(-36)\div (-9)=9+6-4=11$

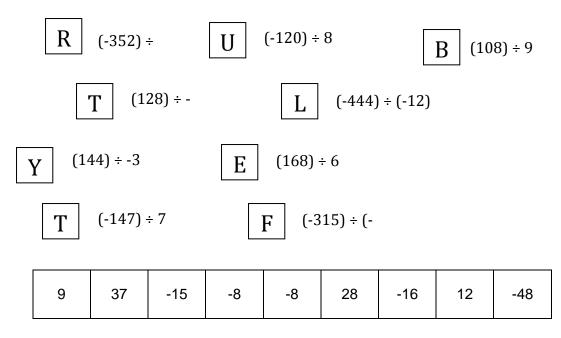
Answers:

1. -1 2. -73 3. 26 4. -4 5. 8

III. Exercises:

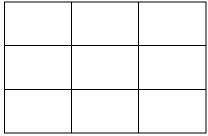
- A. Compute the following
 - 1. $(10+15)-4\times3+7\times(-2)$
 - 2. $22 \times 9 \div (-6) 5 \times 8$
 - 3. $36 \div 12 + 53 + (-30)$
 - 4. $(30 + 26) \div [(-2) \times 7]$
 - 5. $(124 5 \times 12) \div 8$
- B. What was the original name for the butterfly?

To find the answer, find the quotient of each of the following. Then write the letter of the problems in the box corresponding to the quotient.



Answer: Flutterby

- C. Solvethe following problems:
 - 1. Vergara's store earned P8,750 a week, How much is her average earning in a day? **Answer: PhP1,250.00 (8750 ÷ 7 = 1250)**
 - 2. Russ worked in a factory and earned P7,875.00 for 15 days. How much is his earning in a day? **Answer: PhP525.00 (7875 ÷ 15 = 525)**
 - 3. There are 336 oranges in 12 baskets. How many oranges are there in 3 baskets? **Answer: 84 oranges (336 ÷ 12 x 3 = 84)**
 - 4. A teacher has to divide 280 pieces of graphing paper equally among his 35 students. How many pieces of graphing paper will each student recieve? **Answer: 8 (280 ÷ 35 = 8)**
 - 5. A father has 976 sq meters lot; he has to divide it among his 4 children. What is the share of each child? **Answer: 244 sq meters** (976 ÷ 4 = 244)
- D. Complete the three-by-three magic square (that is, the sums of the numbers in each row, in each column and in each of the diagonals are the same) using the numbers -10, -7, -4, -3, 0, 3, 4, 7, 10. What is the sum for each row, column, and diagonal line?



Answer: The sum of all the numbers is 0. Hence each column/row/diagonal will have a sum of $0 \div 3 = 0$. Put 0 in the middle square. Put each number and its negative on either side of 0. A possible solution is

7	10	3
-4	0	4
-3	-10	- 7

Summary

Division is the reverse operation of multiplication. Using this definition, it is easy to see that the quotient of two integers with the same signs is a positive integer, and the quotient of two integers having unlike signs is a negative integer.

Lesson 5: Properties of the Operations on Integers

Prerequisite Concepts: Addition, Subtraction, Multiplication and Division of Integers

Time: 1.5 hours

Objectives

In this lesson, you are expected to:

1. state and illustrate the different properties of the operations on integers

a. closureb. commutativec. associatived. distributivee. identitiyf. inverse

2. rewrite given expressions according to the given property.

NOTE TO THE TEACHER:

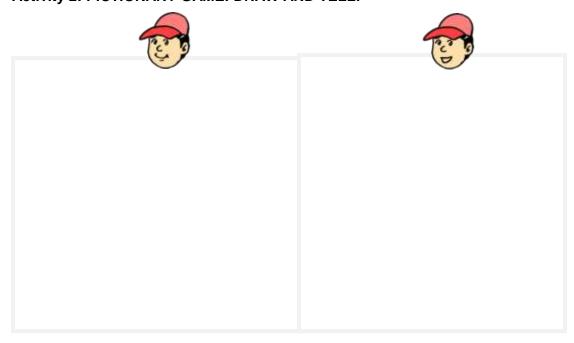
Operations on integers are some of the difficult topics in elementary algebra, and one of the least mastered skills of students based on research. The different activities presented in this lesson will hopefully give the students a tool for creating their own procedures in solving equations involving operations on integers. These are the basic rules of our system of algebra and they will be used in all succeeding mathematics. It is very important that students understand how to apply each property when solving math problems.

In activities 1 and 2, try to test the students' ability to give corresponding meaning to the different words exhibited and later on relate said terms to the lesson. In addition, students can show some creativity in activity 2.

Lesson Proper:

- I. A. Activity 1: Try to reflect on these . . .
 - 1. Give at least 5 words synonymous to the word "property".

Activity 2: PICTIONARY GAME: DRAW AND TELL!



The following questions will be answered as you go along to the next activity.

- What properties of real numbers were shown in the Pictionary Game?
 - Give one example and explain.
- How are said properties seen in real life?

NOTE TO THE TEACHER

Activity 3 gives a visual presentation of the properties.

Activity 3: SHOW AND TELL!

Determine what kind of property of real numbers is illustrated in the following images:

A. Fill in the blanks with the correct numerical values of the motorbike and bicycle riders.











If a represents the number of motorbike riders and b represents the number of bicycle riders, show the mathematical statement in a the diagram below.

____ + ___ = ___ + ___ Expected Answer: a + b = b + a

Guide Questions:

- What operation is used in illustrating the diagram? Addition
- What happened to the terms in both sides of the equation? The terms were interchanged.
- Based on the previous activity, what property is applied?

Commutative Property of Addition: For integers a, b, a + b = b + a

• If the operation is replaced by multiplication, will the same property be applicable? Give an example to prove your answer.

$$2 \cdot 3 = 3 \cdot 2$$

 $6 = 6$

Commutative Property of Multiplication: For integers a, b, ab = ba

Define the property.

Commutative Property

Changing the order of two numbers that are either being added or multiplied does not change the result.

 Give a real life situation in which the commutative property can be applied.

An example is preparing fruit juices - even if you put the powder first before the water or vice versa, the product will still be the same. It's still the same fruit juice.

• Test the property on subtraction and division operations by using simple examples. What did you discover?

Commutative property is not applicable to subtraction and division as shown in the following examples:

$$6 - 2 = 2 - 6$$

 $4 \neq -4$

$$6 \div 2 = 2 \div 6$$
$$3 \neq$$

B. Fill in the blanks with the correct numerical values of the set of cellphones, ipods, and laptops.



equals



If **a** represents the number of cellphones, **b** represents the ipods, and **c** represents the laptops, show the mathematical statement in the diagram below.

Expected Answer: (a + b) + c = a + (b + c)

Guide Questions:

What operation is used in illustrating the diagram? Addition

- What happened to the groupings of the given sets that correspond to both sides of the equation? **The groupings were changed.**
- Based on the previous activity, what property is applied?

Associative Property of Addition For integers a, b and c, (a + b) + c = a + (b + c)

• If the operation is replaced by multiplication, will the same property be applicable? Give an example to prove your answer.

$$(2 \cdot 3) \cdot 5 = 3 \cdot (2 \cdot 5)$$

 $6 \cdot 5 = 3 \cdot 10$
 $30 = 30$

Associative Property of Multiplication For integers a, b and c, (a• b)c = a(b• c)

Define the property.

Associative Property

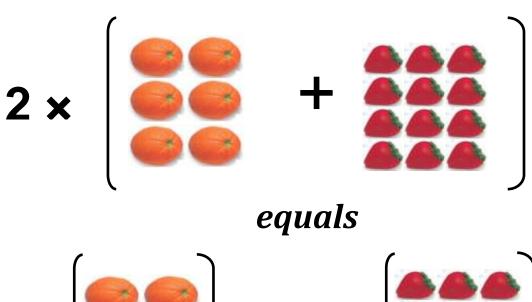
Changing the grouping of numbers that are either being added or multiplied does not change its value.

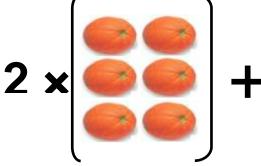
- Give a real life situation wherein associative property can be applied.
 An example is preparing instant coffee even if you combine coffee and creamer then sugar or coffee and sugar then creamer the result will be the same 3-in-1coffee.
- Test the property on subtraction and division operations by using simple examples. What did you discover?

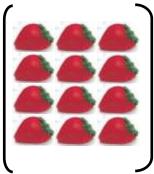
Associative property is not applicable to subtraction and division as shown in the following examples:

2)
$$(6-2)-1=6-(2-1) \qquad (12 \div 2) \div 2 = 12 \div (2 \div 2)$$
$$4-1=6-1 \qquad 6 \div 2 = 12 \div 1$$
$$3 \neq 5 \qquad 3 \neq 12$$

C. Fill in the blanks with the correct numerical values of the set of oranges and the set of strawberries.







If **a** represents the multiplier in front, **b** represents the set of oranges and **c** represents the set of strawberries, show the mathematical statement in the diagram below.

______ (____+____) = _____ • _____+ _____• _____

Answer: a(b + c) = ab + ac

Guide Questions:

 Based on the previous activity, what property is applied in the images presented?

Distributive Property

For any integers a, b, c, a(b + c) = ab + acFor any integers a, b, c, a(b - c) = ab - ac

Define the property.

Distributive Property

When two numbers have been added / subtracted and then multiplied by a factor, the result will be the same

when each number is multiplied by the factor and the products are then added / subtracted.

 In the said property can we add/subtract the numbers inside the parentheses and then multiply or perform multiplication first and then addition/subtraction? Give an example to prove your answer.

In the example, we can either add or subtract the numbers inside the parentheses first and then multiply the result; or, we can multiply each term separately and then add/ subtract the two products together. The answer is the same in both cases as shown below.

$$-2(4+3) = (-2 \cdot 4) + (-2 \cdot 3)$$

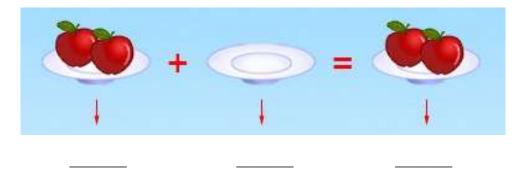
$$-2(7) = (-8) + (-6)$$

$$-14 = -14$$
or
$$-2(4+3) = -2(7)$$

$$-2(7) = -14$$

$$-14 = -14$$

- Give a real life situation where the distributive property can be applied.
 Your mother gave you four 5-peso coins and your grandmother gave you four 20-peso bills. You now have PhP20 worth of 5-peso coins and PhP80 worth of 20-peso bill. You also have four sets of PhP25 each consisting of a 5-peso coin and a 20-peso bill.
- D. Fill in the blanks with the correct numerical representation of the given illustration.



Answer: a + 0 = a

Guide Questions:

 Based on the previous activity, what property is applied in the images presented?

Identity Property for Addition a + 0 = a

- What is the result if you add something represented by any number to nothing represented by zero? The result is the non-zero number.
- What do you call zero "0" in this case? **Zero**, "0" is the additive identity.
- Define the property.
 - **Identity Property for Addition** states that 0 is the additive identity, that is, the sum of any number and 0 is the given number.
- Is there a number multiplied to any number that will result to that same number? Give examples.

Yes, the number is 1.

Examples: 1-2=2 1-3=2 1-4=2

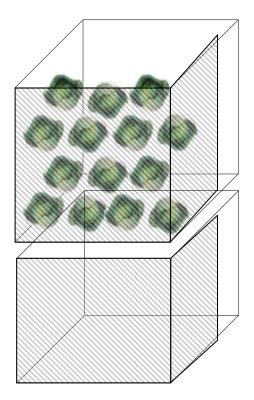
What property is illustrated? Define.

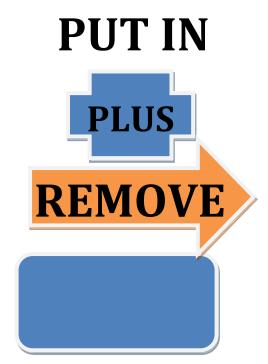
Identity Property for Multiplication says that 1 is the Multiplicative Identity

- the product of any number and 1 is the given number, $a \cdot 1 = a$.
- What do you call one "1" in this case?

One, "1" is the multiplicative identity

E. Give the correct mathematical statement of the given illustrations. To do this, refer to the guide questions below.





Guide Questions:

How many cabbages are there in the crate? 14 cabbages

- Using integers, represent "put in 14 cabbages" and "remove 14 cabbages"? What will be the result if you add these representations?
 (+14) + (-14) = 0
- Based on the previous activity, what property is applied in the images presented?

Inverse Property for Addition

$$a + (-a) = 0$$

- What is the result if you add something to its negative? The result is always zero.
- What do you call the opposite of a number in terms of sign? What is the opposite of a number represented by a?

Additive Inverse. The additive inverse of the number a is -a.

Define the property.

Inverse Property for Addition

- states that the sum of any number and its additive inverse or its negative, is zero.
- What do you mean by reciprocal, and what is the other term used for it?

The reciprocal is 1 divided by that number or the fraction $\frac{1}{a}$, where a represents the number.

The reciprocal of a number is also known as its multiplicative inverse.

- If you multiply a number say 5 by its multiplicative inverse $\frac{1}{5}$, what is the result? $5 \cdot \frac{1}{5} = 1$
- What property is illustrated? Define this property.

Inverse Property for Multiplication

- states that the product of any number and its multiplicative inverse or reciprocal, is 1.

For any number a, the multiplicative inverse is $\frac{1}{a}$.

Important Terms to Remember

The following are terms that you must remember from this point on.

1. Closure Property

Two integers that are added and multiplied remain as integers. The set of integers is closed under addition and multiplication.

2. Commutative Property

Changing the order of two numbers that are either added or multiplied does not change the value.

3. Associative Property

Changing the grouping of numbers that are either added or multiplied does not change its value.

4. Distributive Property

When two numbers are added / subtracted and then multiplied by a factor, the result is the same when each number is multiplied by the factor and the products are then added / subtracted.

5. Identity Property

Additive Identity

- states that the sum of any number and 0 is the given number. Zero, "0" is the additive identity.

Multiplicative Identity

- states that the product of any number and 1 is the given number, a • 1 = a. One, "1" is the multiplicative identity.

6. Inverse Property

In Addition

- states that the sum of any number and its additive inverse, is zero. The additive inverse of the number a is –a.

In Multiplication

- states that the product of any number and its multiplicative inverse or reciprocal, is 1. The multiplicative inverse of the number a is $\frac{1}{a}$.

Notations and Symbols

In this segment, you will learn some of the notations and symbols pertaining to properties of real number applied in the operations of integers.

Closure Property under addition and multiplication	$a, b \in I$, then $a+b \in I$, $a \cdot b \in I$
Commutative property of addition	a + b = b + a
Commutative property of multiplication	ab = ba
Associative property of addition	(a + b) + c = a + (b + c)
Associative property of multiplication	(ab) c = a (bc)
Distributive property	a(b+c) = ab + ac
Additive identity property	a + 0 = a
Multiplicative identity property	a • 1 = a
Multiplicative inverse property	$\frac{1}{a} \cdot a = 1$
Additive inverse property	a + (-a) = 0

NOTE TO THE TEACHER:

It is important for you to examine and discuss the responses of your students to the questions posed in every activity and exercise in order to practice what they have learned for themselves. Remember application as part of the learning process is essential to find out whether the learners gained knowledge of the concept or not. It is also appropriate to encourage brainstorming, dialogues and arguments in class. After the exchanges, see to it that all questions are resolved.

III. Exercises

A. Complete the Table: Which property of real number justifies each statement?

Given	Property
1. 0 + (-3) = -3	Additive Identity Property
2. 2(3 - 5) = 2(3) - 2(5)	Distributive Property
3. (-6) + (-7) = (-7) + (-6)	Commutative Property
4. 1 x (-9) = -9	Multiplicative Identity Property
5. $-4 \times -\frac{1}{4} = 1$	Multiplicative Inverse Property
6. $2 \times (3 \times 7) = (2 \times 3) \times 7$	Associative Property
7. 10 + (-10) = 0	Additive Inverse Property
8. 2(5) = 5(2)	Commutative Property
9. $1 \times (-\frac{1}{4}) = -\frac{1}{4}$	Multiplicative Identity Property
10. $(-3)(4+9) = (-3)(4) + (-3)(4)$	Distributive Property
3)(9)	

B. Rewrite the following expressions using the given property.

1. 12a – 5a	Distributive Property	(12-5)a
2. (7a)b	Associative Property	7 (ab)
3. 8 + 5	Commutative Property	5 + 8
44(1)	Identity Property	-4
5. 25 + (-25)	Inverse Property	0

C. Fill in the blanks and determine what properties were used to solve the equations.

1.	$5 \times (-2 + 2) = 0$	Additive Inverse, Zero Property
2.	-4 + 4 = 0	Additive Inverse
3.	-6 + 0 = -6	Additive Identity
4.	(-14 + 14) + 7 = 7	Additive Inverse, Additive Identity
5.	$7 \times (0 + 7) = 49$	Additive Identity

NOTE TO THE TEACHER

Try to give more of the type of exercises in Exercise C. Combine properties so that you can test how well your students have understood the lesson.

Summary

The lesson on the properties or real numbers explains how numbers or values are arranged or related in an equation. It further clarifies that no matter how these numbers are arranged and what processes are used, the composition of the equation and the final answer is still the same. Our society is much like these equations - composed of different numbers and operations, different people with varied personalities, perspectives and experiences. We can choose to look at the differences and forever highlight our advantage or superiority over the others. Or we can focus on the commonality among people and altogether work for the common good. A peaceful society and harmonious relationship start with recognizing, appreciating and fully maximizing the positive traits that we, as a people, have in common.

Lesson 6: Rational Numbers in the Number Line

Time: 1 hour

Prerequisite Concepts: Subsets of Real Numbers, Integers

Objective:

In this lesson, you, the students, are expected to

- 1. define rational numbers;
- 2. illustrate rational numbers on the number line;
- 3. arrange rational numbers on the number line.

NOTE TO THE TEACHER:

Ask students to recall the relationship of the set of rational numbers to the set of integers and the set of non-integers (Lesson 4). This lesson gives students a challenge in their numerical estimation skills. How accurately can they locate rational numbers between two integers, perhaps, or between any two numbers?

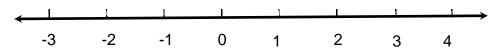
Lesson Proper

I. Activity

Determine whether the following numbers are rational numbers or not.

- 2,
$$\pi$$
, $\frac{1}{11}$, $\sqrt[3]{4}$, $\sqrt{16}$, -1.89,

Now, try to locate them on the real number line below by plotting:



NOTE TO THE TEACHER:

Give as many rational numbers as class time can allow. Give them in different forms: integers, fractions, mixed numbers, decimals, repeating decimals, etc.

II. Questions to Ponder

Consider the following examples and answer the questions that follow:

a.
$$7 \div 2 = 3 \frac{1}{2}$$
,

b.
$$(-25) \div 4 = -6 \%$$

c.
$$(-6) \div (-12) = \frac{1}{2}$$

- 1. Are quotients integers? Not all the time. Consider $\frac{01}{7}$.
- 2. What kind of numbers are they? Quotients are rational numbers.
- 3. Can you represent them on a number line? Yes. Rational numbers are real numbers and therefore, they are found in the real number line.

Recall what rational numbers are...

3 ½, -6 ¼, ½, are rational numbers. The word *rational* is derived from the word "ratio" which means quotient. Rational numbers are numbers which can be written as a quotient of two integers, $\frac{a}{b}$ where b \neq 0.

The following are more examples of rational numbers:

$$5 = \frac{5}{1} \qquad 0.06 = \frac{6}{100} \qquad 1.3 = \frac{13}{10}$$

From the example, we can see that an integer is also a rational number and therefore, integers are a subset of rational numbers. Why is that?

Let's check on your work earlier. Among the numbers given, - 2, π , $\frac{1}{11}$, $\sqrt[3]{4}$, $\sqrt{16}$, -

1.89, the numbers π and $\sqrt[3]{4}$ are the only ones that are not rational numbers. Neither can be expressed as a quotient of two integers. However, we can express the remaining ones as a quotient of two intergers:

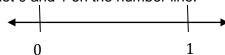
$$-2 = \frac{-2}{1}$$
, $\sqrt{16} = 4 = \frac{4}{1}$, $-1.89 = \frac{-189}{100}$

Of course, $\frac{1}{11}$ is already a quotient by itself.

We can locate rational numbers on the real number line.

Example 1. Locate ½ on the number line.

a. Since $0 < \frac{1}{2} < 1$, plot 0 and 1 on the number line.

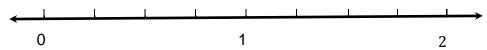


b. Get the midpoint of the segment from 0 to 1. The midpoint now corresponds to $\frac{1}{2}$

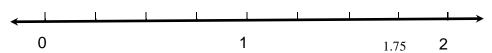


Example 2. Locate 1.75 on the number line.

a. The number 1.75 can be written as $\frac{7}{4}$ and, $1 < \frac{7}{4} < 2$. Divide the segment from 0 to 2 into 8 equal parts.

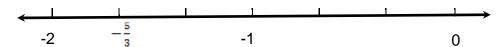


b. The 7th mark from 0 is the point 1.75.



Example 3. Locate the point $-\frac{5}{3}$ on the number line.

Note that $-2 < -\frac{5}{3} < -1$. Dividing the segment from -2 to 0 into 6 equal parts, it is easy to plot $-\frac{5}{3}$. The number $-\frac{5}{3}$ is the 5th mark from 0 to the left.



Go back to the opening activity. You were asked to locate the rational numbers and plot them on the real number line. Before doing that, it is useful to arrange them in order from least to greatest. To do this, express all numbers in the same form -either as similar fractions or as decimals. Because integers are easy to locate, they need not take any other form. It is easy to see that

$$-2 < -1.89 < \frac{1}{11} < \sqrt{16}$$

Can you explain why?

Therefore, plotting them by approximating their location gives

III. Exercises

Locate and plot the following on a number line (use only one number line).

NOTE TO THE TEACHER: You are given a number line

to work on. Plot the numbers on this number line to serve as

your answer key.

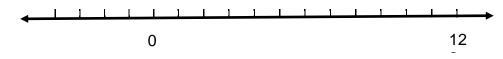
a.
$$\frac{-10}{3}$$

f.
$$7\frac{1}{9}$$

c.
$$\frac{2}{5}$$

f.
$$7\frac{1}{9}$$

h.
$$-\frac{1}{6}$$



2. Name 10 rational numbers that are greater than -1 but less than 1, and arrange them from least to greatest on the real number line?

Examples are: $\frac{1}{10}$, $-\frac{3}{10}$, $-\frac{1}{2}$, $-\frac{1}{5}$, $-\frac{1}{100}$, 0, $\frac{1}{8}$, $\frac{2}{11}$, $\frac{8}{37}$, $\frac{9}{10}$

3. Name one rational number x that satisfies the descriptions below:

a.
$$-10 \le x < -9$$
, Possible answers: $x = -\frac{46}{5}, -\frac{48}{5}, -9.75, -9\frac{8}{9}, -9.99$

b.
$$\frac{1}{10} < x < \frac{1}{2}$$

Possible answers:

$$x = -\frac{46}{5}, -\frac{48}{5}, -9.75, -9\frac{8}{9}, -9.99$$

c.
$$3 < x < \pi$$

Possible answers: $x = 3.1, 3.01, 3.001, 3.12$

d.
$$\frac{1}{4} < x < \frac{1}{3}$$

Possible answers:

$$x = \frac{13}{50}$$
, 0.27, 0.28, $\frac{299}{1000}$, $\frac{3}{10}$

e.
$$-\frac{1}{8} < x < -\frac{1}{9}$$

$$x = -\frac{3}{25}$$
, -0.124, $-\frac{17}{144}$, -0.112

NOTE TO THE TEACHER:

In this exercise, you may allow students to use the calculator to check that their choice of x is within the range given. You may, as always also encourage them to use mental computation strategies if calculators are not readily available. The important thing is that students have a way of checking their answers and will not only rely on you to give the correct answers.

NOTE TO THE TEACHER:

End this lesson with a summary as well as a preview of what students are expected to learn about rational numbers, their properties, operations and uses.

Summary

In this lesson, you learned more about what rational numbers are and where they can be found in the real number line. By changing all rational numbers to equivalent forms, it is easy to arrange them in order, from least to greatest or vice versa.

Lesson 7: Forms of Rational Numbers and Addition and Subtraction of **Rational Numbers**

Time: 2 hours

Prerequisite Concepts: definition of rational numbers, subsets of real numbers, fractions, decimals

Objectives:

In this lesson, you are expected to:

- 1. express rational numbers from fraction form to decimal form (terminating and repeating and non-terminating) and vice versa;
- 2. add and subtract rational numbers;
- 3. solve problems involving addition and subtraction of rational numbers.

NOTE TO THE TEACHER:

The first part of this module is a lesson on changing rational numbers from one form to another, paying particular attention to changing rational numbers in non-terminating and repeating decimal form to fraction form. It is assumed that students know decimal fractions and how to operate on fractions and decimals.

Lesson Proper:

A. Forms of Rational Numbers

- I. Activity
- 1. Change the following rational numbers in fraction form or mixed number form to decimal form:

a.
$$-\frac{1}{4} = -0.25$$

b. $\frac{3}{10} = 0.3$

d.
$$\frac{5}{2}$$
 = **2.5**

b.
$$\frac{3}{10} = 0.3$$

e.
$$-\frac{17}{10} = -1.7$$

c.
$$3\frac{5}{100} = 3.05$$

f.
$$-2\frac{1}{5} = -2.2$$

NOTE TO THE TEACHER:

These should be treated as review exercises. There is no need to spend too much time on reviewing the concepts and algorithms involved here.

2. Change the following rational numbers in decimal form to fraction form.

a.
$$1.8 = \frac{9}{5}$$

d.
$$-0.001 = -\frac{1}{1000}$$

b.
$$-3.5 = -\frac{7}{2}$$

e.
$$10.999 = \frac{10999}{1000}$$

c.
$$-2.2 = -\frac{11}{5}$$

f.
$$0.\overline{11} = \frac{1}{9}$$

NOTE TO THE TEACHER:

The discussion that follows assumes that students remember why certain fractions are easily converted to decimals. It is not so easy to change fractions to decimals if they are not decimal fractions. Be aware of the fact that this is the time when the concept of a fraction becomes very different. The fraction that students remember as indicating a part of a whole or of a set is now a number (rational) whose parts (numerator and denominator) can be treated separately and can even be divided! This is a major shift in concept, and students have to be prepared to understand how these concepts are consistent with what they have learned from elementary level mathematics.

II. Discussion

Non-decimal Fractions

There is no doubt that most of the above exercises were easy for you. This is because all except item 2f are what we call decimal fractions. These numbers are all parts of powers of 10. For example, $-\frac{1}{4} = \frac{25}{100}$ which is easily convertible to a decimal form, 0.25. Likewise, the number -3.5 = $-3\frac{5}{10} = -\frac{35}{10}$.

What do you do when the rational number is not a decimal fraction? How do you convert from one form to the other?

Remember that a rational number is a quotient of 2 integers. To change a rational number in fraction form, you need only to divide the numerator by the denominator.

Consider the number $\frac{1}{8}$. The smallest power of 10 that is divisible by 8 is 1000. But, $\frac{1}{8}$ means you are dividing 1 whole unit into 8 equal parts. Therefore, divide 1 whole unit first into 1 000 equal parts, and then take $\frac{1}{8}$ of the thousandths part. That is equal to $\frac{125}{1000}$ or 0.125.

Example: Change $\frac{1}{16}$, $\frac{9}{11}$, and $-\frac{1}{3}$ to their decimal forms.

The smallest power of 10 that is divisible by 16 is 10,000. Divide 1 whole unit into 10,000 equal parts and take $\frac{1}{16}$ of the ten thousandths part. That is equal to $\frac{625}{10000}$ or 0.625. You can obtain the same value if you perform the long division $1 \div 16$.

Do the same for $\frac{9}{11}$. Perform the long division $9 \div 11$, and you should obtain $0.\overline{81}$. Therefore, $\frac{9}{11} = 0.\overline{81}$. Also, $-\frac{1}{3} = -0.\overline{3}$. Note that both $\frac{9}{11}$ and $-\frac{1}{3}$ are nonterminating but repeating decimals.

To change rational numbers in decimal forms, express the decimal part of the numbers as a fractional part of a power of 10. For example, -2.713 can be changed initially to $-2\frac{713}{1000}$ and then changed to $-\frac{2173}{1000}$.

What about non-terminating but repeating decimal forms? How can they be changed to fraction form? Study the following examples:

Example 1: Change $0\overline{2}$ to its fraction form.

Solution: Let

$$r = 0.222...$$
 Since there is only 1 repeated digit, multiply the first equation by 10.

Then subtract the first equation from the second equation and obtain

$$9r = 2.0$$

$$r = \frac{2}{9}$$
 Therefore, $0\overline{2} = \frac{2}{9}$.

Example 2. Change
$$-1.\overline{35}$$
 to its fraction form. Solution: Let

$$r = -1.353535...$$

 $100r = -135.353535...$

Since there are 2 repeated digits, multiply the first equation by 100. In general, if there are n repeated digits, multiply the first equation by 10^n .

Then subtract the first equation from the second equation and obtain 99r = -134

$$r = -\frac{134}{99} = -1\frac{35}{99}$$

Therefore,
$$-1.\overline{35} = -\frac{135}{99}$$
.

NOTE TO THE TEACHER:

Now that students are clear about how to change rational numbers from one form to another, they can proceed to learning how to add and subtract them. Students will soon realize that these skills are the same skills that they have learned in elementary mathematics.

B. Addition and Subtraction of Rational Numbers in Fraction Form

I. Activity

Recall that we added and subtracted whole numbers by using the number line or by using objects in a set.

Using linear or area models, find the sum or difference.

a.
$$\frac{3}{5} + \frac{1}{5} =$$

c.
$$\frac{10}{11} - \frac{3}{11} =$$

b.
$$\frac{1}{8} + \frac{5}{8} =$$

d.
$$3\frac{6}{7} - 1\frac{2}{7} =$$

Without using models, how will you get the sum or difference?

Consider the following examples:

1.
$$\frac{1}{6} + \frac{1}{2} = \frac{1}{6} + \frac{3}{6} = \frac{4}{6} \text{ Or } \frac{2}{3}$$

2.
$$\frac{6}{7} + \left(-\frac{2}{3}\right) = \frac{18}{21} + \left(-\frac{14}{21}\right) = \frac{4}{21}$$

3.
$$-\frac{4}{3} + \left(-\frac{1}{5}\right) = -\frac{20}{15} + \left(-\frac{3}{15}\right) = -\frac{23}{15} \quad or -1\frac{8}{15}$$

4.
$$\frac{14}{5} - \frac{4}{7} = \frac{98}{35} - \frac{20}{35} = \frac{78}{35}$$
 or $2\frac{8}{35}$

5.
$$-\frac{7}{12} - \left(-\frac{2}{3}\right) = -\frac{7}{12} - \left(-\frac{8}{12}\right) = \frac{-7+8}{12} = \frac{1}{12}$$

6.
$$-\frac{1}{6} - \left(-\frac{11}{20}\right) = -\frac{10}{60} - \left(-\frac{33}{60}\right) = \frac{-10+33}{60} = \frac{23}{60}$$

Answer the following questions:

- 1. Is the common denominator always the same as one of the denominators of the given fractions?
- 2. Is the common denominator always the greater of the two denominators?
- 3. What is the least common denominator of the fractions in each example?
- 4. Is the resulting sum or difference the same when a pair of dissimilar fractions is replaced by any pair of similar fractions?

Problem: Copy and complete the fraction magic square. The sum in each row, column, and diagonal must be 2.

а	1/2	b
⁷ / ₅	1/3	С
d	е	² / ₅

» What are the values of a, b, c, d and e?
$$\mathbf{a} = \frac{1}{6}$$
, $\mathbf{b} = \frac{4}{3}$, $\mathbf{c} = \frac{4}{15}$, $\mathbf{d} = \frac{13}{30}$, $\mathbf{e} = \frac{7}{6}$

NOTE TO THE TEACHER:

The following pointers are not new to students at this level. However, if they had not mastered how to add and subtract fractions and decimals well, this is the time for them to do so.

Important things to remember

To Add or Subtract Fraction

With the same denominator,

If a, b and c denote integers, and b \neq 0, then

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

and

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

With different denominators, $\frac{a}{b}$ and $\frac{c}{d}$, where $b \neq 0$ and $d \neq 0$

If the fractions to be added or subtracted are dissimilar

- » Rename the fractions to make them similar whose denominator is the least common multiple of b and d.
- » Add or subtract the numerators of the resulting fractions.
- » Write the result as a fraction whose numerator is the sum or difference of the numerators, and whose denominator is the least common multiple of b and d.

Examples:

To Add:

To Subtract:

a.
$$\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{2}{7}$$

a.
$$\frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \frac{3}{7}$$

b.
$$\frac{2}{5} + \frac{1}{4}$$

b.
$$\frac{4}{5} - \frac{1}{4}$$

LCM/LCD of 5 and 4 is 20

$$\frac{2}{5} + \frac{1}{4} = \frac{8}{20} + \frac{5}{20} = \frac{8+5}{20} = \frac{13}{20}$$
 $\frac{4}{5} - \frac{1}{4} = \frac{16}{20} - \frac{5}{20} = \frac{16-5}{20} = \frac{11}{20}$

$$\frac{4}{5} - \frac{1}{4} = \frac{16}{20} - \frac{5}{20} = \frac{16-5}{20} = \frac{11}{20}$$

NOTE TO THE TEACHER:

Below are the answers to the activity. Make sure that students clearly understand the answers to all the questions and the concepts behind each question.

II. Questions to Ponder (Post –Activity Discussion)

Let us answer the questions posed in activity.

You were asked to find the sum or difference of the given fractions.

a.
$$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

c.
$$\frac{10}{11} - \frac{3}{11} = \frac{7}{11}$$

b.
$$\frac{1}{8} + \frac{5}{8} = \frac{6}{8} \text{ or } \frac{3}{4}$$

d.
$$3\frac{6}{7} - 1\frac{2}{7} = 2\frac{4}{7}$$

Without using the models, how would you get the sum or difference? You would have to apply the rule for adding or subtracting similar fractions. 1. Is the common denominator always the same as one of the denominators of the given fractions?

Not always. Consider $\frac{2}{5} + \frac{3}{4}$. Their least common denominator is 20 not 5 or 4.

- 2. Is the common denominator always the greater of the two denominators? Not always. The least common denominator is always greater than or equal to one of the two denominators and it may not be the greater of the two denominators.
 - 3. What is the least common denominator of the fractions in each example?

(1) 6

(5) 12

4. Is the resulting sum or difference the same as when a pair of dissimilar fractions is replaced by any pair of similar fractions?

Yes, for as long as the replacement fractions are equivalent to the original fractions.

NOTE TO THE TEACHER:

Answers in simplest form or lowest terms could mean both mixed numbers with the fractional part in simplest form or an improper fraction whose numerator and denominator have no common factor except 1. Both are acceptable as simplest forms.

III. Exercises

Do the following exercises.

a. Perform the indicated operations and express your answer in simplest form.

1.
$$\frac{2}{9} + \frac{3}{9} + \frac{1}{9} = \frac{2}{3}$$

2.
$$\frac{6}{5} + \frac{3}{5} + \frac{4}{5} = \frac{13}{5}$$

3.
$$\frac{2}{5} + \frac{7}{10} = \frac{11}{10} = 1\frac{1}{10}$$

$$4. \ \frac{16}{24} - \frac{6}{12} = \frac{1}{6}$$

$$5. \ \ 2\frac{5}{12} - \frac{2}{3} = \frac{7}{4}$$

6.
$$8\frac{1}{4} + \frac{2}{7} = \frac{239}{28} = 8\frac{15}{28}$$

$$7. \ 3\frac{1}{4} + 6\frac{2}{3} = 9\frac{11}{12}$$

$$8. \ 9\frac{5}{7} - 3\frac{2}{7} = 6\frac{3}{7}$$

9.
$$\frac{7}{9} - \frac{1}{12} = \frac{25}{36}$$

10.
$$11\frac{5}{9} - 7\frac{5}{6} = \frac{67}{18} = 3\frac{13}{18}$$

11.
$$\frac{1}{4} + \frac{2}{3} - \frac{1}{2} = \frac{5}{12}$$

12.
$$10 - 3\frac{5}{11} = \frac{72}{11} = 6\frac{6}{11}$$

13.
$$\frac{7}{20} + \frac{3}{8} + \frac{2}{5} = \frac{9}{8}$$

14.
$$\frac{5}{12} + \frac{4}{9} - \frac{1}{4} = \frac{11}{18}$$

15.
$$2\frac{5}{8} + \frac{1}{2} + 7\frac{3}{4} = \frac{87}{8} = 10\frac{7}{8}$$

NOTE TO THE TEACHER:

You should give more exercises if needed. You, the teacher should probably use the calculator to avoid computing mistakes.

- b. Give the number asked for.
- 1. What is three more than three and one-fourth? $6\frac{1}{4}$
- 2. Subtract from $15\frac{1}{2}$ the sum of $2\frac{1}{3}$ and $4\frac{2}{5}$. What is the result? $\frac{263}{30} = 8\frac{23}{30}$
- 3. Increase the sum of $6\frac{3}{14}$ and $2\frac{2}{7}$ by $3\frac{1}{2}$. What is the result?
- 4. Decrease $21\frac{3}{8}$ by $5\frac{1}{5}$. What is the result? $\frac{647}{40} = 16\frac{7}{40}$
- 5. What is $-8\frac{4}{5}$ minus $3\frac{2}{7}$? $-\frac{423}{35} = -12\frac{3}{35}$

NOTE TO THE TEACHER:

Note that the language here is crucial. Students need to translate the English phrases to the correct mathematical phrase or equation.

- c. Solve each problem.
- 1. Michelle and Corazon are comparing their heights. If Michelle's height is $120 \frac{3}{4}$ cm and Corazon's height is $96\frac{1}{3}$ cm. What is the difference in their heights?

Answer: $24\frac{5}{12}$ **cm**

- 2. Angel bought $6\frac{3}{4}$ meters of silk, $3\frac{1}{2}$ meters of satin and $8\frac{2}{5}$ meters of velvet. How many meters of cloth did she buy? **Answer:** $18\frac{13}{20}$ **m**
- 3. Arah needs $10\frac{1}{4}$ kg of meat to serve 55 guests. If she has $3\frac{1}{2}$ kg of chicken, a $2\frac{3}{4}$ kg of pork, and $4\frac{1}{4}$ kg of beef, is there enough meat for 55 guests? **Answer: Yes,** she has enough. She has a total of $10\frac{1}{2}$ kilos.
- 4. Mr. Tan has $13\frac{2}{5}$ liters of gasoline in his car. He wants to travel far so he added $16\frac{1}{2}$ liters more. How many liters of gasoline is in the tank? **Answer:** $29\frac{9}{10}$ **liters**

5. After boiling, the $17\frac{3}{4}$ liters of water is reduced to $9\frac{2}{3}$ liters. How much water has evaporated? **Answer:** $8\frac{1}{12}$ **liters**

NOTE TO THE TEACHER:

The last portion of this module is on the addition and subtraction of rational numbers in decimal form. This is mainly a review. Emphasize that they are not just working on decimal numbers, but also with rational numbers. Emphasize that these decimal numbers are a result of the numerator divided by the denominator of a quotient of two integers.

C. Addition and Subtraction of Rational Numbers in Decimal Form

There are 2 ways of adding or subtracting decimals.

 Express the decimal numbers in fractions then add or subtract as described earlier.

Example:

Add: 2.3 + 7.21 Subtract:: 9.6 – 3.25

$$2\frac{3}{10} + 7\frac{21}{100}$$
 $9\frac{6}{10} - 3\frac{25}{100}$

$$2\frac{30}{100} + 7\frac{21}{100}$$
 $9\frac{60}{100} - 3\frac{25}{100}$

$$(2+7) + \left(\frac{30+21}{100}\right)$$
 $(9-3) + \frac{60-25}{100}$

$$9 + \frac{51}{100} = 9\frac{51}{100}$$
 or 9.51 $6 + \frac{35}{100} = 6\frac{35}{100}$ or 6.35

Arrange the decimal numbers in a column such that the decimal points are aligned, then add or subtract as with whole numbers.

Example:

Add: 2.3 + 7.21 Subtract: 9.6- 3.25

Exercises:

1. Perform the indicated operation.

6)
$$700 - 678.891 =$$

7)
$$7.3 - 5.182 = 2.118$$

- 3) 900 + 676.34 + 78.003 = **1 654.343 29.5459**
- 8) 51.005 21.4591 =
- 4) 0.77 + 0.9768 + 0.05301 = **1.79981 5.78**
- 9) (2.45 + 7.89) 4.56 =

5) 5.44 - 4.97 = **0.47 12.099**

10) (10 – 5.891) + 7.99 =

- 2. Solve the following problems:
 - a. Helen had P7,500 for shopping money. When she got home, she had P132.75 in her pocket. How much did she spend for shopping?
 P7,367.25
 - b. Ken contributed P69.25, while John and Hanna gave P56.25 each for their gift to Teacher Daisy. How much were they able to gather altogether? P181.75
 - c. Ryan said, "I'm thinking of a number N. If I subtract 10.34 from N, the difference is 1.34." What is Ryan's number? **11.68**
 - d. Agnes said, "I'm thinking of a number N. If I increase my number by 56.2, the sum is 14.62." What is Agnes number? **–41.58**
 - e. Kim ran the 100-meter race in 135.46 seconds. Tyron ran faster by 15.7 seconds. What is Tyron's time for the 100-meter dash? **119.76**

NOTE TO THE TEACHER:

The summary is important especially because this is a long module. This lesson provided students with plenty of exercises to help them master addition and subtraction of rational numbers.

SUMMARY

This lesson began with some activities and instruction on how to change rational numbers from one form to another and proceeded to discuss addition and subtraction of rational numbers. The exercises given were not purely computational. There were thought questions and problem solving activities that helped in deepening one's understanding of rational numbers.

Lesson 8: Multiplication and Division of Rational Numbers

Time: 2 hours

Prerequisite Concepts: addition and subtraction of rational numbers, expressing rational numbers in different forms

Objectives:

In this lesson, you are expected to:

- 1. multiply rational numbers:
- 2. Divide rational numbers;
- 3. solve problems involving multiplication and division of rational numbers.

NOTE TO THE TEACHER:

This lesson reinforces what students learned in elementary mathematics. It starts with the visualization of the multiplication and division of rational numbers using the area model. Use different, yet appropriate shapes when illustrating using the area model. The opening activity encourages the students to use a model or drawing to help them solve the problem. Although, some students will insist they know the answer, it is a whole different skill to teach them to visualize using the area model.

Lesson Proper

A. Models for the Multiplication and Division

I. Activity:

- Make a model or a drawing to show the following:

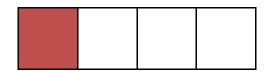
 1. A pizza is divided into 10 equal slices. Kim ate $\frac{3}{5}$ of $\frac{1}{2}$ of the pizza. What part of the whole pizza did Kim eat?
- 2. Miriam made 8 chicken sandwiches for some street children. She cut up each sandwich into 4 triangular pieces. If a child can only take a piece, how many children can she feed?

Can you make a model or a drawing to help you solve these problems?

A model that we can use to illustrate multiplication and division of rational numbers is the area model.

What is $\frac{1}{4} \times \frac{1}{3}$? Suppose we have one bar of chocolate represent 1 unit.

Divide the bar first into 4 equal parts vertically. One part of it is $\frac{1}{4}$



Then, divide each fourth into 3 equal parts, this time horizontally to make the divisions easy to see. One part of the horizontal division is $\frac{1}{3}$.

1 1			
$\frac{1}{3} \times \frac{1}{4} =$	12		
5 1			

There will be 12 equal-sized pieces and one piece is $\frac{1}{12}$. But, that one piece is $\frac{1}{3}$ of $\frac{1}{4}$, which we know from elementary mathematics to mean $\frac{1}{3} \times \frac{1}{4}$.

NOTE TO THE TEACHER

The area model is also used in visualizing division of rational numbers in fraction form. This can be helpful for some students. For others, the model may not be easily understandable. But, do not give up. It is a matter of getting used to the model. In fact, this is a good way to help them use a non-algorithmic approach to dividing rational numbers in fraction form: by using the idea that division is the reverse of multiplication.

What about a model for division of rational numbers?

Take the division problem: $\frac{4}{5} \div \frac{1}{2}$. One unit is divided into 5 equal parts and 4 of them are shaded.



Each of the 4 parts now will be cut up in halves

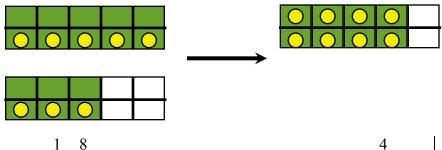


Since there are 2 divisions per part (i.e. $\frac{1}{5}$) and there are 4 of them (i.e. $\frac{4}{5}$), then there will be 8 pieces out of 5 original pieces or $\frac{4}{5} \div \frac{1}{2} = \frac{8}{5}$.

NOTE TO THE TEACHER

The solution to the problem $\frac{4}{5} \div \frac{1}{2}$ can be easily checked using the area

model as well. Ask the students, what is $\frac{1}{2} \times \frac{8}{5}$. The answer can be obtained using the area model



$$\frac{1}{2} \times \frac{8}{5} \qquad = \qquad$$

$$\frac{4}{5}$$

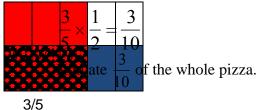
NOTE TO THE TEACHER:

It is important for you to go over the answers of your students to the questions posed in the opening activity in order to process what they have learned for themselves. Encourage discussions and exchanges in class. Do not leave questions unanswered.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions posed in the opening activity.

1. A pizza is divided into 10 equal slices. Kim ate $\frac{3}{5}$ of $\frac{1}{2}$ of the pizza. What part of the whole pizza did Kim eat?



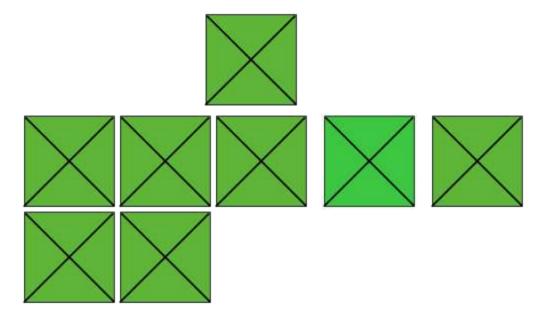
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NOTE TO THE TEACHER

The area model works for multiplication of rational numbers because the operation is binary, meaning it is an operation done on two elements. The area model allows for at most "shading" or "slicing" in two directions.

2. Miriam made 8 chicken sandwiches for some street children. She cut up each sandwich into 4 triangular pieces. If a child can only take a piece, how many children can she feed?



The equation is $8 \div \frac{1}{4} = 32$. Since there are 4 fourths in one sandwich, there will be 4 x 8 = 32 triangular pieces and hence, 32 children can be fed.

How then can you multiply or divide rational numbers without using models or drawings?

NOTE TO THE TEACHER:

Below are important rules or procedures that the students must remember. From here on, be consistent in your rules so that your students will not be confused. Give plenty of examples.

Important Rules to Remember

The following are rules that you must remember. From here on, the symbols to be used for multiplication are any of the following: \bullet , x, \times , or x.

1. To multiply rational numbers in fraction form, simply multiply the numerators and multiply the denominators.

In symbol,
$$\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$$
 where: b and d are NOT equal to zero, (b \neq 0; d \neq 0)

2. To divide rational numbers in fraction form, you take the reciprocal of the second fraction (called the divisor) and multiply it by the first fraction.

In symbol,
$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$$
 zero.

where: b, c, and d are NOT equal to

Example:

Multiply the following and write your answer in simplest form

a.
$$\frac{3}{7} \bullet \frac{2}{5}$$

$$\frac{3}{7} \bullet \frac{2}{5} = \frac{3 \times 2}{7 \times 5} = \frac{6}{35}$$

b.
$$4\frac{1}{3} \cdot 2\frac{1}{4}$$

$$\frac{13}{3} \cdot \frac{9}{4} = \frac{13 \cdot 3 \cdot 3}{3 \cdot 4} = \frac{13 \cdot 3}{4}$$

$$= \frac{39}{4} \text{ or } 9\frac{3}{4}$$

The easiest way to solve for this number is to change mixed numbers to an improper fraction and then multiply it. Or use prime factors or the greatest common factor, as part of the multiplication process.

Divide:
$$\frac{8}{11} \div \frac{2}{3}$$

 $\frac{8}{11} \div \frac{2}{3} = \frac{8}{11} \bullet \frac{3}{2}$
 $= \frac{2 \bullet 4}{11} \bullet \frac{3}{2}$
 $\frac{4 \bullet 3}{11} = \frac{12}{11} \text{ or } 1\frac{1}{11}$

Take the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$ then multiply it with the first fraction. Using prime factors, it is easy to see that 2 can be factored out of the numerator then cancelled out with the denominator, leaving 4 and 3 as the remaining factors in the numerator and 11 as the remaining factors in the denominator.

III. Exercises.

Do the following exercises. Write your answer on the spaces provided:

1. Find the products. Express in lowest terms (i.e. the numerator and denominators do not have a common factor except 1). Mixed numbers are acceptable as well:

a.
$$\frac{5}{6} \bullet \frac{2}{3} = \frac{5}{9}$$

b.
$$7 \bullet \frac{2}{3} = \frac{14}{3} = 4\frac{2}{3}$$

c.
$$\frac{4}{20} \bullet \frac{2}{5} = \frac{2}{25}$$

d.
$$10\frac{5}{6} \bullet 3\frac{1}{3} = \frac{325}{9} = 36\frac{1}{9}$$
 i. $-\frac{5}{6} \bullet \frac{2}{3} \bullet \left(-\frac{12}{15}\right) = \frac{4}{9}$

e.
$$-\frac{9}{20} \bullet \frac{25}{27} = -\frac{5}{12}$$

f.
$$4\frac{1}{2} \bullet 5\frac{2}{3} = \frac{51}{2} = 25\frac{1}{2}$$

g.
$$\frac{2}{15} \bullet \frac{3}{4} = \frac{1}{10}$$

h.
$$\frac{1}{6} \bullet \frac{2}{3} \bullet \frac{1}{4} = \frac{1}{36}$$

i.
$$-\frac{5}{6} \bullet \frac{2}{3} \bullet \left(-\frac{12}{15}\right) = \frac{4}{9}$$

j.
$$\frac{9}{16} \bullet \frac{4}{15} \bullet (-2) = -\frac{3}{10}$$

B. Divide:

1.
$$20 \div \frac{2}{3} = 30$$

2.
$$\frac{5}{12} \div \left(-\frac{3}{4}\right) = -\frac{5}{9}$$

$$3. \ \frac{5}{50} \div \frac{20}{35} = \frac{7}{40}$$

4.
$$5\frac{3}{4} \div 6\frac{2}{3} = \frac{69}{80}$$

$$5. \frac{9}{16} \div \frac{3}{4} \div \frac{1}{6} = \frac{9}{2} = 4\frac{1}{2}$$

6.
$$\frac{8}{15} \div \frac{12}{25} = \frac{10}{9} = 1\frac{1}{9}$$

2.
$$\frac{5}{12} \div \left(-\frac{3}{4}\right) = -\frac{5}{9}$$
 7. $13\frac{1}{6} \div (-2) = -\frac{79}{12} = -6\frac{7}{12}$

8.
$$-\frac{5}{6} \div \left(-\frac{10}{14}\right) = \frac{7}{6} = 1\frac{1}{6}$$

9.
$$-\frac{2}{9} \div \frac{11}{15} = -\frac{10}{33}$$

$$10. \ \frac{15}{6} \div \frac{2}{3} \div \frac{5}{8} = 6$$

C. Solve the following:

- 1. Julie spent $3\frac{1}{2}$ hours doing her assignment. Ken did his assignment for $1\frac{2}{3}$ times as many hours as Julie did. How many hours did Ken spend doing his assignment? $\frac{35}{6} = 5\frac{5}{6}$ hours
- 2. How many thirds are there in six-fifths? $\frac{18}{5} = 3\frac{3}{5}$
- 3. Hanna donated $\frac{2}{5}$ of her monthly allowance to the Iligan survivors. If her monthly allowance is P3,500, how much did she donate? P1,400.00
- 4. The enrolment for this school year is 2 340. If $\frac{1}{6}$ are sophomores and $\frac{1}{4}$ are seniors, how many are freshmen or juniors? 1 365 students are freshmen or
- 5. At the end of the day, a store had 2/5 of a cake leftover. The four employees each took home the same amount of leftover cake. How much of the cake did each employee take home? $\frac{1}{10}$ of the cake.

B. Multiplication and Division of Rational Numbers in Decimal Form

NOTE TO THE TEACHER

The emphasis here is on what to do with the decimal point when multiplying or dividing rational numbers in decimal form. Do not get stuck on the rules. Give a deeper explanation. Consider:

$$6.1 \times 0.08 = 6\frac{1}{10} \times \frac{8}{100} = \frac{488}{1000} = 0.488$$

The decimal places indicate the powers of 10 used in the denominators, hence the rule for determining where to place the decimal point in the product.

This unit will draw upon your previous knowledge of multiplication and division of whole numbers. Recall the strategies that you learned and developed when working with whole numbers.

Activity:

 Give students several examples of multiplication sentences with the answers given. Place the decimal point in an incorrect spot and ask students to explain why the decimal place should not be placed there and explain where it should be placed and why. Example:

 $215.2 \times 3.2 = 68.864$

2. Five students ordered buko pie, and the total cost is P135.75. How much did each student have to pay if they shared the cost equally?

Questions and Points to Ponder:

- In multiplying rational numbers in decimal form, note the importance of knowing where to place the decimal point in a product of two decimal numbers. Do you notice a pattern? Take the sum of the decimal places in each of the multiplicand and the multiplier and that is the number of places in the product.
- 2. In dividing rational numbers in decimal form, how do you determine where to place the decimal point in the quotient? The number of decimal places in the quotient depends on the number of decimal places in the divisor and the dividend.

NOTE TO THE TEACHER

Answer to the Questions and Points to Ponder is to be elaborated when you discuss the rules below.

Rules in Multiplying Rational Numbers in Decimal Form

- 1. Arrange the numbers in a vertical column.
- 2. Multiply the numbers, as if you are multiplying whole numbers.
- 3. Starting from the rightmost end of the product, move the decimal point to the left the same number of places as the sum of the decimal places in the multiplicand and the multiplier.

Rules in Dividing Rational Numbers in Decimal Form

- 1. If the divisor is a whole number, divide the dividend by the divisor applying the rules of a whole number. The position of the decimal point is the same as that in the dividend.
- 2. If the divisor is not a whole number, make the divisor a whole number by moving the decimal point in the divisor to the rightmost end, making the number seem like a whole number.

- 3. Move the decimal point in the dividend to the right the same number of places as the decimal point was moved to make the divisor a whole number.
- 4. Lastly divide the new dividend by the new divisor.

Exercises:

A. Perform the indicated operation

1.
$$3.5 \div 2 = 1.75$$

2.
$$78 \times 0.4 = 31.2$$

$$3. 9.6 \times 13 = 124.8$$

4.
$$3.24 \div 0.5 = 6.48$$

5.
$$1.248 \div 0.024 = 52$$

6.
$$27.3 \times 2.5 = 68.25$$

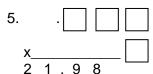
7.
$$9.7 \times 4.1 = 39.77$$

$$8.3.415 \div 2.5 = 1.366$$

9.
$$53.61 \times 1.02 = 54.6822$$

10.
$$1948.324 \div 5.96 = 326.9$$

B. Finds the numbers that when multiplied give the products shown.





Answers: (1) 5.3×2 ; (2) 8.4×2 or 5.6×3 ; (3) 5.4×4 ; (4) 3.5×3 ; (5) 3.14×7 **NOTE TO THE TEACHER: These are only some of the possible pairs. Be open to accept or consider other pairs of numbers.**

NOTE TO THE TEACHER

Give a good summary to this lesson emphasizing how this lesson was meant to deepen their understanding of rational numbers and develop better their skills in multiplying and dividing rational numbers.

Summary

In this lesson, you learned to use the area model to illustrate multiplication and division of rational numbers. You also learned the rules for multiplying and dividing rational numbers in both the fraction and decimal forms. You solved problems involving multiplication and division of rational numbers.

Lesson 9: Properties of the Operations on Rational Numbers

Time: 1 hour

Pre-requisite Concepts: Operations on rational numbers

Objectives:

In this lesson, you are expected to

- 1. Describe and illustrate the different properties of the operations on rational numbers.
- 2. Apply the properties in performing operations on rational numbers.

NOTE TO THE TEACHER:

Generally, rational numbers appear difficult among students. The following activity should be fun and could help your students realize the importance of the properties of operations on rational numbers.

Lesson Proper:

I. Activity

Pick a Pair						
	2	3	0	1	13	
	$\frac{\overline{14}}{14}$	5	U	'	$\overline{40}$	
		13	1		3	
		$\overline{12}$	$\frac{-}{3}$		$\overline{20}$	

From the box above, pick the correct rational number to be placed in the spaces provided to make the equation true.

1.
$$\frac{3}{14} + \left[\frac{2}{14}\right] = \frac{5}{14}$$

6.
$$\left(\frac{1}{2} + \frac{1}{4}\right) + \frac{1}{3} = \underline{\qquad} \left[\frac{13}{12}\right]$$

2.
$$\left[\frac{2}{14}\right] + \frac{3}{14} = \frac{5}{14}$$

7.
$$\frac{1}{2} + \left(\frac{1}{4} + \ldots\right) = \frac{13}{12} \left[\frac{1}{3}\right]$$

3.
$$\frac{1}{3} x = 0$$
 [0]

4. 1 x
$$\left[\frac{3}{5}\right] = \frac{3}{5}$$

9.
$$\left(\frac{2}{5} \ x \ \frac{1}{2}\right) x \ \frac{3}{4} = \underline{\qquad} \left[\frac{3}{20}\right]$$

5.
$$\frac{2}{3}$$
 + $[0] = \frac{2}{3}$

10.
$$\frac{1}{2} x \left(\frac{2}{5} + \frac{1}{4}\right) = \left(\frac{1}{2} x \frac{2}{5}\right) + \left(\frac{1}{2} x \frac{1}{4}\right) = \left[\frac{13}{40}\right]$$

Answer the following questions:

- 1. What is the missing number in item 1?
- 2. How do you compare the answers in items 1 and 2?
- 3. What about item 3? What is the missing number?

- 4. In item 4, what number did you multiply with 1 to get $\frac{3}{5}$?
- 5. What number should be added to $\frac{2}{3}$ in item 5 to get the same number?
- 6. What is the missing number in items 6 and 7?
- 7. What can you say about the grouping in items 6 and 7?
- 8. What do you think are the answers in items 8 and 9?
- 9. What operation did you apply in item 10?

NOTE TO THE TEACHER

The follow-up problem below could make the points raised in the previous activity clearer.

Problem:

Consider the given expressions:

a.
$$\frac{1}{4} + \frac{1}{8} + \frac{1}{2} + \frac{2}{3} = \frac{1}{4} + \frac{1}{2} + \frac{2}{3} + \frac{1}{8}$$

b.
$$\frac{2}{15} \bullet \frac{5}{6} = \frac{5}{6} \bullet \frac{2}{15}$$

* Are the two expressions equal? If yes, state the property illustrated. Yes, the expressions in item (a) are equal and so are the expressions in item (b). This is due to the Commutative Property of Addition and of Multiplication. The Commutative Property allows you to change the order of the addends or factors and the resulting sum or product, respectively, will not change.

NOTE TO THE TEACHER

Discuss among your students the following properties. These properties make adding and multiplying of rational numbers easier to do.

PROPERTIES OF RATIONAL NUMBERS (ADDITION & MULTIPLICATION)

1. CLOSURE PROPERTY: For any two defined rational numbers. $\frac{a}{b}$ and $\frac{c}{d}$, their sum $\frac{a}{b} + \frac{c}{d}$ and product $\frac{a}{b}$ $x + \frac{c}{d}$ is also rational.

For example:

a.
$$\frac{3}{4} + \frac{2}{4} = \left(\frac{3+2}{4}\right) = \frac{5}{4}$$

b.
$$\frac{3}{4} \bullet \frac{2}{4} = \frac{6}{16} \text{ or } \frac{3}{8}$$

2. COMMUTATIVE PROPERTY: For any two defined rational numbers $\frac{a}{b}$ and $\frac{c}{d}$,

i.
$$\frac{a}{b} + \frac{c}{d} = \frac{b}{d} + \frac{a}{b}$$

ii.
$$\frac{a}{b} \bullet \frac{c}{d} = \frac{c}{d}$$
 and $\frac{a}{b}$

For example:

a.
$$\frac{2}{7} + \frac{1}{3} = \frac{1}{3} + \frac{1}{7}$$

b.
$$\frac{6}{7} \bullet \frac{2}{3} = \frac{3}{3} \bullet \frac{6}{7}$$

3. ASSOCIATIVE PROPERTY: For any three defined rational numbers $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$

i.
$$\frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f}$$

ii.
$$\frac{a}{b} \bullet \left(\frac{c}{d} \bullet \frac{e}{f}\right) = \left(\frac{a}{b} \bullet \frac{c}{d}\right) \bullet \frac{e}{f}$$

For example:

a.
$$\frac{3}{5} + \left(\frac{2}{3} + \frac{1}{4}\right) = \left(\frac{3}{5} + \frac{2}{3}\right) + \frac{1}{4}$$

b.
$$\frac{1}{4} \bullet \left(\frac{3}{4} \bullet \frac{2}{3}\right) = \left(\frac{1}{4} \bullet \frac{3}{4}\right) \bullet \frac{2}{3}$$

4. DISTRIBUTIVE PROPERTY of multiplication over addition for rational numbers.

If $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$ are any defined rational numbers, then $\frac{a}{b} \bullet \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} \bullet \frac{c}{d}\right) + \left(\frac{a}{b} \bullet \frac{e}{f}\right)$

For example:
$$\frac{3}{7} \bullet \left(\frac{2}{3} + \frac{7}{8}\right) = \left(\frac{3}{7} \bullet \frac{2}{3}\right) + \left(\frac{3}{7} \bullet \frac{7}{8}\right)$$

5. DISTRIBUTIVE PROPERTY of multiplication over subtraction for rational numbers.

If $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$ are any defined rational numbers, then $\frac{a}{b} \bullet \left(\frac{c}{d} - \frac{e}{f}\right) = \left(\frac{a}{b} \bullet \frac{c}{d}\right) - \left(\frac{a}{b} \bullet \frac{e}{f}\right)$

For example:
$$\frac{3}{10} \bullet \left(\frac{2}{3} - \frac{2}{8}\right) = \left(\frac{3}{10} \bullet \frac{2}{3}\right) - \left(\frac{3}{10} \bullet \frac{2}{8}\right)$$

6. IDENTITY PROPERTY

Addition: Adding 0 to a number will not change the identity or value of that number.

$$\frac{a}{b} + 0 = \frac{a}{b}$$

For example: $\frac{1}{2} + 0 = \frac{1}{2}$

Multiplication: Multiplying a number by 1 will not change the identity or value of that number.

$$\frac{a}{b} \bullet 1 = \frac{a}{b}$$

For example: $\frac{3}{5} \bullet 1 = \frac{3}{5}$

7. ZERO PROPERTY OF MULTIPLICATION: Any number multiplied by zero

equals 0, i. e.
$$\frac{a}{b} \bullet 0 = 0$$

For example: $\frac{2}{7} \bullet 0 = 0$

II. Question to Ponder (Post-Activity Discussion)

NOTE TO THE TEACHER

Answer each question in the opening Activity thoroughly and discuss the concepts clearly. Allow students to express their ideas, their doubts and their questions. At this stage, they should really be able to verbalize what they understand or do not understand so that can properly address any misconceptions they have. Give additional examples, if necessary.

Let us answer the questions posed in the opening activity.

- 1. What is the missing number in item1? » $\frac{2}{14}$
- 2. How do you compare the answers in items 1 and 2? » The answer is the same, the order of the numbers is not important.
- 3. What about item 3? What is the missing number? » The missing number is 0. When you multiply a number with zero the product is zero.
- 4. In item 4, what number did you multiply with 1 to get $\frac{3}{5}$? $\Rightarrow \frac{3}{5}$, When you multiply a number by one the answer is the same.
- 5. What number should be added to $\frac{2}{3}$ in item 5 to get the same number? » 0, When you add zero to any number, the value of the number does not change.
- 6. What do you think is the missing number in items 6 and 7?» $\frac{13}{12}$
- 7. What can you say about the grouping in items 6 and 7? » The groupings are different but they do not affect the sum.
- 8. What do you think are the answers in items 8 and 9? » The answer is the same in both items, $\frac{3}{20}$.
- 9. What operation did you apply in item 10? » The Distributive Property of Multiplication over Addition

III. Exercises:

Do the following exercises. Write your answer in the spaces provided. A. State the property that justifies each of the following statements.

1.
$$\frac{2}{3} + \frac{5}{8} = \frac{5}{8} + \frac{2}{3}$$
Commutative Property

2. 1 x
$$\frac{9}{35} = \frac{9}{35}$$

2. 1 x $\frac{9}{35} = \frac{9}{35}$ Identity Property for Multiplication

3.
$$\frac{4}{5} \bullet \left(\frac{3}{4} + \frac{2}{3}\right) = \left(\frac{4}{5} \bullet \frac{3}{4}\right) + \left(\frac{4}{5} \bullet \frac{2}{3}\right)$$
Distributive Property of Multiplication over Addition

4.
$$\frac{3}{5} + \left(\frac{1}{2} + \frac{1}{4}\right) = \left(\frac{3}{5} + \frac{1}{2}\right) + \frac{1}{4}$$
Associative Property

5.
$$\left(\frac{2}{7} + \frac{1}{5} + \frac{2}{3}\right) \bullet 1 = \left(\frac{2}{7} + \frac{1}{5} + \frac{2}{3}\right)$$

Identity Property for Multiplication

6.
$$\left(\frac{3}{4} + 0\right) = \frac{3}{4}$$

6. $\left(\frac{3}{4} + 0\right) = \frac{3}{4}$ Identity Property for Addition

7.
$$\frac{1}{2} + \frac{5}{6} = \frac{4}{3}$$
 Closure Property

8.
$$\frac{3}{8} \bullet \frac{1}{4} \bullet \frac{2}{3} \bullet \frac{1}{2} = \frac{3}{8} \bullet \frac{2}{3} \bullet \frac{1}{4} \bullet \frac{1}{2}$$
Commutative Property

9.
$$\frac{1}{4} \bullet \left(\frac{5}{7} - \frac{2}{3}\right) = \left(\frac{1}{4} \bullet \frac{5}{7}\right) - \left(\frac{1}{4} \bullet \frac{2}{3}\right)$$

Distributive Property of Multiplication over Subtraction

10.
$$\left(\frac{2}{15} \bullet \frac{5}{7}\right) \bullet 0 = 0$$

Zero Property for Multiplication

B. Find the value of N in each expression

1. N +
$$\frac{1}{45} = \frac{1}{45}$$

$$N = 0$$

2.
$$\left(\frac{1}{4} \bullet N\right) \bullet \frac{2}{3} = \frac{1}{4} \bullet \left(\frac{6}{7} \bullet \frac{2}{3}\right)$$
 $N = \frac{6}{7}$

$$N = \frac{6}{7}$$

3.
$$\left(\frac{2}{15} + \frac{12}{30}\right) + \frac{1}{5} = \frac{2}{15} + \left(N + \frac{1}{5}\right)$$
 N = $\frac{12}{30}$

$$N = \frac{12}{30}$$

4.
$$0 + N = \frac{5}{18}$$
 $N = \frac{5}{18}$

6. N •
$$\left(\frac{6}{14} + \frac{2}{7}\right) = \left(\frac{1}{6} \cdot \frac{6}{14}\right) + \left(\frac{1}{6} \cdot \frac{2}{7}\right)$$
 N = $\frac{1}{6}$

7.
$$\frac{8}{23} \bullet 1 = N$$
 $N = \frac{8}{23}$

8.
$$\frac{2}{9} + \frac{2}{3} = N$$
 N = $\frac{8}{9}$

NOTE TO THE TEACHER

You might want to add more exercises. When you are sure that your students have mastered the properties, do not forget to end your lesson with a summary.

Summary

This lesson is about the properties of operations on rational numbers. The properties are useful because they simplify computations of rational numbers. These properties are true under the operations addition and multiplication. Note that for the Distributive Property of Multiplication over Subtraction, subtraction is considered part of addition. Think of subtraction as the addition of a negative rational number.

Lesson 10: Principal Roots and Irrational Numbers

Time: 2 hours

Prerequisite Concepts: Set of rational numbers

Objectives:

In this lesson, you are expected to:

- 1. describe and define irrational numbers:
- 2. describe principal roots and tell whether they are rational or irrational:
- 3. determine between what two integers the square root of a number is;
- 4. estimate the square root of a number to the nearest tenth;
- 5. illustrate and graph irrational numbers (square roots) on a number line with and without appropriate technology.

NOTE TO THE TEACHER

This is the first time that students will learn about irrational numbers. Irrational numbers are simply numbers that are not rational. However, they are not easy to determine, hence we limit our discussions to principal *nth* roots, particularly square roots. A lesson on irrational numbers is important because these numbers are often encountered. While the activities are meant to introduce these numbers in a non-threatening way, try not to deviate from the formal discussion on principal *nth* roots. The definitions are precise so be careful not to overextend or over generalize.

Lesson Proper:

I. Activities

A. Take a look at the unusual wristwatch and answer the questions below.

- 1. Can you tell the time?
- 2. What time is shown in the wristwatch?
- 3. What do you get when you take the $\sqrt{1}$? $\sqrt{4}$? $\sqrt{9}$? $\sqrt{16}$?
- 4. How will you describe the result?
- 5. Can you take the exact value of $\sqrt{130}$?
- 6. What value could you get?

NOTE TO THE TEACHER

In this part of the lesson, the square root of a number is used to introduce a new set of numbers called the irrational numbers. Take note of the two ways by which irrational numbers are described and defined.

Taking the square root of a number is like doing the reverse operation of squaring a number. For example, both 7 and -7 are square roots of 49 since $7^2 = 49$ and $(-7)^2 = 49$. Integers such as 1, 4, 9, 16, 25 and 36 are called perfect squares.

Rational numbers such as 0.16, $\frac{4}{100}$ and 4.84 are also, perfect squares. Perfect squares are numbers that have rational numbers as square roots. The square roots

of perfect squares are rational numbers, while the square roots of numbers that are not perfect squares are *irrational numbers*.

Any number that cannot be expressed as a quotient of two integers is an irrational number. The numbers $\sqrt{2}$, π , and the special number e are all irrational numbers. Decimal numbers that are non-repeating and non-terminating are irrational numbers.

NOTE TO THE TEACHER

It does not hurt for students at this level to use a scientific calculator in obtaining principal roots of numbers. With the calculator, it becomes easier to identify irrational numbers.

B. Activity Use the $\sqrt[q]{}$ button of a scientific calculator to find the following values: 1. $\sqrt[q]{64}$ 2. $\sqrt[q]{-16}$ 3. $\sqrt[q]{90}$ 4. $\sqrt[q]{-3125}$ 5. $\sqrt{24}$

II. Questions to Ponder (Post-Activity Discussions)

Let us answer the questions in the opening activity.

- 1. Can you tell the time? Yes
- 2. What time is it in the wristwatch? 10:07
- 3. What do you get when you take the $\sqrt{1}$? $\sqrt{4}$? $\sqrt{9}$? $\sqrt{16}$? 1, 2, 3, 4
- 4. How will you describe the result? They are all positive integers.
- 5. Can you take the exact value of $\sqrt{130}$? *No.*
- 6. What value do you get? Since the number is not a perfect square you could estimate the value to be between $\sqrt{121}$ and $\sqrt{144}$, which is about 11.4.

Let us give the values asked for in Activity B. Using a scientific calculator, you probably obtained the following:

- 1. $\sqrt[6]{64} = 2$
- 2. $\sqrt[4]{-16}$ Math Error, which means not defined
- 3. $\sqrt[3]{90} = 4.481404747$, which could mean non-terminating and non-repeating since the calculator screen has a limited size
- 4. $\sqrt[5]{-3125} = -5$
- 5. $\sqrt{24} = 4.898979486$, which could mean non-terminating and non-repeating since the calculator screen has a limited size

NOTE TO THE TEACHER

The transition from the concept of two square roots of a positive number to that of the principal *nth* root has always been a difficult one for students. The important and precisely stated concepts are in bold so that students pay attention to them. Solved problems that are meant to illustrate certain procedures and techniques in determining whether a principal root is rational or irrational, finding two consecutive integers between which the

irrational number is found, estimating the value of irrational square roots to the nearest tenth, and plotting an irrational square root on a number line.

On Principal nth Roots

Any number, say a, whose n^{th} power (n, a positive integer), is b is called the n^{th} root of b. Consider the following: $(-7)^2 = 49$, $2^4 = 16$ and $(-10)^3 = -1000$. This means that -7 is a 2nd or square root of 49. 2 is a 4th root of 16 and -10 is a 3rd or cube root of -1000.

However, we are not simply interested in any n^{th} root of a number; we are more concerned about the principal n^{th} root of a number. The principal n^{th} root of a positive number is the positive n^{th} root. The principal n^{th} root of a negative number is the negative n^{th} root if n is odd. If n is even and the number is negative, the principal n^{th} root is not defined. The notation for the principal n^{th} root of a number b is $\sqrt[n]{b}$. In this expression, n is the index and b is the radicand. The n^{th} roots are also called radicals.

Classifying Principal nth Roots as Rational or Irrational Numbers To determine whether a principal root is a rational or irrational number, determine if the radicand is a perfect n^{th} power of a number. If it is, then the root is rational. Otherwise, it is irrational.

Problem 1. Tell whether the principal root of each number is rational or irrational.

(a)
$$\sqrt[3]{225}$$

(b)
$$\sqrt{0.04}$$

(c)
$$\sqrt[5]{-111}$$

(a)
$$\sqrt[3]{225}$$
 (b) $\sqrt{0.04}$ (c) $\sqrt[5]{-111}$ (d) $\sqrt{10000}$

(e)
$$\sqrt[4]{625}$$

Answers:

- a) $\sqrt[3]{225}$ is irrational
- (b) $\sqrt{0.04} = 0.2$ is rational
- (c) $\sqrt[4]{-111}$ is irrational
- (d) $\sqrt{10000} = 100$ is rational
- (e) $\sqrt[4]{625} = 5$ is rational

If a principal root is irrational, the best you can do for now is to give an estimate of its value. Estimating is very important for all principal roots that are not roots of perfect *n*th powers.

Problem 2. The principal roots below are between two integers. Find the two closest such integers.

(a)
$$\sqrt{19}$$

(b)
$$\sqrt[3]{101}$$

(c)
$$\sqrt{300}$$

Solution:

(a)
$$\sqrt{19}$$

16 is a perfect integer square and 4 is its principal square root. 25 is the next perfect integer square and 5 is its principal square root. Therefore, $\sqrt{19}$ is between 4 and 5.

(b) $\sqrt[3]{101}$

64 is a perfect integer cube and 4 is its principal cube root. 125 is the next perfect integer cube and 5 is its principal cube root. Therefore, $\sqrt[3]{101}$ is between 4 and 5.

(c) $\sqrt{300}$

289 is a perfect integer square and 17 is its principal square root. 324 is the next perfect integer square and 18 is its principal square root. Therefore, $\sqrt{300}$ is between 17 and 18.

Problem 3. Estimate each square root to the nearest *tenth*.

(a)
$$\sqrt{40}$$

(b)
$$\sqrt{12}$$

(c)
$$\sqrt{175}$$

Solution:

(a) $\sqrt{40}$

The principal root $\sqrt{40}$ is between 6 and 7, principal roots of the two perfect squares 36 and 49, respectively. Now, take the square of 6.5, midway between 6 and 7. Computing, $(6.5)^2 = 42.25$. Since 42.25 > 40 then $\sqrt{40}$ is closer to 6 than to 7. Now, compute for the squares of numbers between 6 and 6.5: $(6.1)^2 = 37.21$, $(6.2)^2 = 38.44$, $(6.3)^2 = 39.69$, and $(6.4)^2 = 40.96$. Since 40 is close to 39.69 than to 40.96, $\sqrt{40}$ is approximately 6.3.

(b) $\sqrt{12}$

The principal root $\sqrt{12}$ is between 3 and 4, principal roots of the two perfect squares 9 and 16, respectively. Now take the square of 3.5, midway between 3 and 4. Computing $(3.5)^2 = 12.25$. Since 12.25 > 12 then $\sqrt{12}$ is closer to 3 than to 4. Compute for the squares of numbers between 3 and 3.5: $(3.1)^2 = 9.61$, $(3.2)^2 = 10.24$, $(3.3)^2 = 10.89$, and $(3.4)^2 = 11.56$. Since 12 is closer to 12.25 than to 11.56, $\sqrt{12}$ is approximately 3.5.

(c) $\sqrt{175}$

The principal root $\sqrt{175}$ is between 13 and 14, principal roots of the two perfect squares 169 and 196. The square of 13.5 is 182.25, which is greater than 175. Therefore, $\sqrt{175}$ is closer to 13 than to 14. Now: $(13.1)^2=171.61$, $(13.2)^2=174.24$, $(13.3)^2=176.89$. Since 175 is closer to 174.24 than to 176.89 then, $\sqrt{175}$ is approximately 13.2.

Problem 4. Locate and plot each square root on a number line.

(a)
$$\sqrt{3}$$

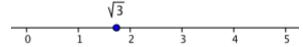
(b)
$$\sqrt{21}$$

(c)
$$\sqrt{87}$$

Solution: You may use a program like Geogebra to plot the square roots on a number line.

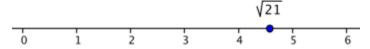
(a) $\sqrt{3}$

This number is between 1 and 2, principal roots of 1 and 4. Since 3 is closer to 4 than to 1, $\sqrt{3}$ is closer to 2. Plot $\sqrt{3}$ closer to 2.



(b) $\sqrt{21}$

This number is between 4 and 5, principal roots of 16 and 25. Since 21 is closer to 25 than to 16, $\sqrt{21}$ is closer to 5 than to 4. Plot $\sqrt{21}$ closer to 5.



(c)
$$\sqrt{87}$$

This number is between 9 and 10, principal roots of 81 and 100. Since 87 is closer to 81, then $\sqrt{87}$ is closer to 9 than to 10. Plot $\sqrt{87}$ closer to 9.



III. Exercises

A. Tell whether the principal roots of each number is rational or irrational.

- 1. $\sqrt{400}$
- 2. $\sqrt{64}$
- 3. $\sqrt{0.01}$
- 4. $\sqrt{26}$
- 5. $\sqrt{\frac{1}{49}}$

- 6. $\sqrt{13,689}$
- 7. $\sqrt{1000}$
- 8. $\sqrt{2.25}$
- 9. $\sqrt{39}$
- 10. $\sqrt{12.1}$

Answers:

- 1. rational
- 2. rational
- 3. rational
- 4. irrational
- 5. rational
- 6. rational
- 7. irrational
- 8. rational
- 9. irrational
- 40 to the state of
- 10.irrational

B. Between which two consecutive integers does the square root lie?

- 1. $\sqrt{77}$
- 2. $\sqrt{700}$
- 3. $\sqrt{243}$
- 4. $\sqrt{444}$
- 5. $\sqrt{48}$

- 6. $\sqrt{90}$
- 7. $\sqrt{2.045}$
- 8. $\sqrt{903}$
- 9. $\sqrt{1899}$
- 10. $\sqrt{100\ 000}$

Answers:

1. 8 and 9	6. 9 and 10
2. 26 and 27	7. 45 and 46
3. 15 and 16	8. 30 and 31
4. 21 and 22	9. 43 and 44
5. 6 and 7	10, 316 and 317

C. Estimate each square root to the nearest *tenth* and plot on a number line.

1. $\sqrt{50}$	6. $\sqrt{250}$
2. $\sqrt{72}$	7 . √5
3. $\sqrt{15}$	8. √ <u>85</u>
4. $\sqrt{54}$	9. √ <u>38</u>
5. $\sqrt{136}$	10. $\sqrt{101}$

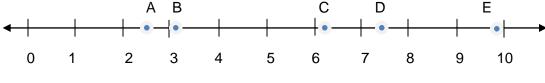
Answers:

1. 7.1	5. 11.7	9. 6.2
2. 8.5	6. 15.8	10. 10.0
3. 3.9	7. 2.2	
4. 7.3	8. 9.2	
T. / L.	(). /.=	

NOTE TO THE TEACHER

You might think that plotting the irrational square roots on a number line is easy. Do not assume that all students understand what to do. Give them additional exercises for practice. Exercise D can be varied to include 2 or 3 irrational numbers plotted and then asking students to identify the correct graph for the 2 or 3 numbers.

D. Which point on the number line below corresponds to which square root?



- 1. √57 D
- 2. $\sqrt{6}$ **A**
- 3. √99 **E**
- 4. √38 **C**
- 5. $\sqrt{11}$ **B**

Summary

In this lesson, you learned about irrational numbers and principal n^{th} roots, particularly square roots of numbers. You learned to find two consecutive integers between which an irrational square root lies. You also learned how to estimate the square roots of numbers to the nearest tenth and how to plot the estimated square roots on a number line.

Lesson 11: The Absolute Value of a Number

Prerequisite Concepts: Set of real numbers

Objectives:

In this lesson, you are expected to describe and illustrate

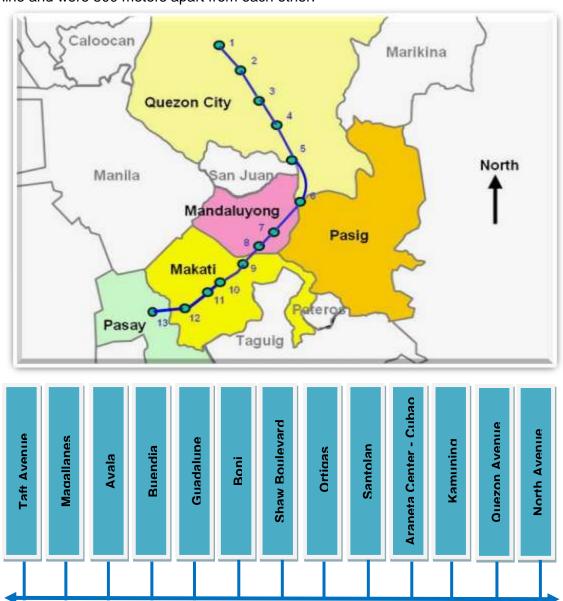
- a. the absolute value of a number on a number line.
- b. the distance of the number from 0.

Lesson Proper:

I. Activity 1: THE METRO MANILA RAIL TRANSIT (MRT) TOUR

Suppose the MRT stations from Pasay City to Quezon City were on a straight line and were 500 meters apart from each other:

Time: 1.5 hours

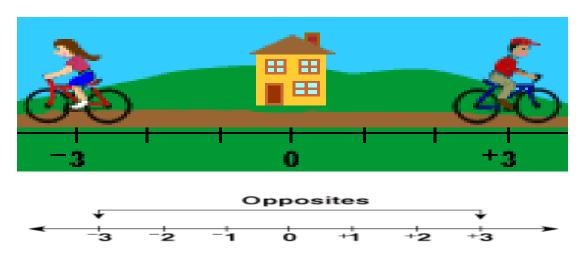


- 1. How far would the North Avenue station be from Taft Avenue? 6 000 meters or 6 kilometers
- 2. What if Elaine took the MRT from North Avenue and got off at the last station? How far would she have travelled? 6 000 meters or 6 kilometers
- 3. Suppose both Archie and Angelica rode the MRT at Shaw Boulevard, and the former got off in Ayala, while the latter got off in Kamuning. How far would each have travelled from the starting point to their destinations? **Archie travelled 2 000 meters from Shaw Boulevard to Ayala. Angelica travelled 2 000 meters from Shaw Boulevard to Kamuning.**
- 4. What can you say about the directions and the distances travelled by Archie and Angelica? They went in opposite directions from the same starting point, but travelled the same distance.

NOTE TO THE TEACHER:

This lesson focuses on the relationship between absolute value and distance. Point out to students that the absolute value of a number as a measure of distance will always be positive or zero since it is simply a magnitude, a measure. Students should realize the importance of the absolute value of a number in contexts such as transportation, weather, statistics, and others.

Activity 2: THE BICYCLE JOY RIDE OF ARCHIE AND ANGELICA



Problem: Archie and Angelica were at Aloys' house. Angelica rode her bicycle 3 miles west of Aloys' house, and Archie rode his bicycle 3 miles east of Aloys' house. Who travelled a greater distance from Aloys' house – Archie or Angelica?

Questions To Ponder:

1. What subsets of real numbers are used in the problem? Represent the trip of Archie and Angelica to the house of Aloys using a number line.

- 2. What are opposite numbers on the number line? Give examples and show on the number line.
- 3. What does it mean for the same distance travelled but in opposite directions? How would you interpret using the numbers -3 and +3?
- 4. What can you say about the absolute value of opposite numbers say -5 and +5?
- 5. How can we represent the absolute value of a number? What notation can we use?

NOTE TO THE TEACHER:

Below are important terminologies, notations and symbols that your students must learn and remember. From here on, be consistent in using these notations so as not to create confusion on the part of the students. Take note of the subtle difference in using the the absolute value bars from the parentheses.

Important Terms to Remember

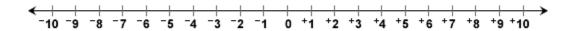
The following are terms that you must remember from this point on.

- 1. **Absolute Value** of a number is the distance between that number and zero on the number line.
- 2. **Number Line** –is best described as a straight line which is extended in both directions as illustrated by arrowheads. A number line consists of three elements:
 - a. set of positive numbers, and is located to the right of zero;
 - b. set of negative numbers, and is located to the left of zero; and
 - c. Zero.

Notations and Symbols

The absolute value of a number is denoted by two bars | |.

Let's look at the number line:



The absolute value of a number, denoted by "| |" is the distance of the number from zero. This is why the absolute value of a number is never negative. In thinking about the absolute value of a number, one only asks "how far?" not "in which direction?" Therefore, the absolute value of 3 and of -3 is the same, which is 3 because both numbers have the same distance from zero.



Warning: The absolute-value notation is *bars*, not parentheses or brackets. Use the proper notation; the other notations do *not* mean the same thing.

It is important to note that the absolute value bars do NOT work in the same way as do parentheses. Whereas -(-3) = +3, this is NOT how it works for absolute value:

Problem: Simplify – | –3 |.

Solution: Given - | -3 |, first find the absolute value of -3.

$$- | -3 | = - (3)$$

Now take the negative of 3. Thus, :

$$- | -3 | = - (3) = -3$$

This illustrates that if you take the negative of the absolute value of a number, you will get a negative number for your answer.

II. Questons to Ponder(Post-Activity Discussion)

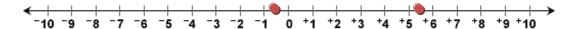
NOTE TO THE TEACHER

It is important for you to examine and discuss the responses by your students to the questions posed in Activity 2. Pay particular attention to how or what they say and write. Always refer to practical examples so they can understand more. Encourage brainstorming, dialogues and arguments in class. After the exchanges, see to it that all questions are answered and resolved.

Let us answer the questions posed in Activity 2.

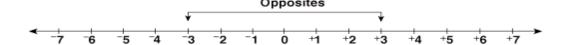
1. What subsets of real numbers are used in the problem? Represent the trip of Archie and Angelica to the house of Aloys using a number line.

The problem uses <u>integers</u>. Travelling 3 miles west can be represented by -3 (pronounced negative 3). Travelling 3 miles east can be represented by +3 (pronounced positive 3). Aloys' house can be represented by the integer 0.



2. What are opposite numbers on the number line? Give examples and show on the number line.

Two integers that are the same distance from zero in opposite directions are called **opposites**. The integers ⁺3 and ⁻3 are opposites since they are each 3 units from zero.



- 3. What does it mean for the same distance travelled but in opposite directions? How would you interpret using the numbers -3 and +3? The absolute value of a number is its distance from zero on the number line. The absolute value of +3 is 3, and the absolute value of -3 is 3.
- 4. What can you say about the absolute value of opposite numbers say -5 and +5?

Opposite numbers have the same absolute values.

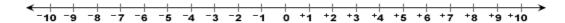
5. How can we represent the absolute value of a number? What notation can we use?

The symbol | | is used for the absolute value of a number.

III. Exercises

Carry out the following tasks. Write your answers on the spaces provided for each number.

1. Find the absolute value of ⁺3, ⁻3, ⁺7, ⁻5, ⁺9, ⁻8, ⁺4, ⁻4. You may refer to the number line below. What should we remember when we talk about the absolute value of a number?



Solution: $|^{+}3| = 3$ $|^{+}9| = 9$

|⁻3| = 3 |⁻8| = 8

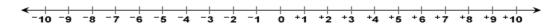
 $|^{+}7| = 7$ $|^{+}4| = 4$

 $|^{-}5| = 5$ $|^{-}4| = 4$

Remember that when we find the absolute value of a number, we are trying to find

its distance from 0 on the number line. Opposite numbers have the same absolute value since they both have the same distance from 0. Also, you will notice that taking the absolute value of a number automatically means taking the positive value of that number.

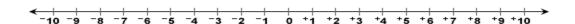
2. Find the absolute value of: +11, -9, +14, -10, +17, -19, +20, -20. You may extend the number line below to help you solve this problem.



Solution:
$$|^{+}11| = 11$$
 $|^{+}17| = 17$

$$|^{-}9| = 9$$
 $|^{-}19| = 19$

3. Use the number line below to find the value of N: |N| = 5.1



Solution: This problem asks us to find all numbers that are a distance of 5.1 units from zero on the number line. We let N represent all integers that satisfy this condition.

The number ⁺5.1 is 5.1 units from zero on the number line, and the number ⁻5.1 is also 5.1 units from zero on the number line. Thus both ⁺5.1 and ⁻5.1 satisfy the given condition.

4. When is the absolute value of a number equal to itself?

Solution: When the value of the number is positive or zero.

5. Explain why the absolute value of a number is never negative. Give an example that will support your answer.

Solution: Let |N| = -4. Think of a number that when you get the absolute value will give you a negative answer. There will be no solution since the distance of any number from 0 cannot be a negative quantity.

Enrichment Exercises:

A. Simplify the following.

3.
$$\left| -\frac{2}{9} \right| = \frac{2}{9}$$

5.
$$|-2\sqrt{2}| - |-3\sqrt{2}| = -\sqrt{2}$$

B. List at least two integers that can replace N such that.

NOTE TO THE TEACHER

There are several possible answers in each item. Be alert to the different answers of your students.

- C. Answer the following.
 - 1. Insert the correct relation symbol(>, =, <): $\left| -7 \right| > \left| -4 \right|$.
 - 2. If |x-7| = 5, what are the possible values of x? $\{2, 12\}$
 - 3. If $|x| = \frac{2}{7}$, what are the possible values of x? $\{-\frac{2}{7}, \frac{2}{7}\}$
 - 4. Evaluate the expression, |x + y| |y x|, if x = 4 and y = 7. {8}
 - 5. A submarine navigates at a depth of 50 meters below sea level while exactly above it; an aircraft flies at an altitude of 185 meters. What is the distance between the two carriers? 235 meters

Summary:

In this lesson you learned about the absolute value of a number, that it is a distance from zero on the number line denoted by the notation |N|. This notation is used for the absolute value of an unknown number that satisfies a given condition. You also learned that a distance can never be a negative quantity, and absolute value pertains to the magnitude rather than the direction of a number.

GRADE 7 MATH TEACHING GUIDE

Lesson 12: SUBSETS OF REAL NUMBERS

Prerequisite Concepts: whole numbers and operations, set of integers, rational numbers, irrational numbers, sets and set operations, Venn diagrams **Objectives**

In this lesson, you are expected to:

- 2. Describe and illustrate the real number system.
- 3. Apply various procedures and manipulations on the different subsets of the set of real numbers.
 - a. Describe, represent and compare the different subsets of real number.
 - b. Find the union, intersection and complement of the set of real numbers and its subsets

NOTE TO THE TEACHER:

Many teachers claim that this lesson is quite simple because we use various kinds of numbers every day. Even the famous theorist of the Pythagorean Theorem, Pythagoras, once said that, "All things are number." Truly, numbers are everywhere! But do we really know our numbers? Sometimes a person exists in our midst but we do not even bother to ask the name or identity of that person. It is the same with numbers. Yes, we are surrounded by these boundless figures, but do we bother to know what they really are?

In Activity 1, try to stimulate the students' interest in the lesson by drawing out their thoughts. The objective of Activities 2 and 3 is for you to ascertain your students' understanding of the different names of sets of numbers.

Lesson Proper:

A.

Activity 1: Try to reflect on these . . .

It is difficult for us to realize that once upon a time there were no symbols or names for numbers. In the early days, primitive man showed how many animals he owned by placing an equal number of stones in a pile, or sticks in a row. Truly our number system underwent the process of development for hundreds of centuries.

Sharing Ideas! What do you think?

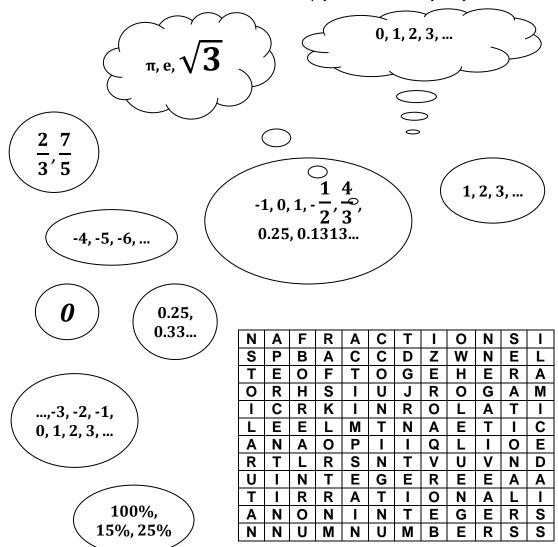
- 1. In what ways do you think did primitive man need to use numbers?
- 2. Why do you think he needed names or words to tell "how many"?
- 3. Was he forced to invent symbols to represent his number ideas?
- 4. Is necessity the root cause that led him to invent numbers, words and symbols?

NOTE TO THE TEACHER:

You need to facilitate the sharing of ideas leading to the discussion of possible answers to the questions. Encourage students to converse, to contribute, and to argue if necessary for better interaction.

Activity 2: LOOK AROUND!

Fifteen different words/partitions of numbers are hidden in this puzzle. How many can you find? Look up, down, across, backward, and diagonally. Figures that are scattered around that will serve as clues to help you locate the mystery words.



Answer the following questions:

- 1. How many words in the puzzle were familiar to you? **Expected Answers: Numbers, Fractions...**
- 2. Which among the word/s have you encountered? **Expected Answer:**Numbers...

 Define and sine executes **Expected Answer:** They are used to see

Define and give examples. **Expected Answer: They are used to count things.**

3. What word/s is/are still strange to you? Expected Answer: Irrational, ...

Activity 3: Determine what numbers/set of numbers will represent the following situations:

- 1. Finding out how many cows are in a barn **Counting Numbers**
- 2. Corresponds to no more apples inside the basket **Zero**
- 3. Describing the temperature in the North Pole Negative Number
- 4. Representing the amount of money each member gets when "P200 prize" is divided among 3 members <u>Fraction</u>, <u>Decimal</u>
- 5. Finding the ratio of the circumference to the diameter of a circle, denoted as π (read "pi") <u>Irrational Number</u>

NOTE TO THE TEACHER:

You need to follow up on the preliminary activity. Students will definitely give varied answers. Be prepared and keep an open mind. Consequently, the next activity below is essential. In this phase, the students should be encouraged to use their knowledge of the real number system.

The set of numbers is called **the real number system** that consists of different partitions/ **subsets** that can be represented graphically on a **number line**.

II. Questions to Ponder

Consider the activities done earlier and recall the different terms you encountered including the set of real numbers. Let us determine the various subsets. Let us go back to the first time we encountered the numbers...

Let's talk about the various subsets of real numbers.

Early Years...

1. What subset of real numbers do children learn at an early stage when they were just starting to talk? Give examples.

Expected Answer: Counting Numbers or Natural Numbers

One subset is the **counting (or natural) numbers**. This subset includes all the numbers we use to count starting with "1" and so on. The subset would look like this: {1, 2, 3, 4, 5...}

In School at an Early Phase...

2. What do you call the subset of real numbers that includes zero (the number that represents nothing) and is combined with the subset of real numbers learned in the early years? Give examples.

Expected Answer: Whole Numbers

Another subset is the **whole numbers**. This subset is exactly like the subset of counting numbers, with the addition of one extra number. This extra number is "0". The subset would look like this:{0, 1, 2, 3, 4...}

In School at Middle Phase...

3. What do you call the subset of real numbers that includes negative numbers (that came from the concept of "opposites" and specifically used in describing debt or below zero temperature) and is united with the whole numbers? Give examples.

Expected Answer: Integers

A third subset is the <u>integers</u>. This subset includes all the whole numbers and their "opposites". The subset would look like this: {... -4, -3, -2, -1, 0, 1, 2, 3, 4...}

Still in School at Middle Period...

4. What do you call the subset of real numbers that includes integers and non-integers and are useful in representing concepts like "half a gallon of milk"? Give examples.

Expected Answer: Rational Numbers

The next subset is the <u>rational numbers</u>. This subset includes all numbers that "come to an end" or numbers that repeat and have a pattern. Examples of rational numbers are: 5.34, 0.131313..., $\frac{6}{7}$, $\frac{2}{3}$, 9

5. What do you call the subset of real numbers that is not a rational number but are physically represented like "the diagonal of a square"?

Expected Answer: Irrational Numbers

Lastly we have the set of <u>irrational numbers</u>. This subset includes numbers that cannot be exactly written as a decimal or fraction. Irrational numbers cannot be expressed as a ratio of two integers. Examples of irrational numbers are:

$$\sqrt{2}$$
, $\sqrt[3]{101}$, and π

NOTE TO THE TEACHER:

Below are important terms that must be remembered by students from here on. You, on the other hand, must be consistent in the use of these terminologies so as not to confuse your students. Give adequate examples and non-examples to further support the learning process of the students. As you discuss these terms, use terms related to sets, such as the union and intersection of sets.

Important Terms to Remember

The following are terms that you must remember from this point on.

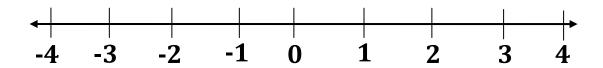
- **1. Natural/Counting Numbers** are the numbers we use in counting things, that is {1, 2, 3, 4, . . . }. The three dots, called ellipses, indicate that the pattern continues indefinitely.
- **2. Whole Numbers** are numbers consisting of the set of natural or counting numbers and zero.
- **3. Integers** are the result of the union of the set of whole numbers and the negative of counting numbers.
- **4.** Rational Numbers are numbers that can be expressed as a quotient $\frac{a}{b}$ of two integers. The integer \boldsymbol{a} is the numerator, while the integer \boldsymbol{b} , which cannot be 0 is the denominator. This set includes fractions and some decimal numbers.
- 5. Irrational Numbers are numbers that cannot be expressed as a quotient $\frac{a}{b}$ of two integers. Every irrational number may be represented by a decimal that neither repeats nor terminates.
- **6. Real Numbers** are any of the numbers from the preceding subsets. They can be found on the real number line. The union of rational numbers and irrational numbers is the set of real numbers.
- **7. Number Line** is a straight line extended on both directions as illustrated by arrowheads and used to represent the set of real numbers. On the real number line, there is a point for every real number and a real number for every point.

III. Exercises

1. Locate the following numbers on the number line by naming the correct point.

$$-2.66...$$
 , $-1\frac{1}{2}$, -0.25 , $\frac{3}{4}$, $\sqrt{2}$, $\sqrt[3]{11}$

Answer:



2. Determine the subset of real numbers to which each number belongs. Use a tick mark $(\sqrt{})$ to answer.

Answer:

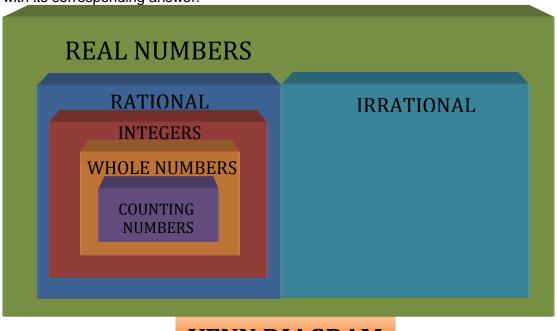
Number	Whole Number	Integer	Rational	Irrational
186		$\sqrt{}$	V	
2. 34.74			V	
3. $\frac{4}{7}$			$\sqrt{}$	
4. √64	V	V	V	
5. $\sqrt{11}$				
60.125			V	
7. -√81		$\sqrt{}$	$\sqrt{}$	
8. e				$\sqrt{}$
945.37				
101.252525			V	

B. Points to Contemplate

It is interesting to note that the set of rational numbers and the set of irrational numbers are disjoint sets; that is, their intersection is empty. The union of the set of rational numbers and the set of irrational numbers yields a set of numbers that is called the set of real numbers.

Exercise:

a. Based on the stated information, show the relationships among *natural or counting numbers, whole numbers, integers, rational numbers, irrational numbers* and *real numbers* using the Venn diagram below. Fill each broken line with its corresponding answer.



VENN DIAGRAM

b. Carry out the task being asked by writing your response on the space provided for each number.

1. Are all real numbers rational numbers? Prove your answer.

Expected Answer: No, because the set of real numbers is composed of two subsets namely, rational numbers and irrational numbers. Therefore, not all real numbers are rational numbers alone.

2. Are all rational numbers whole numbers? Prove your answer.

Expected Answer: No, because rational numbers is composed of two subsets; namely, integers, where whole numbers are included, and non-integers. Therefore, it is impossible that all rational numbers are whole numbers alone.

3. Are $-\frac{1}{4}$ and $-\frac{2}{5}$ negative integers? Prove your answer.

Expected Answer: They are negative numbers but not integers. An integer is composed of positive and negative whole numbers and not a signed fraction.

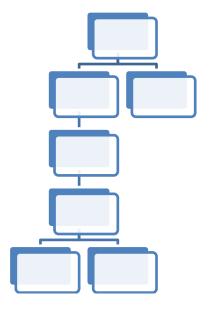
4. How is a rational number different from an irrational number?

Expected Answer: Rational Numbers can be expressed as a quotient of two integers with a nonzero denominator while Irrational numbers cannot be written in this form.

5. How do natural numbers differ from whole numbers?

Expected Answer: Natural numbers are also known as counting numbers that will always start with 1. Once you include 0 to the set of natural numbers that becomes the set of whole numbers.

c. Complete the details in the Hierarchy Chart of the Set of Real Numbers.



THE REAL NUMBER SYSTEM

NOTE TO THE TEACHER:

Make sure you summarize this lesson because there are many terms and concepts to remember.

Summary

In this lesson, you learned different subsets of real numbers that enable you to name numbers in different ways. You also learned to determine the hierarchy and relationship of one subset to another that leads to the composition of the real number system using the Venn diagram and hierarchy chart. You also learned that necessity led man to invent numbers, words, and symbols.

Lesson 13: Significant Digits and the Scientific Notation

OPTIONAL

Prerequisite Concepts: Rational numbers and powers of 10

Objectives:

In this lesson, you are expected to:

- 1. determine the significant digits in a given situation.
- 2. write very large and very small numbers in scientific notation

NOTE TO THE TEACHER

This lesson may not be familiar to your students. The primary motivation for including this lesson is that they need these skills in their science course/s. You the teacher should make sure that you are clear about the many rules they need to learn.

Lesson Proper:

I. A. Activity

The following is a list of numbers. The number of significant digits in each number is written inside the parentheses after the number.

234 (3)	0.0122 (3)
745.1 (4)	0.00430 (3)
6007 (4)	0.0003668 (4)
1.3 X 10 ² (2)	10000 (1)
7.50 X 10 ⁻⁷ (3)	1000. (4)
0.012300 (5)	2.222 X 10 ⁻³ (4)
100.0 (4)	$8.004 \times 10^5 (4)$
100 (1)	6120. (4)
7890 (3)	120.0 (4)
4970.00 (6)	530 (2)

Describe what digits are not significant.

NOTE TO THE TEACHER

If this is the first time that your students will encounter this lesson, you have to be patient in explaining and drilling them on the rules. Give many of examples and exercises.

Important Terms to Remember

Significant digits are the digits in a number that express the precision of a measurement rather than its magnitude. The number of significant digits in a given

measurement depends on the number of significant digits in the given data. In calculations involving multiplication, division, trigonometric functions, for example, the number of significant digits in the final answer is equal to the least number of significant digits in any of the factors or data involved.

Rules for Determining Significant Digits

A. All digits that are not zeros are significant.

For example: 2781 has 4 significant digits 82.973 has 5 significant digits

- B. Zeros may or may not be significant. Furthermore,
 - 1. Zeros appearing between nonzero digits are significant.

For example: 20.1 has 3 significant digits 79002 has 5 significant digits

2. Zeros appearing in front of nonzero digits are not significant.

For example: 0.012 has 2 significant digits 0.0000009 has 1 significant digit

3. Zeros at the end of a number and to the right of a decimal are significant digits. Zeros between nonzero digits and significant zeros are also significant.

For example: 15.0 has 3 significant digits 25000.00 has 7 significant digits

4. Zeros at the end of a number but to the left of a decimal may or may not be significant. If such a zero has been measured or is the first estimated digit, it is significant. On the other hand, if the zero has not been measured or estimated but is just a place holder it is not significant. A decimal placed after the zeros indicates that they are significant

For example: 560000 has 2 significant digits 560000, has 6 significant digits

Significant Figures in Calculations

- 1. When multiplying or dividing measured quantities, round the answer to as many significant figures in the answer as there are in the measurement with the least number of significant figures.
- 2. When adding or subtracting measured quantities, round the answer to the same number of decimal places as there are in the measurement with the least number of decimal places.

For example:

a. $3.0 \times 20.536 = 61.608$

Answer: 61 since the least number of significant digits is 2, coming from 3.0

b. 3.0 + 20.536 = 23.536

Answer: 23.5 since the addend with the least number of decimal places is 3.0

II. Questions to Ponder (Post-Activity Discussion)

NOTE TO THE TEACHER

The difficult part is to arrive at a concise description of non-significant digits. Do not give up on this task. Students should be able to describe and define significant digits as well as non-significant digits.

Describe what digits are not significant. The digits that are not significant are the zeros before a non-zero digit and zeros at the end of numbers without the decimal point.

Problem 1. Four students weigh an item using different scales. These are the values they report:

- a. 30.04 g
- b. 30.0 g
- c. 0.3004 kg
- d. 30 g

How many significant digits are in each measurement?

Answer: 30.04 has 4 significant; 30.0 has 3 significant digits; 0.3004 has 4 significant digits; 30 has 1 significant digit

Problem 2. Three students measure volumes of water with three different devices. They report the following results:

Device	Volume
Large graduated cylinder	175 mL
Small graduated cylinder	39.7 mL
Calibrated buret	18.16 mL

If the students pour all of the water into a single container, what is the total volume of water in the container? How many digits should you keep in this answer?

Answer: The total volume is 232.86 mL. Based on the measures, the final answer should be 232.9 mL.

On the Scientific Notation

The speed of light is 300 000 000 m/sec, quite a large number. It is cumbersome to write this number in full. Another way to write it is 3.0×10^8 . How about a very small number like 0.000 000 089? Like with a very large number, a very small number may be written more efficiently, 0.000 000 089 may be written as 8.9×10^{-8} .

Writing a Number in Scientific Notation

1. Move the decimal point to the right or left until after the first significant digit, and copy the significant digits to the right of the first digit. If the number is a whole number and has no decimal point, place a decimal point after the first significant digit and copy the significant digits to its right.

For example, 300 000 000 has 1 significant digit, which is 3. Place a decimal point after 3.0

The first significant digit in 0.000 000 089 is 8 and so place a decimal point after 8, (8.9).

2. Multiply the adjusted number in step 1 by a power of 10, the exponent of which is the number of digits that the decimal point moved, positive if moved to the left and negative if moved to the right.

For example, 300 000 000 is written as 3.0 x 10⁸ because the decimal point was moved past 8 places.

0.0 000 089 is written as 8.9 x 10⁻⁸ because the decimal point was moved 8 places to the right past the first significant digit 8.

III. Exercises

A. Determine the number of significant digits in the following measurements. Rewrite the numbers with at least 5 digits in scientific notation.

1. 0.0000056 L 2. 4.003 kg 3. 350 m	6. 8207 mm 7. 0.83500 kg 8. 50.800 km	Answers: 1) 2; 2) 4; 3) 2; 4) 7; 5) 4; 6) 4;
4. 4113.000 cm 5. 700.0 mL	9. 0.0010003 m ³ 10. 8 000 L	7) 5; 8) 5; 9) 5; 10) 1

B. a. Round off the following quantities to the specified number of significant figures.

- 1. 5 487 129 m to three significant figures
- 2. 0.013 479 265 mL to six significant figures
- 3. 31 947.972 cm² to four significant figures
- 4. 192.6739 m² to five significant figures
- 5. 786.9164 cm to two significant figures

Answers: 1) 5 490 000 m; 2) 0.0134793 mL; 3) 31 950 cm²; 4) 192.67 m2; 5) 790 cm

b. Rewrite the answers in (a) using the scientific notation

Answers: 1) 5.49 x 10^6 ; 2) 1.34793 x 10^{-2} ; 3) 3.1950 x 10^4 ; 4) 1.9267 x 10^2 ; 5) 7.9 x 10^2

C. Write the answers to the correct number of significant figures

```
1. 4.5 \times 6.3 \div 7.22 = 3.9
2. 5.567 \times 3.0001 \div 3.45 = 4.84
3. (37 \times 43) \div (4.2 \times 6.0) = 63
4. (112 \times 20) \div (30 \times 63) = 1
5. 47.0 \div 2.2 = 21
```

D. Write the answers in the correct number of significant figures

- E. Answer the following.
 - 1. A runner runs the last 45m of a race in 6s. How many significant figures will the runner's speed have? **Answer: 2**
 - A year is 356.25 days, and a decade has exactly 10 years in it. How many significant figures should you use to express the number of days in two decades? **Answer: 1**
 - 3. Which of the following measurements was recorded to 3 significant digits: 50 mL, 56 mL, 56.0 mL or 56.00 mL? **Answer: 56.0 mL**
 - 4. A rectangle measures 87.59 cm by 35.1 mm. Express its area with the proper number of significant figures in the specified unit: **a.** in cm² **b.** in mm² **Answer: a. 307 cm² b. 30 700 mm²**
 - 5. A 125 mL sample of liquid has a mass of 0.16 kg. What is the density of the liquid in g/mL? **Answer: 1.3 g/mL**

Summary

In this lesson, you learned about significant digits and the scientific notation. You learned the rules in determining the number of significant digits. You also learned how to write very large and very small numbers using scientific notation.

Lesson 14: More Problems Involving Real Numbers

Prerequisite Concepts: Whole numbers, Integers, Rational Numbers, Real Numbers. Sets

Objectives:

In this lesson, you are expected to:

- 1. Apply the set operations and relations to sets of real numbers
- 2. Describe and represent real-life situations which involve integers, rational numbers, square roots of rational numbers, and irrational numbers
- 3. Apply ordering and operations of real numbers in modeling and solving reallife problems

Time: 1.5 hours

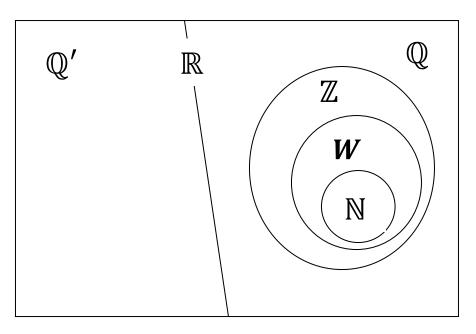
NOTE TO THE TEACHER:

This module provides additional problems involving the set of real numbers. There will be no new concepts introduced, merely reinforcement of previously learned properties of sets and real numbers.

Lesson Proper:

Recall how the set of real numbers was formed and how the operations are performed. Numbers came about because people needed and learned to count. The set of counting numbers was formed. To make the task of counting easier, addition came about. Repeated addition then got simplified to multiplication. The set $\mathbb N$ of counting numbers is <u>closed</u> under both the operations of addition and multiplication. When the need to represent zero arose, the set $\mathbf W$ of whole numbers was formed. When the operation of subtraction began to be performed, the $\mathbf W$ was extended to the set $\mathbb Z$ or integers. $\mathbb Z$ is closed under the operations of addition, multiplication and subtraction. The introduction of division needed the expansion of $\mathbb Z$ to the set $\mathbb Q$ of rational numbers. $\mathbb Q$ is closed under all the four arithmetic operations of addition, multiplication, subtraction and division. When numbers are used to represent measures of length, the set $\mathbb Q$ or rational numbers no longer sufficed. Hence, the set $\mathbb R$ of real numbers came to be the field where properties work.

The above is a short description of the way the set of real numbers developed to accommodate applications to counting and measurement and performance of the four arithmetic operations. We can also explore the set of real numbers by dissection – beginning from the big set, going into smaller subsets. We can say that $\mathbb R$ is the set of <u>all</u> decimals (positive, negative and zero). The set $\mathbb Q$ includes all the decimals which are repeating (we can think of terminating decimals as decimals in which all the digits after a finite number of them are zero). The set $\mathbb Z$ comprises all the decimals in which the digits to the right of the decimal point are all zero. This view gives us a clearer picture of the relationship among the different subsets of $\mathbb R$ in terms of inclusion.



We know that the *n*th root of any number which is not the *n*th power of a rational number is irrational. For instance, $\sqrt{2}$, $\sqrt{5}$, and $\sqrt[3]{9}$ are irrational.

Example 1. Explain why $3\sqrt{2}$ is irrational.

We use an argument called an indirect proof. This means that we will show why $3\sqrt{2}$ becoming rational will lead to an absurd conclusion. What happens if $3\sqrt{2}$ is rational? Because $\mathbb Q$ is closed under multiplication and $\frac{1}{3}$ is rational, then $3\sqrt{2} \times \frac{1}{3}$ is rational. However, $3\sqrt{2} \times \frac{1}{3} = \sqrt{2}$, which we know to be irrational. This is an absurdity. Hence we have to conclude that $3\sqrt{2}$ must be irrational.

Example 2. A deep-freeze compartment is maintained at a temperature of 12°C below zero. If the room temperature is 31°C, how much warmer is the room temperature than the temperature in the deep-freeze compartment?

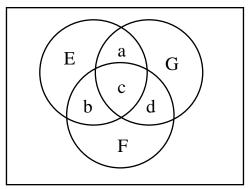
Get the difference between room temperature and the temperature inside the deep-freeze compartment

31 - (-12) = 43. Hence, room temperature is 43°C warmer than the compartment.

Example 3. Hamming Code

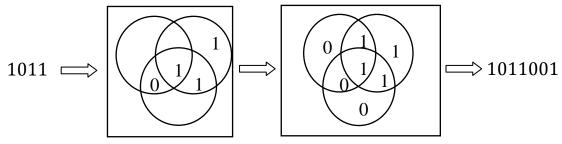
A mathematician, Richard Hamming developed an error detection code to determine if the information sent electronically is transmitted correctly. Computers store information using bits (binary digits, that is a 0 or a 1). For example, 1011 is a four-bit code. Hamming uses a Venn diagram with three "sets" as follows:

1. The digits of the four-bit code are placed in regions a, b, c, and d, in this order.



- 2. Three additional digits of 0's and 1's are put in the regions E, F, and G so that each "set" has an even number of 1's.
- 3. The code is then extended to a 7-bit code using (in order) the digits in the regions a, b, c, d, E, F, G.

For example, the code 1011 is encoded as follows:



Example 4. Two students are vying to represent their school in the regional chess competition. Felix won 12 of the 17 games he played this year, while Rommel won 11 of the 14 games he played this year. If you were the principal of the school, which student would you choose? Explain.

The Principal will likely use fractions to get the winning ratio or percentage of each player. Felix has a $\frac{12}{17}$ winning ratio, while Rommel has a $\frac{11}{14}$ winning ratio. Since $\frac{11}{14} > \frac{12}{17}$, Rommel will be a logical choice.

Example 5. A class is holding an election to decide whether they will go on a fieldtrip or not. They will have a fieldtrip if more than 50% of the class will vote Yes. Assume that every member of the class will vote. If 34% of the girls and 28% of the boys will vote Yes, will the class go on a fieldtrip? Explain.

Note to the Teacher

This is an illustration of when percentages cannot be added. Although 38 + 28 = 64 > 50, less than half of the girls and less than half the boys voted Yes. This means that less than half of the students voted Yes. Explain that the percentages given are taken from two different bases (the set of girls and the set of boys in the class), and therefore cannot be added.

Example 6. A sale item was marked down by the same percentage for three years in a row. After two years the item was 51% off the original price. By how much was the price off the original price in the first year?

Since the price after 2 years is 51% off the original price, this means that the price is then 49% of the original. Since the percentage ratio must be multiplied to the original price twice (one per year), and $0.7 \times 0.7 = 0.49$, then the price per year is 70% of the price in the preceding year. Hence the discount is 30% off the original.

Note to the Teacher

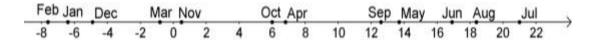
This is again a good illustration of the non-additive property of percent. Some students will think that since the discount after 2 years is 51%, the discount per year is 25.5%. Explain the changing base on which the percentage is taken.

Exercises:

1. The following table shows the mean temperature in Moscow by month from 2001 to 2011

January	-6.4°C	May	13.7°C	September	12.6°C
February	−7.6°C	June	16.9℃	October	6.0°C
March	−0.8°C	July	21.0°C	November	0.5°C
April	6.8°C	August	18.4°C	December	−4.9°C

Plot each temperature point on the number line and list from lowest to highest. **Answer: List can be generated from the plot.**



- **2.** Below are the ingredients for chocolate oatmeal raisin cookies. The recipe yields 32 cookies. Make a list of ingredients for a batch of 2 dozen cookies.
 - 1 ½ cups all-purpose flour
 - 1 tsp baking soda
 - 1 tsp salt
 - 1 cup unsalted butter
 - 34 cup light-brown sugar
 - 34 cup granulated sugar
 - 2 large eggs
 - 1 tsp vanilla extract
 - 2 ½ cups rolled oats
 - 1 ½ cups raisins
 - 12 ounces semi-sweet

chocolate chips

Answer: Since $24/32 = \frac{3}{4}$, we get $\frac{3}{4}$ of each item in the ingredients

1 1/8 cups all-purpose flour

34 tsp baking soda

34 tsp salt

34 cup unsalted butter

9/16 cup light-brown sugar

9/16 cup granulated sugar

1 ½ large eggs

34 tsp vanilla extract

17/8 cups rolled oats

1 1/8 cups raisins

9 ounces semi-sweet

chocolate chips

3. In high-rise buildings, floors are numbered in increasing sequence from the ground-level floor to second, third, etc, going up. The basement immediately below the ground floor is usually labeled B1, the floor below it is B2, and so on. How many floors does an elevator travel from the 39th floor of a hotel to the basement parking at level B6?

Answer: We need to find the solution to 39 - N = -5. Hence N = 39 - (-5) = 44. Note that Level B6 is -5, not -6. This is because B1 is 0.

4. A piece of ribbon 25 m long is cut into pieces of equal length. Is it possible to get a piece with irrational length? Explain.

Answer: It is not possible to get an irrational length because the length is $\frac{25}{N}$, where *N* is the number of pieces. This is clearly rational as it is the quotient of two integers.

5. Explain why $5 + \sqrt{3}$ is irrational. (See Example 1.)

Solution:

What will happen if $5+\sqrt{3}$ is rational. Then since 5 is rational and the set of rationals is closed under subtraction, $5+\sqrt{3}-5=\sqrt{3}$ will become rational. This is clearly not true. Therefore, $5+\sqrt{3}$ cannot be rational.

Lesson 15: Measurement and Measuring Length

Prerequisite Concepts: Real Numbers and Operations

Objective

At the end of the lesson, you should be able to:

- 1. Describe what it means to measure:
- 2. Describe the development of measurement from the primitive to the present international system of unit:

Time: 2.5 hours

- 3. Estimate or approximate length;
- 4. Use appropriate instruments to measure length;
- 5. Convert length measurement from one unit to another, including the English system;
- 6. Solve problems involving length, perimeter and area.

NOTE TO THE TEACHER:

This is a lesson on the English and Metric System of Measurement and using these systems to measure length. Since these systems are widely used in our community, a good grasp of this concept will help your students be more accurate in dealing with concepts involving length such as distance, perimeter and area. This lesson on measurement tackles concepts which your students have most probably encountered and will continue to deal with in their daily lives. Moreover, concepts and skills related to measurement are prerequisites to topics in Geometry as well as Algebra.

Lesson Proper

A.

I. Activity:

Instructions: Determine the dimension of the following using only parts of your arms. Record your results in the table below. Choose a classmate and compare your results.

	SHEET OF INTERMEDIATE PAPER		TEACHER	S'S TABLE	CLASSROOM		
	Length	Width	Length	Width	Length	Width	
Arm part used*							
Measure ment							
Comparison to: (classmate's name)							

^{*} For the arm part, please use any of the following only: the palm, the handspan, and the forearm length

Important Terms to Remember:

- >palm the width of one's hand excluding the thumb
- > handspan the distance from the tip of the thumb to the tip of the little finger of one's hand with fingers spread apart.
- > forearm length the length of one's forearm: the distance from the elbow to the tip of the middle finger.

NOTE TO THE TEACHER:

The activities in this module involve measurement of actual objects and lengths found inside the classroom, but you may modify the activity and include objects and distances outside the classroom. Letting the students use non-standard units of measurement first will give them the opportunity to appreciate our present measuring tools by emphasizing on the discrepancy of their results vis-a-vis their partner's results.

Answer the following questions:

- 1. What was your reason for choosing which arm part to use? Why?
- 2. Did you experience any difficulty when you were doing the actual measuring?
- 3. Were there differences in your data and your classmate's data? Were the differences significant? What do you think caused those differences?

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the opening activity:

- 1. What is the appropriate arm part to use in measuring the length and width of the sheet of paper? of the teacher's table? of the classroom? What was your reason for choosing which arm part to use? Why?
 - While all of the units may be used, there are appropriate units of measurement to be used depending on the length you are trying to measure.
 - For the sheet of paper, the palm is the appropriate unit to use since the handspan and the forearm length is too long.
 - For the teacher's table, either the palm or the handspan will do, but the forearm length might be too long to get an accurate measurement.
 - For the classroom, the palm and handspan may be used, but you may end up with a lot of repetitions. The best unit to use would be the forearm length.
- 2. Did you experience any difficulty when you were doing the actual measuring? The difficulties you may have experienced might include having to use too many repetitions.
- 3. Were there differences in your data and your classmate's data? Were the differences significant? What do you think caused those differences?

If you and your partner vary a lot in height, then chances are your forearm length, handspan, and palm may also vary, leading to different measurements of the same thing.

NOTE TO THE TEACHER:

This is a short introduction to the History of Measurement. Further research would be needed to get a better understanding of the concept. The questions that follow will help in enriching the discussion on this particular topic.

History of Measurement

One of the earliest tools that human beings invented was the unit of measurement. In the olden times, people needed measurement to determine how long or wide things are; they needed to build their houses or make their clothes. Later, units of measurement were used in trade and commerce. In the 3rd century BC in Egypt, people used their body parts to determine measurements of things; the same body parts that you used to measure the assigned things to you.

The forearm length, as described in the table below, was called a cubit. The handspan was considered a half cubit, while the palm was considered 1/6 of a cubit. Go ahead, check out how many handspans your forearm length is. The Egyptians came up with these units to be more accurate in measuring different lengths.

However, using these units of measurement had a disadvantage. Not everyone had the same forearm length. Discrepancies arose when the people started comparing their measurements to one another because measurements of the same thing differed, depending on who was measuring it. Because of this, these units of measurement are called non-standard units of measurement which later on evolved into what is now the inch, foot, and yard -- the basic units of length in the English system of measurement.

III. Exercise:

- 1. Can you name other body measurements which could have been used as a non-standard unit of measurement? Do some research on other non-standard units of measurement used by people other than the Egyptians.
- 2. Can you relate an experience in your community where a non-standard unit of measurement was used?

В.

I. Activity

NOTE TO THE TEACHER:

In this activity, comparisons of their results will underscore the advantages of using standard units of measurement as compared to using non-standard units of measurement. However, this activity may also provide a venue to discuss the limitations of actual measurements. Emphasize on the differences of their results, however small they may be.

Instructions: Determine the dimension of the following using the specified English units only. Record your results in the table below. Choose a classmate and compare your results.

	SHEET OF INTERMEDIATE PAPER			HER'S BLE	CLASSROOM		
	Length	Width	Length	Width	Length	Width	
Unit used*							
Measurement							
Comparison to: (classmate's name)							

For the unit used, choose which of the following SHOULD be used: inch or foot.

Answer the following questions:

- 1. What was your reason for choosing which unit to use? Why?
- 2. Did you experience any difficulty when you were doing the actual measuring?
- 3. Were there differences in your data and your classmate's data? Were the differences as big as the differences when you used non-standard units of measurement? What do you think caused those differences?

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

- 1. What was your reason for choosing which unit to use? Why?
 - For the sheet of paper, the appropriate unit to use is inches since its length and width might be shorter than a foot.
 - For the table and the classroom, a combination of both inches and feet may be used for accuracy and convenience of not having to deal with a large number.
- 2. What difficulty, if any, did you experience when you were doing the actual measuring?
- 3. Were there differences in your data and your classmate's data? Were the differences as big as the differences when you used non-standard units of measurement? What do you think caused those differences?
 - If you and your partner used the steel tape correctly, both your data should have little or no difference at all. The difference should not be as big or as significant as the difference when non-standard units of measurement were used. The slight difference might be caused by how accurately you tried to measure each dimension or by how you read the ticks on the steel tape. In doing actual measurement, a margin of error should be considered.

NOTE TO THE TEACHER:

The narrative that follows provides continuity to the development of the English system of measurement. The conversion factors stated herein only involve common units of length. Further research may include other English units of length.

History of Measurement (Continued)

As mentioned in the first activity, the inch, foot, and yard are said to be based on the cubit. They are the basic units of length of the English System of Measurement, which also includes units for mass, volume, time, temperature and angle. Since the inch and foot are both units of length, each can be converted into the other. Here are the conversion factors, you learned from previous lessons:

1 foot = 12 inches

1 yard = 3 feet

For long distances, the mile is used:

1 mile = 1,760 yards = 5,280 feet

Converting from one unit to another might be tricky at first, so an organized way of doing it would be a good starting point. As the identity property of multiplication states, the product of any value and 1 is the value itself. Consequently, dividing a value by the same value would be equal to one. Thus, dividing a unit by its equivalent in another unit is equal to 1. For example:

These conversion factors may be used to convert from one unit to another. Just remember that you're converting from one unit to another so cancelling same units would guide you in how to use your conversion factors. For example:

1. Convert 36 inches into feet:

36 inches
$$x \frac{1 \text{ foot}}{12 \text{ inches}} = 3 \text{ feet}$$

2. Convert 2 miles into inches:
$$2 \text{ miles } x \frac{5280 \text{ feet}}{1 \text{ mite}} x \frac{12 \text{ inches}}{1 \text{ foot}} = \frac{2x5280x12}{1x1} \text{ inches} = 126,720 \text{ inches}$$

Again, since the given measurement was multiplied by conversion factors which are equal to 1, only the unit was converted but the given length was not changed. Try it yourself.

III. Exercise:

Convert the following lengths into the desired unit:

- 1. Convert 30 inches to feet

 Solution: $30 inches x \frac{1 foot}{12 inches} = 2.5 feet$ 2. Convert 130 yards to inches

 Solution: $130 yards x \frac{3 feet}{1 yard} x \frac{12 inches}{1 foot} = 4,680$

3. Sarah is running in a 42-mile marathon. How many more feet does Sarah need to run if she has already covered 64,240 yards?

Solution:

Step 1:42 miles
$$x = \frac{5,280 \text{ feet}}{1 \text{ mile}} = 221,760 \text{ feet}$$

Step 1:42 miles
$$x \frac{5,280 \ feet}{1 \ mile} = 221,760 \ feet$$

Step 2:64,240 yards $x \frac{3 \ feet}{1 \ yard} = 192,720 \ feet$

Step 3: 221,760 feet - 192,720 feet = 29,040 feet

Answer: Sarah needs to run 29,040 feet to finish the marathon

NOTE TO TEACHER:

In item 3, disregarding the units and not converting the different units of measurement into the same units of measurement is a common error.

I. Activity:

NOTE TO THE TEACHER:

This activity introduces the metric system of measurement and its importance. This also highlights how events in Philippine and world history determined the systems of measurement currently used in the Philippines.

Answer the following questions:

- 1. When a Filipina is described as 1.7 meters tall, would she be considered tall or short? How about if the Filipina is described as 5 ft, 7 inches tall, would she be considered tall or short?
- 2. Which particular unit of height are you more familiar with? Why?

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

- 1. When a Filipina is described as 1.7 meters tall, would she be considered tall or short? How about if the Filipina is described as 5 ft, 7 inches tall, would she be considered tall or short?
 - Chances are, you would find it difficult to answer the first question. As for the second question, a Filipina with a height of 5 feet, 7 inches would be considered tall by Filipino standards.
- 2. Which particular unit of height are you more familiar with? Why?
 - Again, chances are you would be more familiar with feet and inches, since feet and inches are still being widely used in measuring and describing height here in the Philippines.

NOTE TO THE TEACHER:

The reading below discusses the development of the Metric system of measurement and the prefixes which the students may use or may encounter later on. Further research may include prefixes which are not commonly used as well as continuing efforts to further standardize the different units.

History of Measurement (Continued)

The English System of Measurement was widely used until the 1800s and the 1900s when the Metric System of Measurement started to gain ground and became the most used system of measurement worldwide. First described by Belgian Mathematician Simon Stevin in his booklet, De Thiende (The Art of Tenths) and proposed by English philosopher, John Wilkins, the Metric System of Measurement was first adopted by France in 1799. In 1875, the General Conference on Weights and Measures (*Conférence générale des poids et mesures or CGPM*) was tasked to define the different measurements. By 1960, CGPM released the International System of Units (SI) which is now being used by majority of the countries with the biggest exception being the United States of America. Since our country used to be a colony of the United States, the Filipinos were schooled in the use of the English instead of the Metric System of Measurement. Thus, the older generation of Filipinos is more comfortable with the English System rather than the Metric System, although the Philippines already adopted the Metric System as its official system of measurement.

The Metric System of Measurement is easier to use than the English System of Measurement since its conversion factors would consistently be in the decimal system, unlike the English System of Measurement where units of lengths have different conversion factors. Check out the units used in your steep tape measure, most likely they are inches and centimeters. The base unit for length is the meter and units longer or shorter than the meter would be achieved by adding prefixes to the base unit. These prefixes may also be used for the base units for mass, volume, time and other measurements. Here are the common prefixes used in the Metric System:

PREFIX	SYMBOL	FACTOR
tera	T	x 1 000 000 000 000
giga	G	x 1 000 000 000
mega	M	x 1 000 000
kilo	k	x 1 000
hecto	h	x 100
deka	da	x 10
deci	d	x 1/10
centi	С	x 1/100
milli	m	x 1/1 000
micro	μ	x 1/1 000 000
nano	n	x 1/1 000 000 000

For example:

1 kilometer = 1,000 meters

1 millimeter = 1/1,000 meter or 1,000 millimeters = 1 meter

These conversion factors may be used to convert from big to small units or vice versa. For example:

1. Convert 3 km to m:

$$3 \lim_{m \to \infty} x \frac{1,000 \, m}{1 \, \text{km}} = 3,000 \, m$$

2. Convert 10 mm to m:

$$10 \text{ mm } x \frac{1 \text{ m}}{1000 \text{ mm}} = \frac{1}{100} \text{ or } 0.01 \text{m}$$

As you can see in the examples above, any length or distance may be measured using the appropriate English or Metric units. In the question about the Filipina whose height was expressed in meters, her height can be converted to the more familiar feet and inches. So, in the Philippines where the official system of measurements is the Metric System, the English System continues to be used, or as long as we have relatives and friends residing in the United States. Knowing how to convert from the English System to the Metric System (or vice versa) would be useful. The following are common conversion factors for length:

1 inch =
$$2.54$$
 cm

For example:

Convert 20 inches to cm:
$$20 \text{ in } x \frac{2.54 \text{ cm}}{1 \text{ in}} = 50.8 \text{ cm}$$

III. Exercise:

NOTE TO THE TEACHER:

Knowing the lengths of selected body parts will help students in estimating lengths and distances by using these body parts and their measurements to estimate certain lengths and distances. Items 5 & 6 might require a review in determining the perimeter and area of common geometric figures.

1. Using the tape measure, determine the length of each of the following in cm. Convert these lengths to meters.

	PALM	HANDSPAN	FOREARM LENGTH
Centimeters			
Meters			

2. Using the data in the table above, estimate the lengths of the following without using the steel tape measure or ruler:

	BALLPEN	LENGTH OF WINDOW PANE	LENGTH OF YOUR FOOT FROM THE TIP OF YOUR HEEL TO THE TIP OF YOUR TOES	HEIGHT OF THE CHALK BOARD	LENGTH OF THE CHALK BOARD
NON- STANDARD UNIT					
METRIC UNIT					

3. Using the data from table 1, convert the dimensions of the sheet of paper, teacher's table, and the classroom into Metric units. Recall past lessons on perimeter and area and fill in the appropriate columns:

	SHEET OF INTERMEDIATE PAPER			TEACHER'S TABLE			CLASSROOM					
	Length	Width	Peri- meter	Area	Length	Width	Peri- meter	Area	Length	Width	Peri- meter	Area
English units												
Metric Units												

4. Two friends, Zale and Enzo, run in marathons. Zale finished a 21-km marathon in Cebu, while Enzo finished a 15-mile marathon in Los Angeles. Who ran a longer distance? By how many meters?

Step 1: 21 km $x \frac{1,000 \text{ m}}{1 \text{ km}} = 21 \ 000 \text{ m}$ Step 2: 15 mi $x \frac{1.6 \text{ km}}{1 \text{ mi}} x \frac{1 \ 000 \text{ m}}{1 \text{ km}} = 24 \ 000 \text{ m}$ Step 3: 24 000 m - 21 000 m = 3 000 m

Answer: Enzo ran a distance of 3 000 meters more.

5. Georgia wants to fence her square garden, which has a side of 20 feet, with two rows of barb wire. The store sold barb wire by the meter at P12/meter. How much money will Georgia need to buy the barb wire she needs?

Step 1: 20 $ft \times 4 \text{ sides } \times 2 \text{ rows } = 160 \text{ } ft$

Step 2: 160 ft $x \frac{1 m}{3.3 ft}$ = 48.48m rounded up to 49 m since the store sells

barb wire by the m

Step 3: 49 m x P12/meter = P 588

Answer: Georgia will need P 588 to buy 49 meters of barb wire

5. A rectangular room has a floor area of 32 square meters. How many tiles, each measuring 50 cm x 50 cm, are needed to cover the entire floor?

Step 1: 50 cm $x \frac{1 m}{100 cm} = 0.5 m$ Step 2: Area of 1 tile: 0.5 m x 0.5 m = 0.25 m²

Step 3: $32 \text{ m}^2 / 0.25 \text{ m}^2 = 128 \text{ tiles}$

Answer: 128 tiles are needed to cover the entire floor

Summary

In this lesson, you learned: 1) that ancient Egyptians used units of measurement based on body parts such as the cubit and the half cubit. The cubit is the length of the forearm from the elbow to the tip of the middle finger; 2) that the inch and foot, the units for length of the English System of Measurement, are believed to be based on the cubit; 3) that the Metric System of Measurement became the dominant system in the 1900s and is now used by most of the countries with a few exceptions, the biggest exception being the United States of America; 4) that it is appropriate to use short base units of length for measuring short lengths and long units of lengths to measure long lengths or distances; 5) how to convert common English units of length into other English units of length using conversion factors; 6) that the Metric System of Measurement is based on the decimal system and is therefore easier to use; 7) that the Metric System of Measurement has a base unit for length (meter) and prefixes to signify long or short lengths or distances; 8) how to estimate lengths and distances using your arm parts and their equivalent Metric lengths; 9) how to convert common Metric units of length into other Metric units of length using the conversion factors based on prefixes; 10) how to convert common English units of length into Metric units of length (and vice versa) using conversion factors; 11) how to solve length, perimeter, and area problems using English and Metric units.

Lesson 16: Measuring Weight/Mass and Volume

Prerequisite Concepts: Basic concepts of measurement, measurement of length

Time: 2.5 hours

About the Lesson:

This is a lesson on measuring volume & mass/weight and converting its units from one to another. A good grasp of this concept is essential since volume & weight are commonplace and have practical applications.

Objectives:

At the end of the lesson, you should be able to:

- 1. estimate or approximate measures of weight/mass and volume;
- 2. use appropriate instruments to measure weight/mass and volume;
- 3. convert weight/mass and volume measurements from one unit to another, including the English system;
- 4. Solve problems involving weight/mass and volume/capacity.

Lesson Proper

Α.

I. Activity:

Read the following passage to help you review the concept of volume.

Volume

Volume is the amount of space an object contains or occupies. The volume of a container is considered to be the capacity of the container. This is measured by the number of cubic units or the amount of fluid it can contain and not the amount of space the container occupies. The base SI unit for volume is the cubic meter (m³). Aside from cubic meter, another commonly used metric unit for volume of solids is the cubic centimeter (cm³ or cc), while the commonly used metric units for volume of fluids are the liter (L) and the milliliter (mL).

Hereunder are the volume formulae of some regularly-shaped objects:

Cube: Volume = edge x edge $(V = e^3)$

Rectangular prism: Volume = length x width x height (V = lwh)

Triangular prism: Volume = $\frac{1}{2}$ x base of the triangular base x height of the triangular base x Height of the prism

$$(V = (\frac{1}{2}bh)H)$$

Cylinder: Volume = πx (radius)² x height of the cylinder (V = $\pi r^2 h$)

Other common regularly-shaped objects are the different pyramids, the cone and the sphere. The volumes of different pyramids depend on the shape of its base. Here are their formulae:

Square-based pyramids: Volume = $1/3 \times (\text{side of base})^2 \times \text{height of pyramid}(V = <math>1/3 \text{ s}^2\text{h})$

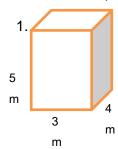
Rectangle-based pyramid: Volume= $1/3 \times length$ of the base x width of the base x height of pyramid($V=1/3 \mid wh$)

Triangle-based pyramid: Volume = $1/3 \times 1/2 \times 1$

$$(V = \frac{1}{3}(\frac{1}{2}bh)H)$$

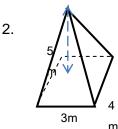
Cone: Volume = $1/3 \times \pi \times (radius)^2 \times height$

Sphere: Volume = $4/3 \text{ x } \pi \text{x} \text{ (radius)}^3 \text{ (V = } 4/3 \text{ } \pi \text{r}^3\text{)}$ Here are some examples:



$$V = lwh = 3 m x 4 m x 5 m$$

= $(3 x 4 x 5) x (m x m x m) = 60 m3$



$$V = 1/3 \text{ lwh} = 1/3 \times 3 \text{ m} \times 4 \text{ m} \times 5 \text{ m}$$

= $(1/3 \times 3 \times 4 \times 5) \times (\text{m} \times \text{m} \times \text{m}) = 20 \text{ m}^3$

Answer the following questions:

- 1. Cite a practical application of volume.
- What do you notice about the parts of the formulas that have been underlined? Come up with a general formula for the volume of all the given prisms and for the cylinder.
- What do you notice about the parts of the formulas that have been shaded?Come up with a general formula for the volume of all the given pyramids and for the cone.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the opening activity:

1. Cite a practical application of volume.

Volume is widely used from baking to construction. Baking requires a degree of precision in the measurement of the ingredients to be used thus measuring spoons and cups are used. In construction, volume is used to measure the size of a room, the amount of concrete needed to create a specific column or beam or the amount of water a water tank could hold.

What do you notice about the parts of the formulas that have been underlined? Come up with a general formula for the volume of all the given prisms and for the cylinder.

The formulas that have been underlined are formulas for area. The general formula for the volume of the given prisms and cylinder is just the area of the base of the prisms or cylinder times the height of the prism or cylinder ($V = A_{base}h$).

What do you notice about the parts of the formulas that have been shaded?Come up with a general formula for the volume of all the given pyramids and for the cone.

The formulas that have been shaded are formulas for the volume of prisms or cylinders. The volume of the given pyramids is just 1/3 of the volume of a prism whose base and height are equal to that of the pyramid while the formula for the cone is just 1/3 of the volume of a cylinder with the same base and height as the cone ($V = 1/3 \ V_{prism \ or \ cylinder}$).

III. Exercise:

Instructions: Answer the following items. Show your solution.

1. How big is a Toblerone box (triangular prism) if its triangular side has a base of 3 cm and a height of 4.5 cm and the box's height is 25 cm?

Volume triangular prism : V=bh/2 H = [(3 cm)(4.5 cm)/2][25 cm] = 168.75 cm3

2. How much water is in a cylindrical tin can with a radius of 7 cm and a height of 20 cm if it is only a quarter full?

Step 1: Volumecylinder: $V = \pi r^2 h$ Step 2: $\frac{1}{4} V = \frac{1}{4} (3080 \text{ cm}3)$ = (22/7)(7 cm)(7 cm)(20 cm)= 770 cm3= 3080 cm3

NOTE TO THE TEACHER

A common error in this type of problem is not noticing that the problem asks for the volume of water in the tank when it's only a quarter full.

3. Which of the following occupies more space, a ball with a radius of 4 cm or a cube with an edge of 60 mm?

Step 1: Vsphere = 4/3 mr3 Step 2: Vcube = e3 Step 3: Since Vball > Vcube, then = 4/3 (22/7)((4 cm)3) = (6 cm)3 the ball occupies more space cm3 = 268.19cube.

NOTE TO THE TEACHER

One of the most common mistakes involving this kind of problem is the disregard of the units used. In order to accurately compare two values, they must be expressed in the same units.

В.

I. Activity

Materials Needed:

Ruler / Steel tape measure

Different regularly-shaped objects (brick, cylindrical drinking glass, balikbayan box)

Instructions: Determine the dimension of the following using the specified metric units only. Record your results in the table below and compute for each object's volume using the unit used to measure the object's dimensions. Complete the table by expressing/converting the volume using the specified units.

		I	BRICK		DRINKING GLASS		BALIKBAYAN BOX		CLASSROOM			
		Length	Width	Height	Radius	Height	Length	Width	Height	Length	Width	Height
Unit used	d *											
Measure	ment											
	cm ³											
Valuma	m^3											
Volume	in ³											
	ft ³											

For the unit used, choose ONLY one: centimeter or meter.

Answer the following questions:

- 1. What was your reason for choosing which unit to use? Why?
- 2. How did you convert the volume from cc to m³ or vice versa?
- 3. How did you convert the volume from cc to the English units for volume?

Volume (continued)

The English System of Measurement also has its own units for measuring volume or capacity. The commonly used English units for volume are cubic feet (ft³) or cubic inches (in³), while the commonly used English units for fluid volume are the pint, quart or gallon. Recall from the lesson on length and area that while the Philippine government has mandated the use of the Metric system, English units are still very much in use in our society so it is an advantage if we know how to convert from the English to the Metric system and vice versa. Recall as well from the previous lesson on measuring length that a unit can be converted into another unit using conversion factors. Below are some of the conversion factors which can help you convert given volume units into the desired volume units:

Since the formula for volume only requires length measurements, another alternative to converting volume from one unit to another is to convert the object's dimensions into the desired unit before solving for the volume.

For example:

1. How much water, in cubic centimeters, can a cubical water tank hold if it has an edge of 3 meters?

Solution 1 (using a conversion factor):

i. Volume =
$$e^3$$
 = $(3 \text{ m})^3$ = 27 m³

i. Volume =
$$e^3$$
 = $(3 \text{ m})^3$ = 27 m³
ii. 27 m³ x ^{1 million cm³} /_{1 m3} = **27 million cm³**

Solution 2 (converting dimensions first):

i. 3 m x
$$^{100 \text{ cm}}$$
 / $_{1 \text{ m}}$ = 300 cm

i. 3 m x
$$^{100 \text{ cm}}$$
 / $_{1 \text{ m}}$ = 300 cm
ii. Volume = e^3 = (300 cm) 3 = **27 million cm** 3

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

1. What was your reason for choosing which unit to use?

Any unit on the measuring instrument may be used but the decision on what unit to use would depend on how big the object is. In measuring the brick, the glass and the balikbayan box, the appropriate unit to use would be centimeter. In measuring the dimensions of the classroom, the appropriate unit to use would be meter.

2. How did you convert the volume from cc to m³ or vice versa?

Possible answer would be converting the dimensions to the desired units first before solving for the volume.

3. How did you convert the volume from cc or m³ to the English units for volume? Possible answer would be by converting the dimensions into English units first before solving for the volume.

III. Exercises:

Answer the following items. Show your solutions.

1. Convert 10 m³ to ft³

10 m3 x 35.94 ft 3 /1 m 3 = 359.4 ft 3

NOTE TO THE TEACHER

A common error in this type of problem is the use of the conversion factor for meter to feet instead of the conversion factor from m³ to ft³. This conversion factor may be arrived at by computing for the number of cubic feet in 1 cubic meter.

2. Convert 12 cups to mL

12 cups
$$x \frac{1 pint}{2 cups} x \frac{1 quart}{2 pints} x \frac{1 gal}{4 quarts} x \frac{3.79 L}{1 gal} x \frac{1000 mL}{1 L} = 2,842.5 mL$$

- 3. A cylindrical water tank has a diameter of 4 feet and a height of 7 feet, while a water tank shaped like a rectangular prism has a length of 1 m, a width of 2 meters and a height of 2 meters. Which of the two tanks can hold more water? By how many cubic meters?
 - Step 1: Vcylinder = π r2h

= (22/7)(0.61 m)2(2.135 m)

= (1 m)(2 m)(2 m)

= 2.5 m3

= 4 m3

The rectangular water tank can hold 1.5 m3 more water than the cylindrical water tank.

NOTE TO THE TEACHER

One of the most common mistakes involving this kind of problem is the disregard of the units used. In order to accurately compare two values, they must be expressed in the same units.

C.

I. Activity:

Problem: The rectangular water tank of a fire truck measures 3 m by 4 m by 5 m. How many liters of water can the fire truck hold?

Volume (Continued)

While capacities of containers are obtained by measuring its dimensions, fluid volume may also be expressed using Metric or English units for fluid volume such as liters or gallons. It is then essential to know how to convert commonly used units for volume into commonly used units for measuring fluid volume.

While the cubic meter is the SI unit for volume, the liter is also widely accepted as a SI-derived unit for capacity. In 1964, after several revisions of its definition, the General Conference on Weights and Measures (CGPM) finally defined a liter as equal to one cubic decimeter. Later, the letter L was also accepted as the symbol for liter.

This conversion factor may also be interpreted in other ways. Check out the conversion factors below:

```
1 L = 1 dm^3

1 mL = 1 cc

1,000 L = 1 m^3
```

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the problem above:

Step 1:
$$V = lwh$$
 Step 2: $60 \text{ m}^3 \text{ x}^{1,000 \text{ L}} / _{1 \text{ m}3} = 60 000 \text{ L}$
= $3m \text{ x} 4m \text{ x} 5m$
= 60 m^3

III. Exercise:

Instructions: Answer the following items. Show your solution.

1. A spherical fish bowl has a radius of 21 cm. How many mL of water is needed to fill half the bowl?

```
Vsphere = 4/3 \text{ mr3}
= (4/3)(22/7)(21 \text{ cm})3
= 38 808 \text{ cm3} or cc
Since 1 cc = 1 mL, then 38 808 \text{ mL} of water is needed to fill the tank
```

2. A rectangular container van needs to be filled with identical cubical balikbayan boxes. If the container van's length, width and height are 16 ft, 4 ft and 6ft, respectively, while each balikbayan box has an edge of 2 ft, what is the maximum number of balikbayan boxes that can be placed inside the van?

```
Step 1: Vvan = lwh
```

3. A drinking glass has a height of 4 in, a length of 2 in and a width of 2 in, while a baking pan has a width of 4 in, a length of 8 in and a depth of 2 in. If the baking pan is to be filled with water up to half its depth using the drinking glass, how many glasses full of water would be needed?

```
Step 1: Vdrinking glass = lwh
= (4 in)(2 in)(2 in)
= 16 in<sup>3</sup>

Step 2: Vbaking pan = lwh
= (4 in)(8 in)(2 in)
= 64 in<sup>3</sup> when full -> 32 in<sup>3</sup> when half full

Step 3: No. of glasses = (1/2)Vpan/Vglass
= 32 in<sup>3</sup>/16 in<sup>3</sup> -> 2 glasses of water are needed to fill half the pan
```

D.

Activity:

Instructions: Fill the table below according to the column headings. Choose which of the available instruments is the most appropriate in measuring the given object's weight. For the weight, choose only one of the given units.

	INSTRUMENT*		WEIGHT	
		Gram	Kilogram	Pound
¢25-coin				
□ 5coin				
Small toy marble				
Piece of brick				
Yourself				

^{*}Available instruments: triple-beam balance, nutrition (kitchen) scale, bathroom scale Answer the following questions:

- 1. What was your reason for choosing which instrument to use?
- 2. What was your reason for choosing which unit to use?
- 3. What other kinds of instruments for measuring weight do you know?
- 4. What other units of weight do you know?

Mass/ Weight

In common language, mass and weight are used interchangeably although weight is the more popular term. Oftentimes in daily life, it is the mass of the given object which is called its weight. However, in the scientific community, mass and weight are two different measurements. Mass refers to the amount of matter an object has, while weight is the gravitational force acting on an object.

Weight is often used in daily life, from commerce to food production. The base SI unit for weight is the kilogram (kg) which is almost exactly equal to the mass of one liter of water. For the English System of Measurement, the base unit for weight is the pound (lb). Since both these units are used in Philippine society, knowing how to convert from pound to kilogram or vice versa is important. Some of the more common Metric units are the gram (g) and the milligram (mg), while another commonly used English unit for weight is ounces (oz). Here are some of the conversion factors for these units:

Use these conversion factors to convert common weight units to the desired unit. For example:

Convert 190 lb to kg: $19\% \, lb \, x \, \frac{1 \, kg}{2 \cancel{Z} \, lb} = 86.18 \, kg$ II. Questions to Ponder (Post-Activity Discussion)

1. What was your reason for choosing which instrument to use?

Possible reasons would include how heavy the object to be weighed to the capacity of the weighing instrument.

2. What was your reason for choosing which unit to use?

The decision on which unit to use would depend on the unit used by the weighing instrument. This decision will also be influenced by how heavy the obiect is.

3. What other kinds of instruments for measuring weight do you know?

Other weighing instruments include the two-pan balance, the spring scale, the digital scales.

4. What other common units of weight do you know?

Possible answers include ounce, carat and ton.

III. Exercise:

Answer the following items. Show your solution.

- 1. Complete the table above by converting the measured weight into the specified units.
- 2. When Sebastian weighed his balikbayan box, its weight was 34 kg. When he got to the airport, he found out that the airline charged \$5 for each lb in excess of the free baggage allowance of 50 lb. How much will Sebastian pay for the excess weight?

```
Step 1: 34 kg -> lb
         34 \text{ kg x } 2.2 \text{ lb } / 1 \text{ kg} = 74.8 \text{ lb}
Step 2: 74.8 \text{ lb} - 50 = 24.8 \text{ lb} in excess
Step 3: Payment = (excess lb)($5)
         = (24.8 lb)($5)
         = $124.00
```

3. A forwarding company charges P1,100 for the first 20 kg and P60 for each succeeding 2 kg for freight sent to Europe. How much do you need to pay for a box weighing 88 lb?

Step 1: 88 lb -> kg 88 lb x 1 kg / 2.2 lb = 40 kg Step 2: (40 - 20)/2 = 10 Step 3: freight charge = □ 1,100 + (10)(□ 60) = □ 1,700.00

Summary

In this lesson, you learned: 1) how to determine the volume of selected regularly-shaped solids; 2) that the base SI unit for volume is the cubic meter; 3) how to convert Metric and English units of volume from one to another; 4) how to solve problems involving volume or capacity; 5) that mass and weight are two different measurements and that what is commonly referred to as weight in daily life is actually the mass; 6) how to use weighing intruments to measure the mass/weight of objects and people; 7) how to convert common Metric and English units of weight from one to another; 8) how to solve problems involving mass / weight.

Lesson 17: Measuring Angles, Time and Temperature

Prerequisite Concepts: Basic concepts of measurement, ratios

About the Lesson:

This lesson should reinforce your prior knowledge and skills on measuring angle, time and temperature as well as meter reading. A good understanding of this concept would not only be useful in your daily lives, but it would also help you in studying geometry and physical sciences.

Time: 2.5 hours

Objectives:

At the end of the lesson, you should be able to:

- 1. estimate or approximate measures of angle, time and temperature;
- 2. use appropriate instruments to measure angles, time and temperature;
- 3. solve problems involving time, speed, temperature and utilities usage (meter reading).

Lesson Proper

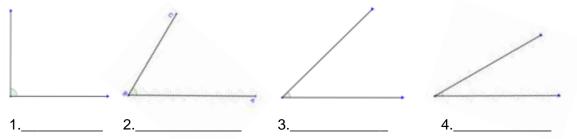
A.

I. Activity:

Material needed:

Protractor

Instruction: Use your protractor to measure the angles given below. Write your answer on the line provided.



Anales

Derived from the Latin word *angulus*, which means corner, an angle is defined as a figure formed when two rays share a common endpoint called the vertex. Angles are measured either in degree or radian measures. A protractor is used to determine the measure of an angle in degrees. In using the protractor, make sure that the cross bar in the middle of the protractor is aligned with the vertex, and one of the legs of the angle is aligned with one side of the line passing through the cross bar. The measurement of the angle is determined by its other leg.

Answer the following items:

1. Estimate the measurement of the angle below. Use your protractor to check your estimate.

	Estimate	
	Measurement using the protractor	
λ	•	

- 2. What difficulties did you meet in using your protractor to measure the angles?
- 3. What can be done to improve your skill in estimating angle measurements?

II. Questions to Ponder (Post-activity discussion):

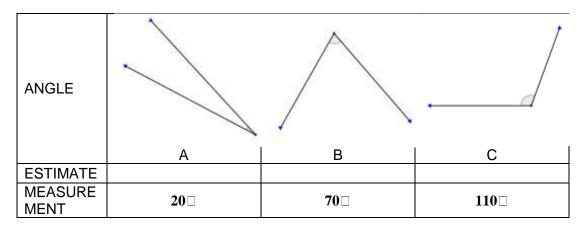
1. Estimate the measurement of the angles below. Use your protractor to check your estimates.

Measurement = 50°

- 2. What difficulties did you meet in using your protractor to measure the angles? One of the difficulties you may encounter would be on the use of the protractor and the angle orientation. Aligning the cross bar and base line of the protractor with the vertex and an angle leg, respectively, might prove to be confusing at first, especially if the angle opens in the clockwise orientation. Another difficulty arises if the length of the leg is too short such that it won't reach the tick marks on the protractor. This can be remedied by extending the leg.
- 3. What can be done to improve your skill in estimating angle measurements? You may familiarize yourself with the measurements of the common angles like the angles in the first activity, and use these angles in estimating the measurement of other angles.

III. Exercise:

Instructions: Estimate the measurement of the given angles, then check your estimates by measuring the same angles using your protractor.



В.

I. Activity

Problem: An airplane bound for Beijing took off from the Ninoy Aquino International Airport at 11:15 a.m. Its estimated time of arrival in Beijing is at1550 hrs. The distance from Manila to Beijing is 2839 km.

- 1. What time (in standard time) is the plane supposed to arrive in Beijing?
- 2. How long is the flight?
- 3. What is the plane's average speed?

Time and Speed

The concept of time is very basic and is integral in the discussion of other concepts such as speed. Currently, there are two types of notation in stating time, the 12-hr notation (standard time) or the 24-hr notation (military or astronomical time). Standard time makes use of a.m. and p.m. to distinguish between the time from 12midnight to 12 noon (a.m. or *ante meridiem*) and from 12 noon to 12 midnight (p.m. or *post meridiem*). This sometimes leads to ambiguity when the suffix of a.m. and p.m. are left out. Military time prevents this ambiguity by using the 24-hour notation where the counting of the time continues all the way to 24. In this notation, 1:00 p.m. is expressed as 1300 hours or 5:30 p.m. is expressed as 1730 hours.

Speed is the rate of an object's change in position along a line. Average speed is determined by dividing the distance travelled by the time spent to cover the distance (Speed = $^{distance}/_{time}$ or S = $^{d}/_{t}$, read as "distance per time"). The base SI unit for speed is meters per second ($^{m}/_{s}$). The commonly used unit for speed is $^{Kilometers}/_{hour}$ (kph or $^{km}/_{h}$) for the Metric system and miles/hour (mph or $^{mi}/_{hr}$) for the English system.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

- 1. What time (in standard time) is the plane supposed to arrive in Beijing? 3:50 p.m.
- 2. How long is the flight?

1555 hrs – 1115 hrs = 4 hrs, 40 minutes or 4.67 hours

3. What is the plane's average speed?

S = d/t

 $= 2839 \, km / 4.67 \, hrs$

=607.92 kph

III. Exercise:

Answer the following items. Show your solutions.

1. A car left the house and travelled at an average speed of 60 kph. How many minutes will it take for the car to reach the school which is 8 km away from the house?

t = d/S

= 8 km / 60 kph

= 2/15 hours = 8 minutes

NOTE TO THE TEACHER

One of the most common mistakes of the students is disregarding the units of the given data as well as the unit of the answer. In this particular case, the unit of time used in the problem is hours, while the desired unit for the answer is in minutes.

2. Sebastian stood at the edge of the cliff and shouted facing down. He heard the echo of his voice 4 seconds after he shouted. Given that the speed of sound in air is 340 m/s, how deep is the cliff?

Let d be the total distance travelled by Sebastian's voice.

d = St = (340 m/s)(4 sec) = 1.360 m

Since Sebastian's voice has travelled from the cliff top to its bottom and back, the cliff depth is therefore half of d. Thus, the depth of the cliff is d/2 = 680 m

$$a / 2 = 680 i$$

NOTE TO THE TEACHER

One of the common mistakes students made in this particular problem is not realizing that 4 seconds is the time it took for Zale's voice to travel from the top of the cliff and back to Zale. Since it took 4 seconds for Sebastian's voice to bounce back to him, 1,360 m is twice the depth of the cliff.

3. Maria ran in a 42-km marathon. She covered the first half of the marathon from 0600 hrs to 0715 hours and stopped to rest. She resumed running and was able to cover the remaining distance from 0720 hrs to 0935 hrs. What was Maria's average speed for the entire marathon?

Since the total distance travelled is 42 km and the total time used is 3:35 or 3 7/12 hrs. If S is the average speed of Maria, then

NOTE TO THE TEACHER

A common error made in problems such as this is the exclusion of the time Maria used to rest from the total time it took her to finish the marathon.

C.

I. Activity:

Problem: Zale, a Cebu resident, was packing his suitcase for his trip to New York City the next day for a 2-week vacation. He googled New York weather and found out the average temperature there is 59°F. Should he bring a sweater? What data should Zale consider before making a decision?

Temperature

Temperature is the measurement of the degree of hotness or coldness of an object or substance. While the commonly used units are Celsius (°C) for the Metric system and Fahrenheit (°F) for the English system, the base SI unit for temperature is the Kelvin (K). Unlike the Celsius and Fahrenheit which are considered degrees, the Kelvin is considered an absolute unit of measure and therefore can be worked on algebraically.

Hereunder are some conversion factors:

$$^{\circ}$$
C = $(^{5}/_{9})(^{\circ}$ F - 32)
 $^{\circ}$ F = $(^{9}/_{5})(^{\circ}$ C) + 32
K = $^{\circ}$ C + 273.15

For example:

Convert 100°C to °F:
$${}^{\circ}F = ({}^{9}/_{5})(100 {}^{\circ}C) + 32$$

= 180 + 32
= 212 °F

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the problem above:

- 1. What data should Zale consider before making a decision?
 In order to determine whether he should bring a sweater or not, Zale needs to compare the average temperature in NYC to the temperature he is used to which is the average temperature in Cebu. He should also express both the average temperature in NYC and in Cebu in the same units for comparison.
- 2. Should Zale bring a sweater?

The average temperature in Cebu is between 24 - 32 °C. Since the average temperature in NYC is 59°F which is equivalent to 15°C, Zale should probably bring a sweater since the NYC temperature is way below the temperature he is used to. Better yet, he should bring a jacket just to be safe.

III. Exercise:

Instructions: Answer the following items. Show your solution.

1. Convert 14°F to K.

2. Maria was preparing the oven to bake brownies. The recipe's direction was to pre-heat the oven to 350°F, but her oven thermometer was in °C. What should be the thermometer reading before Maria puts the baking pan full of the brownie mix in the oven?

D.

Activity:

Instructions: Use the pictures below to answer the questions that follow.



Initial electric meter reading at 0812 hrs on 14 Feb 2012



Final electric meter reading at 0812 hrs on 15 Feb 2012

- 1. What was the initial meter reading? Final meter reading?
- 2. How much electricity was consumed during the given period?
- 3. How much will the electric bill be for the given time period if the electricity charge is P9.50 / kiloWatt hour?

Reading Your Electric Meter

Nowadays, reading the electric meter would be easier considering that the newly-installed meters are digital, but most of the installed meters are still dial-based. Here are the steps in reading the electric meter:

- a. To read your dial-based electric meter, read the dials from left to right.
- b. If the dial hand is between numbers, the smaller of the two numbers should be used. If the dial hand is on the number, check out the dial to the right. If the dial hand has passed zero, use the number at which the dial hand is pointing. If the dial hand has not passed zero, use the smaller number than the number at which the dial hand is pointing.
- c. To determine the electric consumption for a given period, subtract the initial reading from the final reading.

NOTE TO THE TEACHER

The examples given here are simplified for discussion purposes. The computation reflected in the monthly electric bill is much more complicated than the examples given here. It is advisable to ask students to bring a copy of the electric bill of their own homes for a more thorough discussion of the topic.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions above:

- 1. What was the initial meter reading? final meter reading? The initial reading is 40493 kWh. For the first dial from the left, the dial hand is on number 4 so you look at the dial immediately to the right which is the second dial. Since the dial hand of the second dial is past zero already, then the reading for the first dial is 4. For the second dial, since the dial hand is between 0 and 1 then the reading for the second dial is 0. For the third dial from the left, the dial hand is on number 5 so you look at the dial immediately to the right which is the fourth dial. Since the dial hand of the fourth dial has not yet passed zero, then the reading for the third dial is 4. The final reading is 40515 kWh.
- 2. How much electricity was consumed during the given period? Final reading – initial reading = 40515 kWh – 40493 kWh = 22 kWh
- 3. How much will the electric bill be for the given time period if the electricity charge is \square 9.50 / kiloWatthour?

```
Electric bill = total consumption x electricity charge
= 22 kWh x P9.50 / kWh
= P209
```

III. Exercise:

Answer the following items. Show your solution.

1. The pictures below show the water meter reading of Sebastian's house.



Initial meter reading at 0726 hrs on 20 February 2012



Final meter reading at 0725 hrs on 21 February 2012

If the water company charges P14 / cubic meter of water used, how much must Sebastian pay the water company for the given period?

Step 1: Water consumption = final meter reading – initial meter reading Step 2:Payment = number of cubic meters of water consumed x rate

- = 2393.5 m3 2392.7 m3
- $= 0.8 \text{ m} 3 \text{ x} \square 14/\text{m} 3$
- = 0.8 m3
- **=** \(\prec11.20
- 2. The pictures below show the electric meter reading of Maria's canteen.



Initial meter reading at 1600 hrs on 20 Feb 2012



Final meter reading @ 1100 hrs on 22 Feb 2012

If the electric charge is P9.50 / kWh, how much will Maria pay the electric company for the given period?

Step 1: consumption = final meter reading - initial meter reading

Step 2: Payment = number of kWh consumed x rate

- = 10860 kWh 10836 kWh
- = 24 kWh x □ 9.50/kWh
- = 24 kWh
- **=** \square 228.00

3. The pictures below show the electric meter reading of a school.



Initial meter reading @1700 hrs on 15 July 2012



Final meter reading @ 1200 hrs on 16 July 2012

Assuming that the school's average consumption remains the same until 1700 hrs of 15 August 2012 and the electricity charge is P9.50 / kWh, how much will the school be paying the electric company?

Average hourly electric consumption

= (final meter reading – initial meter reading) / time
= (911.5 kWh – 907.7 kWh) / 19 hrs
= 0.2 kW

Electric consumption from 15 July 2012 to 15 August 2012 = average hourly consumption x number of hours

= 0.2 kW x 744 hrs = 148.8 kWh

Payment = number of kWh consumed x rate

= 148.8 kWh x □ 9.50/kWh

= \(1,413.60

Summary

In this lesson, you learned:

- 1. how to measure angles using a protractor;
- 2. how to estimate angle measurement;
- 3. express time in 12-hr or 24-hr notation;
- 4. how to measure the average speed as the quotient of distance over time;
- 5. convert units of temperature from one to the other;
- 6. solve problems involving time, speed and temperature;
- 7. read utilities usage.

Lesson 18: Constants, Variables and Algebraic Expressions

Prerequisite Concepts: Real Number Properties and Operations

Objectives:

At the end of the lesson, you should be able to:

- differentiate between constants and variables in a given algebraic expression; and
- 2. evaluate algebraic expressions for given values of the variables.

NOTE TO THE TEACHER

This lesson is an introduction to the concept of constants, unknowns and variables and algebraic expressions. Familiarity with this concept is necessary as a foundation for learning Algebra and in understanding and translating mathematical phrases and sentences, solving equations and algebraic word problems, and the concept of functions. In this lesson, it is important that you do not assume too much. Many misconceptions have arisen from a hurried up discussion of these basic concepts. Take care in introducing the concept of a letter and its different uses in algebra and the concept of a term in an algebraic expression.

Lesson Proper

I. Activity

A. Instructions: Complete the table below according to the pattern you see.

	TABLE A							
ROW	1 ST TERM	2 ND TERM						
a.	1	5						
b.	2	6						
C.	3	7						
d.	4							
e.	5							
f.	6							
g.	59							
h.	Any number							
	n							

- B. Using Table A as your basis, answer the following questions:
 - 1. What did you do to determine the 2nd term for rows d to f?
 - 2. What did you do to determine the 2nd term for row g?
 - 3. How did you come up with your answer in row h?
 - 4. What is the relation between the 1st and 2nd terms?
 - 5. Express the relation of the 1st and 2nd terms in a mathematical sentence.

NOTE TO THE TEACHER

Encourage your students to talk about the task and to verbalize whatever pattern they see. Again, do not hurry them up. Many students do not get the same insight as the fast ones.

II. Questions to Ponder (Post-Activity Discussion)

A. The 2^{nd} terms for rows d to f are 8, 9 and 10, respectively. The 2^{nd} term in row g is 63. The 2^{nd} term in row h is the sum of a given number n and 4.

B.

1. One way of determining the 2^{nd} terms for rows d to f is to add 1 to the 2^{nd} term of the preceding row (e.g. 7 + 1 = 8). Another way to determine the 2^{nd} term would be to add 4 to its corresponding 1^{st} term (e.g. 4 + 4 = 8).

NOTE TO TEACHER:

Most students would see the relation between terms in the same column rather than see the relation between the 1st and 2nd terms. Students who use the relation within columns would have a hard time determining the 2nd terms for rows g & h.

- 2. Since from row f, the first term is 6, and from 6 you add 53 to get 59, to get the 2^{nd} term of row g, 10 + 53 = 63. Of course, you could have simply added 4 to 59
- 3. The answer in row h is determined by adding 4 to *n*, which represents any number.
- 4. The 2nd term is the sum of the 1st term and 4.
- 5. To answer this item better, we need to be introduced to Algebra first.

Algebra

We need to learn a new language to answer item 5. The name of this language is Algebra. You must have heard about it. However, Algebra is not entirely a new language to you. In fact, you have been using its applications, and some of the terms have been used for a long time already. You just need to see it from a different perspective.

Algebra comes from the Arabic word, *al-jabr* (which means restoration), which in turn was part of the title of a mathematical book written around the 820 AD by the Arab mathematician, *Muhammad ibn Musa al-Khwarizmi*. While this book is widely considered to have laid the foundation of modern Algebra, history shows that ancient Babylonian, Greek, Chinese and Indian mathematicians were discussing and using algebra long before this book was published.

Once you've learned this new language, you'll begin to appreciate how powerful it is and how its applications have drastically improved our way of life.

III. Activity

NOTE TO THE TEACHER

It is crucial that students begin to think algebraically rather than arithmetically. Thus, emphasis is placed on how one reads algebraic expressions. This activity is designed to allow students to realize the two meanings of some signs and symbols used in both Arithmetic and Algebra, such as the equal sign and the operators +, -, and now x, which has become a variable and not a multiplication symbol. Tackling these double meanings will help your students transition comfortably from Arithmetic to Algebra.

Instructions: How do you understand the following symbols and expressions?

SYMBOLS /	MEANING
EXPRESSIONS	
1. <i>x</i>	
2. 2 + 3	
3. =	

IV. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the previous activity:

- 1. You might have thought of x as the multiplication sign. From here on, x will be considered a symbol that stands for any value or number.
- 2. You probably thought of 2 + 3 as equal to 5 and must have written the number 5. Another way to think of 2 + 3 is to read it as the sum of 2 and 3.
- 3. You must have thought, "Alright, what am I supposed to compute?" The sign "=" may be called the equal sign by most people but may be interpreted as a command to carry out an operation or operations. However, the equal sign is also a symbol for the relation between the expressions on its left and right sides, much like the less than "<" and greater than ">" signs.

The Language Of Algebra

The following are important terms to remember.

- a. **constant** a constant is a number on its own. For example, 1 or 127;
- b. variable a variable is a symbol, usually letters, which represent a value or a number. For example, a or x. In truth, you have been dealing with variables since pre-school in the form of squares (\square), blank lines (___) or other symbols used to represent the unknowns in some mathematical sentences or phrases:
- c. term a term is a constant or a variable or constants and variables multiplied together. For example, 4, xy or 8yz. The term's number part is called the numerical coefficient while the variable or variables is/are called the literal coefficient/s. For the term 8yz, the numerical coefficient is 8 and the literal coefficients are yz;
- d. expression an Algebraic expression is a group of terms separated by the plus or minus sign. For example, x - 2 or $4x + \frac{1}{2}y - 45$

Problem: Which of the following is/are equal to 5?

a. 2 + 3

b. 6 - 1 c. $\frac{10}{2}$ d. 1+4 e. all of these

Discussion: The answer is e since 2 + 3, 6 - 1, $\frac{10}{2}$ and 1 + 4 are all equal to 5.

NOTE TO TEACHER:

One of the most difficult obstacles is the transition from seeing say, an expression such as 2 + 3, as a sum rather than an operation to be carried out. A student of arithmetic would feel the urge to answer 5 instead of seeing 2 + 3 as an expression which is another way of writing the number 5. Since the ability to see expressions as both a process and a product is essential in grasping Algebraic concepts, more exercises should be given to students to make them comfortable in dealing with expressions as products as well as processes.

Notation

Since the letter x is now used as a variable in Algebra, it would not only be funny but confusing as well to still use x as a multiplication symbol. Imagine writing the product of 4 and a value x as 4xx! Thus, Algebra simplifies multiplication of constants and variables by just writing them down beside each other or by separating them using only parentheses or the symbol " •" . For example, the product of 4 and the value x (often read as four x) may be expressed as 4x, 4(x) or 4•x. Furthermore, division is more often expressed in fraction form. The division sign ÷ is now seldom used.

NOTE TO TEACHER:

A common misconception is viewing the equal sign as a command to execute an operational sign rather than regard it as a sign of equality. This may have been brought about by the treatment of the equal sign in arithmetic (5 + 2 = 7, 3 - 2 = 1, etc.). This misconception has to be corrected before proceeding to the discussion on the properties of equality and solving equations since this will pose as an obstacle in understanding these concepts.

Problem: Which of the following equations is true?

a. 12 + 5 = 17

b. 8 + 9 = 12 + 5

c. 6 + 11 = 3(4 + 1) + 2

Discussion: All of the equations are true. In each of the equations, both sides of the equal sign give the same number though expressed in different forms. In a) 17 is the same as the sum of 12 and 5. In b) the sum of 8 and 9 is 17 thus it is equal to the sum of 12 and 5. In c) the sum of 6 and 11 is equal to the sum of 2 and the product of 3 and the sum of 4 and 1.

NOTE TO THE TEACHER

The next difficulty is what to do with letters when values are assigned to them or when no value is assigned to them. Help students understand that letters or variables do not always have to have a value assigned to them, but that they should know what to do when letters are assigned numerical values.

On Letters and Variables

Problem: Let x be any real number. Find the value of the expression 3x (the product of 3 and x, remember?) if

a)
$$x = 5$$

b)
$$x = \frac{1}{2}$$

b)
$$x = \frac{1}{2}$$
 c) $x = -0.25$

Discussion: The expression 3x means multiply 3 by any real number x. Therefore,

- a) If x = 5, then 3x = 3(5) = 15.
- b) If $x = \frac{1}{2}$, then $3x = 3(\frac{1}{2}) = \frac{3}{2}$
- c) If x = -0.25, then 3x = 3(-0.25) = -0.75

The letters such as x, y, n, etc. do not always have specific values assigned to them. When that is the case, simply think of each of them as any number. Thus, they can be added (x + y), subtracted (x - y), multiplied (xy), and divided (x/y) like any real number.

Problem: Recall the formula for finding the perimeter of a rectangle, P = 2L + 2W. This means you take the sum of twice the length and twice the width of the rectangle to get the perimeter. Suppose the length of a rectangle is 6.2 cm and the width is $^{1}/_{8}$ cm, what is the perimeter?

Discussion: Let L = 6.2 cm and W =
$$^{1}/_{8}$$
 cm. Then,
P = 2(6.2) + 2($^{1}/_{8}$) = 12.4 + $^{1}/_{4}$ = 12.65 cm

V. Exercises:

Note to teacher: Answers are in bold characters.

- 1. Which of the following is considered a constant?
- a. f b. □ c. 500
- 2. Which of the following is a term?
- a. 23m + 5 b. (2)(6x)
- c. x y + 2 d. $\frac{1}{2}x y$

d. 42x

d. 11(5)

- 3. Which of the following is equal to the product of 27 and 2?
 - a. 29 b. 49 + 6 **c. 60 6**
- 4. Which of the following makes the sentence 69 3 = ____ + 2 true?
 a. 33 b. **64** c. 66 d. 68
- a. 33 b. **64** 5. Let y = 2x + 9. What is y when x = 5?
- 5. Let *y* = 2*x* + 9. What is *y* when *x* = 5? a. 118 b. 34 c. 28 **d. 19**

Let us now answer item B.5. of the initial problem using Algebra:

1. The relation of the 1st and 2nd terms of Table A is "the 2nd term is the sum of the 1st term and 4". To express this using an algebraic expression, we use the letters n and y as the variables to represent the 1st and 2nd terms, respectively. Thus, if n represents the 1st term and y represents the 2nd term, then

$$y = n + 4$$
.

FINAL PROBLEM:

A. Fill the table below:

	TABLE B				
ROW	1 ST TERM	2 ND TERM			
a.	10	23			
b.	11	25			
C.	12	27			
d.	13	29			
e.	15	33			

f.	18	39
g.	37	77
h.	n	2(n) + 3

- B. Using Table B as your basis, answer the following questions:
 - 1. What did you do to determine the 2nd term for rows d to f?
 - 2. What did you do to determine the 2nd term for row g?
 - 3. How did you come up with your answer in row h?
 - 4. What is the relation between the 1st and 2nd terms? The 2nd term is the sum of twice the 1st term and 3.
 - 5. Express the relation of the 1st and 2nd terms using an algebraic expression. Let y be the 2nd term and x be the 1st term, then y = 2x + 3.

Summary

In this lesson, you learned about constants, letters and variables, and algebraic expressions. You learned that the equal sign means more than getting an answer to an operation; it also means that expressions on either side have equal values. You also learned how to evaluate algebraic expressions when values are assigned to letters.

Lesson 19: Verbal Phrases and Mathematical Phrases

Time: 2 hours

Prerequisite Concepts: Real Numbers and Operations on Real Numbers

Objectives

In this lesson, you will be able to translate verbal phrases to mathematical phrases and vice versa.

NOTE TO THE TEACHER

Algebra is a language that has its own "letter", symbols, operators and rules of "grammar". In this lesson, care must be taken when translating because you still want to maintain the correct grammar in the English phrase without sacrificing the correctness of the equivalent mathematical expression.

Lesson Proper

I. Activity 1

Directions: Match each verbal phrase under Column A to its mathematical phrase under Column B. Each number corresponds to a letter which will reveal a quotation if answered correctly. A letter may be used more than once.

	Column A	Column B					
2. Fo 3. Or 4. A 5. Fo 6. A 7. Th 8. Tv 9. A 10. Th 11. Th 12. Th 13. A	pur times a certain number decreased by one me subtracted from four times a number certain number decreased by two pur increased by a certain number certain number decreased by three decreased by th	A. $x + 3$ B. $3 + 4x$ E. $4 + x$ A. $x + 4$ A. $x + 4$ A. $x - 1$ A. $x - 2$ A. $x - 3$ B. $x - 3$ C. $x - 3$					
NOTE TO THE TEACHER Make sure that all phrases in both columns are clear to the students. II. Question to Ponder (Post-Activity Discussion) Which phrase was easy to translate? Translate the mathematical expression 2(x-3) in at least two ways.							

III. Activity 2

Directions: Choose the words or expressions inside the boxes, and write it under its respective symbol.

plus increased by	more than subtracted from	times multiplied by	divided by ratio of	is less than is greater than or equal to
is greater than	the quotient of	of	is at most	is less than or equal to
the sum of is at least	the difference of the product of	diminished by decreased by	less than is not equal to	added to minus

+	_	х	÷	<	>	<	>	≠
increased	decreased	multiplied	ratio of	is	is	is less	is	is not
by	by	by		less than	greater than	than or equal to	greater than or equal to	equal to
added to	subtracted from	of	the quotient of			is at most	is at least	
the sum of	the difference of	the product of						
more than	less than							
	diminished by							

IV. Question to Ponder (Post-Activity Discussion)

- 1. Addition would indicate an increase, a putting together, or combining. Thus, phrases like *increased by* and *added to* are addition phrases.
- 2. Subtraction would indicate a lessening, diminishing action. Thus, phrases like *decreased by, less, diminished by* are subtraction phrases.
- 3. Multiplication would indicate a multiplying action. Phrases like *multiplied by* or *n times* are multiplication phrases.
- 4. Division would indicate partitioning, a quotient, and a ratio. Phrases such as *divided by, ratio of,* and *quotient of* are common for division.
- 5. The inequalities are indicated by phrases such as *less than*, *greater than*, *at least*, and *at most*.
- 6. Equalities are indicated by phrases like the same as and equal to.

NOTE TO THE TEACHER

Emphasize to students that these are just some common phrases. They should not rely too much on the specific phrase, but instead on the meaning of the phrases.

V. THE TRANSLATION OF THE "=" SIGN

Directions: The table below shows two columns, A and B. Column A contains mathematical sentences, while Column B contains their verbal translations. Observe the items under each column and compare. Answer the proceeding questions.

Column A Mathematical Sentence	Column B Verbal Sentence	
x + 5 = 4	The sum of a number and 5 is 4.	
2x - 1 = 1	Twice a number decreased by 1 is equal to 1.	
7 + x = 2x + 3	Seven added by a number x is equal to twice the same number increased by 3.	
3x = 15	Thrice a number x yields 15.	
x - 2 = 3	Two less than a number x results to 3.	

VI. Question to Ponder (Post-Activity Discussion)

- 1) Based on the table, what do you observe are the common verbal translations of the "=" sign? "is", "is equal to"
- 2) Can you think of other verbal translations for the "=" sign? "results in", "becomes"
- 3) Use the phrase "is equal to" on your own sentence.
- **4)** Write your own pair of mathematical sentence and its verbal translation on the last row of the table.
 - 4 x < 5: Four decreased by a certain number is less than 5.

VII. Exercises:

A. Directions: Write your responses on the space provided.

1. Write the verbal translation of the formula for converting temperature from

Celsius (C) to Fahrenheit (F) which is
$$F = \frac{9}{5}C + 32$$
.

The temperature in Fahrenheit (F) is nine-fifths of the temperature in Celsius (C) increased by (plus) 32.

The temperature in Fahrenheit (F) is 32 more than nine-fifths of the temperature in Celsius (C).

2. Write the verbal translation of the formula for converting temperature from

Fahrenheit (F) to Celsius (C) which is
$$C = \frac{5}{9} F - 32$$
.

The temperature in Celsius (C) is five-ninths of the difference of the temperature in Fahrenheit (F) and 32.

3. Write the verbal translation of the formula for simple interest: I = PRT, where I is simple interest, P is Principal Amount, R is Rate and T is time in years.

The simple interest (I) is the product of the Principal Amount (P), Rate (R) and time (T) in years.

4. The perimeter (P) of a rectangle is twice the sum of the length (L) and width (W). Express the formula of the perimeter of a rectangle in algebraic expressions using the indicated variables.

Answer: P = 2(L + W)

5. The area (A) of a rectangle is the product of length (L) and width (W).

Answer: A = LW

6. The perimeter (P) of a square is four times its side (S).

Answer: P = 4S

7. Write the verbal translation of the formula for Area of a Square (A): $A = s^2$, where s is the length of a side of a square.

The Area of a Square (A) is the square of side (s).

8. The circumference (C) of a circle is twice the product of π and radius (r).

Answer: $C = 2\pi r$

9. Write the verbal translation of the formula for Area of a Circle (A): $A = \pi r^2$, where r is the radius.

The Area of a Circle (A) is the product π and the square of radius (r).

10. The midline (k) of a trapezoid is half the sum of the bases (a and b) or the sum of the bases (a and b) divided by 2.

Answer: $k = \frac{1}{2} a + b$

11. The area (A) of a trapezoid is half the product of the sum of the bases (a and b) and height (h).

$$A = \frac{1}{2}(a+b)h$$

12. The area (A) of a triangle is half the product of the base (b) and height (h).

$$\mathbf{A} = \frac{1}{2}bh$$

13. The sum of the angles of a triangle (A, B and C) is 180°.

 $A + B + C = 180^{\circ}$

14. Write the verbal translation of the formula for Area of a Rhombus (A): A =

 $\frac{1}{2}d_1d_2$, where d_1 and d_2 are the lengths of diagonals.

The Area of a Rhombus (A) is half the product of the diagonals, d_1 and d_2 .

15. Write the verbal translation of the formula for the Volume of a rectangular parallelepiped (V): A = lwh, where l is the length, w is the width and h is the height.

The Volume of a regular parallelepiped (V) is the product of the length (I), width (w) and height (h).

16. Write the verbal translation of the formula for the Volume of a sphere (V): V =

 $\frac{4}{3}\pi r^3$, where *r* is the radius.

The Volume of a sphere (V) is four-thirds of the product of π and the square of radius (r).

17. Write the verbal translation of the formula for the Volume of a cylinder (V): $V = \pi r^2 h$, where r is the radius and h is the height.

The Volume of a cylinder (V) is the product of π , the square of radius (r) and height (h).

18. The volume of the cube (V) is the cube of the length of its edge (a). Or the volume of the cube (V) is the length of its edge (a) raised to 3. Write its formula. $V = a^3$

NOTE TO THE TEACHER

Allow students to argue and discuss, especially since not all are well versed in the English language.

B. Directions: Write as many verbal translations as you can for this mathematical sentence.

$$3x - 2 = -4$$

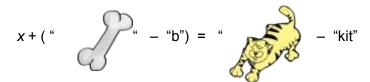
Possible answers are

- 1. Three times (Thrice) a number x decreased by (diminished by) two is (is equal to/results to/yields to) 4.
- 2. 2 less than three times (Thrice) a number x is (is equal to/results to/yields to) 4.
- 3. 2 subtracted from three times (Thrice) a number x is (is equal to/results to/yields to) 4.
- 4. The difference of Three times (Thrice) a number x and two is (is equal to/results to/yields to) 4.

C. REBUS PUZZLE

Try to answer this puzzle!

What number must replace the letter x?



Answer: $x + 1 = 10 \rightarrow x = 9$

SUMMARY

In this lesson, you learned that verbal phrases can be written in both words and in mathematical expressions. You learned common phrases associated with addition, subtraction, multiplication, division, the inequalities and the equality. With this lesson, you must realize by now that mathematical expressions are also meaningful.

Lesson 20: Polynomials Time: 1.5 hours

Pre-requisite Concepts: Constants, Variables, Algebraic expressions

Objectives:

In this lesson, the students must be able to:

- 1) give examples of polynomials, monomials, binomials, and trinomials:
- 2) identify the base, coefficient, terms, and exponents in a given polynomial.

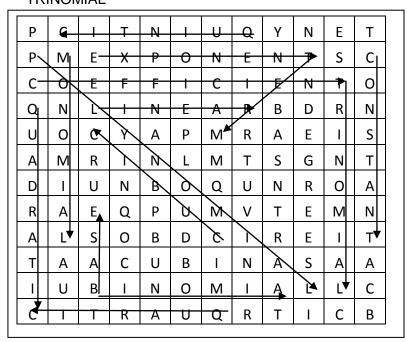
Lesson Proper:

I. A. Activity 1: Word Hunt

Find the following words inside the box.

BASE CUBIC
COEFFICIENT LINEAR
DEGREE QUADRATIC
EXPONENT QUINTIC
TERM QUARTIC

CONSTANT BINOMIAL MONOMIAL POLYNOMIAL TRINOMIAL



NOTE TO THE TEACHER:

These words may be given as assignment before the teaching day so that the students can participate actively during the activity. The easier way of defining the terms is by giving an example.

Definition of Terms

In the algebraic expression $3x^2 - x + 5$, $3x^2$, -x and 5 are called the terms.

Term is a constant, a variable or a product of constant and variable. In the term $3x^2$, 3 is called the numerical coefficient and x^2 is called the **literal** coefficient.

In the term –x has a **numerical coefficient** which is -1 and a literal coefficient which is x.

The term 5 is called the *constant*, which is usually referred to as the term without a variable.

Numerical coefficient is the constant/number.

Literal coefficient is the variable including its exponent.

The word **Coefficient** alone is referred to as the numerical coefficient.

In the literal coefficient x^2 , x is called the **base** and 2 is called the **exponent**.

Degree is the highest exponent or the highest sum of exponents of the variables in a term.

In $3x^2 - x + 5$, the degree is 2.

In $3x^2y^3 - x^4y^3$ the degree is 7.

Similar Terms are terms having the same literal coefficients.

 $3x^2$ and $-5x^2$ are similar because their literal coefficients are the same.

5x and $5x^2$ are NOT similar because their literal coefficients are NOT the same.

 $2x^3y^2$ and $-4x^2y^3$ are NOT similar because their literal coefficients are NOT the same.

NOTE TO THE TEACHER:

Explain to the students that a constant term has no variable, hence the term <u>constant</u>. Its value does not change.

A *polynomial* is a kind of algebraic expression where each term is a constant, a variable or a product of a constant and variable in which the variable has a whole number (non-negative number) exponent. A polynomial can be a monomial, binomial, trinomial or a multinomial.

An algebraic expression is **NOT** a polynomial if

- 1) the exponent of the variable is NOT a whole number {0, 1, 2, 3..}.
- 2) the variable is inside the radical sign.
- 3) the variable is in the denominator.

NOTE TO THE TEACHER:

Explain to the students the difference between multinomial and polynomial. Give emphasis on the use of the prefixes mono, bi, tri and multi or poly.

Kinds of Polynomial according to the number of terms

- 1) Monomial is a polynomial with only one term
- 2) Binomial is polynomial with two terms
- 3) Trinomial is a polynomial with three terms
- 4) Polynomial is a polynomial with four or more terms

B. Activity 2

Tell whether the given expression is a polynomial or not. If it is a polynomial, determine its degree and tell its kind according to the number of terms. If it is NOT, explain why.

6)
$$x^{\frac{1}{2}} - 3x + 4$$

2)
$$x^2 - 5xy$$

7)
$$\sqrt{2} x^4 - x^7 + 3$$

8)
$$3x^2\sqrt{2x}-1$$

4)
$$3x^2 - 5xy + x^3 + 5$$

9)
$$\frac{1}{3}x - \frac{3x^3}{4} + 6$$

5)
$$x^3 - 5x^{-2} + 3$$

10)
$$\frac{3}{x^2} - x^2 - 1$$

NOTE TO THE TEACHER:

We just have to familiarize the students with these terms so that they can easily understand the different polynomials. This is also important in solving polynomial equations because different polynomial equations have different solutions.

Kinds of Polynomial according to its degree

- 1) Constant a polynomial of degree zero
- 2) Linear a polynomial of degree one
- 3) Quadratic a polynomial of degree two
- 4) Cubic a polynomial of degree three
- 5) Quartic a polynomial of degree four
- 6) Quintic a polynomial of degree five

A polynomial is in **Standard Form** if its terms are arranged from the term with the highest degree, up to the term with the lowest degree.

^{*} The next degrees have no universal name yet so they are just called "polynomial of degree ____."

If the polynomial is in standard form the first term is called the *Leading Term*, the numerical coefficient of the leading term is called the *Leading Coefficient* and the exponent or the sum of the exponents of the variable in the leading term the *Degree* of the polynomial.

The standard form of $2x^2 - 5x^5 - 2x^3 + 3x - 10$ is $-5x^5 - 2x^3 + 2x^2 + 3x - 10$. The terms $-5x^5$ is the leading term, -5 is its leading coefficient and 5 is its degree. It is a quintic polynomial because its degree is 5.

C. Activity 3
Complete the table.

Given	Leading Term	Leading Coefficient	Degree	Kind of Polynomial according to the no. of terms	Kind of Polynomial According to the degree	Standard Form
1) 2x + 7	2x	2	1	monomial	linear	2x + 7
2) $3 - 4x + 7x^2$	7x²	7	2	trinomial	quadratic	$7x^2 - 4x + 3$
3) 10	10	10	0	monomial	constant	10
4) $x^4 - 5x^3 + 2x - x^2 - 1$	x ⁴	1	4	multinomial	quartic	$x^4 - 5x^3$ - $x^2 + 2x$ - 1
5) $5x^5 + 3x^3 - x$	5x ⁵	5	5	Trinomial	Quintic	$5x^5 + 3x^3 - x$
6) 3 – 8x	-8x	-8	1	Binomial	Linear	- 8x + 3
7) x ² – 9	x ²	1	2	Binomial	Quadratic	$x^2 - 9$
8) $13 - 2x + x^5$	x ⁵	1	5	Trinomial	Quintic	x ⁵ – 2x + 13
9) 100x ³	100x ³	100	3	Monomial	Cubic	100x ³
10) $2x^3$ - $4x^2$ + $3x^8$ - 6	3x ⁸	3	8	Multinomial	Polynomial of degree 8	3x ⁸ +2x3 - 4x ² - 6

Summary

In this lesson, you learned about the terminologies in polynomials: term, coefficient, degree, similar terms, polynomial, standard form, leading term, and leading coefficient.

Lesson 21: Laws of Exponents

Pre-requisite Concepts:

The students have mastered the multiplication.

Objectives:

In this lesson, the students must be able to:

- 1) define and interpret the meaning of aⁿ where n is a positive integer:
- 2) derive inductively the Laws of Exponents (restricted to positive integers)
- 3) illustrate the Laws of Exponents.

Lesson Proper

I. Activity 1

Give the product of each of the following as fast as you can.

1) 3 x 3 =	Ans. 9
2) 4 x 4 x 4 =	Ans. <i>64</i>
3) 5 x 5 x 5 =	Ans. 125
4) 2 x 2 x 2 =	Ans. 8
5) 2 x 2 x 2 x 2 =	Ans. 16
6) 2 x 2 x 2 x 2 x 2 =	Ans. 32

II. Development of the Lesson

Discovering the Laws of Exponent

NOTE TO THE TEACHER:

You can follow up this activity by telling the students that $3 \times 3 \times 3 =$ 3^3 , $4 \times 4 \times 4 = 4^3$ and so on. From here, you can now explain the very first and basic law of exponent. The elementary teachers have discussed this already.

A)
$$a^n = a \times a \times a \times a \dots$$
 (n times) In a^n , a is called the base and n is called the exponent

Time: 1.5 hours

NOTE TO THE TEACHER:

We have to emphasize that violation of a law means a wrongdoing. So tell them that there is no such thing as multiplying the base and the exponent as stated in the very first law.

Exercises

1) Which of the following is/are correct?

a)
$$4^2 = 4 \times 4 = 16$$
 b) $2^4 = 2 \times 2 \times 2 \times 2 = 8$ c) $2^5 = 2 \times 5 = 10$ d) $3^3 = 3 \times 3 \times 3 = 27$

Sample Ans. CORRECT INCORRECT INCORRECT CORRECT

2) Give the value of each of the following as fast as you can.

a) 2³

b) 2⁵

c) 3⁴

d) 10⁶

Sample Ans.

8

32

81

1,000,000

NOTE TO THE TEACHER:

It is important to tell the students to use a "dot" or "parenthesis" as a symbol for multiplication because at this stage, we are already using x as a variable.

Let the students explore the next activities. If they can't figure out what you want them to see, guide them. Throw more questions. If it won't work, do the lecture. The "What about these" are follow-up questions. The students should be the one to answer it.

Activity 2

Evaluate the following by applying the law that we have discussed. Investigate the result. Make a simple conjecture on it. The first two are done for you.

1)
$$(2^3)^2 = 2^3 \cdot 2^3 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$$

3)
$$(3^2)^2 =$$

Ans. 81

4)
$$(2^2)^3 =$$

Ans. 64

5)
$$(a^2)^5 =$$

Ans. a¹⁰

Did you notice something?

What can you conclude about $(a^n)^m$? What will you do with a, n and m?

$$B)\left(a^{n}\right)^{m}=a^{nm}$$

What about these?

1)
$$(x^{100})^3 =$$

2) $(y^{12})^5 =$

Ans. *x*³⁰⁰ Ans. *y*⁶⁰

Activity 3

Evaluate the following. Notice that the bases are the same. The first example is done for you.

1)
$$(2^3)(2^2) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$$

2)
$$(x^5)(x^4) =$$
 Ans. x^9

3)
$$(3^2)(3^4) =$$
 Ans. 729

4)
$$(2^4)(2^5) =$$
 Ans. 512

5)
$$(x^3)(x^4) =$$
 Ans. x^7

Did you notice something?

What can you conclude about $a^n \cdot a^m$? What will you do with a, **n** and **m**?

C)
$$a^n \cdot a^m = a^{n+m}$$

What about these?

Activity 4

Evaluate each of the following. Notice that the bases are the same. The first example is done for you.

1)
$$\frac{2^7}{2^3} = \frac{128}{8} = 16$$
 \rightarrow remember that 16 is the same as 2⁴

2)
$$\frac{3^5}{3^3}$$
 =

3)
$$\frac{4^3}{4^2}$$
 =

4)
$$\frac{2^8}{2^6}$$
 =

Did you notice something?

What can you conclude about $\frac{a^n}{a^m}$? What will you do with **a**, **n**

and *m*?

$$D) \frac{a^n}{a^m} = a^{n-m}$$

What about these?

1)
$$\frac{x^{20}}{x^{13}}$$

2)
$$\frac{y^{105}}{y^{87}}$$

NOTE TO THE TEACHER:

After they finished the discovery of the laws of exponent, it is very important that we summarize those laws. Don't forget to tell them that there are still other laws of exponent, which they will learn in the next stage (second year).

Laws of exponents

1)
$$a^n = a \cdot a \cdot a \cdot a \cdot a \cdot a$$
..... (n times

4)
$$\frac{a^n}{a^m} = a^{n-m}$$
 quotient of a power

NOTE TO THE TEACHER:

The next two laws of exponent are for you to discuss with your students.

5) $a^0 = 1$ where $a \ne 0$ **law for zero exponent** Ask the students. "If you divide a number by itself, what is the answer?"

Follow it up with these: (Do these one by one)

No.	Result	Applying a law of Exponent	GIVEN (Start here)	ANSWER	REASON
1)	5° ←	5 ¹⁻¹	$\frac{5}{5}$	_ 1	-
2)	100 ⁰ ◀	1001-1	100 100 ←	1 -	Any
3)	x ⁰ ←	X ¹⁻¹	$\frac{x}{x}$	1 -	divided by →itself is equal to 1.
4)	a ⁰ ←	a ⁵⁻⁵	$\frac{a^5}{a^5} \leftarrow$	_ 1 -	-

You can draw the conclusion from the students. As they will see, all numbers that are raised to zero is equal to 1. But take note, the *base* should not be equal to zero because division by zero is not allowed.

What about these?

a)
$$(7,654,321)^0$$

b) $3^0 + x^0 + (3y)^0$

6)
$$a^{-n} = \frac{1}{a^n}$$
 and $\frac{1}{a^{-n}} = a^n$ law for negative exponent

You can start the discussion by showing this to the students.

a)
$$\frac{2}{4} = \frac{1}{2}$$
 then show that $\frac{2}{4} = \frac{2^1}{2^2} = 2^{1-2}$ which means $2^{1-2} = 2^{-1} = \frac{1}{2}$

b) $\frac{4}{32} = \frac{1}{8}$ then show that $\frac{4}{32} = \frac{2^2}{2^5} = 2^{2-5}$ which means $2^{2-5} = 2^{-3} = \frac{1}{8}$

c) $\frac{27}{81} = \frac{1}{3}$ then show that $\frac{27}{81} = \frac{3^3}{3^4} = 3^{3-4}$ which means $3^{3-4} = 3^{-1} = \frac{1}{3}$

Now ask them.

What did you notice? What about these?

d)
$$x^{-2}$$
 Ans. $\frac{1}{x^2}$ e) 3^{-3} Ans. $\frac{1}{27}$ Ans. $\frac{1}{4}$

Now, explain them the rule. If you can draw it from them, better.

III. Exercises

A. Evaluate each of the following.

1)
$$2^8$$
 Ans. **256** 6) $(2^3)^3$ Ans. **512**
2) 8^2 Ans. **64** 7) $(2^4)(2^3)$ Ans. **128**
3) 5^{-1} Ans. **1/5** 8) $(3^2)(2^3)$ Ans. **72**
4) 3^{-2} Ans. **1/9** 9) $x^0 + 3^{-1} - 2^2$ Ans. **-8/3**
5) 18^0 Ans. 1 10) $[2^2 - 3^3 + 4^4]^0$ Ans. 1

B. Simplify each of the following.

1)
$$(x^{10})(x^{12})$$
 Ans. x^{22}
2) $(y^{-3})(y^8)$ Ans. y^5
3) $(m^{15})^3$ Ans. m^{45}
4) $(d^{-3})^2$ Ans. $1/d^6$
5) $(a^{-4})^{-4}$ Ans. a^{16}
6) $\frac{z^{23}}{z^{15}}$ Ans. z^8
7) $\frac{b^8}{b^{12}}$ Ans. $1/b^4$
8) $\frac{c^3}{c^{-2}}$ Ans. c^5
9) $\frac{x^7y^{10}}{x^3y^5}$ Ans. x^4y^5
10) $\frac{a^8b^2c^0}{a^5b^5}$ Ans. a^{3}/b^3
11) $\frac{a^8a^3b^{-2}}{a^{-1}b^{-5}}$ Ans. $a^{12}b^3$

Summary:

In these lessons, you have learned some laws of exponent.

Lesson 22: Addition and Subtraction of Polynomials

Pre-requisite Concepts: Similar Terms, Addition and Subtraction of Integers

Time: 2 hours

About the Lesson: This lesson will teach students how to add and subtract polynomials using tiles at first and then by paper and pencil after.

Objectives:

In this lesson, the students are expected to:

- 1) add and subtract polynomials;
- 2) solve problems involving polynomials.

NOTE TO THE TEACHER

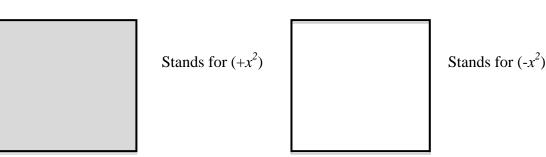
It is possible that at this point, some of your students still cannot relate to x's and y's. If that is so, then they will have difficulty moving on with the next lessons. The use of Tiles in this lesson is a welcome respite for students who are struggling with variables, letters, and expressions. Take advantage and use these tiles to the full. You may make your own tiles.

Lesson Proper:

I. Activity 1

Stands for $(+1)$	S	tands for $(+x)$

Stands for (-1)	Stands for $(-x)$



Can you represent the following quantities using the above tiles?

- 1. x-2
- 2. 4x + 1

Activity 2.

Use the tiles to find the sum of the following polynomials;

- 1. 5x + 3x
- 2. (3x-4)-6x

Familiarize yourself with the tiles below:

3. $(2x^2 - 5x + 2) + (3x^2 + 2x)$

Can you come up with the rules for adding polynomials?

II. Questions/Points to Ponder (Post-Activity Discussion) The tiles can make operations on polynomials easy to understand and do. Let us discuss the first activity. 1. To represent x - 2, we get one (+x) tile and two (-1) tiles. 2. To represent 4x + 1, we get four (+x) tiles and one (+1) tile. What about the second activity? Did you pick out the correct tiles? 5x + 3x1. Get five (+x tiles) and three more (+x) tiles. How many do you have in all? There are eight (+x) altogether. Therefore, 5x + 3x = 8x. 2. (3x - 4) - 6xGet three (+x) tiles and four (-1) tiles to represent (3x - 4). Add six (-x)[Recall that subtraction also means adding the negative of the quantity.]

Now, recall further that a pair of one (+x) and one (-x) is zero. What tiles do you have left?

That's right, if you have with you three (-x) and four (-1), then you are correct. That means the sum is (-3x - 4).

NOTE TO THE TEACHER

At this point, encourage your students to work on the problems without using Tiles if they are ready. Otherwise, let them continue using the tiles.

3.
$$(2x^2 - 5x + 2) + (3x^2 + 2x)$$

What tiles would you put together? You should have two $(+x^2)$, five (-x) and two (+1) tiles then add three $(+x^2)$ and two (+x) tiles. Matching the pairs that make zero, you have in the end five $(+x^2)$, three (-x), and two (+1) tiles. The sum is $5x^2 - 3x + 2$.

Or, using your pen and paper, you will have:

$$(2x^2 - 5x + 2) + (3x^2 + 2x) = (2x^2 + 3x^2) + (-5x + 2x) + 2 = 5x^2 - 3x + 2$$

NOTE TO THE TEACHER

Make sure your students can verbalize what they do to add polynomials so that it is easy for them to remember the rules.

Rules for Adding Polynomials

To add polynomials, simply combine similar terms. To combine similar terms, get the sum of the numerical coefficients and annex the same literal coefficients. If there is more than one term, for convenience, write similar terms in the same column.

NOTE TO THE TEACHER:

You may give as many examples as you want if you think that your students need it. Your number of examples may vary depending on the ability of your students. If you think that the students understand it after two examples, you may let them work on the next examples.

Do you think you can add polynomials now without the tiles? Perform the operation.

1) Add
$$4a - 3b + 2c$$
, $5a + 8b - 10c$ and $-12a + c$.
 $4a - 3b + 2c$
 $5a + 8b - 10c$
 $+ -12a + c$
 $-3a + 5b - 7c$
2) Add $13x^4 - 20x^3 + 5x - 10$ and $-10x^2 - 8x^4 - 15x + 10$.
 $13x^4 - 20x^3 + 5x - 10$
 $+ -8x^4 - 10x^2 - 15x + 10$
 $5x^4 - 20x^3 - 10x^2 - 10x$

Rules for Subtracting Polynomials

To subtract polynomials, change the sign of the subtrahend then proceed to the addition rule. Also, remember what subtraction means. It is adding the negative of the quantity.

Perform the operation.

1)
$$5x - 13x = 5x + (-5x) + (-8x) = -8x$$

1)
$$5x - 13x = 5x + (-5x) + (-8x) = -8x$$

2) $2x^2 - 15x + 25$ $2x^2 - 15x + 25$ $-3x^2 + 12x - 18$ $+-3x^2 - 12x + 18$

3)
$$(30x^3 - 50x^2 + 20x - 80) - (17x^3 + 26x + 19)$$

$$30x^3 - 50x^2 + 20x - 80$$

+ $-17x^3$ - $26x - 19$

III. Exercises

A. Perform the indicated operation by first using the tiles when applicable, then using paper and pen.

1)
$$3x + 10x$$

7)
$$20x^2y^2 + 30x^2y^2$$

3)
$$14x^3 + (-16x^3)$$

8)
$$-9x^2y + 9x^2y$$

4)
$$-5x^3 - 4x^3$$

9)
$$10x^2y^3 - 10x^3y^2$$

5)
$$2x - 3y$$

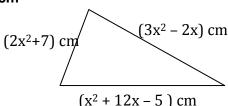
10)
$$5x - 3x - 8x + 6x$$

Answers: 1) 13x; 2) -6y; 3) $-2x^3$; 4) $-9x^3$; 5) 2x - 3y; 6) 2xy; 7) $50x^2y^2$; 8) 0; 9) $10x^2y^3 - 10x^3y^2$; 10) 0

NOTE TO THE TEACHER: You may do this in the form of a game.

B. Answer the following questions. Show your solution.

- 1) What is the sum of $3x^2 11x + 12$ and $18x^2 + 20x 100$? **21x² +**
- 2) What is $12x^3 5x^2 + 3x + 4$ less than $15x^3 + 10x + 4x^2 10$? $3x^3$ $+9x^2+7x-14$
- 3) What is the perimeter of the triangle shown at the right? $(6x^2 + 10x)$ + 2) cm



- 4) If you have $(100x^3 5x + 3)$ pesos and you spent $(80x^3 2x^2 + 9)$ pesos in buying foods how much money is left? $(20x^3 + 2x^2 5x 6)$ pesos
- 5) What must be added to 3x + 10 to get a result of 5x 3? 2x 13

NOTE TO THE TEACHER:

The summary of the lesson should be drawn from the students (as much as possible). Let the students re-state the rules. This is a way of checking what they have learned and how they understand the lesson.

Summary

In this lesson, you learned about tiles and how to use them to represent algebraic expressions. You learned how to add and subtract terms and polynomials using these tiles. You were also able to formulate/deduce the rules in adding and subtracting polynomials. To add polynomials, simply combine similar terms. To combine similar terms, get the sum of the numerical coefficients and annex the same literal coefficients. If there is more than one term, for convenience, write similar terms in the same column. To subtract polynomials, change the sign of the subtrahend then proceed to the addition rule.

Lesson 23: Multiplying Polynomials

Pre-requisite Concepts: Laws of exponents, Adding and Subtracting Polynomials, Distributive Property of Real Numbers

Objectives:

In this lesson, you should be able to:

- 1) multiply polynomials such as;
 - a) monomial by monomial,
 - b) monomial by polynomial with more than one term,
 - c) binomial by binomial,
 - d) polynomial with more than one term to polynomial with three or more terms.
- 2) solve problems involving multiplying polynomials.

NOTE TO THE TEACHER

Give students a chance to work with the Tiles. These tiles not only help provide a context for multiplying polynomials, but they also help students learn special products in the future. Give your students time to absorb and process the many steps and concepts involved in multiplying polynomials.

Lesson Proper

I. Activity

Familiarize yourself with the following tiles:

Stands for (+x2)	Stands for (+x)
	Stands for (-x)
Stands for (-x ²)	Stands for (+1)
	Stands for (-1)

Now, find the following products and use the tiles whenever applicable:

1) (3*x*) (*x*)

2) (-x)(1+x)

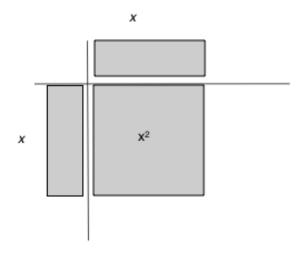
3) (3 - x)(x + 2)

Time: 3 hours

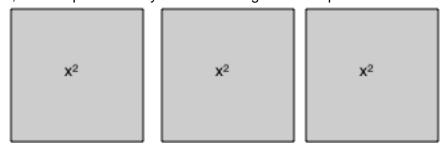
Can you tell what the algorithms are in multiplying polynomials?

II. Questions/Points to Ponder (Post-Activity Discussion)

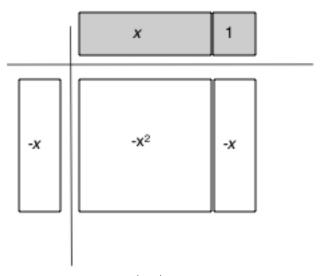
Recall the Laws of Exponents. The answer to item (1) should not be a surprise. By applying the Laws of Exponents, $(3x)(x) = 3x^2$, can you use the tiles to show this product?



So, $3x^2$ is represented by three of the big shaded squares.

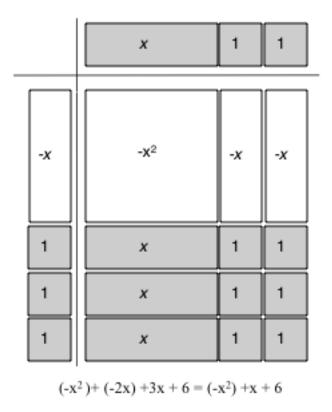


What about item (2)? The product (-x)(1+x) can be represented by the following.



The picture shows that the product is $(-x^2)+(-x)$. Can you explain what happened? Recall the sign rules for multiplying.

For the third item is (3 - x)(x + 2), how can you use the Tiles to show the product?



Rules in Multiplying Polynomials

NOTE TO THE TEACHER:

Emphasize to the students that the most important thing that they have to remember in multiplying polynomials is the "distributive property."

A. To multiply a monomial with another monomial, simply multiply the numerical coefficients then multiply the literal coefficients by applying the basic laws of exponents.

Examples:

1)
$$(x^3)(x^5) = x^8$$

2)
$$(3x^2)(-5x^{10}) = -15x^{12}$$

1)
$$(x^3)(x^5) = x^8$$

2) $(3x^2)(-5x^{10}) = -15x^{12}$
3) $(-8x^2y^3)(-9xy^8) = 72x^3y^{11}$

NOTE TO THE TEACHER:

You may give first the examples and let them think of the rule or do it the other way around. Also, if you think that they can easily understand it, let them do the next few examples. Ask for volunteers. Give additional exercises for them to do on the board.

B. To multiply a monomial with a polynomial, simply apply the distributive property and follow the rule in multiplying monomial by a monomial.

Examples:

1)
$$3x(x^2 - 5x + 7) = 3x^3 - 15x^2 + 21x$$

1)
$$3x (x^2 - 5x + 7) = 3x^3 - 15x^2 + 21x$$

2) $-5x^2y^3 (2x^2y - 3x + 4y^5) = -10x^4y^4 + 15x^3y^3 - 20x^2y^8$

C. To multiply a binomial with another binomial, simply distribute the first term of the first binomial to each term of the other binomial then distribute the second term to each term of the other binomial and simplify the results by combining similar terms. This procedure is also known as the F-O-I-L method or Smile method. Another way is the vertical way of multiplying which is the conventional one.

Examples

1)
$$(x + 3)(x + 5) = x^2 + 8x + 15$$

Outer terms

$$F \rightarrow (x)(x) = x^2$$

$$O -> (x)(5) = 5x$$

$$I \rightarrow (3)(x) = 3x$$

$$L \rightarrow (3)(5) = 15$$

Since 5x and 3x are similar terms we can combine them. 5x + 3x =8x. The final answer is $x^2 + 8x + 15$

2)
$$(x-5)(x+5) = x^2 + 5x - 5x - 25 = x^2 - 25$$

3)
$$(x + 6)^2 = (x + 6)(x + 6) = x^2 + 6x + 6x + 36 = x^2 + 12x + 36$$

4)
$$(2x + 3y)(3x - 2y) = 6x^2 - 4xy + 9xy - 6y^2 = 6x^2 + 5xy - 6y^2$$

5)
$$(3a - 5b)(4a + 7) = 12a^2 + 21a - 20ab - 35b$$

There are no similar terms so it is already in simplest form.

Guide questions to check whether the students understand the process or not If you multiply (2x + 3) and (x - 7) by F-O-I-L method,

- a) the product of the first terms is $2x^2$.
- b) the product of the outer terms is -14x.
- c) the product of the inner terms is 3x.
- d) the product of the last terms is -21.
- e) Do you see any similar terms? What are they? -14x and 3x
- f) What is the result when you combine those similar terms? -11x
- g) The final answer is 2x2 -11x -21

Another Way of Multiplying Polynomials

1) Consider this example.

2) Now, consider this:

NOTE TO THE TEACHER:

Be very careful in explaining the second example because the aligned terms are not always similar.

Consider the example below.

$$3a - 5b$$

 $4a + 7$
 $21a - 35b$

In this case, although 21a and -20ab are aligned, you cannot combine them because they are not similar.

D. To multiply a polynomial, with more than one term, with a polynomial with three or more terms, simply distribute the first term of the first polynomial to each term of the other polynomial. Repeat the procedure up to the last term and simplify the results by combining similar terms.

Examples:

1)
$$(x + 3)(x^2 - 2x + 3) = x(x^2 - 2x + 3) - 3(x^2 - 2x + 3)$$

 $= x^3 - 2x^2 + 3x - 3x^2 + 6x - 9$
 $= x^3 - 5x^2 + 9x - 9$
2) $(x^2 + 3x - 4)(4x^3 + 5x - 1) = x^2(4x^3 + 5x - 1) + 3x(4x^3 + 5x - 1) - 4(4x^3 + 5x - 1)$
 $= 4x^5 + 5x^3 - x^2 + 12x^4 + 15x^2 - 3x - 16x^3 - 20x + 4$
 $= 4x^5 + 12x^4 - 11x^3 + 14x^2 - 23x + 4$
3) $(2x - 3)(3x + 2)(x^2 - 2x - 1) = (6x^2 - 5x - 6)(x^2 - 2x - 1)$
 $= 6x^4 - 17x^3 - 22x^2 + 17x + 6$

*Do the distribution one by one.

NOTE TO THE TEACHER:

We cannot finish this lesson in one day. The first two (part A and B) can be done in one session. We can have one or two sessions (distributive property and FOIL method) for part C because if the students can master it, they can easily follow part D. Moreover, this is very useful in factoring.

III. Exercises

A. Simplify each of the following by combining like terms.

1)
$$6x + 7x$$
 = $13x$
2) $3x - 8x$ = $-5x$
3) $3x - 4x - 6x + 2x$ = $-5x$
4) $x^2 + 3x - 8x + 3x^2$ = $4x^2 - 5x$
5) $x^2 - 5x + 3x - 15$ = $x^2 - 2x - 15$

B. Call a student or ask for volunteers to recite the basic laws of exponents. Focus more on the "product of a power" or "multiplying with the same base." Give follow up exercises through flashcards.

1)
$$x^{12} \div x^{5}$$
 = x^{7}
2) $a^{-10} \cdot a^{12}$ = a^{2}
3) $x^{2} \cdot x^{3}$ = x^{5}
4) $2^{2} \cdot 2^{3}$ = 2^{5}
5) $x^{100} \cdot x$ = x^{101}

C. Answer the following.

1) Give the product of each of the following.

a)
$$(12x^2y^3z)(-13ax^3z^4)$$
 = -156ax⁵y³z⁵
b) $2x^2(3x^2 - 5x - 6)$ = 6x⁴ - 10x³ - 12x²
c) $(x - 2)(x^2 - x + 5)$ = x³ - 3x² + 7x - 10

2) What is the area of the square whose side measures (2x - 5) cm? (*Hint:* Area of the square = s^2) $(4x^2 - 20x + 25)$ cm²

- 3) Find the volume of the rectangular prism whose length, width and height are (x + 3) meter, (x 3) meter and (2x + 5) meter. (Hint: Volume of rectangular prism = $l \times w \times h$) (2x³ + 5x² 18x 45) cubic meters
- 4) If I bought (3x + 5) pencils which cost (5x 1) pesos each, how much will I pay for them? $(15x^2 + 22x 5)$ pesos

Summary

In this lesson, you learned about multiplying polynomials using different approaches: using the Tiles, using the FOIL, and using the vertical way of multiplying numbers.

Pre-requisite Concepts: Addition, Subtraction, and Multiplication of Polynomials

About the Lesson: In this lesson, students will continue to work with Tiles to help reinforce the association of terms of a polynomial with some concrete objects, hence helping them remember the rules for dividing polynomials.

Objectives:

In this lesson, the students must be able to:

- 1) divide polynomials such as:
 - a) polynomial by a monomial and
 - b) polynomial by a polynomial with more than one term.
- 2) solve problems involving division of polynomials.

Lesson Proper

I. Activity 1:

Decoding

"I am the father of Archimedes." Do you know my name? Find it out by decoding the hidden message below.

Match Column A with its answer in Column B to know the name of Archimedes' father. Put the letter of the correct answer in the space provided below.

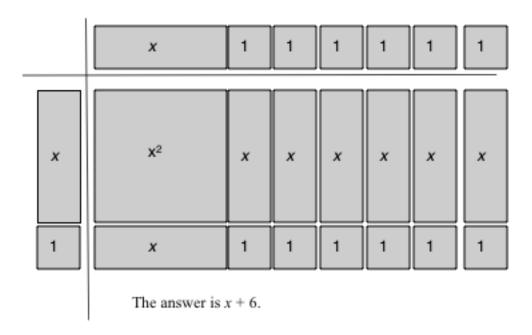
Column A (Perform the indicated operation)

1)
$$(3x^2 - 6x - 12) + (x^2 + x + 3)$$

2) $(2x - 3)(2x + 3)$
3) $(3x^2 + 2x - 5) - (2x^2 - x + 5)$
4) $(3x^2 + 4) + (2x - 9)$
5) $(x + 5)(x - 2)$
6) $3x^2 - 5x + 2x - x^2 + 6$
7) $(2x + 3)(2x + 3)$
S $4x^2 + 12x + 9$
H $4x^2 - 9$
I $x^2 + 3x - 10$
P $4x^2 - 5x - 9$
A $2x^2 - 3x + 6$
E $4x^2 - 6x - 9$
D $3x^2 + 2x - 5$
V $5x^3 - 5$

Activity 2.

Recall the Tiles. We can use these tiles to divide polynomials of a certain type. Recall also that division is the reverse operation of multiplication. Let's see if you can work out this problem using Tiles: $(x^2 + 7x + 6) \div (x + 1)$



II. Questions/Points to Ponder (Post-Activity Discussion)

The answer to Activity 1 is PHIDIAS. Did you get it? If you did not, find out where you committed a mistake in deciphering the problem.

In Activity 2, note that the dividend is under the horizontal bar similar to the long division process on whole numbers.

Rules in Dividing Polynomials

To divide a polynomial by a monomial, simply divide each term of the polynomial by the given divisor.

Examples:

1) Divide $12x^4 - 16x^3 + 8x^2$ by $4x^2$

a)
$$\frac{12x^4 - 16x^3 + 8x^2}{4x^2}$$
$$= \frac{12x^4}{4x^2} - \frac{16x^3}{4x^2} + \frac{8x^2}{4x^2}$$
$$= 3x^2 - 4x + 2$$

b.
$$4x^{2} \overline{\smash{\big)} 12x^{4} - 16x^{3} + 8x^{2}}$$

$$\underline{12x^{4}} - 16x^{3}$$

$$\underline{-16x^{3}}$$

$$\underline{8x^{2}}$$

$$\underline{8x^{2}}$$

$$0$$

2) Divide
$$15x^4y^3 + 25x^3y^3 - 20x^2y^4$$
 by $-5x^2y^3 = \frac{15x^4y^3}{-5x^2y^3} + \frac{25x^3y^3}{-5x^2y^3} - \frac{20x^2y^4}{-5x^2y^3} = -3x^2 - 5x + 4y$

To divide a polynomial by a polynomial with more than one term (by long division), simply follow the procedure in dividing numbers by long division.

These are some suggested steps to follow:

- 1) Check the dividend and the divisor to see if they are in standard form.
- 2) Set-up the long division by writing the division symbol where the divisor is outside the division symbol and the dividend inside it.
- 3) You may now start the Division, Multiplication, Subtraction and Bring Down cycle.
- 4) You can stop the cycle when:
 - a) the quotient (answer) has reached the constant term.
- b) the exponent of the divisor is greater than the exponent of the dividend

NOTE TO THE TEACHER:

Better start with whole numbers, but you have to be very cautious with the differences in procedure in bringing down numbers or terms. With whole numbers, you can only bring down numbers one at a time. With polynomials, you may or you may not bring down all terms altogether. It is also important that you familiarize the students with the divisor, dividend and quotient.

Examples:

2) Divide
$$x^2 - 3x - 10$$
 by $x + 2$

$$x + 2 \sqrt{x^2 - 3x - 10}$$

$$x + 2 \sqrt{x^2 - 3x - 10}$$

$$x + 2 \sqrt{x^2 - 3x - 10}$$
1) divide x^2 by x and put the result on top
2) multiply that result to $x + 2$
3) subtract the product to the dividend

5) repeat the procedure from 1.

4) bring down the remaining term/s

3) Divide
$$x^3 + 6x^2 + 11x + 6$$
 by $x - 3$

$$x - 3) x^3 - 6x^2 + 11x - 6$$

$$x - 3) x^3 - 6x^2 + 11x - 6$$

$$x - 3x + 11x$$

$$-3x + 9x$$

$$2x - 6$$

$$-2x - 6$$

$$0$$

4) Divide
$$2x^3 - 3x^2 - 10x - 4$$
 by $2x - 1$

$$x^2 - 2x - 4 - \frac{2}{2x+1}$$

$$2x+1 \sqrt{2x^3 - 3x^2 - 10x - 6}$$

$$2x^3 + x^2 - 4x^2 - 10x$$

$$-4x^2 - 2x$$

$$-8x - 6$$

$$-8x - 4$$

$$-2$$

5) Divide
$$x^4 - 3x^2 + 2$$
 by $x^2 - 2x + 3$

NOTE TO THE TEACHER:

In this example, it is important that we explain to the students the importance of inserting missing terms.

$$x^{2} + 2x - 2 + \frac{-10x + 18}{x^{2} - 2x + 3}$$

$$x^{2} - 2x + 3$$

$$x^{4} + 0x^{3} - 3x^{2} + 0x + 12$$

$$x^{4} - 2x^{3} + 3x^{2}$$

$$2x^{3} - 6x^{2} + 0x$$

$$2x^{3} - 4x^{2} + 6x$$

$$-2x^{2} - 6x + 12$$

$$-2x^{2} + 4x - 6$$

$$-10x + 18$$

III. Exercises

Answer the following.

1) Give the quotient of each of the following.

a)
$$30x^3y^5$$
 divided by $-5x^2y^5$

a)
$$30x^3y^5$$
 divided by $-5x^2y^5$ = -6x
b) $\frac{13x^3 - 26x^5 - 39x^7}{13x^3}$ = 1 - 2x² - 3x⁴

c) Divide
$$7x + x^3 - 6$$
 by $x - 2 = x^2 + 2x + 11$ r. 16

- 2) If I spent $(x^3 + 5x^2 2x 24)$ pesos for $(x^2 + x 6)$ pencils, how much does each pencil cost? (x + 4) pesos
 - 3) If 5 is the number needed to be multiplied by 9 to get 45, what polynomial is needed to be multiplied to x + 3 to get $2x^2 + 3x - 9$? (2x - 3)
- 4) The length of the rectangle is x cm and its area is $(x^3 x)$ cm². What is the measure of its width? $(x^2 - 1)$ cm

NOTE TO THE TEACHER:

If you think that the problems are not suitable to your students, you may construct a simpler problem that they can solve.

Summary:

In this lesson, you have learned about dividing polynomials by first using the Tiles then using the long way of dividing.

Lesson 25: Special Products

Pre-requisite Concepts: Addition and Multiplication of Polynomials

Time: 3.5 hours

Objectives:

In this lesson, you are expected to:

find (a) inductively, using models and (b) algebraically the

- 1. product of two binomials
- 2. product of a sum and difference of two terms
- 3. square of a binomial
- 4. cube of a binomial
- 5. product of a binomial and a trinomial

Lesson Proper:

A. Product of two binomials

I. Activity

Prepare three sets of algebra tiles by cutting them out from a page of newspaper or art paper. If you are using newspaper, color the tiles from the first set black, the second set red and the third set yellow.

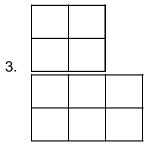
This activity uses algebra tiles to find a general formula for the product of two binomials. Have the students bring several pages of newspaper and a pair of scissors in class. Ask them to cut at least 3 sheets of paper in the following pattern. Have them color the pieces from one sheet black, the second red and the last one yellow.

Problem:

- 1. What is the area of a square whose sides are 2cm?
- 2. What is the area of a rectangle with a length of 3cm and a width of 2cm?
- 3. Demonstrate the area of the figures using algebra tiles.

Solution:

- 1. $2cm \times 2cm = 4cm^2$
- 2. $3 \text{cm} \times 2 \text{cm} = 6 \text{cm}^2$



Tell the students that the large squares have dimensions of x units, the rectangles are x units by 1 unit and the small squares have a side length of 1 unit.

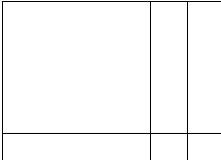
Review with the students the area of a square and a rectangle. Have them determine the area of the large square, the rectangle, and the small square.

Problem:

- 1. What are the areas of the different kinds of algebra tiles?
- 2. Form a rectangle with a length of x + 2 and a width of x + 1 using the algebra tiles. What is the area of the rectangle?

Solution:

1. x^2 , x and 1 square units.



2. The area is the sum of all the areas of the algebra tiles.

Area =
$$x^2 + x + x + x + 1 + 1 = x^2 + 3x + 2$$

Ask the students what the product of x + 1 and x + 2 is. Once they answer $x^2 + 3x + 2$, ask them again, "why is it the same as the area of the rectangle?" Explain that the area of a rectangle is the product of its length and its width, and if the dimensions are represented by binomials, then the area of the rectangle is equivalent to the product of the two binomials.

Problem:

1. Use algebra tiles to find the product of the following:

a.
$$x + 2x + 3$$

b.
$$2x + 1 \bar{x} + 4$$

c.
$$2x+1 - 2x+3$$

2. How can you represent the difference x - 1 using algebra tiles?

Solution:

1.

a.
$$x^2 + 5x + 6$$

b.
$$2x^2 + 9x + 4$$

c.
$$4x^2 + 8x + 3$$

2. You should use black colored tiles to denote addition and red colored tiles to denote subtraction.



Problem:

1. Use algebra tiles to find the product of the following:

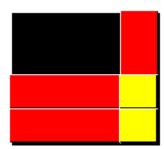
a.
$$x-1\bar{x}-2$$

b.
$$2x^{-1}x^{-1}$$

c.
$$x-2\bar{x}+3$$

d.
$$2x-1\bar{x}+4$$

Solution:



1. $x^2 - 3x + 2$

The students should realize that the yellow squares indicate that they have

subtracted that area twice using the red figures, and they should "add them" back again to get the product.

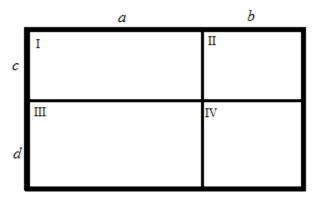
2.
$$2x^2 - 3x + 1$$

3.
$$x^2 + x - 6$$

4.
$$2x^2 + 7x - 4$$

II. Questions to Ponder

1. Using the concept learned in algebra tiles what is the area of the rectangle shown below?



2. Derive a general formula for the product of two binomials $a+b \ c+d$.

The area of the rectangle is equivalent to the product of $a+b \ c+d$ which is ac+ad+bc+cd. This is the general formula for the product of two binomials $a+b \ c+d$. This general form is sometimes called the **FOIL** method where the letters of **FOIL** stand for first, outside, inside, and last.

Example: Find the product of (x + 3) (x + 5)

$$\begin{array}{ccc}
\mathbf{F} & \mathbf{L} \\
(x+3)(x+5) \\
\mathbf{O}
\end{array}$$

First: $x \cdot x = x^2$

Outside: $x \cdot 5 = 5x$

Inside: $3 \cdot x = 3x$

Last: 3 . 5 = 15

 $(x + 3) (x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$

III. Exercises

Find the product using the **FOIL** method. Write your answers on the spaces provided:

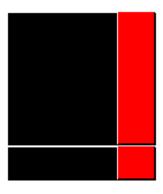
- 1. $(x + 2) (x + 7) x^2 + 9x + 14$
- 2. $(x + 4) (x + 8) x^2 + 12x + 32$
- 3. $(x-2)(x-4)x^2-6x+24$
- 4. $(x-5)(x+1)x^2-4x-5$
- 5. $(2x + 3)(x + 5) 2x^2 + 13x + 15$
- 6. $(3x-2)(4x+1)12x^2-5x-2$
- 7. $(x^2 + 4) (2x 1) 2x^3 x^2 + 8x 4$
- 8. $(5x^3 + 2x)(x^2 5)5x^5 23x^3 10x$
- 9. $(4x + 3y) (2x + y) 8x^2 + 10xy + 3y^2$
- 10. $(7x 8y) (3x + 5y) 21x^2 + 11xy 40y^2$

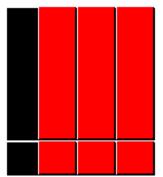
B. product of a sum and difference of two terms

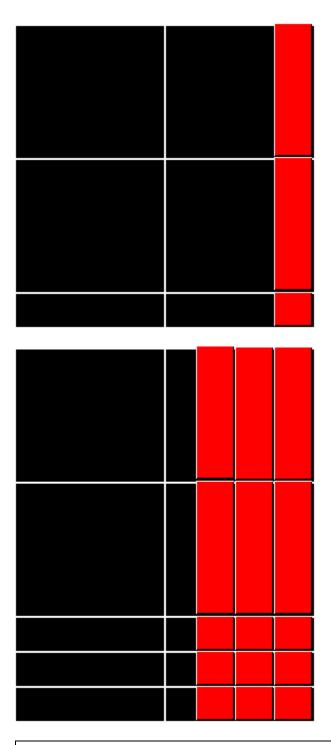
I. Activity

- 1. Use algebra tiles to find the product of the following:
 - a. (x + 1)(x 1)
 - b. (x + 3) (x 3)
 - c. (2x-1)(2x+1)
 - d. (2x-3)(2x+3)
- 2. Use the **FOIL** method to find the products of the above numbers.

The algebra tiles should be arranged in this form.







The students should notice that, for each multiplication, there is an equal number of black and red rectangles. This means that they "cancel" out each other. Also, the red small squares form a bigger square whose dimensions are equal to the last term in the factors.

Answers:

- 1. $x^2 + x x 1 = x^2 1$
- 2. $x^2 + 3x 3x 9 = x^2 9$
- 3. $4x^2 + 2x 2x 1 = 4x^2 1$
- 4. $4x^2 + 6x 6x 9 = 4x^2 9$

Questions to Ponder

- 1. What are the products?
- 2. What is the common characteristic of the factors in the activity?
- 3. Is there a pattern for the products for these kinds of factors? State the rule. *Concepts to Remember*

The factors in the activity are called the sum and difference of two terms. Each binomial factor is made up of two terms. One factor is the **sum** of the terms, and the other factor being their **difference**. The general form is (a + b) (a - b).

The product of the sum and difference of two terms is given by the general formula

$$(a + b) (a - b) = a^2 - b^2$$
.

Exercises

Find the product of each of the following:

- 1. $(x-5)(x+5)x^2-25$
- 2. $(x + 2) (x 2) x^2 4$
- 3. $(3x-1)(3x+1)9x^2-1$
- 4. $(2x + 3)(2x 3)4x^2 9$
- 5. $(x + y^2) (x y^2) x^2 y^4$
- 6. $(x^2 10)(x^2 + 10) x^4 100$
- 7. $(4xy + 3z^3) (4xy 3z^3) 16x^2y^2 9z^6$
- 8. $(3x^3 4)(3x^3 + 4) 9x^6 16$
- 9. $[(x + y) 1][(x + y) + 1](x + y)^2 1 = x^2 + 2xy + y^2 1$

10.
$$(2x + y - z) (2x + y + z) (2x + y)^2 - z^2 = 4x^2 + 4xy + y^2 - z^2$$

C. square of a binomial

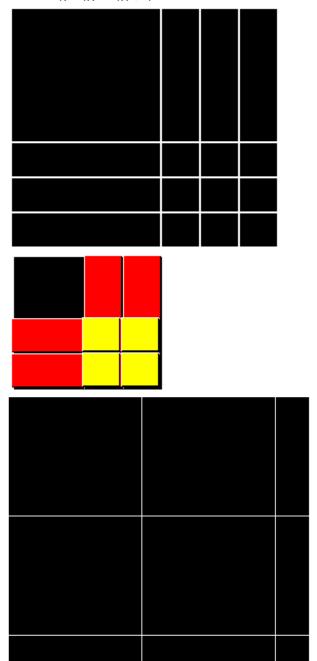
Activity

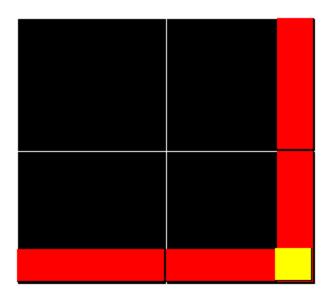
- 1. Using algebra tiles, find the product of the following:
 - a. (x + 3) (x + 3)
 - b. (x-2)(x-2)
 - c. (2x + 1)(2x + 1)

d.
$$(2x-1)(2x-1)$$

2. Use the ${f FOIL}$ method to find their products. Answers:

- 1. $x^2 + 6x + 9$ 2. $x^2 4x + 4$ 3. $4x^2 + 4x + 1$ 4. $4x^2 4x + 1$





Questions to Ponder

- 1. Find another method of expressing the product of the given binomials.
- 2. What is the general formula for the square of a binomial?
- 3. How many terms are there? Will this be the case for all squares of binomials? Why?
- 4. What is the difference between the square of the sum of two terms from the square of the difference of the same two terms?

Concepts to Remember

The square of a binomial $a\pm b^2$ is the product of a binomial when multiplied to itself. The square of a binomial has a general formula, $a\pm b^2=a^2\pm 2ab\pm b^2$.

The students should know that the outer and inner terms using the FOIL method are identical and can be combined to form one term. This means that the square of a binomial will always have three terms. Furthermore, they should realize that the term b^2 is always positive, while the sign of the middle term 2ab depends on whether or not the binomials are sums or differences.

Exercises

Find the squares of the following binomials.

1.
$$(x + 5)^2 x^2 + 10x + 25$$

2.
$$(x-5)^2 x^2 - 10x + 25$$

3.
$$(x + 4)^2 x^2 + 8x + 16$$

4.
$$(x-4)^2 x^2 - 8x + 16$$

5.
$$(2x + 3)^2 4x^2 + 12x + 9$$

6.
$$(3x - 2)^2 9x^2 - 12x + 4$$

7.
$$(4-5x)^2$$
 16 – 40x + 25x²

8.
$$(1 + 9x)^2 1 + 18x + 81x^2$$

9.
$$(x^2 + 3y)^2 x^4 + 6x^2y + 9y^2$$

10.
$$(3x^3 - 4y^2)^2 9x^6 - 24x^6y^4 + 16y^4$$

D. Cube of a binomial

I. Activity

A. The cube of the binomial (x + 1) can be expressed as $(x + 1)^3$. This is equivalent to

$$(x + 1)(x + 1)(x + 1)$$
.

- 1. Show that $(x + 1)^2 = x^2 + 2x + 1$.
- 2. How are you going to use the above expression to find $(x + 1)^3$?
- 3. What is the expanded form of $(x + 1)^3$?

Answers:

1. By using special products for the square of a binomial, we can show that $(x + 1)^2 = x^2 + 2x + 1$.

2.
$$(x + 1)^3 = (x + 1)^2(x + 1) = (x^2 + 2x + 1)(x + 1)$$

3.
$$(x + 1)^3 = x^3 + 3x^2 + 3x + 1$$

B. Use the techniques outlined above, to find the following:

1.
$$(x + 2)^2$$

2.
$$(x-1)^2$$

3.
$$(x-2)^2$$

Answers:

1.
$$x^3 + 6x^2 + 12x + 8$$

2.
$$x^3 - 3x^2 + 3x - 1$$

3.
$$x^3 - 6x^2 + 12x - 8$$

This activity is meant to present the students with several simple examples of finding the cube of a binomial. They should then analyze the answers to identify the pattern and the general rule in finding the cube of a binomial.

II. Questions to Ponder

- 1. How many terms are there in each of the cubes of binomials?
- 2. Compare your answers in numbers 1 and 2.
 - a. What are similar with the first term? How are they different?
 - b. What are similar with the second term? How are they different?
 - c. What are similar with the third term? How are they different?
 - d. What are similar with the fourth term? How are they different?

- 3. Formulate a rule for finding the cube of the binomial in the form $(x + a)^3$. Use this rule to find $(x + 3)^3$. Check by using the method outlined in the activity.
- 4. Compare numbers 1 and 3 and numbers 2 and 4.
 - a. What are the similarities for each of these pairs?
 - b. What are their differences?
- 5. Formulate a rule for finding the cube of a binomial in the form $(x a)^3$. Use this rule to find $(x 4)^3$.
- 6. Use the method outlined in the activity to find $(2x + 5)^3$. Can you apply the rule you made in number 3 for getting the cube of this binomial? If not, modify your rule and use it to find $(4x + 1)^3$.

Answers:

- 1. The cube of a binomial has four terms.
- 2. First, make sure that the students write the expanded form in standard form.
 - a. The first terms are the same. They are both x^3 .
 - b. The second terms have the same degree, x². Their coefficients are different. (3 and 6).
 - c. The third terms have the same degree, x. Their coefficients are 3 and 12.
- d. The fourth terms are both constants. The coefficients are 1 and 8. Make sure that the students notice that the ratio of the coefficients of the terms are 1, 2, 4 and 8. These correspond to the powers of the second term 2^0 , 2^1 , 2^2 , and 2^3 .

3.
$$x + a^3 = x^3 + 3ax^2 + 3a^2x + a^3$$
. Thus,
 $(x+3)^3 = x^3 + 3(3)x^2 + 33^2x + 3^3 = x^3 + 9x^2 + 27x + 27$

4. The pairs have similar terms, except that the second and fourth terms of (x-a)³ are negative, while those of (x+a)³ are positive.

5.
$$x-a^3 = x^3 - 3ax^2 + 3a^2x - a^3$$
. Thus,
 $x-4^3 = x^3 - 34x^2 + 34^2x - 4^3 = x^3 - 12x^2 + 48x - 64$.

6. From numbers, 3 and 5, we can generalize the formula to $a \pm b^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$. In $(2x + 5)^3$, a = 2x and b = 5. Thus, $2x + 5^3 = 2x^3 + 32x^2 + 5 + 32x + 5^3 = 8x^3 + 60x^2 + 150x + 125$

Concepts to Remember

The cube of a binomial has the general form, $a \pm b^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$.

III. Exercises

Expand.

1.
$$x+5^{3}$$

2.
$$x-5^{3}$$

3.
$$x + 7^{\frac{3}{2}}$$

4.
$$x-6^{3}$$

5.
$$2x+1^3$$

6.
$$3x-2^{-3}$$

7.
$$x^2 - 1^{\frac{3}{3}}$$

8.
$$x + 3y^{3}$$

9.
$$4xy+3^{3}$$

10.
$$2p-3q^2$$

Answers

1.
$$x^3 + 15x^2 + 75x + 125$$

2.
$$x^3 - 15x^2 + 75x - 125$$

3.
$$x^3 + 21x^2 + 147x + 343$$

4.
$$x^3 - 18x^2 + 108x - 216$$

5.
$$8x^3 + 12x^2 + 6x + 1$$

6.
$$27x^3 - 54x^2 + 36x - 8$$

7.
$$x^6 - 3x^4 + 3x^2 - 1$$

8.
$$x^3 + 9x^2y + 27xy^2 + 27y^3$$

9.
$$64x^3y^3 + 144x^2y^2 + 108xy + 27$$

10.
$$8p^3 - 36p^2q^2 + 54pq^4 - 27q^6$$

E. Product of a binomial and a trinomial

I. Activity

In the previous activity, we have tried multiplying a trinomial with a binomial. The resulting product then has four terms. But, the product of a trinomial and a binomial **does not always** give a product of four terms.

- 1. Find the product of $x^2 x + 1$ and x + 1.
- 2. How many terms are in the product?

Answers:

The product is $x^3 + 1$, and it has two terms. Tell the students that the product is a sum of two cubes and can be written as $x^3 + 1^3$.

3. What trinomial should be multiplied to x=1 to get x^3-1 ?

Answers

The other factor should be $x^2 + x + 1$. This question can be done step-by-step analytically. First, ask the students what the first term should be and why. They should realize that the first term can only be x^2 , since multiplying it by x from (x - 1) is the only way to get x^3 . Then, ask them what the last term should be and why. The only possible answer is 1, since that is the only way to get -1 in $(x^3 - 1)$ by multiplying by -1 in (x - 1). They should then be able to get that the middle term should be +x.

4. Is there a trinomial that can be multiplied to x - 1 to get $x^3 + 1$? Answers

There is none. To get the sum of two cubes, one of the factors should be the sum of the terms. Similarly, explain that to get the difference of two cubes, one of the factors should be the difference of the terms.

 Using the methods outlined in the previous problems, what should be multiplied to x + 2 to get x³ + 8? Multiplied to x - 3 to get x³ - 27?

II. Questions to Ponder

Answers:

$$(x^2 - 2x + 4)(x + 2) = x^3 + 8$$
 and $(x^2 + 3x + 9)(x - 3) = x^3 - 27$

1. What factors should be multiplied to get the product $x^3 + a^3$? $x^3 - a^3$? Answers:

$$x \mp ax + a^2$$
 $x \pm a = x^3 \pm a^3$

2. What factors should be multiplied to get $27x^3 + 8$? Answers:

Make the students discover that the previous formula can be generalized to $a^2 \mp ab + b^2$ $a \pm b = a^3 \pm b^3$. $27x^3 + 8 = (3x)^3 + 2^3$; a = 3x and b = 2. Thus, $[(3x)^2 - (3x)(2) + 2^2](3x + 2) = (9x^2 - 6x + 4)(3x + 2) = 27x^3 + 8$

Concepts to Remember

The product of a trinomial and a binomial can be expressed as the sum or difference of two cubes if they are in the following form.

$$a^2 - ab + b^2$$
 $a + b = a^3 + b^3$

$$a^2 + ab + b^2$$
 $a - b = a^3 - b^3$

III. Exercises

A. Find the product.

1.
$$x^2 - 3x + 9 x + 3$$

2.
$$x^2 + 4x + 16 x - 4$$

3.
$$x^2 - 6x + 36 x + 6$$

4.
$$x^2 + 10x + 100 x - 10$$

5.
$$4x^2 + 10x + 25 2x - 5$$

6.
$$9x^2 + 12x + 16 3x - 4$$

B. What should be multiplied to the following to get the sum/difference of two cubes? Give the product.

1.
$$x-7$$

2.
$$x + 8$$

3.
$$4x + 1$$

4.
$$5x-3$$

5.
$$x^2 + 2x + 4$$

6.
$$x^2 - 11x + 121$$

7.
$$100x^2 + 30x + 9$$

8.
$$9x^2 - 21x + 49$$

Answers

A.

1.
$$x^3 + 27$$

2.
$$x^3 - 64$$

3.
$$x^3 + 216$$

4.
$$x^3 - 1000$$

5.
$$8x^3 - 125$$

6.
$$27x^3 - 64$$

В.

1.
$$x^2 + 7x + 49$$
; $x^3 - 343$

2.
$$x^2 - 8x + 64$$
; $x^3 + 512$

3.
$$16x^2 - 4x + 1$$
; $64x^3 + 1$

4.
$$25x^2 + 15x + 9$$
; $125x^3 - 27$

5.
$$x-2$$
; x^3-8

6.
$$x+11$$
; x^3+1331

7.
$$10x-3$$
; $1000x^3-27$

8.
$$3x + 7$$
; $27x^3 + 343$

Summary: You learned about special products and techniques in solving problems that require special products.