Grade Level/Course: Math 6 and Math 7

Lesson/Unit Plan Name: Mean Absolute Deviation (MAD)

Rationale/Lesson Abstract: The objective of this lesson is to give students an understanding of Mean Absolute Deviation (MAD) as a single value of variability. Students will be able to calculate the MAD from data and know when and why to do so.

Timeframe: 1-2 60 minute periods

Common Core Standard(s):

Develop understanding of statistical variability.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

6.SP.5 Summarize numerical data sets in relation to their context, such as by:

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reverence to the context in which the data were gathered.

Instructional Resources/Materials:

Warm-up (p.9), one deck of playing cards or dice per pair of students, paper, pencil, student note taking guide (p.11-14) and Exit Ticket (p.7)

Activity/Lesson:

Ask: What do you know or what can you tell me about any of these three words…?
Mean-
Absolute-
Deviation-

Sample Responses:

Mean- An average, adding up all of the numbers and dividing by how many numbers you have.
Absolute- A reference may be made to absolute value, the distance a number is from zero.
Deviation- A distance.

Deviation may be a word that is unfamiliar to most students. An example is when you deviate from a path. If you deviate from a path, you are straying or veering off from the path leaving you at a certain distance from the original path.

Activity/Game: Understanding Deviation

This activity can be done with either cards or dice. The goal of the game is to get a sum of 21 (teachers can use any number they like) or closest to it. Students will work with a partner.

Cards	Dice
• Each student draws 3 cards and records the values. (Similar to Blackjack an ace can be a 1 or an 11, Jack, Queen and King are worth 10, and cards 2-10 are the face value of the card) • Students then add up the 3 face values of the cards.	• Each student rolls a pair of dice 3 times and records the values. • Students then add up the 3 rolls of the dice.

- If a student gets a sum of 21, that student wins the round.
- If the sum is a different number, even if it is over 21, students should say…

"I was off or deviated from 21 by _____"

(Teachers should walk around the room and make sure that students are using the provided sentence stem. Teachers should also note that deviation is a distance so answers should always be a positive value)

- The player with the lowest deviation wins the round! (Play at least 3 to 5 rounds)

Activity/Lesson continued: **Pass out the Student Note-Taking Guide**

Students at this point have looked at measures of central tendency (mean median and mode) and some measures of variation or spread (range and interquartile range). Another measure of variation is mean absolute deviation.

Definition: The **mean absolute deviation (aka MAD)** of a set of data is the average distance between each data value and the mean.

Example 1:
The boy's basketball team recorded their scores from 10 games this season. The scores are shown in the table below. Find the mean absolute deviation of the set of data. Describe what the mean absolute deviation represents in this situation.

Basketball Scores				
41	37	50	38	46
54	42	56	49	47

Step 1: Find the mean.

$$\frac{41+37+50+38+46+54+42+56+49+47}{10}$$

$$= \frac{460}{10}$$

$$= 46$$

$\therefore$ The mean of the data is 46 points.

Step 2: Find the absolute value of the difference between each value in the data set and the mean.

$|41-46| = |-5| = \boxed{5}$

$|37-46| = |-9| = \boxed{9}$

$|50-46| = |4| = \boxed{4}$

$|38-46| = |-8| = \boxed{8}$

$|46-46| = |0| = \boxed{0}$

$|54-46| = |8| = \boxed{8}$

$|42-46| = |-4| = \boxed{4}$

$|56-46| = |10| = \boxed{10}$

$|49-46| = |3| = \boxed{3}$

$|47-46| = |1| = \boxed{1}$

Step 3: Find the average of the absolute values of the differences between each value in the data set and the mean.

$$\frac{5+8+4+1+9+0+10+4+8+3}{10}$$
$$=\frac{52}{10}$$
$$=5.2$$

$\therefore$ The mean absolute deviation is 5.2. This means that the average distance each data value is from the mean is 5.2 points.

Example 2 (You try!):

The number of runs allowed by a pitcher on the baseball team was recorded for his last 9 starts in a game. The results are shown in the table below. Find the mean absolute deviation of the set of data. Describe what the mean absolute deviation represents in this situation.

Number of Runs Allowed		
0	8	6
4	5	5
2	6	0

Step 1: Find the mean.

$$\frac{0+8+6+4+5+5+2+6+0}{9}$$
$$=\frac{36}{9}$$
$$=4$$

$\therefore$ The mean of the data is 4 runs allowed per game.

Step 2: Find the absolute value of the difference between each value in the data set and the mean.

$|0-4|$ $|8-4|$ $|6-4|$ $|4-4|$ $|5-4|$

$=|-4|$ $=|4|$ $=|2|$ $=|0|$ $=|1|$

$=\boxed{4}$ $=\boxed{4}$ $=\boxed{2}$ $=\boxed{0}$ $=\boxed{1}$

$|5-4|$ $|2-4|$ $|6-4|$ $|0-4|$

$=|1|$ $=|-2|$ $=|2|$ $=|-4|$

$=\boxed{1}$ $=\boxed{2}$ $=\boxed{2}$ $=\boxed{4}$

Step 3: Find the average of the absolute values of the differences between each value in the data set and the mean.

$$\frac{4+4+2+0+1+1+2+2+4}{9}$$
$$=\frac{20}{9}$$
$$=2.\overline{2}$$
$$\approx 2.2$$

$\therefore$ The mean absolute deviation is 2.2. This means that the average distance each data value is from the mean is 2.2 runs.
(Teachers should mention that the smaller the MAD, the more consistant the pitcher is because they do not stray far from their mean)

Example 3 (You try!):
The table below shows the high temperatures in the Bay Area for one week during the Month of March. Find the mean absolute deviation of the set of data. Describe what the mean absolute deviation represents in this situation.

High Temperatures (°F)						
60	64	62	68	70	72	66

Step 1: Find the mean.

$$\frac{60+64+62+68+70+72+66}{7}$$
$$=\frac{462}{7}$$
$$=66$$

$\therefore$ The mean of the data is 66 °F.

Step 2: Find the absolute value of the difference between each value in the data set and the mean.

$$|60-66| \quad |64-66| \quad |62-66| \quad |68-66|$$
$$=|-6| \quad =|-2| \quad =|-4| \quad =|2|$$
$$=\boxed{6} \quad =\boxed{2} \quad =\boxed{4} \quad =\boxed{2}$$

$$|70-66| \quad |72-66| \quad |66-66|$$
$$=|4| \quad =|6| \quad =|0|$$
$$=\boxed{4} \quad =\boxed{6} \quad =\boxed{0}$$

Step 3: Find the average of the absolute values of the differences between each value in the data set and the mean.

$$\frac{6+2+4+2+4+6+0}{7}$$

$$= \frac{24}{7}$$

$$\approx 3.43$$

$\therefore$ The mean absolute deviation is approximately 3.43. This means that the average distance each data value is from the mean is about 3.43 °F.

After doing all of this work you might ask yourself when and why you would take the time to find the Mean Absolute Deviation of a data set! This answer is made clear in the Common Core Progression Documents in statistics and probability for 6-8th grade mathematics.

Look at a set of data and choose the best **measure of center**.

If the **median** is a more useful measure of center (there are extreme values) then the **interquartile range (IQR)** is a more meaningful measure of spread.

If the **mean** is a more useful measure of center (there are no extreme values) then the **mean absolute deviation (MAD)** is a more meaningful measure of spread.

Exit Ticket/Assessment:

1) **In your own words describe the steps for finding the mean absolute deviation of a data set.**

_____
_____
_____
_____
_____
_____
_____

2) **What is the mean absolute deviation of the data: 10, 17, 20, 12 and 16 ?**

3) **<u>Challenge Problem</u>**

From the list of numbers below, write one number in each box. You may use each number exactly once.

3
4
7
8
9
14

Three unique numbers with mean = 8 and MAD = 4		

Three unique numbers with mean = 7 and MAD = 2		

Exit Ticket/Assessment: **Answer Key**

1) **In your own words describe the steps for finding the mean absolute deviation of a data set.**

[See students' work]

_____
_____
_____
_____
_____
_____
_____

2) **What is the mean absolute deviation of the data: 10, 17, 20, 12 and 16 ?**

[MAD = 3.2]

3) **<u>Challenge Problem</u>**

From the list of numbers below, write one number in each box. You may use each number exactly once.

3
4
7
8
9
14

Three unique numbers with mean = 8 and MAD = 4		
3	7	14

Three unique numbers with mean = 7 and MAD = 2		
4	8	9

Warm-Up

CCSS: 6.RP.3c

60% of what number is 105?

A) 63

B) 175

C) 630

D) 1750

- **Solve using two different methods**

CCSS: 5.NBT.7

Find the sum:

6.2 + 31.59 + 11.11 + 19.85 =

CCSS: 6.SP.3

Find the mean, median and mode(s) of the following data:

8, 14, 22, 7, 2, 11, 25, 7, 5, 9

- **What are mean, median and mode also known as?**

CCSS: 6.SP.3

Find the range and interquartile range of the following data:

8, 14, 22, 7, 2, 11, 25, 7, 5, 9

- **What are range and interquartile range also known as?**

Warm-Up: Answer Key

CCSS: 6.RP.3c

60% of what number is 105?

A) 63

B) 175

C) 630

D) 1750

[B) 175]

- **Solve using two different methods**

CCSS: 5.NBT.7

Find the sum:

6.2 + 31.59 + 11.11 + 19.85 =

[68.75]

CCSS: 6.SP.3

Find the mean, median and mode(s) of the following data:

8, 14, 22, 7, 2, 11, 25, 7, 5, 9

[mean = 11]

[median = 8.5]

[mode = 7]

- **What are mean, median and mode also known as?**

[measures of center]

CCSS: 6.SP.3

Find the range and interquartile range of the following data:

8, 14, 22, 7, 2, 11, 25, 7, 5, 9

[range = 23]

[IQR = 7]

- **What are range and interquartile range also known as?**

[measures of variability]

Mean Absolute Deviation: Note-Taking Guide

Definition: The **mean absolute deviation (aka MAD)** _____
_____
_____

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After doing all of this work you might ask yourself when and why you would take the time to find the Mean Absolute Deviation of a data set! This flow chart will help you decide!