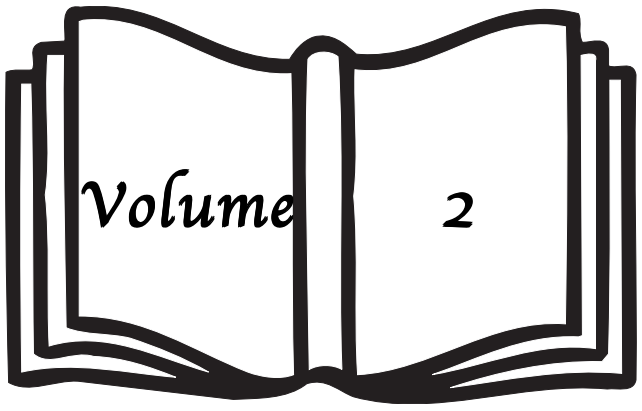
Grade Six

Classroom

Strategies



The learner will understand and compute with rational numbers.

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Notes and textbook references

1.01 Develop number sense for negative rational numbers.

a) Connect the model, number word, and number using a variety of representations, including the number line.

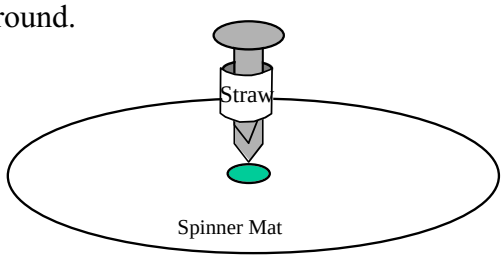
b) Compare and order.

c) Make estimates in appropriate situations.

A. **James Bond Game** (Blackline Master I - 21)

Students use the spinner to race from sea level to the bottom of the sea and then to the top of a cliff. Students will use integers to measure above or below sea level and also to indicate a move upwards or downwards.

An easy way to make spinners is to punch a paper brad through the center of the spinner, and place a large paper clip over the brad. To improve this technique, cut off a small section of straw to pass the brad through before you punch it through the paper. The straw forms a smooth wall for the paper clip to spin around.



Notes and textbook references

B. **Thermometers** (Blackline Master I - 22)
Students will use the thermometer model to investigate some temperatures in the colder parts of the U.S.

C. **Enrichment Activity: Temperature on the Beach** (Blackline Master I - 23) Students complete a data table to show what happens to temperatures of beverages in various containers.

Blackline Master I - 1 contains a problem-solving guide which may be useful to share with students as they begin their academic year.

D. **Life on the Line** Make a personal lifeline (number line) using the month and year of your birth as 0. Encourage students to think of events that happened before their birth (the birth of a sibling or the marriage of their parents would be good examples) to represent negative numbers and the events happening after their birth (when they walked or began school) to represent positive numbers. This is an excellent way to incorporate fraction use for example, if you began to crawl at 9 months old, you would place that event on the number line at 3/4, since 9 months out of 12 would be the fraction 9/12 which would simplify to 3/4. This is a fun activity, which involves the whole family and can allow students to connect mathematics to real life situations.

E. **Secret Message Game** (Blackline Master I - 24)
When the students list the integers in order, the corresponding letters spell out a message.

F. **Integer War** (Blackline Masters I - 25 and I - 26)
Students work in pairs or in two teams. Students are given the blackline masters and instructed to cut the number cards apart. They may wish to keep the number line diagram for reference. The cards are placed face down on a table and shuffled. A dealer deals 14 cards to each player/team and 14 are left face down on the table as the draw area. On any play, each team will choose one of his cards to turn face up in a play area. These two cards are the comparison cards. Now the player/team member whose turn it is will turn up one of the cards from the draw area. If the card drawn is higher than either of the comparison cards, the player/team wins one point. If the card drawn is between the two comparison cards, the player/team wins five points. If the card drawn is less than both the comparison cards, the opponent of the player/ team wins one point. Play alternates until each player/team has had seven plays. High score wins.

Notes and textbook references

G. **Classroom Number Line** Use a brightly colored rope held by two students to demonstrate comparing and ordering rational numbers. Begin by having a student at one end hold a card numbered zero and a student at the other end holding the card one. Hand other students cards with a variety of rational numbers written on them such as, fractions, decimals, mixed numbers, or improper fractions. It would be helpful if while students were seated they would change all the numbers to one form such as, decimals. Then allow one student at a time to place his/her card on the number line using a clothespin. This activity involves not only using number sense, but it encourages communication, reasoning, and justification. You can also use just the specific number topic you are working on; for example, you could use only decimals or fractions. This is also an excellent way to involve every student and help him or her to be successful.

H. **Using Butcher Paper** Have groups of students draw a number line from -5 to 5 on a sheet of butcher paper six feet long. Have students use a ruler to mark the numbers. Assign each group a set of numbers to plot on their number line. Each group should share its number line with the class. This encourages mathematical discourse.

I. **Connection to Meteorology** Have students use the almanac to record the highest and lowest temperatures for selected states. Then have them list the temperatures from least to greatest. This is a good way to show students that negative numbers get smaller the farther they are away from zero. Students could place their temperatures on a number line or you could use a class size thermometer.

J. **Checkbook Activity** (Blackline Master I - 95)
Give the students the following scenario: Bill's dad has devised a method of making sure he always has extra money in his checking account. In his check register, he rounds each check amount to the next highest whole dollar. For example, if he wrote a check to the dry cleaners for $18.34, he would put it in the check register as $19.00. This would give him an extra $0.66 in his account. Use the following checks to determine how much extra money Bill's dad has in his account.

Suggested Classroom Accommodations for Students with Specific Learning Disabilities

Cognitive Strategies	Behavior	Accommodations
Remembering	forgets order of steps	chart of steps displayed
Self-managing	cannot explain concept	self-questioning taught
Information gathering	does not understand on first listening	frequent summaries paraphrasing strategy
Organizing	cannot make visual representation	vocabulary recorded with both words and a visualization strategy
Analyzing	cannot locate errors	verbal rehearsal strategy
Problem solving	cannot shift strategies	demonstrate each problem using two strategies
Time managing	poor assignment completion	prioritize assignments; required time chart for increased awareness of time demands
Integrating	poor notes	note taking strategy organized by concepts, not textbook chapters
Generating	weak concept connecting	prediction strategies pattern awareness
Evaluating	poor test taking	alternate tests; frequent assessment; test taking strategies

Some Additional Accommodations

- Modify original task to meet the needs of handicapped students.
- Provide taped material to listen to, rather than read.
- Emphasize higher use of objective test in contrast to subjective tests.
- Provide highlighted text for student use.
- Provide large print materials.
- Increase allowable time for completion.
- Reduce weight of test importance.
- Change fill-in-the-blank to multiple-choice format.
- Offer three choices instead of four in multiple-choice formats.

1.02 Develop meaning for percents.

a) Connect the model, number word, and number using a variety of representations.

Notes and textbook references

A. **Fill a Grid** (Blackline Master I - 29)

This activity provides models for percents based on a 100-cell grid.
Preparation: Laminate the black line sheet or slide it into a sheet protector.
Materials: Paper clip for spinner, dry erase markers, tissue for eraser.
Instructions: Students take turns spinning the spinner and coloring the indicated percentage on their grid. Each spinner section has two percents; the student may choose either of those or the sum of the two. The round is over when a student has colored in his grid completely. The winner's score is the percentage of his opponent's grid not yet colored. Play continues for five rounds. The player with the highest score wins.

B. **Mini Review – Percents** (Blackline Master I - 2)

This mini review covers most of the percentage skills from this unit. Allow students to work in pairs to share strategies and skills.

C. **Percent Models** (Blackline Master I - 27 and I - 28)

Preparation : Copy the Blackline Master I-27 on transparency film to make enough copies for each student to have a small grid. Students may work in pairs. Discuss with the students how to find rectangles on the grid to represent 1%, 10%, 5%, 20%, 25%, etc. Then direct the students to use percents to estimate quantities illustrated on Blackline Master I - 28 by placing the hundreds grid over the pictures on Blackline Master I - 28.

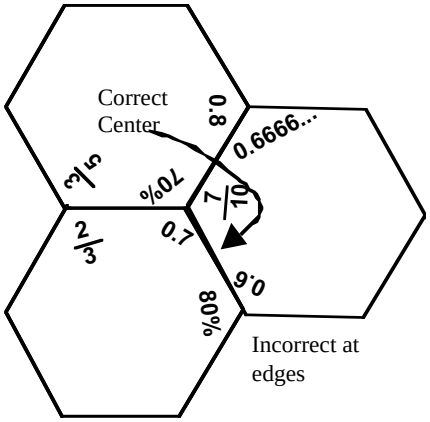
Notes and textbook references

D. Fractions, Decimals and Percent Hexagon Puzzle

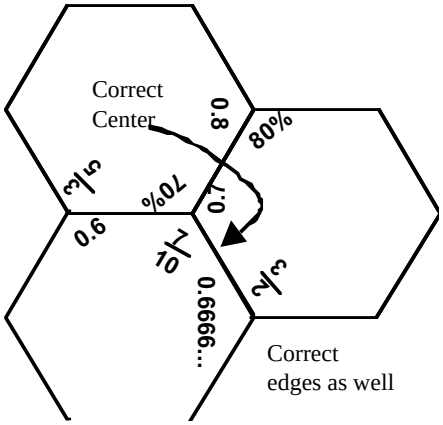
(Blackline Master I - 30) Students work in small groups to assemble a puzzle with hexagonal pieces. Where three hexagonal corners meet, the pieces will show a fraction, decimal, and percentage that are equivalent.

Students should be warned that an arrangement of hexagons might have three equivalent numbers at the center, but it could still have incorrect matches at the edges as shown below. This is corrected by exchanging the position of two of the hexagons.

Teachers: You may want to write a letter in each box before copying the puzzle as this will make it easier to check your students' work. The Blackline Master shows the puzzle assembled correctly. Teachers may wish to cut the puzzle pieces and place them in envelopes.



Incorrect at edges



Correct edges as well

b) Make estimates in appropriate situations.

Notes and textbook references

A. **Homework Percents** Have students keep a one week record of their homework (or other daily grade). Ask the students to make a fraction with the numerator represented by the number they answered correctly, and the denominator represented by the total number of questions. Have them estimate a percent using the fraction, and give an explanation for their estimation. Then students can check their estimate with a calculator. As the week goes on, they should get better at estimating their grade. Teacher should ask questions of the students such as "Was your fraction close to one half? What percent would that be? Did you get almost all of the problems correct? What percent would be almost 100%?"

B. **Smart Shoppers** Students work in cooperative pairs. Teacher can distribute sales flyers from Wal-mart™, Walgreens™, CVS™, etc (most stores will give you a class set of flyers that are about to go out of date). Teacher chooses a few items from the flyer, and tells the students that these items are 20% off. The students than estimate what the new sales cost would be by rounding the price in the flyer and subtracting 20%. When activity is finished, the class could have a discussion about which items you would save the most on and why.

C. **Dinner Out** When Joe and his friends go to dinner they always want to be sure and give the waiter at least a 15% tip. When the check comes, the total amount of their dinner bill is $59.28. Teacher should use this scenario to model how to estimate the amount of tip. First, round the dinner bill to the nearest dollar. Second, estimate 10% by moving the decimal point one place value to the left .(Multiplying by one-tenth). Estimate half of the 10% and add that to the 10% to arrive at the tip.

After modeling, the teacher will give each pair (or group) of students a menu from a local restaurant and have students select their dinner including beverage and dessert. Students should then estimate the tip amount for their dinner.

Notes and textbook references

1.03 Compare and order rational numbers.

The concept of rational numbers is one that students have difficulty mastering. Using activities that promote peer communication allows students to develop and confirm their own understanding of fractions and decimals.

A. Rational Race (Blackline Masters I - 3 through I - 12)

Materials (per group): Playing mat, deck of problem cards, paper clip for spinner, chips or other playing pawns.

Procedure: Students work in teams of two to race around the track. Each team has a pawn at the starting block. On a team's turn, the other team draws a card and covers the answer with their thumb. They then show the card to the team in play. The team in play attempts to find the answer. If the team finds the correct solution, they spin the spinner to see how far they advance. If the team misses the solution, they move back one space (but no farther back than the start). The first team to reach the finish wins.

Alternate whole class play: Print the playing grid and cards on transparency film. Cover the answers with sticky dots. Divide the class into several teams and within those teams have consulting pairs of students who work together. On a team's turn, one of the pairs will be designated as the ones to produce an answer for the team. If this designated team fails to produce a correct answer, a pair from the opposing team may be called upon to give the answer and move forward for their team. Note: If the class is divided into more than three groups, the teacher may wish to use various colors of dry erase markers to signify the teams instead of pawns.

B. Chalk Tray Racko (Blackline Masters I - 31 through I - 38)

Preparation: Print the masters onto heavy paper or copy the numbers onto index cards. Only one deck is needed. Divide the class into two teams. Shuffle the deck thoroughly and "deal" each team eight cards. Display the cards in the order drawn in a row on the chalk tray. Display the cards for one team on the left side of the tray and the other on the right. The object of the game is to be the first team to display eight cards in numerical order.

Playing the Game: On a team's turn, a student is selected to come to the board. He may choose the discarded card showing, or he may draw from the top of the deck. The student may replace one of the cards in his team's display with his drawn card if that improves his team's chance of winning. After playing, the student shows the unused card as the new discarded card.

Example: The first team has these cards showing:

32%, 0.2999, 0.049, 5.5%, 1, 0.3902, 0.69, 72.2%

The discarded card is 103%. That card is not helpful, so the player chooses a card from the pile. He draws 1.01%.

This card is less than 0.049, so the player chooses to replace 0.2999 with 1.01%. His teams' display now reads:

32%, 1.01%, 0.049, 5.5%, 1, 0.3902, 0.69

The new discard is 0.2999.

Notes and textbook references

C. Mathematical Message (Blackline Master I - 39)

Students put the numbers in order to decode a secret message. Spaces are provided on the blackline master for students to put the numbers in decimal form in order to arrange them more easily. You may wish to provide other strategies to the students as well.

D. Patterns for Repeating Decimals (Blackline Master I - 40)

This worksheet enables students to discover for themselves how some repeating decimals can be changed into rational numbers.
(Note: Part II of the Blackline Master is considered to be enrichment)

E. Fraction Card Games (Blackline Master I - 41 through I - 47)

Concentration – Deal all cards face down in five rows of 14. Players take turns turning over two cards at a time. If the fractions are equivalent, the student keeps the pair. The winner is the person with the most cards when all have been taken.

Go Fishing – Deal five cards to each player. Stack the remainder face down in the middle of the table. The object is to get pairs of two equivalent fractions. At each turn players may ask others in the group for a certain fraction. As long as someone gives the person a card, the player may keep asking. When no one has an equivalent fraction to give the player, the person "goes fishing" by drawing from the deck. At the end of the game, the player with the most pairs wins.
Note: You may adapt other card games to use with your equivalent fraction deck.

Notes and textbook references

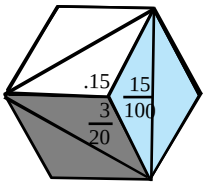
F. Converting Repeating Decimals. Show students an algebraic way to convert repeating decimals to ratios.
Example 1: x = 0.44444...

Use equations:
$$10x = 4.44444...$$
$$-\ x = -\ 0.44444 \quad \text{and now subtract}$$
$$9x = 4$$
$$x = 4/9$$

Example 2: x = 0.2333333. . .

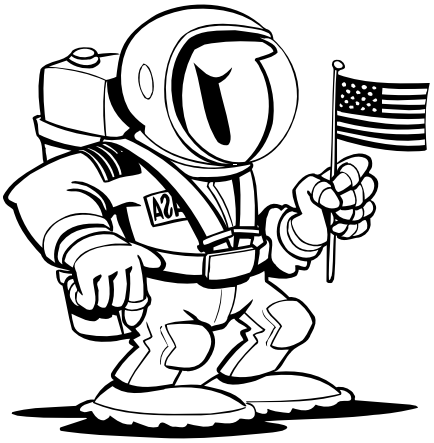
Use equations:
$$100x = 23.33333...$$
$$-10x = -2.33333... \quad \text{and now subtract}$$
$$90x = 21$$
$$x = 21/90 = 7/30$$

(Teachers: This is an enrichment activity that should be done after teaching equations.)



G. Spaceship Shape Up (Blackline Masters I - 49 and I - 50) Materials needed: scissors, glue sticks, and blackline masters for each group. Students work in groups of three to complete the design of a space ship by filling in equivalent fractions and decimals.

A dark, light, and medium-shaded rhombus will be needed to fill in each hexagon. Equivalent fractions will meet where the rhombuses meet. Each hexagon should be filled in with the dark rhombus at the bottom, light rhombus at the top, and medium-shaded rhombus at the right side as shown in the diagram here. A practical suggestion is to let one student in each group be responsible for pieces with the same shading.



Notes and textbook references

H. Rational Race (Blackline Masters I - 51 through I - 54)

Materials needed: Rational number cards, playing mat, pawns for each team, paper clip spinner.

Students compete in teams to complete a racetrack. Rational number cards are divided between two teams. On a given turn, the spinner is spun to determine a target number. Then each team turns over a card. The team whose card shows a rational number closest to the target advances one square. If the team card shows exactly the target number, the team advances three squares. If the two teams are equidistant from the target number, a new target number is spun until one of the cards showing is a clear winner. The first team to reach finish is the winner. While playing this game, the students will have an opportunity to challenge each other and discuss strategies for comparing rational numbers. Communication about math concepts helps students develop and confirm their own understanding of these topics.

I. Order-Up (Blackline Masters I - 52 through I - 54)

Materials needed: rational number cards. Two teams compete in this game. The cards are shuffled, and five cards are dealt to each team. The team places the cards in order as dealt from left to right (the order battery). The remaining cards are placed in a stack face down (the draw pile). The top card is turned face up beside the draw pile (the discard pile).

The object of the game is to get five cards in the order battery in numerical order from smallest to largest. On a team's turn, the player can choose to take the top card from the draw pile, or the top card from the discard pile. They then decide if they wish to replace a card in the order battery with the newly drawn card. By replacing, the students are working toward getting five cards in numerical order. If they choose to replace, the card removed from the battery becomes the top card on the discard pile. If they wish, they may keep the battery as is, and place the newly drawn card on the discard pile. If the draw pile is exhausted, the discard pile is reshuffled and turned over to refill the draw pile.

J. Robot Packing Company (Blackline Master I - 55)

Two students or teams can play opposite each other on one playing mat. Each team is attempting to fill in the four crates. One of them is divided into fourths, one into sixths, and two are divided into twelfths. On a player's turn, he will spin the spinner to determine which amount of a crate he can fill. Students should be able to justify their selection when the model is not identical to the fraction named. Play stops when a player spins and cannot fill in that amount in his crates. At the end of play, the player with the smallest amount empty is the winner.

Notes and textbook references

1.04 Develop fluency in addition, subtraction, multiplication, and division of non-negative rational numbers.

a) Analyze computational strategies.

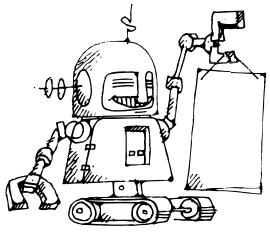
b) Describe the effect of operations on size.

c) Estimate the results of computations.

d) Judge the reasonableness of solutions.

A. **Bode's Number Patterns** (Blackline Master I - 17)
AU is the abbreviation for an Astronomical Unit which represents the distance from the Earth to the Sun. Bode used the AU to find an interesting number pattern relating the orbits of the planets. Students will explore, expand and evaluate this number pattern.

B. **Alien Test Taking** (Blackline Master I - 18)
Materials: Paper reinforcement rings or sticky dots.
Allow students to work in pairs to answer the questions on this test. The students are not allowed to use pencils or calculators to do the computation but should rely on their understanding of numbers. Students often rely on memorized procedures and fail to recall that the numbers should make sense. The purpose of this activity is to connect computational algorithms with meaning. When students work in pairs, they will have an opportunity to share strategies with each other. Some strategies you may want to point out are estimation, elimination, and checking the units or tens digit of the answer. For example, when multiplying 103 x 3.5, the least digit in the answer must be a five. This eliminates many of the solutions.

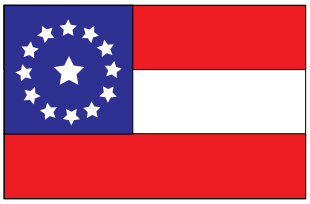


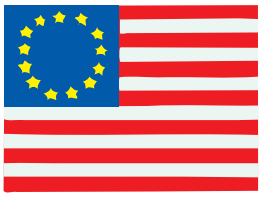
C. **Mini-Review – Fractions** (Blackline Masters I - 13 and I - 14) This mini review covers most of the fraction skills from this unit. Allow students to work in pairs to share strategies and skills.

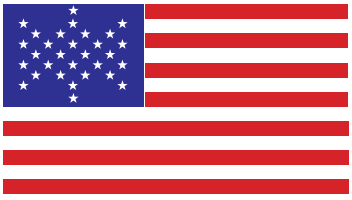
Notes and textbook references

D. **Alien Flags** (Blackline Master I - 19)

Teachers may want to begin this exercise by showing students pictures of previous flags of the United States with fewer than 50 stars. In each flag arrangement an appealing design was used to display the stars. The stars were often arranged in rows of the same size, but what happened with the design when the number of stars could not be evenly divided? In this activity, students will use rational numbers to compute the number of symbols that will be used from each Alien culture. Finally, they will use the comets and stars to create a pleasing and symmetric design for a new flag of the **Alien Federation.**







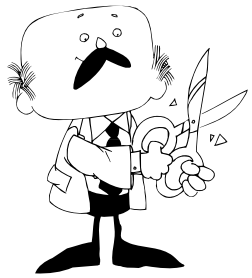
E. **Division of Mixed Numbers with Models** (Blackline Master I - 56 through I - 59)

Divide students into groups and provide a deck of cards for each group. Be sure the cards are shuffled well. Have the students arrange the cards so that a problem, a model, and an answer card match in each case. As the teacher models during the discussion, this task will help students understand the concept of dividing mixed numbers.

When dividing a mixed number by a whole number, it is often convenient to think of the whole number as a number of groups. Example: If I have 4 1/2 apples, how much will each person get if I divide them into three equal parts? When dividing a mixed number by a fraction, it is often more convenient to think of the problem by thinking of how many groups would be needed if each group contained the same amount. Example: I have 1 1/2 cups of paint. Each board I paint requires 1/2 cup. How many boards can I paint?

F. **Mini-Review – Decimals** (Blackline Masters I - 15 and I - 16) This mini review covers most of the decimal skills from this unit. Allow students to work in pairs to share strategies and skills.

Notes and textbook references



You may wish to cut the puzzle pieces apart and place them in envelopes as the Blackline Master gives the solution.

G. Multiplication and Division of Fractions Square Puzzle

(Blackline Master I - 60) With this puzzle, students will match multiplication and division of fraction problems with their solutions. Students should work in groups. At the start of the puzzle, each student should have some of the pieces in his possession. Have the students work out the problems on their squares and then match sides. Students can use this group activity to share mental math strategies with each other. (Teachers: You may give the students the starting square if necessary)

H. Dominos for Multiplication and Division of Mixed Numbers

(Blackline Master I - 61) With this puzzle, students will match problems involving multiplication and division of mixed numbers with their solutions. One domino will match another at a star.

I. Addition and Subtraction of Fractions Square Puzzle

(Blackline Master I – 63) Students should work in groups to complete this puzzle. At the beginning of the activity, each student should be in possession of some of the small squares. To match the squares, the students should place a problem next to its solution. The group activity gives the students the opportunity to discuss their strategies for solving the puzzle.

J. Decimal Dice (Blackline Master I - 62)

Materials: Playing mat, four dice, calculator.
Instructions: On a player's turn, he will roll the four dice. Using the digits rolled, the player creates a division problem. The problem should be written in a fraction format.

For example, if the student rolls 4, 2, 2, 1, he may write $\frac{42}{21}$.

If the target 2 is open, this would be a good problem for the round. If 2 is not available, but 200 is, he may insert a decimal in any position desired such as $\frac{42}{.21}$

Each problem should be constructed to have an answer as close as possible to one of the target numbers. "Big Boy" means the largest possible answer. Each player will get a total of nine turns in which he will attempt to get close to each of the nine targets. Once he has used one of the target numbers, he cannot reuse that same target in a future round. At the end of the nine rounds, the players use a calculator to determine who came closest to each target. One point is awarded for each target winner; highest score is the winner.

Notes and textbook references

K. Fraction Blocks (Blackline Master I - 64)

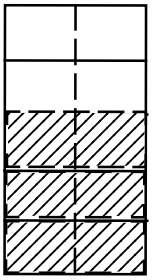
Materials – Scissors. Students have worked with pattern blocks since the early grades. The standard blocks make good models for 1/2, 1/3, and 1/6. In this activity, we have added some additional blocks to represent 1/4 and 1/12. Students should work in groups to complete the chart by filling in the shaded boxes. Manipulating the blocks will help them find the solutions.

L. Modeling Fraction of a Number

Use paper-folding or shading to model the process. A paper rectangle can be used to represent the whole number. Waxed paper is good because the divisions are easily seen.

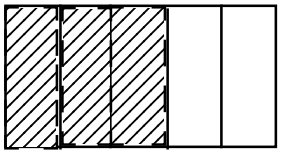
Example: A sheet represents 30. You need to find 3/5 of 30 or the product of

3/5 x 30 = H



Fold the paper into five equal parts. Rulers are helpful, or scissors, to trim the paper to an easily divided size. Shade three-fifths. The five large rectangles each represent six, and three of them make 18. Other foldings are possible. Giving students a tactile model and allowing them to represent the parts is a powerful tool in a problem-solving approach to this concept.

Other examples: 1/4 x 20
2/3 x 27



M. Modeling Tenths (Blackline Master I - 65)

Using the "Tenths" models ,have students shade in a decimal fraction (such as 0.3) a given number of times (such as 5) using a different color for each new shading of the decimal number. Shade on the same model until it is full and continue on a second and third when necessary. Record the total area shaded in an equation like this: 5 x 0.3 = 1.5. Use problems like these:

4 x 0.6 = _____ 3 x 0.7 = _____
9 x 0.2 = _____ 10 x 0.3 = _____

Discuss why the solutions are less than the whole number factors.

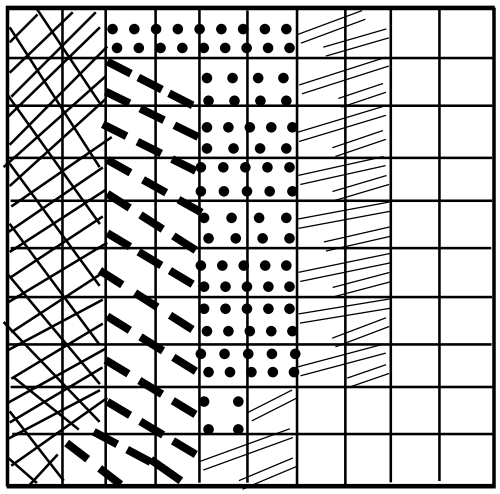
N. Decimals on the Number Line (Blackline Masters I - 68 and I - 69)

On a number line divided into tenths, have students show "jumps" of given decimal lengths and record the results in a multiplication equation. For example: 6 x 0.7 = 4.2.

Notes and textbook references

O. Modeling Hundredths (Blackline Master I - 66)

Using the "Hundreds Grids" have student shade in a decimal fraction (0.19) a given number of times - four. Use a different color for each new shading of the decimal number. Shade on the same grid until full, continuing onto other grids as necessary.



Record the total shaded in an equation like this:
4 x 0.19 = 0.76.

The North Carolina Manipulatives Kit contains tools for understanding fractions. Decimal Squares and Fraction Bars are two such manipulative tools.
.

Use problems like these :

4 x 0.63 = _____ 6 x 0.19 = _____
2 x 0.94 = _____ 7 x 0.51 = _____

P. Multiplication as Repeated Addition

Remind students that multiplication is repeated addition.

For example: **5 x 3 = 3 + 3 + 3 + 3 + 3**

Show this problem:
6 x 0.5 = _____

Ask how to show this as repeated addition.
(0.5 + 0.5 + 0.5 + 0.5 + 0.5 + 0.5)

Discuss solving multiplication as addition. Relate to solutions on the tenths circles and hundredths grids. See 1.04 M and 1.04 O for examples.

Notes and textbook references

Q. **Base 10 Activity** (Blackline Masters I - 70 and I - 71)
Students need several ten rods and unit cubes from the Base 10 Blocks (or orange and white Cuisenaire™ rods, or graph paper cut into one 1x10 rectangle and 1x1 squares), a regular die, and a set of cards prepared with the following decimal numbers: 3.2, 2.6, 1.9, 0.8, 2.1, 1.5, 0.3, 0.2, 3.6, 1.2. In pairs, students draw a card and show that number with blocks, using the 10 rod as a **one** rod and the unit cube as **tenths**. They then roll the die and make as many sets of the decimal number as shown on the die. They then combine the sets, regrouping as necessary, recording the results as an equation, such as 2.1 x 4 = 8.4. Discuss the pattern and placement of the decimal in solutions.

R. **Building Decimals** Base ten blocks and hundreds grids supply helpful models for developing this objective. Determine a decimal number such as 0.12. Roll a decahedron die or spin a spinner to determine a whole number, for example five. Then build 0.12 five times with base ten blocks or color 0.12 on a hundredths grid five times. If the base ten blocks are built onto a flat using the longs and units, the total is more obvious. The five longs, which represent tenths, are placed side by side and then the five groups of two units, which represent hundredths, are lined up into a long. This models how 5 x 0.12 = 0.60. Ask students to generate problems and model solutions with a partner, taking turns determining the decimal and whole numbers.

5 x 0.12

first, build 0.12 |. .

then build four more |. . |. . |. . |. .

When combined,

the model shows 0.60 or | | | | |. . . . . .

S. **Recipe Workout** (Blackline Master I - 67)
Students complete a chart to find quantities needed to increase or decrease the number of servings that a recipe will make. This activity shows a need to multiply and divide fractions that connects to a student's real world experiences. Patterns found in the rows and columns of the chart can help students better understand why the fraction algorithms work the way they do. The teacher may substitute any recipe that promotes interest.

Notes and textbook references

T. Rational Number Operations I (Blackline Master I - 98)

Students will work in pairs to discover the effects of multiplication and division on rational numbers. Each student will take a turn spinning the spinners. On their chart the students will write the two numbers and then multiply and divide the two numbers.
After completing the chart, the teacher will have a class discussion about what students have learned regarding computation of rational numbers.

1. Which situations produced an answer that was larger than the numbers spun?

Multiplication – Multiplication of two whole numbers gives a product that is larger than either the muliplicand or multiplier but what happens when numbers are less than one?
Why do you think this is true?

2. Which situations produced an answer that was smaller than at least one number spun?

Division – Division of two whole numbers gives a quotient smaller than the dividend but what happens when numbers are less than one?
Why do you think this is true?

U. Rational Number Operations II (Blackline Master I - 97)

Students will work in pairs or groups to discover the effects of addition, subtraction, multiplication, and division of fractions. The teacher can follow the same procedures used in **Rational Number Operations I.** Students will complete the activity and follow it up with a classroom discussion.
Questions:

1. When you add fractions, what happens to the sum?

Answer: The sum is larger than either of the individual fractions. Why: Because addition is combining the numbers.

2. When you subtract fractions, what happens to the difference?

Answer: The difference is smaller because you are taking away from the first number (minuend).

3. When you multiply fractions, what happens to the product?

Answer: The product is smaller because your product is part of the original group.

4. When you divide fractions, what happens to the quotient?

Answer: The quotient is larger because you are dividing the first fraction into groups of the second fraction.

Discuss how fraction operations are alike and different from whole number operations.

Notes and textbook references

C. Decimal Operations Students will work in pairs to discover the effects of addition, subtraction, multiplication and division of decimals. Students will draw two cards from a deck of cards. Using their two cards the students will each make a decimal. (Example: If a student picks a 7 and a 2 they will make either 0.72 or 0.27) The students will chart their decimal and their partner's decimal and then complete the computations.

Questions:

1. When you add decimals what happens to the sum?

Answer: The sum is larger than either of the individual decimals,

2. When you subtract decimals what happens to the difference?

Answer: The difference is smaller because you are taking away from the first number.

3. When you multiply decimals what happens to the product?

Answer: The product is smaller because your product is part of the original group.

4. When you divide decimals what happens to the product?

Answer: The quotient is larger because you are dividing the first decimal into groups of the second decimal.

Discuss how decimal operations are alike and different from fraction operations and whole number operations.

V. Shopping Trip Materials Needed: Catalogs or store fliers (Example: Walmart™, Best Buy™, etc.)

Students work in groups of three or four. One student is the shopper and the others will be the checkers. The student who is the shopper chooses 5 items to buy in the flier. They name the items and the actual price. The checkers write down the list of items and the price of each. The checkers estimate the cost of each item and compare estimates. Students will discuss their estimates and support their answers. The next student takes their turn as shopper and play continues until each student has had 2 turns as the shopper.

Notes and textbook references

W. Space Weights (Blackline Master I - 20)
Students use facts about planetary gravity to determine weights on various planets. They will use a fraction value and then a more precise decimal value to do the calculations.

X. Is it reasonable? The teacher will use a variety of student-related situations to discuss reasonableness of solutions.
Example: The teacher will ask the students for a reasonable average of the age of the students. Would 15 be a reasonable average? Why or why not? Would 11 be a reasonable average? Why or why not? Then, have the students volunteer their ages and figure the exact average. Then compare that to the answers that were offered as a reasonable average and discuss how they differ and whether or not they were reasonable.

Other possible topics:
Shoe size, hand span, height, arm length, number of televisions in your home, number of students who have cell phones, etc.

Y. Day at the Park (Blackline Masters I - 100 and I - 101)
This is a whole class game. The students will play against each other in teams of two. Give each team a 3 x 5 card and have them write a large U for unreasonable on one side and a large R for reasonable on the other. Teams will also need paper and pencil. The teacher will show on the overhead and read situation with an answer. Teams should determine the reasonableness of the answer. Then they hold up their card with the correct letter facing the teacher. Teams should be prepared to defend their answer and a point will be given to each team who correctly answers the question.

Z. You're the Teacher The teacher will select a quiz or short assessment from a current unit. The teacher will fill in answers (making some reasonable and some not). The students will work in pairs to "correct" the quiz by marking each answer reasonable or unreasonable. They should be prepared to defend their answer in class discussion as a follow-up to the activity.

Notes and textbook references

Tips for Problem Solving in Your Class

- Set the expectation that everyone thinks! State a problem and then give everyone a moment to think about it.
- Use think-pair-share to jumpstart your students' problem-solving processes. First they think over the question, then they talk it over in pairs, then each pair shares with a larger group.
- Don't let textbooks or other published supplementary materials thwart the problem-solving process. Be wary of texts that give many drill problems with one word problem that is solved the same way as the previous problems. Also watch out for problem sets that are all basically identical.
- Incorporate group problem solving into your lessons, so students have a chance to observe their peers.
- Use problems from a variety of sources. Ask questions in a variety of ways.
- Ask a variety of questions from the same problem source data. Students begin to anticipate what a question will be without having really read the problem. Keep them flexible in their expectations.
- Expose students to problems in which the numbers they read in the problem are not necessarily the ones they will "crunch" to solve the problem. Use price lists, menus and other materials so that students will search out meaning and not just begin to crunch numbers.

Notes and textbook references

1.05 Develop fluency in the use of factors, multiples, exponential notation, and prime factorization.

A. **Problem Discussion Cards** (Blackline Master I - 72) Students should work in groups to solve the given problem. The teacher may wish to have all students in the class working on the same problem, or she may prefer to use different problems in the various groups. After the students have had an opportunity to brainstorm, the groups should share their solutions with the rest of the class.

B. **Exponent Dice (**Blackline Master I - 96)

Ask the students to work in pairs. Give each pair of students dice and a calculator. The students decide who will roll for the "base" and who will roll for the "exponent". Roll the dice. The first die is the base number. The second die is the exponent. You may wish to model the first one for the students.

Students continue for 10 rounds. Look for patterns. Ask the students to answer the following questions:

1. What happens when 1 is the exponent?
2. How could you write 12? Find the product. Will your calculator do this? Try lattice multiplication as the numbers get very large.

C. **Exponents with the Calculator** Give each student a calculator that uses algebraic logic and a list of problems similar to these. Ask the students to write down what they input into the calculator and the solution.

Problem	Input into Calculator	Solution
$4^3 + 4^2$	4 x 4 x 4 + 4 x 4	80
$3^2 + 4^3$	3 x 3 + 4 x 4 x 4	73
$3^3 + 2^3$	3 x 3 x 3 + 2 x 2 x 2	35

Notes and textbook references

D. **Exponent Steps** Using exponents of 1 through 10 and a base of 2, ask the students to write the steps as begun below:

$2^1 = 1 \times 2$
$2^2 = 1 \times 2 \times 2$
$2^3 = 1 \times 2 \times 2 \times 2$
$2^4 = 1 \times 2 \times 2 \times 2 \times 2$
•
•
•

Then "climb" the steps by finding the solutions. (Use a calculator when appropriate.) Repeat the activity with a base of 3, of 4, and of 8. Culminate the activity by asking the students to write the answers to the following questions.

1. What happens when a number has an exponent of 1?
2. What happens when a number has an exponent of 2?
3. What happens when a number has an exponent of 5?

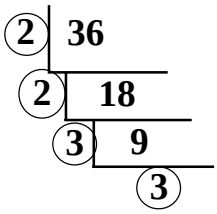
E. **Game of POWERS** Introduce POWERS. In order to play this game, students need two dice or spinners and a calculator. The goal is to create the largest possible number. The two dice are rolled. Students decide which number to use as the base and which to use as the exponent in an effort to create the largest number. For example, the dice show a two and a five. Students decide whether to create a 2^5 or 5^2. After writing the base and exponent, students can use their calculators to compute the value using repeated multiplication. Two to the fifth power is$1 \times 2 \times 2 \times 2 \times 2 \times 2 = 32$ while five squared is $1 \times 5 \times 5 = 25$. Using these materials, what is the largest possible number? What is the smallest? Is there a strategy for deciding which digit becomes the base and which becomes the exponent? Play **Powers** by establishing a target number. Here the goal is to get as close as possible to the target, such as 250. Students might be asked to build models of the numbers created while playing **Powers**. How might students create a model showing 2^5? If they keep in mind that $2^5 = 1 \times 2 \times 2 \times 2 \times 2 \times 2$, this might suggest an approach. If $2^1 = 1 \times 2$, this might be modeled with two cubes linked. By doubling this, the model is increased to two sets of two cubes, or four cubes. This model is then doubled to become 8 cubes. This model is then doubled again to show 16 cubes. Finally, this set of 16 cubes is doubled to 32. Students might cut rectangles from grid paper showing this progression and labeling each. If each resulting rectangle is treated as a new unit to be doubled, an interesting pattern results. This same pattern occurs as other numbers are raised by consecutive powers. Students might be asked to create this series of models for smaller numbers.

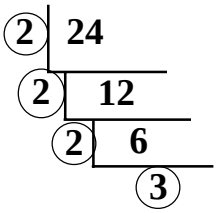
Notes and textbook references

F. Factors with the Calculator Encourage the students to use their calculators. By ***chaining***, the students can generate the multiples of several numbers as shown on the chart. Remind the students that zero is a multiple of every number and then discuss other like multiples. Examine the list for patterns. Students can shade the multiples and locate prime numbers.

Number	Multiples
3	0, 3, 6, 9, 12, 15, 18, 21, 24, 27, . . .
5	0, 5, 10, 15, 20, 25, 30, 35, 40, 45, . . .
7	0, 7, 14, 21, 28, 35, 42, 49, 56, 63, . . .
9	0, 9, 18, 27, 36, 45, 54, 63, 72, 81, . . .

G. Factor Ladders Find the prime factors of a number using the division method. Students should begin with the least prime number that is a factor and proceed to the next factors in ascending order.





Open ended prompt: "When is the product in multiplication less than the larger factor?

H. Multiples and Factors Provide each student with a hundred board and a set of linking cubes or connecting cubes or some other stacking blocks that come in a variety of colors. Have students choose one color of cube such as blue. Students count by 2's and place a blue cube on each number named. This places a blue cube on every even number. Then a second color of cube is chosen, perhaps red, and placed on multiples of 3. At this point, any number that is a multiple of both 2 and 3 has a tower of two blocks, blue and red. Continue this process as long as practical. (Note: at some point, students might take a second hundred board and begin coloring this pattern. A blue stripe could be colored on each even number. Then the blue cubes could be removed before continuing to add more colors. Then a red stripe is added to each multiple of three, etc.) What kinds of patterns exist? Are there numbers with only one block? Are there numbers with an even number of blocks? Do some have an odd number of blocks? Why? Have students record their results by writing numbers and their factors. For example: 12 has factors of 1, 2, 3, 4, 6, and 12. Ask students which numbers would have a cube if they had counted by ones? This provides opportunity to discuss the fact that 1 is also a factor of all whole numbers. Ask students to work with a partner to write definitions for the terms "multiple" and "factor".

Notes and textbook references

I. **Tax Collector** (Blackline Masters I - 73 through I - 75) Provide an opportunity for students to play Tax Collector, page 67 of THE FAMILY MATH BOOK, Lawrence Hall of Science, University of California, Berkeley.

J. **Perfect, Abundant, and Deficient Numbers** Explore the results of adding all the proper divisors of a given number. A proper divisor is a divisor (factor) that is less than the given number.
The results will yield numbers that are perfect, abundant or deficient!

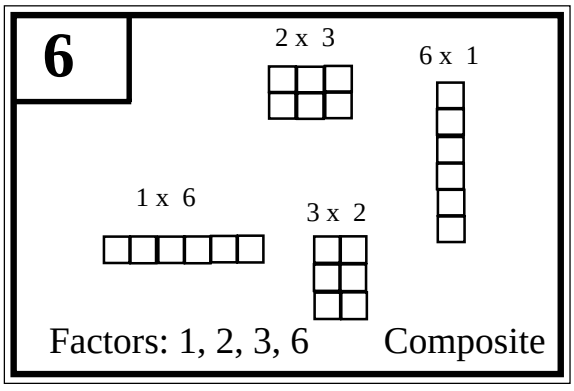
Perfect Numbers are numbers which are equal to the sum of their divisors.
Ex. 6 is **perfect** because 6 = 1 + 2 + 3;
28 is **perfect** because 28 = 1 + 2 + 4 + 7 + 14.

Abundant Numbers are numbers that are greater than the sum of their proper divisors (factors).
Ex. 36 is **abundant** because 1 + 2 + 3 + 4 + 6 + 9 + 12 + 18 = 55, and 55 > 36;
24 is **abundant** because 1 + 2 + 3 + 4 + 6 + 8 + 12 = 36 and 36 > 24.

Deficient Numbers are numbers that are less than the sum of their proper divisors (factors).
Ex. 8 is **deficient** because 1 + 2 + 4 = 7 and 7 < 8;
39 is **deficient** because 1 + 3 + 13 = 17 and 17 < 39.

These explorations give a purpose to a factor search and also keep addition skills sharp. Note: This is an enrichment activity.

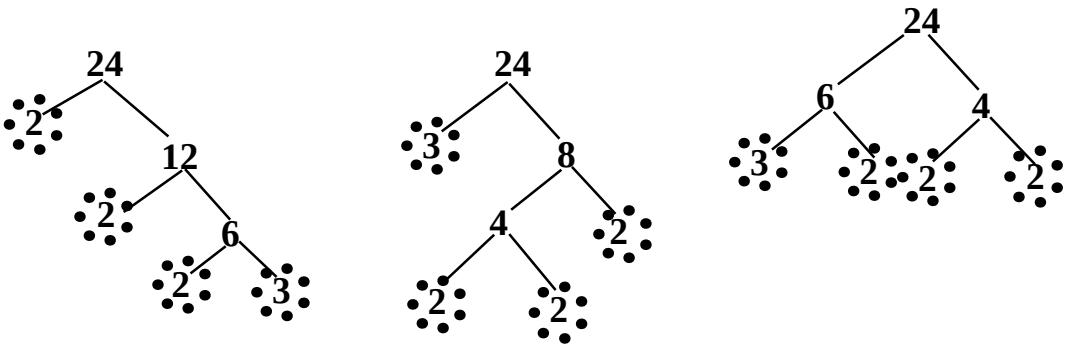
K. **Calendar Factors** Using each date on the calendar, make area models with graph paper of that number. List the factors of that number and tell whether the number is prime or composite. Students may take turns making the chart of each date's number. *Example:* On the sixth of the month, this might be the chart:



Notes and textbook references

L. Prime Numbers This activity should be used after students have developed the definition of prime numbers. Use a factor tree to find the prime factors of a number.

To find all of the factors of 24, write down all of the product combinations of the prime factors and 1. Remind your students that 1 and the number itself are always factors. List the factors of 24. Try the activity again with 36. Note that there are several possible "trees" for each number and that a branch ends when a prime "blooms".



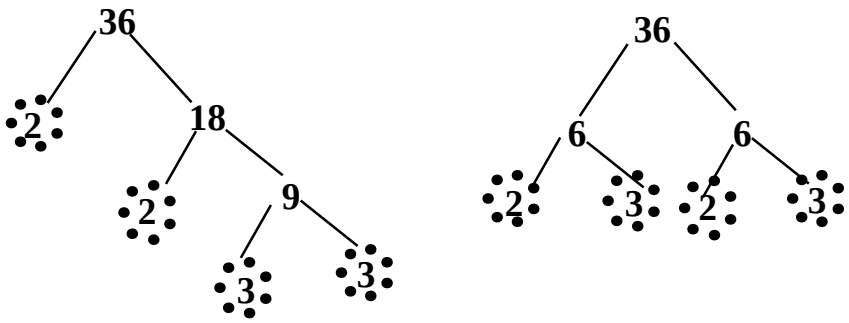
Prime Factorization $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

Product Combinations of Prime Factors

2 x 1	= 2
2 x 2	= 4
2 x 2 x 2	= 8
2 x 2 x 3	= 12
2 x 2 x 2 x 3	= 24
3 x 1	= 3
3 x 2	= 6

Ask students to answer the following questions:

1. What is a factor of a number?
2. How can you determine the factors of a number?



Notes and textbook references

Product Combinations of Prime Factors

2 x 1	= 2
2 x 2	= 4
2 x 2 x 3	= 12
2 x 2 x 3 x 3	= 36
3 x 1	= 3
3 x 2	= 6
3 x 3	= 9
3 x 3 x 2	= 18

In January, 1994 a team of scientists using a Cray computer reported finding the largest prime number to date. The number is 2^{859433}-1. If printed in a newspaper, its 258,716 digits would take eight pages!

M. Writing about Primes and Composites Review what prime and composite numbers are and then ask the students to work in pairs and write the answers to the following questions:

1. What can be said about the number 1?
2. What can be said about the number 2?
3. What can be said about <u>any</u> composite number?
4. What can be said about <u>any</u> prime number?

N. Prime Number Dice Roll a pair of dice and add. Is the sum prime or composite? Suppose you had multiplied, would the product be prime or composite? Suppose you decide to play a game: one person scores a point for each prime number (obtained by either adding or multiplying) and the other person earns a point for each composite number he or she is able to obtain at a turn. Would this be a fair game? Why or why not?

Notes and textbook references

O. Prime Numbers with the Hundred Board (Blackline Master I - 76) In these hundred board activities students identify numbers less than 100 with only two factors, one and the number. These, of course, are prime and students can record these as such in their math journals. There should be 25 numbers in this list of primes less than 100. All other positive numbers, except 1 which is neither prime nor composite, are composite. Ask students to imagine that they don't have their list of primes and need to decide whether a given number is prime or composite. How might they approach this task? Would square tiles be useful? Would calculators be helpful? Allow students time to discuss this problem.

P. Rectangles with Primes and Composites Write some composite and prime numbers on the chalkboard. Ask students to choose one of the numbers and build all the possible rectangles using square tiles for this number. Ask them to also record these rectangles on grid paper or by drawing appropriate grids. Students might work in small groups to build rectangles for all the numbers on the chalkboard. Ask students to compare results and look for patterns. What conclusions can be drawn about prime and composite numbers?

Q. Rules of Exponents Triangle Puzzle (Blackline Master I - 77) Let students work in groups to put the puzzle together. Each pair of touching edges should show equivalent expressions. When the puzzle is completed correctly it will be in the shape shown in miniature on the page. Note: This is an enrichment activity.

R. Exponent Experts Game (Blackline Masters I - 78 and I - 79)

Materials: Each group needs a spinner and a set of cards that have been cut apart.

Directions: Students play in groups of two to four students. The cards are shuffled and distributed among the students. On a player's turn, he spins the spinner and gives the answer that results when substituting the spun number for the variable in the expression on one of the cards. One point is awarded for each correct answer. At the end of the game, individual points and team points are totaled.

Notes and textbook references

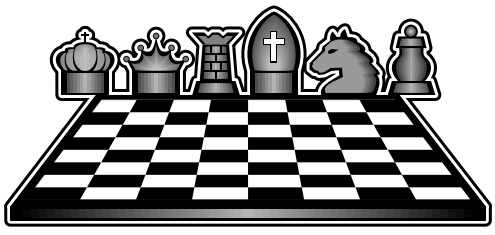
S. Cooperative Problem-Solving Cards - Exponents
(Blackline Masters I - 80 and I - 81) Let students work in groups of four to solve these two problems. Give each person in the group one of the cards. The students may share the information on the cards with the group, but they cannot give the card to anyone else. This gives each student something to contribute to the group, and each student gets an opportunity to observe the thought processes of his peers.

T. Lucy Bakes Beans! In an episode of ***The Lucy Show***, Lucy needs $5000 from her banker to buy new furniture. He tells her that if she can start with a penny on day 1, and then double the amount she has each following day, before a month is over she will have enough money. How many days would it actually take her to get at least $5000?

Lucy finds out that a bean company is offering "double your money back" if the beans are not the best the buyer ever tasted. Lucy knows her Grandmother's baked beans are the best ever, so she sees this as a chance to double her money. She starts buying beans and then returning them for double the money back. She uses that money to buy twice as many cans as the day before. She plans to continue buying more cans and returning them for double the amount until she has enough to buy the furniture. If the beans cost $0.50 a can, and she makes one buy-and-return transaction per day, how many days would it take her to have enough for her furniture? Note: Lucy finally tastes the beans and decides she can't accept the money. However, the bean company owner decides to pay her for her testimonial, so there is a happy ending for all.

U. The King and the Gamemaker The story is told that the King of Persia was so thrilled with the game of chess that he offered the creator of the game anything he wished. The proud chessman asked for something seemingly simple. He asked for one grain of rice to be placed on the first square of a chessboard, twice as much on the 2nd square, twice as much again on the 3rd square and so on until all 64 squares had been filled with each square having twice as much as the one before.

The king was puzzled, but decided to grant the request. However, this turns out to be enough rice to cover the country of Persia with a blanket of rice one meter thick (or the state of California with a blanket of rice 1 foot thick). We are not told what reward the chessman finally received.



Notes and textbook references

V. The Towers of Hanoi (Blackline Masters I - 82 through I - 84) There is an ancient legend that in the great tower of Hanoi there are three diamond spindles. On the middle one there is a stack of 64 disks of different sizes, each one smaller than the one below it. Monks in the temple have the task of moving the disks from one spindle to another, but they can move only one disk at a time, and they can never place a larger disk on top of a smaller one. The legend says that when this task is complete, the temple will disappear in a clap of thunder and the world will end. If the monks are very efficient and move these disks in the quickest way possible with each move lasting only one second, how long do we have until the world ends?

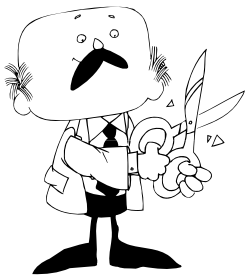
Models of such towers with seven disks can be purchased or made from wooden blocks, nails, and washers. Computer graphics are also useful in solving the problem. A suggested strategy is to start with a smaller number of disks and find the smallest number of moves to transfer all the disks. Gradually increase the number of disks in the puzzle and look for a pattern.

The solution is $2^{64} - 1$ moves. If each move takes a second, this is well over 500 billion years. Note: This is an enrichment activity.

1.06 Use exponential, scientific, and calculator notation to write very large and very small numbers.

Notes and textbook references

A. **Modeling Exponentials** (Blackline Masters I - 85 and I - 86) Teachers should show their students the base-10 blocks at the beginning of this exercise. Small blocks may be used to show the students what the blocks would look like if there were only two on a side instead of ten. Explain to the students that *x* represents the length of the long block and the edge of each cube. Point out that x^3 can represent a cube of any size and x^2 can represent a flat of any size. Note; this is an enrichment activity.



B. **Scientific Notation Square Puzzle** (Blackline Master I - 87)
This is intended to be a cooperative activity in which students assemble 16 small squares to form a larger square. Edges of the puzzle pieces should match a number in scientific notation to its decimal equivalent. Students will benefit from listening to each other discuss strategies for solving the puzzle.

You may wish to cut the puzzle pieces apart and place them in envelopes as the Blackline Master gives the solution.

C. **Mathematical Message - I** (Blackline Master I - 88) Students will convert each number from exponential notation into decimal notation. When the numbers are placed in numerical order, a message is spelled out.

D. **Population Study** Integrating mathematics with other subjects is one way to help students understand the importance of the math they study. In the 6^{th} grade, students are studying Europe and South America. Have students find the population of each country in the two continents. They should round each number to the nearest million and then write them in scientific notation.

E. **Powers of Ten** A book and short video entitled ***Powers of Ten*** was written in 1977 by Charles and Ray Eames. The Powers of Ten website, http://www.powersof10.com/, has many activities related to the topic and ordering information for an interactive CD. The essence of the film shows a picture of a person lying on a blanket which represents one square meter. Each picture following zooms out or in by a power of ten.

Notes and textbook references

F. Scientific Notation Web Pages The following list of websites deal with the topic of scientific notation. Most allow students to practice converting decimal numbers to scientific notation with feedback.
http://www.nyu.edu/pages/mathmol/textbook/scinot.html
http://janus.astro.umd.edu/astro/scinot
http://www.ieer.org/clssroom/scinote.html
http://memers.aol.com/profchm/sci_not.htm/
www.chempractice.com/drills/java_sci_notation.shtml
www.edinformatics.com/math_science/scinot.htm

G. Scientific Notation Square Puzzle (Blackline Master I - 89) Students work in groups to rearrange the small squares back into a large square. Two touching edges must contain equivalent expressions.
Note: It would be best to cut out the small squares and place them in an envelope before giving the puzzle to the students as the blackline gives the "answer".
This puzzle should be worked by pairs or small groups of students. Each group member should be in possession of some of the puzzle pieces at the start of the activity.

H. Scientific Notation Team Game (Blackline Master I - 90)
Materials: Transparency or laminated sheet of the playing mat. Two colors of dry erase markers or two objects with different shapes are used to mark the position of each team. A large paper clip is needed for the spinner.
Directions: Divide the class into two teams, or let students play against each other in teams. The leader begins the game by writing a number in scientific notation in the top rectangle on the board. On a team's turn, they spin and change the number according to the instructions on the spinner. If they are correct, the team advances one square, and the number in play is changed to the number the team just constructed. If they are incorrect, the number in play remains the same and the team is moved backwards one square. The winner is the first team to reach the finish.

I. Scientifico (Blackline Masters I - 102 and I - 103)
Students practice translating numbers expressed in scientific notation into standard notation. Students take turns rolling three number cubes and constructing a number in scientific notation. Example: 3, 4, 6 can be written as 3.6×10^4. After recording the number on the recording chart, the student places a marker in the proper place on the game board. The student who completes a row, diagonal, or column is the winner.
Teacher Note: There are two game boards on the Blackline Master. Use one for very large numbers and the other for very small numbers.

1.07 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

Notes and textbook references

A. Interpreting Problems (Blackline Master I - 91)
Students are given fraction and decimal problems to solve with the instructions to draw a picture of each situation. The purpose of this activity is to help students remember to understand a problem before they choose an algorithm to solve it. A similar idea would be to have students draw a picture of each word problem before they try to solve it. Once the problem is drawn, have them identify where the numbers are in the problem. Are they price tags? Numbers of items? Once a student understands the problem well enough to draw it, the solution becomes much easier.

B. Weighty Problems (Blackline Master I - 92)
In sixth grade health classes, students are studying nutrition and weight management. These problems integrate with that topic. Students should work in groups to solve each problem. The teacher may wish to use the same card for each group or to let the groups work on different problems. A class discussion of the results will allow groups to share their strategies.

How thick is a sheet of toilet paper? How can you find out?

C. Problems From a Brochure Provide students with a sales flier, brochure, or menu in which there is a variety of numerical data. Challenge the students to make up problems from this brochure for other students. Then have the students exchange problems and solve. Can one group stump the others? Can the group providing a challenging problem and defend their solution?

D. How a Simple Telescope Works (Blackline Master I - 93) Two sixth-grade science topics are the solar system and technology used to explore space. In this activity, students are told how a simple telescope works. They can use this information to solve problems that involve decimal addition and division.

Notes and textbook references

E. **Acme Science Supply** (Blackline Master I - 94)

Students solve problems from a chart that provides information on the sale of scientific supplies.

F. **Comets in the News** (Blackline Master I - 99)

Students use reports of comets in the news to get an idea of the size of comets and the frequency of their appearance. Useful materials for this activity are a state road map and a foam Hefty™ plate.