Grade Level/Course: 7-8

Lesson/Unit Plan Name: Solving Two-Step Equations with Number Lines

Rationale/Lesson Abstract: A picture is worth a thousand words. This lesson includes number lines, bar models and decomposition as illustrations for solving equations.

Timeframe: 50 minutes

Common Core Standard(s): 8.EE.7 Solve linear equations in one variable.
7.EE.4a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$ where p, q, and r are specific rational numbers.

Instructional Resources/Materials: Paper, pencil, copies of Ticket Out the Door

Activity/Lesson: Warm-up in groups of four. Have students count off one through four in their groups. "Looking at the structure of these equations, how many steps do you think they will take?" [2] "Ones will start with equation #1) $2x + 4 = 16$, Twos start with equation #2) $3x - 14 = 16$, Threes start with #3) $4 + \frac{x}{5} = 6$ and Fours start with #4) $\frac{x}{6} - 7 = -2$. Do just the first step of your equation using any method you choose. Pass your paper to the right when I tell you to switch and do the next step on the equation you receive. If the equation is solved by the time it is your turn, your job is to check or double check the work. Line up equal signs and use proper syntax!" Possible group solutions are listed below.

#1)
$$2x + 4 = 16$$
$$2x + 4 - 4 = 16 - 4$$
$$2x = 12$$
$$\frac{2x}{2} = \frac{12}{2}$$
$$x = 6$$

#2)
$$3x - 14 = 16$$
$$3x - 14 = 16 - 14 + 14$$
$$3x = 16 + 14$$
$$3x = 30$$
$$x = 10$$

#3)
$$4 + \frac{x}{5} = 6$$
$$4 - 4 + \frac{x}{5} = 6 - 4$$
$$\frac{x}{5} = 2$$
$$x = 10$$

#4)
$$\frac{x}{6} - 7 = -2$$
$$\frac{x}{6} - 7 = -2 - 7 + 7$$
$$\frac{x}{6} = -2 + 7$$
$$\frac{x}{6} = 5$$
$$x = 30$$

"What do you think is the most common mistake made with these equations?" [Adding or subtracting the wrong number on both sides. Errors with integers.]

Activity/Lesson continued:

Ex 1) Solve: $2x+5=13$

Method 1

A bar model shows the left and right sides of the equation taking equal space.

"Looking at the structure of this equation, how many steps do you think it will take to solve?" [2 or more] "Today, we will focus on visual models of two-step equations."

$2x+5$
13

$2x$	5
8	5

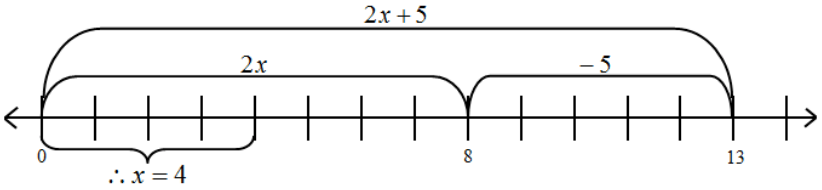
x	x	5
4	4	5

$\therefore x=4$

Method 2

A number line shows the left and right sides of the equation as the same distance.

Solve: $2x+5=13$ "Mark 13 on the number line 13 spaces from zero. Now go five less than that to mark 2*x*. Subtracting five means five spaces to the left of 13 on the number line. "Where do we stop?" [8] "Now we'll split the length 2*x* in half to find *x*."



Method 3

Decomposing shows the left and right sides of the equation as equal parts.

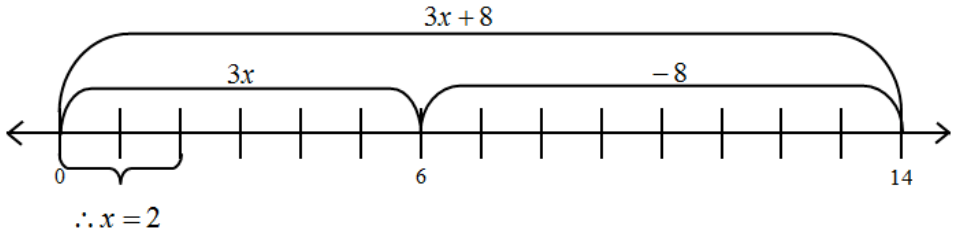
Solve: $2x+5=13$

$$2x+5=13$$
$$2x+5=8+5$$
$$2x=8$$
$$x+x=4+4$$
$$\therefore x=4$$

"What similarities do you notice between the different methods?"

You Try:

Solve using a number line: $3x+8=14$



"Did anyone draw this equation a different way? Let's compare."

Ex 2) Solve: $4x-3=9$

Method 1
A bar model shows the left and right sides of the equation taking equal space.

$4x-3$
9

$4x-3$
$9+3-3$

$4x$	-3
12	-3

x	x	x	x	-3
3	3	3	3	-3

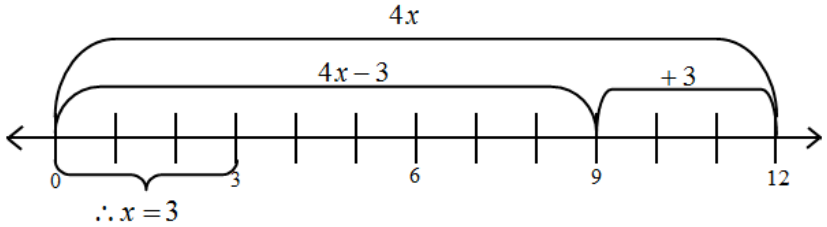
$\therefore x=3$

Showing negative numbers in a bar model can be a challenge. Reminder: we can add a zero pair.

Method 2
A number line shows the left and right sides of the equation as the same distance.

Solve: $4x-3=9$

"Four *x* minus three is nine. Mark 9 on the number line. Then move three spaces to the right to undo subtracting three. What is three to the right of 9?" [12] "We'll have to cut the 12 into four equal lengths to find one *x*."

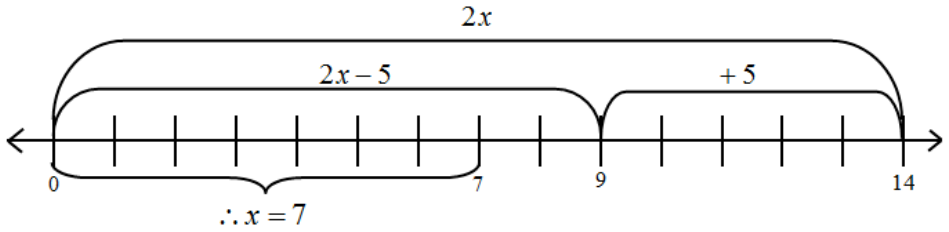


Method 3
Using inverse operations emphasizes the Addition Property of Equality.

Solve: $4x-3=9$

$$4x-3=9$$
$$4x-3+3=9+3$$
$$4x=12$$
$$x+x+x+x=3+3+3+3$$
$$\therefore x=3$$

You Try:
Solve using a number line: $2x-5=9$



"Did anyone draw this equation a different way? Let's compare."

Ex 3) Solve: $\frac{x}{4}+3=8$ Method 1: Bar Model

$\frac{1}{4}x+3$
8

$\frac{1}{4}x$	3
5	3

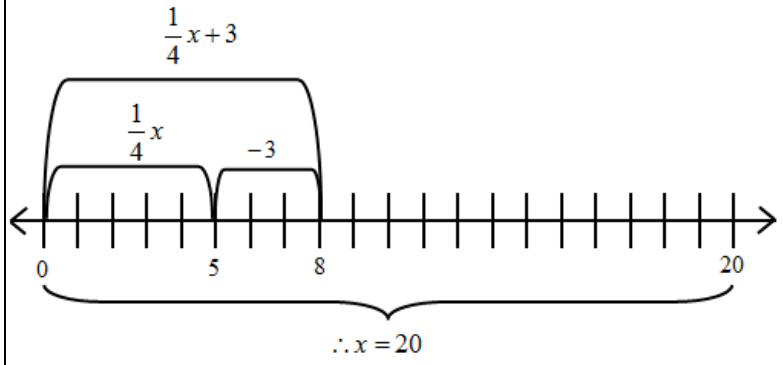
$\frac{1}{4}x$	$\frac{1}{4}x$	$\frac{1}{4}x$	$\frac{1}{4}x$	3
5	5	5	5	3

$\therefore x=20$

"If one quarter of *x* is 5, then *x* has to be 20." Making a separate bar model for $\frac{1}{4}x=20$ is an option.

Solve: $\frac{x}{4}+3=8$

Method 2: "Starting 8 spaces from zero, we'll move back three spaces to undo adding three. Where do we stop?" [5] "Another way of saying "*x* divided by four" is "one quarter of *x*". Once we've found the length of one quarter of *x*, we can repeat that distance four times to get the length of one whole *x*."



$\therefore x=20$

Solve: $\frac{x}{4}+3=8$

Method 3: Using inverse operations to subtract three from both sides mirrors the number line movement to the left. Repeated addition of one fourth *x* to get to one whole also correlates with the number line.

$$\frac{x}{4}+3=8$$

$$\frac{x}{4}+3-3=8-3$$

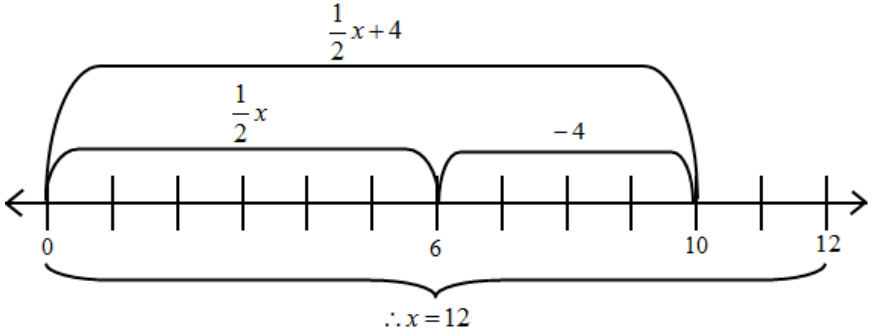
$$\frac{x}{4}=5$$

$$\frac{1}{4}x+\frac{1}{4}x+\frac{1}{4}x+\frac{1}{4}x=5+5+5+5$$

$$\therefore x=20$$

You Try:

Solve using a number line: $\frac{x}{2}+4=10$



$\therefore x=12$

Assessment:

Ticket Out the Door: For A through E, choose Yes or No to indicate whether it could be a correct step in solving $3x + 6 = 21$.

A

$3x$	6
15	6

○ Yes ○ No

B $x + 2 = 7$

○ Yes ○ No

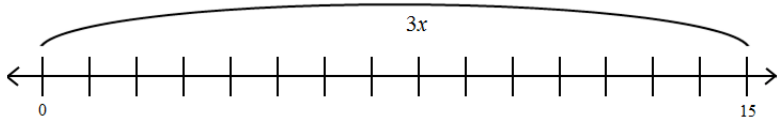
C $3x + 6 = 21 + 6 - 6$

○ Yes ○ No

D $3x = 27$

○ Yes ○ No

E



○ Yes ○ No

For any "No" responses, explain the error that was made.