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FOR TEACHING & LEARNING

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NEW JERSEY CENTER
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# Algebra I

## Graphing Linear Equations

**2015-11-04**

**www.njctl.org**

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Teacher Notes

# Linear Equations

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# Linear Equations

Any equation must have at least one variable.

Linear equations have either one or two variables and may also have a constant.

The variables in a linear equation are not raised to any power (beyond one): they are not squared, cubed, etc.

The **standard form** of a linear equation is

$$Ax + By = C$$

Where:

- x and y are variables
- A and B are coefficients and C is a constant
- A, B, and C are integers
- $A \geq 0$

# Linear Equations

There are an infinite number of solutions to a linear equation.

In general, each solution is an ordered pair of numbers representing the values for the variables that make the equation true.

For each value of one variable, the value of the other variable is determined.

# Linear Equations

The fact that the solutions of linear equations are an infinite set of ordered pairs helped lead to the idea that those could be treated as points on a graph, and that those points would then form a line.

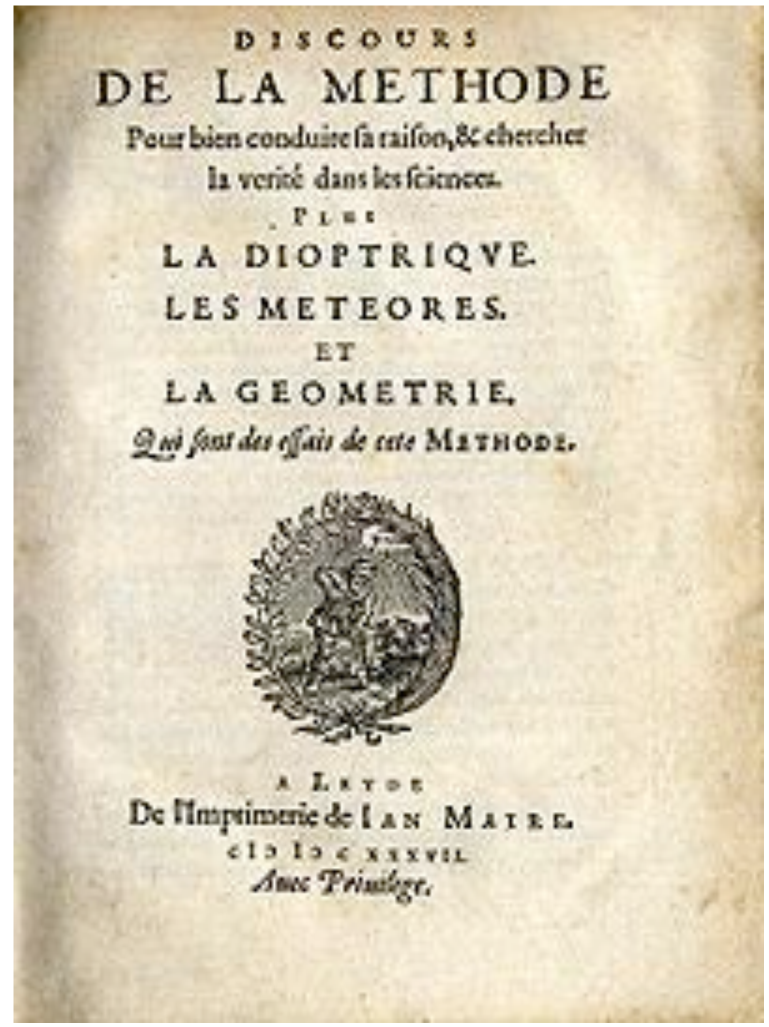
Which is why these are called "Linear Equations."

The idea of merging algebra and geometry led to analytic geometry in the mid 1600's.

# Graphing Equations

Analytic Geometry

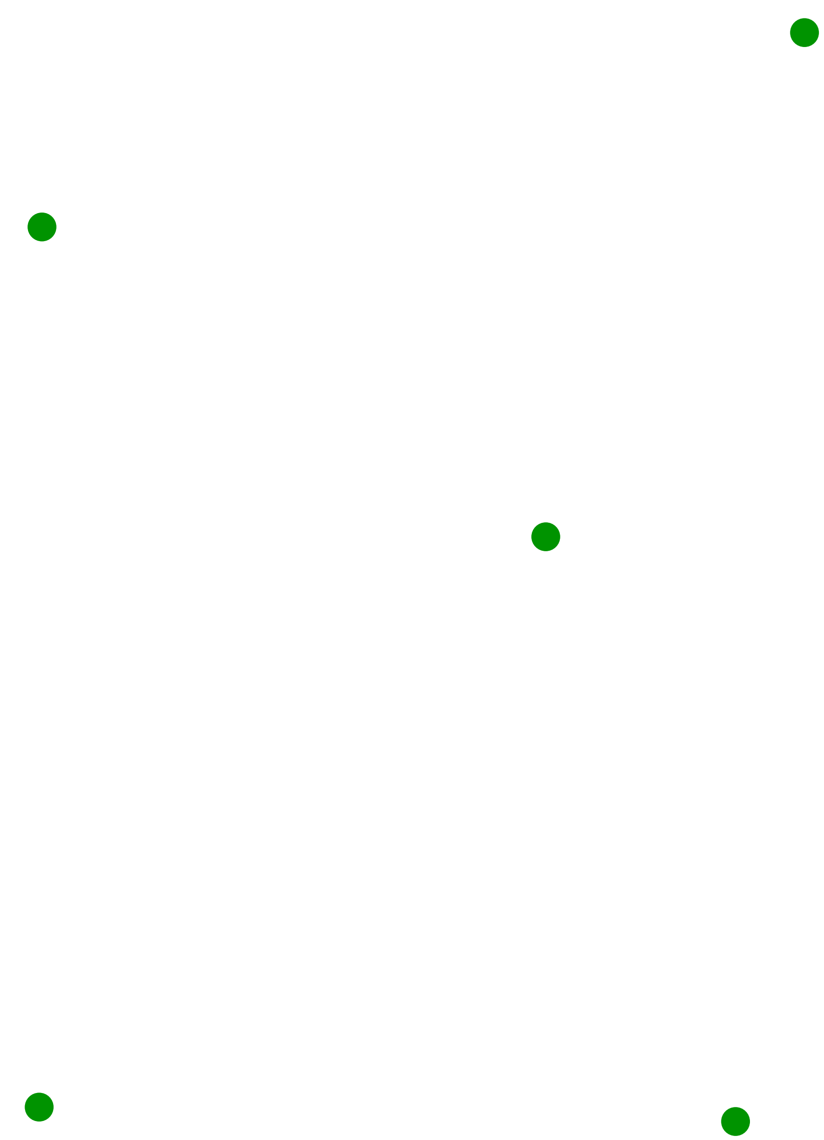
- A powerful combination of algebra and geometry.
- Independently developed, and published in 1637, by Rene Descartes and Pierre de Fermat in France.
- The Cartesian Plane is named for Descartes.


DISCOURS
DE LA METHODE
Pour bien conduire ſa raiſon, & chercher
la verité dans les ſciences.
PLUS
LA DIOPTRIQVE.
LES METEORES.
ET
LA GEOMETRIE.
Qui ſont des eſſais de cete METHODE.
A LEYDE
De l'Imprimerie de IAN MAIRE.
CIϽ IϽ C XXXVII.
Avec Privilege.

# Graphing Equations

How would you describe to someone the location of these five points so they could draw them on another piece of paper without seeing your drawing?

Discuss.

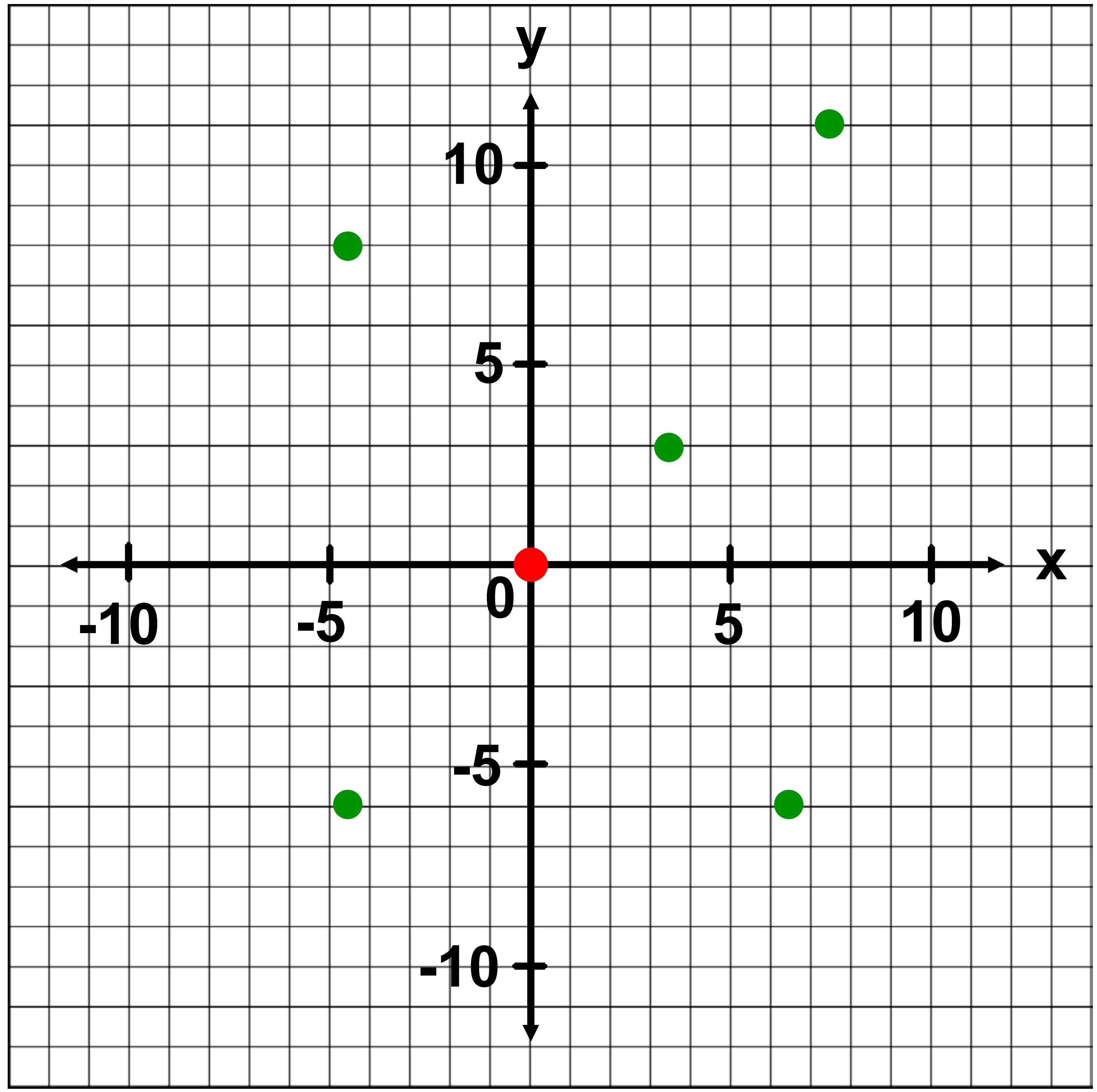


Math Practice

# Graphing Equations

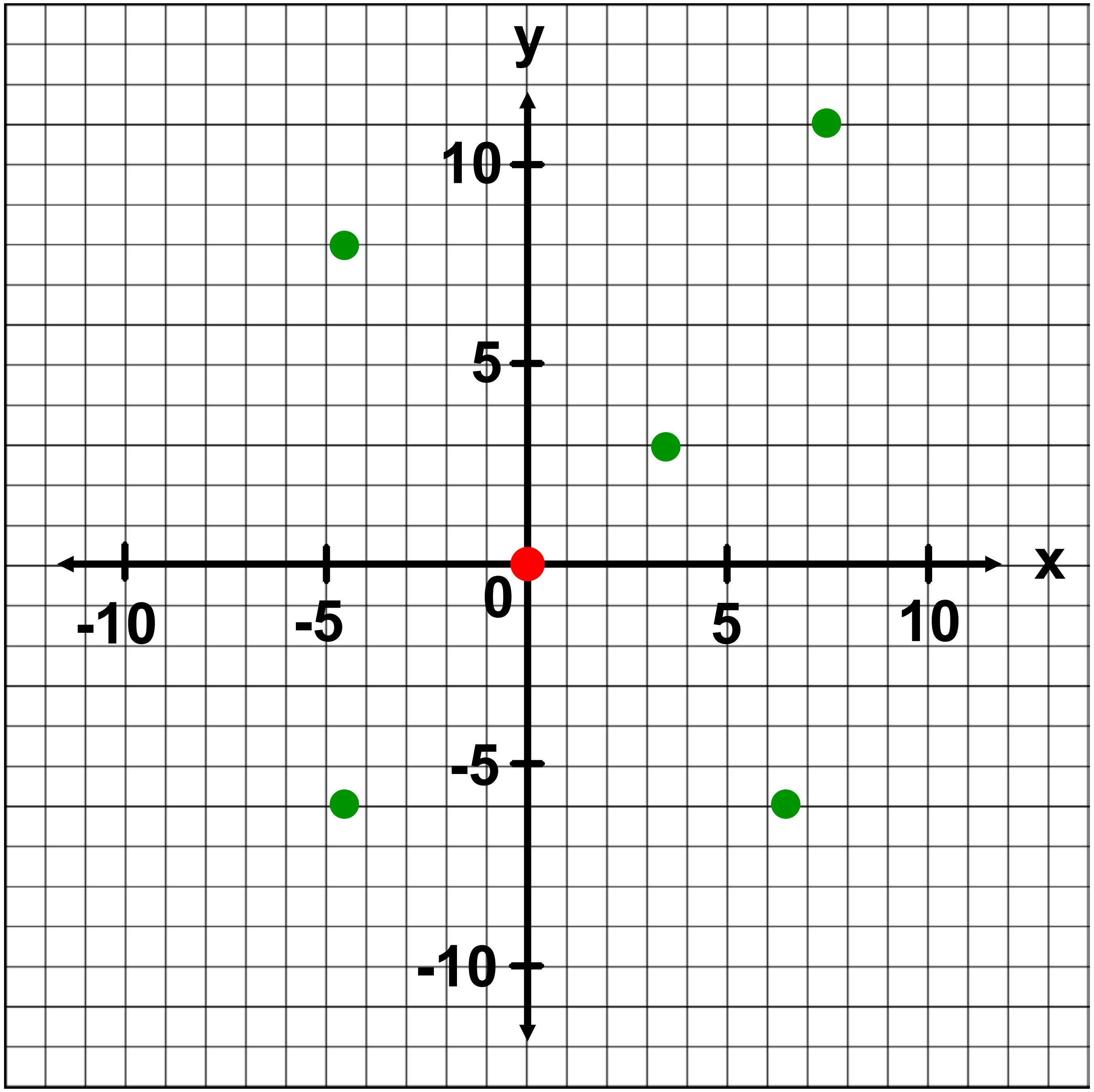
Adding this Cartesian coordinate plane makes that task simple since the location of each point can be given by just two numbers:

an x- and y-coordinate, written as the ordered pair (x, y)



# Graphing Equations

With the Cartesian Plane providing a graphical description of locations on the plane, solutions of equations (as ordered pairs) can be analyzed using algebra.

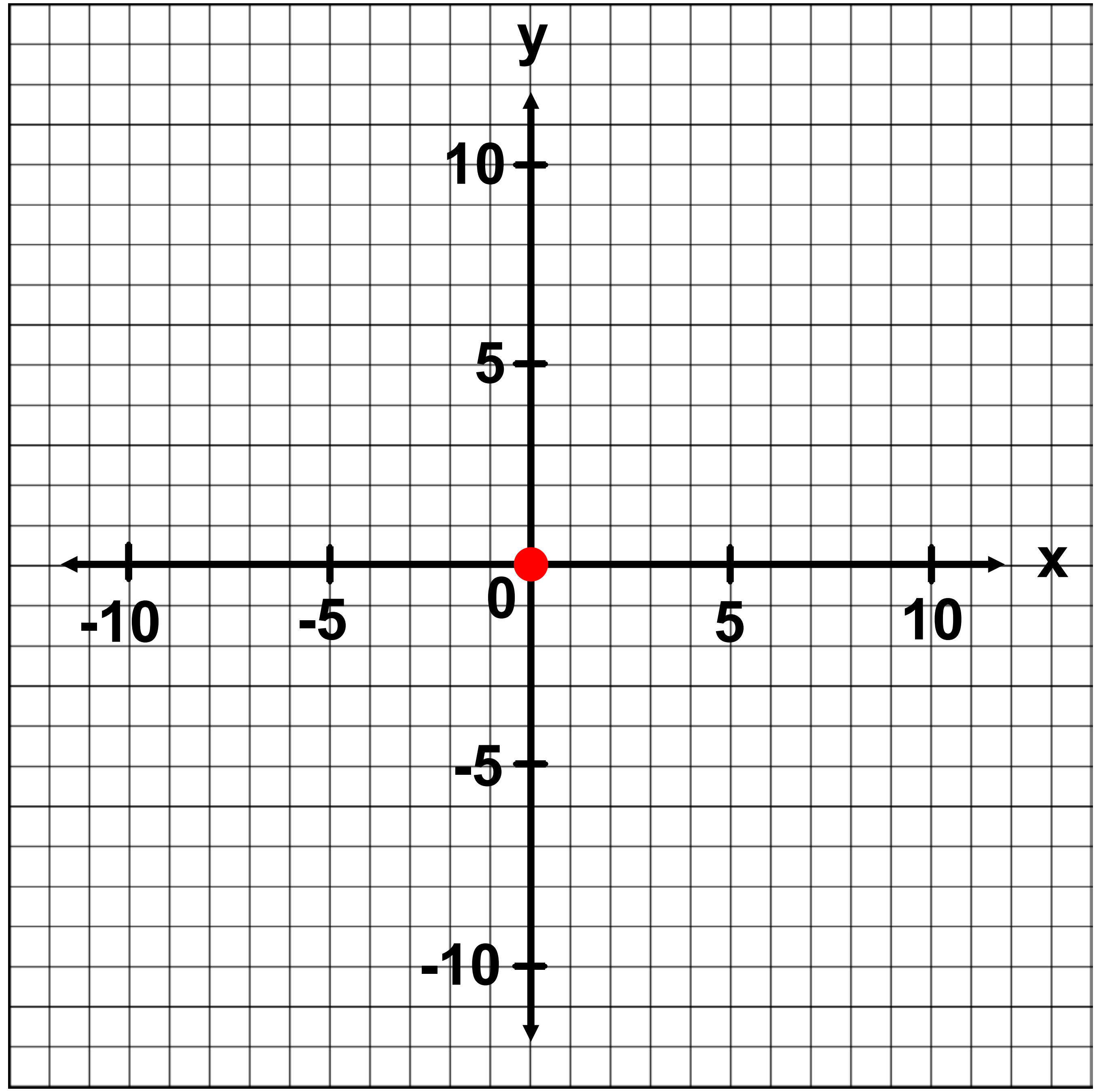


# Graphing Equations

The Cartesian Plane is formed by the intersection of the x-axis and y-axis, which are perpendicular. It's also called a Coordinate Plane or an XY Plane.

The x-axis is horizontal (side-to-side) and the y-axis is a vertical (up and down).

The axes intersect at the origin.

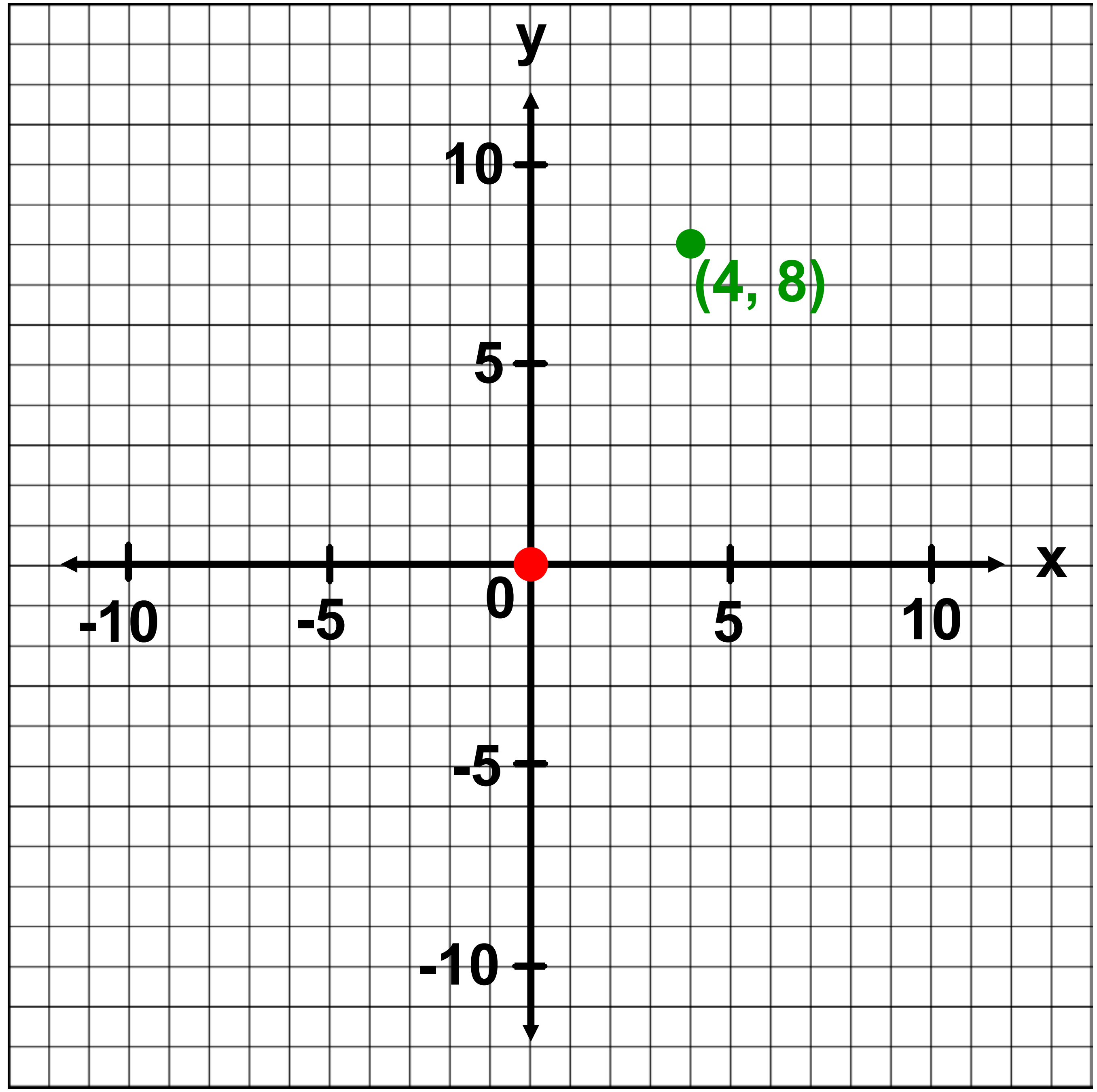


# Graphing Equations

An ordered pair represents a solution to a linear equation, and a point on the plane.

The numbers represent the x- and y- coordinates: (x,y).

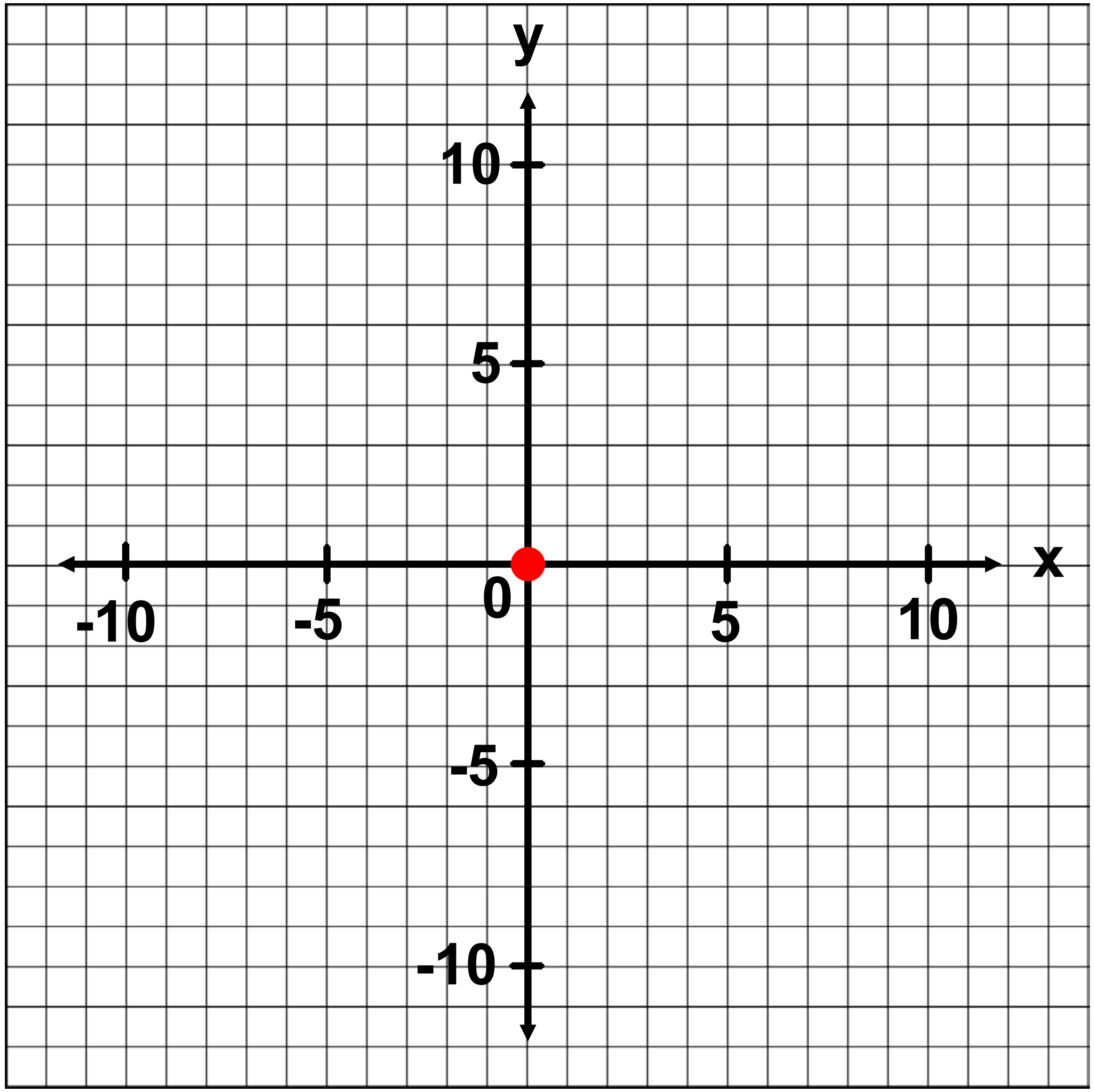
The point (4, 8) is shown.



# Graphing Equations

A <u>linear equation</u> has an infinite set of solutions.

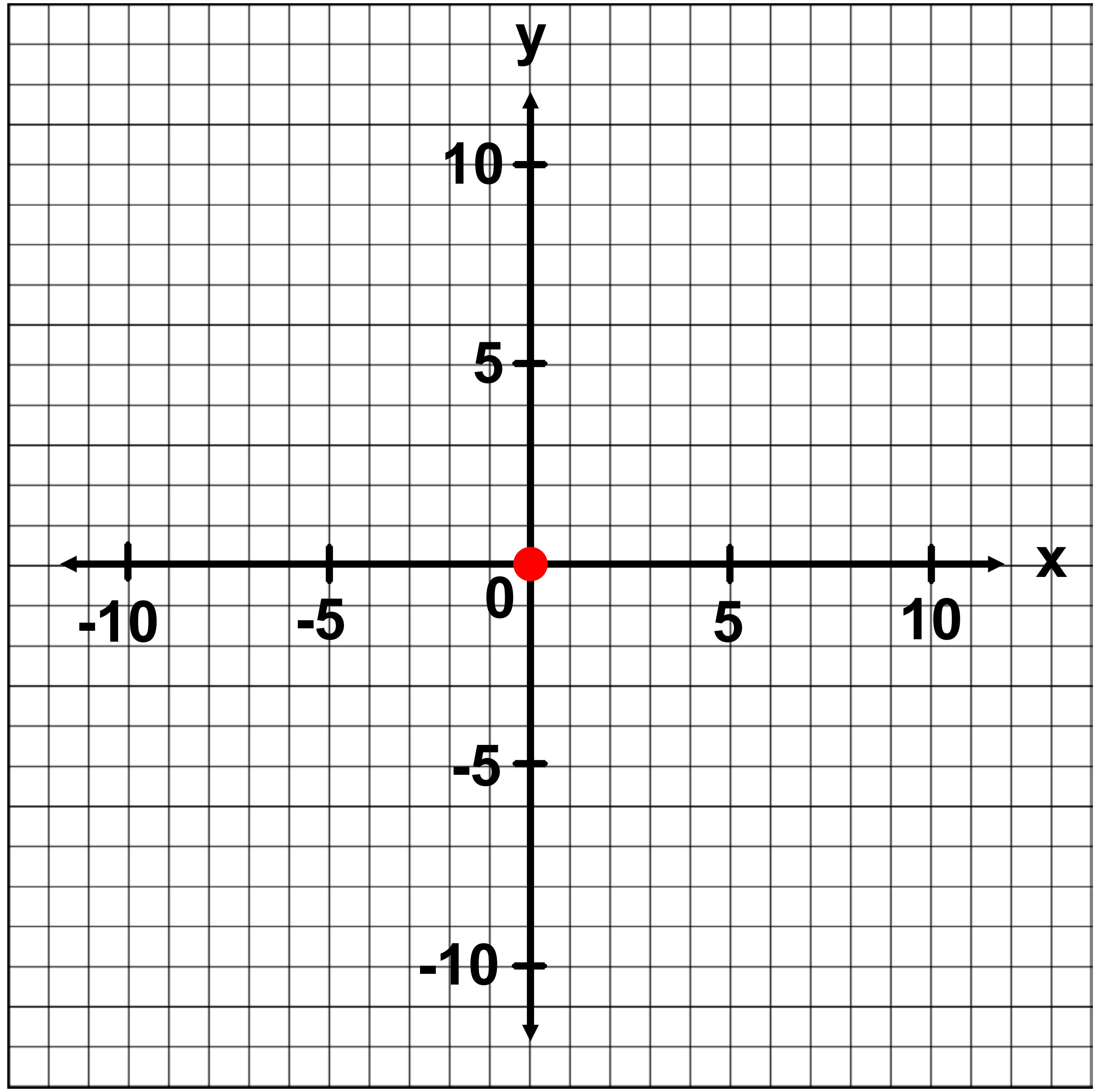
Graphing the pairs of x and y values which satisfy a linear equation forms a line (hence the name "linear" equation).



# Graphing Equations

One way to graph the line that represents the solutions to a linear equation is to use a table to find a few sets of solutions.

Since a line is uniquely defined by any two points, finding three or more points provides the line, and a check to make sure the points are correct.



# Graphing Equations

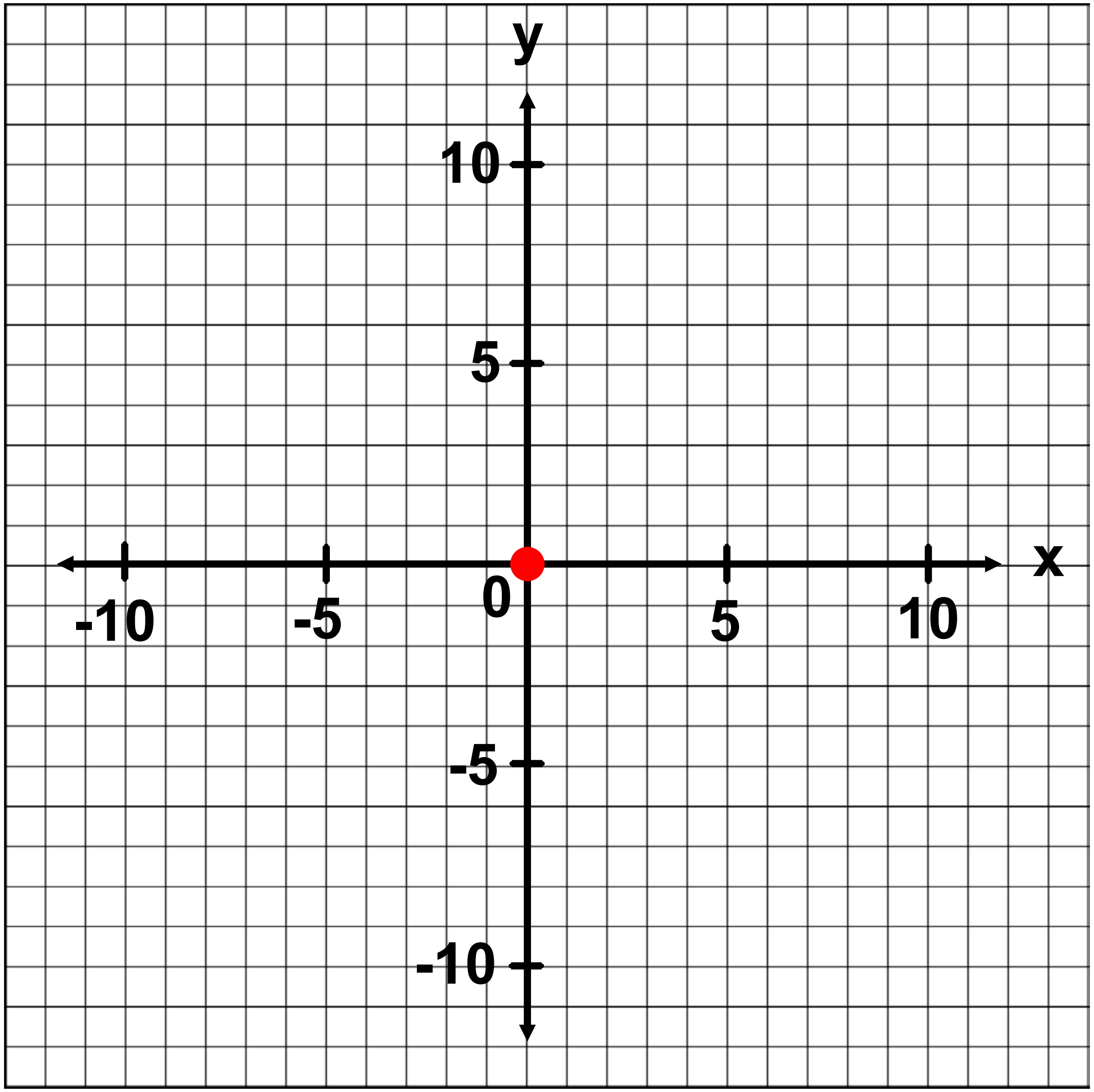
Let's graph the line:

$$y = 2x + 3$$

We'll make a table, pick some x-values and then calculate the matching y-values to create ordered pairs to graph.

We can pick any values for x, but will choose them so that the resulting points:

- Are easy to plot.
- Are far enough apart to allow us to draw an accurate line.



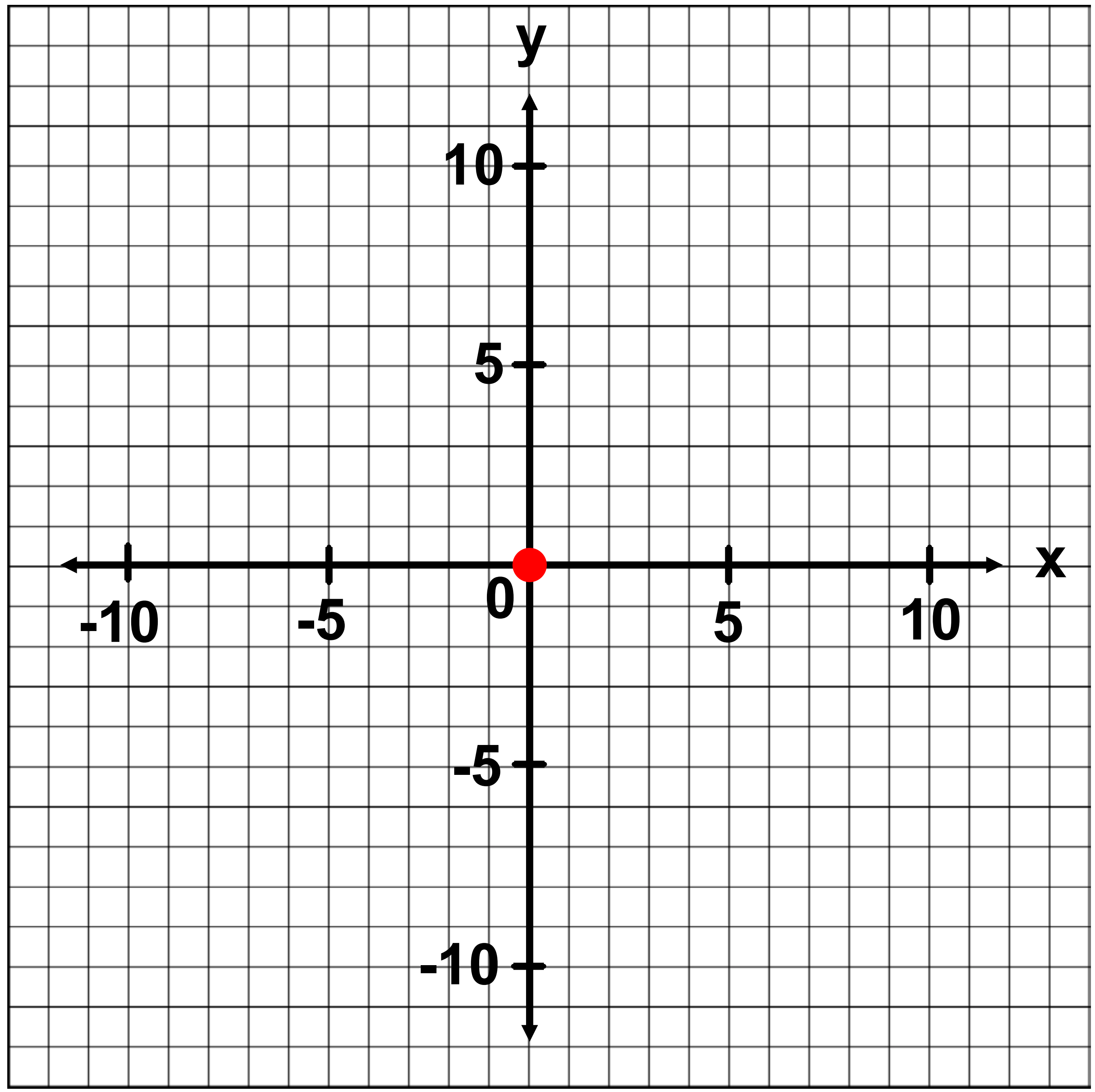
# Graphing Equations

$y = 2x + 3$

| x | y |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 0 | |
| -3 | |

While we only need two points to determine the line, it's good to check with some extra points.

Use the equation to fill in the y-values in the table.



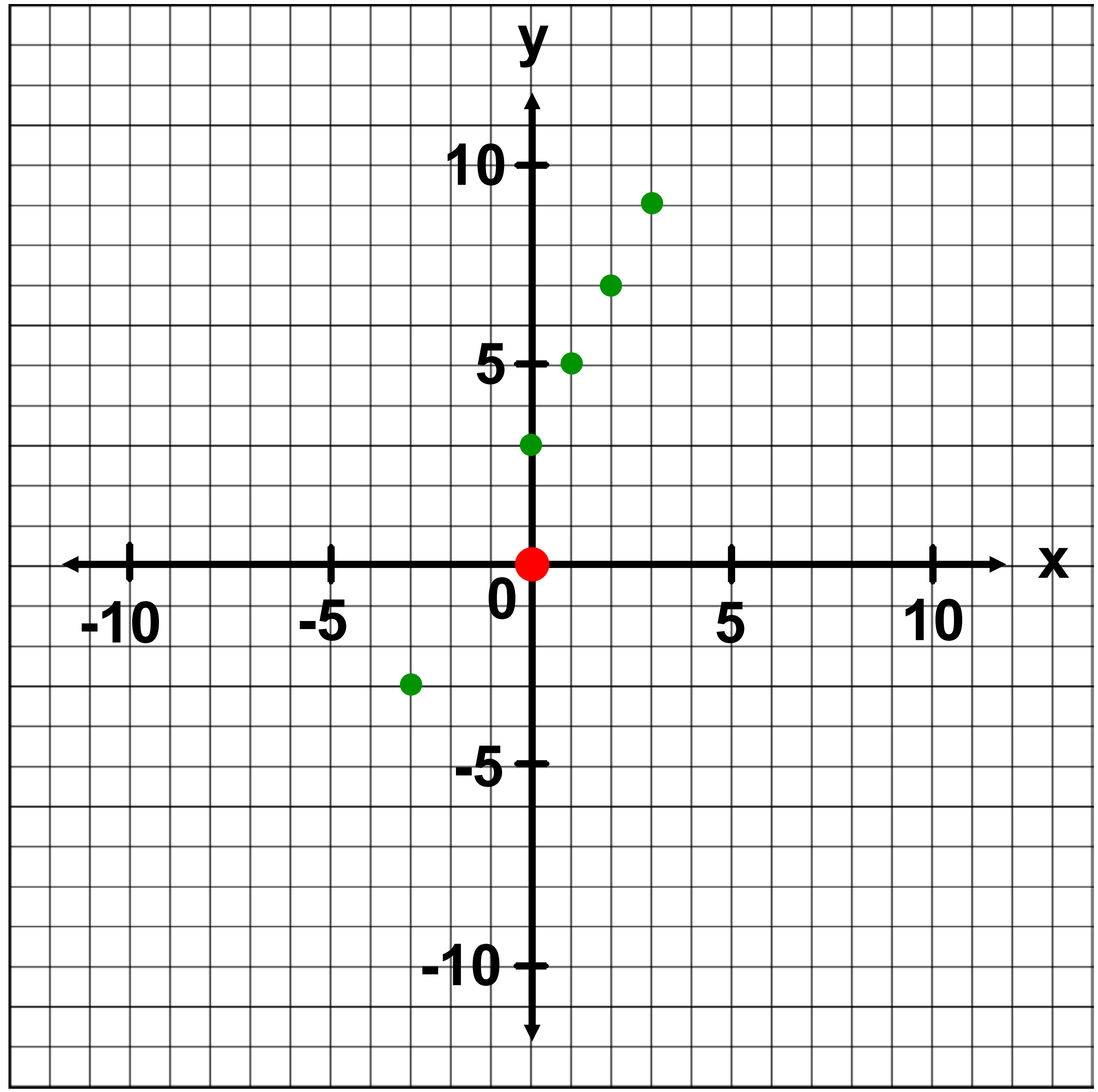
# Graphing Equations

$y = 2x + 3$

| x | y |
|---|---|
| 1 | 5 |
| 2 | 7 |
| 3 | 9 |
| 0 | 3 |
| -3 | -3 |

These are just a few points on the line.

There are an infinite number of ordered pairs that satisfy the equation.



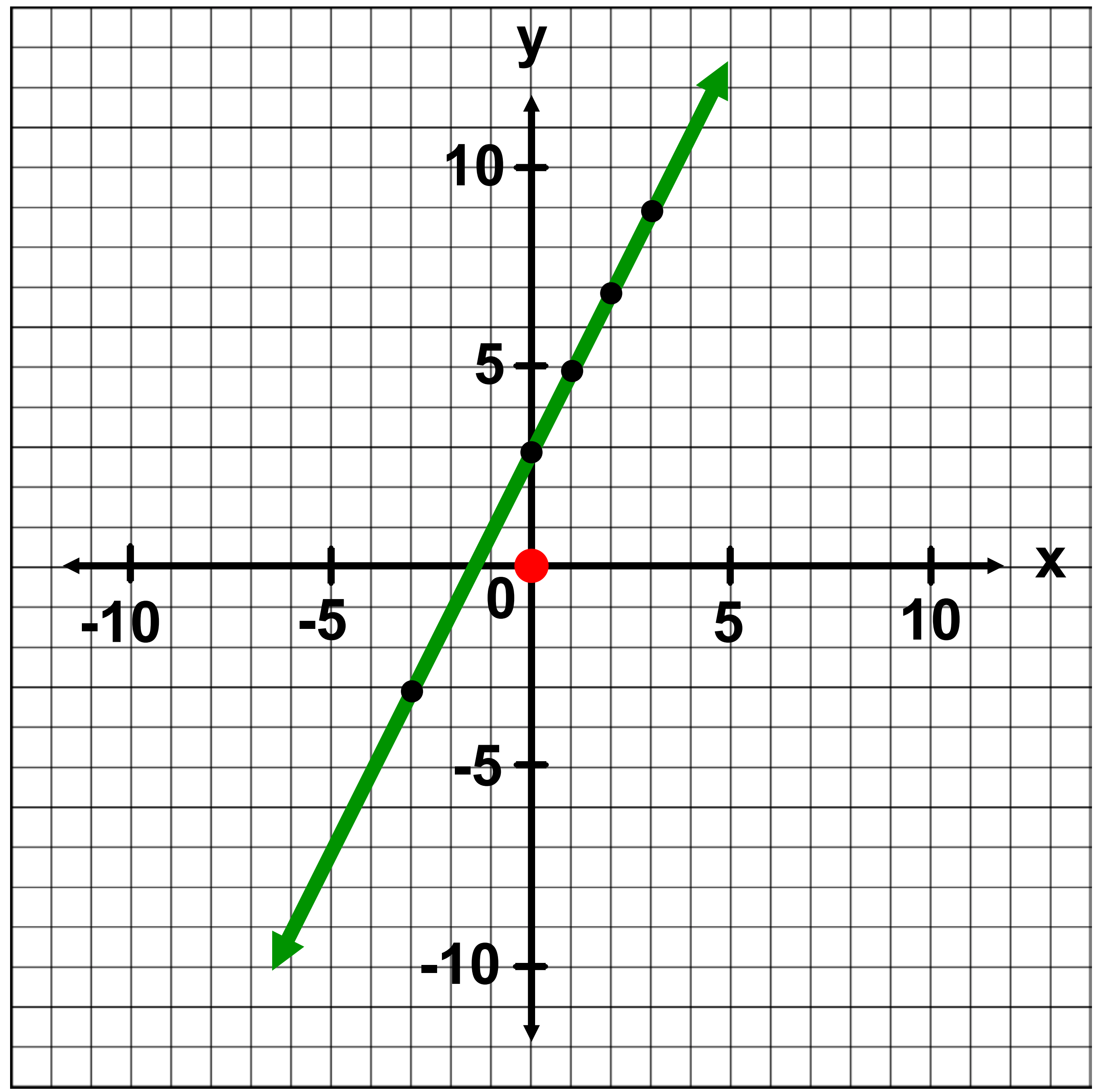

Let's draw the line that represents the infinite set of solutions to this equation.

# Graphing Equations

$y = 2x + 3$

The arrows on both ends of the line indicate that it continues forever in both directions.

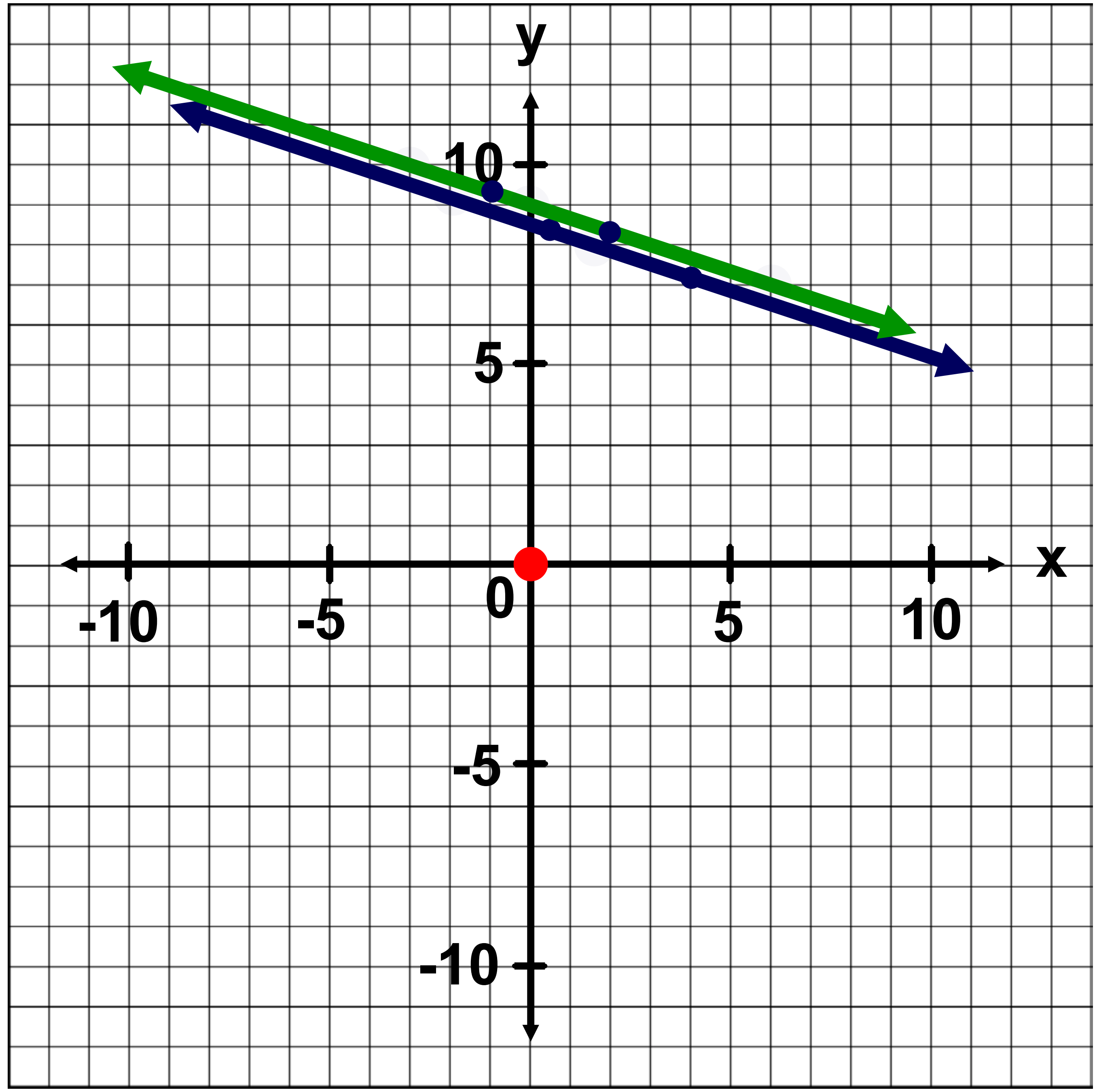
Because it is a line, it includes an infinity of points representing all the real numbers.



# Graphing Equations

$$y = -\frac{1}{3}x + 9$$

| x | y |
|---|---|
| -3 | *click* |
| -1 | *click* |
| 0 | *click* |
| +2 | *click* |
| +6 | *click* |




*Note: click to reveal*

Click on the points that are integers & the line to graph

1 Given the equation, $y = 2x - 5$, what is $y$ when $x = 0$?

A -7

B -5

C -3

D 0

Answer

2 Given an equation of $y = 2x - 5$, what is $y$ if $x = \frac{1}{2}$ ?

A -5

B -4

C -1

D 7

Answer

3 Is (3, -5) on the line $y = 2x - 12$ ?

A yes

B no

C not enough information

Answer

4 Which point is on the line $4y - 2x = 0$ ?

A $(-2, 1)$

B $(0, 1)$

C $(-2, -1)$

D $(1, 2)$

Answer

5 Which point lies on the line whose equation is $2x - 3y = 9$ ?

A (0, 3)

B (-3, 1)

C (-3, 0)

D (6, 1)

Answer

6 Point (k, -3) lies on the line whose equation is x - 2y = -2.

What is the value of k?

A -8

B -6

C 6

D 8

Answer

# Graphing Linear Equations Using Intercepts

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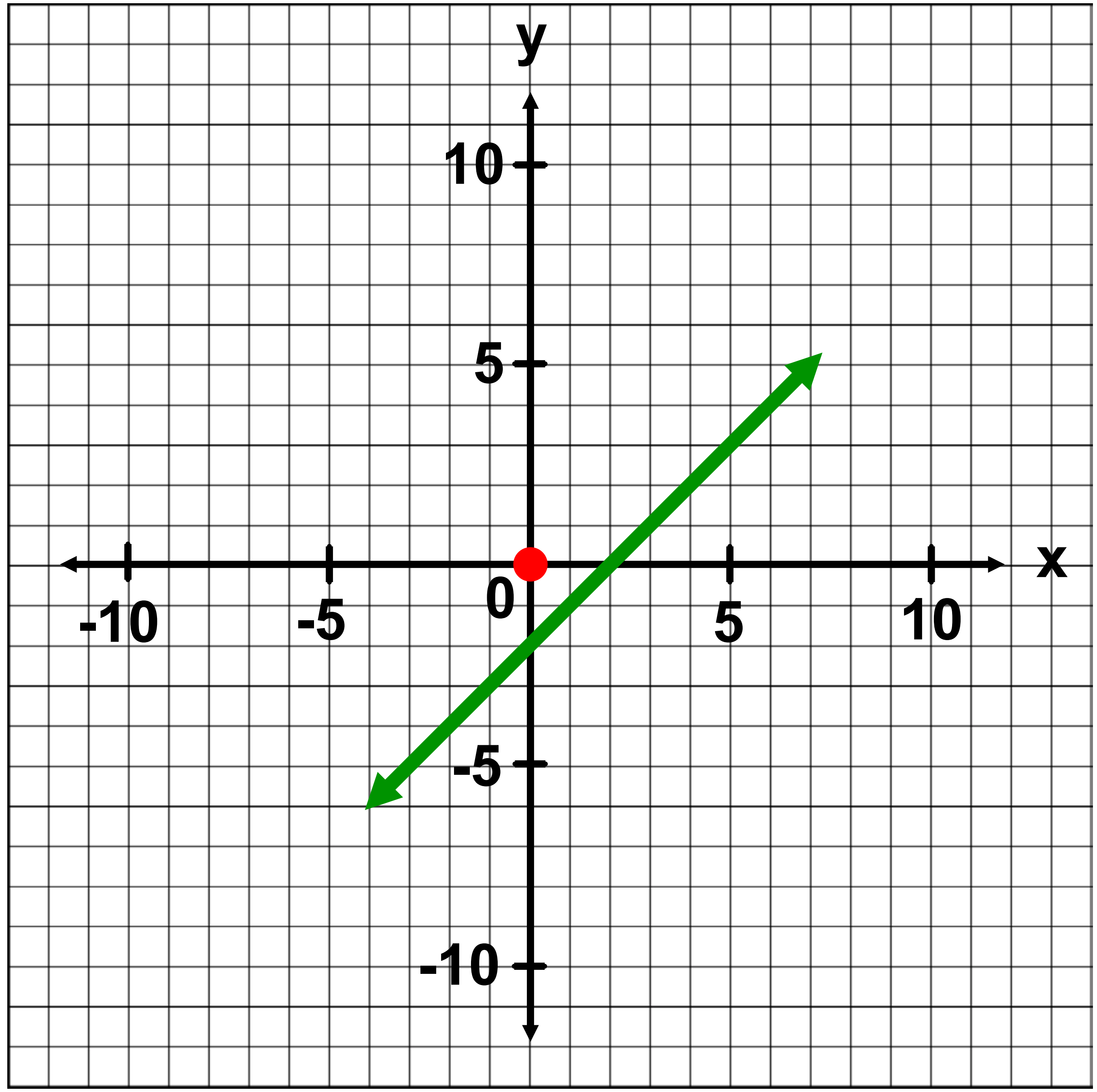
# x- and y-intercepts

To graph a line, two points are required. One technique uses the x- and y- intercepts.

The **x-intercept** is where a graph of an equation passes through the x-axis. The coordinates of the x-intercept are (a, 0), where a is any real number.

The x-intercept of the linear equation shown is (2, 0)

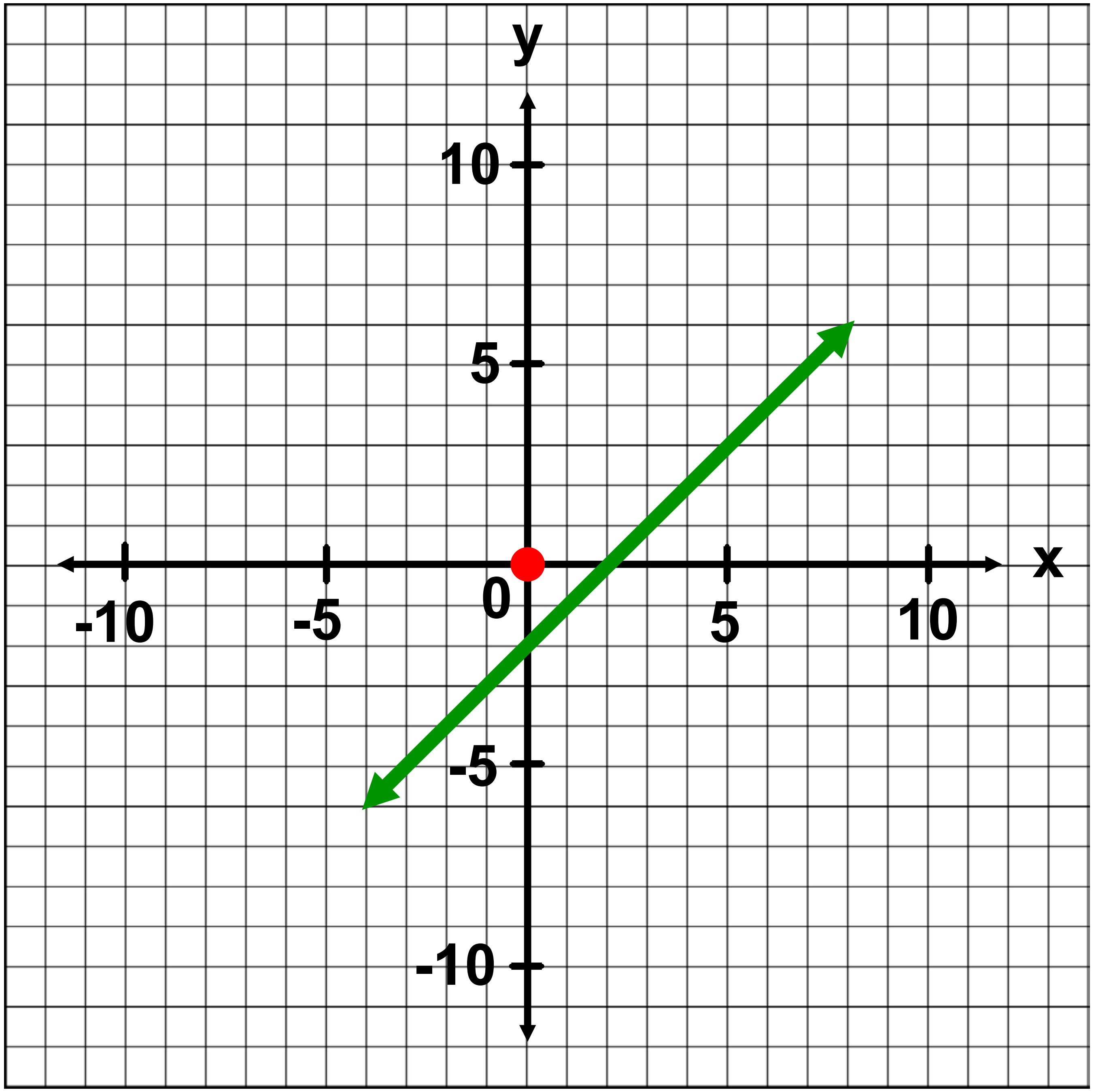


# x- and y-intercepts

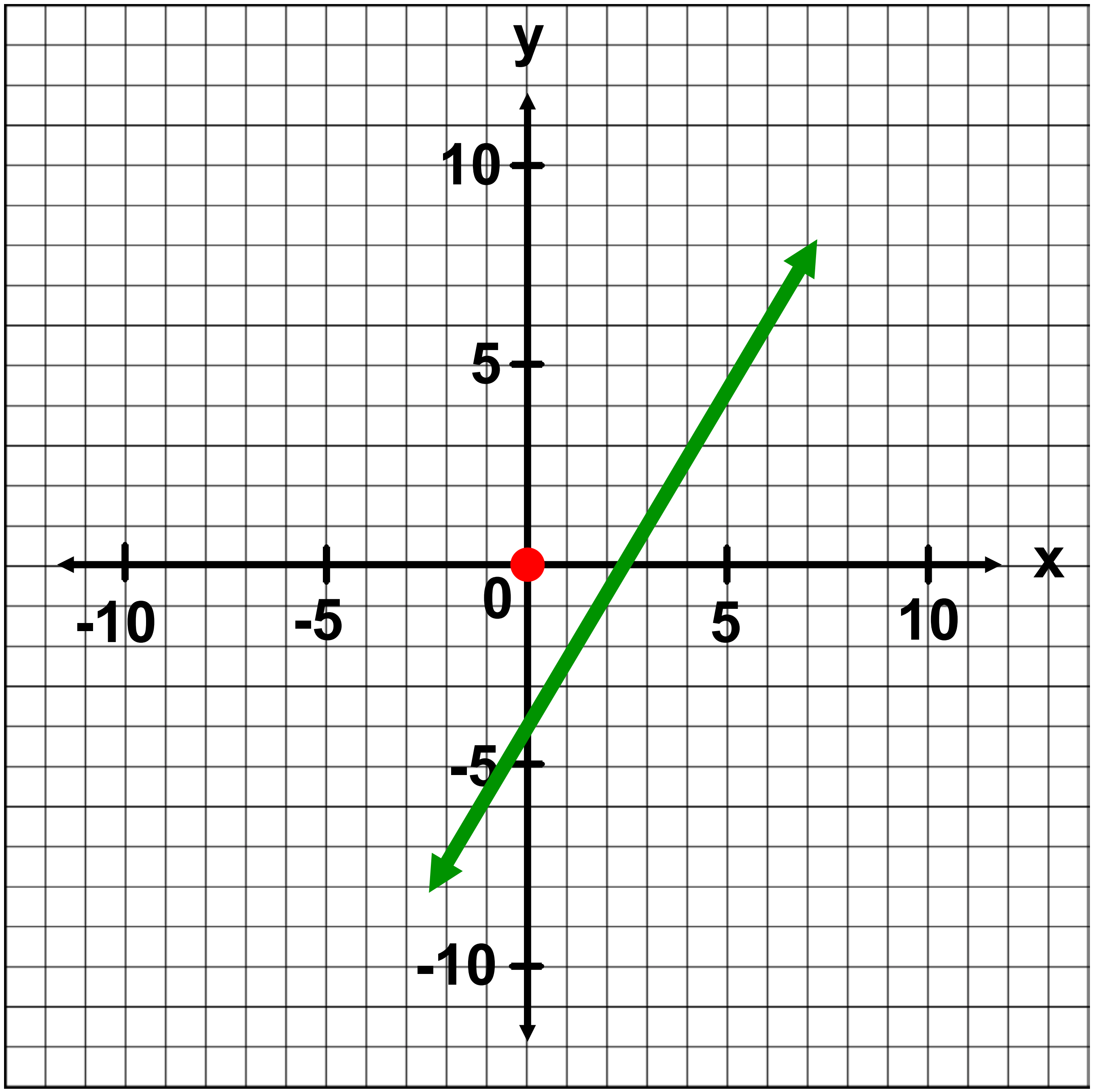
To graph a line, two points are required.  One technique uses the x- and y- intercepts.

The **y-intercept** is where a graph of an equation passes through the y-axis.  The coordinates of the y-intercept are (0, b), where b is any real number.

The y-intercept of the linear equation shown is (0, -2)

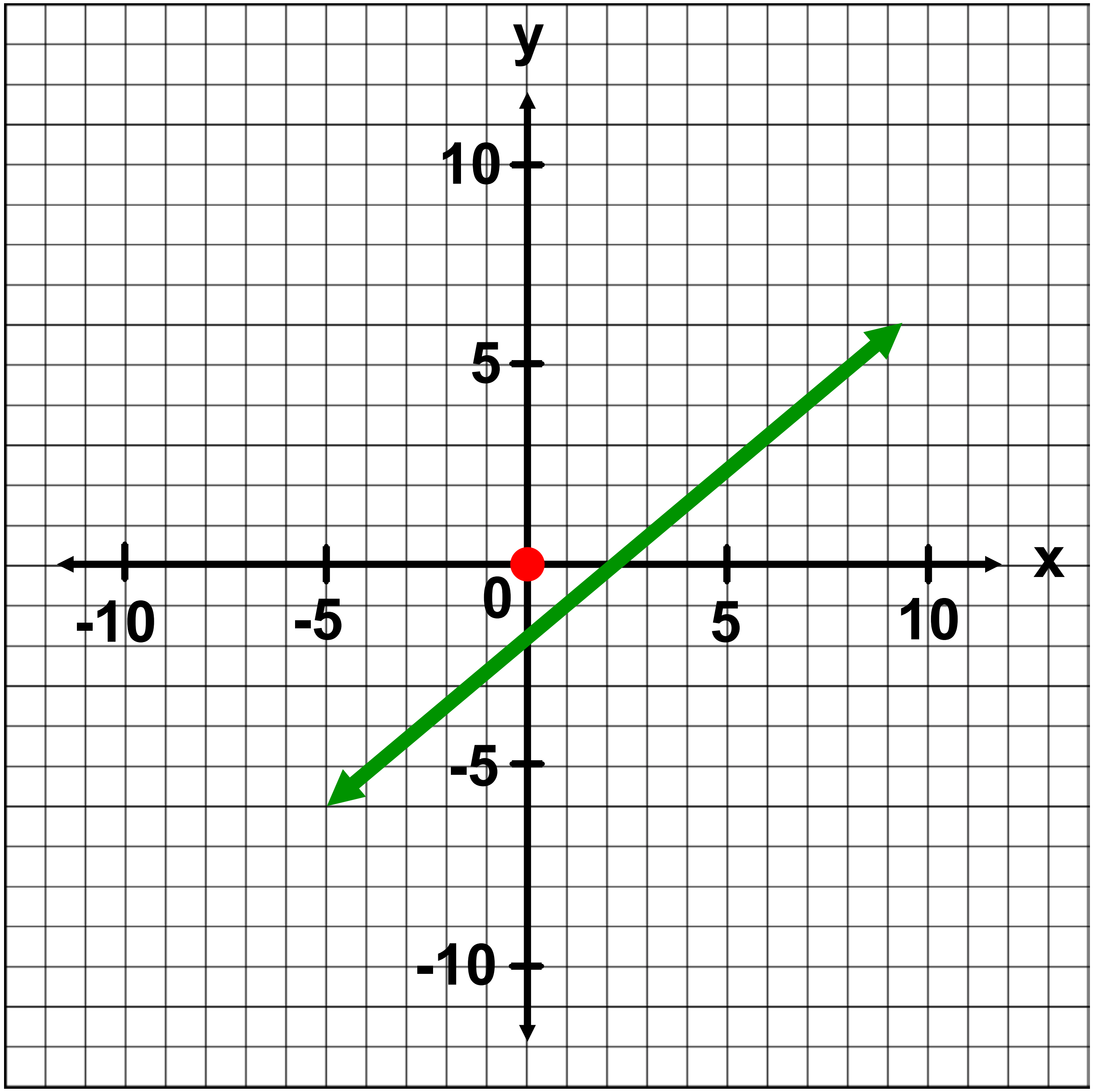


7 What is the y-intercept of this line?



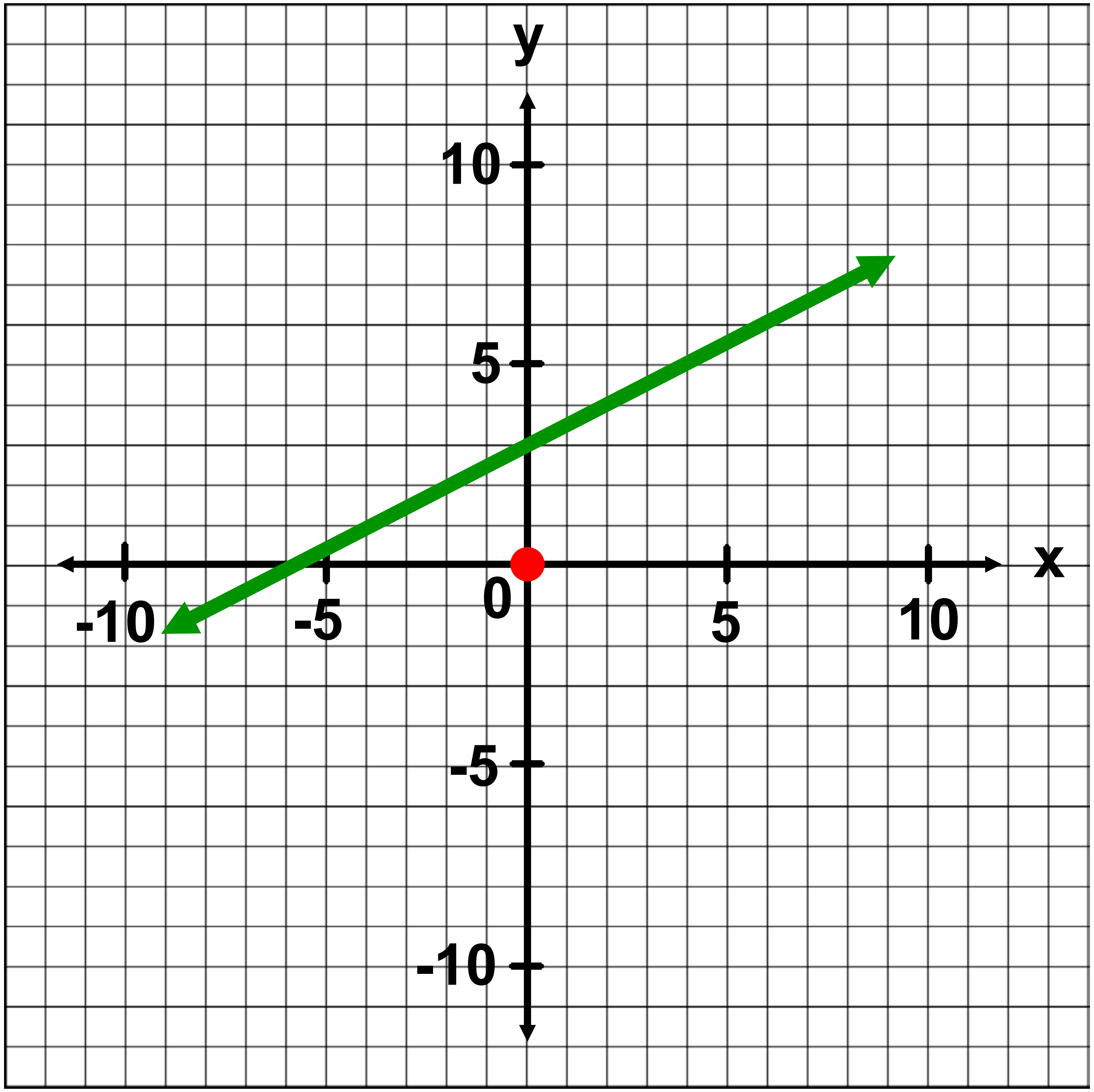

Answer

8 What is the x-intercept of this line?



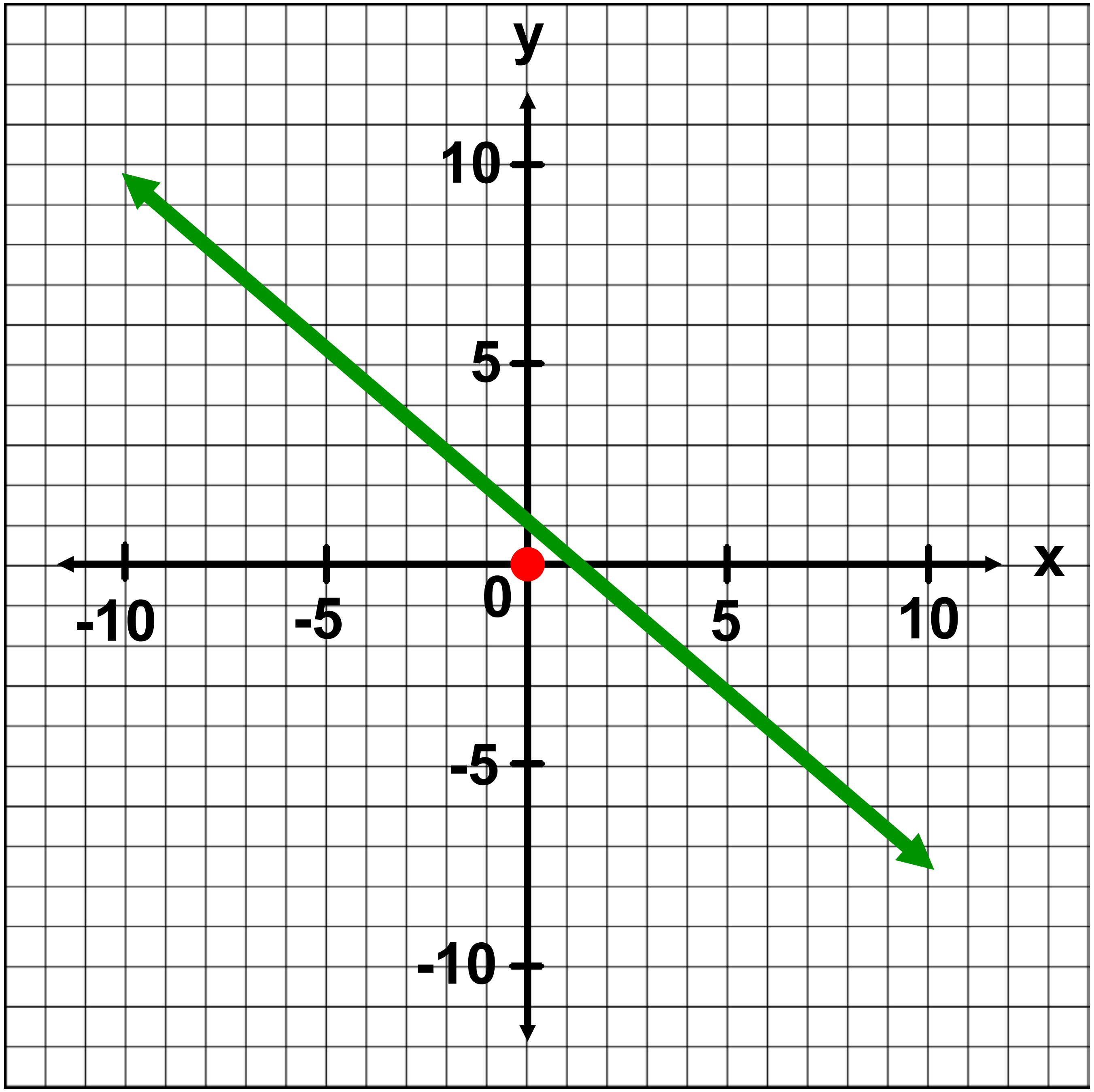

Answer

9 What is the y-intercept of this line?



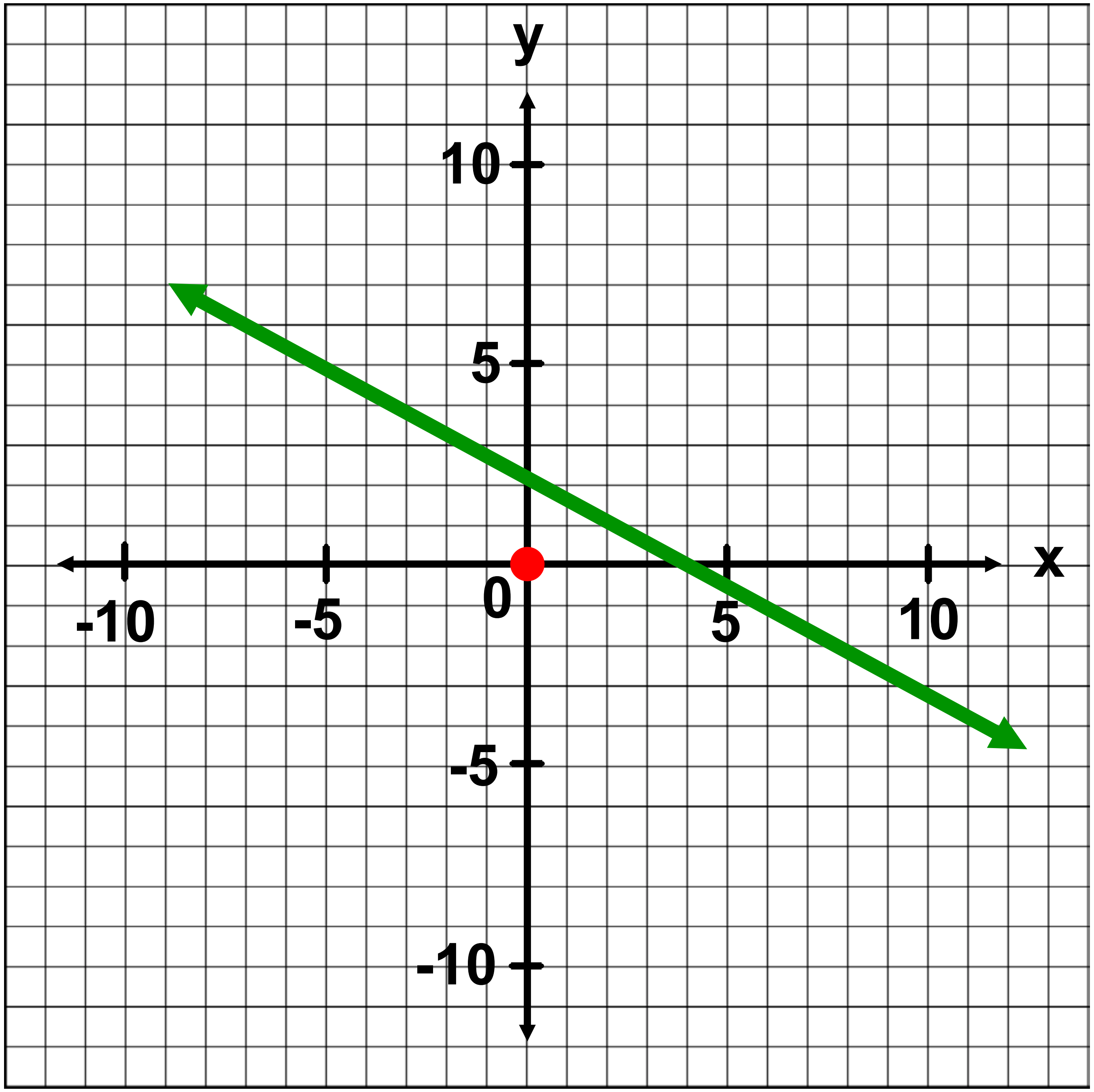

Answer

10 What is the y-intercept of this line?



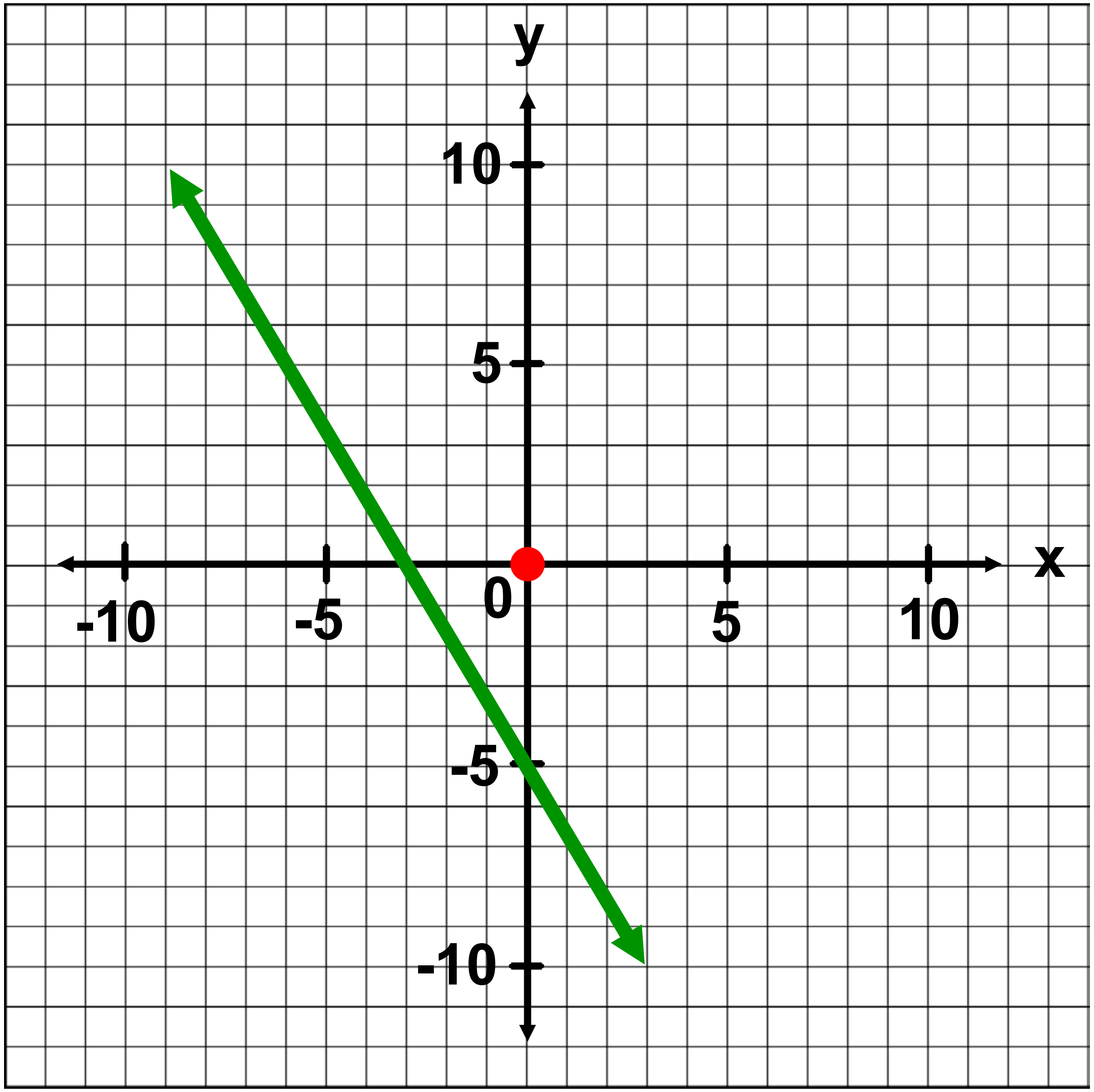

Answer

11 What is the x-intercept of this line?




Answer

12 What is the x-intercept of this line?




Answer

# Graphing Linear Equations Using Intercepts

The technique of using intercepts works well when an equation is written in Standard Form. Recall that a linear equation written in standard form is $Ax + By = C$, where A, B, and C are integers and $A \geq 0$.

# Graphing Linear Equations Using Intercepts

Example: Find the x- and y-intercepts in the equation $3x + 5y = 15$. Then graph the equation.

x-intercept: Let $y = 0$:

$$3x + 5(0) = 15$$
$$3x + 0 = 15$$
$$3x = 15$$
$x = 5$ so x-intercept is $(5, 0)$

y-intercept: Let $x = 0$:

$$3(0) + 5y = 15$$
$$0 + 5y = 15$$
$$5y = 15$$
$y = 3$ so y-intercept is $(0, 3)$

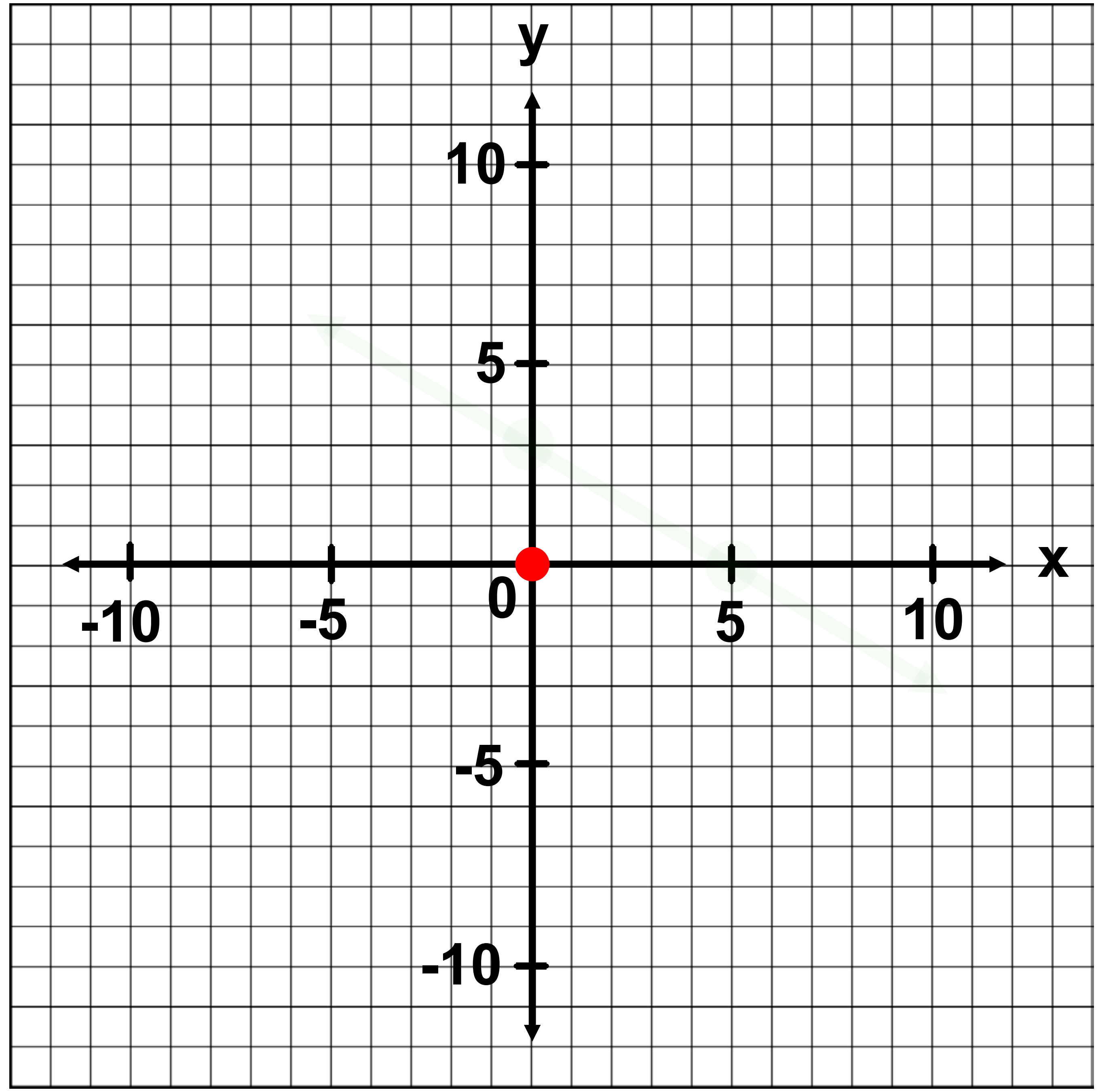
# Graphing Linear Equations Using Intercepts

Example: Find the x- and y-intercepts in the equation $3x + 5y = 15$. Then graph the equation.

x-intercept is ________
*Click*

y-intercept is ________
*Click*

Click on the points & the line in the coordinate plane to reveal.



# Graphing Linear Equations Using Intercepts

Example: Find the x- and y-intercepts in the equation $4x - 3y = 12$. Then graph the equation.

x-intercept: Let $y = 0$:

y-intercept: Let $x = 0$:

***slide down***

# Graphing Linear Equations Using Intercepts

Example: Find the x- and y-intercepts in the equation 4x - 3y = 12. Then graph the equation.

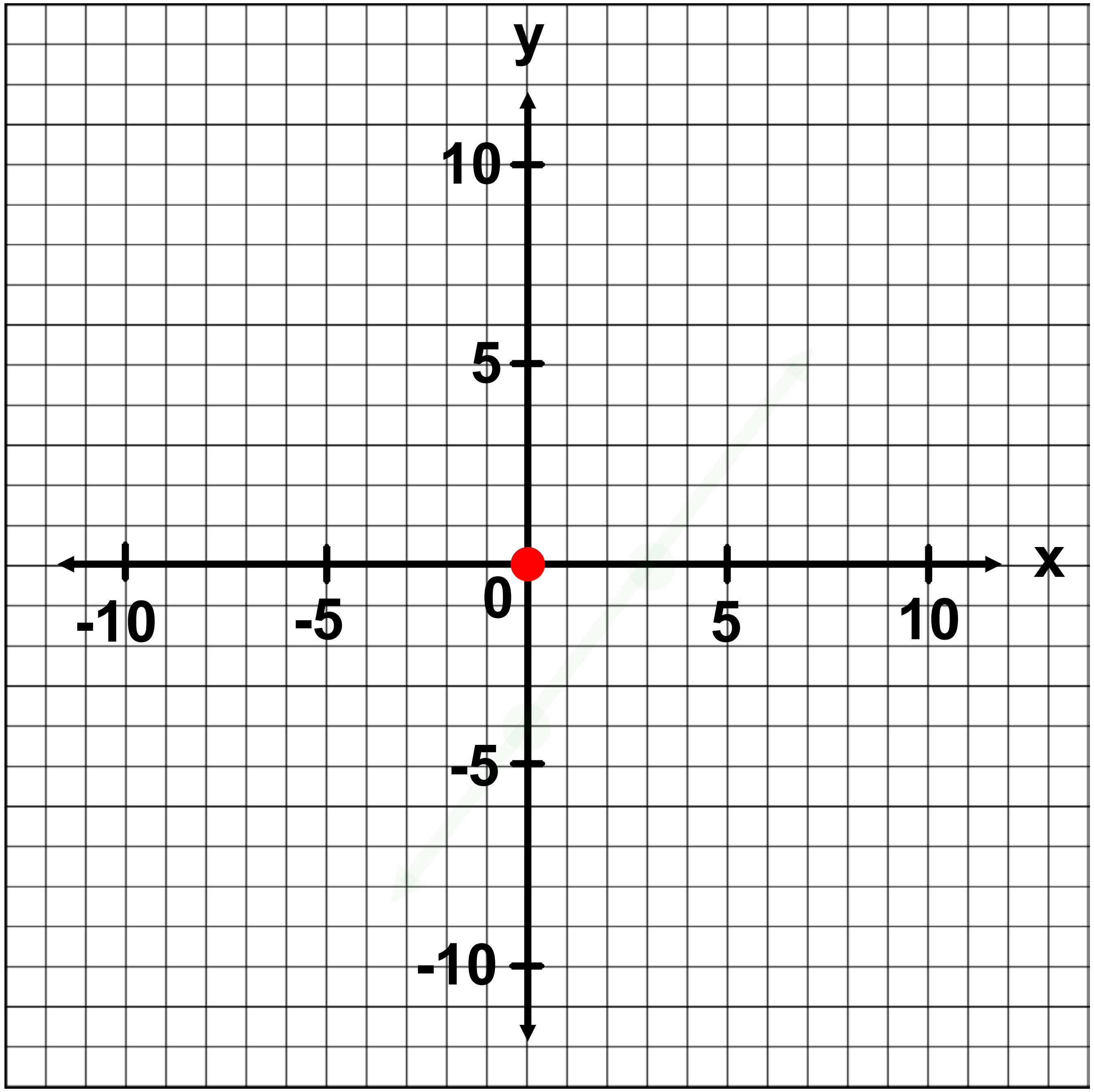
x-intercept is ________

*Click*

y-intercept is ________

*Click*

Click on the points & the line in the coordinate plane to reveal.



# Graphing Linear Equations Using Intercepts

Does anyone see a shortcut to finding the x- and y-intercepts? How could your shortcut make the problem easier?

Math Practice

# Graphing Linear Equations Using Intercepts

Given the equation 4x - 3y = 12, another way to look at the intercept method is called the "cover-up method."

If y = 0, we can cover -3y up (because zero times anything is 0) and solve the remaining equation.

4x - 3y = 12

**press -3y**

that leaves us with ________
*Click*

solve for x

the x-intercept is ________
*Click*

# Graphing Linear Equations Using Intercepts

If $x = 0$, we can cover that up and solve the remaining equation.

**press 4x** → $4x - 3y = 12$

leaves us with ________

*Click*

solve for y

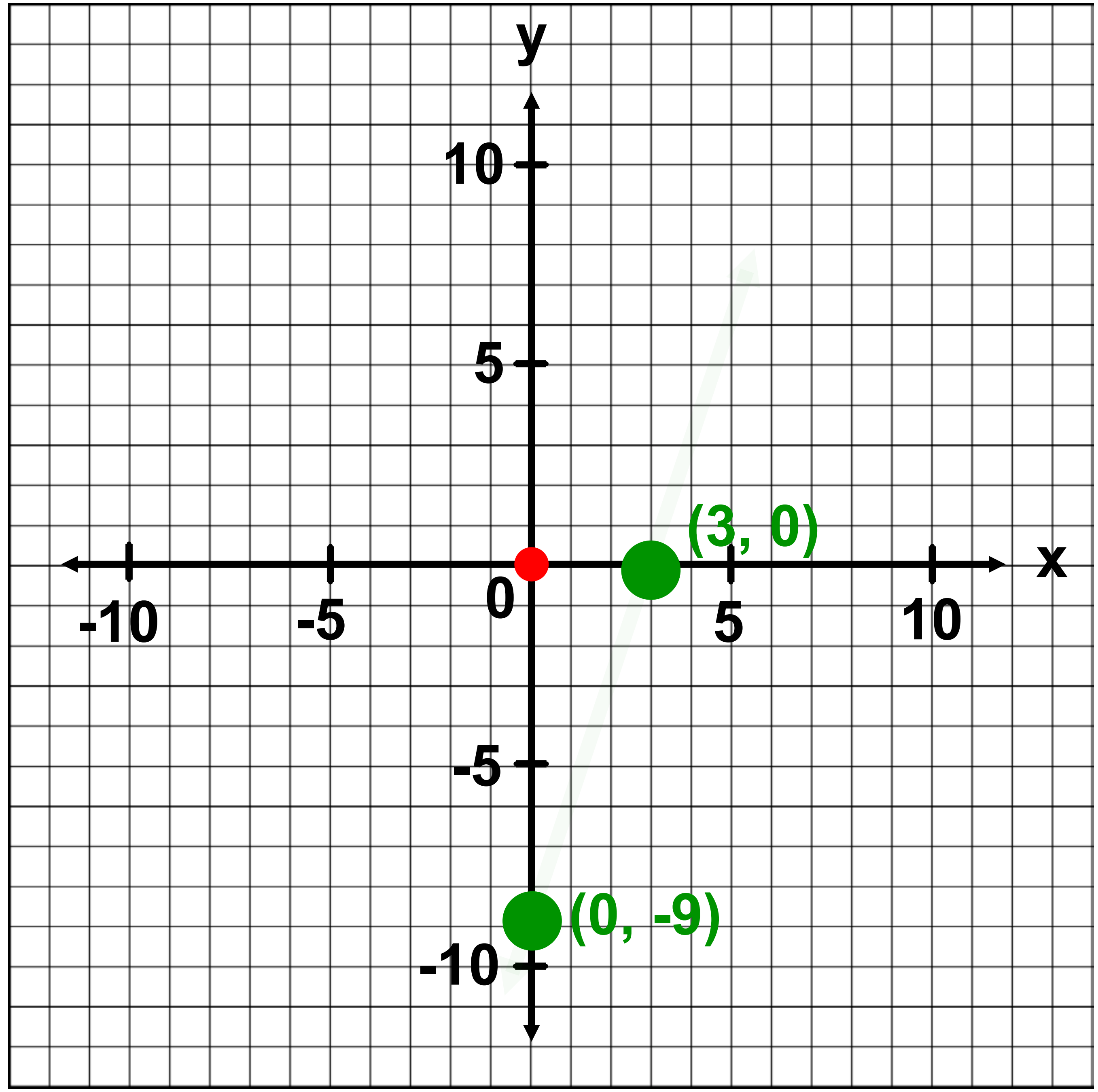
the y -intercept is ________

*Click*

# Graphing Linear Equations Using Intercepts

Try This:

Find the x- and y-intercepts of y = 3x - 9. Then graph the equation.




Answer

Click on the points & the line in the coordinate plane to reveal.

13 Given the equation $y = \frac{1}{2}x - 7$, what is the x-intercept?

Answer

14 Given the equation $y = \frac{1}{2}x - 7$, what is the y-intercept?

Answer

15 Given the equation $y - 3 = 4(x + 2)$, what is the x intercept?

Answer

16 Given the equation $y - 3 = 4(x + 2)$, what is y-intercept?

Answer

17 Given the equation $x + 3y = 3$, what is the y-intercept?

A (3, 0)

B (0, 1)

C (0, 4)

D (0, 3)

Answer

18 Given the equation $x + 3y = 3$, what is the x-intercept?

A (3, 0)

B (0, 1)

C (0, 4)

D (0, 3)

Answer

# Horizontal and Vertical Lines

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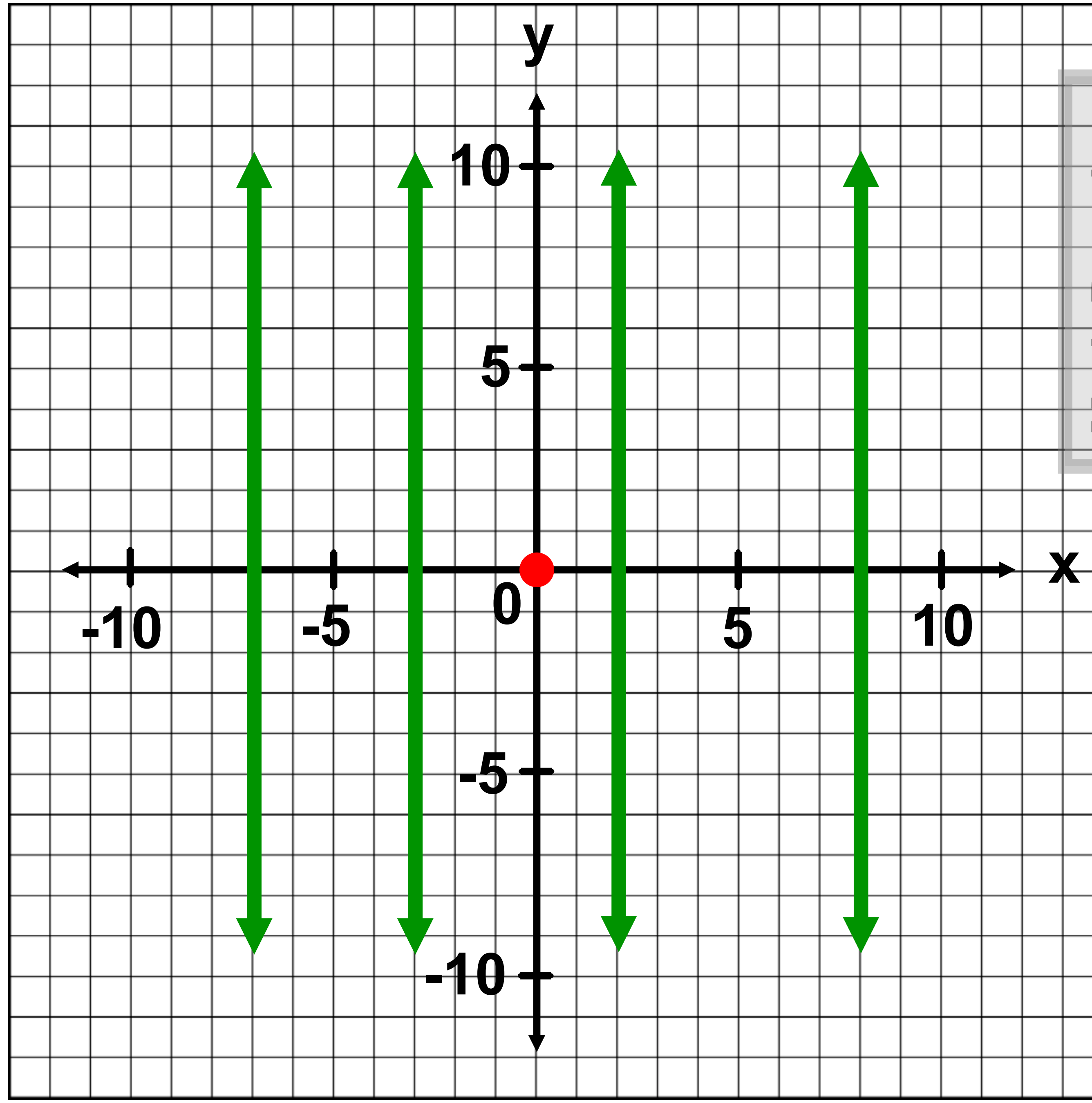
# Horizontal & Vertical Lines

Horizontal and vertical lines are different from slanted lines in the coordinate plane.

A vertical line goes "up and down".

Select random points on each line shown to the left. What are the similarities and differences between the points on the vertical lines?

Discuss!



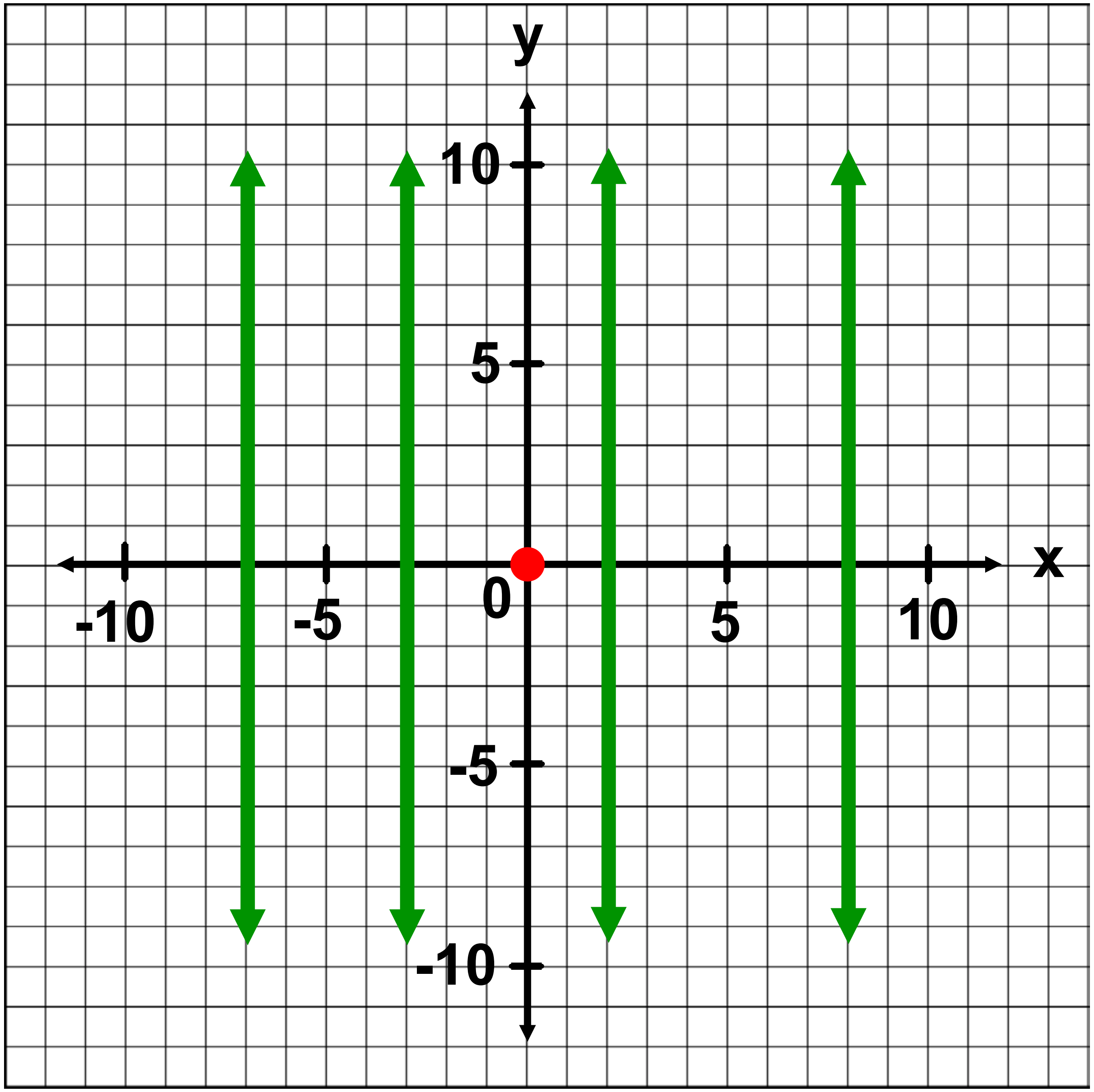

Math Practice

# Horizontal & Vertical Lines

Notice that each point on the line furthest to the left all have x-coordinates of -7. Examples of points on this line are (-7, 2), (-7, 0), (-7, -3), etc.

The same holds true for the points on all of the vertical lines that follow. What is the common x-coordinate shared on the remaining lines?

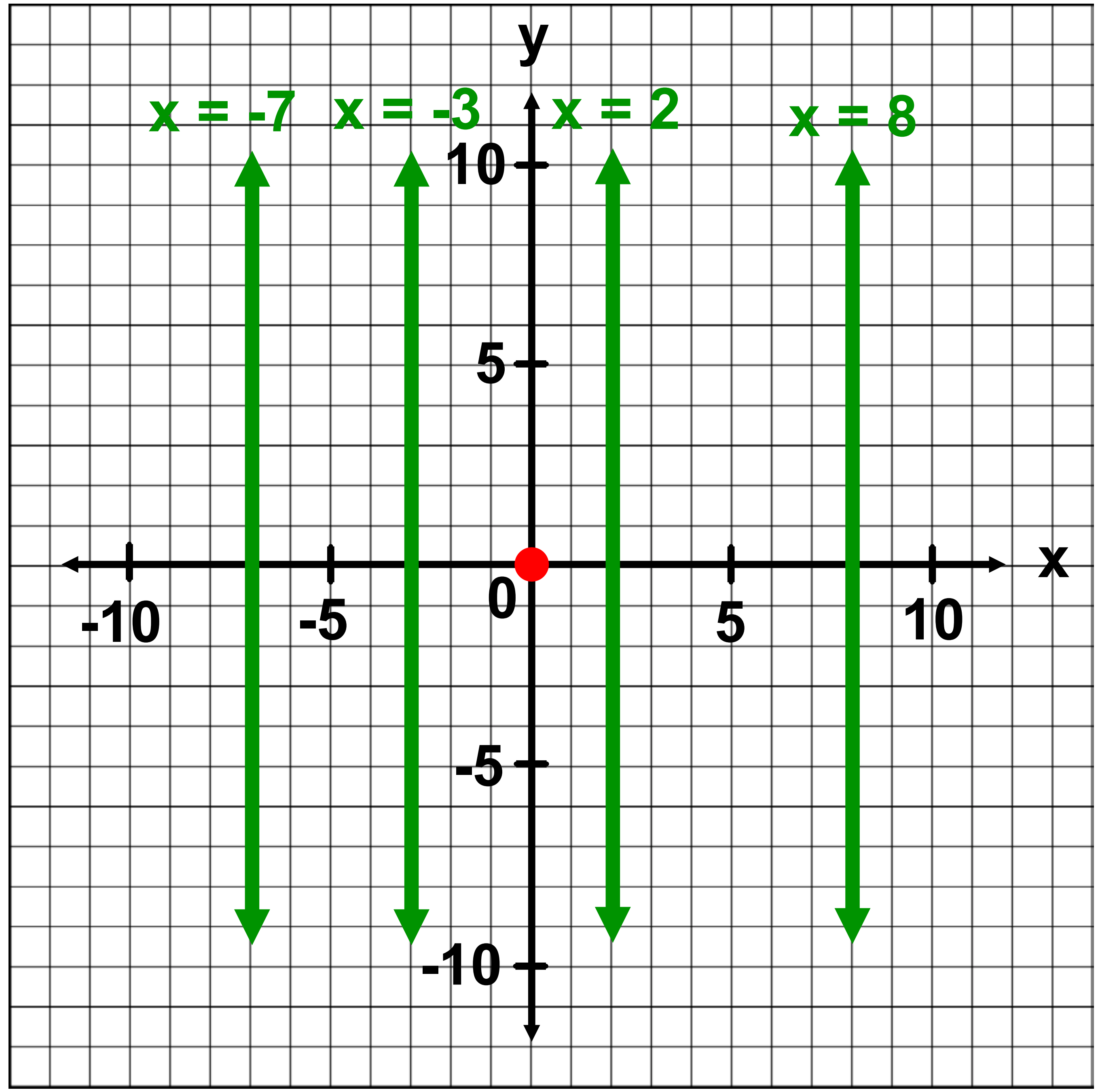
*slide to view*



# Horizontal & Vertical Lines

A **vertical line** has the equation x = a, where a is the x-intercept and the common x-coordinate shared by all of the points on the line.

Notice no y is contained in the equation.

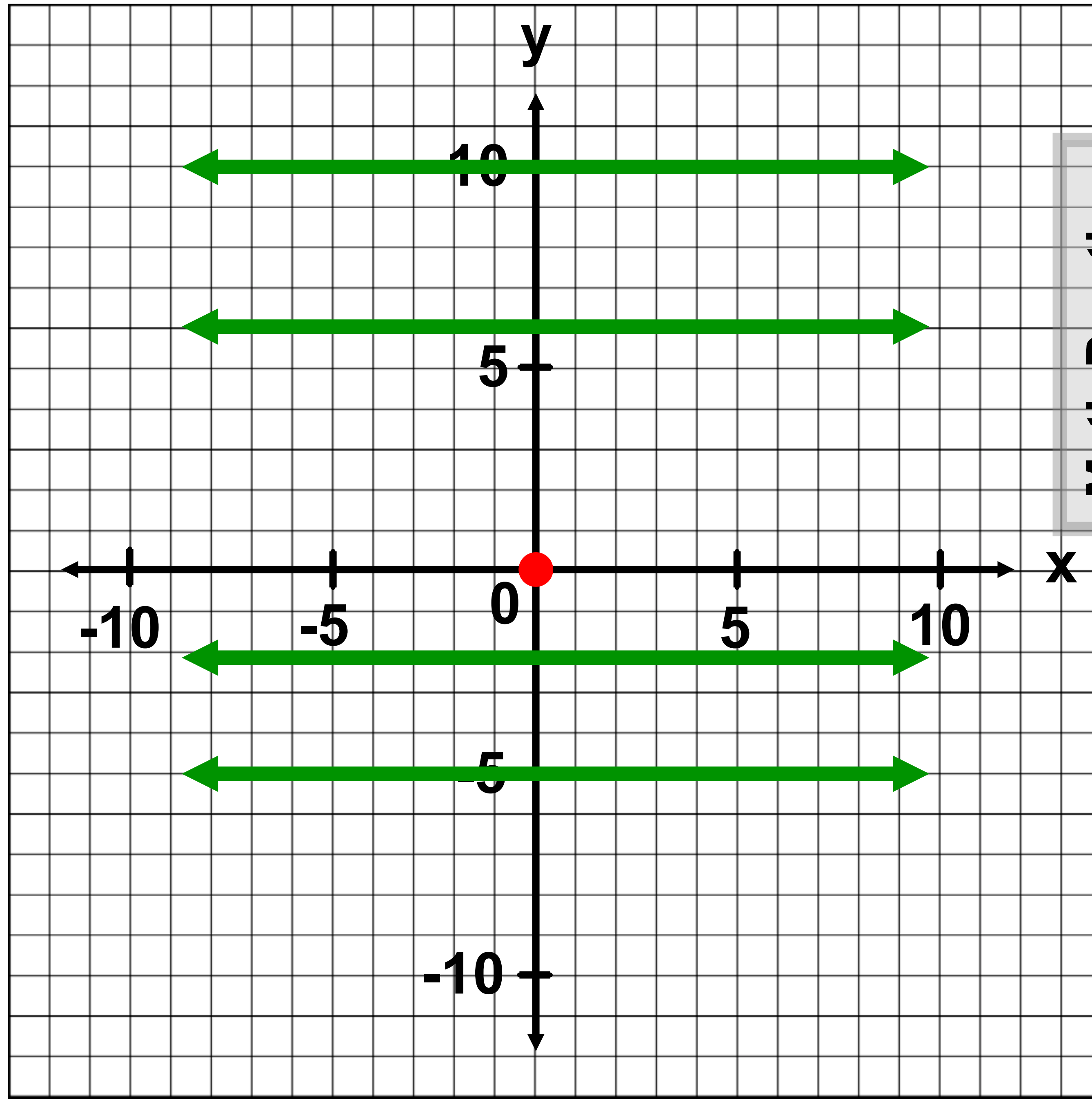


# Horizontal & Vertical Lines

A horizontal line goes "sideways".

Select random points on each line shown to the left. What are the similarities and differences between the points on the horizontal lines.

Discuss!



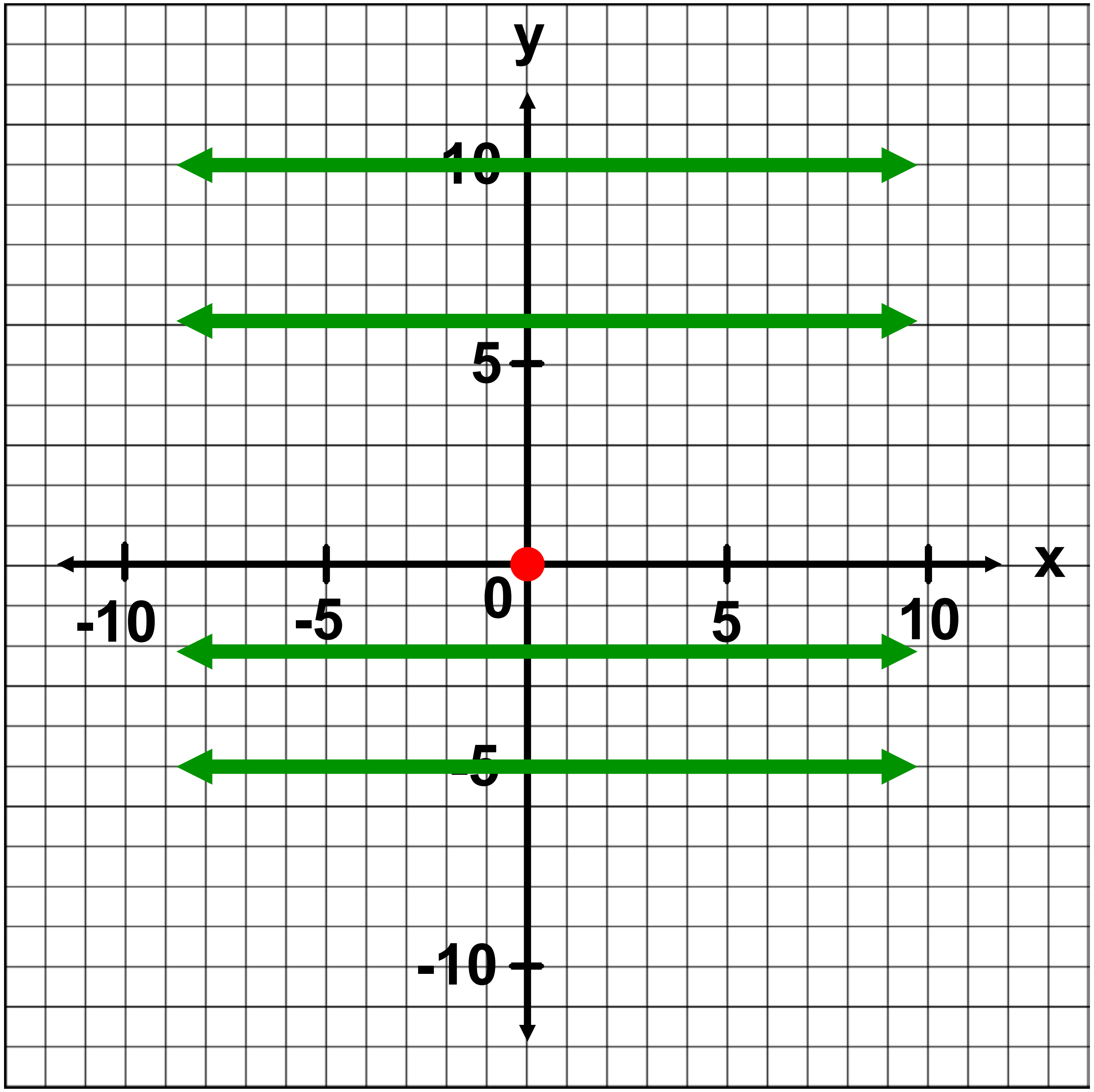

Math Practice

# Horizontal & Vertical Lines

Notice that each point on the top line have y-coordinates of 10.  Examples of points on this line are (-5, 10), (-2, 10), (0, -10), etc.

The same holds true for the points on all of the horizontal lines that follow.  What is the common y-coordinate shared on the remaining lines?

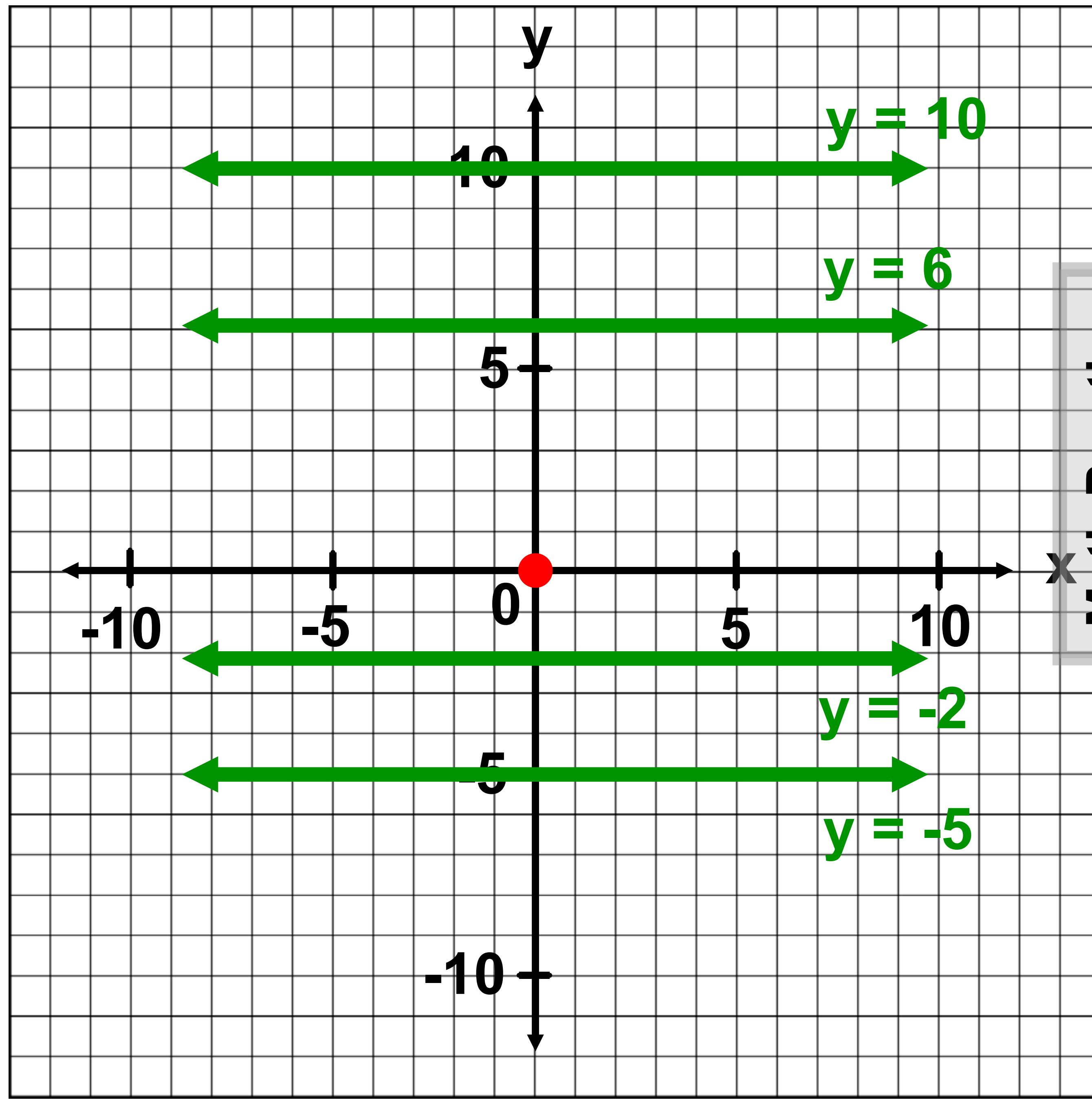
*slide to view*



# Horizontal & Vertical Lines

A **horizontal line** has the equation y = b, where b is the y-intercept and the common y-coordinate shared by all of the points on the line.

Notice no x is contained in the equation.




Math Practice

19 Is the following equation that of a vertical line, a horizontal line, neither, or cannot be determined:

$$y = 4$$

A Vertical

B Horizontal

C Neither

D Cannot be determined

Answer

20 Is the following equation that of a vertical line, a horizontal line, neither, or cannot determine:

$$x + 2y = 9$$

A Vertical

B Horizontal

C Neither

D Cannot be Determined

Answer

21 Is the following line that of a vertical, a horizontal, neither, or cannot be determined:

$$x = -23$$

A Vertical

B Horizontal

C Neither

D Cannot be Determined

Answer

22 Is the following equation that of a vertical line, a horizontal line,neither, or cannot be determined:

$$2x - 3 = 0$$

A Vertical

B Horizontal

C Neither

D Cannot be Determined

Answer

23 Which statement describes the graph of $x = 3$?

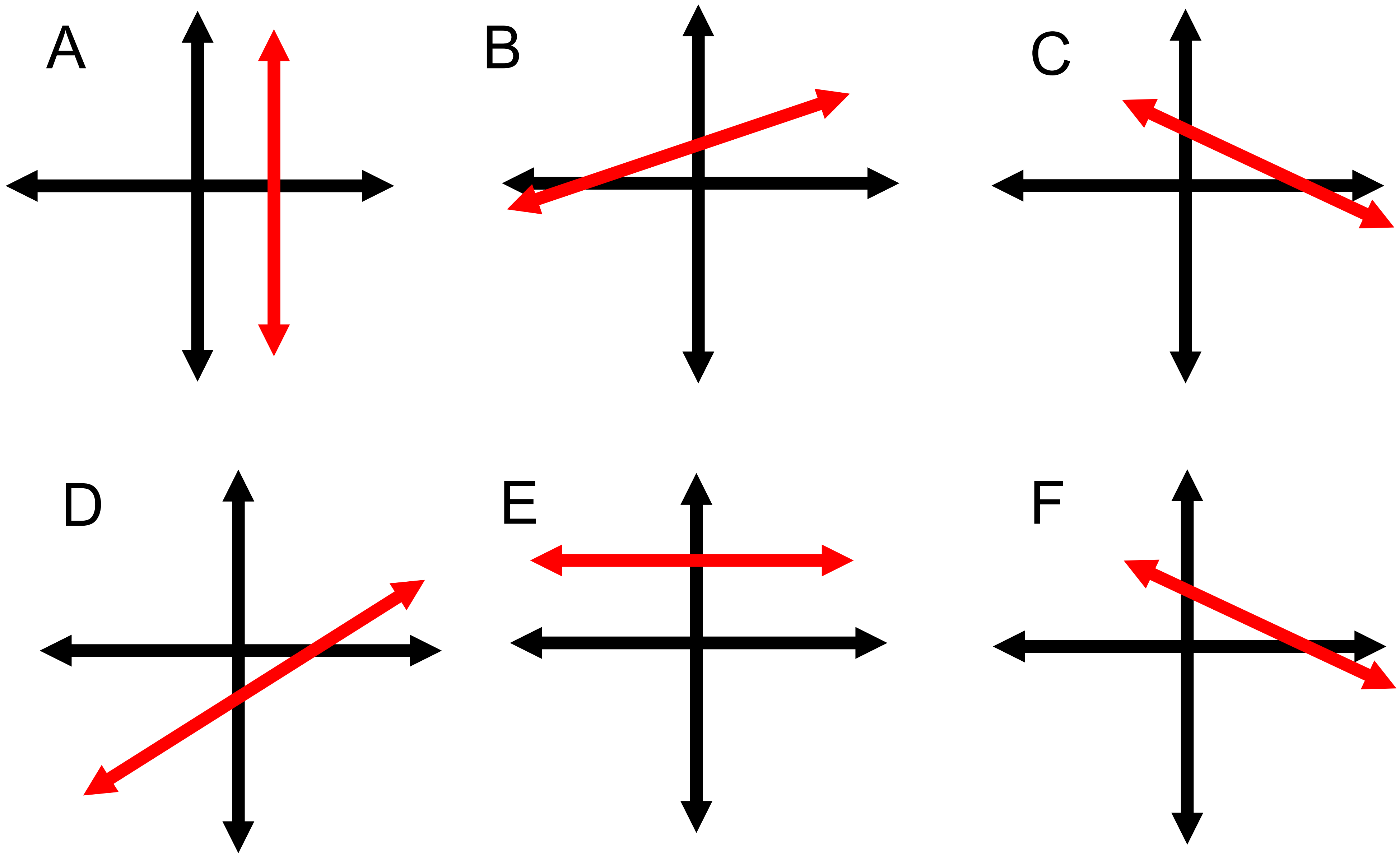
A It passes through the point (0, 3)

B It is parallel to the y-axis

C It is parallel to the x-axis

Answer

24 The intercepts method (cover-up method) of graphing could NOT have been used to graph which of the following graphs? Select all that apply.




Answer

25 Which of the following equations can't be graphed using the intercepts method? Select all that apply.

A $y = -3$

B $y - 2 = \frac{1}{2}(x + 9)$

C $y = -3x$

D $x = -4$

E $y = 4x + 7$

F $3x - 4y = 12$

G $x = 2y - 8$

H $y = x$

Answer

# Slope of a Line

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"Steepness" of a Line

# Slopes and Points

It's possible, and often easier, to graph lines using a slope and a point as opposed to a table.

Also, it's not difficult to write an equation for a line from finding the slope and a point from a graph.

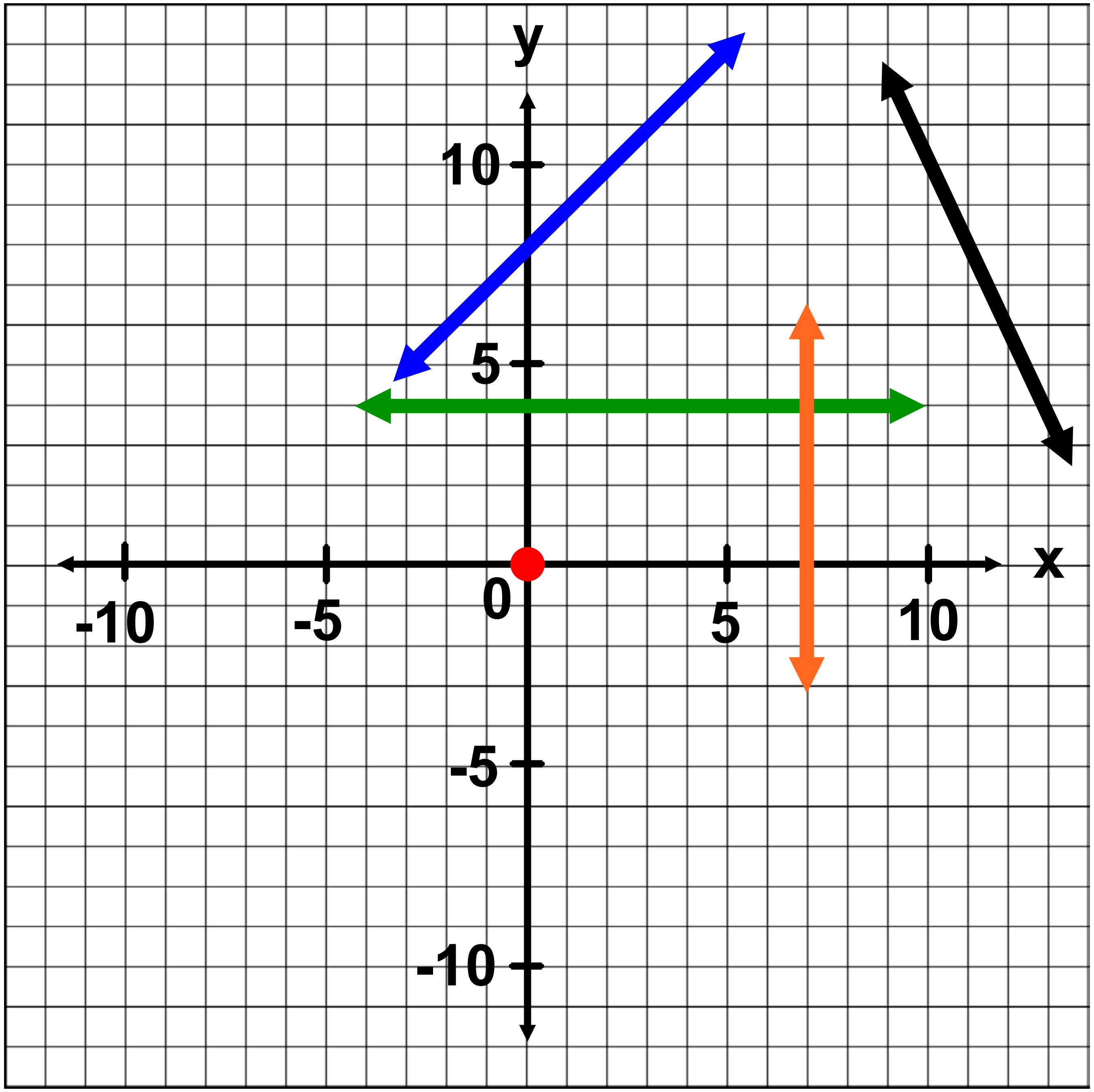
Let's first define slope, and then we can use that idea.

# Slope

The **slope** of a line is a number that describes both the direction and steepness of a line. The letter m is typically used as the variable for slope.

The slope can have 4 types of direction:

- **positive**: rising from left to right
- **negative:** falling from left to right
- **zero:** horizontal
- **undefined:** vertical




To measure the steepness of a line, we use the ratio of "rise" over "run."

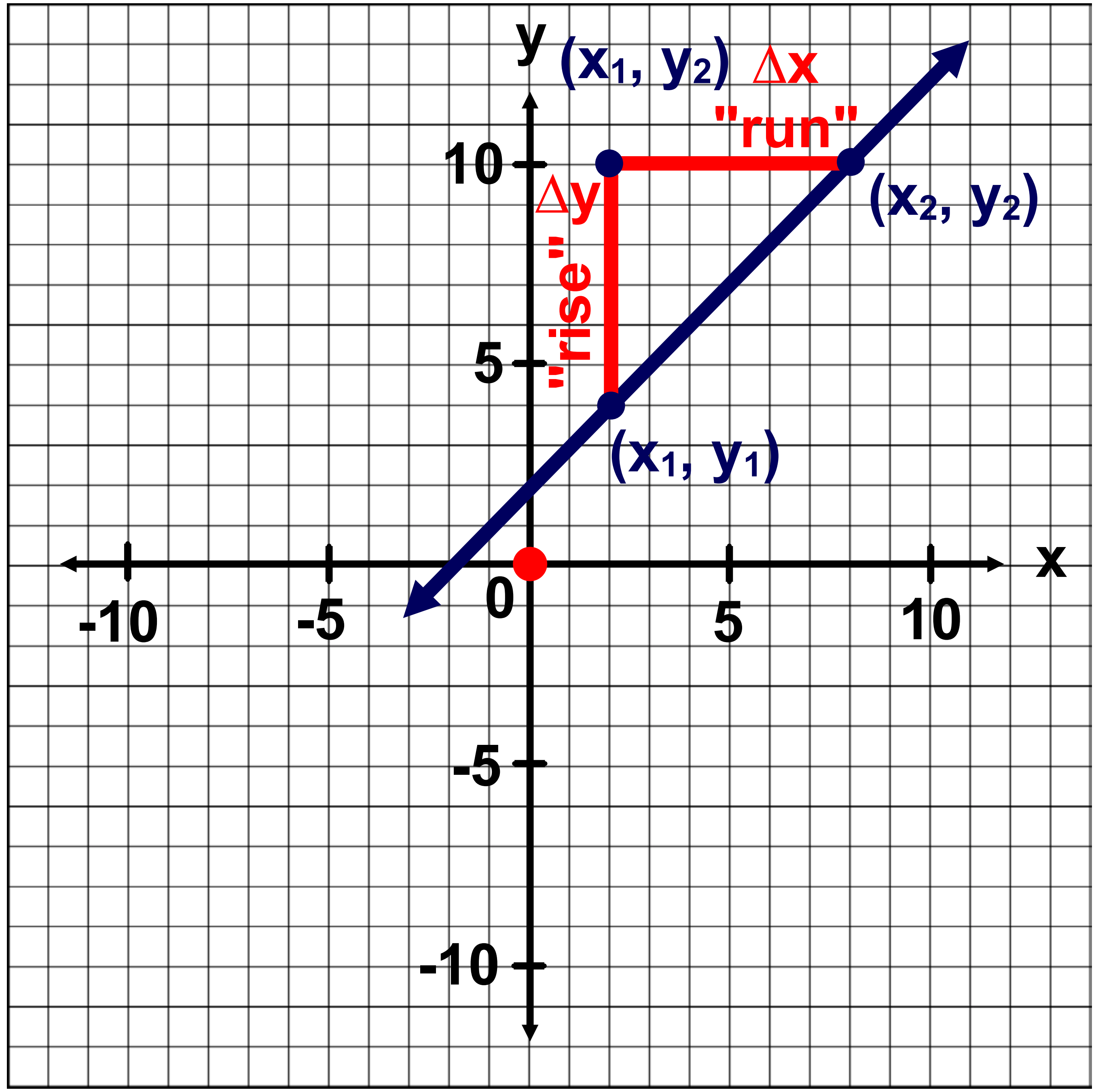
# Slope

The "rise" is the change in the value of the y-coordinate while the "run" is the change in the value of the x coordinate.

The symbol for "change" is the Greek letter delta, "Δ," which just means "change in."

So the slope is equal to the change in y divided by the change in x, or Δy divided by Δx... delta y over delta x.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



# Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

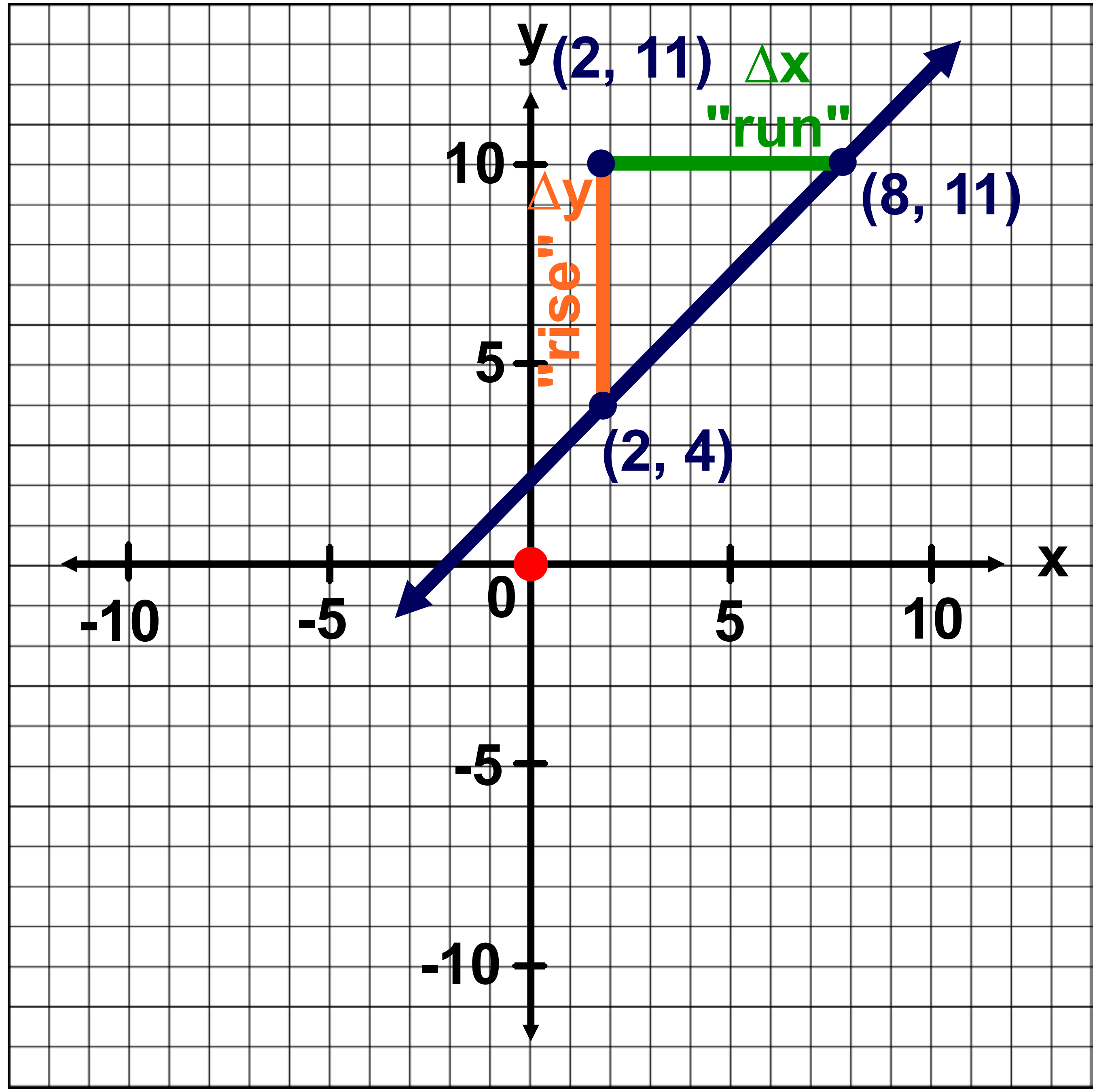
In this case:

The rise is from 4 to 11
$\Delta y = 11 - 4 = 7$

And the run is from 2 to 8,
$\Delta x = 8 - 2 = 6$

So the slope is

$$m = \frac{\Delta y}{\Delta x} = \frac{7}{6}$$



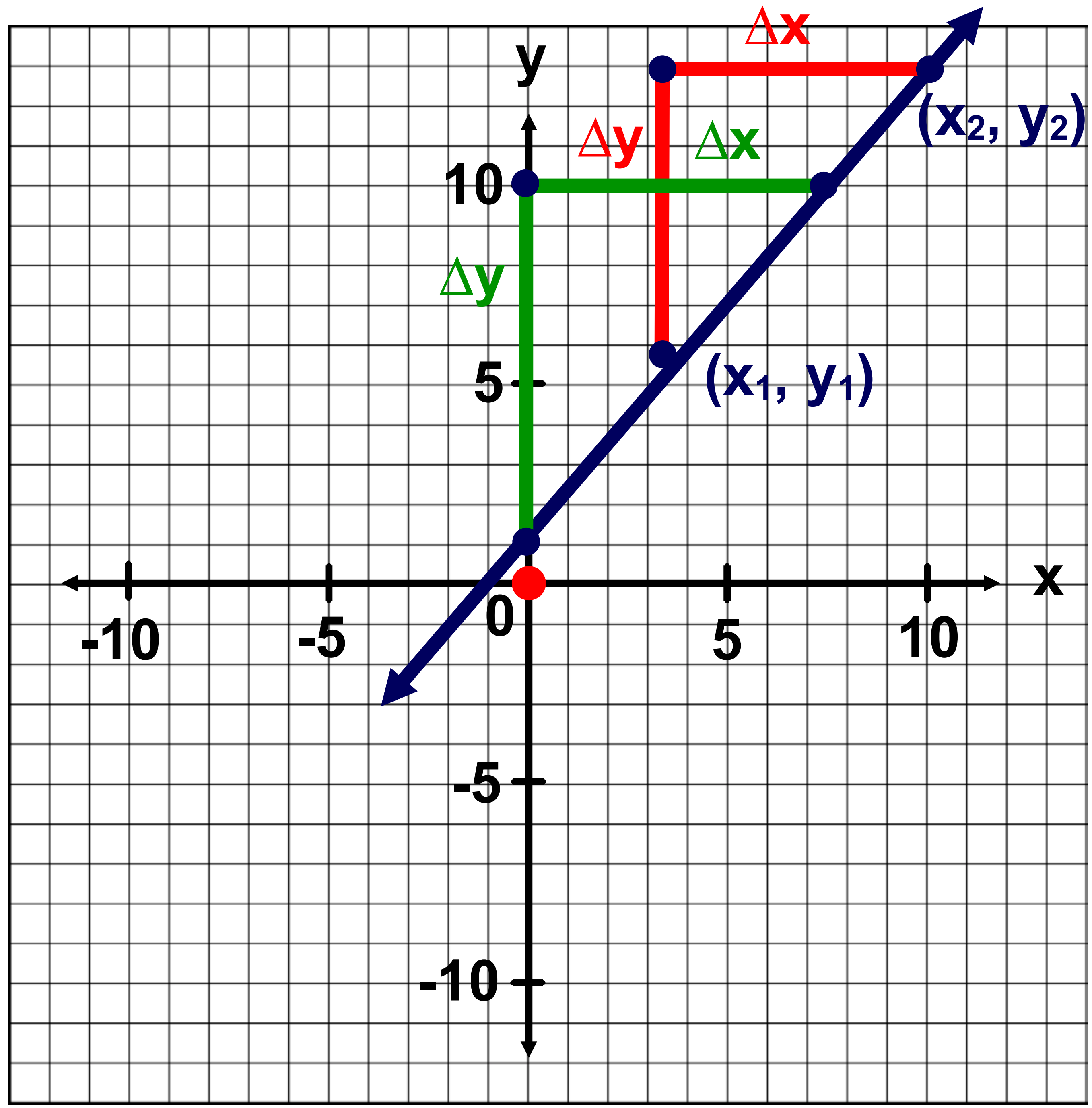
# Slope

Any points on the line can be used to calculate its slope, since the slope of a line is the same everywhere.

The values of $\Delta y$ and $\Delta x$ may be different for other points, but their ratio will be the same.

You can check that with the red and green triangles shown here.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



# Slope

Slope is also referred to as a constant rate of change.

Here is an application of slope:

A road might rise 1 foot for every 10 feet of horizontal distance.

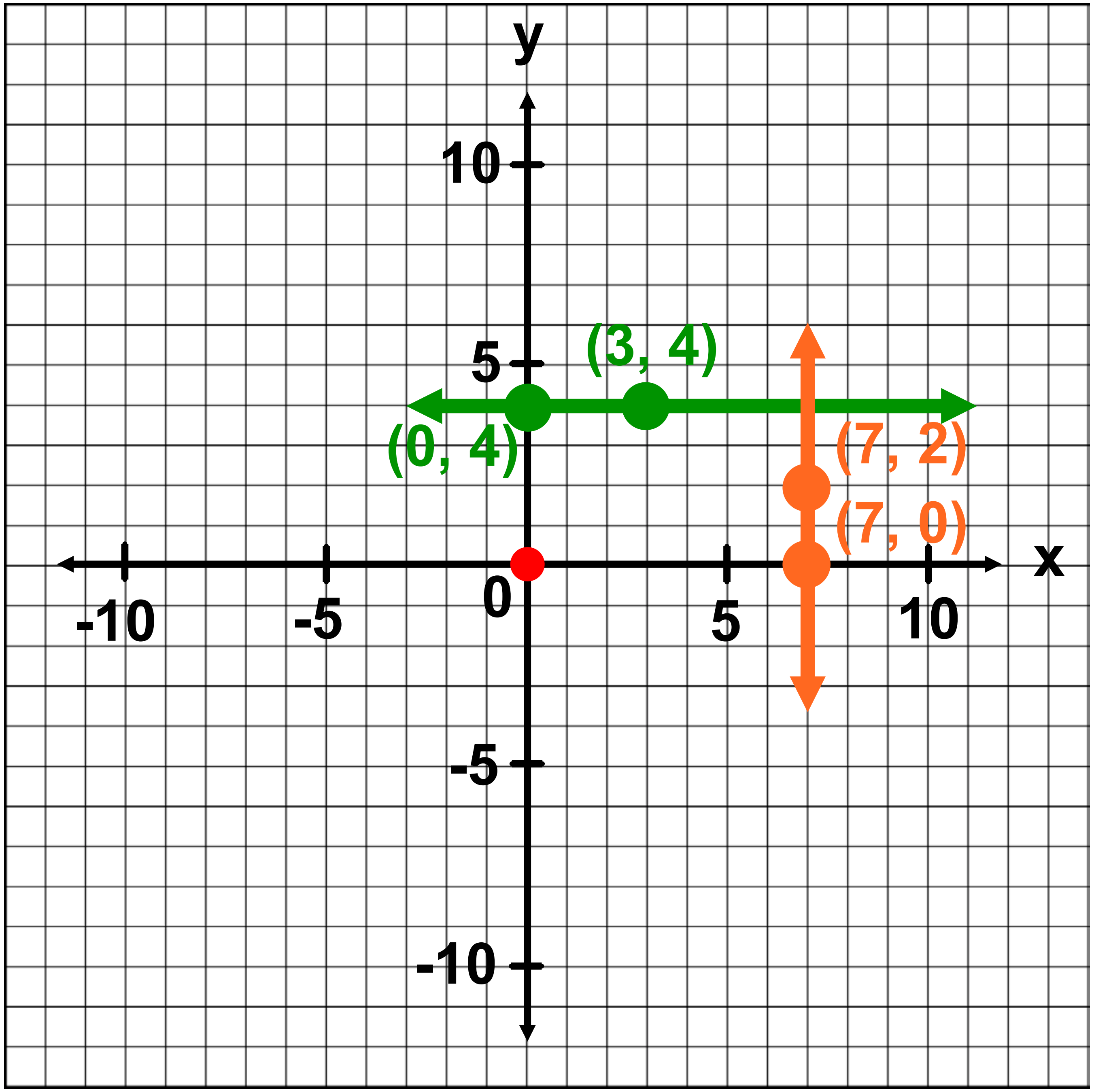

The ratio, 1/10, which is called slope, is a measure of the steepness of the hill. Engineers call this use of slope **grade** and measure the grade with percentages. The grade of the road above is 10%.

# Slope

Horizontal and vertical lines have special slopes. A horizontal line has a slope of 0, and a vertical line has an undefined slope. Let's see what makes these slopes special.

2 points on the horizontal line are (0, 4) and (3, 4). If we look at the graph, the $\Delta y$ is 0 and the $\Delta x$ is 3, so $m = \frac{0}{3} = 0$

Slope of horizontal & vertical lines using Rise & Run



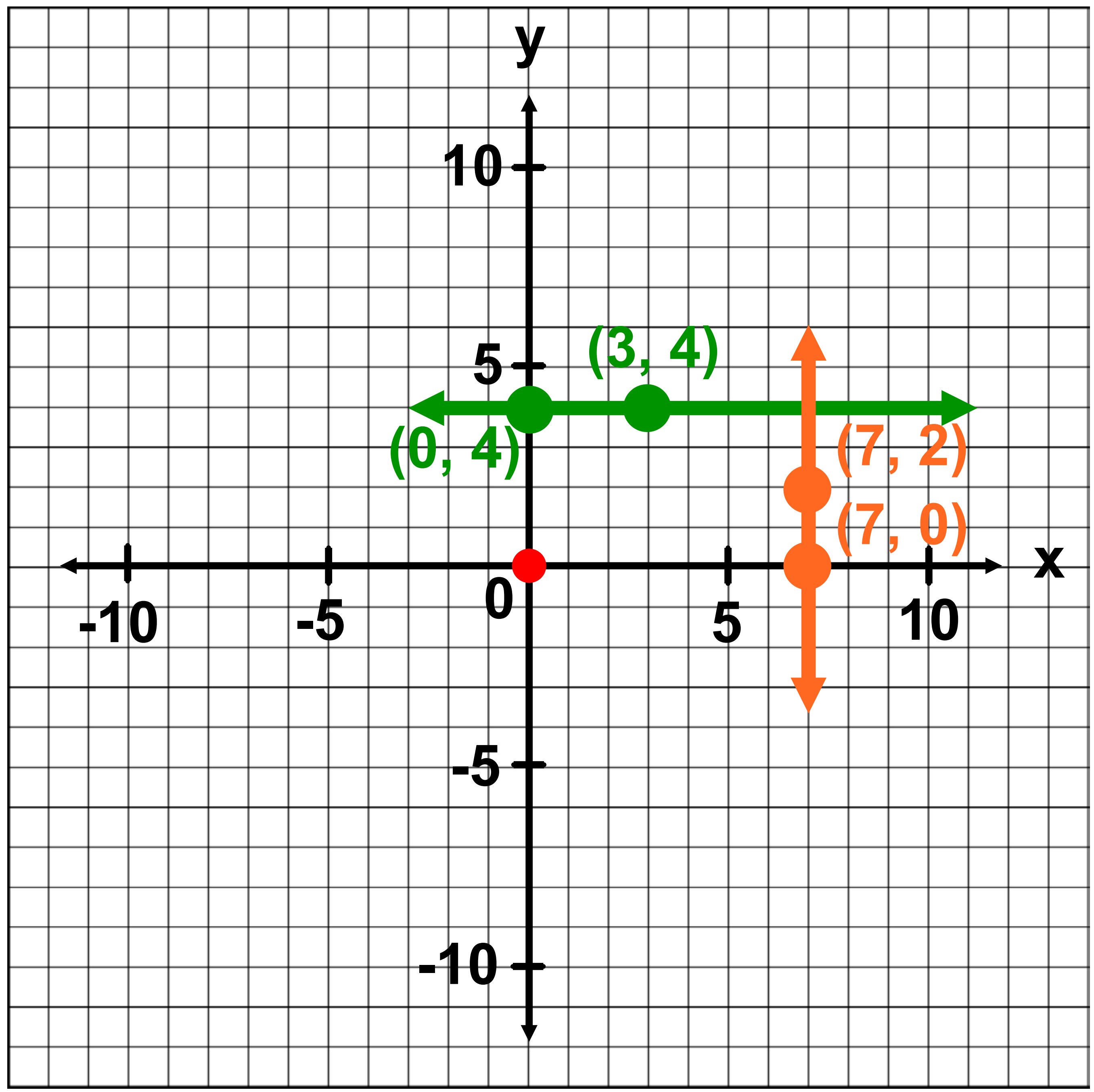
# Slope

Horizontal and vertical lines have special slopes. A horizontal line has a slope of 0, and a vertical line has an undefined slope. Let's see what makes these slopes special.

2 points on the vertical line are (7, 0) and (7, 2). If we look at the graph, the $\Delta y$ is 2 and the $\Delta x$ is 0, so...

$m = \frac{2}{0}$ = undefined because you can't divide by 0.

Slope of horizontal & vertical lines using Rise & Run



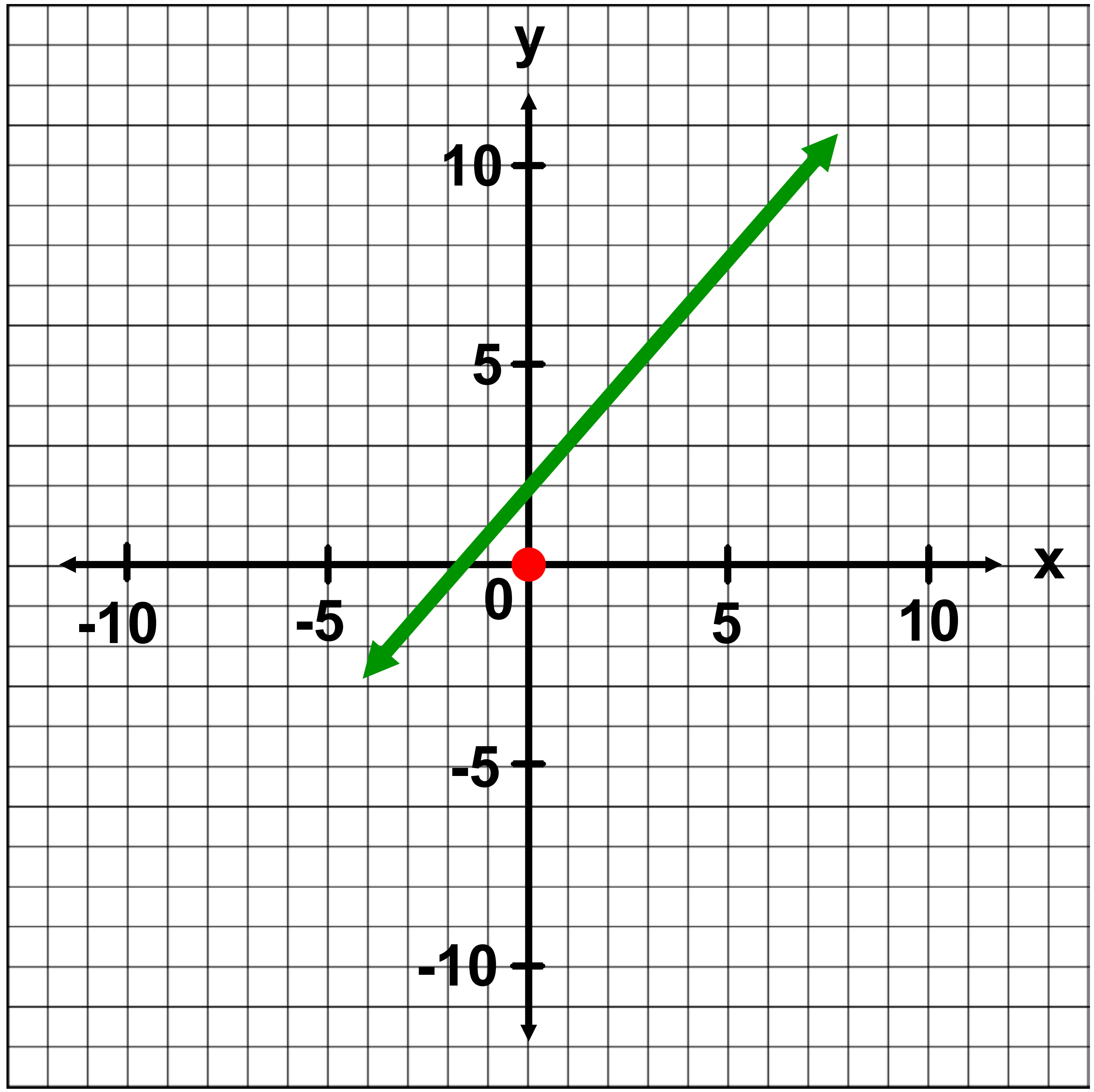
26 The slope of the indicated line is:

A negative

B positive

C zero

D undefined




Answer

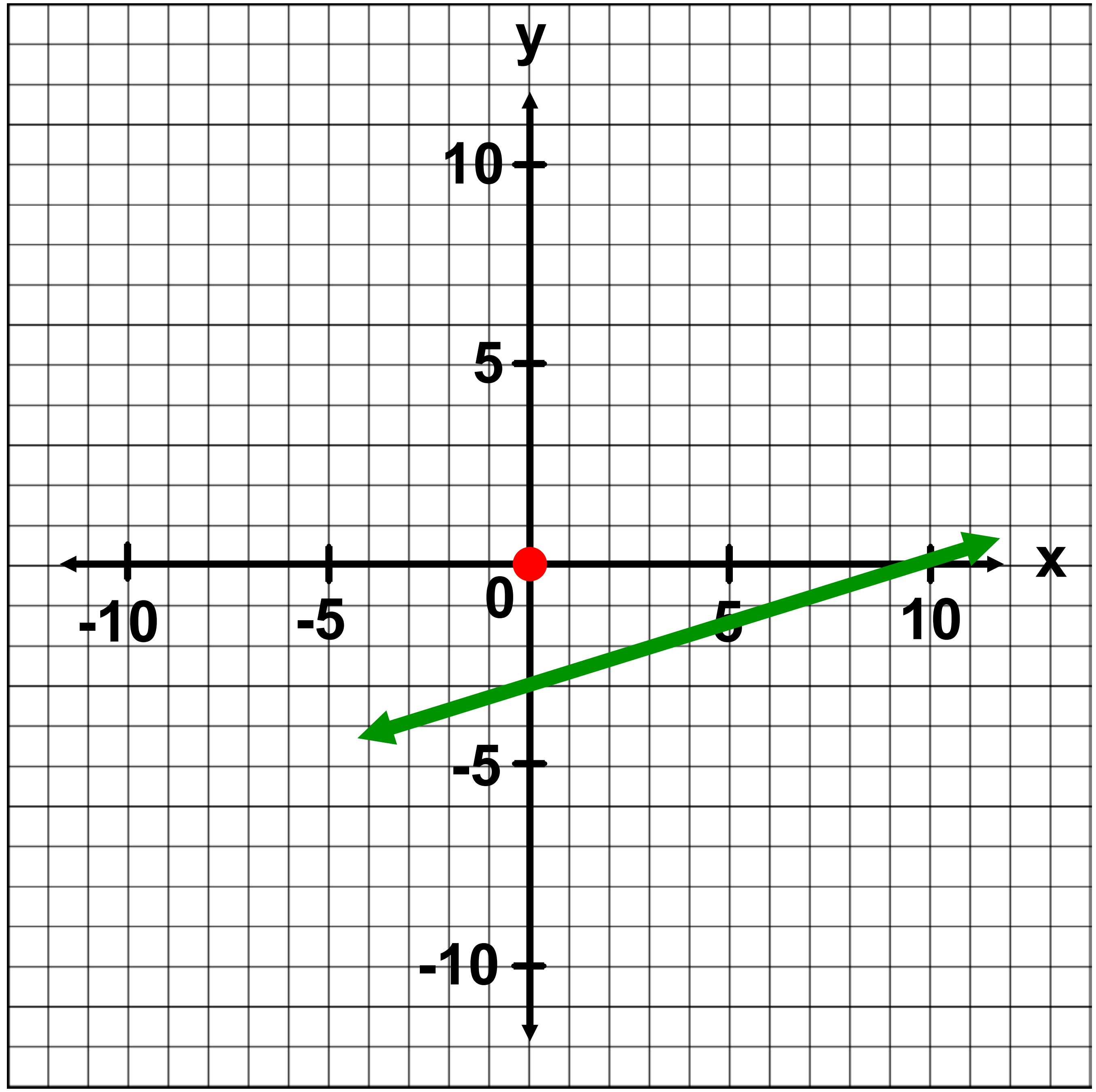
27 The slope of the indicated line is:

A negative

B positive

C zero

D undefined




Answer

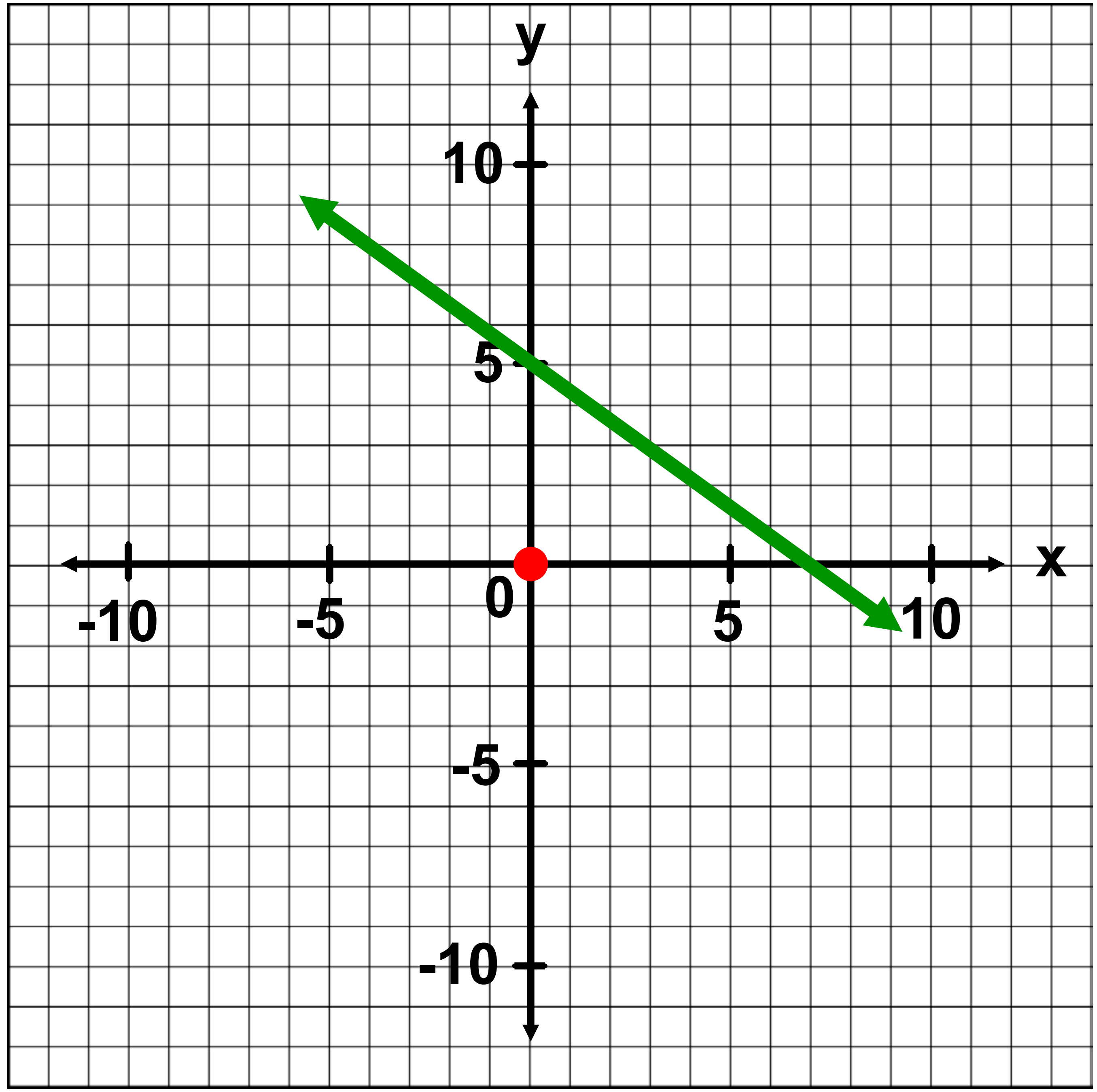
28 The slope of the indicated line is:

A negative

B positive

C zero

D undefined




Answer

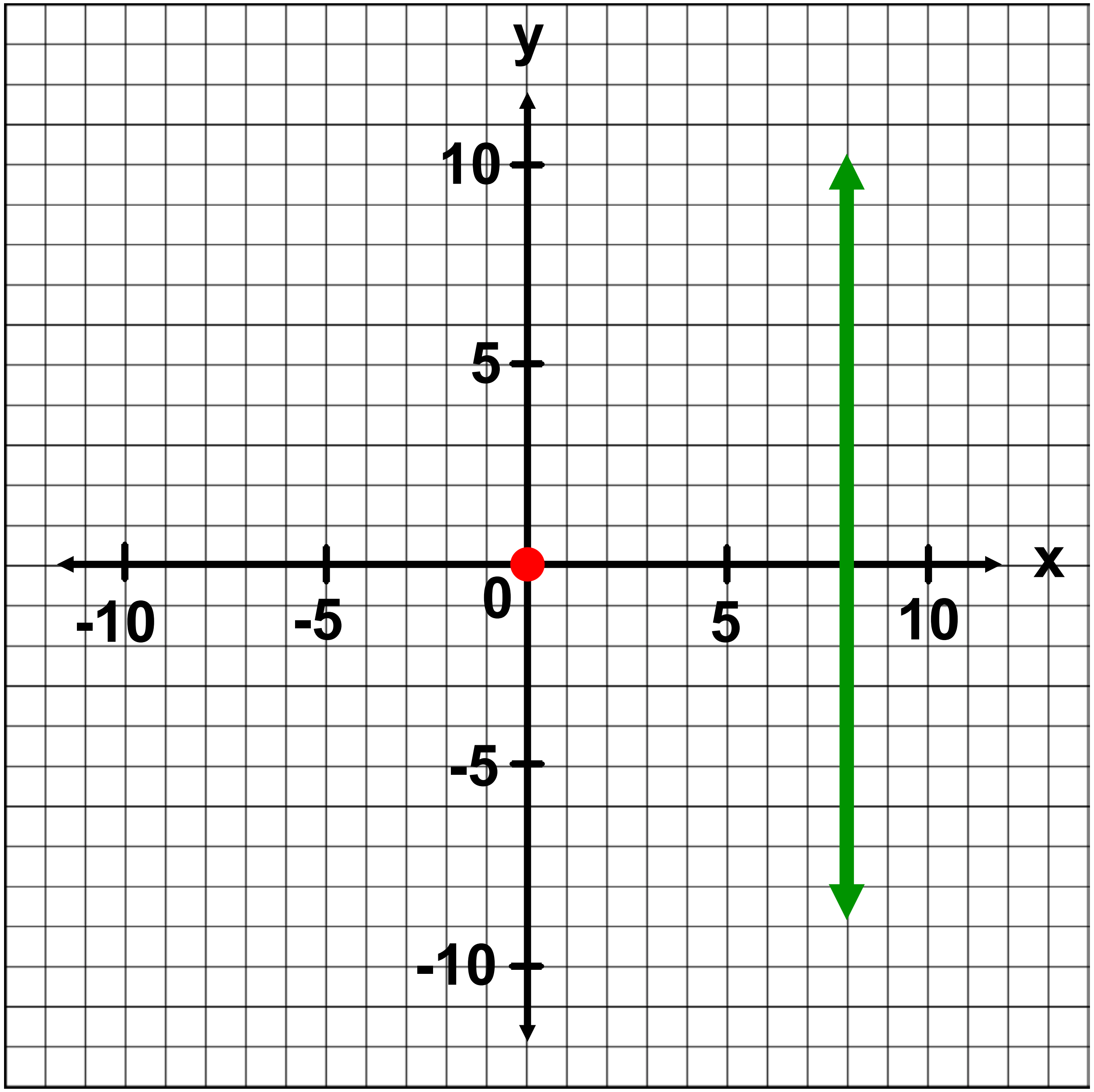
29 The slope of the indicated line is:

A negative

B positive

C zero

D undefined




Answer

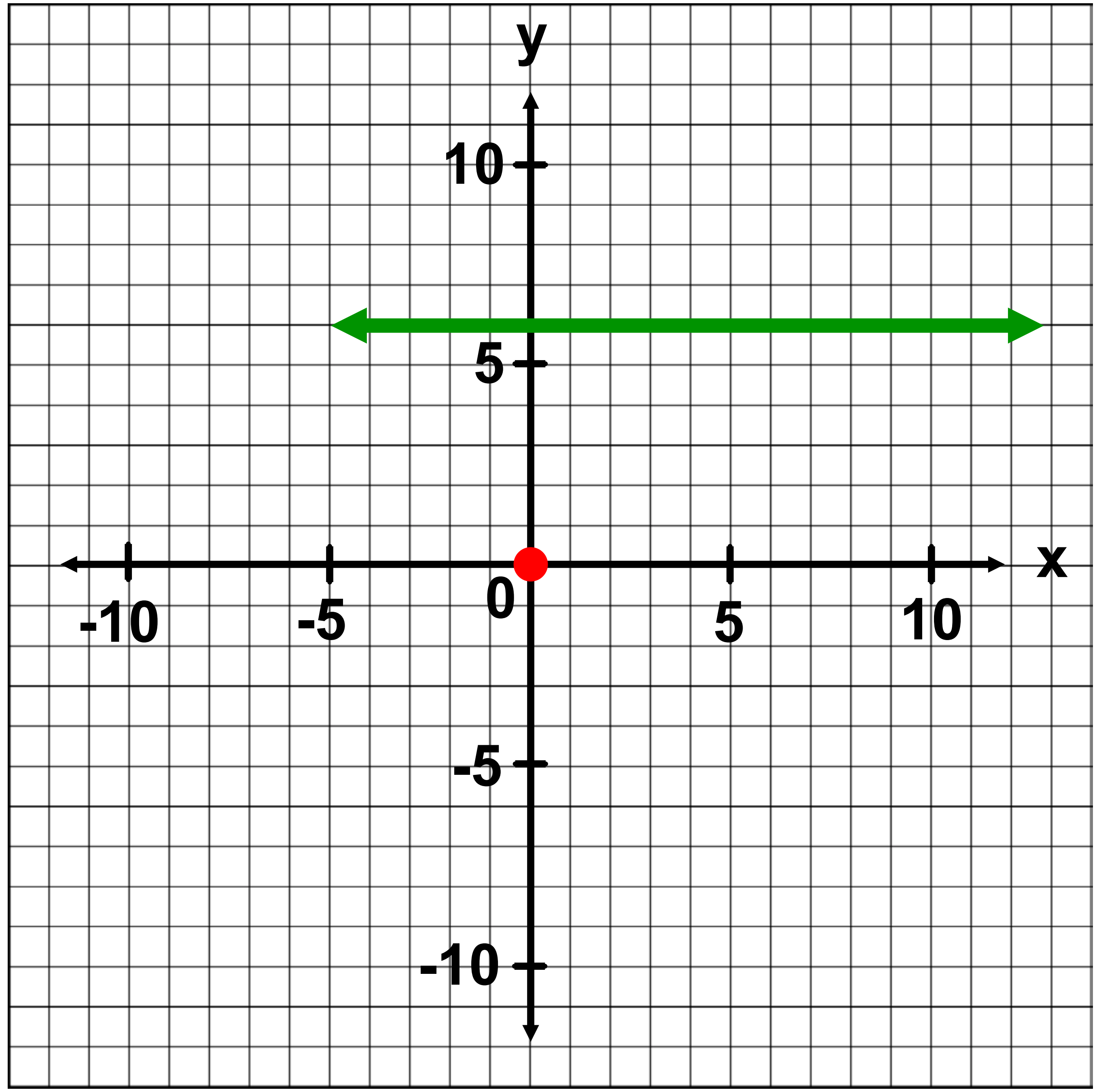
30 The slope of the indicated line is:

A negative

B positive

C zero

D undefined




Answer

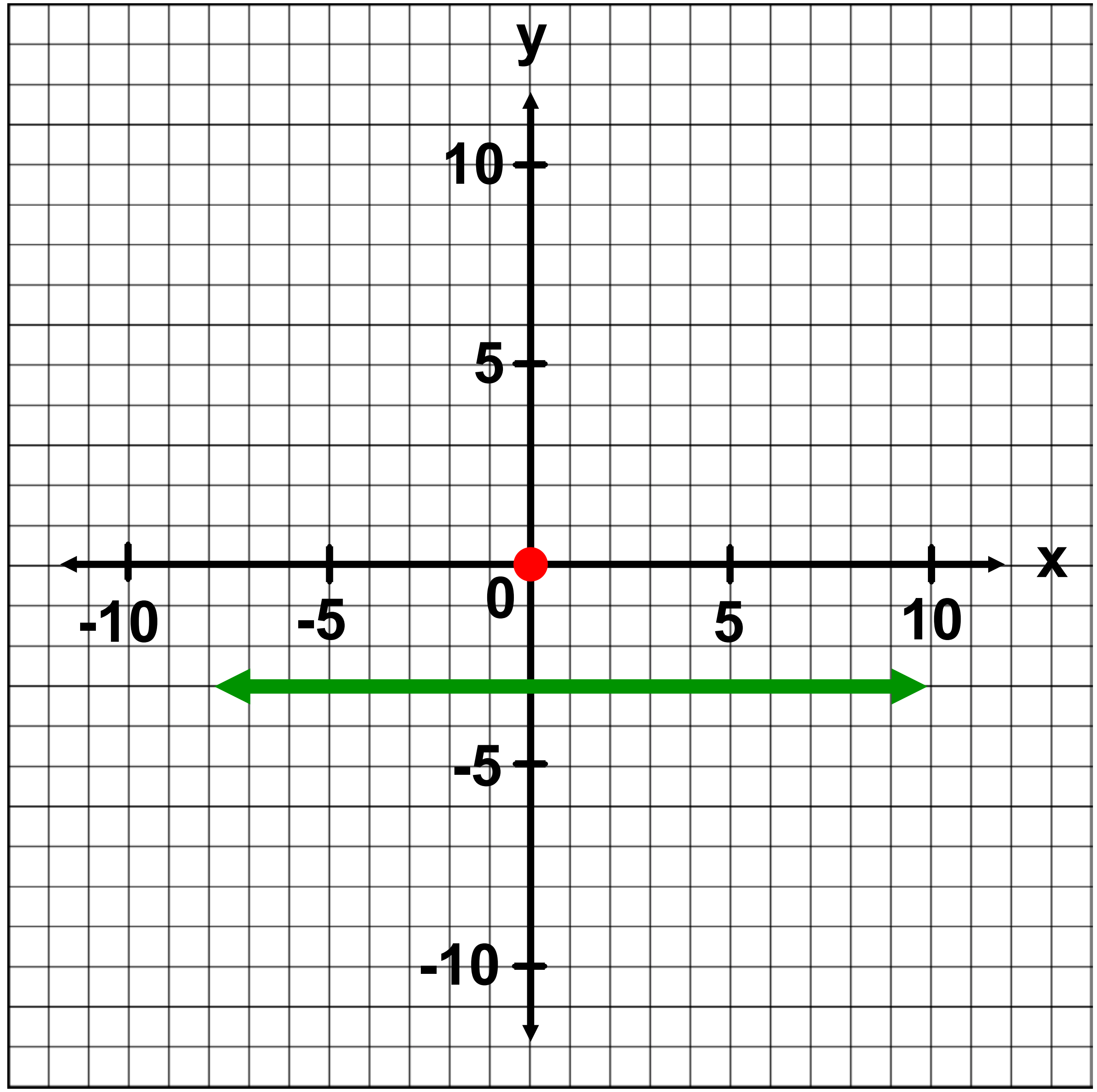
31 The slope of the indicated line is:

A negative

B positive

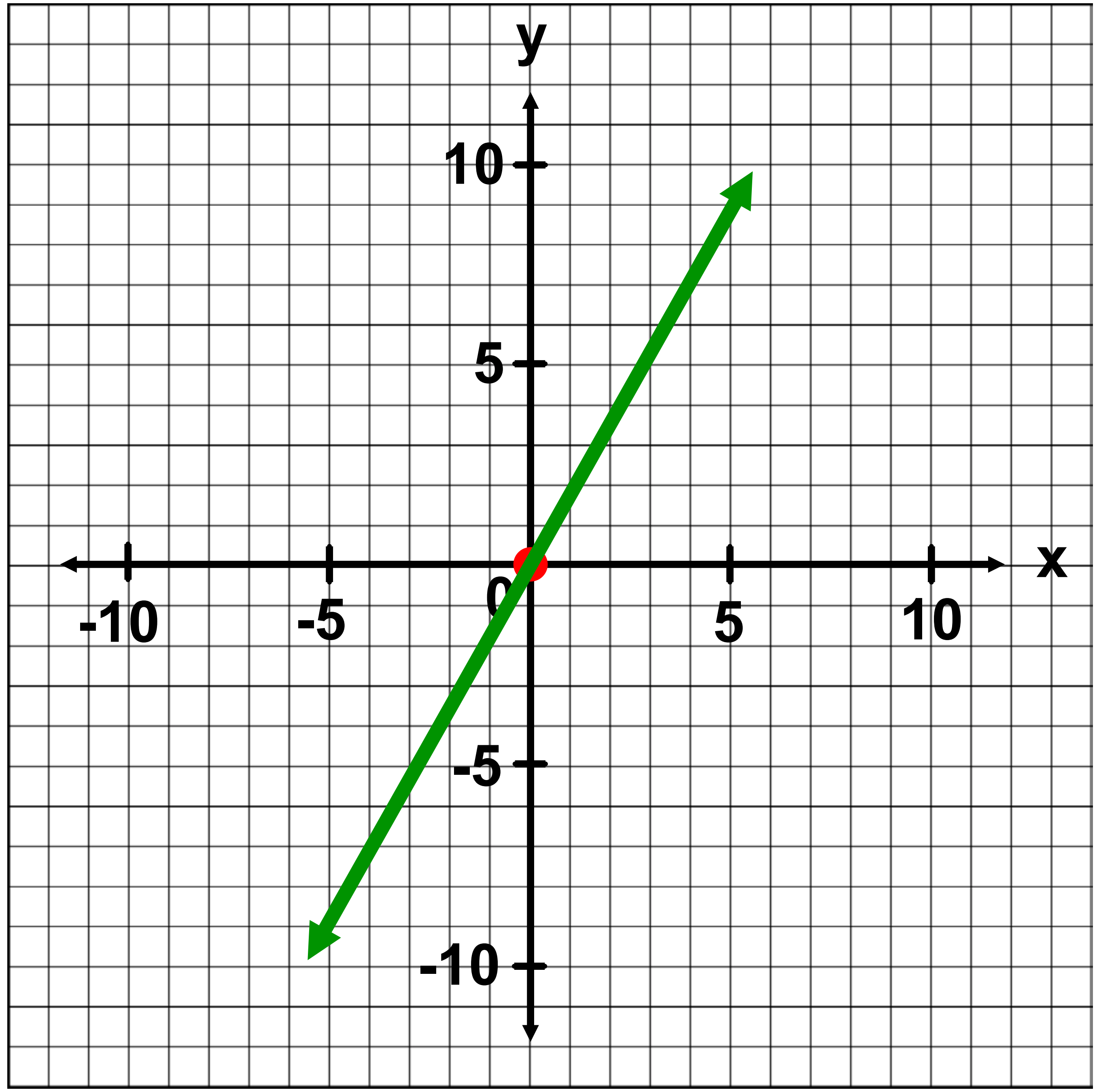
C zero

D undefined



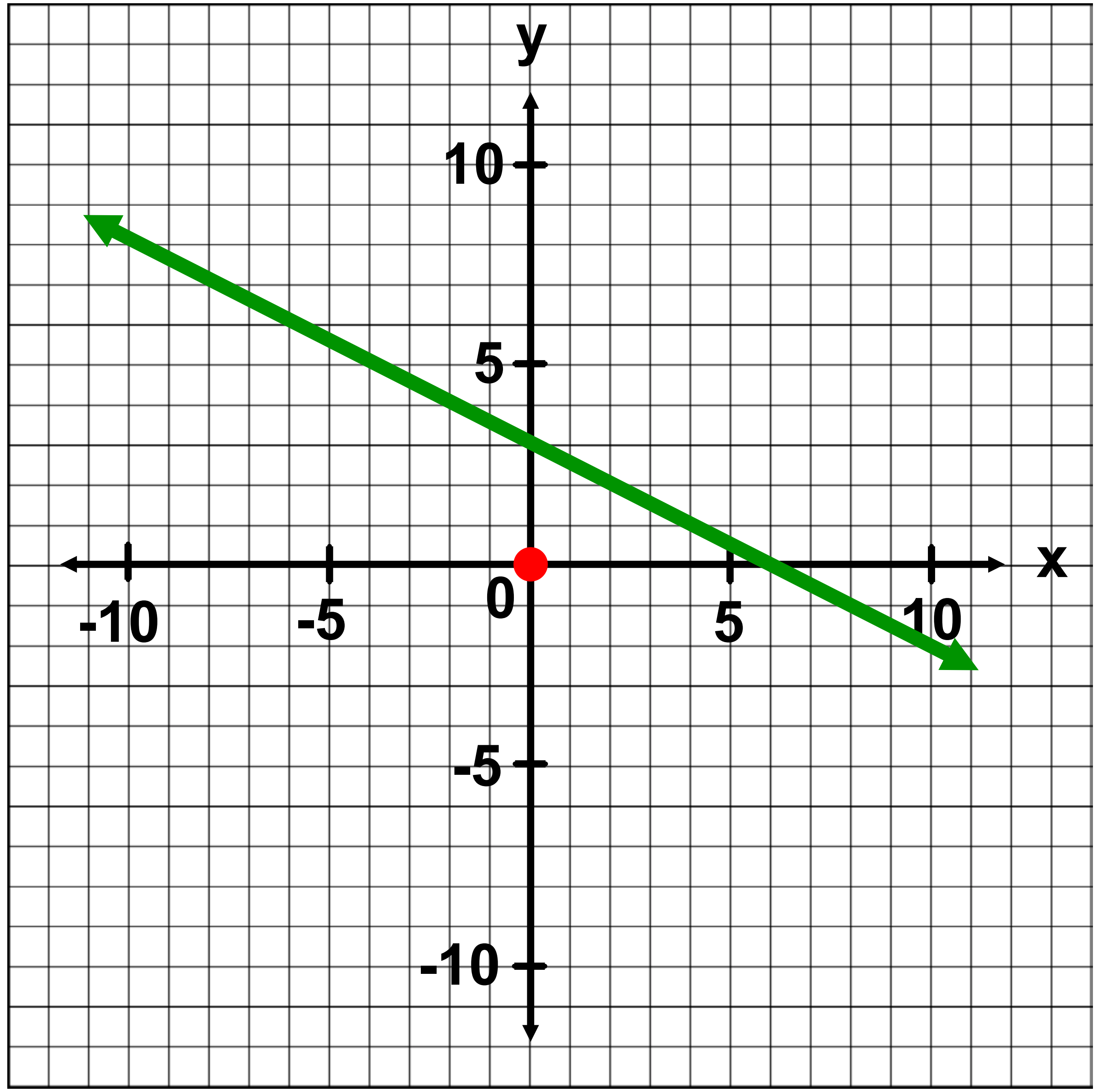

Answer

32 What's the slope of this line?



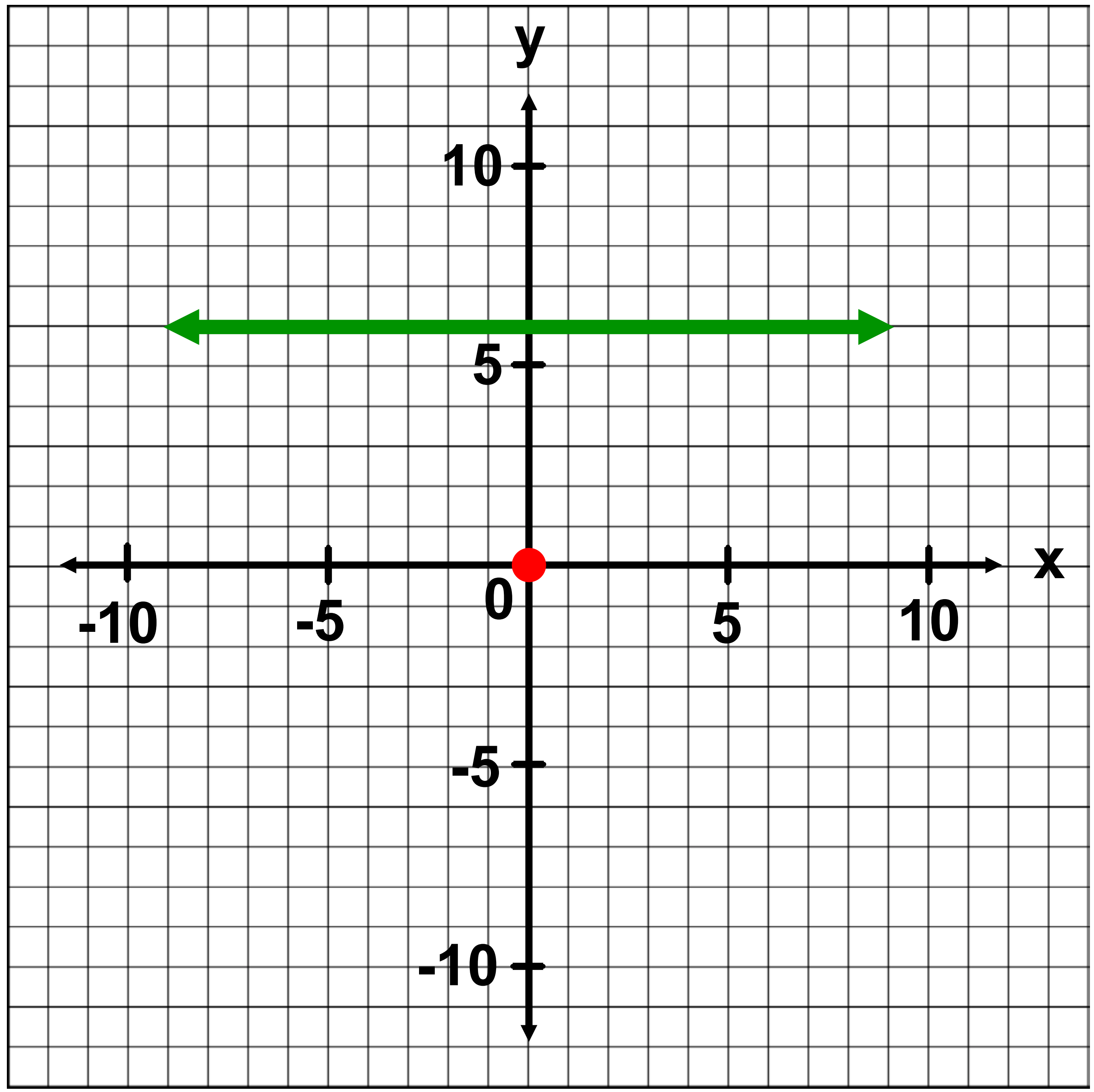

Answer

33 What's the slope of this line?



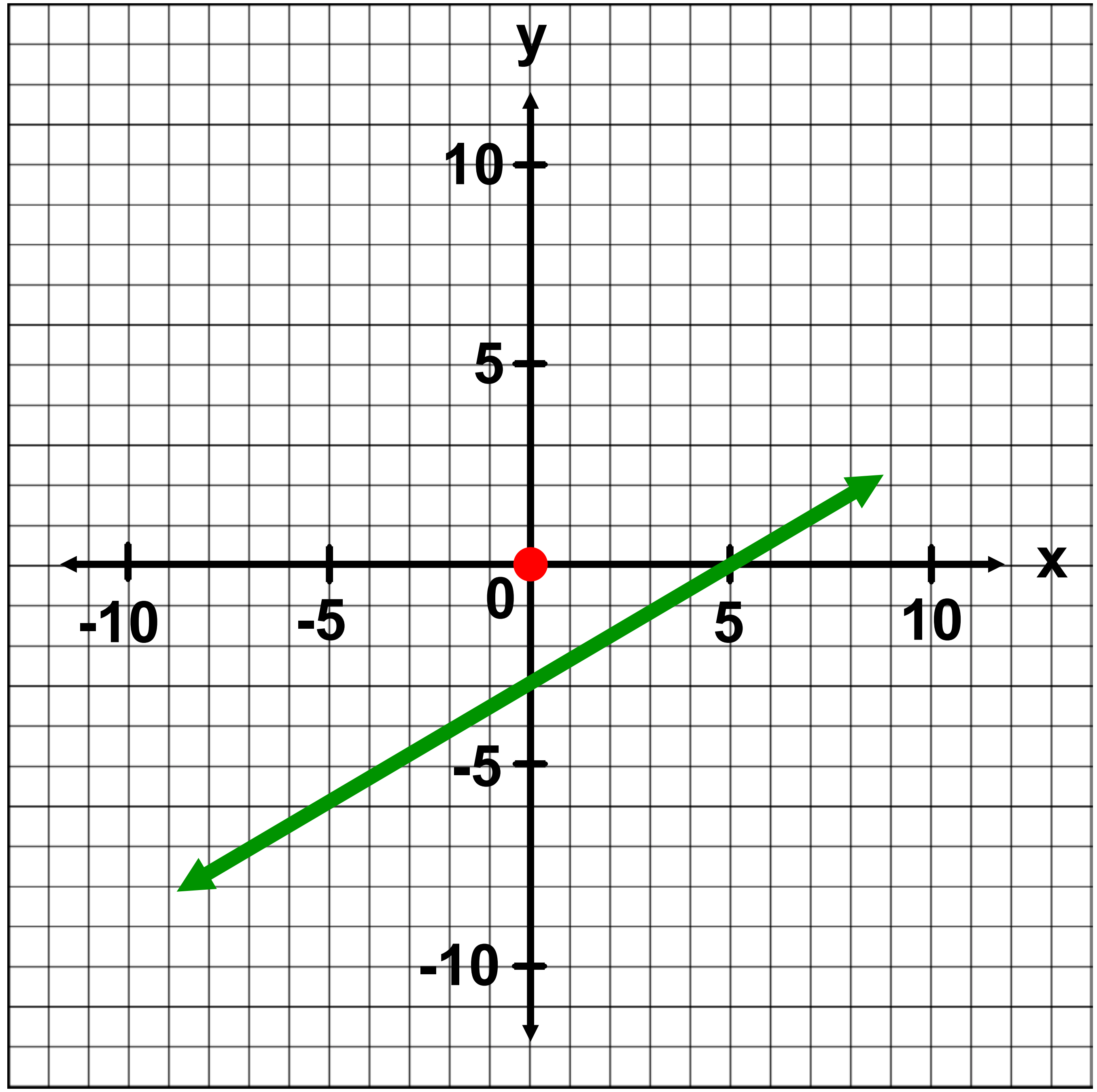

Answer

34 What's the slope of this line?



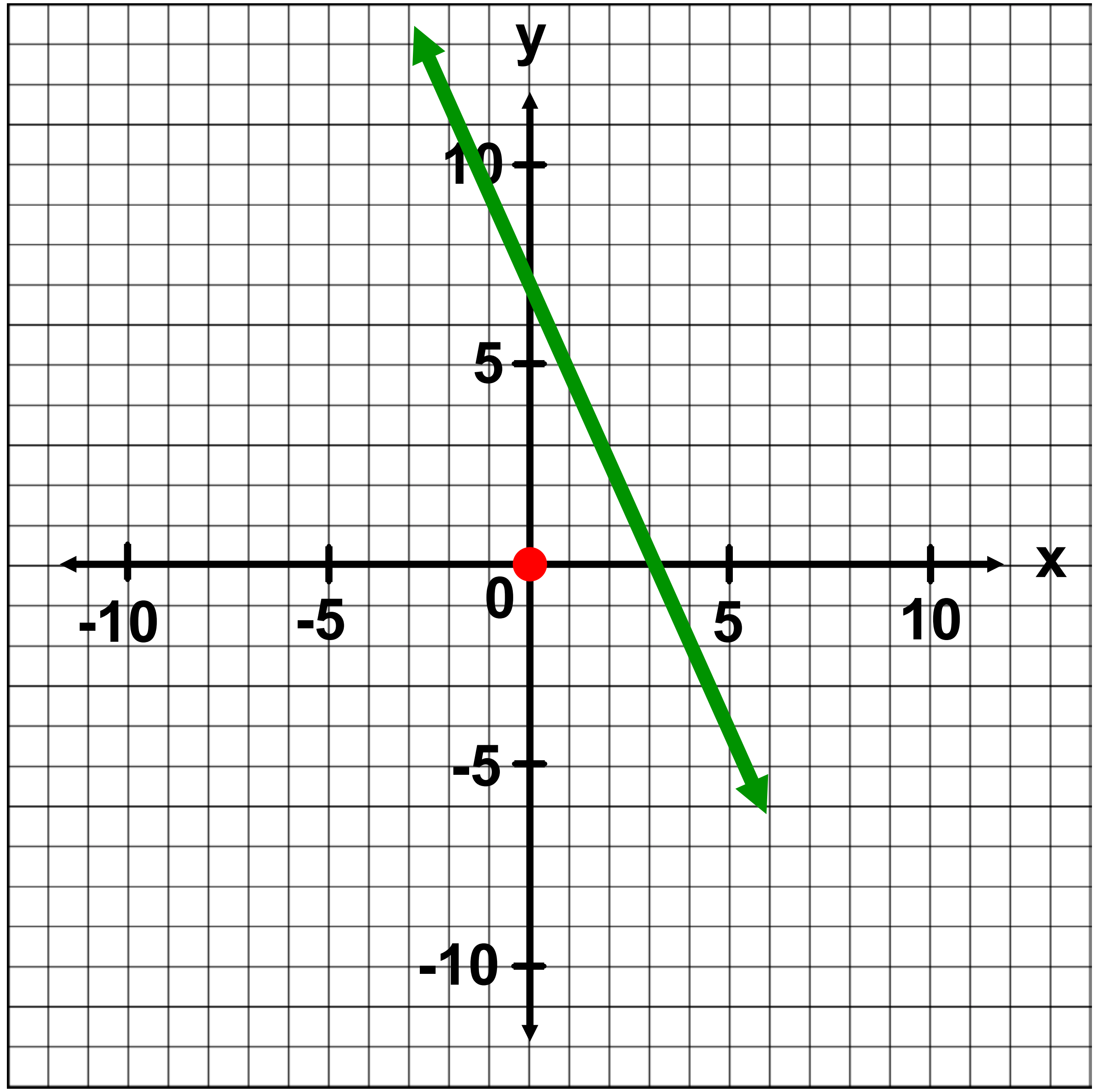

Answer

35 What's the slope of this line?



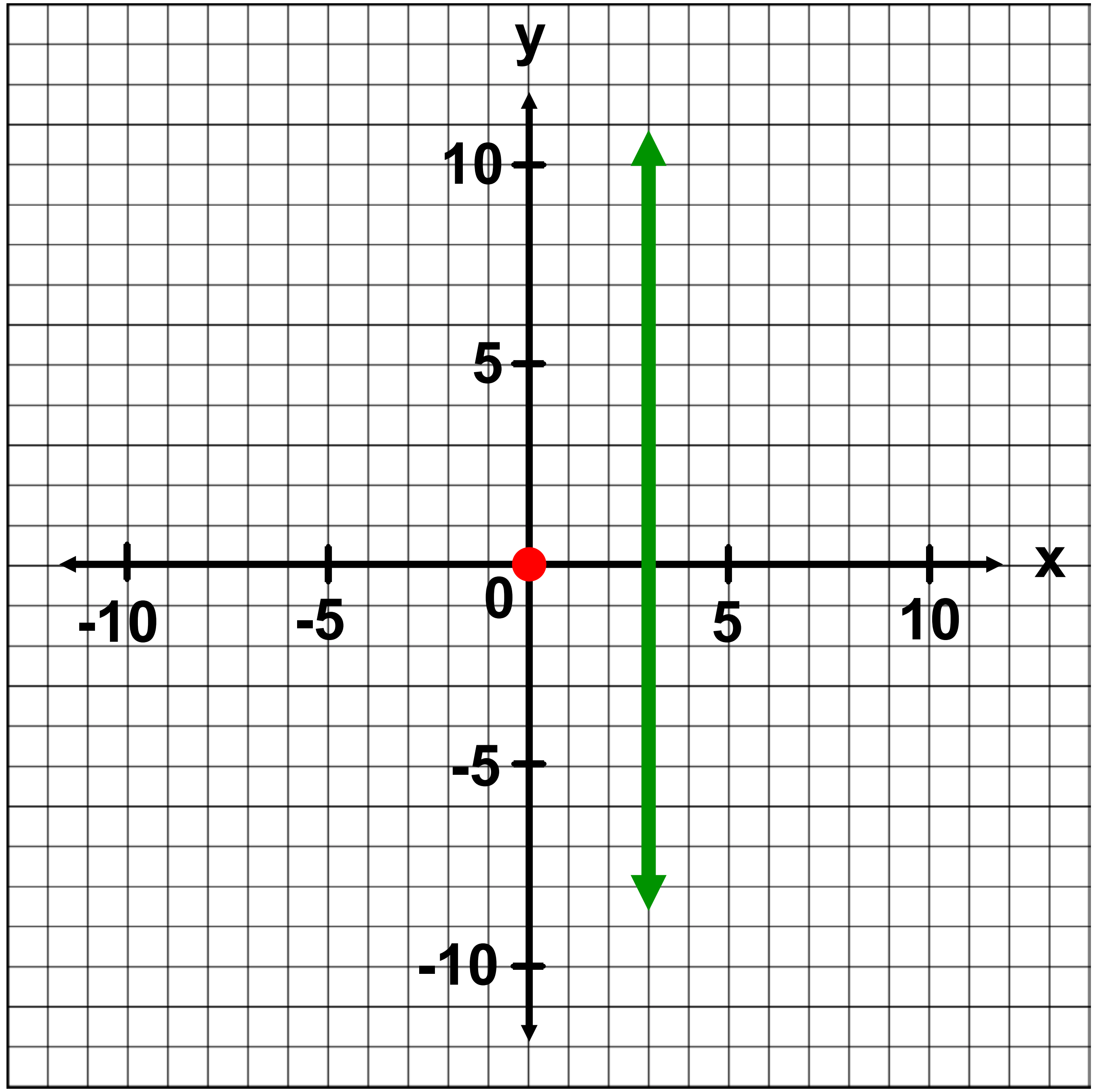

Answer

36 What's the slope of this line?




Answer

37 What is the slope of this line?




Answer

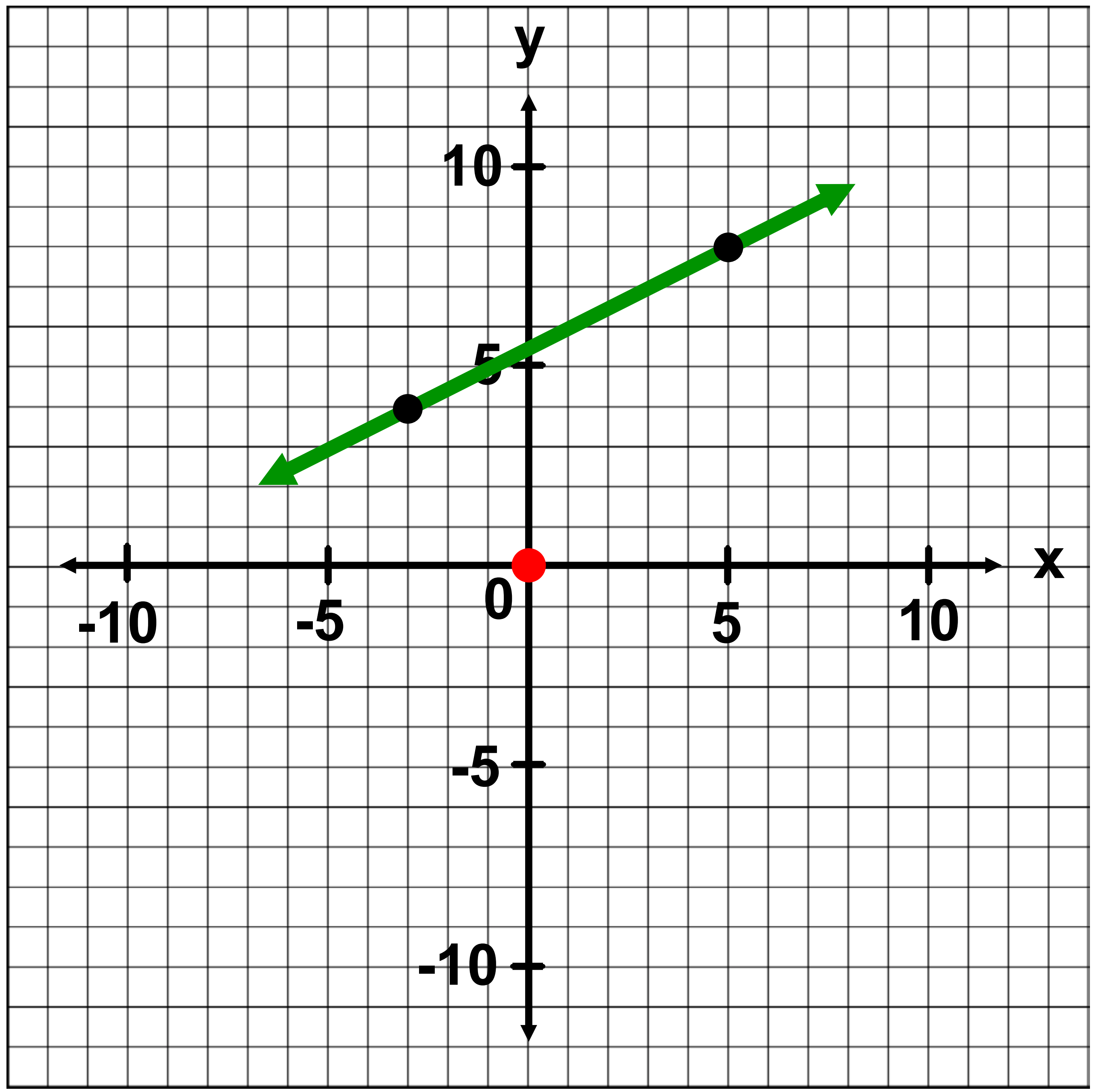
38 What is the slope of the line passing through the indicated points?

A $-2$

B $2$

C $\frac{-1}{2}$

D $\frac{1}{2}$




Answer

# Slope

Let's try an example that does not have a graph.

Calculate the slope of the line that passes through (-5, 4) and (5, 0).

First identify (-5, 4) as your $(x_1, y_1)$ and (5, 0) as your $(x_2, y_2)$.

Second, substitute your numbers into the slope formula for their assigned variables.

$$\frac{0 - 4}{5 - (-5)} = \frac{0 - 4}{5 + 5} = \frac{-4}{10} = \frac{-2}{5}$$

39 What is the slope of the line through A(-2, 1) and B(3, -1)?

Answer

40 What is the slope of line MN if M(1, 7) and N(3, -4)?

Answer

41 What is the slope of the line containing (-1, 7) and (3, -7)?

Answer

42 What is the slope of the line that passes through the points (3, 5) and (−2, 2)?

A $\frac{1}{5}$

B $\frac{3}{5}$

C $\frac{5}{3}$

D 5

Answer

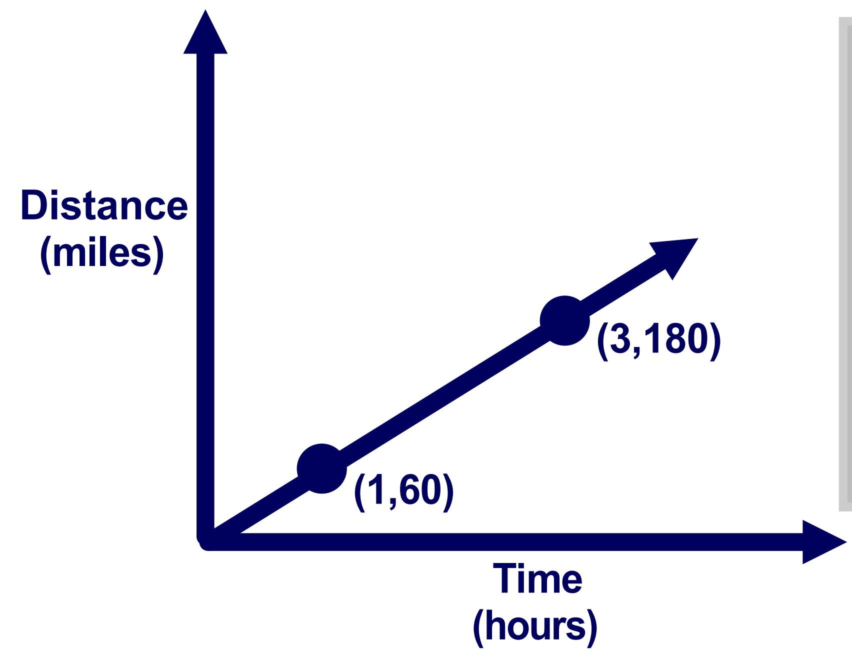
43 A straight line with slope 5 contains the points (1, 2) and (3, k). Find the value of k.

Answer

# Constant Rate of Change

Slope formula can be used to find the **constant rate of change** in a "real world" problem.

When traveling on the highway, drivers will set the cruise control and travel at a constant speed this means that the distance traveled is a constant increase.




The graph at the right represents such a trip. The car passed mile-marker 60 at 1 hour and mile-marker 180 at 3 hours. Find the slope of the line and what it represents.

$$m = \frac{180 \text{ miles} - 60 \text{ miles}}{3 \text{ hours} - 1 \text{ hours}} = \frac{120 \text{ miles}}{2 \text{ hours}} = \frac{60 \text{ miles}}{\text{hour}}$$

So the slope of the line is 60 and the rate of change of the car is 60 miles per hour.

Math Practice

# Constant Rate of Change

If a car passes mile-marker 100 in 2 hours and mile-marker 200 in 4 hours, how many miles per hour is the car traveling?

Use the information to write ordered pairs (2, 100) and (4, 200).

***slide to move***

44 If a car passes mile-marker 90 in 1.5 hours and mile-marker 150 in 3.5 hours, how many miles per hour is the car traveling?

Answer

45 How many meters per second is a person running if they are at 10 meters in 3 seconds and 100 meters in 15 seconds?

Answer

# Point-Slope Form

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of Contents

# Point-Slope Form

A useful form of the equation of a line is the point-slope form. It's equation is $y - y_1 = m(x - x_1)$

It's based on the use of the slope and any point that is on the line.

This equation is the most effective when you are given the slope and a point on the line because you can use it to write the equation in multiple forms.

Let's get started.

Math Practice

# Point-Slope Form

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-slope form starts using the definition of slope, in which two points on a line are given by the ordered pairs

$(x_1, y_1)$ and $(x_2, y_2)$

As a first step, let's just name the coordinates for the second point $(x, y)$ rather than $(x_2, y_2)$.

That will be true for any point on the line, not just one point, and will allow us to write an equation for all points on the line.

Then our slope formula becomes:

$$m = \frac{y - y_1}{x - x_1}$$

# Point-Slope Form

$$m = \frac{y - y_1}{x - x_1}$$

Now let's solve that equation for y using what we've learned about solving equations.

Try it yourself, before we show you our answer.

(Hint: Remember to treat the denominator $(x - x_1)$ like it's in parentheses.)

# Point-Slope Form

$$m = \frac{y - y_1}{x - x_1}$$

Multiply both sides by $(x - x_1)$ to get rid of the fraction.

$$m(x - x_1) = y - y_1$$

The last step is to switch the expressions to the opposite sides of the equals sign.

$$y - y_1 = m(x - x_1)$$

# Point-Slope Form

$$y - y_1 = m(x - x_1)$$

**Point-Slope Form** Where:

- $m$ is the slope,
- $(x_1, y_1)$ is any of the infinite points that satisfy the equation.

# Point-Slope Form

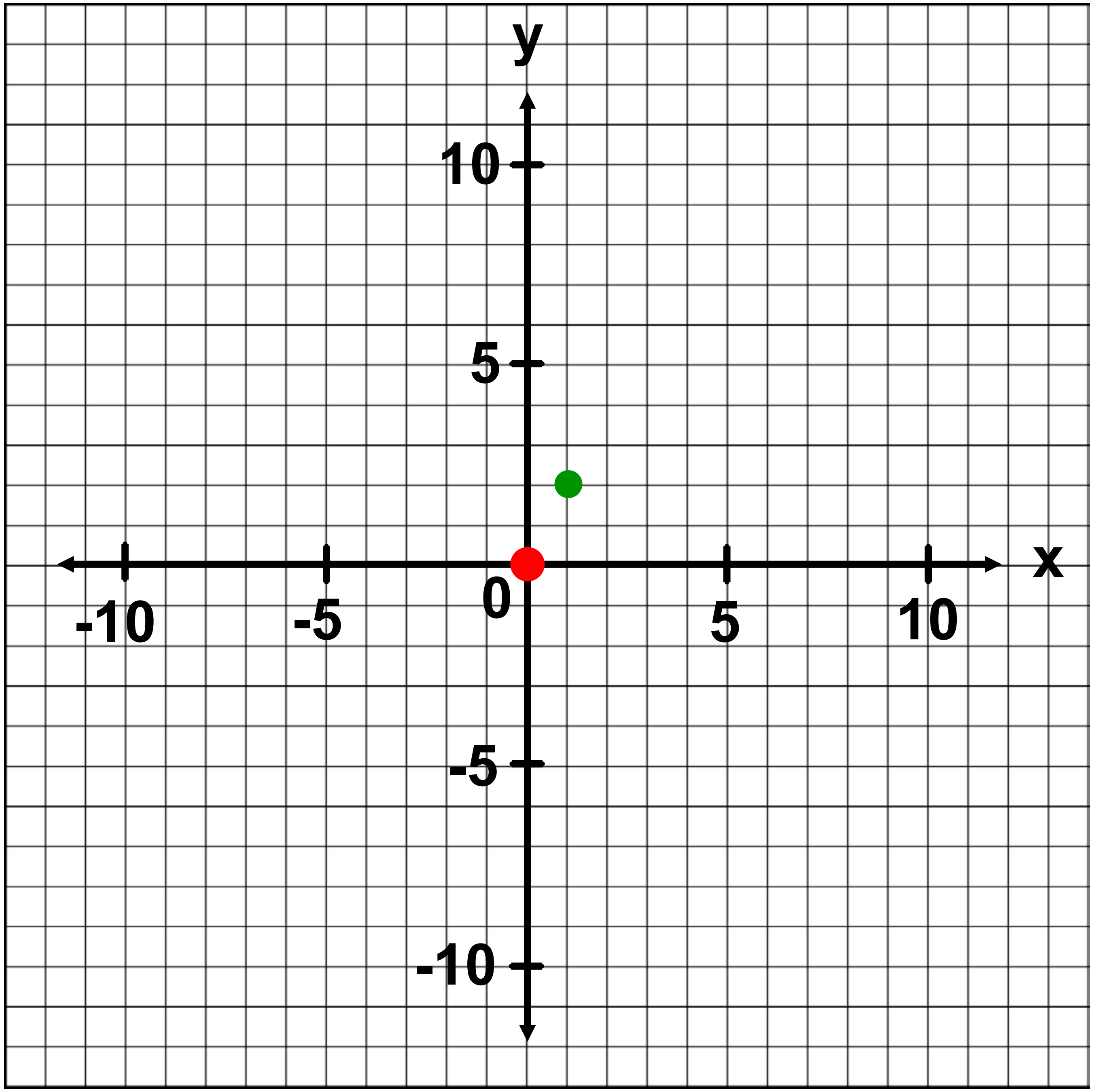
$$y - y_1 = m(x - x_1)$$

If you are provided a graph of a line, you can calculate m and locate a point directly from the graph.

That allows you to write the equation of the line directly, which you can then use to find any other needed points.

# Using Point-Slope to Draw a Line

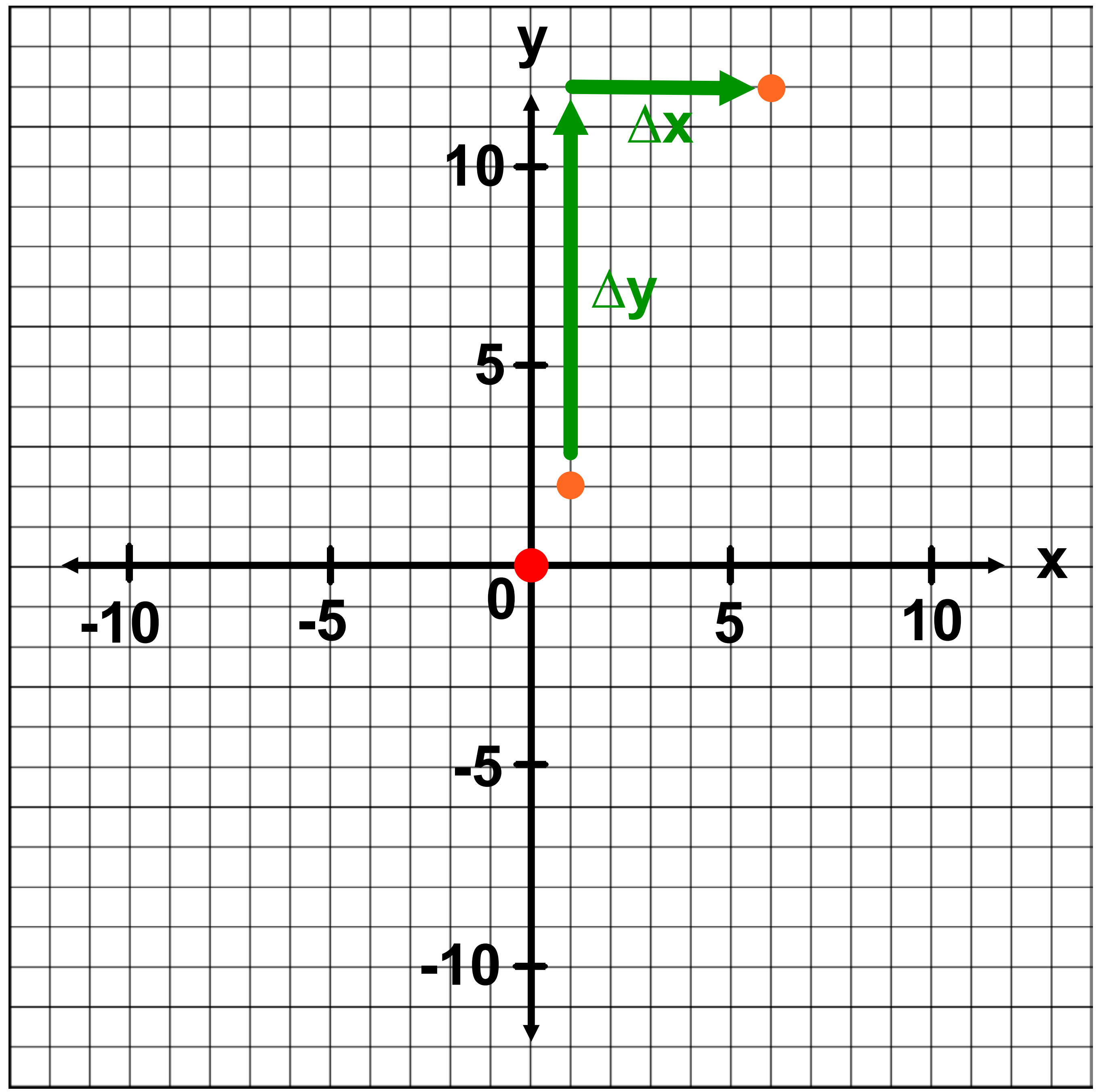
For instance, if I know that the equation of a line is $y - 2 = 2(x - 1)$, then one point on the line is (1, 2) and the slope of the line is 2. Using this information, I can find a second point, and then draw the line.



# Using Point-Slope to Draw a Line

I do this by recognizing that the slope of 2 means that if I go up 2 units on the y-axis I have to go 1 unit to the right on the x-axis.

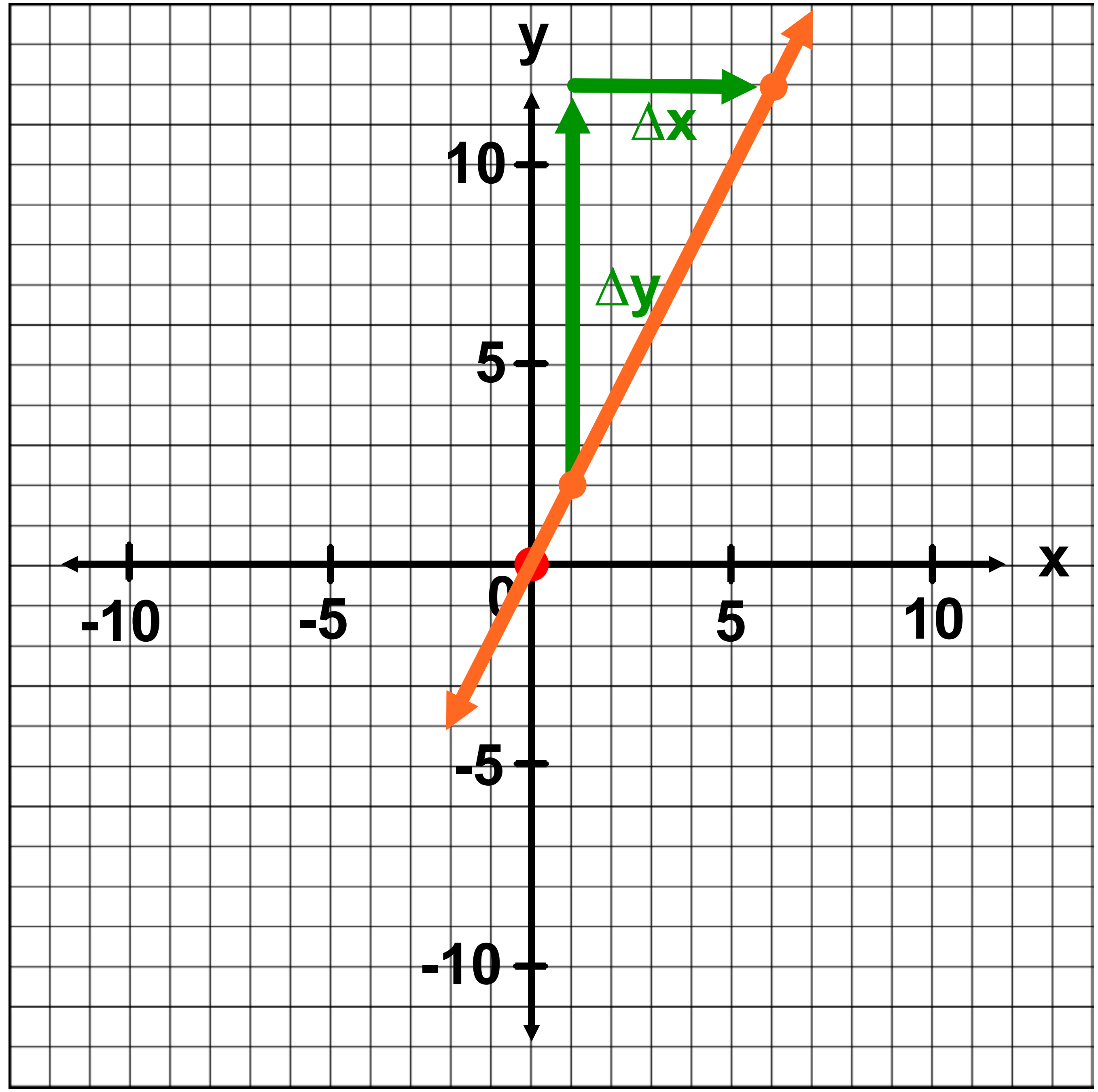
Or if I go up 10, I have to go over 5 units, etc.



# Using Point-Slope to Draw a Line

Then I draw the line through any two of those points.

This method is the easiest to use if you just have to draw a line given a point and slope.



# Using Point-Slope to Draw a Line

Graph the equation

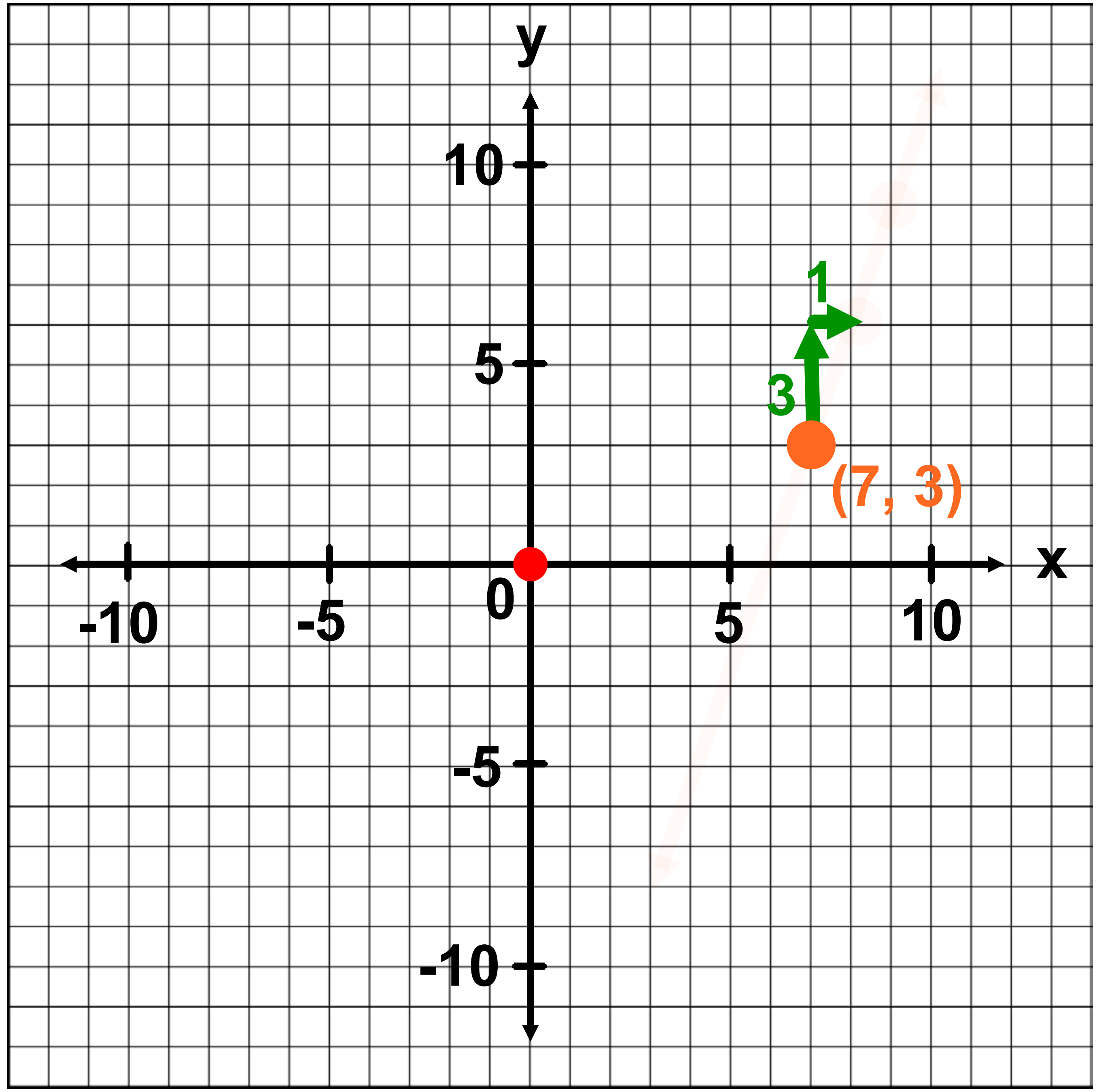
$$y - 3 = 3(x - 7)$$

Based on the equation, we know that the line passes through ______
*click*

and has a slope of ______
*click*

Now the graph can be drawn.

Click on the point to reveal it. Click slightly above it to show the rise & slightly to the right to show the run. Click on the new point. Click in between to reveal the line.



# Using Point-Slope to Draw a Line

Example: Given the equation

$$y + 4 = \frac{1}{3}(x + 2)$$

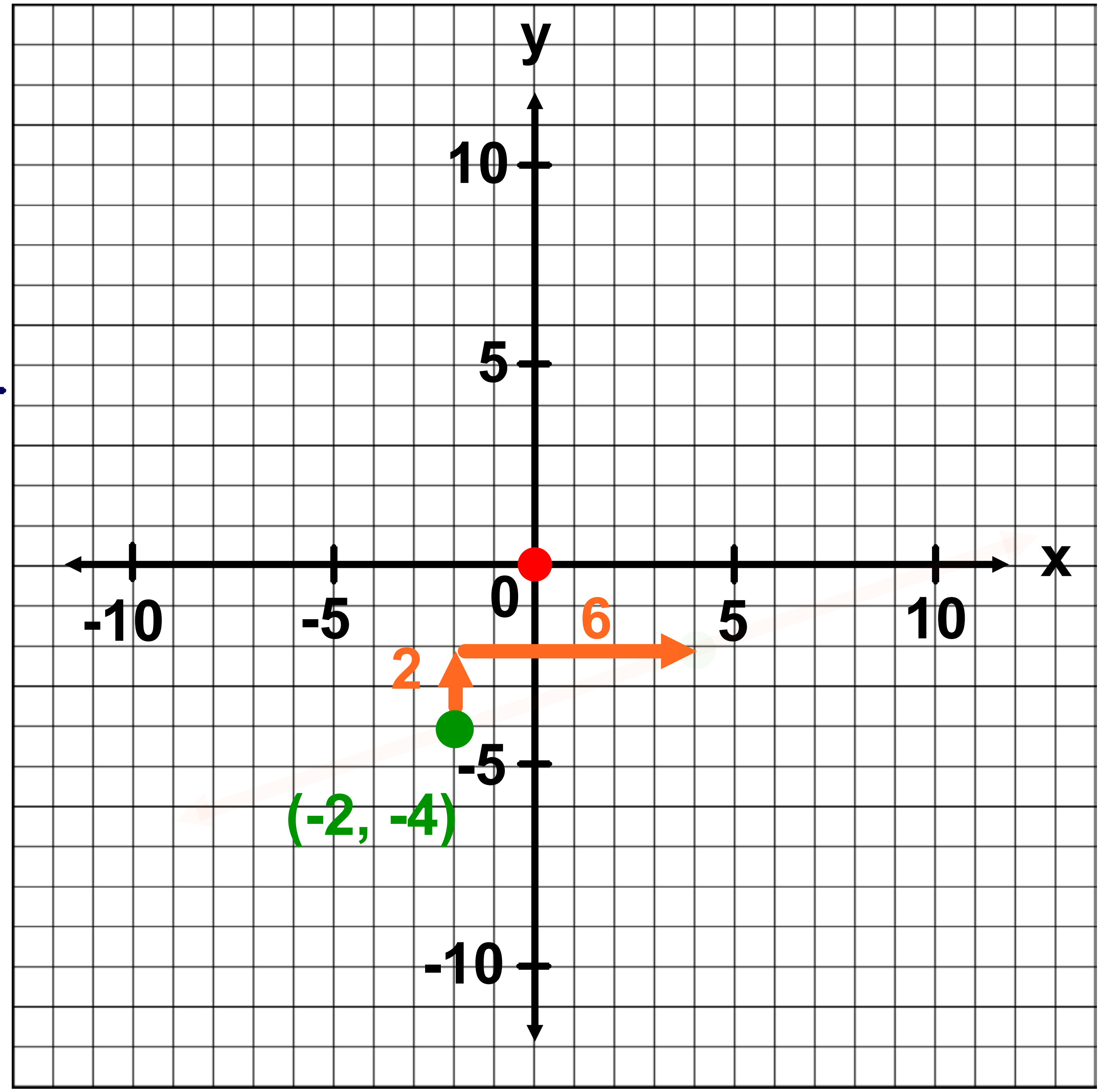
Determine the point on the line and the slope.

The point on the line is ________
*click*

The slope is ________
*click*

Graph the line representing the equation.

Click on the point to reveal it. Click slightly above it to show the slope. Click on the new point. Click in between to reveal the line.



46 What is the slope of $y - 3 = 4(x + 2)$?

Answer

47 Which point is on the line $y - 3 = 4(x + 2)$?

A (-3, 2)

B (3, -2)

C (2, -3)

D (-2, 3)

Answer

48 What is the slope and a point on the line $y + 5 = -3(x - 4)$?

A $m = -3$; $(4, -5)$

B $m = -3$; $(-4, 5)$

C $m = 3$; $(4, -5)$

D $m = 3$; $(-4, 5)$

Answer

49 Which is the slope and a point on the line

$y - 1 = \frac{1}{3}(x)$?

A $m = \frac{1}{3}$ ; (-1, 0)

B $m = -\frac{1}{3}$ ; (0, -1)

C $m = \frac{1}{3}$ ; (0, 1)

D m is undefined; (0, 1)

Answer

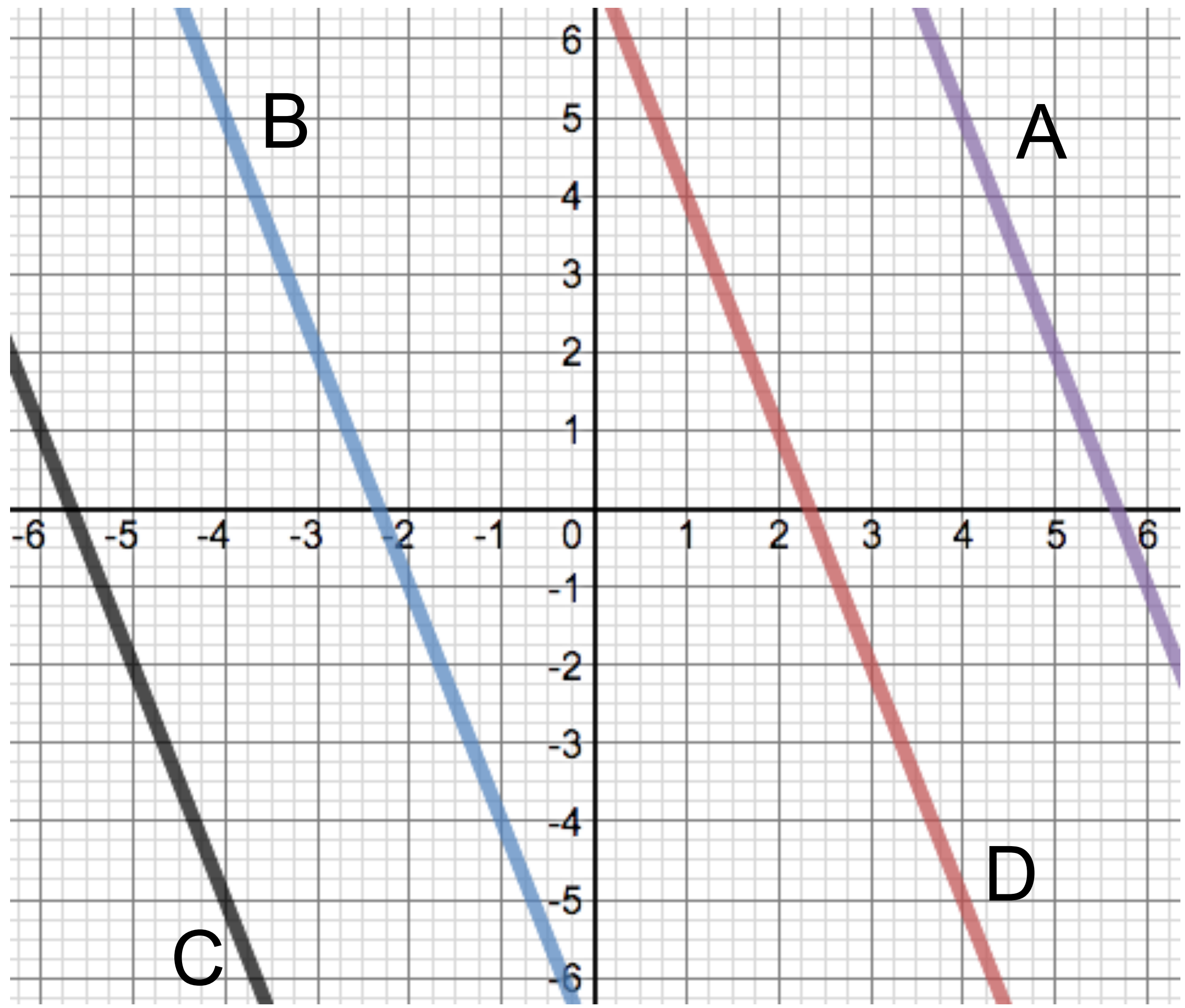
50 Which line represents $y + 5 = -3(x - 4)$?

A line A

B line B

C line C

D line D




Answer

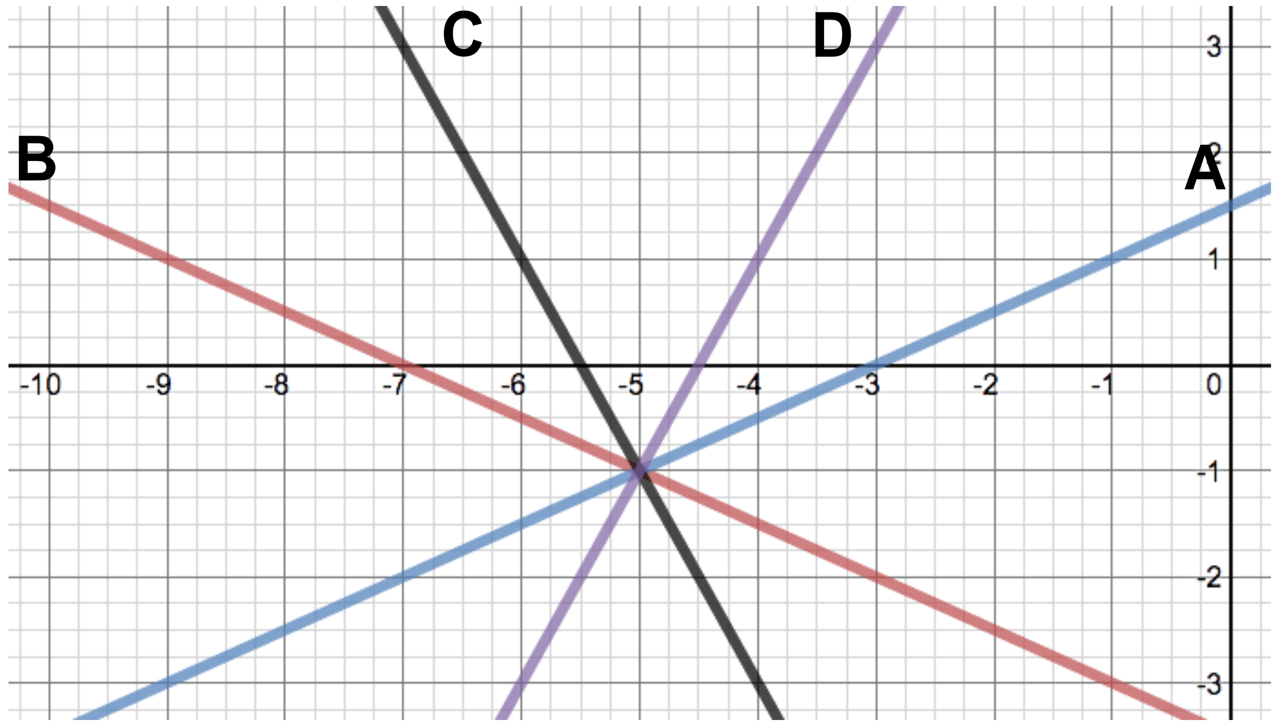
51 Which line represents $y + 1 = -\frac{1}{2}(x + 5)$?

A line A

B line B

C line C

D line D




Answer

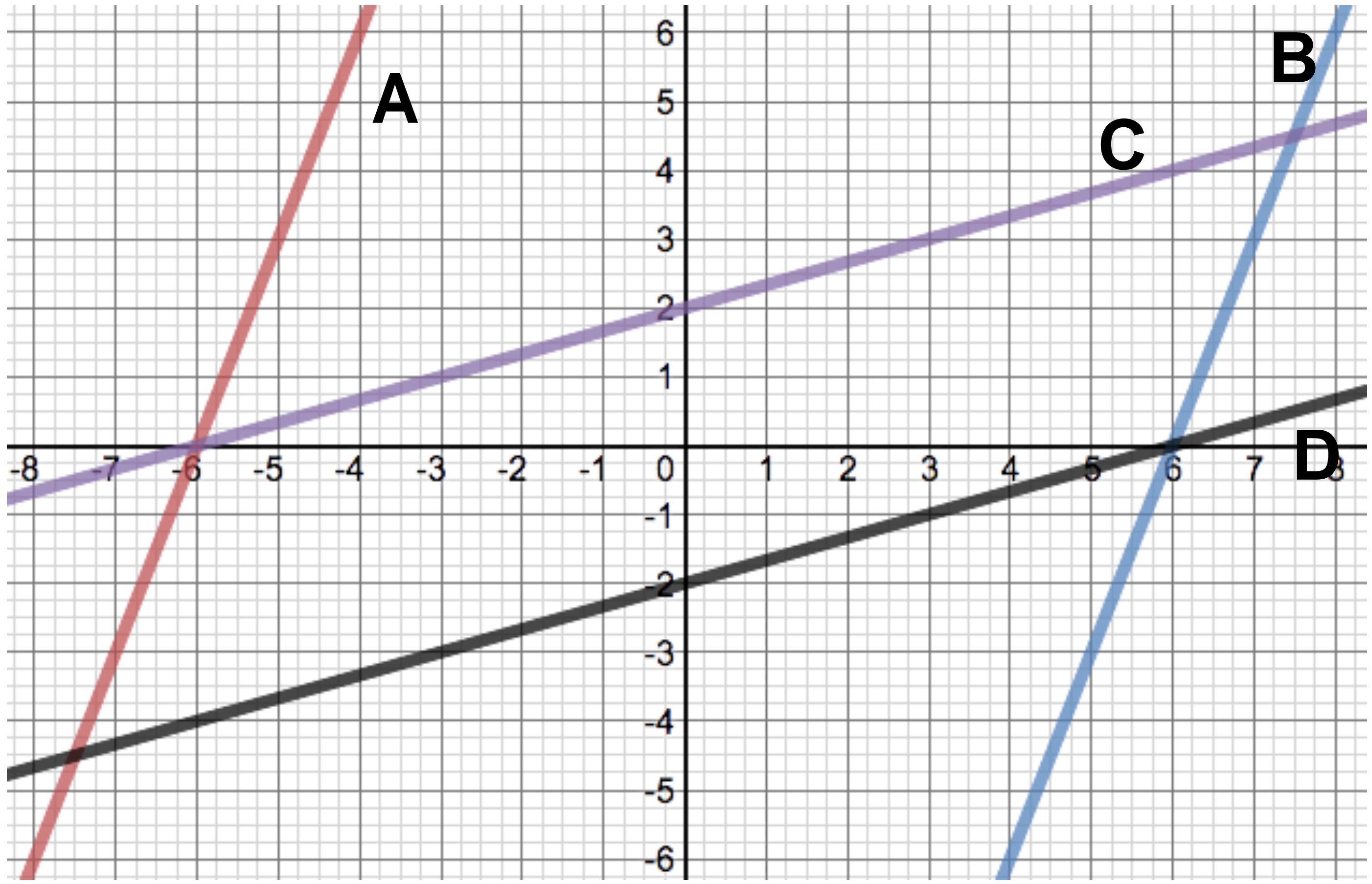
52 Which line represents y - 6 = 3(x + 4)?

A line A

B line B

C line C

D line D




Answer

# Point-Slope Form

$$y - y_1 = m(x - x_1)$$

You can determine and graph equations in point-slope form even when you are given limited information.

For example, if you are given the slope (m) and any point ($x_1$, $y_1$), then by substituting the point into the equation for $x_1$ and $y_1$ & the slope for m, you have the equation of the line with no additional work required.

# Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Or, if you are given any two points, it's always possible to determine the slope, m.

By then substituting one of those points in for $x_1$ and $y_1$, write the equation of a line from two points.

Let's clarify the steps required by doing some examples for both cases.

# Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Example:

Write the equation of a line in point-slope form that has a slope of 7 and passes through the point (-9, 3).

We already know that the slope is 7, or m = 7.

We also know that a point that the line passes through is (-9, 3), which represent ($x_1$, $y_1$) respectfully.

By substituting these numbers into our equation, we have:

$y - 3 = 7(x - (-9))$

$y - 3 = 7(x + 9)$, which is our equation in point-slope form

# Point-Slope Form

Example:

$$y - y_1 = m(x - x_1)$$

Write the equation of a line in point-slope form that passes through the points (5, -9) and (3, 0).

This time, we do not know the slope, so we need to calculate it.

$$m = \frac{0 - (-9)}{3 - 5} = \frac{9}{-2} = -\frac{9}{2}$$

We also know that a point that the line passes through is (5, -9), which represent $(x_1, y_1)$ respectfully.

By substituting these numbers into our equation, we have:

$$y - (-9) = -\frac{9}{2}(x - 5) \text{ which simplifies to } y + 9 = -\frac{9}{2}(x - 5)$$

# Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Example:

Write the equation of a line in point-slope form that passes through the points (5, -9) and (3, 0).

You might be asking if the equation on the previous slide is the only answer. Actually, you can also write the equation in point-slope form using the other point (3, 0) as $(x_1, y_1)$. Therefore the equation could also be:

$$y - 0 = -\frac{9}{2}(x - 3) \text{ which simplifies to } y = -\frac{9}{2}(x - 3)$$

# Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Example:

Write the equation of a line in point-slope form that passes through the points (5, -9) and (3, 0).

The reason that we have two possible answers is because they would both simplify into the same equation, in standard form. Below is the work for each case.

$$y + 9 = -\frac{9}{2}(x - 5)$$

$$2y + 18 = -9(x - 5)$$
$$2y + 18 = -9x + 45$$
$$9x + 2y + 18 = 45$$
$$9x + 2y = 27$$

$$y = -\frac{9}{2}(x - 3)$$

$$2y = -9(x - 3)$$
$$2y = -9x + 27$$
$$9x + 2y = 27$$

53 What is the equation of the line in point-slope form if its slope is -2 and it passes through the point (9, -7).

A $y - 7 = -2(x - 9)$

B $y + 7 = -2(x + 9)$

C $y + 7 = -2(x - 9)$

D $y - 7 = -2(x + 9)$

Answer

54 What is the equation of the line in point-slope form if its slope is 5 and it passes through the point (-1, -4).

A $y - 4 = 5(x - 1)$

B $y + 4 = 5(x + 1)$

C $y + 4 = 5(x - 1)$

D $y - 4 = 5(x + 1)$

Answer

55 What is the equation of the line in point-slope form if its slope is $-\frac{4}{9}$ and it passes through the point (3, 8).

A $y - 8 = -\frac{4}{9}(x - 3)$

B $y + 8 = -\frac{4}{9}(x + 3)$

C $y + 8 = -\frac{4}{9}(x - 3)$

D $y - 8 = -\frac{4}{9}(x + 3)$

Answer

56 What is the equation of the line in point-slope form that passes through the points (3, 8) and (-2, -2).

A $y + 8 = 2(x + 3)$

B $y - 8 = 2(x - 3)$

C $y + 2 = \frac{1}{2}(x + 2)$

D $y - 2 = \frac{1}{2}(x - 2)$

Answer

57 What is the equation of the line in point-slope form that passes through the points (7, 9) and (5, -1).

A $y - 9 = 5(x - 7)$

B $y - 1 = 5(x - 5)$

C $y + 1 = \frac{1}{5}(x - 5)$

D $y - 9 = \frac{1}{5}(x - 7)$

Answer

58 What is the equation of the line in point-slope form that passes through the points (4, -7) and (-2, -3).

A $y - 7 = -\frac{3}{2}(x + 4)$

B $y - 3 = -\frac{3}{2}(x - 2)$

C $y + 3 = -\frac{2}{3}(x + 2)$

D $y - 7 = -\frac{2}{3}(x - 4)$

Answer

# Slope-Intercept Form

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# Slope-Intercept Form

Another very useful form of the equation of a line is the slope-intercept form.

It's based on the use of the slope and the y-intercept of a line.

Similar to point-slope, we are going to start by showing a derivation proof of slope-intercept form.

# Slope-Intercept Form

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Let's start with the definition of slope, in which two points on a line are given by the ordered pairs

$$(x_1,y_1) \text{ and } (x_2, y_2)$$

As a first step, let's just name the coordinates for the second point $(x,y)$ rather than $(x_2, y_2)$.

That will be true for any point on the line, not just one point, and will allow us to write an equation for all points on the line.

Then our slope formula becomes:

$$m = \frac{y - y_1}{x - x_1}$$

# Slope-Intercept Form

$$m = \frac{y - y_1}{x - x_1}$$

Now let's solve that equation for y using what we've learned about solving equations.

Try it yourself, before we show you our answer.

(Hint: Remember to treat the denominator $(x - x_1)$ like it's in parentheses.)

# Slope-Intercept Form

$$m = \frac{y - y_1}{x - x_1}$$

Multiply both sides by $(x - x_1)$ to get rid of the fraction.

$$m(x - x_1) = y - y_1$$

Now, add $y_1$ to both sides.

$$m(x - x_1) + y_1 = y$$

Now switch the sides

$$y = m(x - x_1) + y_1$$

This is fine, and allows us to graph a line given any point $(x_1, y_1)$.

But, one additional step is taken to get the most useful equation for a line.

# Slope-Intercept Form

$y = m(x - x_1) + y_1$

Use the y-intercept for $(x_1,y_1)$.

The y-intercept is the point where the line crosses the y-axis.

The coordinates of that point are (0, b).

- x is zero anywhere on the y-axis
- We name the y-intercept "b."

Then, this becomes:

$y = mx + b$

# Slope-Intercept Form

$$y = mx + b$$

**Slope-Intercept Form** is where:

- $m$ is the slope,
- $b$ is the y-intercept $(0, b)$
- $(x, y)$ is any of the infinite points that satisfy the equation.

# Slope-Intercept Form

$$y = mx + b$$

This is the form of the equation of a line that is most often used.

If you are provided a graph of a line, you can calculate m and read b directly from the graph.

That allows you to write the equation of the line directly.

Which you can then use to find any other needed points.

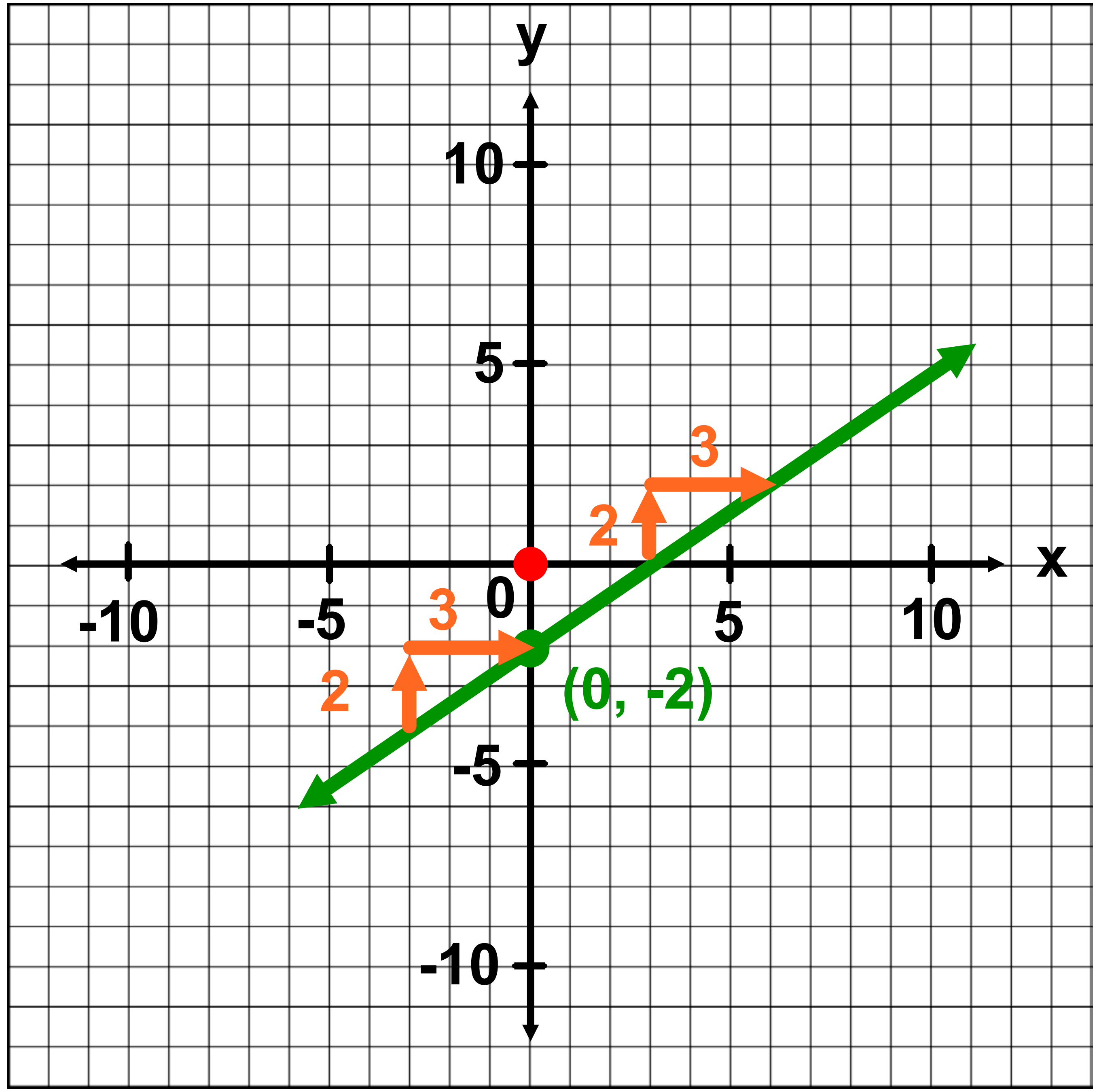
# Slope-Intercept Form

The line shown in the coordinate plane to the left has a y-intercept at the point (0, -2) and has a slope

of $\frac{2}{3}$. Therefore, the

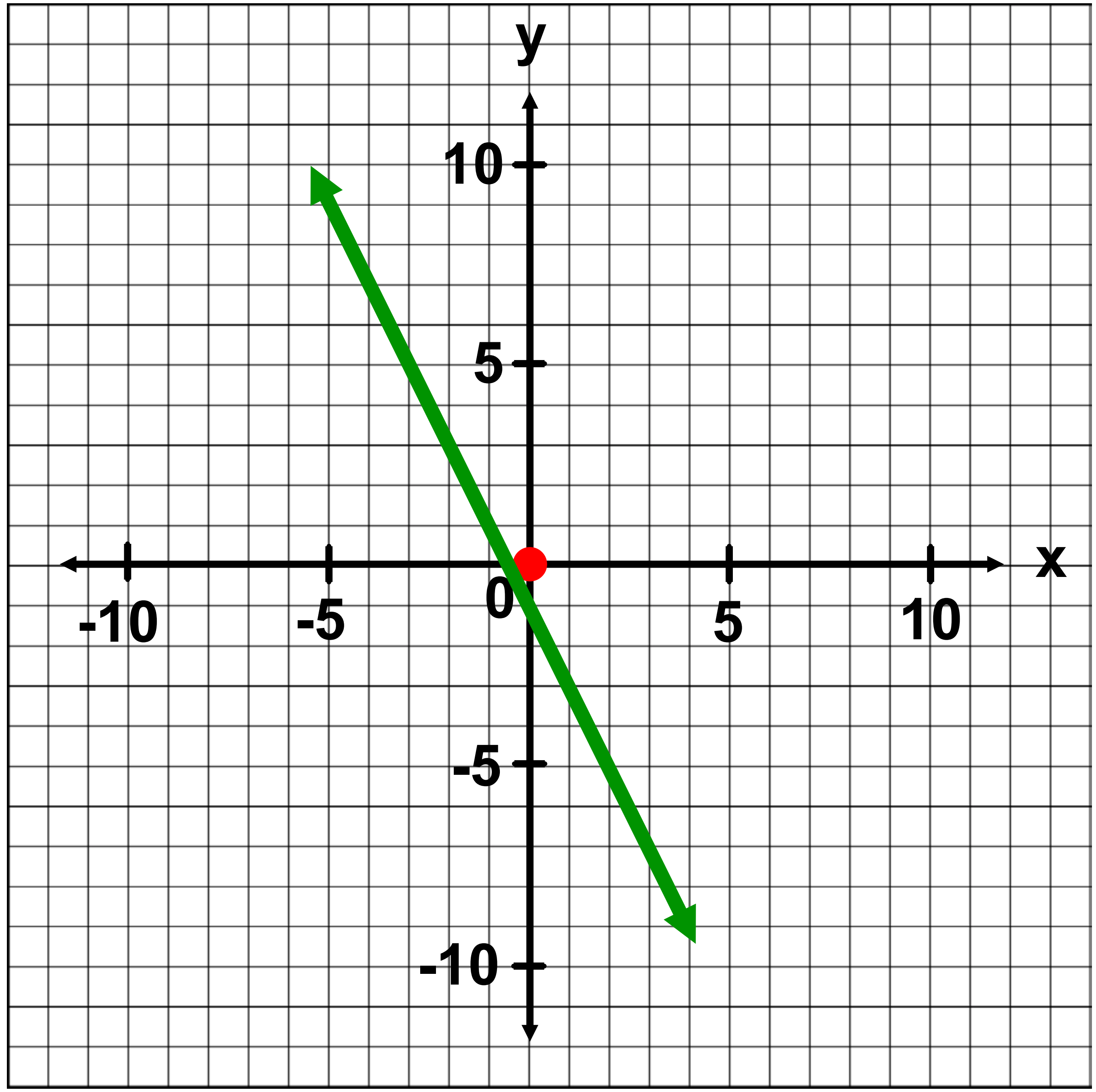
equation of this line is

$$y = \frac{2}{3}x - 2$$



# Slope-Intercept Form

Example: Write the equation for the line shown in the graph to the right.




The slope of the line (m) is

click

The y-intercept of the line (b) is

click

Therefore the equation of the line in slope intercept form is

click

# Slope-Intercept Form

Example: Write the equation for the line shown in the graph to the right.

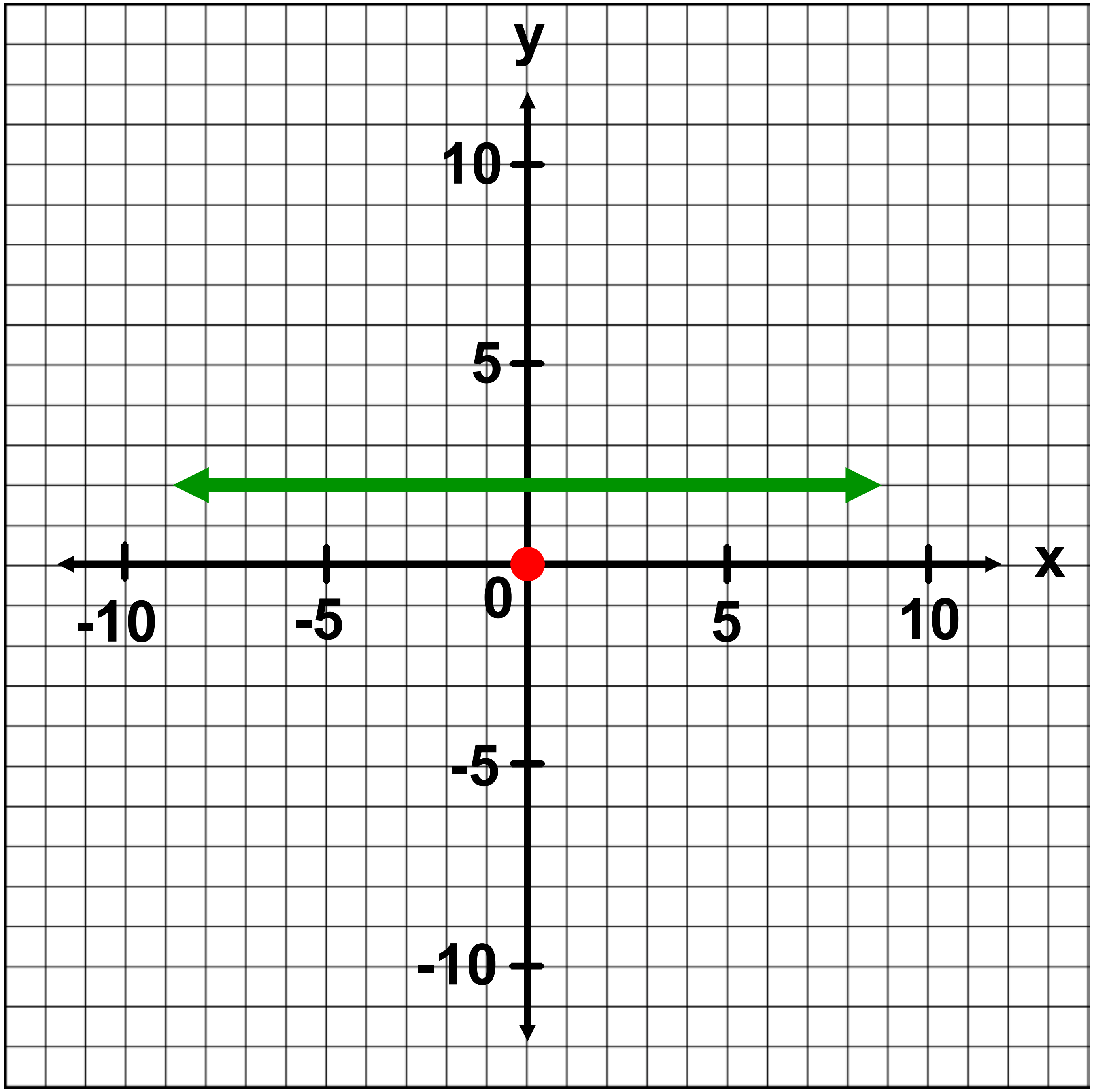
The slope of the line (m) is

______
*click*

The y-intercept of the line (b) is ______
*click*

Therefore the equation of the line in slope intercept form is

______
*click*



59 Which equation is that of the graphed line?

A $y = 3x - 1$

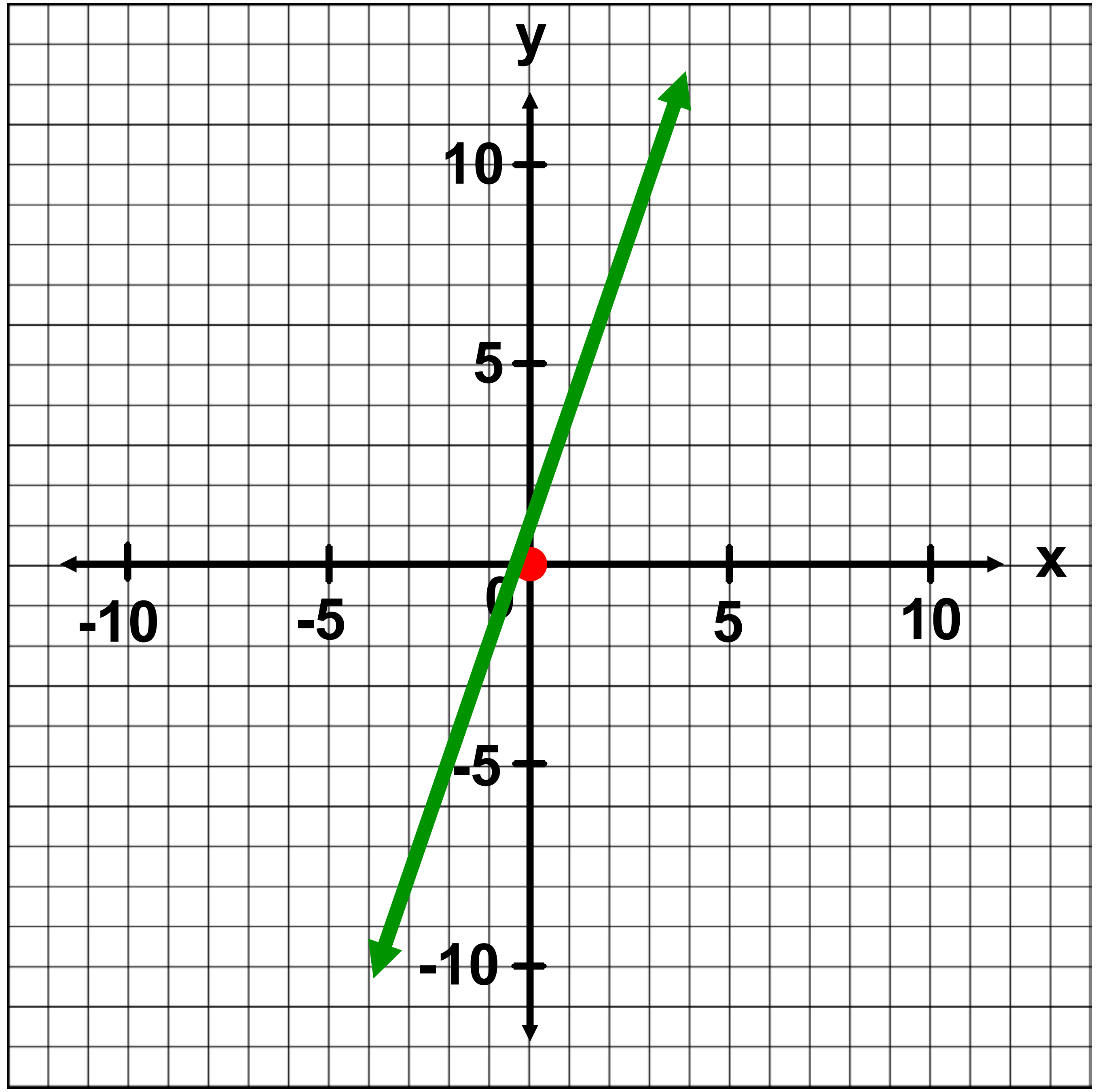
B $y = 3x$

C $y = 3x + 1$

D $y = \frac{1}{3}x - 1$

E $y = \frac{1}{3}x$

F $y = \frac{1}{3}x + 1$




Answer

60 Which equation is that of the graphed line?

A $y = 3x - 1$

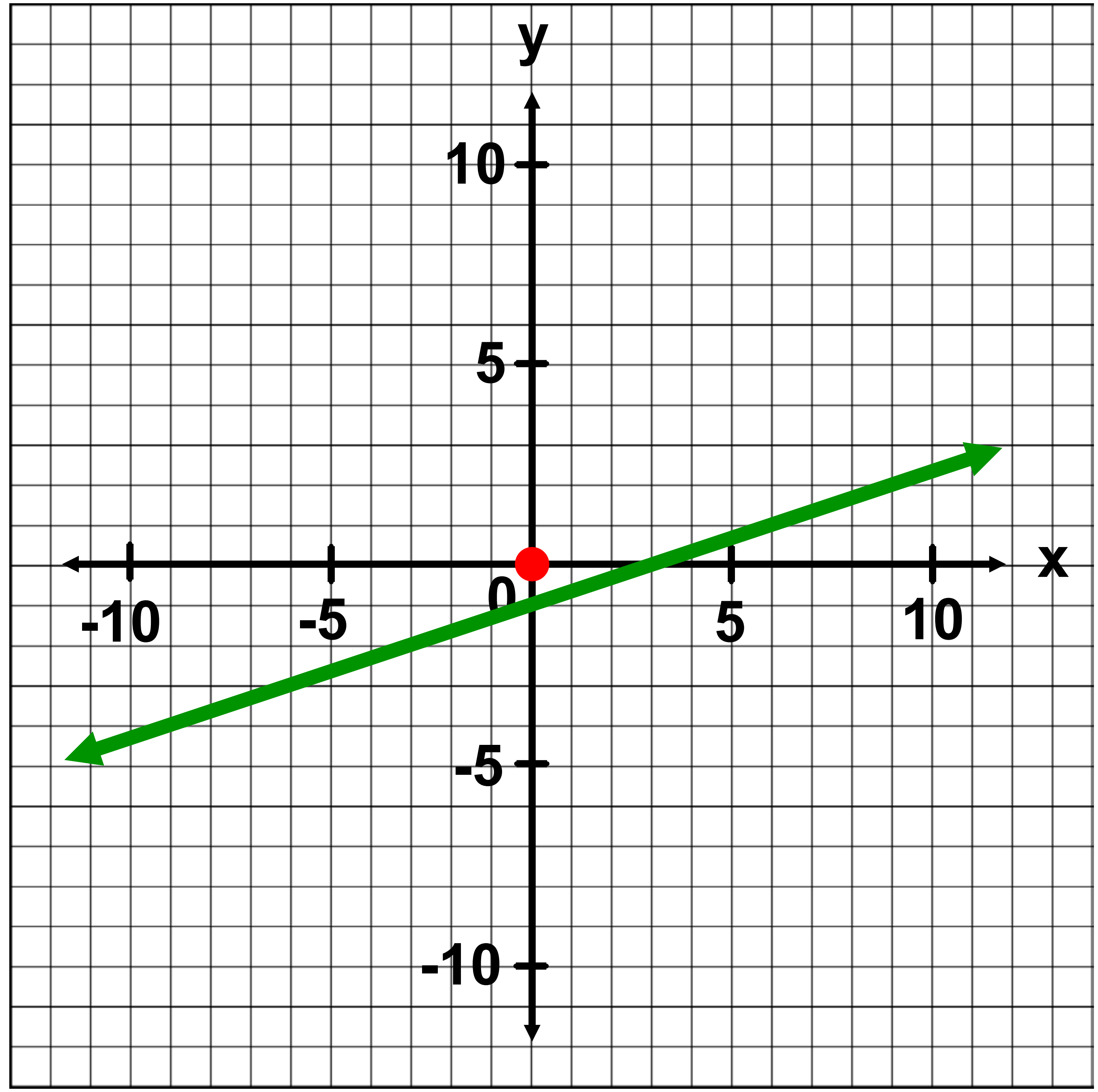
B $y = 3x$

C $y = 3x + 1$

D $y = \frac{1}{3}x - 1$

E $y = \frac{1}{3}x$

F $y = \frac{1}{3}x + 1$




Answer

61 Which equation is that of the graphed line?

A $y = 3x - 1$

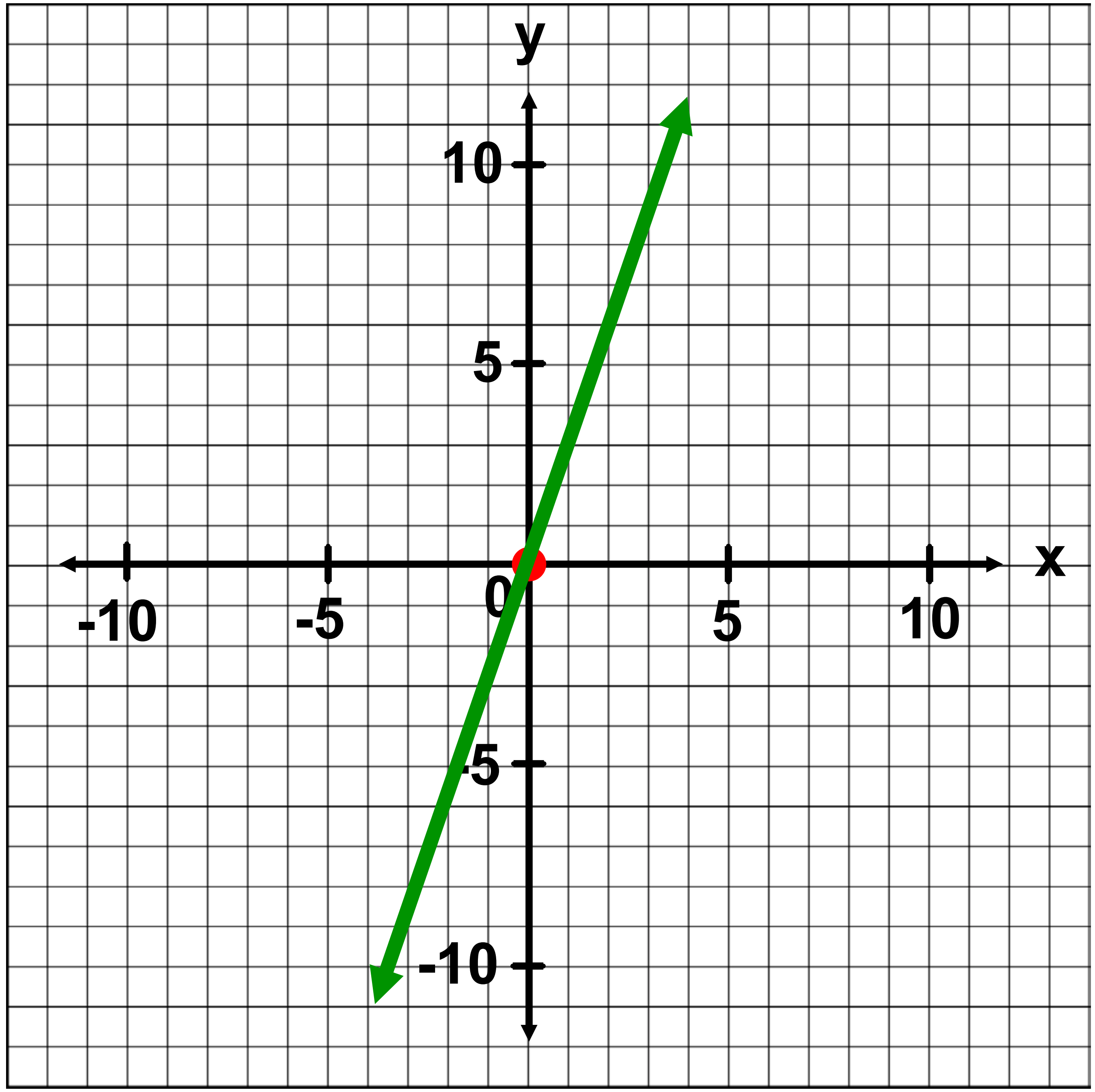
B $y = 3x$

C $y = 3x + 1$

D $y = \frac{1}{3}x - 1$

E $y = \frac{1}{3}x$

F $y = \frac{1}{3}x + 1$




Answer

62 What is the equation of this line?

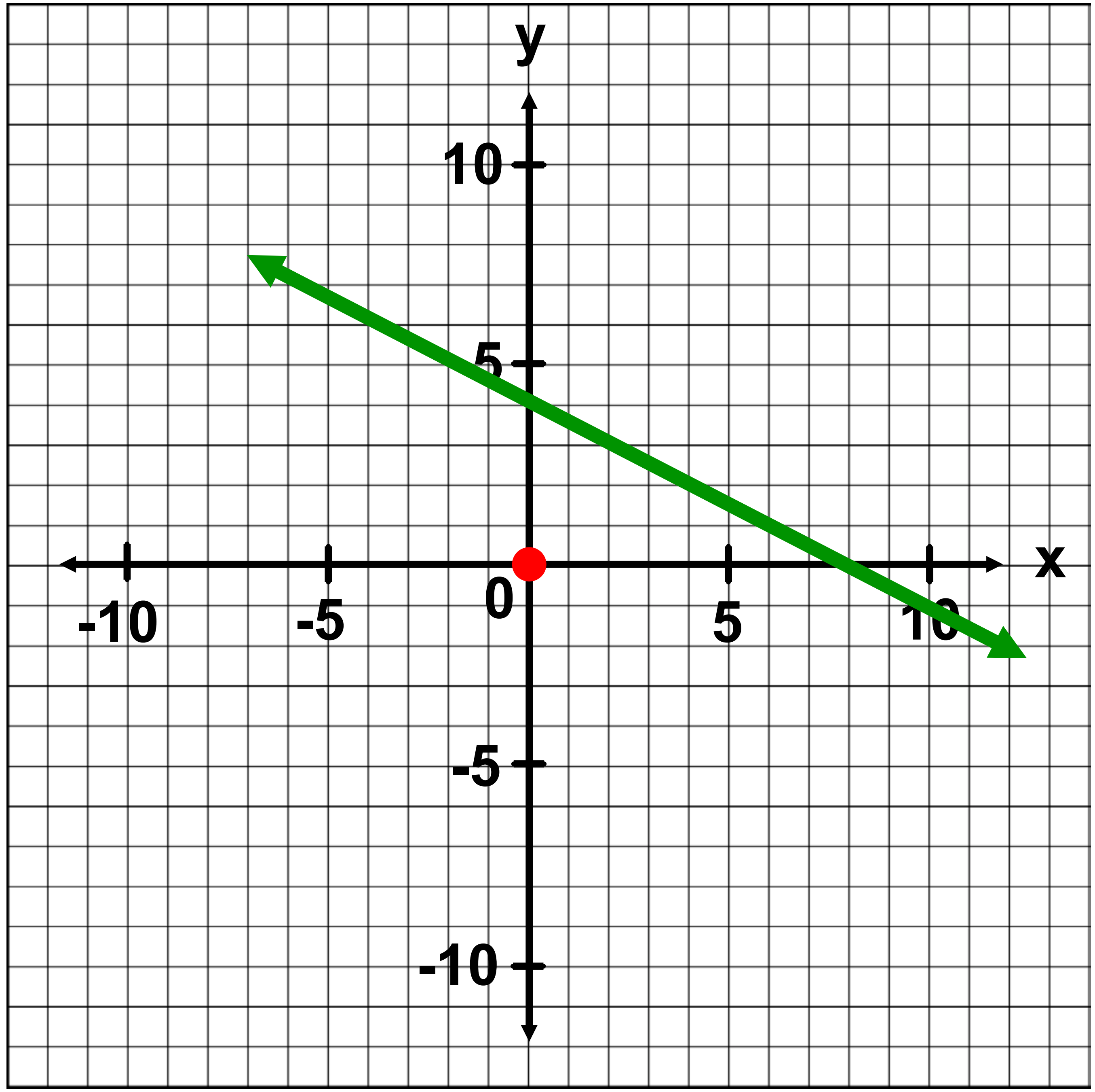
A $y = 4x - 2$

B $y = -\frac{1}{2}x - 2$

C $y = -\frac{1}{2}x + 4$

D $y = -2x + 4$

E $y = -2x + 8$




Answer

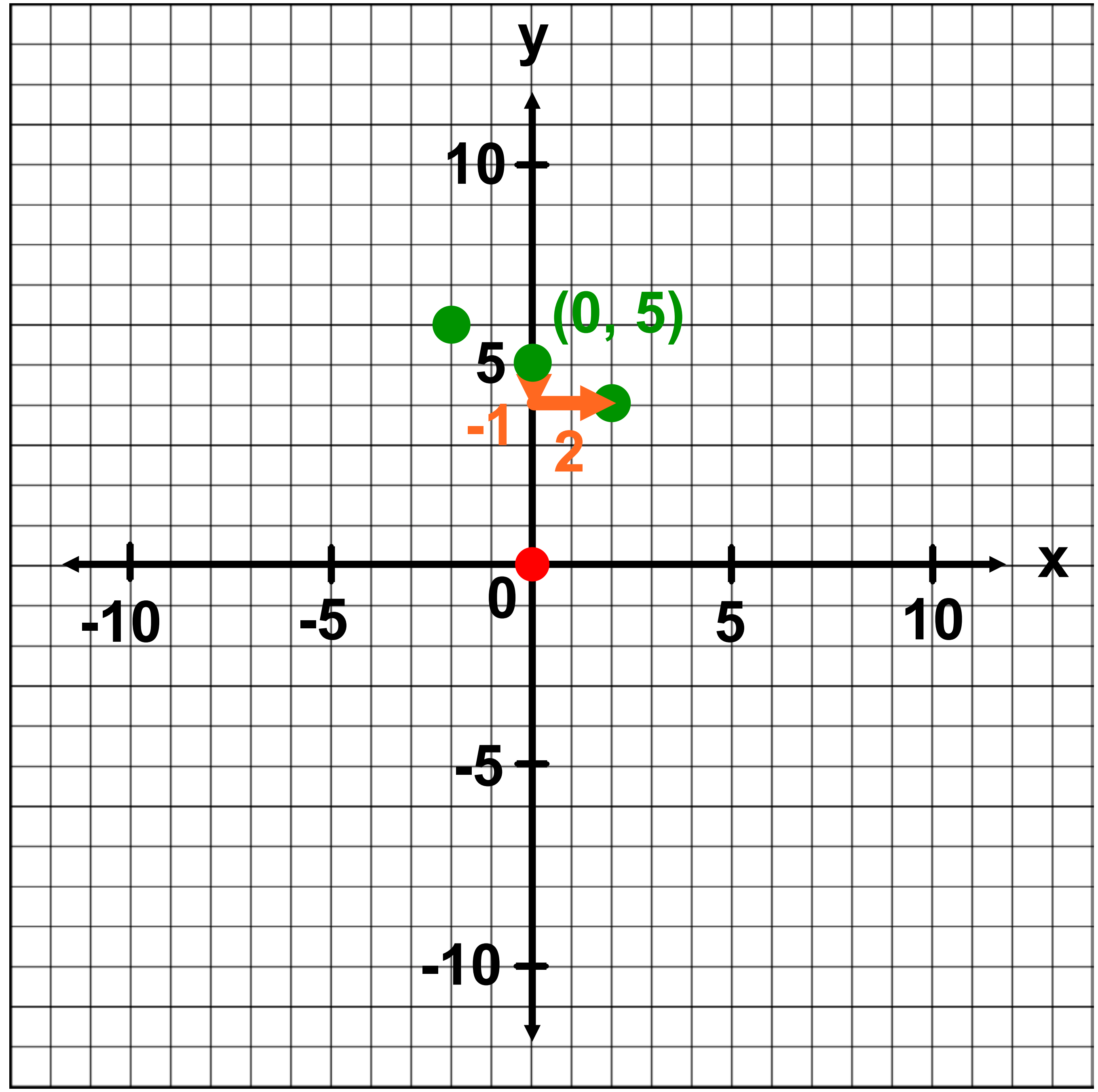
# Graphing a Line Using Slope-Intercept Form

Having the equation of a line in slope-intercept form also allows us to quickly graph a line, using the y-intercept and the slope.

For instance, if we know that the equation of the line is

$$y = -\frac{1}{2}x + 5$$

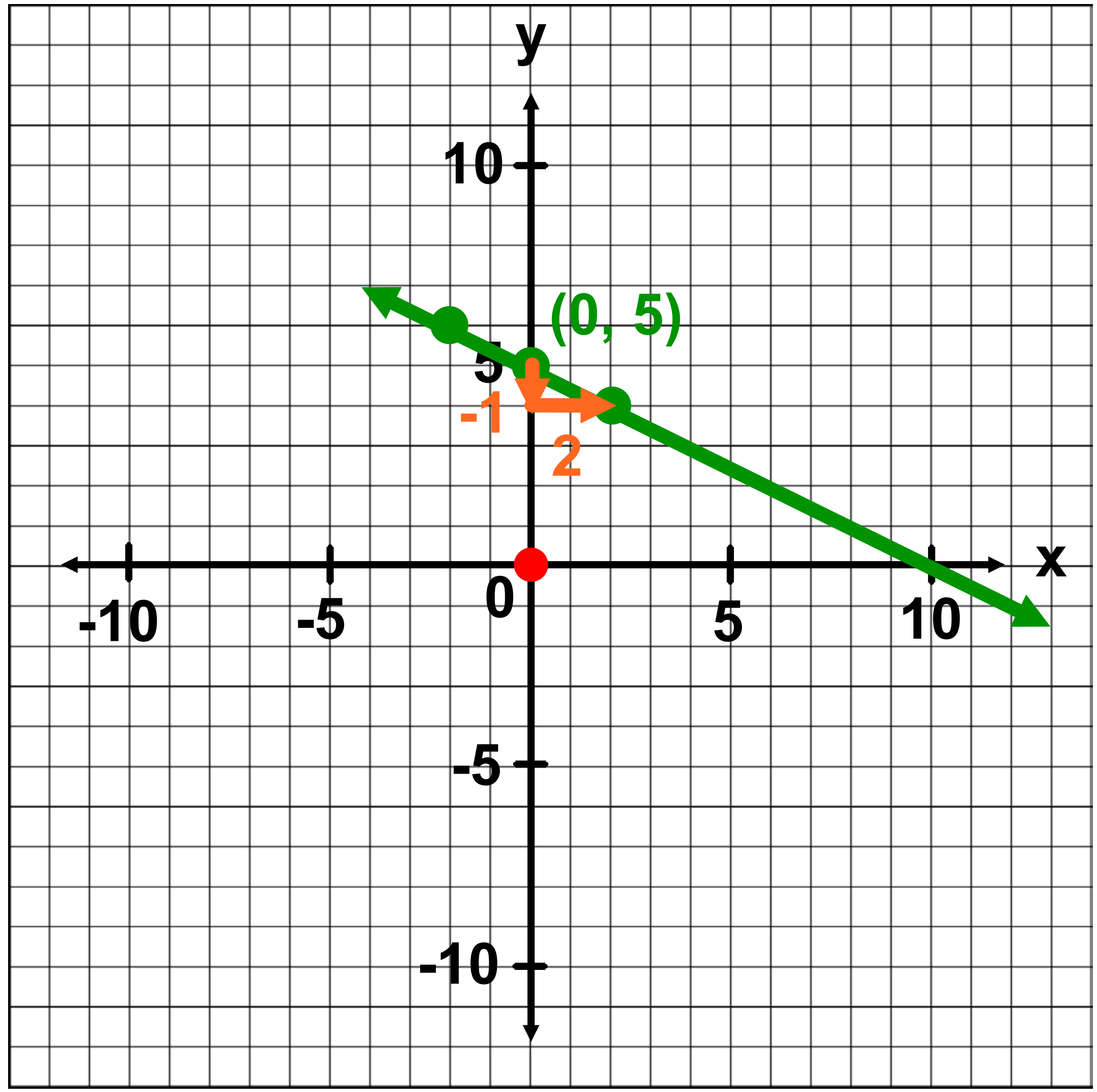
then we can graph the y-intercept (0, 5) and use the slope of $-\frac{1}{2}$ to count the number of spaces, down 1 unit and right 2 units (or up one and left 2 units), to get an additional point on the line.



# Graphing a Line Using Slope-Intercept Form

Then we draw the line through any two of those points.

This method is the easiest to use if you just have to draw a line given the y-intercept and slope.



63 What is the slope of the linear equation

$$y = -\frac{3}{2}x - 5?$$

A $5$

B $-5$

C $\frac{3}{2}$

D $-\frac{3}{2}$

Answer

64 What is the y-intercept of the linear equation

$$y = -\frac{3}{2}x - 5?$$

A $(0, 5)$

B $(0, -5)$

C $(0, \frac{3}{2})$

D $(0, -\frac{3}{2})$

Answer

65 Which line in the graph below represents the linear equation

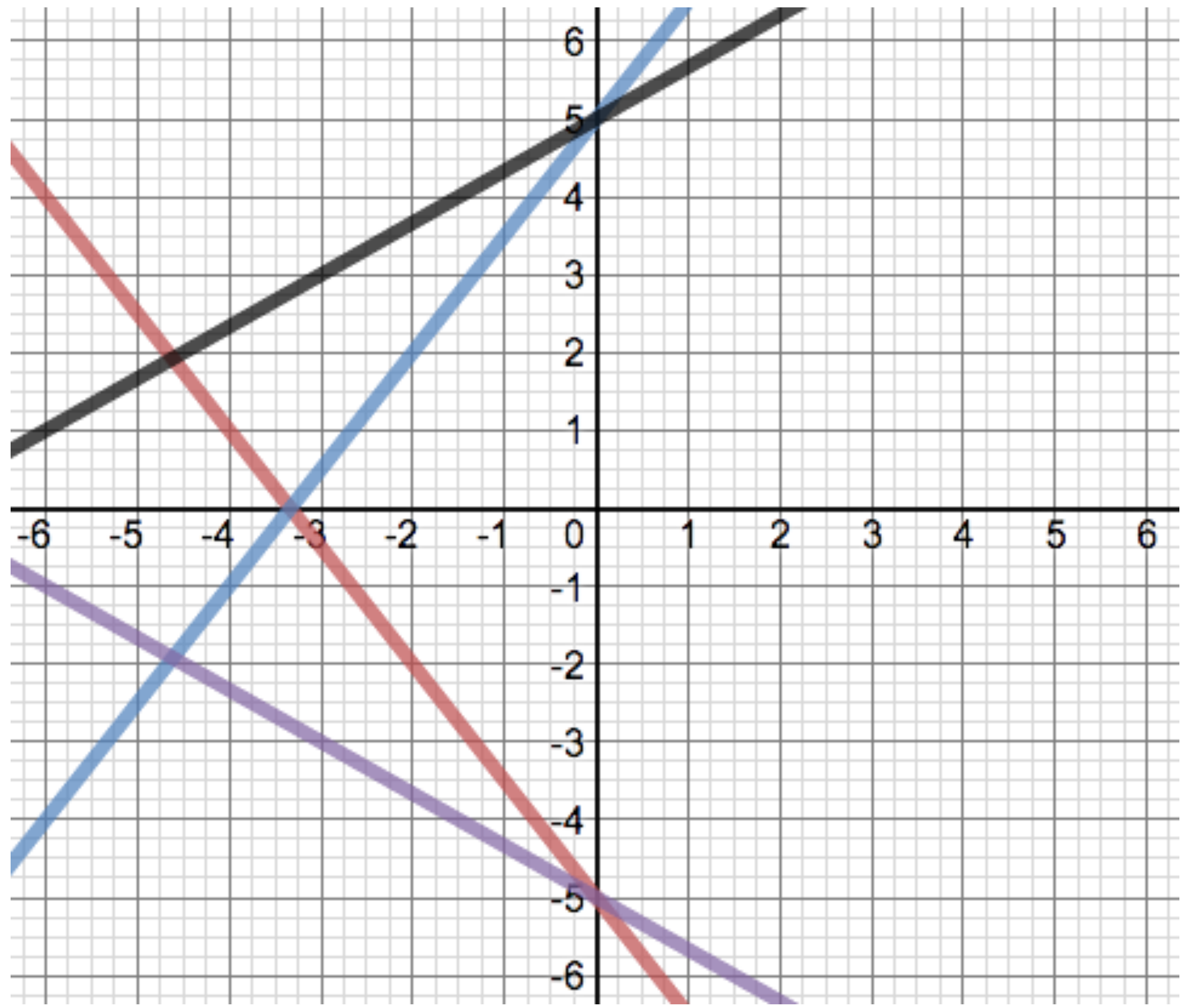
$$y = -\frac{3}{2}x - 5?$$

A Red

B Blue

C Purple

D Black




Answer

66 What is the slope of the linear equation

$$y = \frac{2}{3}x + 5?$$

A 5

B -5

C $\frac{2}{3}$

D $-\frac{2}{3}$

Answer

67 What is the y-intercept of the linear equation

$y = \frac{2}{3}x + 5$?

A (0, 5)

B (0, -5)

C $(0, \frac{2}{3})$

D $(0, -\frac{2}{3})$

Answer

68 Which line in the graph below represents the linear equation

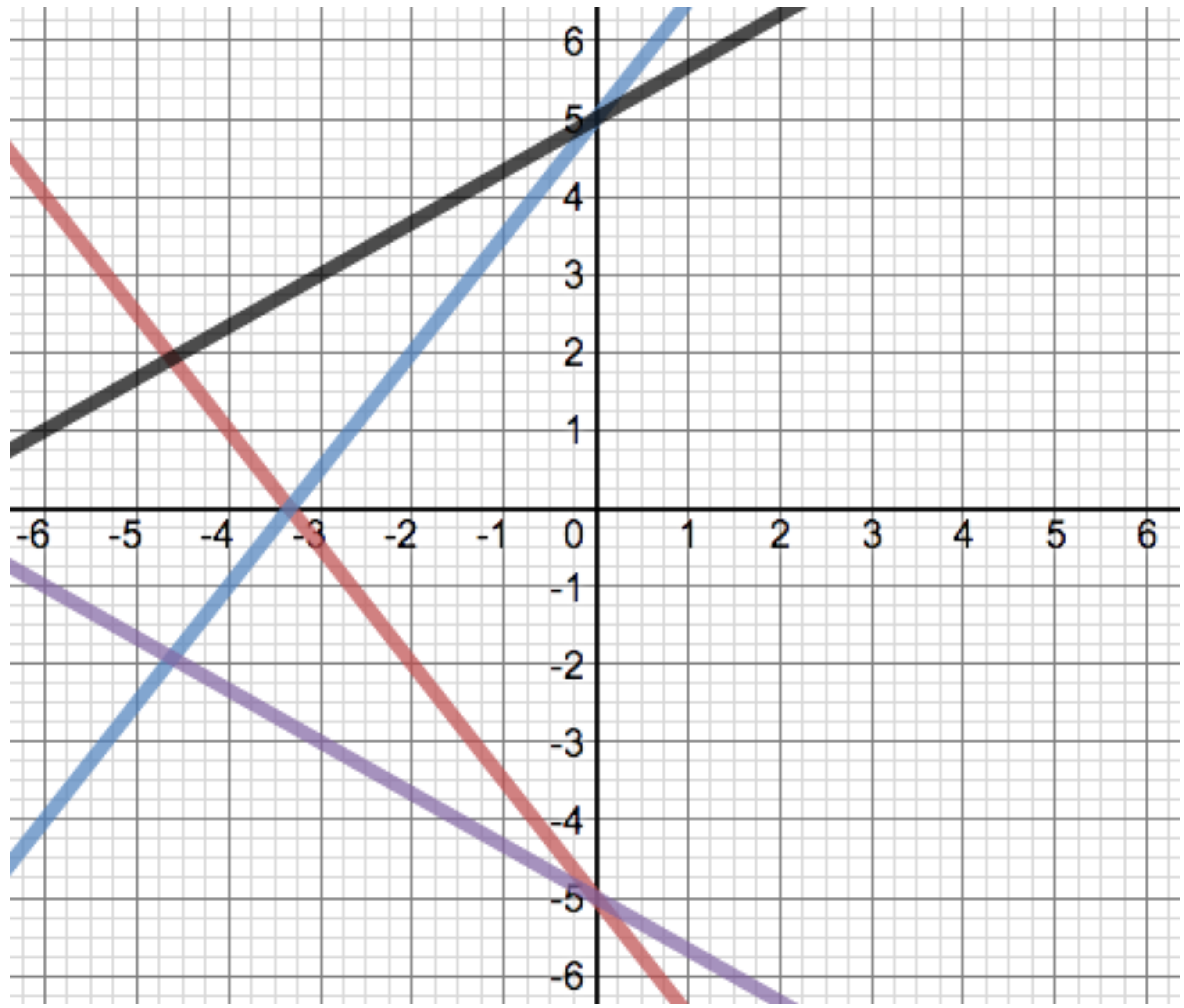
$$y = \frac{2}{3}x + 5?$$

A Red

B Blue

C Purple

D Black




Answer

# Slope-Intercept Form

$$y = mx + b$$

You can also determine and graph equations in slope-intercept form even when you are given limited information.

For example, if you are given the slope (m) and any point (x, y), then by substituting the point into the equation for x and y, you can solve for b.

Then, you can write the equation of a line when given the slope and a point that the line passes through.

# Slope-Intercept Form

$$y = mx + b$$

Or, if you are given any two points, it's always possible to determine the slope, m.

By then substituting one of those points in for x and y, you can solve for b.

Then, you can write the equation of a line from two points.

Let's clarify the steps required by doing some examples for both cases.

# Slope-Intercept Form

$$y = mx + b$$

Example:

Write the equation of a line in slope-intercept form that has a slope of 8 and passes through the point (3, 10).

We already know that the slope is 8, or $m = 8$.

We also know that a point that the line passes through is (3, 10), which represent (x, y) respectfully.

By substituting these numbers into our equation, we have: $10 = 8(3) + b$

# Slope-Intercept Form

$$y = mx + b$$

Now, we can solve for b.

$10 = 8(3) + b$

Multiply 8 and 3

$10 = 24 + b$

Subtract 24 from both sides

$-14 = b$

Switch the order

$b = -14$

Our equation is then $y = 8x - 14$.

# Slope-Intercept Form

Example: $y = mx + b$

Write the equation of a line in slope-intercept form that has a slope of 8 and passes through the point (3, 10).

This problem can also be solved using point-slope and using the rules of algebra to get it into slope-intercept form. We have our point (3, 10) as ($x_1$, $y_1$) and our slope of 8. If I substitute the numbers into point-slope, we will get this equation:

$$y - 10 = 8(x - 3)$$

Distribute the 8 to x and -3

$$y - 10 = 8x - 24$$

Addition of 10 to both sides

$$y = 8x - 14$$

# Slope-Intercept Form

$$y = mx + b$$

Example:

Write the equation of a line in slope-intercept form that passes through the points (4, 0) and (6, 5).

This time, we do not know the slope, so we need to calculate it.

$$m = \frac{5 - 0}{6 - 4} = \frac{5}{2}$$

We also know that a point that the line passes through is (4, 0), which represent (x, y) respectfully.

By substituting these numbers into our equation, we have:

$$0 = \frac{5}{2}(4) + b$$

# Slope-Intercept Form

$$y = mx + b$$

Now, we can solve for b.

$0 = \frac{5}{2}(4) + b$

Multiply $\frac{5}{2}$ and 4

$0 = 10 + b$

Subtract 10 from both sides

$-10 = b$

Switch the order

$b = -10$

Our equation is then $y = \frac{5}{2}x - 10$.

# Slope-Intercept Form

$$y = mx + b$$

Example:

Write the equation of a line in slope-intercept form that passes through the points (4, 0) and (6, 5).

This problem can also be solved using point-slope and using the rules of algebra to get it into slope-intercept form. We can select one of our points as $(x_1, y_1)$ and our slope that we calculated was $\frac{5}{2}$. Let's use (4, 0). If we substitute the numbers into point-slope, we will get this equation:

$$y - 0 = \frac{5}{2}(x - 4)$$

Distribute the slope & drop the 0

$$y = \frac{5}{2}x - 10$$

69 What is the equation of the line that passes through the point (-3, -7) and has a slope of 4?

A $y = 4x + 25$

B $y = 4x + 5$

C $y = -3x + 5$

D $y = 7x - 17$

Answer

70 What is the equation of the line that passes through the point (6, -2) and has a slope of $\frac{5}{6}$?

A $y = \frac{5}{6}x - 7$

B $y = \frac{5}{6}x + 7.67$

C $y = 6x - 7$

D $y = -2x + 7.67$

Answer

71 What is the equation of the line that passes through the point (-1, -8) and has a slope of $-\frac{1}{2}$?

A $y = -\frac{1}{2}x - 8.5$

B $y = -\frac{1}{2}x + 4$

C $y = -\frac{1}{2}x - 5$

D $y = -\frac{1}{2}x - 7.5$

Answer

72 What is the equation of the line that passes through the points (-1, -8) and (2, 1)?

A $y = \frac{1}{3}x - \frac{1}{3}$

B $y = \frac{1}{3}x + \frac{5}{3}$

C $y = 3x - 5$

D $y = 3x - 1$

Answer

73 What is the equation of the line that passes through the points (7, 0) and (3, 2)?

A $y = -\frac{1}{2}x + \frac{7}{2}$

B $y = -\frac{1}{2}x + 7$

C $y = -2x + 7$

D $y = -2x + 8$

Answer

74 What is the equation of the line that passes through the points (-3, -5) and (3, 2)?

A $y = \frac{6}{7}x + \frac{17}{7}$

B $y = \frac{6}{7}x - \frac{4}{7}$

C $y = \frac{7}{6}x - 8.5$

D $y = \frac{7}{6}x - 1.5$

Answer

# Lab: Marble Masters

Students use a linear equation, slope, and X-Y intercept to aim a marble launch tube so the marble will cross a specified set of Cartesian coordinates and hit the "target"!

raft
resource area for teaching

# Proportional Relationships

Lab: Proportional Relationships

Math Practice

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# Proportional Relationships

Complete the items below each table.

### Family Z

| Time (hr.) | Distance (mi.) from home |
|---|---|
| 0 | 0 |
| 3 | 210 |
| 5 | 350 |

Slope (m) = ____________
*click*

y-intercept (b) = ____________
*click*

equation ____________
*click*

### Family A

| Time (hr.) | Distance (mi.) from home |
|---|---|
| 0 | 10 |
| 3 | 220 |
| 5 | 360 |

Slope (m) = ____________
*click*

y-intercept (b) = ____________
*click*

equation ____________
*click*

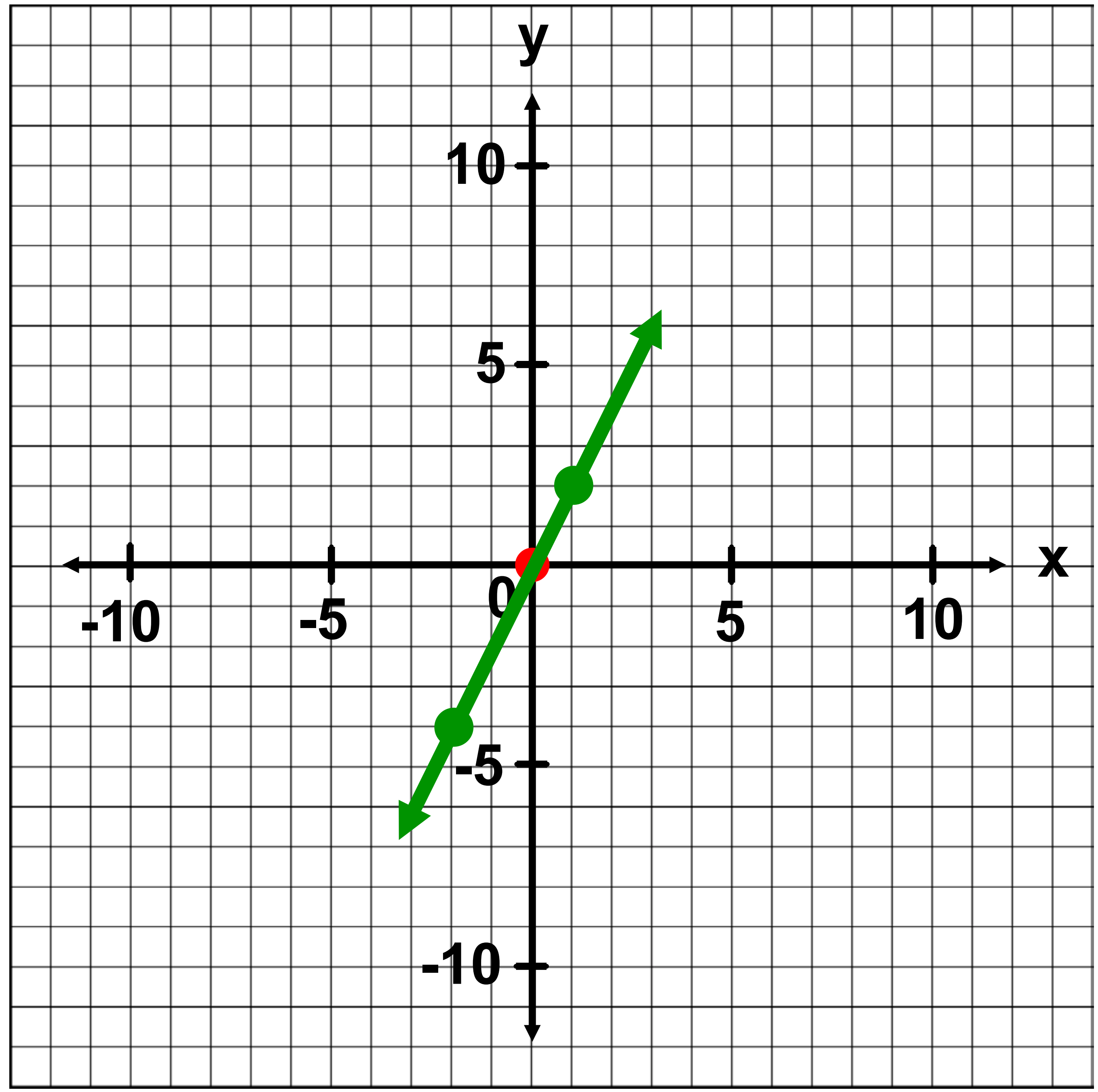
If this data from both tables were graphed on the same coordinate plane, what would you notice?

# Proportional Relationships

A **Proportional Relationship** is a relationship in which the ratios of related terms are equal.

Linear equations that have a proportional relationship are known as **direct variation** equations, which take the form y = mx.

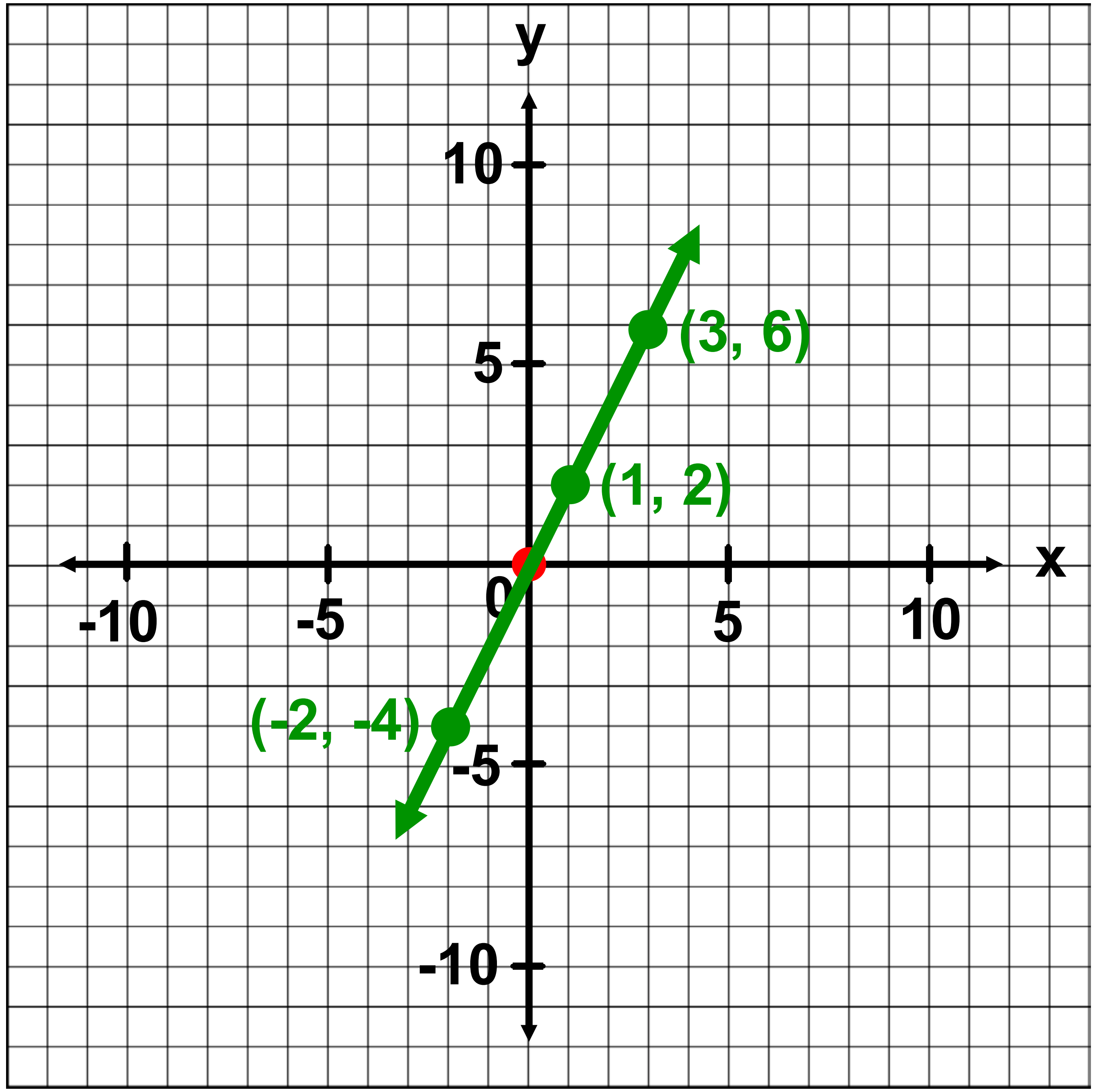
The graph to the right is the graph of y = 2x



# Proportional Relationships

The proportional relationship is expressed in the slope and the ratio of every y-coordinate to every x-coordinate with the exception of the origin.

Why is the proportional relationship not shown with the origin?




Answer

# Proportional Relationships

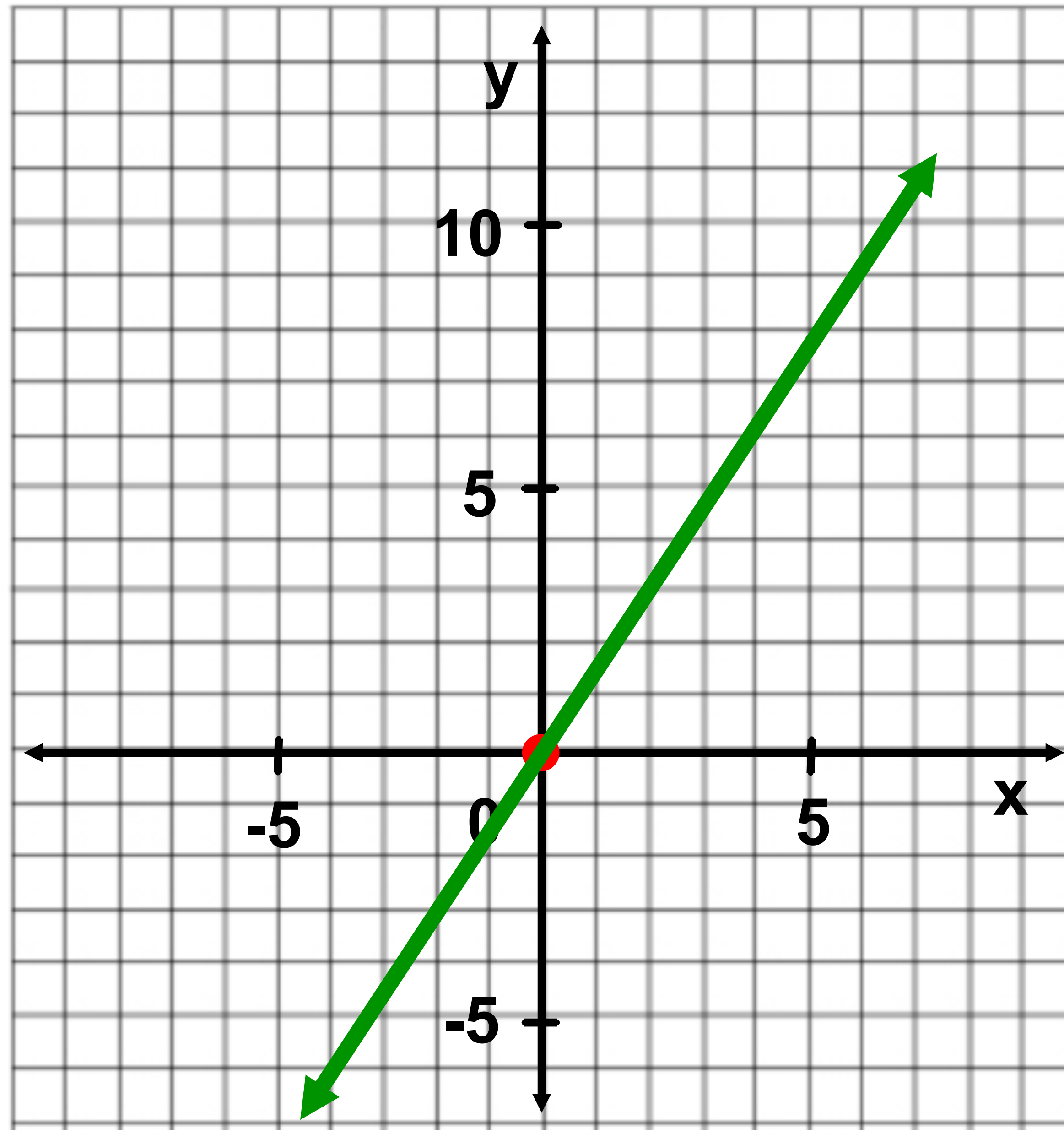
Example:

What is the proportional relationship shown in the graph?

*click to reveal*

What is the equation of the line?

*click to reveal*




Math Practice

# Proportional Relationships

Example:

What is the proportional relationship shown in the table?

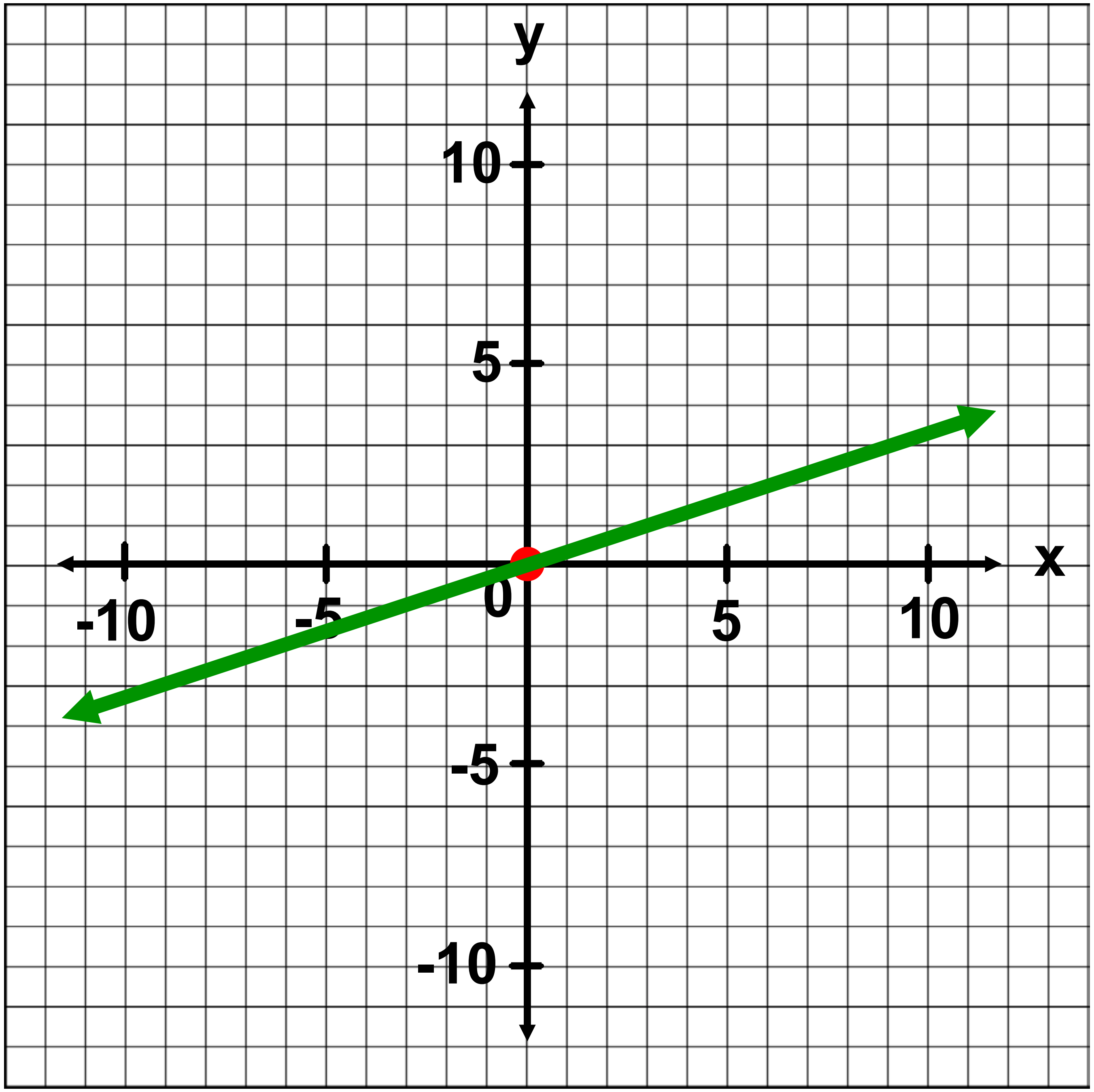
*click to reveal*

What is the equation of the line?

*click to reveal*

| x | y |
|---|---|
| 3 | 5 |
| 6 | 10 |
| 9 | 15 |

75 What is the proportional relationship shown in the graph?



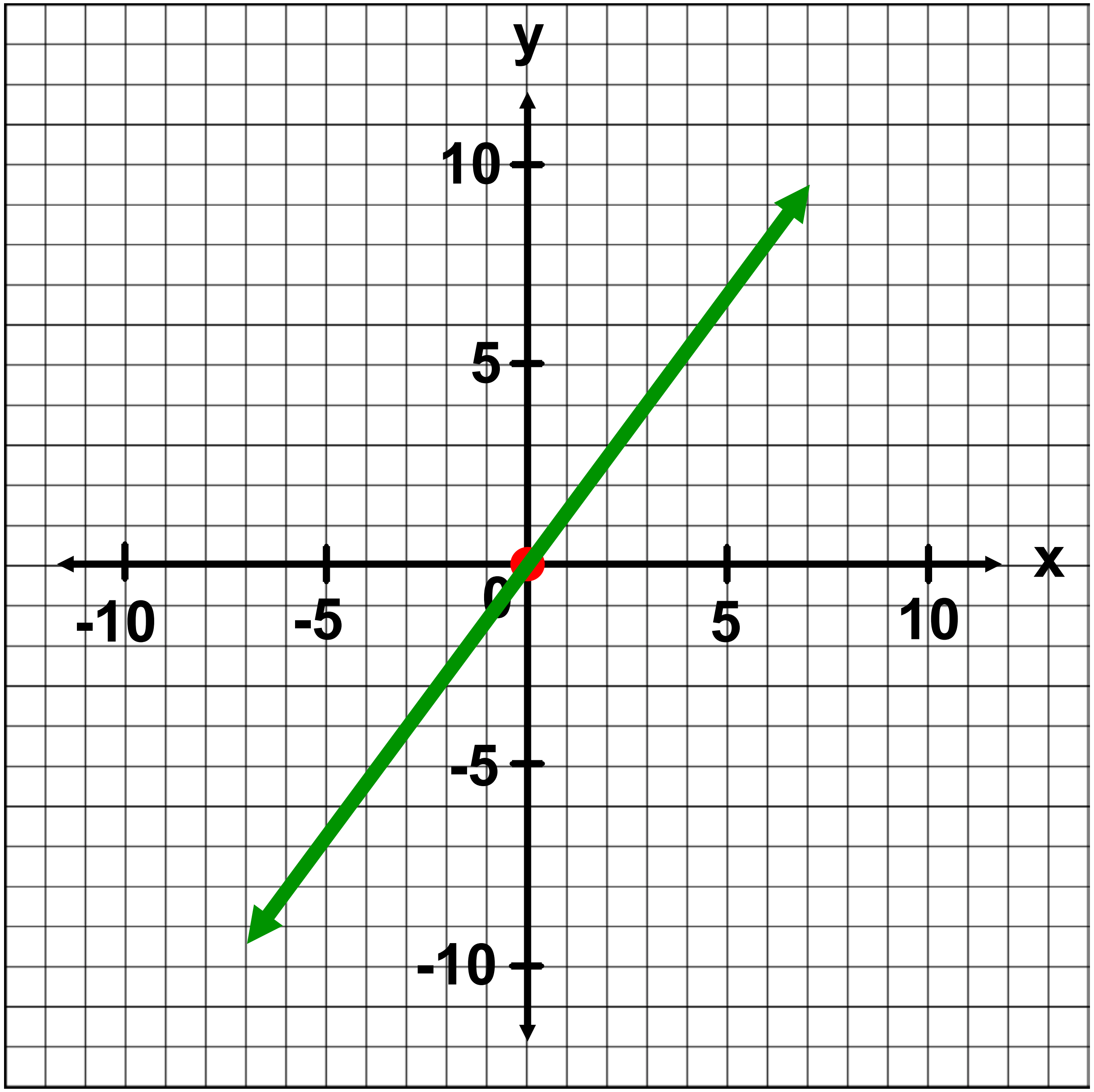

Answer

76 What is the proportional relationship shown in the table?

| x | y |
|---|---|
| 4 | -7 |
| 8 | -14 |
| 12 | -21 |

Answer

77 What is the proportional relationship shown in the graph?




Answer

78 What is the proportional relationship shown in the equation $y = -11x$?

Answer

79 What is the proportional relationship shown in the table?

| x | y |
|---|---|
| -5 | -12 |
| 0 | 0 |
| 10 | 24 |

Answer

# Proportional Relationships

A family drove their car 225 miles in 5 hours.

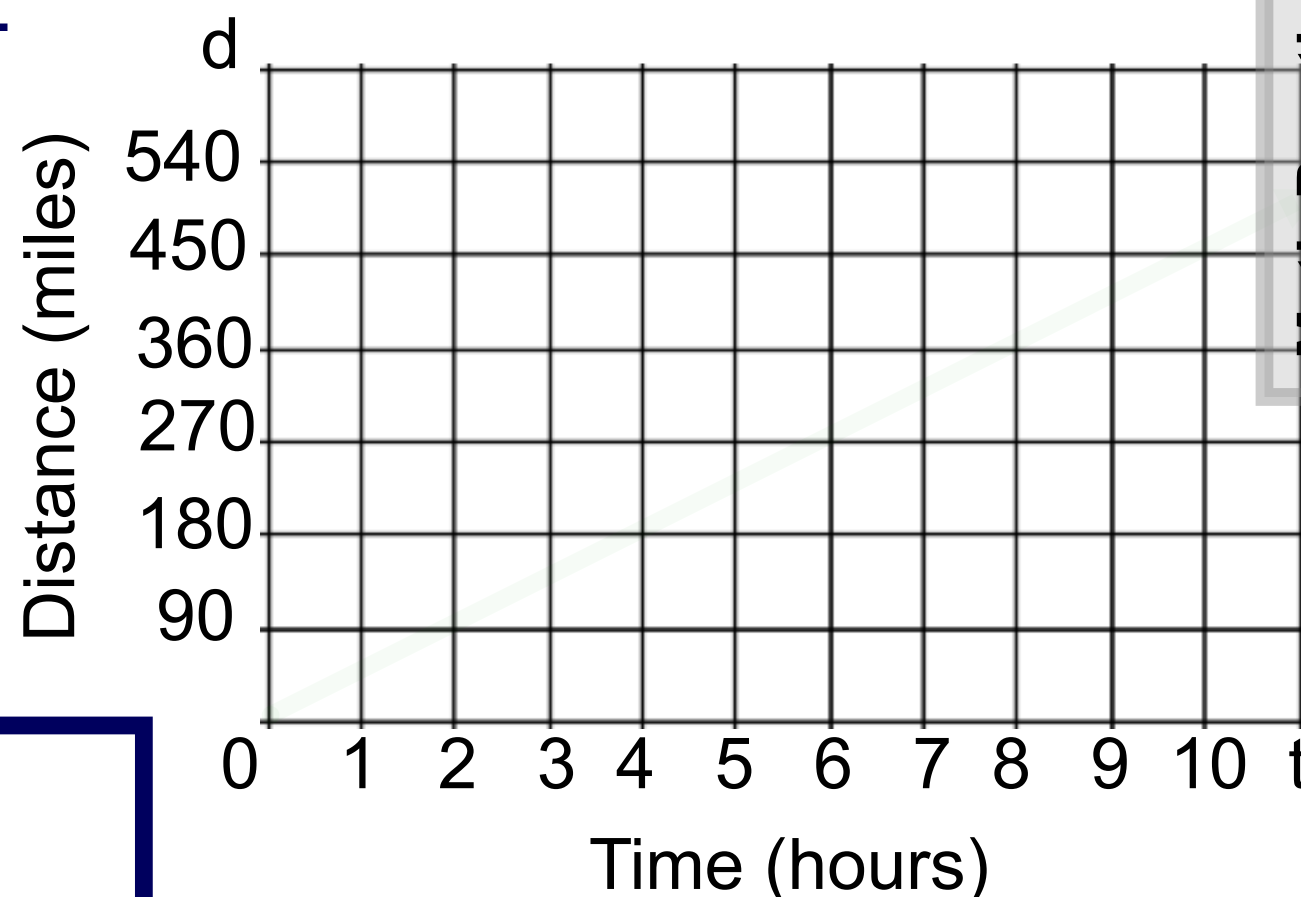
a) Write a direct variation equation that relates the distance traveled in respect to time.

*click to reveal*

b) Graph the equation.

c) Predict about how long it will take the family to drive 360 miles.

*slide to reveal*




click on the line to reveal

Math Practice

# Proportional Relationships

Do all lines have proportional relationships?

Create the equation of a line in the form
$y = mx + b$, where $b \neq 0$. Test out the relationship between the x- and y-coordinates in each ordered pair.

DISCUSS!

Answer

# Proportional Relationships

Example:

A line passes through the points (3, -1), (18, -6), and (-24, 8).

- What is the equation of the line?

- Use the equation of the line to explain why the ratio of the y-coordinate to the x-coordinate is the same for any point on the line except the y-intercept. Explain why this is not true for the y-intercept.

Math Practice

# Proportional Relationships

Example:

A line passes through the points (3, -1), (18, -6), and (-24, 8).

- What is the equation of the line?

Using two of our 3 points, we can calculate the slope of the line.

$$m = \frac{-6 - (-1)}{18 - 3} = \frac{-6 + 1}{18 - 3} = \frac{-5}{15} = -\frac{1}{3}$$

Next, using the point and our slope we can find the equation of the line.

$$y - (-1) = -\frac{1}{3}(x - 3)$$

$$y + 1 = -\frac{1}{3}x + 1$$

$$y = -\frac{1}{3}x$$

# Proportional Relationships

Example:

A line passes through the points (3, -1), (18, -6), and (-24, 8).

- Use the equation of the line to explain why the ratio of the y-coordinate to the x-coordinate is the same for any point on the line except the y-intercept. Explain why this is not true for the y-intercept.

Answer

80 A line passes through the points (5, 2), (11, 5), and (-13, -7). What is the equation of the line?

A $y = 0.5x$

B $y = 2x$

C $y = 0.5x - 0.5$

D $y = 2x - 8$

Answer

81 A line passes through the points (5, 2), (11, 5), and (-13, -7). Does this line represent a proportional relationship? Explain.

Yes

No

Answer

82 A line passes through the points (4, -8), (10, -20), and (-15, 30). What is the equation of the line?

A $y = -0.5x$

B $y = -2x$

C $y = -0.5x - 2$

D $y = -2x + 2$

Answer

83 A line passes through the points (4, -8), (10, -20), and (-15, 30). Does this line represent a proportional relationship? Explain.

Yes

No

Answer

# Solving Linear Equations

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# Solving Linear Equations

Throughout this unit, you have learned how to calculate the slope of a line and write the equation of a line in three different forms. What if you are given the equation of a line, and they ask you to find the slope? or an intercept?

Depending on what form the equation is in, the information might be easy to find. Sometimes, it might require some algebraic steps to manipulate the equation into the desired form.

# Solving Linear Equations

Example:

What is the slope of the equation $8x - 10y = 40$?

This equation is in standard form, which can be used to find the intercepts. It doesn't tell you the slope, though. So we have to manipulate it using what we know from algebra.

Which form(s) of a linear equation can help us find the slope of this line?

***click to reveal***

Which form of a linear equation would be the most appropriate for this situation? Why?

***click to reveal***

Math Practice

# Solving Linear Equations

Example:

What is the slope of the equation $8x - 10y = 40$?

$8x - 10y = 40$

Subtract 8x from both sides

$-10y = -8x + 40$

Divide both sides (or all terms) by -10

$y = \frac{4}{5}x - 4$

Therefore, the slope of the equation is ______

*click*

# Solving Linear Equations

Example: Write the equation of the line with an x-intercept of 5 and a y-intercept of 10.

Since two points are given, we can use the slope formula. The 2 points are (5, 0) and (0, 10).

m =

*click* *click*

Since this problem does not specify a form for the equation, you can select any form that you desire. The equation of the line is in point-slope form, using the x-intercept, is

*click to reveal*

Or you could also use the y-intercept to write the equation in point-slope form:

*click to reveal*

Or since you have the y-intercept and you calculated the slope, you could have written the equation in slope-intercept form:

*click to reveal*

Math Practice

# Solving Linear Equations



Example: Write the equation $y - 7 = \frac{3}{4}(x + 4)$ in standard form. Then determine the x- and y-intercepts.

We need to get this equation into $Ax + By = C$, where $A \geq 0$. Therefore, we need to use our inverse operations.

$$y - 7 = \frac{3}{4}(x + 4)$$

Multiply both sides of the equation by the LCD

$$4(y - 7) = 3(x + 4)$$

Distributive Property

$$4y - 28 = 3x + 12$$

Subtract 3x from both sides

$$-3x + 4y - 28 = 12$$

Add 28 to both sides

$$-3x + 4y = 40$$

Multiply all terms by -1, because $A \geq 0$

$$3x - 4y = -40$$

# Solving Linear Equations

Example: Write the equation $y - 7 = \frac{3}{4}(x + 4)$ in standard form. Then determine the x- and y-intercepts.

We need to get this equation into $Ax + By = C$, where $A \geq 0$. Therefore, we need to use our inverse operations.

$$3x - 4y = -40$$

***move to reveal***

***move to reveal***

84 What is the slope of the line whose equation is $3x - 7y = 9$?

A $-\frac{3}{7}$

B $\frac{3}{7}$

C $-\frac{7}{3}$

D $\frac{7}{3}$

Answer

85 What is the slope of the line whose equation is $5x + 9y = 45$

A $-\frac{9}{5}$

B $\frac{9}{5}$

C $-\frac{5}{9}$

D $\frac{5}{9}$

Answer

86 What is the equation of a line that has an x-intercept of 2 and a y-intercept of 7?

A $y = -\frac{7}{2}(x - 2)$

B $y = -\frac{7}{2}x + 7$

C $7x + 2y = 14$

D All of the above

Answer

87 Which equation represents $y + 6 = 2(x - 1)$ in standard form?

A $2x - y = 8$

B $-2x + y = -8$

C $y = 2x - 8$

D $y = -2x + 8$

Answer

88 Which equation represents $y = -\frac{3}{5}x + 9$ in standard form?

A $3x - 5y = -45$

B $3x + 5y = 45$

C $5y = -3x + 9$

D $y - 2 = -\frac{3}{5}(x - 5)$

Answer

89 What is the equation of a line that has an x-intercept of 5 and a y-intercept of -7?

A $y = \frac{7}{5}x + 5$

B $y = \frac{7}{5}x - 7$

C $7x - 5y = 70$

D All of the above

Answer

# Function Notation

Sometimes, you will see an equation in the form f(x) = mx + b instead of y = mx + b. This can occur because every linear equation is a function. A function is a relationship that exists when every x value has exactly one y value. When this happens, nothing changes mathematically. Yet, instead of writing y mathematicians use f(x), read "f of x."

For example: $y \Leftrightarrow f(x)$ given that y is a function.

$y = 5x + 7$ becomes $f(x) = 5x + 7$

*$f(x) = 5x + 7$ is still a line with a slope of 5 and a $y$-intercept of (0, 7).

Since we haven't yet covered the material for functions, it would be easiest for you, at this point, to switch the f(x) to y and solve the equation from there.

# Solving Linear Equations

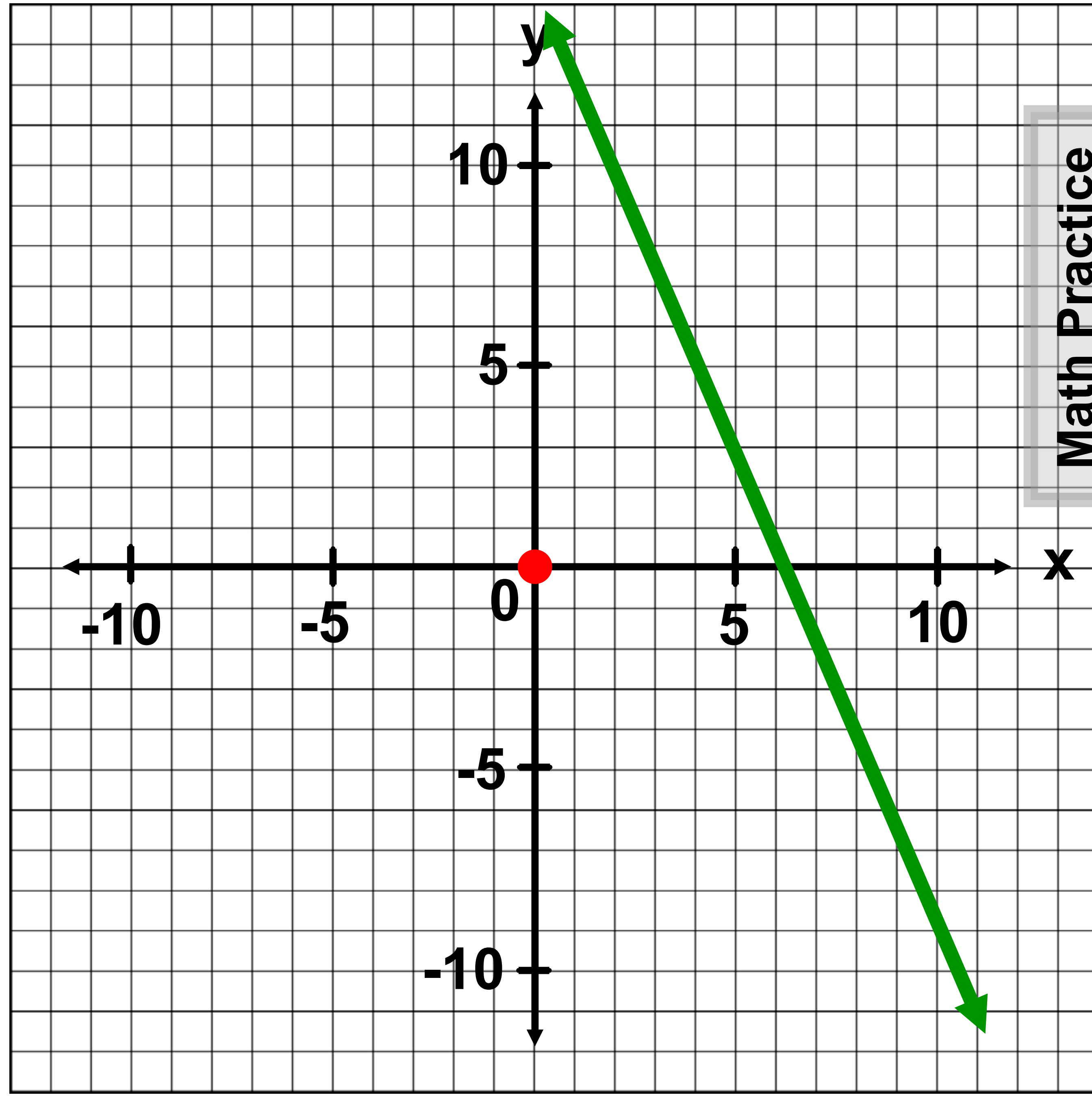
Example: The graph shown below represents the function

$$f(x) = -\frac{7}{3}x + 14.$$

For what value of x does f(x) = 0?

If we start with the advice that we started with, substituting y in for f(x), we get the equation

$$y = -\frac{7}{3}x + 14$$



# Solving Linear Equations

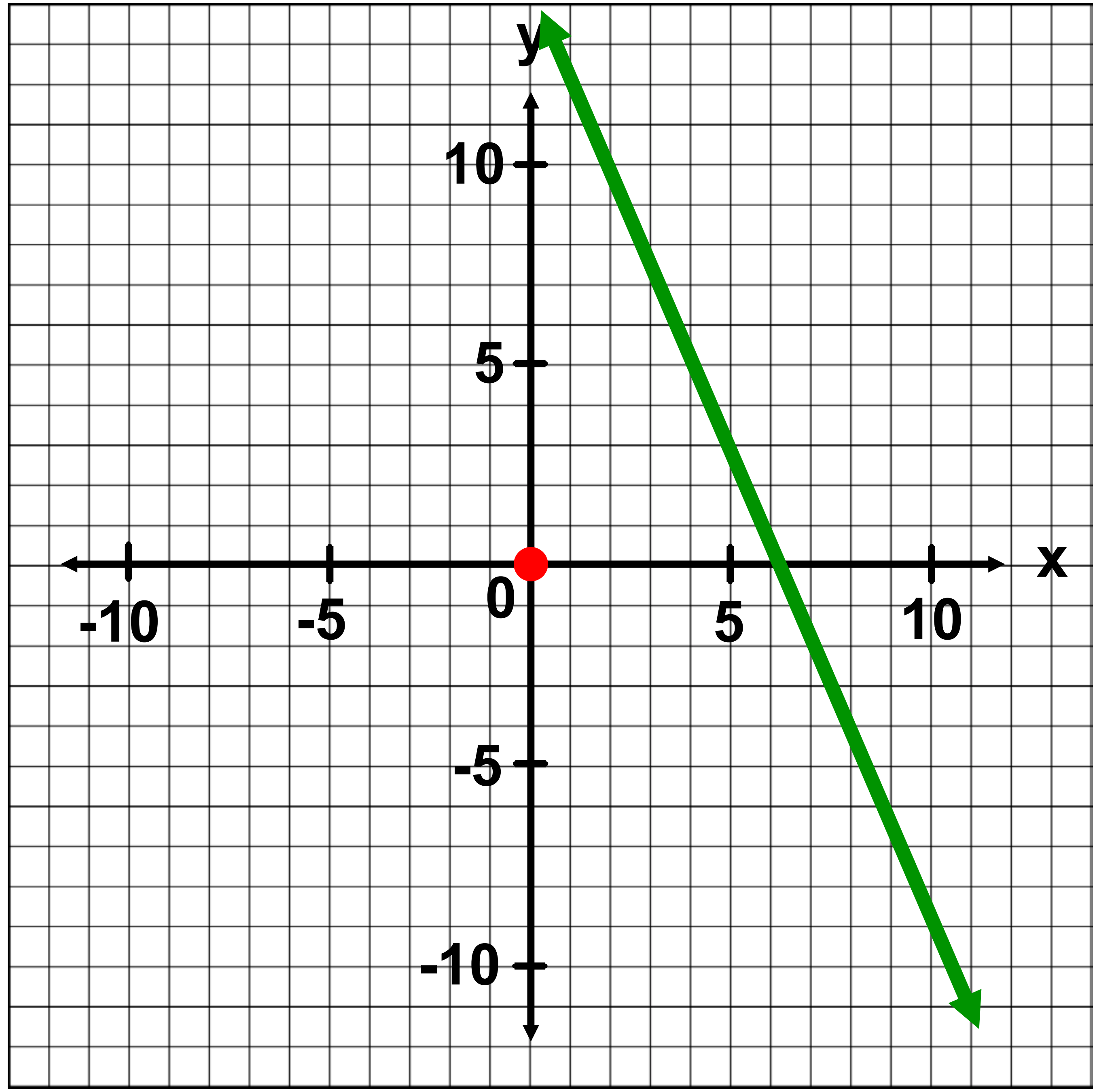
Example: The graph shown below represents the function

$f(x) = -\frac{7}{3}x + 14.$

For what value of x does f(x) = 0?

$y = -\frac{7}{3}x + 14$

If f(x) = 0, that means that y = 0. This occurs when the graph crosses the x-axis, giving us our x-intercept. Looking at the graph, we see that our x-intercept is (6, 0), so the value of x is 6.



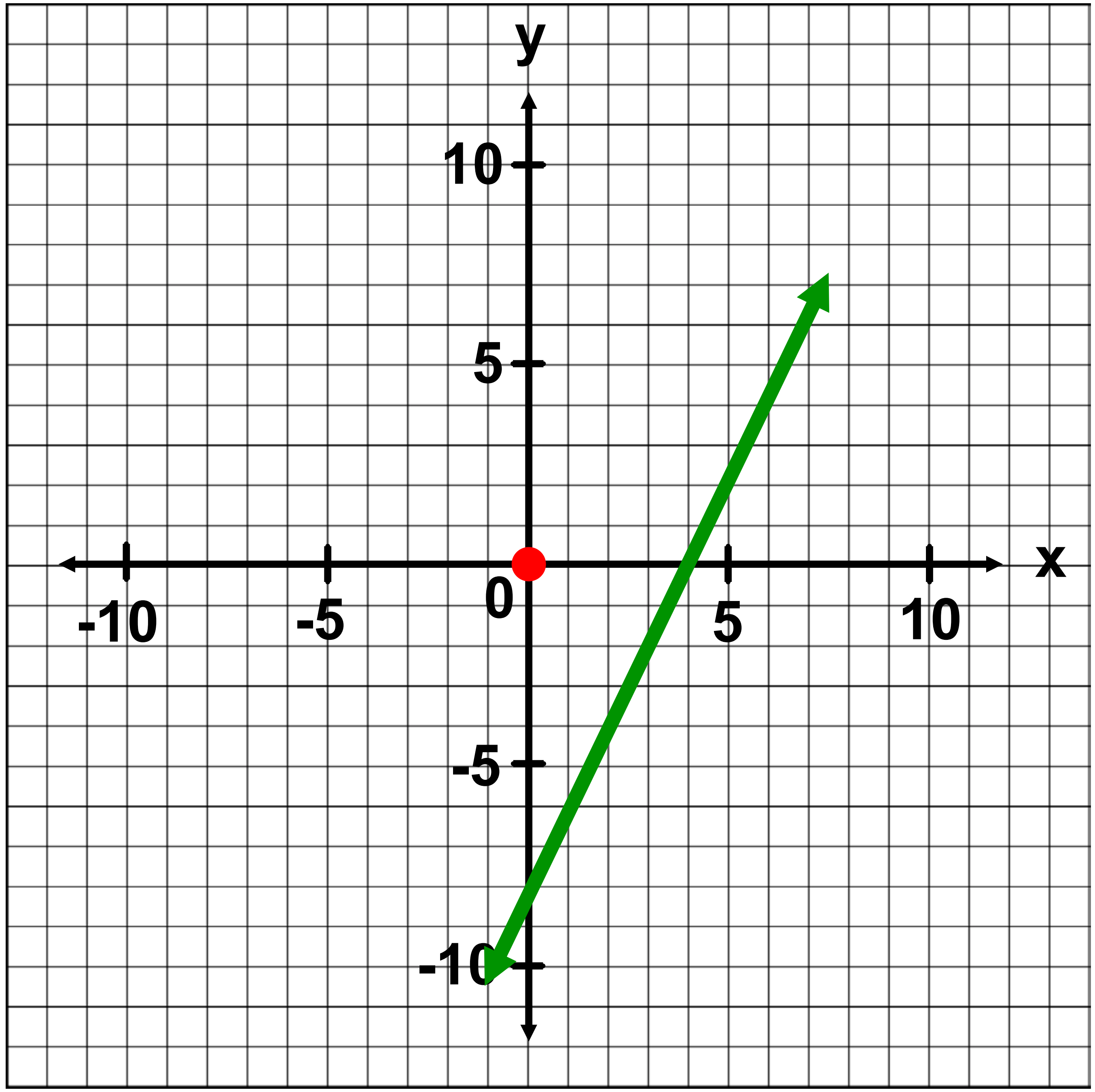
90 The graph shown below represents the function $g(x) = 2x - 8$. For what value of x does $g(x) = 0$?

A -8

B 8

C 4

D -4




Answer

91 The graph shown below represents the function

$h(x) = -\frac{1}{3}x + 2.$

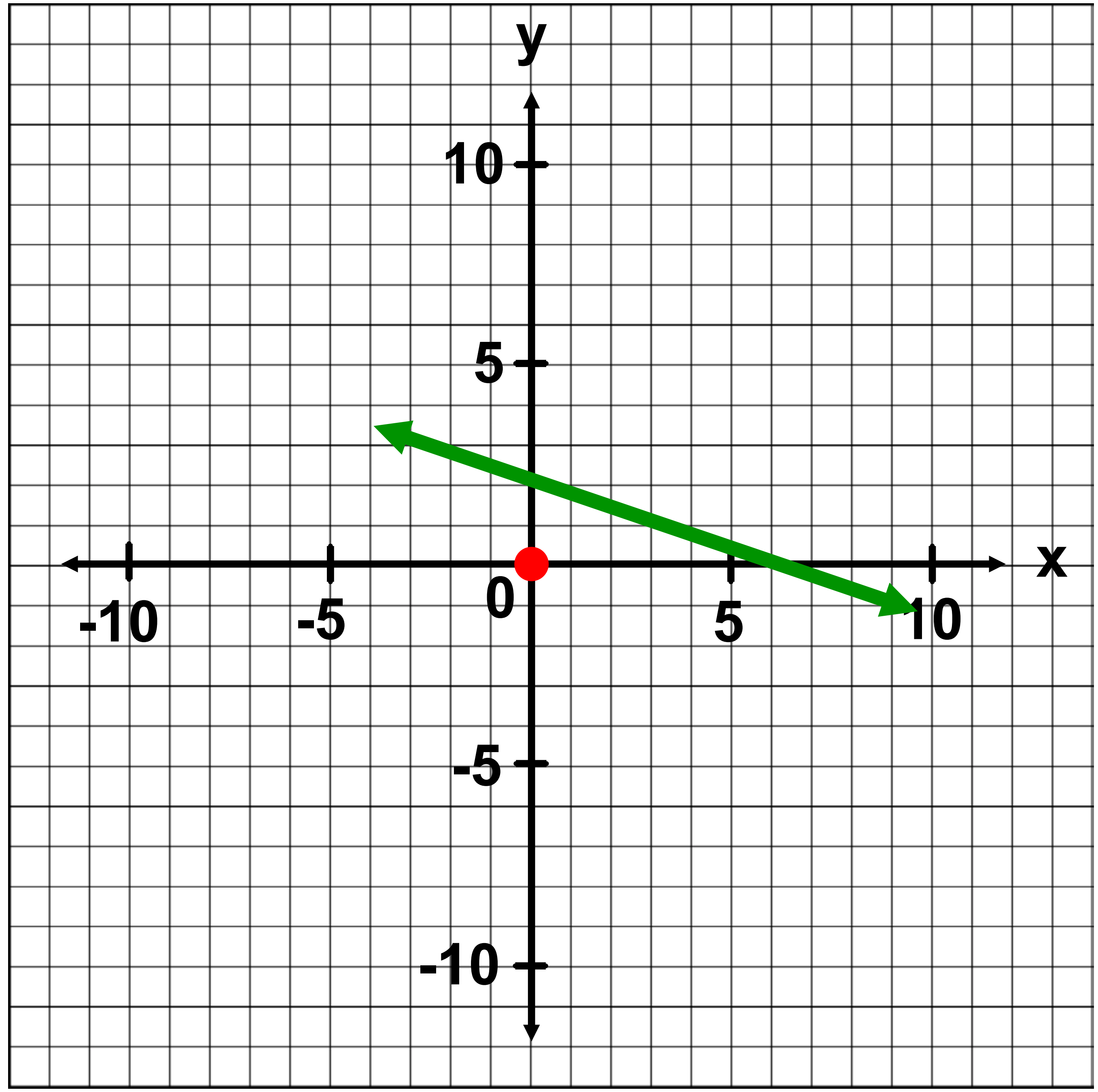
For what value of x does $h(x) = 0$?

A 6

B -6

C 2

D -2




Answer

92 The graph shown below represents the function

$$j(x) = -\frac{3}{5}x - 3.$$

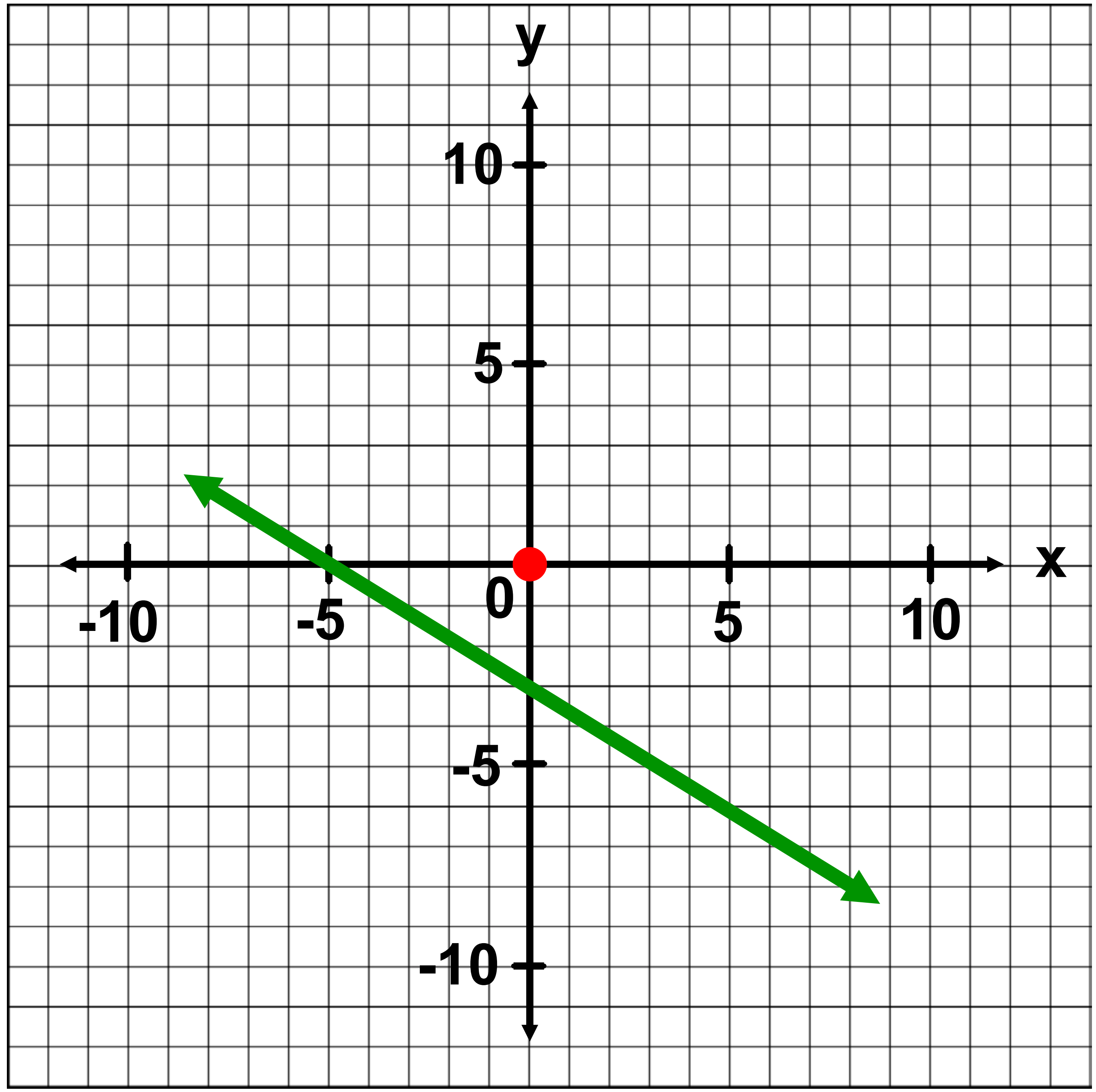
For what value of x does $j(x) = 0$?

A 3

B -3

C 5

D -5




Answer

# Scatter Plots and the Line of Best Fit

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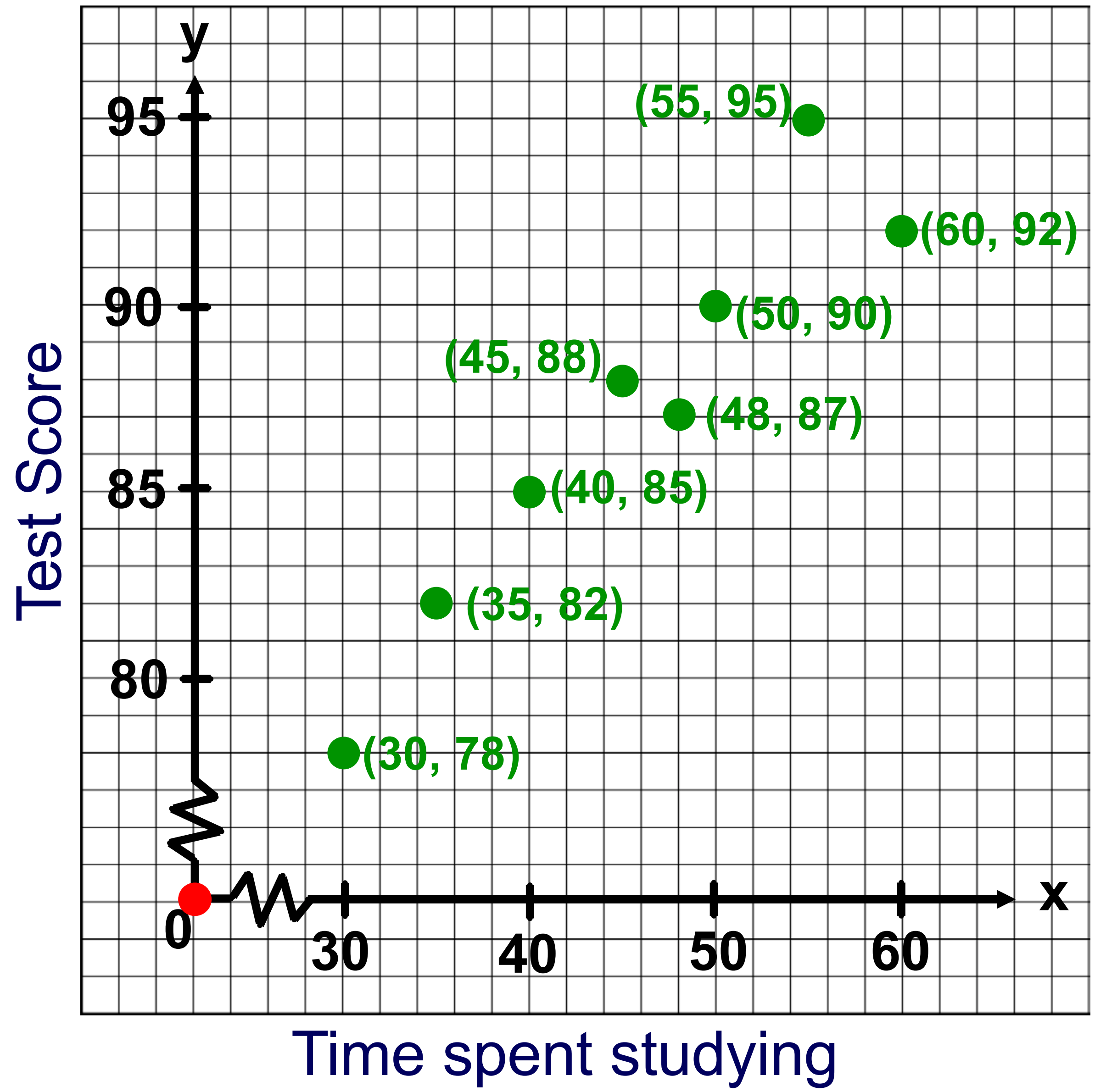
# Scatter Plot

A **scatter plot** is a graph that shows a set of data that has two variables.

| Time Studying | Test Score |
|---|---|
| 45 | 89 |
| 30 | 78 |
| 50 | 90 |
| 60 | 92 |
| 40 | 85 |
| 48 | 87 |
| 55 | 95 |
| 35 | 82 |



# Scatter Plot

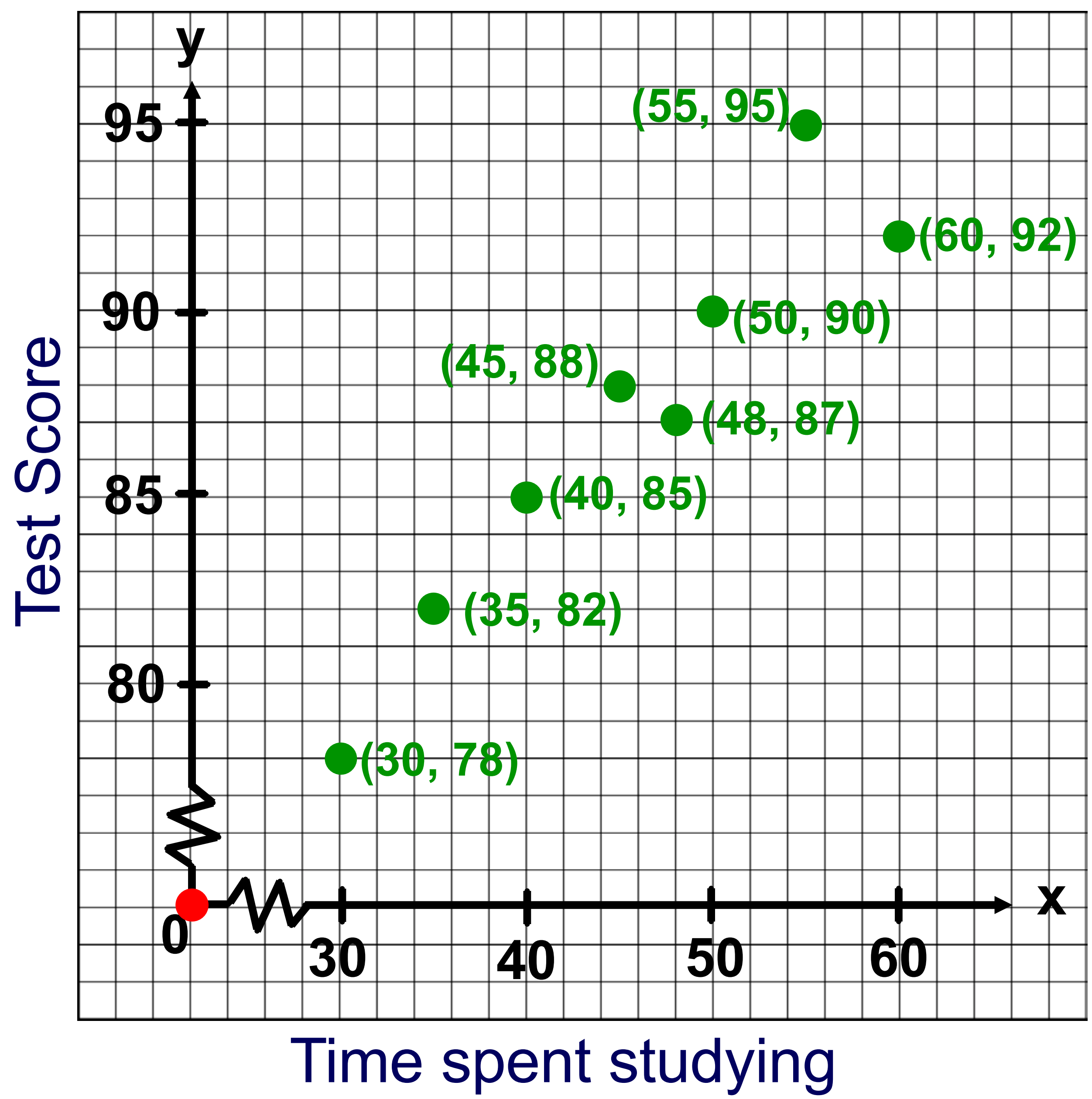
There are three types of linear association that are possible for scatter plots.  What are they?

***click to reveal***

What type of linear association does the graph have to the right?

***click to reveal***

**Test Score vs. Time Spent Studying**

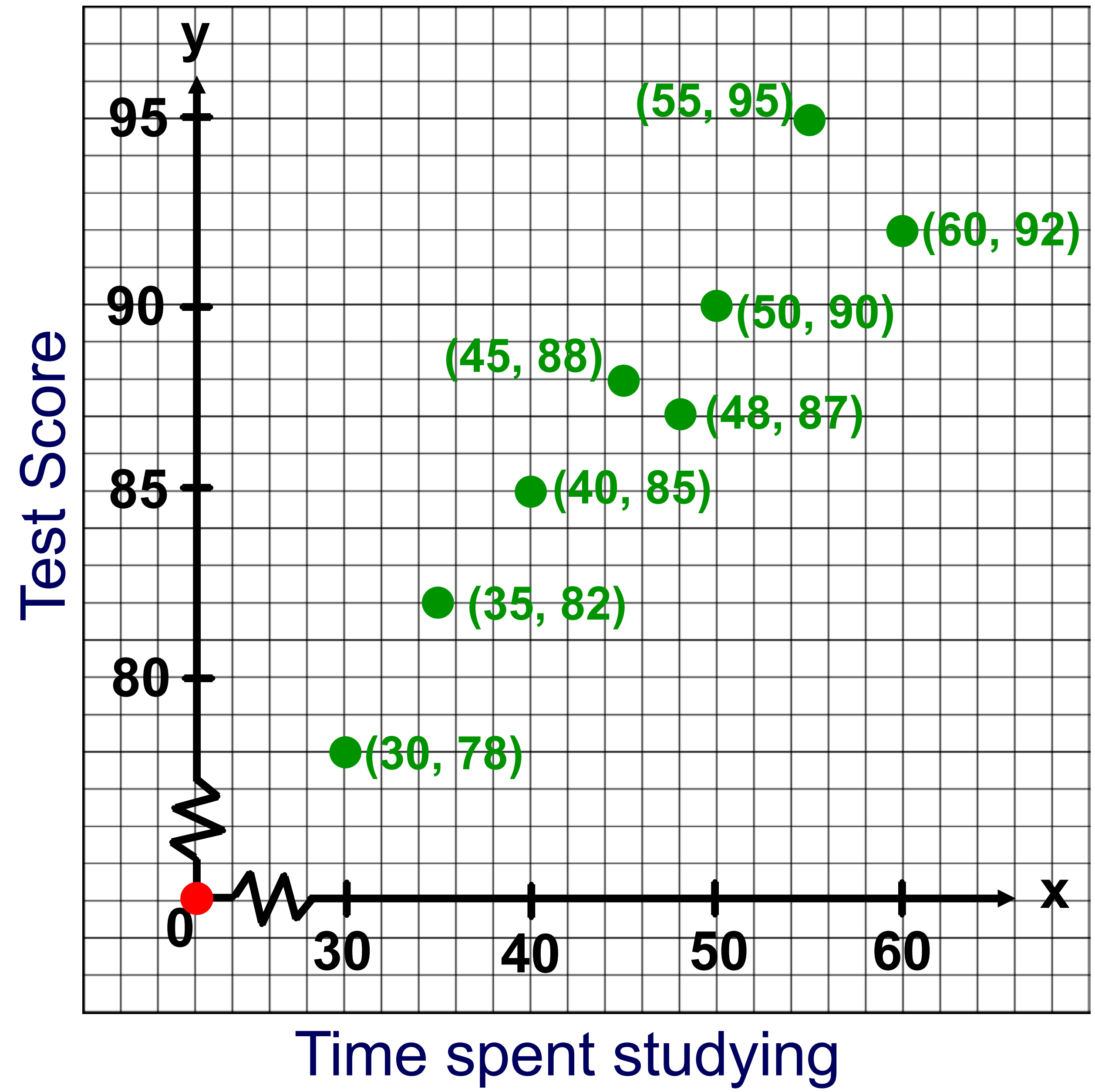



Teacher Notes

# Scatter Plot

What connection do you see between linear association and the slope of a line?

**Test Score vs. Time Spent Studying**




***click to reveal***

Math Practice

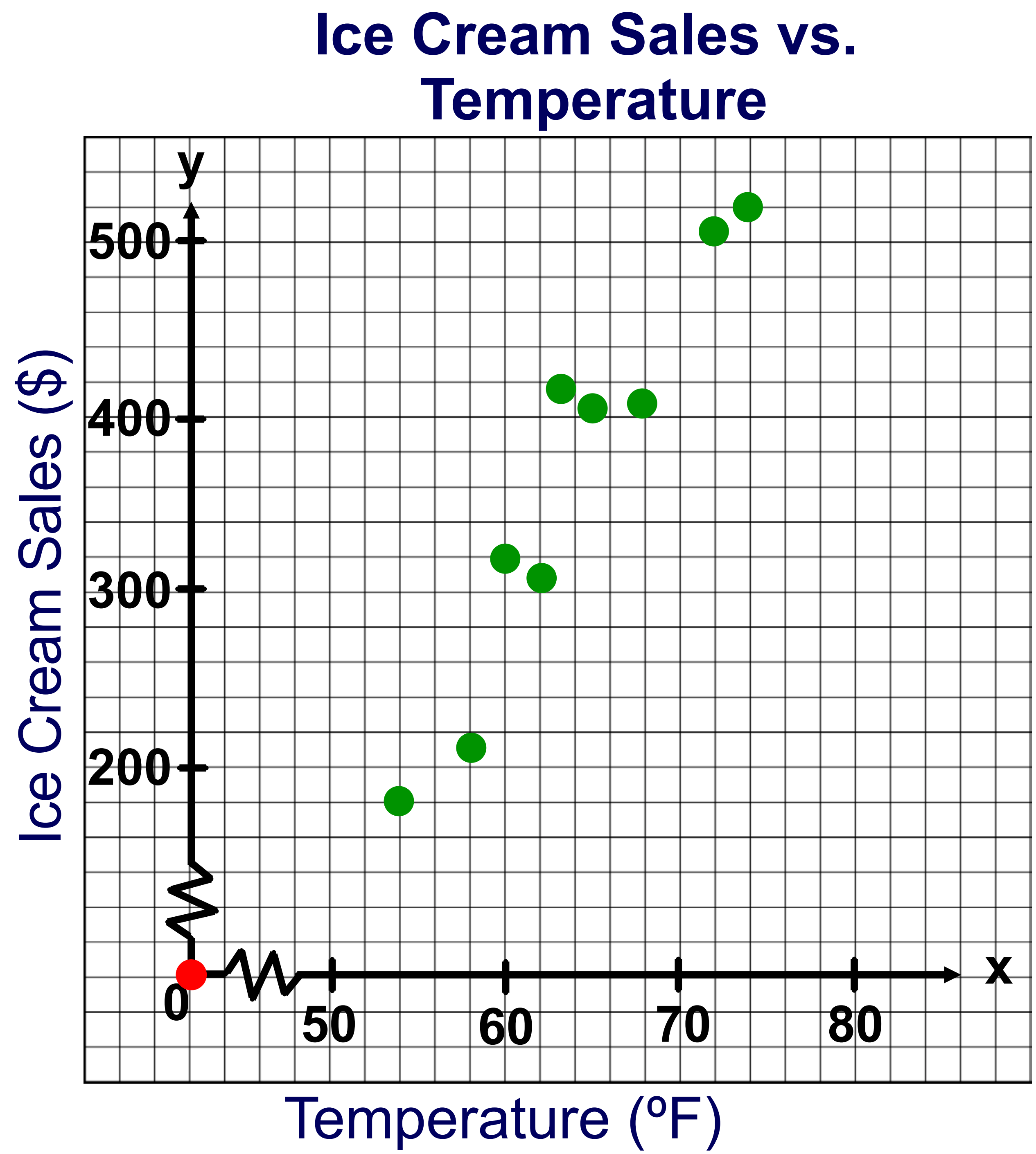
93 What type of scatter plot is shown in the graph below?

A non-linear

B linear, positive association

C linear, negative association

D linear, no association




Answer

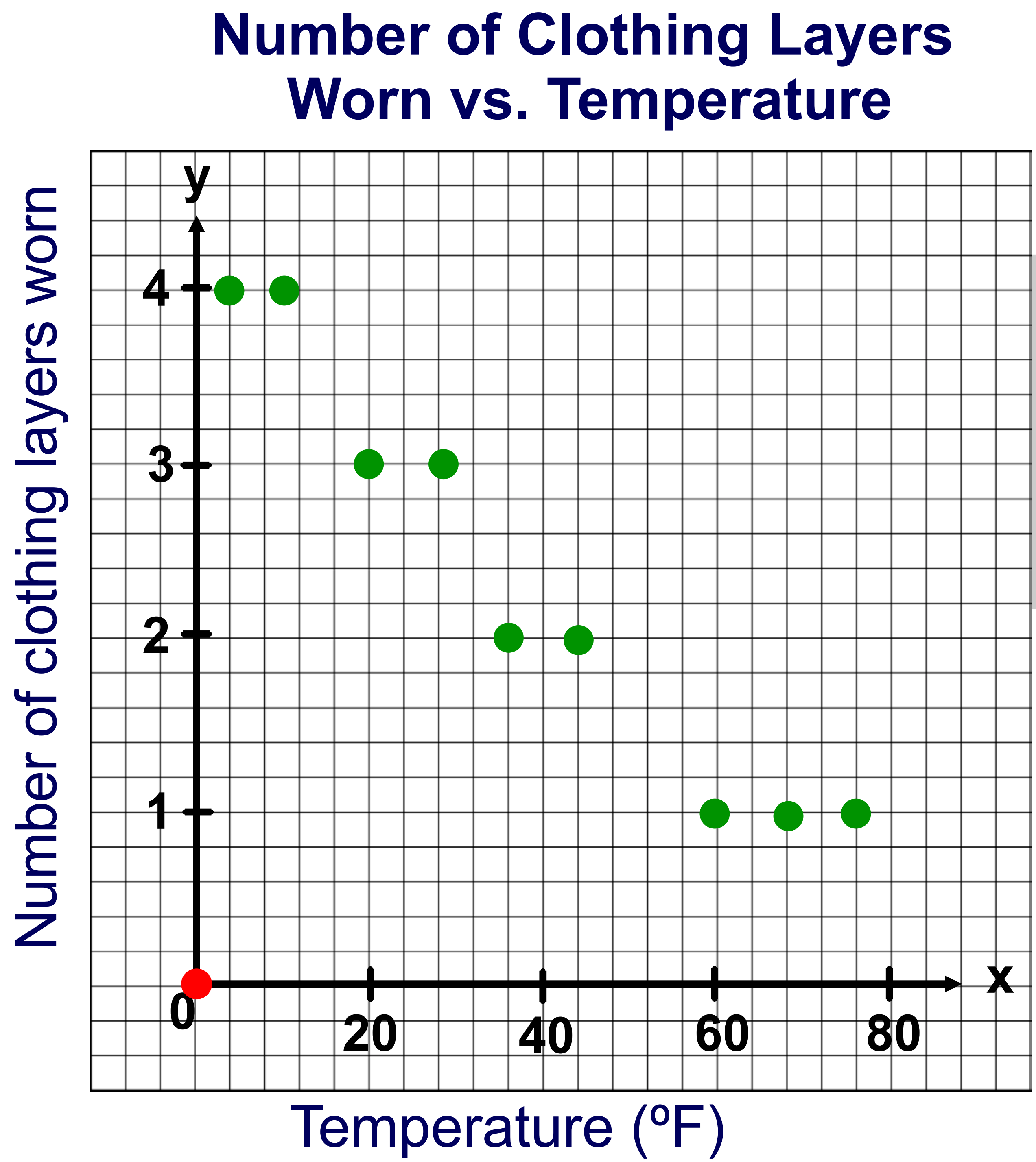
94 What kind of association is shown in the graph?

A non-linear

B linear, positive association

C linear, negative association

D linear, no association




Answer

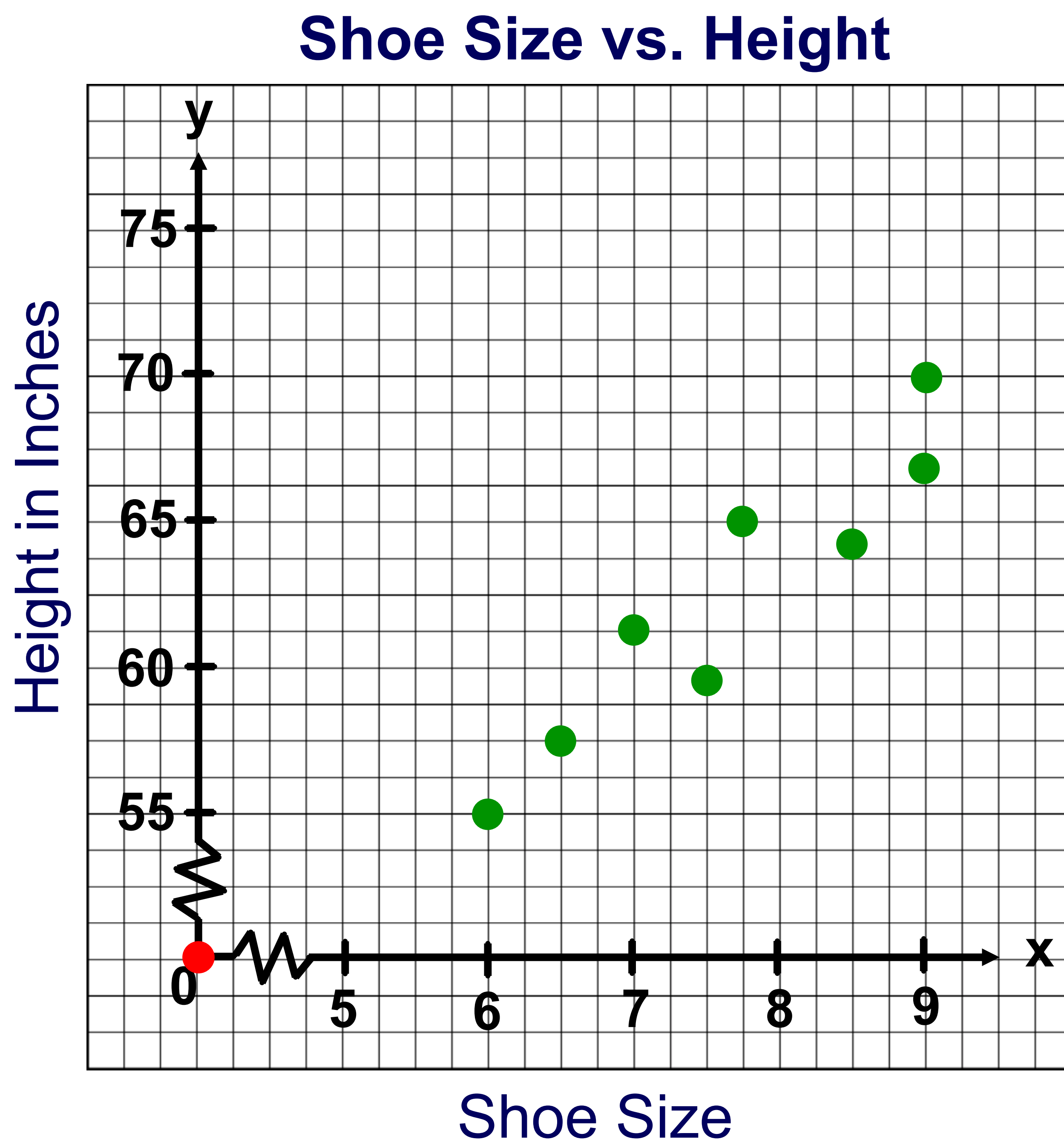
95 What kind of association is shown in the graph ?

A non-linear

B linear, positive association

C linear, negative association

D linear, no association




Answer

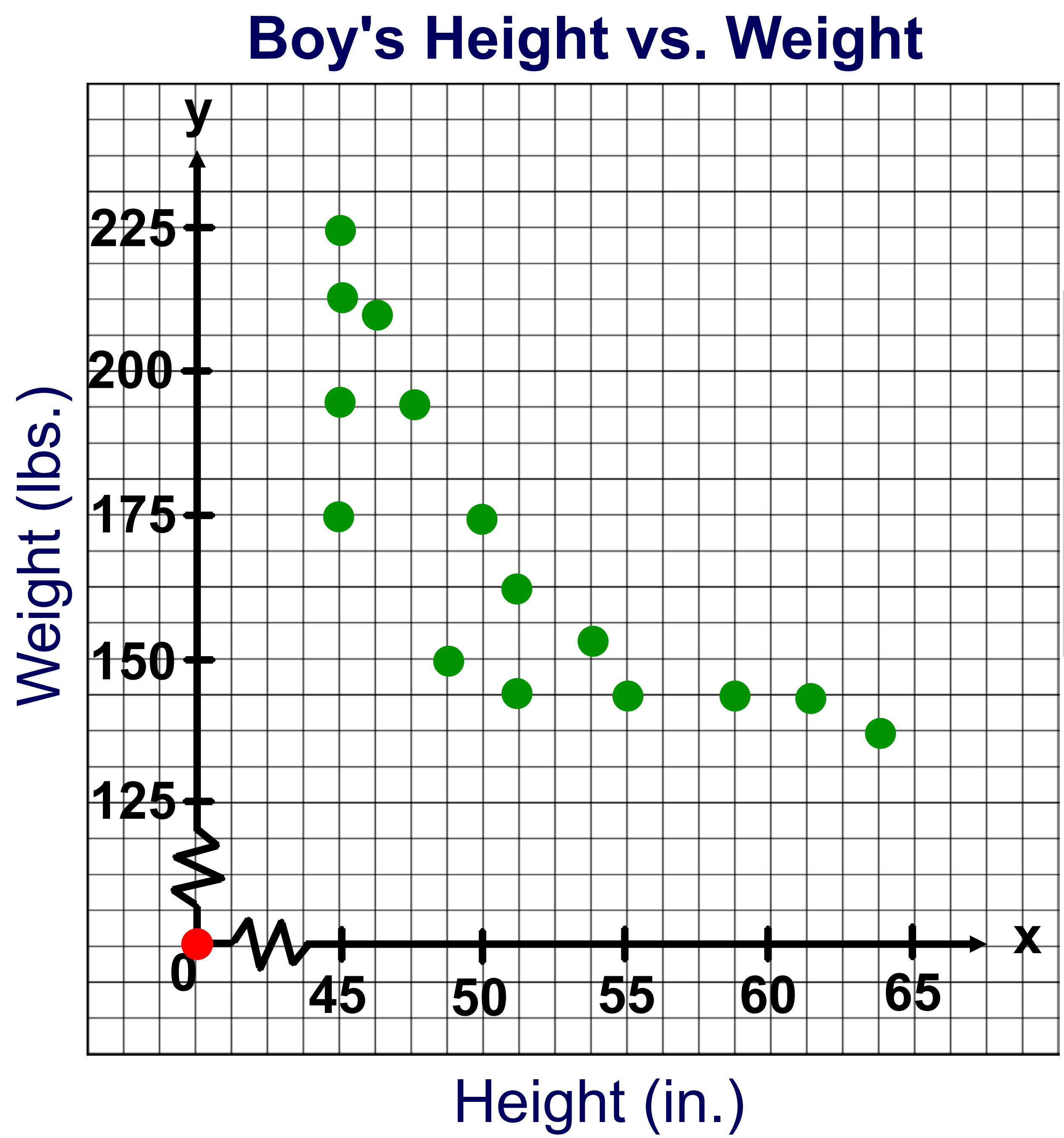
96 What association is shown in this graph?

A non-linear

B linear, positive correlation

C linear, negative correlation

D linear, no correlation




Answer

97 Which of the following scenarios would produce a linear scatter plot with a positive association?

A Miles driven and money spent on gas

B Number of pets and how many shoes you own

C Work experience and income

D Time spent studying and number of bad grades

Answer

98 Which of the following would have no association if plotted on a scatter plot?

A Number of toys and calories consumed in a day

B Number of books read and reading scores

C Length of hair and amount of shampoo used

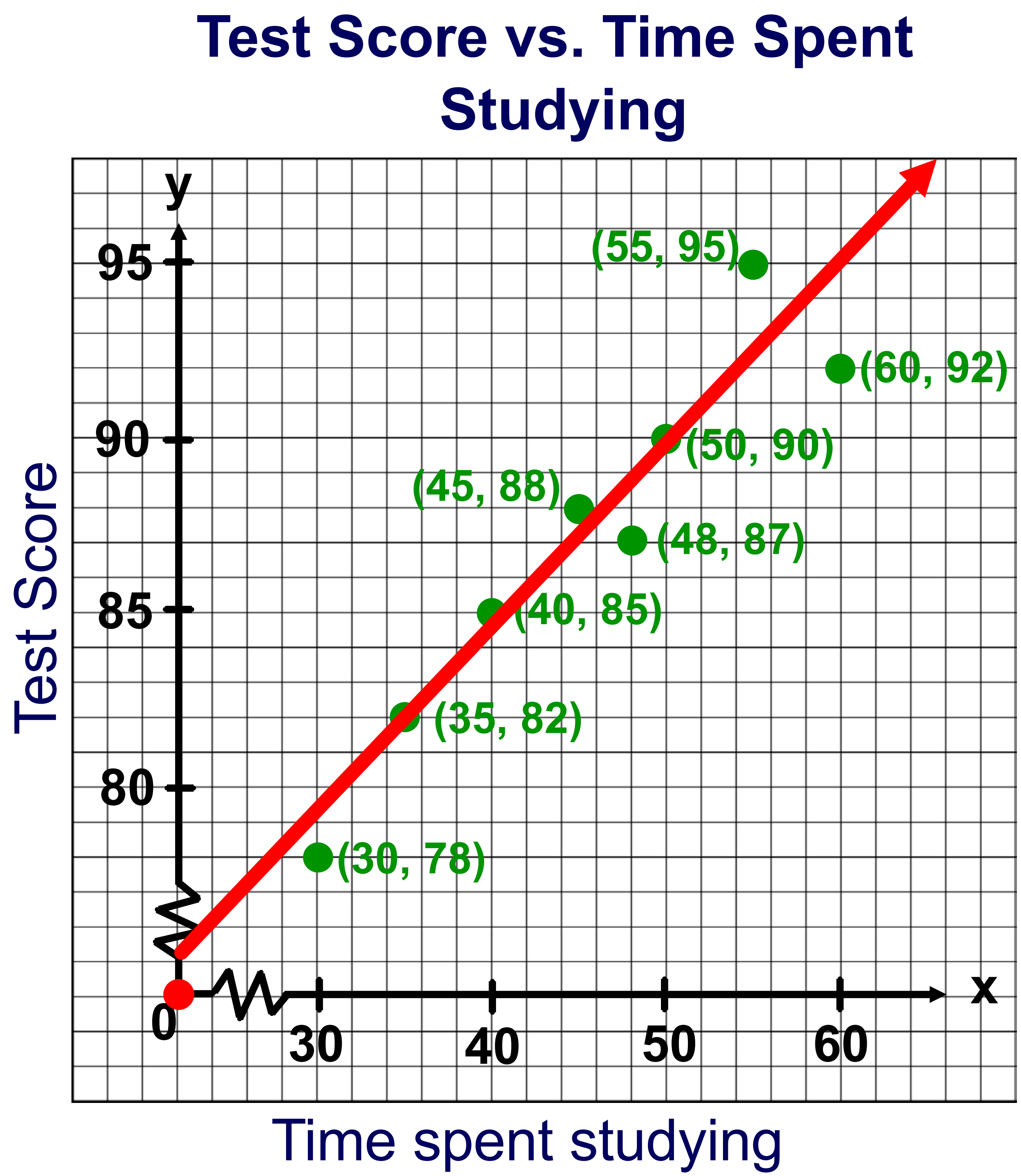
D Person's weight and calories consumed in a day

Answer

# Draw a Line

Notice that the points form a linear like pattern.  To draw a **line of best fit**, use two points so that the line is as close as possible to the data points.

Our line is drawn so that it fits as close as possible to the data points. The number of points above and below the line should be about the same.  There are 3 points above our line and 3 points below our line.  This line was drawn through (35, 82) and (50, 90).

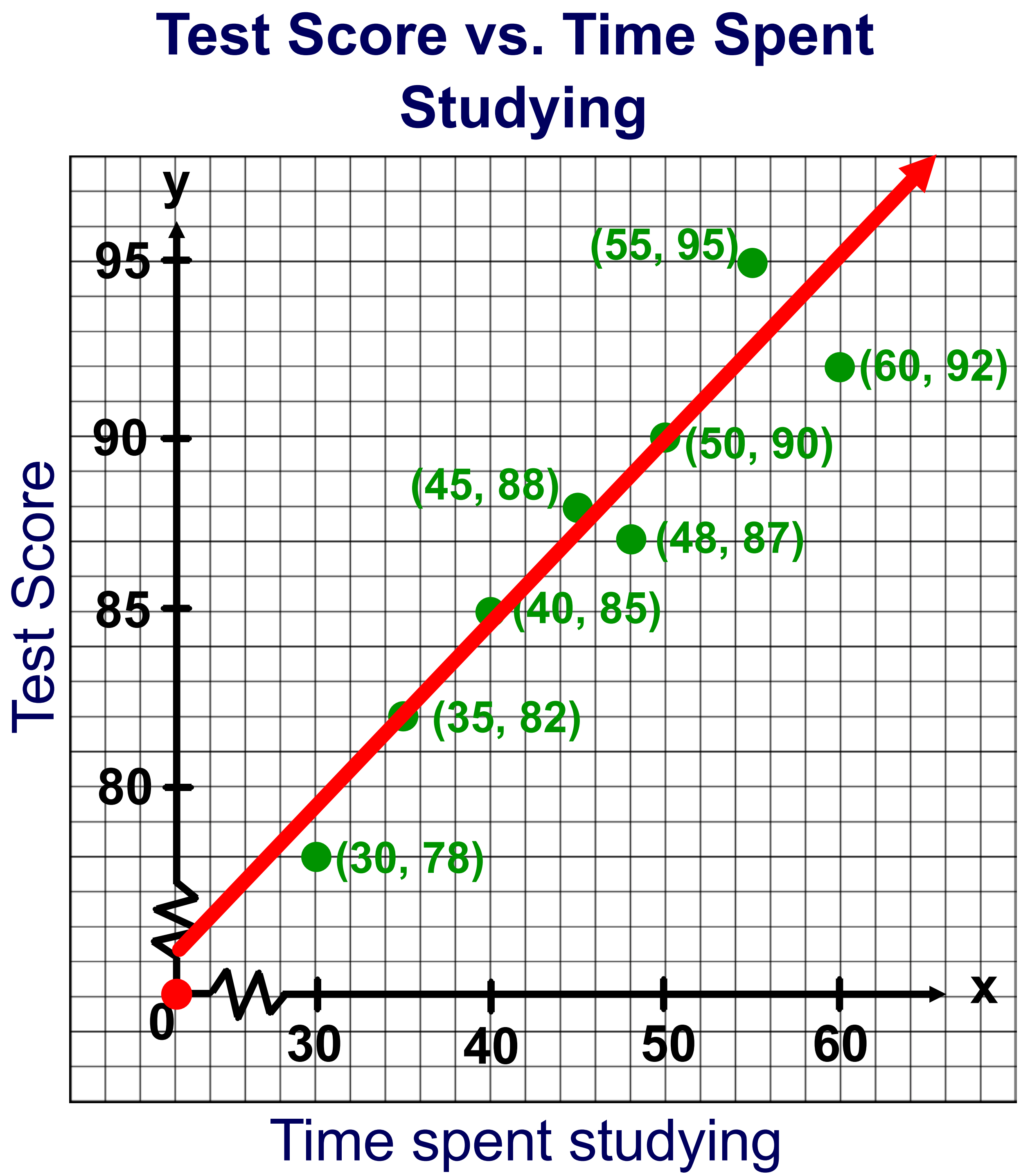


# Scatter Plot

Using the line of best fit shown in the graph:

Predict the test score of someone who spends 32 minutes studying.

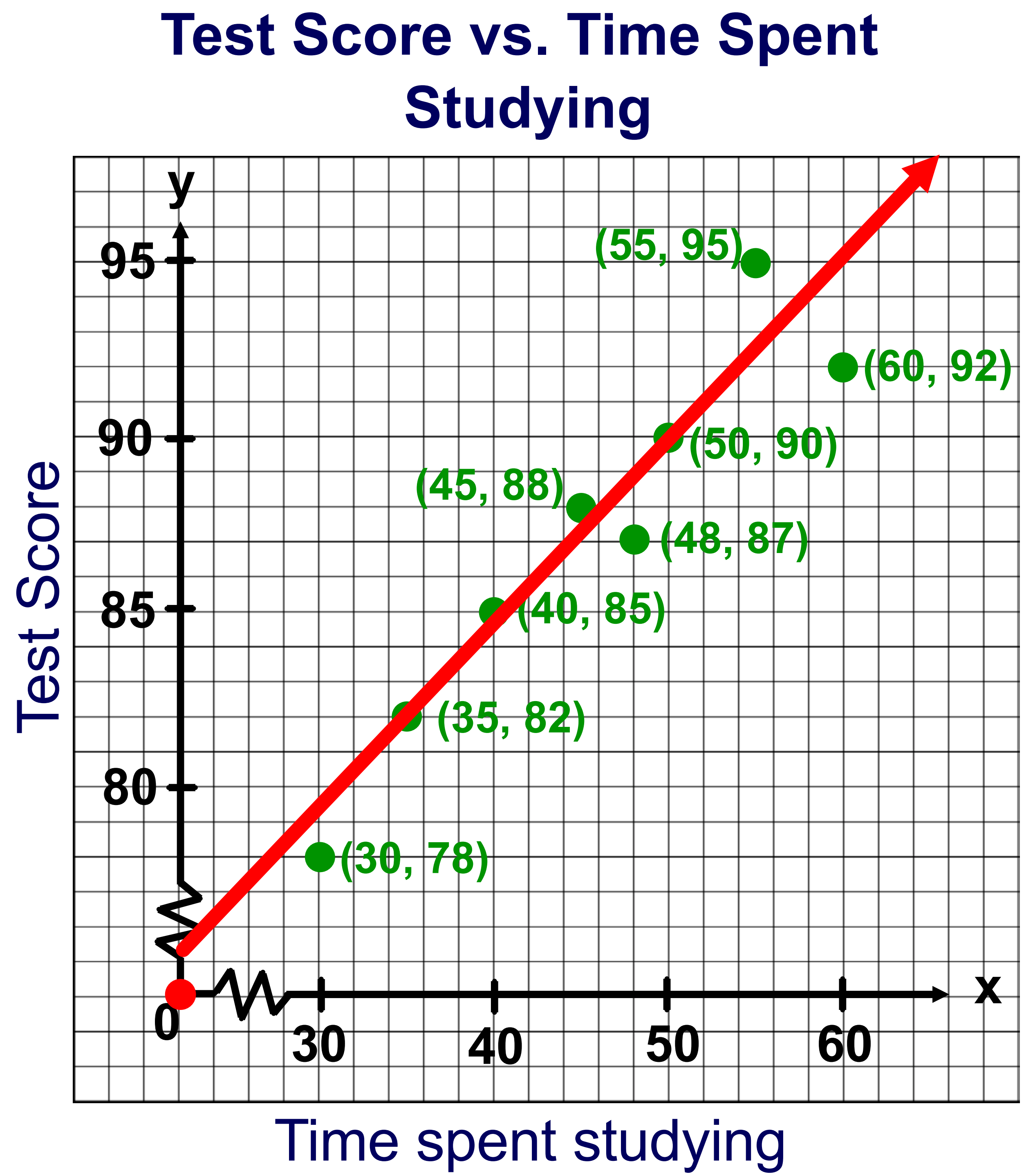
Predict the test score of someone who spends 58 minutes studying.




Answer & Math Practices

# Prediction Equation

Use the two points that formed the line to write an equation for the line.




Math Practice

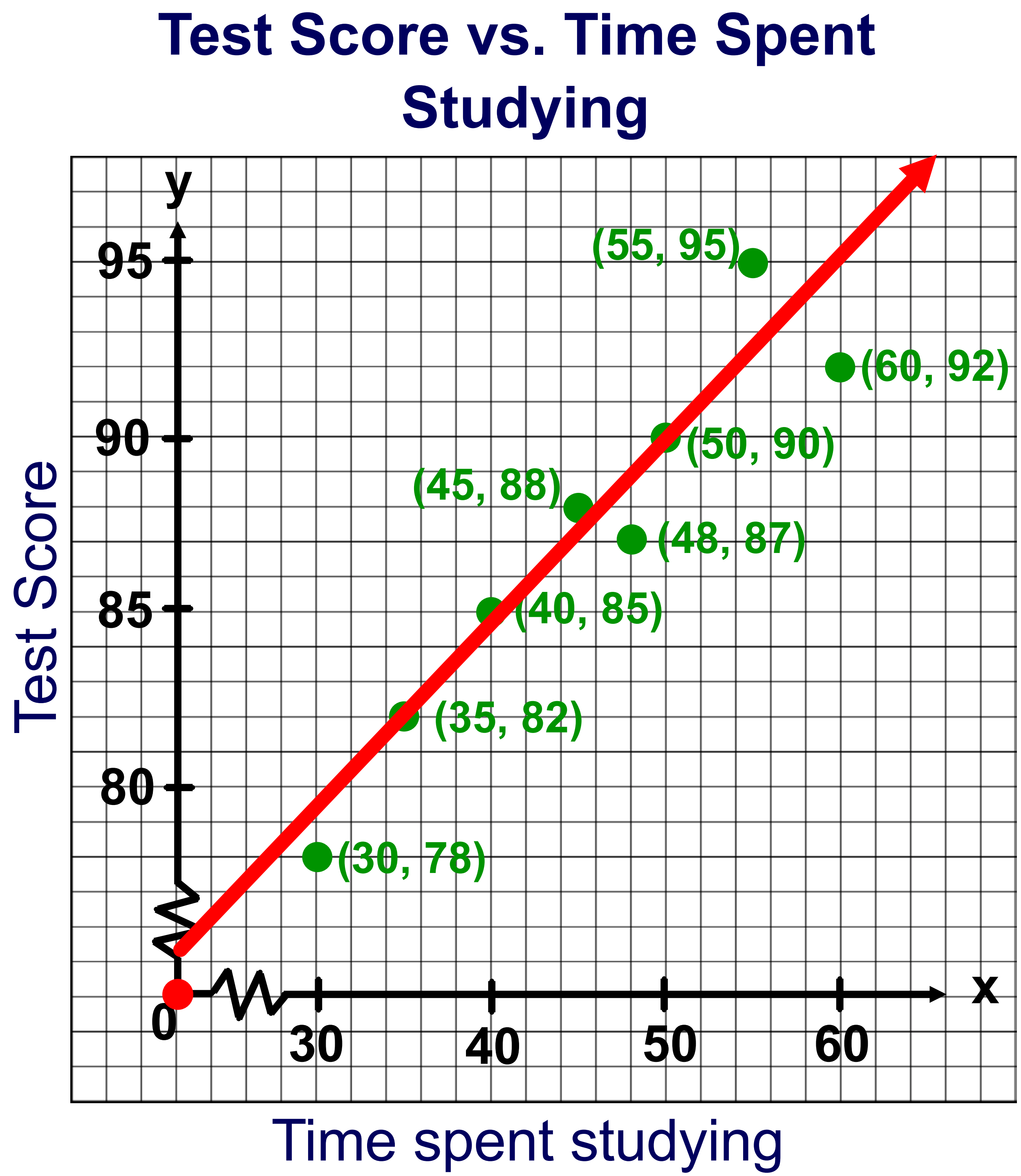
# Prediction Equation

Use the two points that formed the line to write an equation for the line.

<u>Find the slope</u>

$$m = \frac{90 - 82}{50 - 35}$$

$$m = \frac{8}{15}$$



# Prediction Equation

Use the two points that formed the line to write an equation for the line.

<u>Find the equation of the line</u>

$$y - 90 = \frac{8}{15}(x - 50)$$

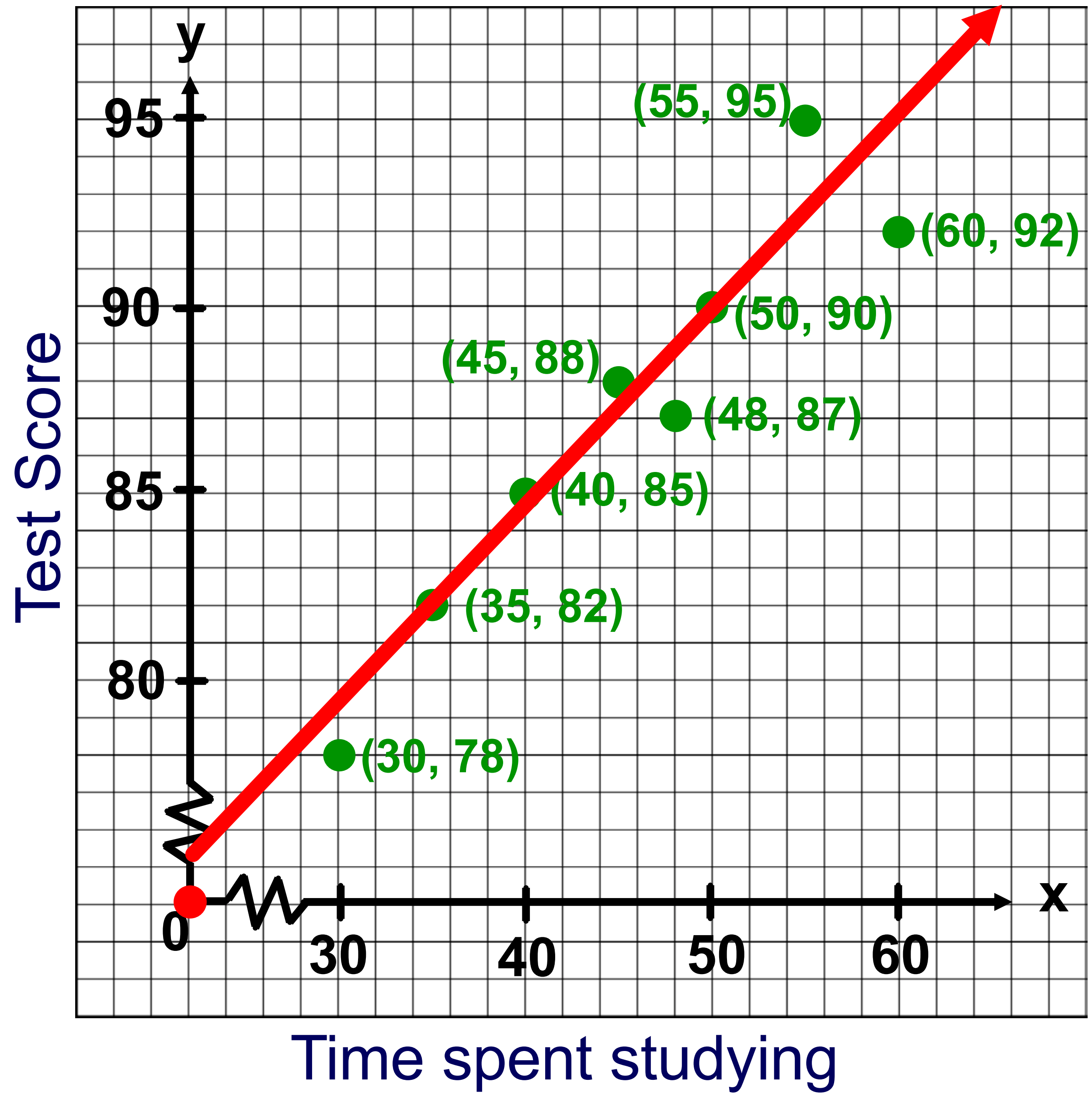
$$y - 90 = \frac{8}{15}x - \frac{80}{3}$$

$$y = \frac{8}{15}x + \frac{190}{3}$$

Where y is the score for x minutes of studying.

This equation is called the **Prediction Equation**.

**Test Score vs. Time Spent Studying**



# Extrapolation

Prediction Equations can be used to predict other related values.

$$y = \frac{8}{15}x + \frac{190}{3}$$

If a person studies 15 minutes, what would be the predicted score?

$$y = \frac{8}{15}(15) - \frac{190}{3} \approx 71.3$$

This is an **extrapolation**, because the time was outside the range of the original times.

# Interpolation

If a person studies 42 minutes, what would be the predicted score?

$$S = \frac{8}{15}(42) - \frac{190}{3} \approx 85.7$$

This is an **interpolation**, because the time was inside the range of the original times.

# What is Wrong?

Interpolations are more accurate because they are within the set.

The farther points are away from the data set the less reliable the prediction.

Using the same prediction equation, consider:

If a person studies 120 minutes, what will be there score?

$$S = \frac{8}{15}(120) + \frac{190}{3} \approx 127.3$$

What is wrong with this prediction?

Answer

# What is the Prediction?

If a student got an 80 on the test,
What would be the predicted length of their study time?

$$80 = \frac{8}{15}x + \frac{190}{3}$$

$$16.7 = \frac{8}{15}x$$

$$31.25 = x$$

The student studied about 31 minutes.

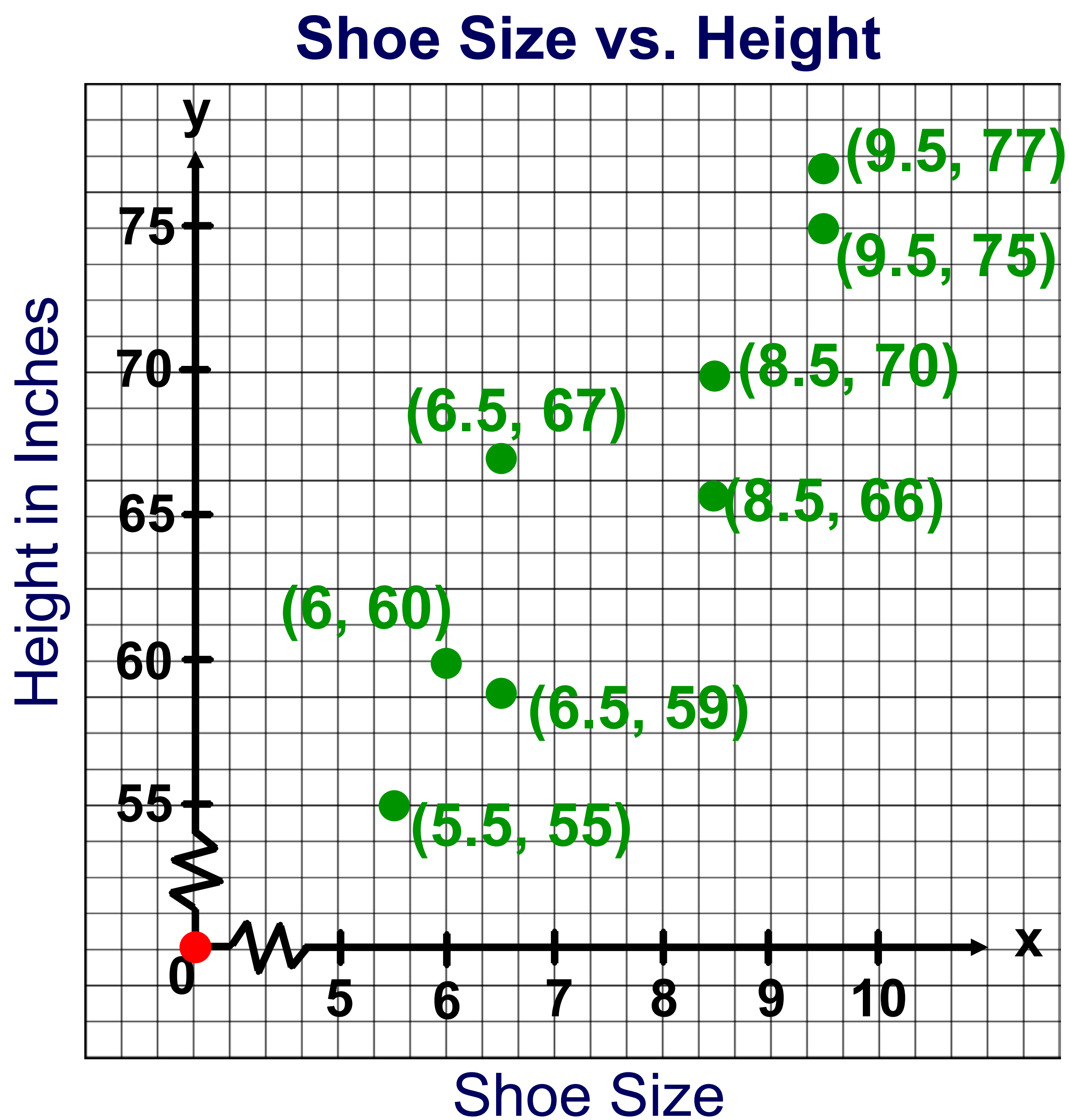
# Shoe Size vs. Height

Draw the line of best fit for our data.

Determine the equation for the line of best fit.

Predict the height of a person who wears a size 8 shoe.

Predict the shoe size of a person who is 50 inches tall.




Math Practice

# Shoe Size vs. Height

Draw the line of best fit for our data.
- Click on the graph to reveal the line of best fit.

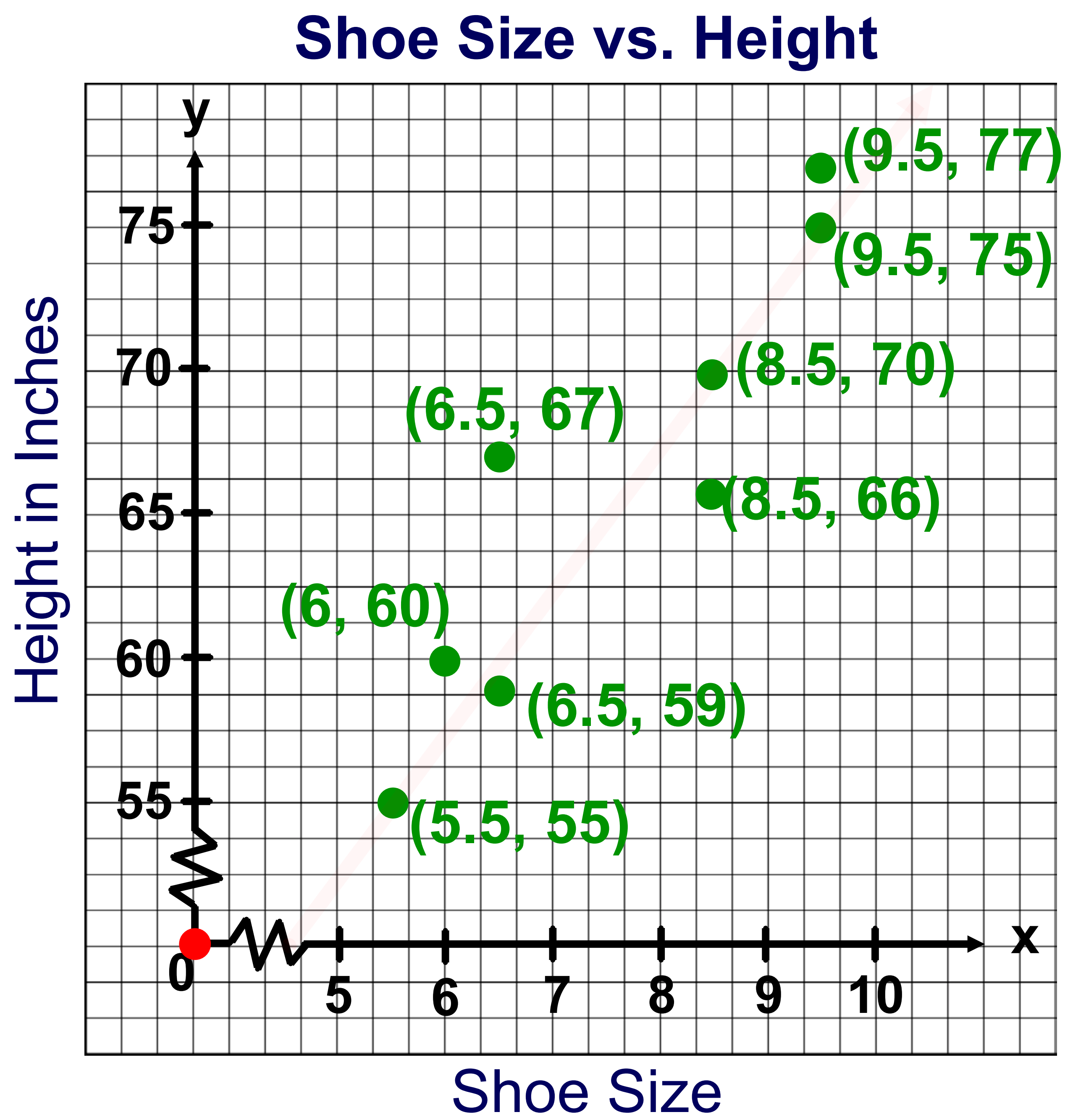
Determine the equation for the line of best fit.

Find the slope

*slide to reveal*

Find the equation of the line

*slide to reveal*



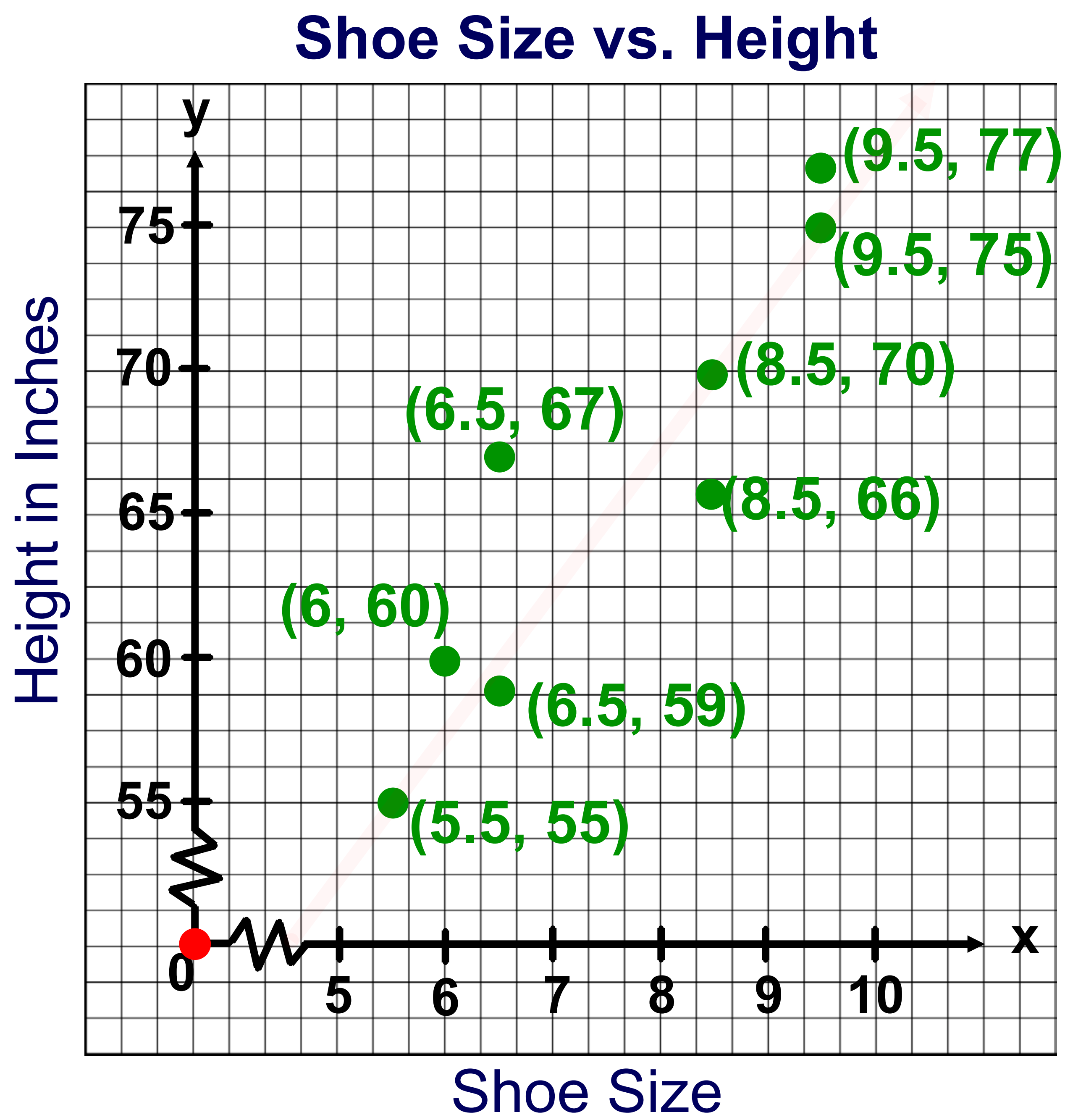
# Shoe Size vs. Height

Predict the height of a person who wears a size 8 shoe.

*slide to reveal*

Predict the shoe size of a person who is 50 inches tall.

*slide to reveal*



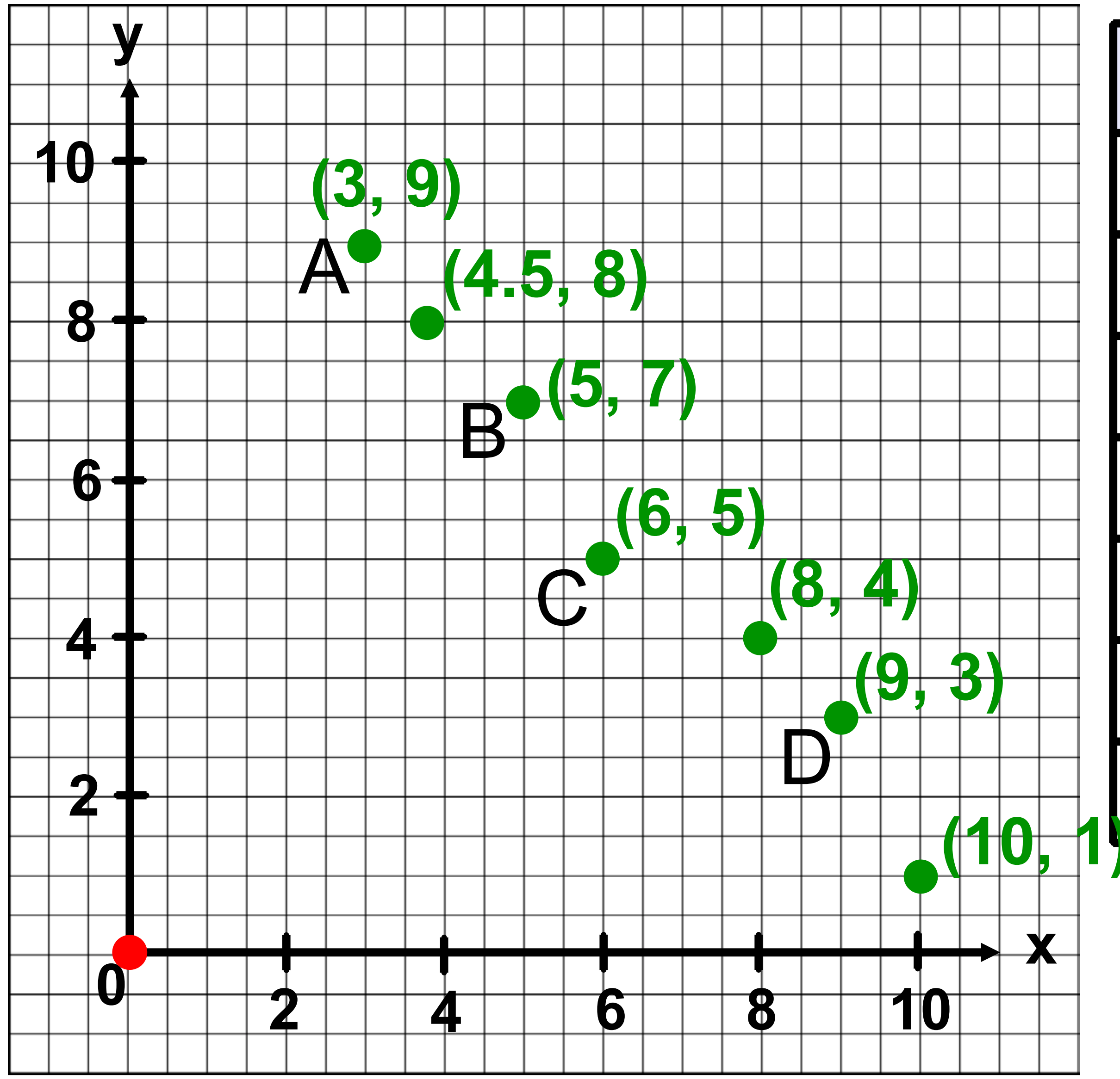
99 Consider the scatter graph to answer the following: Which 2 points would give the best line of fit?

A A and D

B B and C

C C and D

D there is no pattern




| X | Y |
|---|---|
| 3 | 9 |
| 4.5 | 8 |
| 5 | 7 |
| 6 | 5 |
| 8 | 4 |
| 9 | 3 |
| 10 | 1 |

Answer

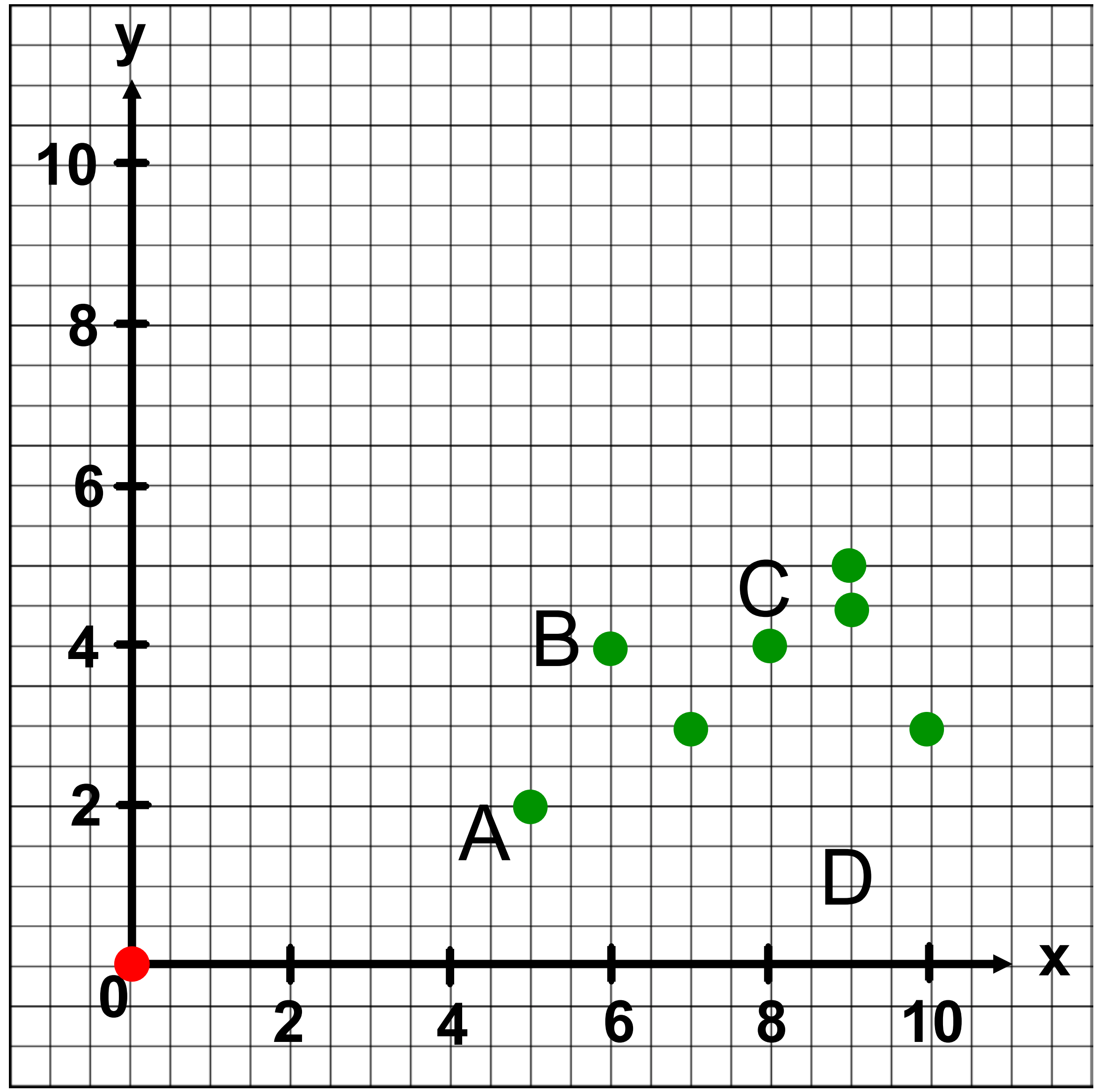
100 Consider the scatter graph to answer the following: Which 2 points would give the best line of fit?

A A and D

B B and C

C C and D

D there is no pattern




| X | Y |
|---|---|
| 5 | 2 |
| 6 | 4 |
| 7 | 3 |
| 8 | 4 |
| 9 | 4.5 |
| 9 | 5 |
| 10 | 3 |

Answer

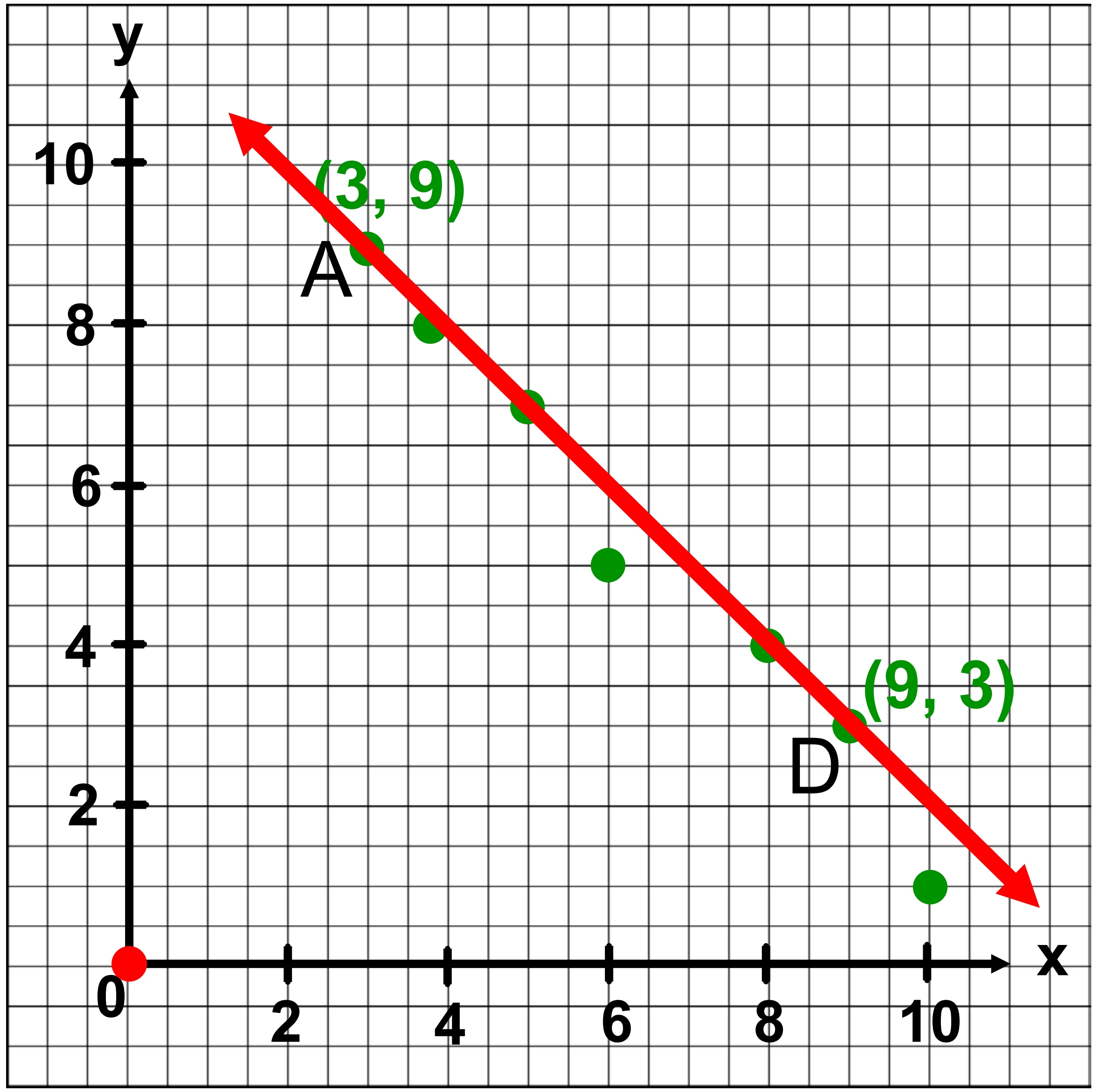
101 Consider the scatter graph to answer the following: What is the slope of the line of best fit going through A and D?

A $\frac{3}{4}$

B $\frac{-3}{4}$

C 1

D $-1$




| X | Y |
|---|---|
| 3 | 9 |
| 5 | 7 |
| 6 | 5 |
| 8 | 4 |
| 9 | 3 |
| 10 | 1 |

Answer

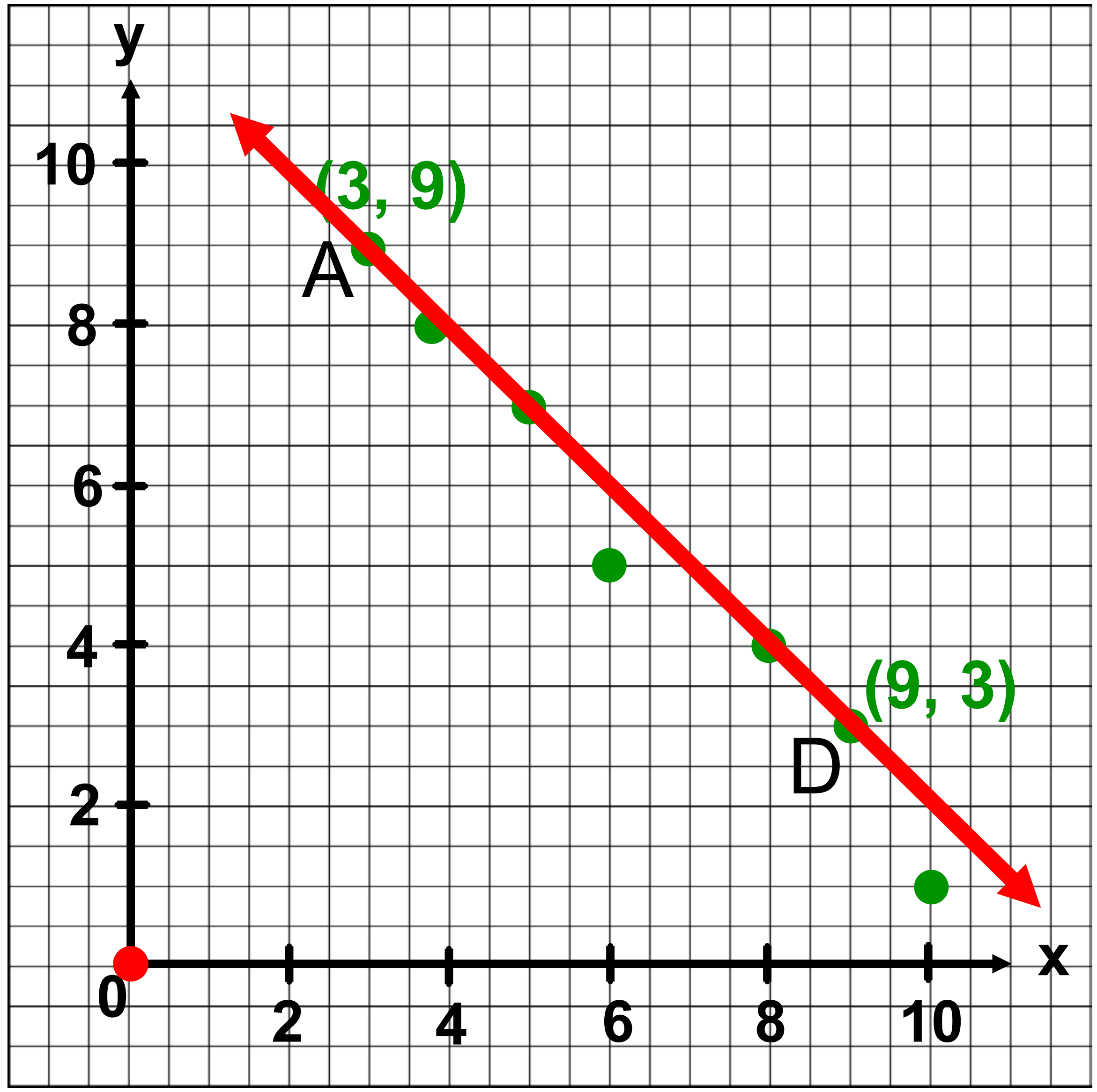
102 Consider the scatter graph to answer the following: What is the y-intercept of the line of best fit going through A and D?

A 9

B 10

C 11

D 12




| X | Y |
|---|---|
| 3 | 9 |
| 5 | 7 |
| 6 | 5 |
| 8 | 4 |
| 9 | 3 |
| 10 | 1 |

Answer

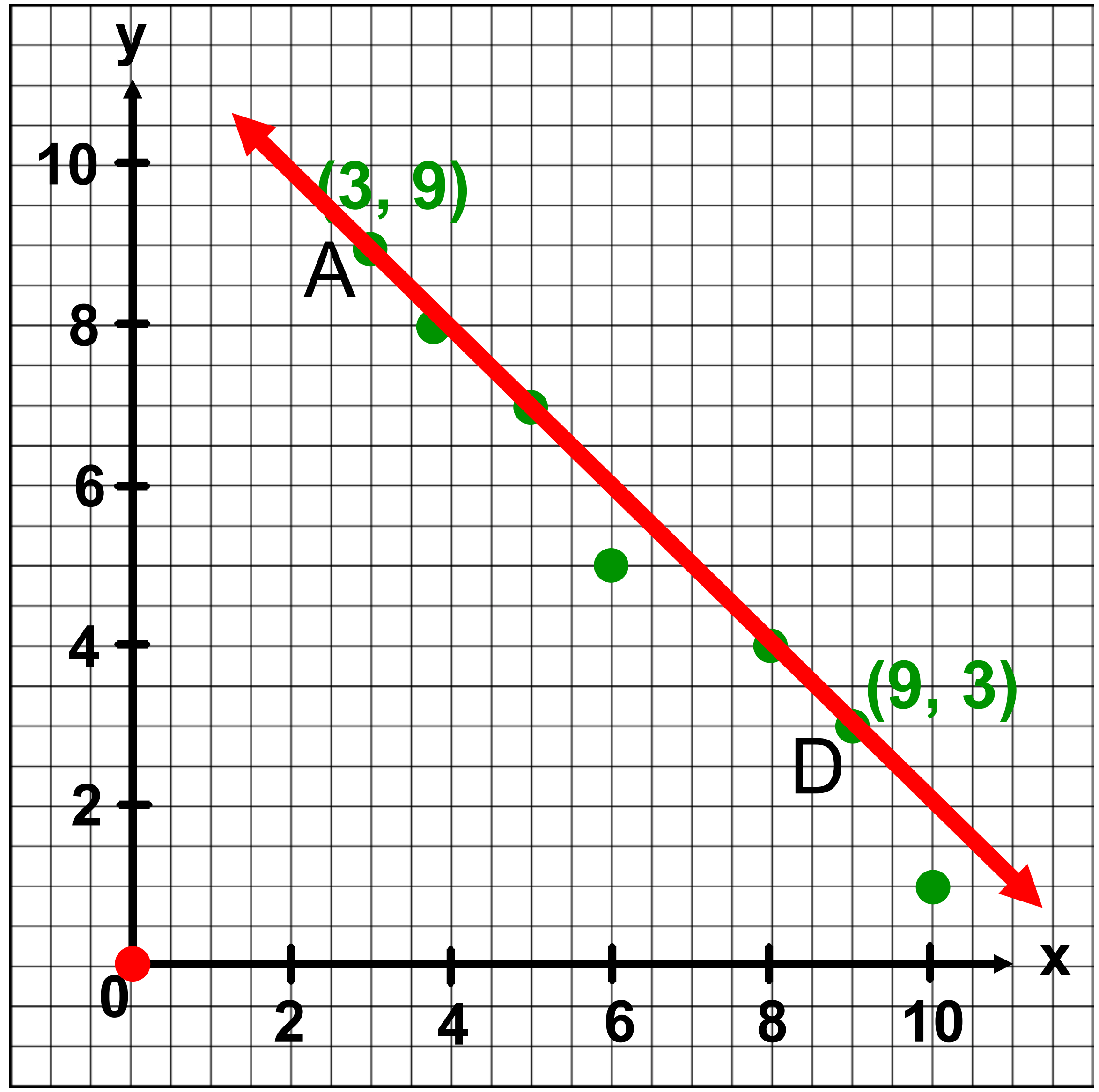
103 Consider the scatter plot to answer the following:
Using the line of best fit shown in the graph below, what would the prediction be if x = 7? Is this an interpolation or extrapolation?

A 5, interpolation

B 5, extrapolation

C 6, interpolation

D 6, extrapolation




Answer

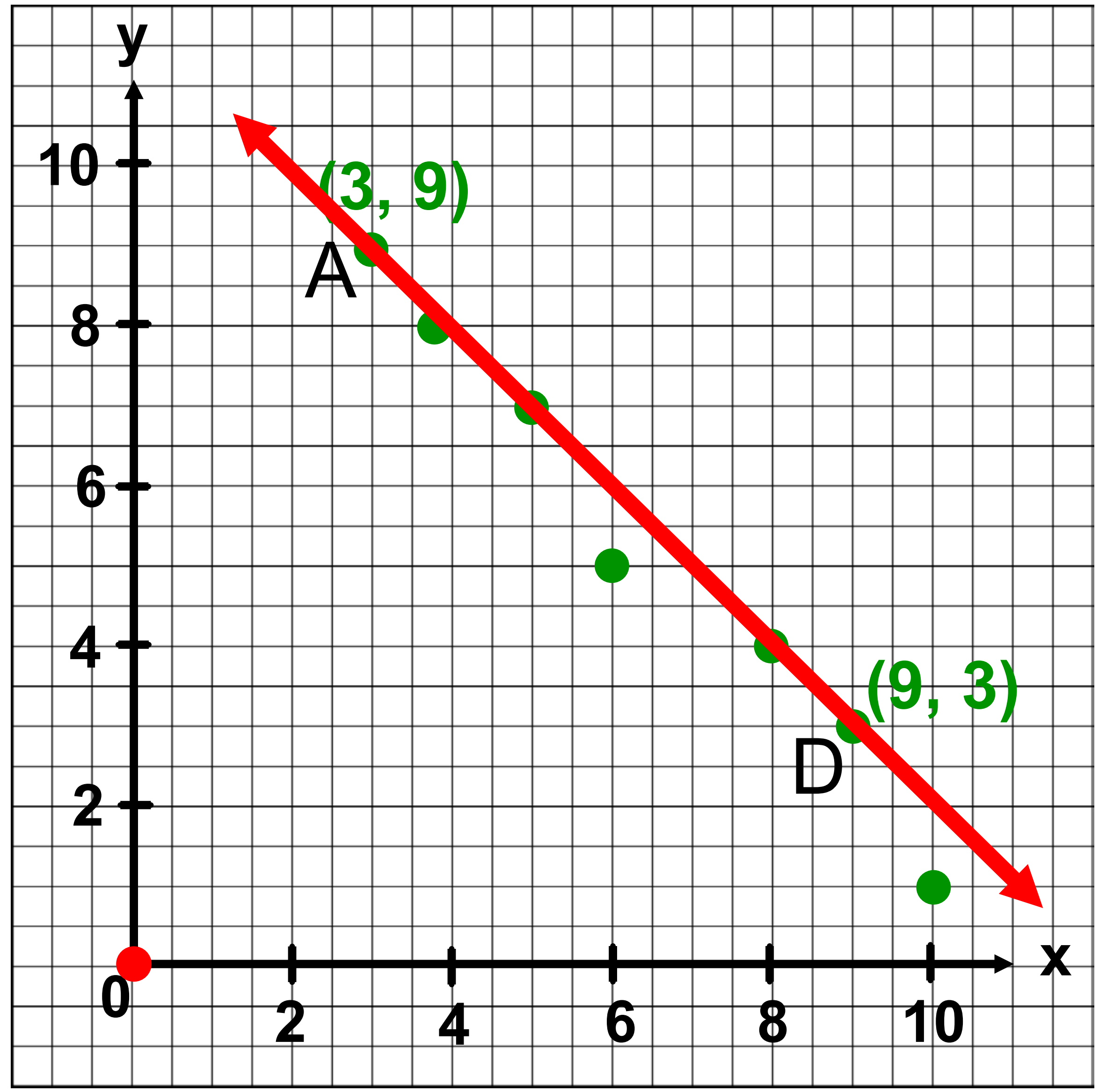
104 Consider the scatter graph to answer the following: Using the line of best fit shown in the graph below, what would the prediction be if x = 14? Is this an interpolation or extrapolation?

A –4, interpolation

B –4, extrapolation

C –2, interpolation

D –2, extrapolation




Answer

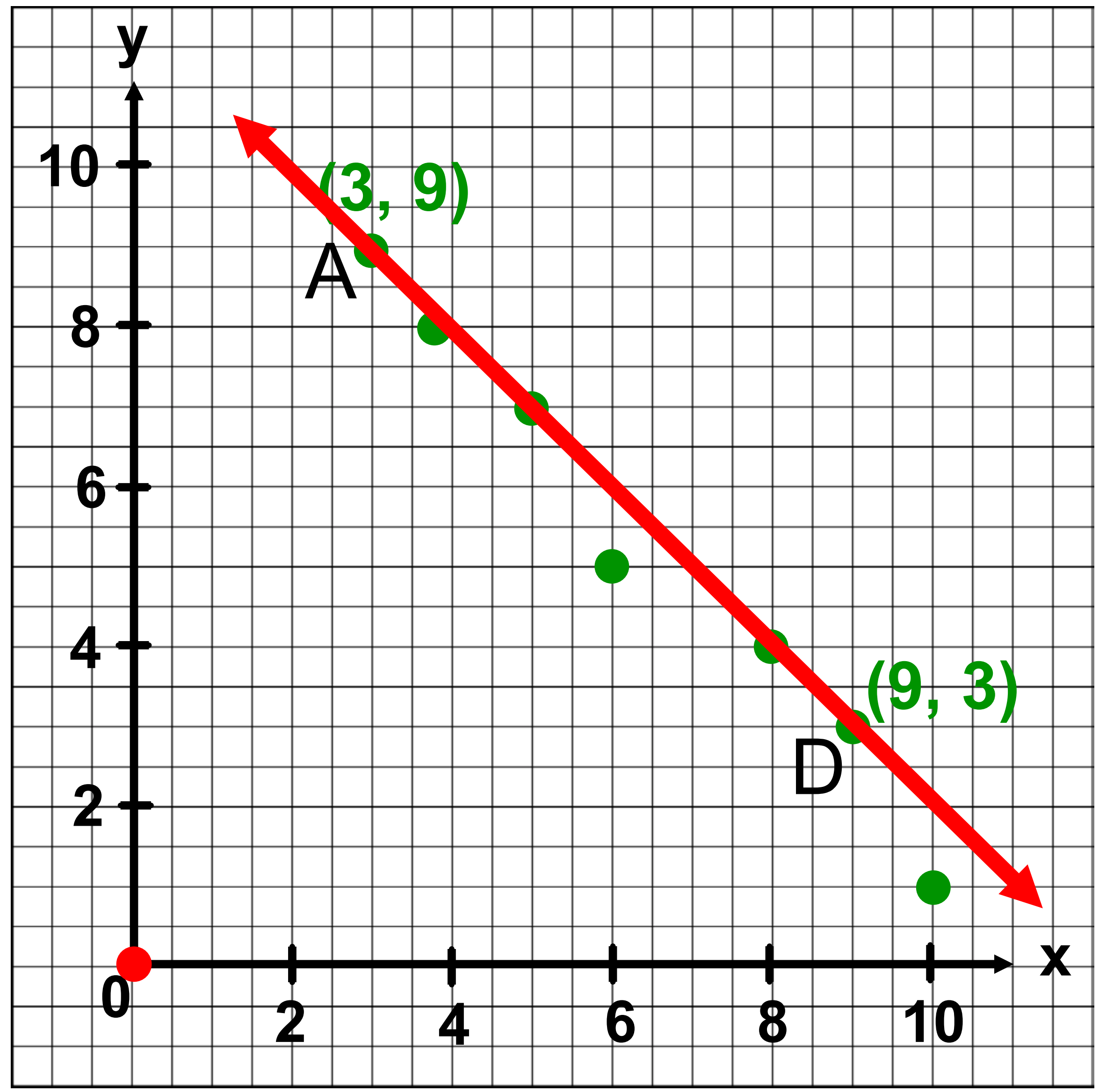
105 Consider the scatter graph to answer the following: Using the line of best fit shown in the graph below, what would the prediction be if y = 11? Is this an interpolation or extrapolation?

A 1, interpolation

B 1, extrapolation

C 2, interpolation

D 2, extrapolation




Answer

106 In the previous questions, we began by using the table at the right. Which of the predicted values (7,5) or (14, –2) will be more accurate and why?

A (7, 5); it is an interpolation

B (7, 5); there already is a 5 and a 7 in the table

C (14, -2) it is an extrapolation

D (14, -2); the line is going down and will become negative

| X | Y |
|---|---|
| 3 | 9 |
| 5 | 7 |
| 6 | 5 |
| 8 | 4 |
| 9 | 3 |
| 10 | 1 |

Answer

## Candle Lab

Students measure the height of a burning candle, graph the data and find the line of best fit.

# PARCC Sample Questions

The remaining slides in this presentation contain questions from the PARCC Sample Test. After finishing this unit, you should be able to answer these questions.

Good Luck!

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107 Which points are on the graph of the equation $-3x + 6y + 5 = -7$? Select all that apply.

A (-3, 6)

B (-2, 0)

C (0, -2)

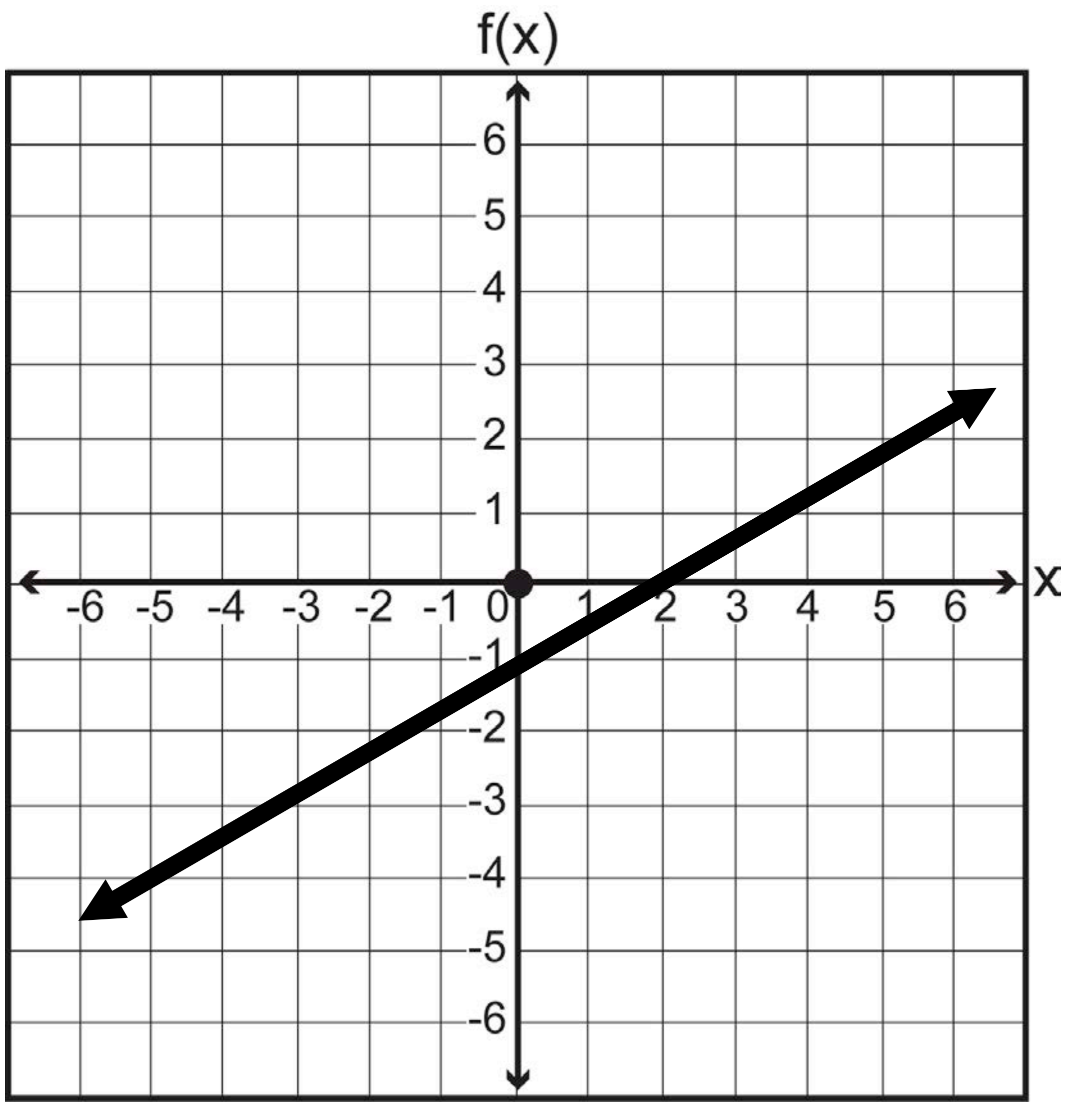
D (6, -3)

E (8, 2)

Answer

**From PARCC PBA sample test non-calculator #7**

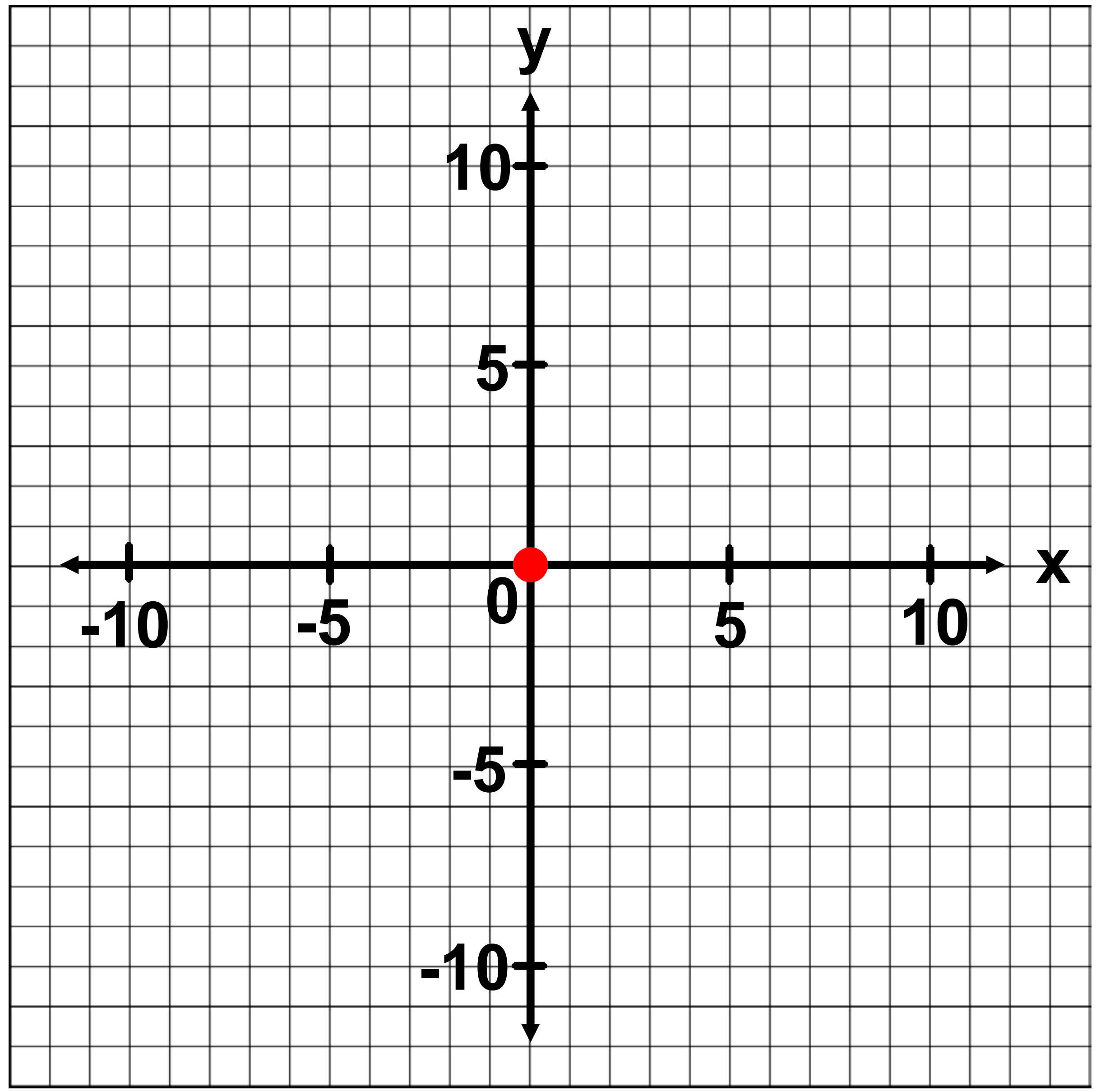
108 The graph of the function $f(x) = -1 + 0.5x$ is shown on the coordinate plane. For what value of x does $f(x) = 0$?




Answer

**From PARCC PBA sample test non-calculator #5**

109 Graph the equation $6x - 4y = 12$ on the xy-coordinate plane. Identify the x-intercept of the graph and the y-intercept of the graph. When you finish, type in the number that represents the x-intercept.



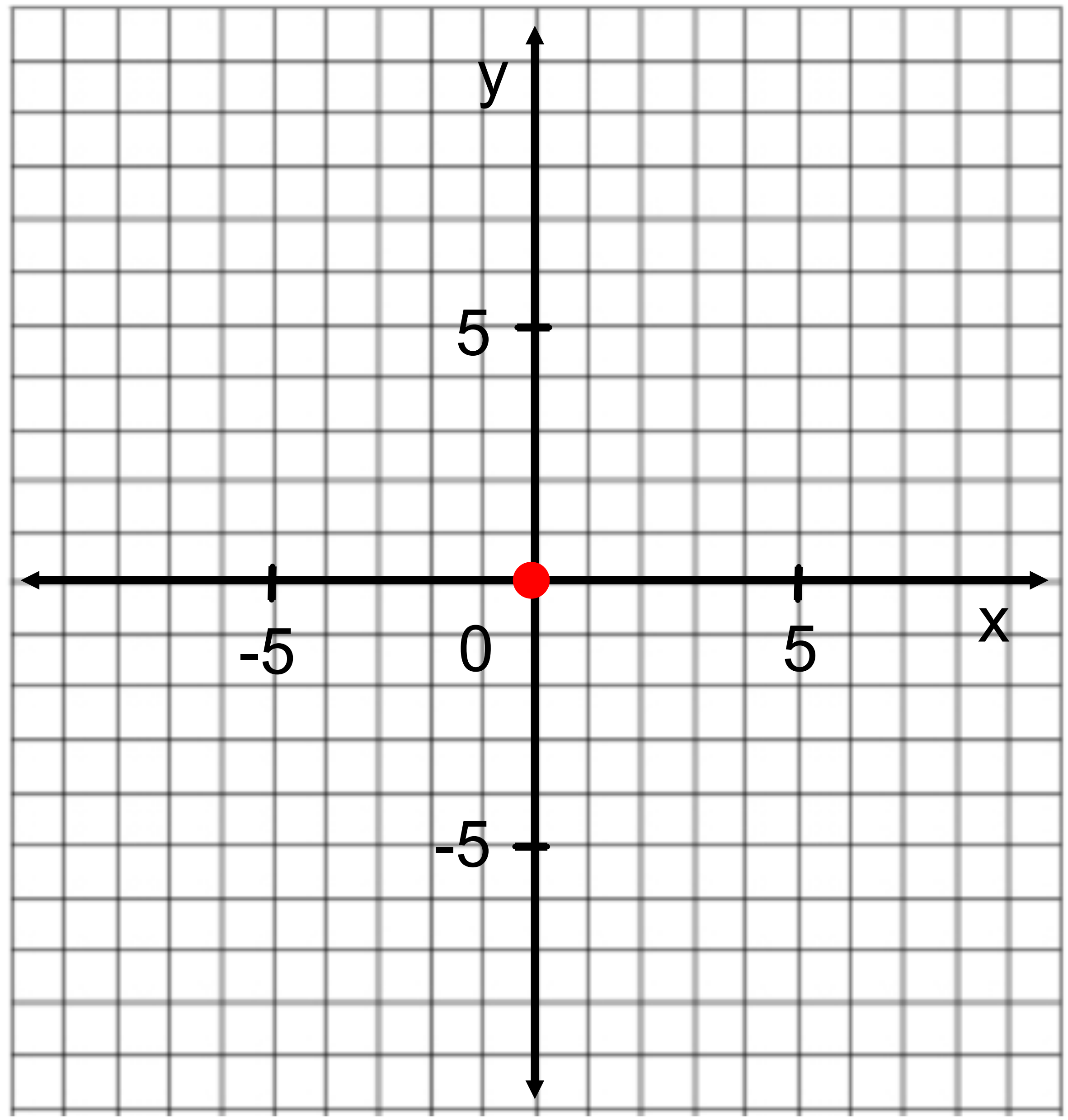

Answer

**From PARCC PBA sample test non-calculator #7**

110 Consider the three points (-4, -3), (20, 15), and (48, 36).

**Part A**

Graph the line that passes through these three points on the coordinate plane. When you finish graphing, type in the number that represents the y-intercept.




Answer

**From PARCC PBA sample test calculator #3**

111 Consider the three points (-4, -3), (20, 15), and (48, 36).

**Part B**

Use the graph in Part A to explain why the ratio of the y-coordinate to the x-coordinate is the same for any point on the line except the y-intercept.

Explain why this is not true for the y-intercept. When you finish writing your answer, type in the number "1" using your SMART Responder.

Answer

**From PARCC PBA sample test calculator #3**

112 Consider the three points (-4, -3), (20, 15), and (48, 36).

**Part C**

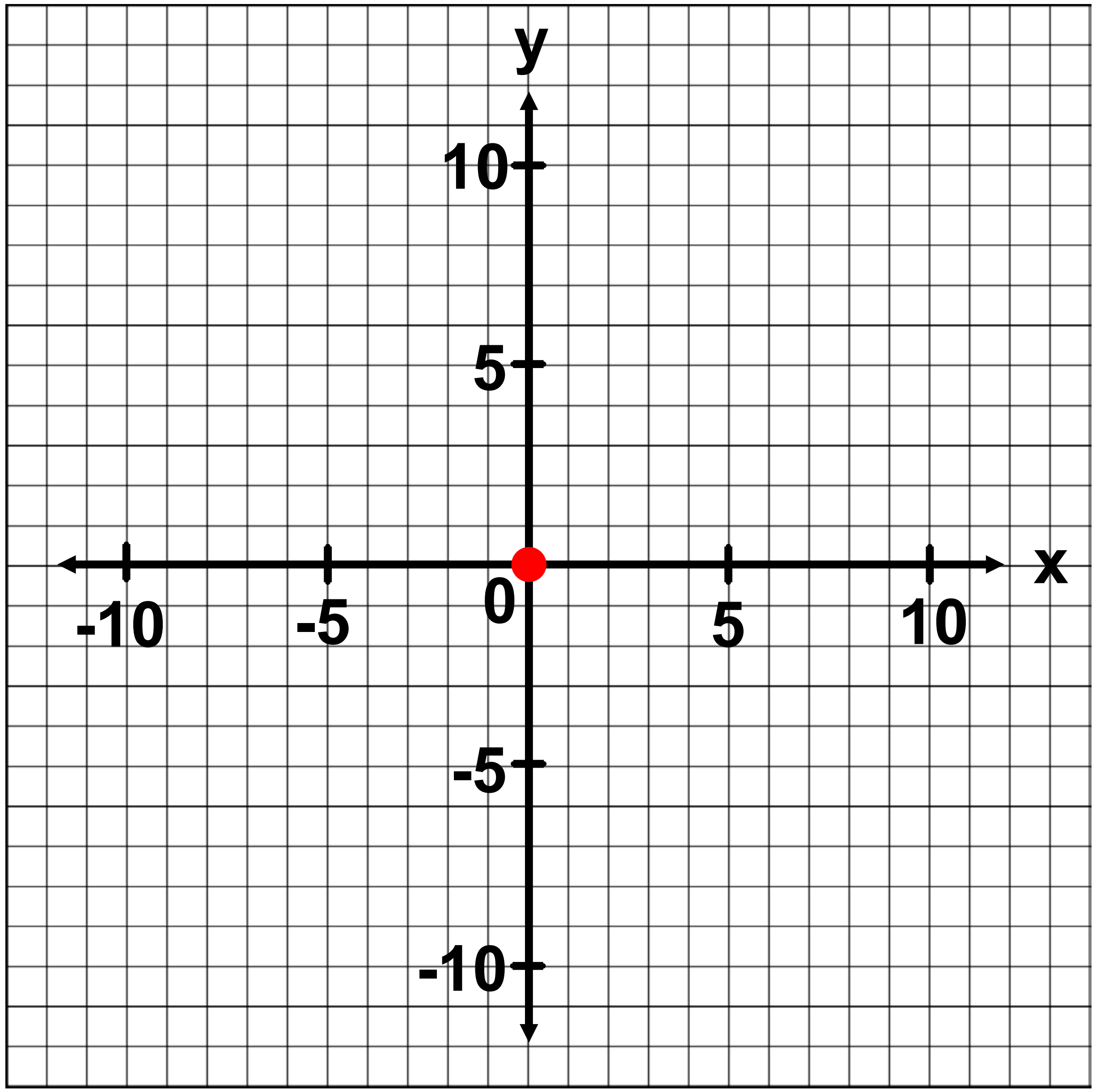
Do the points on the line $y = 3x - 2$ have a constant ratio of the y-coordinate to the x-coordinate for any point on the line except the y-intercept? Explain your answer. When you finish writing your answer, type in the number "1" using your SMART Responder.

Answer

**From PARCC PBA sample test calculator #3**

113 The ordered pairs (20, -29.5), (21, -31) and (22, -32.5) are points on the graph of a linear equation. Graph the line that shows all of the ordered pairs in the solution set of this linear equation.

After you finish graphing the line, type in the number "1" in your SMART Responder.




Answer

**From PARCC EOY sample test calculator #10**

# Glossary and Standards

Teacher Notes

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Contents

# Constant Rate of Change

A rate that describes how one quantity changes in relation to another. This rate never changes.

| | | |
|---|---|---|
| $\frac{\Delta y}{\Delta x}$ | $\frac{\text{60 miles}}{\text{hour}}$ | $\frac{\$2.35}{\text{gallon}}$ |

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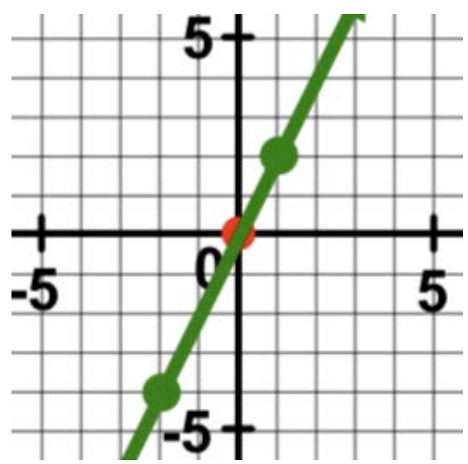
# Direct Variation

A relationship between two variables in which one is a constant multiple of the other. When one variable changes the other changes in proportion to the first.

| | | |
|---|---|---|
| y = mx |  | Goes through the origin.<br>(0,0) |

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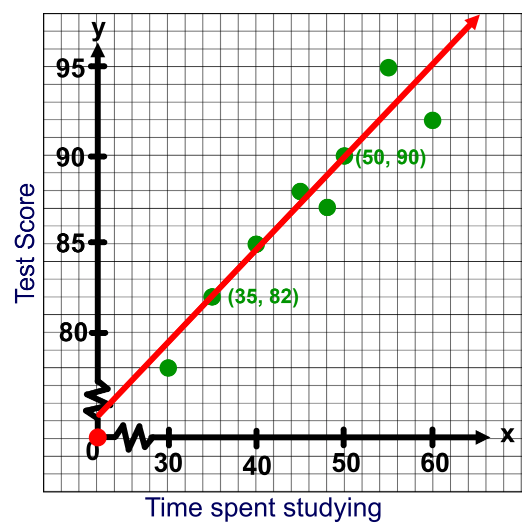
# Extrapolation

A data point that is outside the range of data.




If you study for 25 minutes, what would be your predicted score?

$$y = \frac{8}{15}(25) + \frac{190}{3}$$

$$y = \frac{40}{3} + \frac{190}{3}$$

$$76.\overline{6} < 80$$

If you study for 65 minutes, what would be your predicted score?

$$y = \frac{8}{15}(65) + \frac{190}{3}$$

$$y = \frac{104}{3} + \frac{190}{3}$$

$$98 > 95$$

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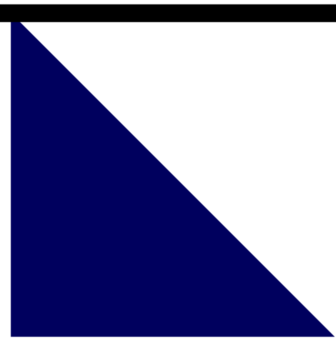
# Grade

A unit engineers use to measure the steepness of a hill.

5 feet

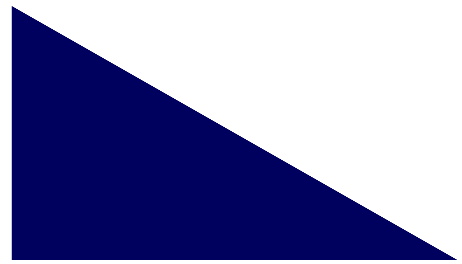


10 feet

$\frac{5}{10} = 0.5 = 50\%$

grade of hill is 50%.

3 meters



25 meters

grade of hill is $\frac{3}{25} = 12\%$

The sign warns cars the hill has a grade of 7%.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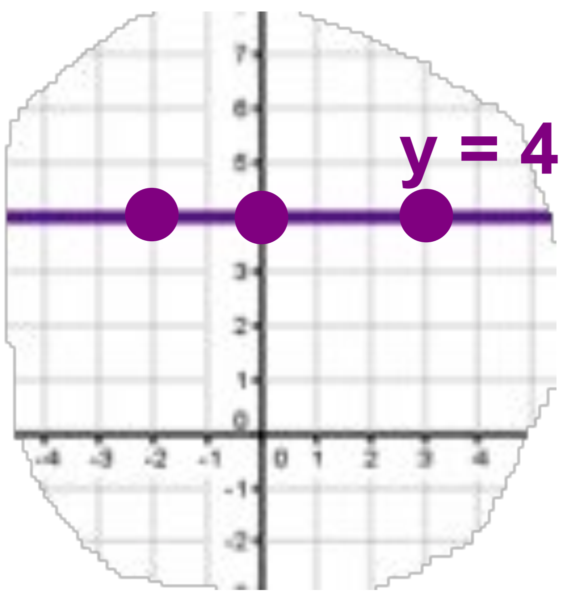
# Horizontal Line

A line whose direction is left and right.

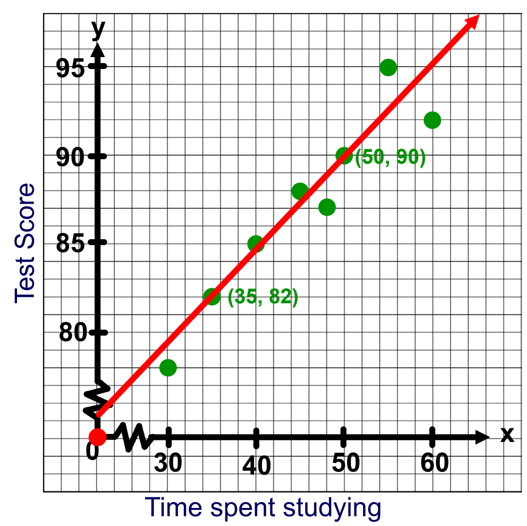
All of the y-coordinates on the line are equal.

| | | |
|---|---|---|
|  | (3, 4)<br>(0, 4)<br>(-2, 4) | $y = 3$<br>$y = -2$<br>$y = \frac{1}{5}$ |

Back to Instruction

# Interpolation

A data point that is inside the range of data.




If you study for 40 minutes, what would be your predicted score?

$$y = \frac{8}{15}(40) + \frac{190}{3}$$

$$y = \frac{64}{3} + \frac{190}{3}$$

$$80 < 84.\overline{6} < 95$$

If you study for 55 minutes, what would be your predicted score?

$$y = \frac{8}{15}(55) + \frac{190}{3}$$

$$y = \frac{88}{3} + \frac{190}{3}$$

$$80 < 92.\overline{6} < 95$$

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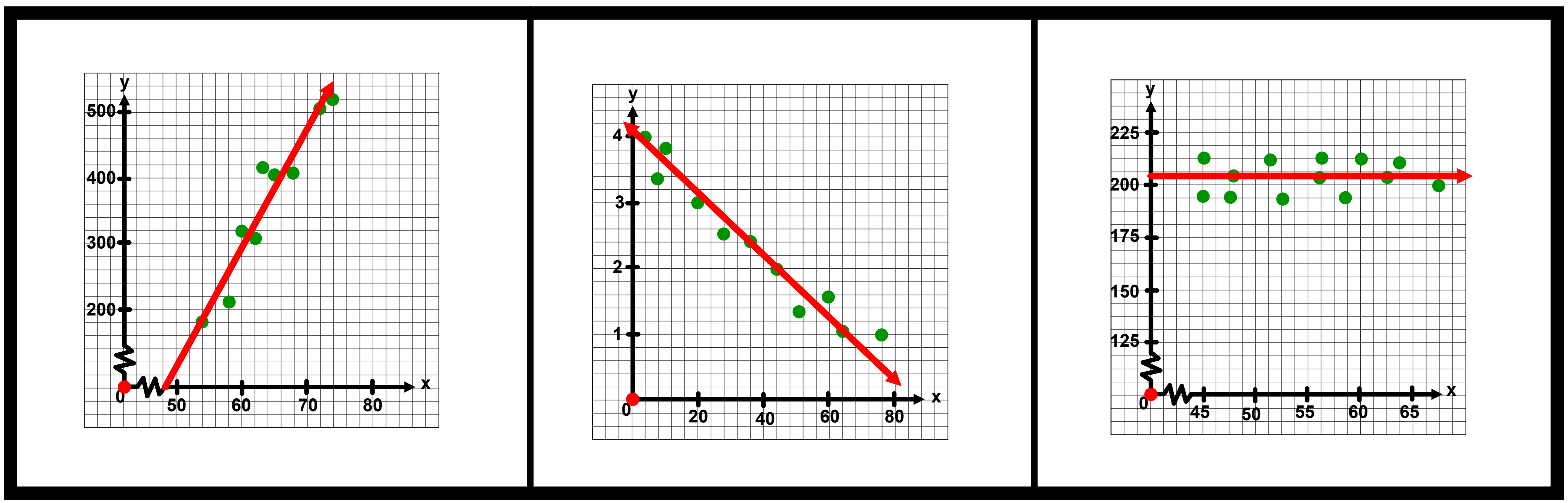
# Line of Best Fit

A line on a graph showing the general direction that a group of points seem to be heading. Trend line.

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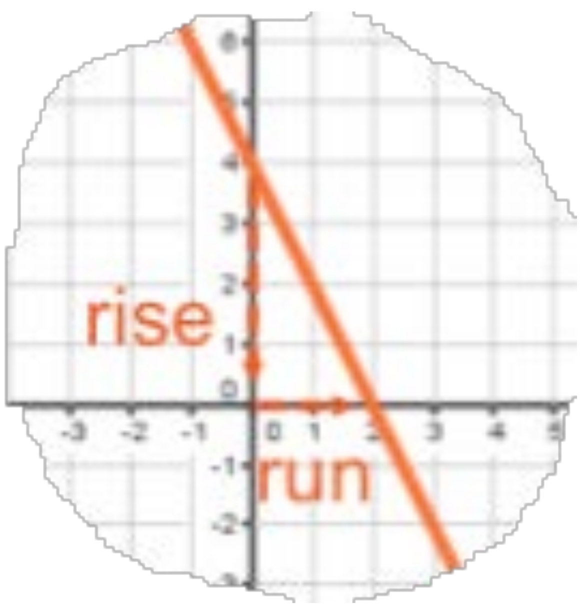
# Negative Slope

When a line falls down from left to right.




negative slope

$$m = \frac{-a}{b} = \frac{a}{-b}$$

$$= -\frac{a}{b}$$

$-\frac{3}{2}$ $\quad$ $-9$

$\frac{-1}{5}$

$\frac{1}{-5}$ $\quad$ $-\frac{3}{2}$

$-4$

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# Point-Slope Form

The point-slope equation for a line is

$$y - y_1 = m\,(x - x_1)$$

where m is the slope and $(x_1, y_1)$ is a point on the line.

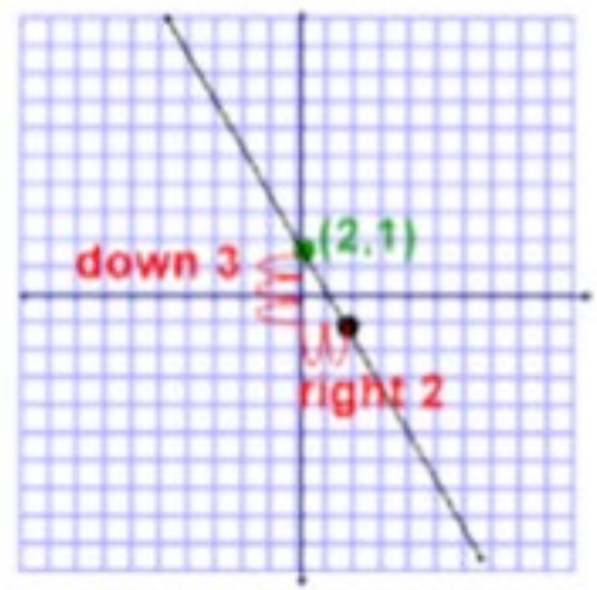
1) Plot $(x_1, y_1)$

$$y - 1 = \frac{-3}{2}(x - 2)$$

(2, 1)

$$y - 1 = \frac{-3}{2}(x - 2)$$

$$m = \frac{-3}{2} : \frac{\textbf{down 3}}{\textbf{right 2}}$$

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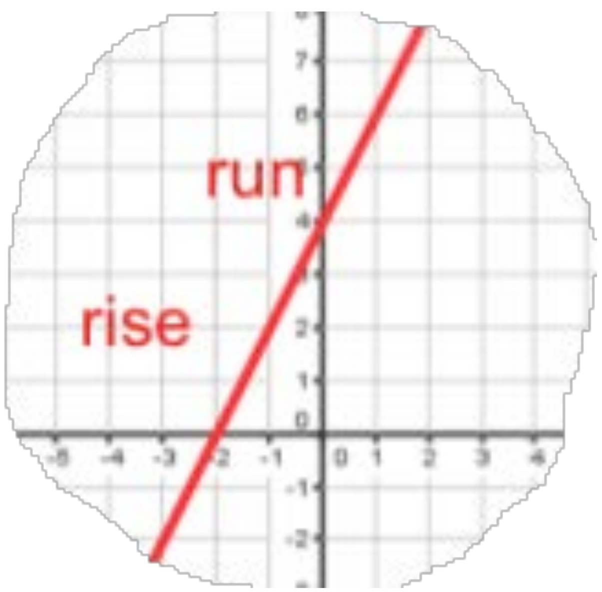
# Positive Slope

When a line rises from left to right.

| | | |
|---|---|---|
|  | positive slope<br>$m = \frac{a}{b} = \frac{-a}{-b}$ | $\frac{3}{2}$ 9 $\frac{1}{5}$ $\frac{-1}{-5}$ $\frac{-3}{-2}$ 4 |

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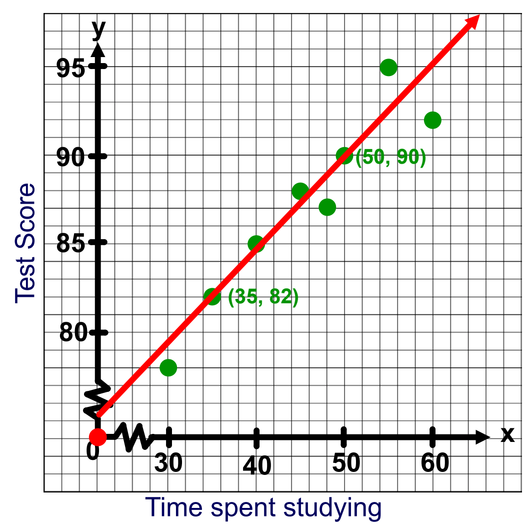
# Prediction Equation

An equation that is created using the line of best fit. A line that can predict outcomes using the given data.




$$m = \frac{90 - 82}{50 - 35}$$

$$m = \frac{8}{15}$$

$$y - 90 = \frac{8}{15}(x - 50)$$

$$y - 90 = \frac{8}{15}x - \frac{80}{3}$$

$$\mathbf{y = \frac{8}{15}x + \frac{190}{3}}$$

**If you study for 55 minutes, what would be your predicted score?**

$$\mathbf{y = \frac{8}{15}(55) + \frac{190}{3}}$$

$$\mathbf{y = \frac{88}{3} + \frac{190}{3}}$$

$$\mathbf{92.\overline{6}}$$

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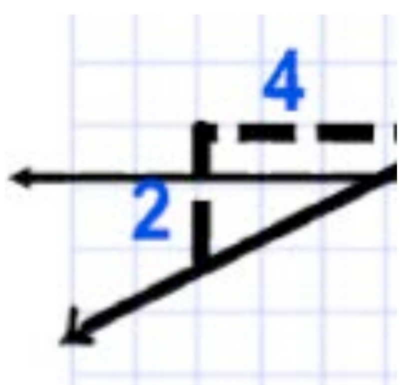
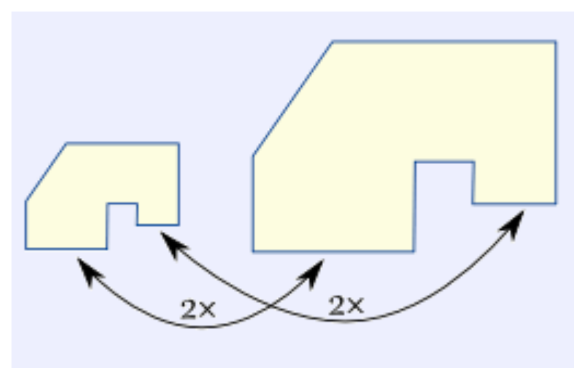
# Proportional Relationship

When two quantities have the same relative size.

$\frac{2}{4} = \frac{1}{2}$




if weight is proportional to age, then a weight of 3kg on the 1st day means it will weigh 6kg on the 2nd day, 9kg on the 3rd day, 30kg on the 10th day, etc.

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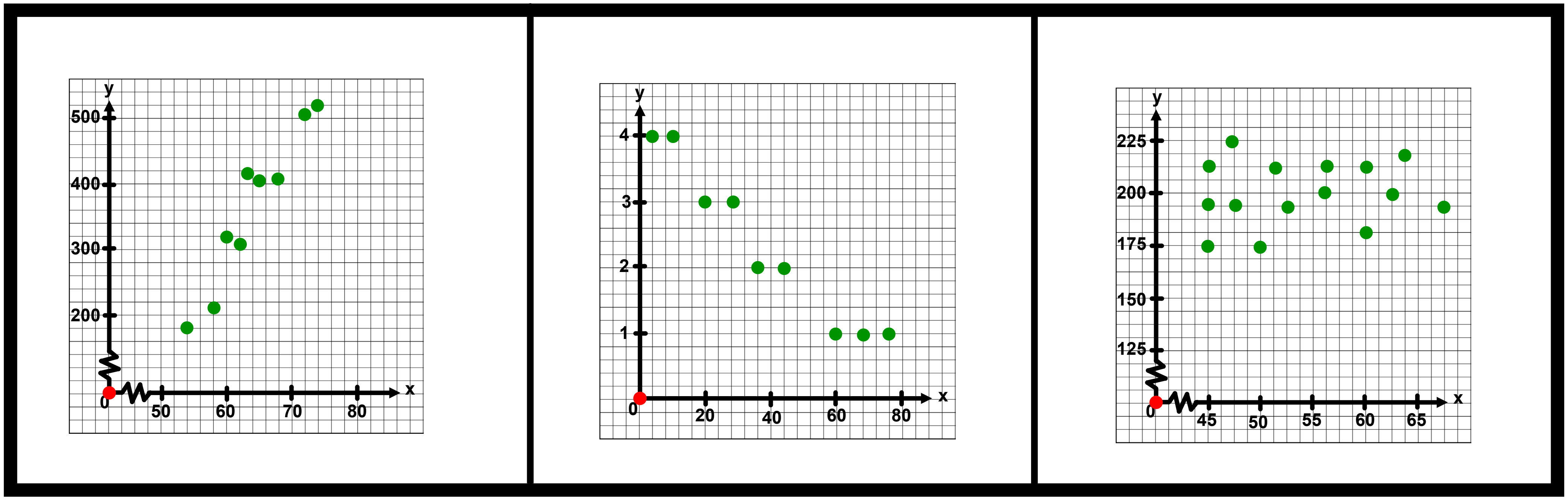
# Scatter Plot

A graph of plotted points that show the relationship between two sets of data.

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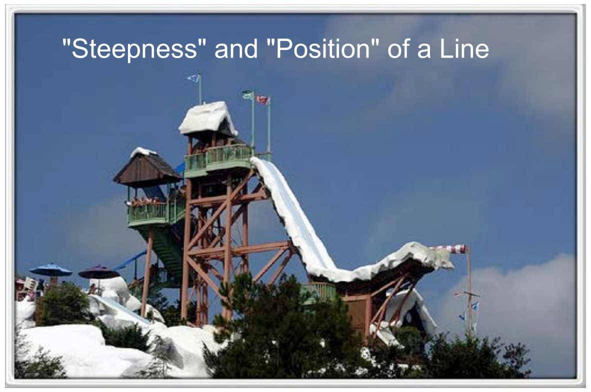
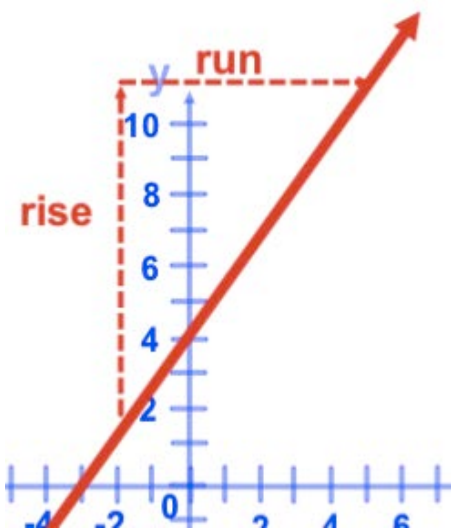
# Slope

How much a line rises or falls.
Steepness of a line.
The ratio of a line's rise over its run.

| | | |
|---|---|---|
| $y = mx + b$<br>"m" = slope or how the line "moves"<br>formula for slope:<br>$m = \frac{y_2 - y_1}{x_2 - x_1}$ | <br> | <br> |

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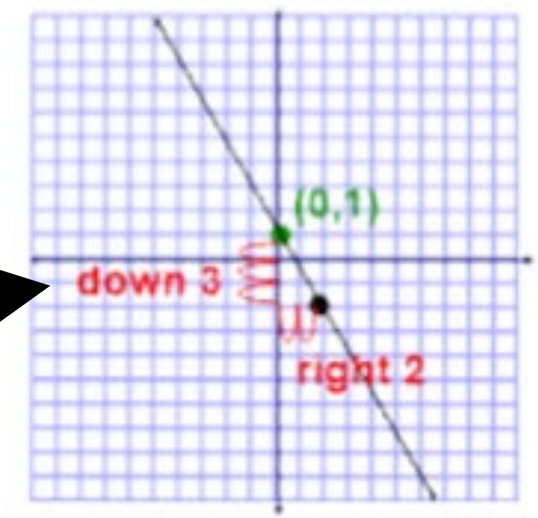
# Slope-Intercept Form

One type of straight line equation that utilizes the slope and y-intercept to graph.

| | |
|---|---|
| $y = \mathbf{m}x + \mathbf{b}$<br>**slope "moves"**<br>**y-int: (0,b) where the line "begins"** | $y = \frac{-3}{2}x + 1$<br>$m = \frac{-3}{2}$ *"down 3, right 2"*<br>y-int: **(0,1)** |

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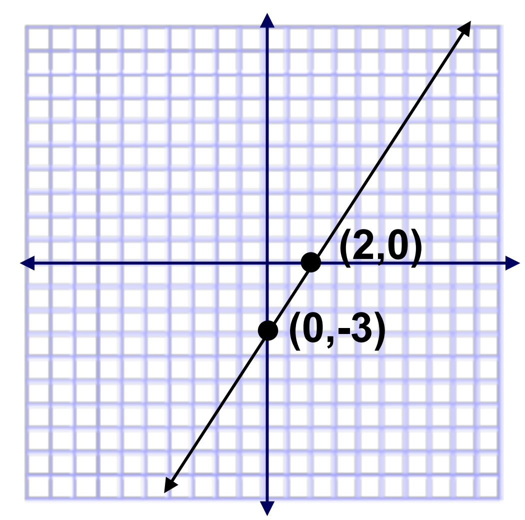
# Standard Form

Standard form looks like

$Ax + By = C$,

where A, B, and C are integers and A > 0.

| | | |
|---|---|---|
| $-3x \boxed{\quad} = -6$<br>$-3x = -6$<br>$x = 2$<br>$(2,0)$ | $\boxed{\quad}\ 2y = -6$<br>$2y = -6$<br>$y = -3$<br>$(0,-3)$ |  |

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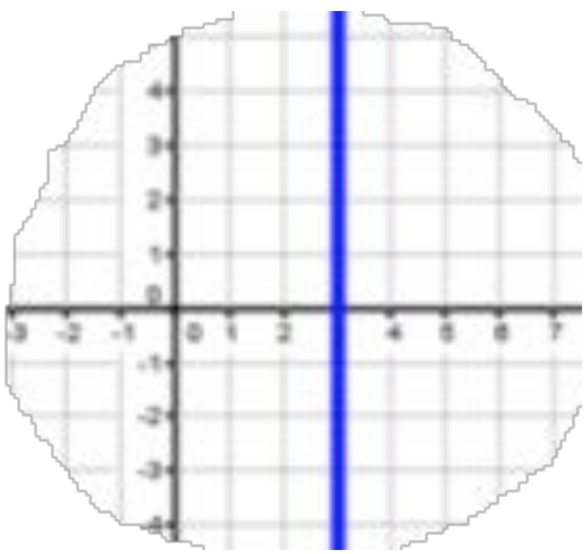
# Undefined Slope

When a line does not run at all as one reads from bottom to top on the y-axis.

| | | |
|---|---|---|
|  | $\frac{\text{rise}}{0}$ = undefined | $x = 3$<br>$x = -2$<br>$x = \frac{1}{5}$ |

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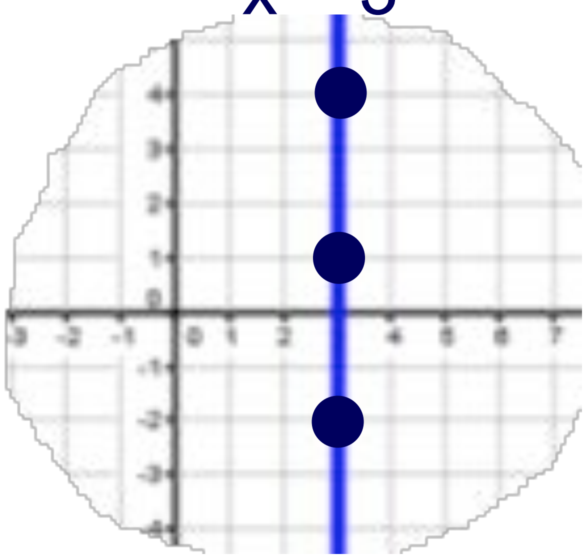
# Vertical Line

A line whose direction is only up and down.

All of the x-coordinates on the line are equal.

| | | |
|---|---|---|
| x = 3  | (3, 4)<br>(3, 1)<br>(3, -2) | $x = 3$<br>$x = -2$<br>$x = \frac{1}{5}$ |

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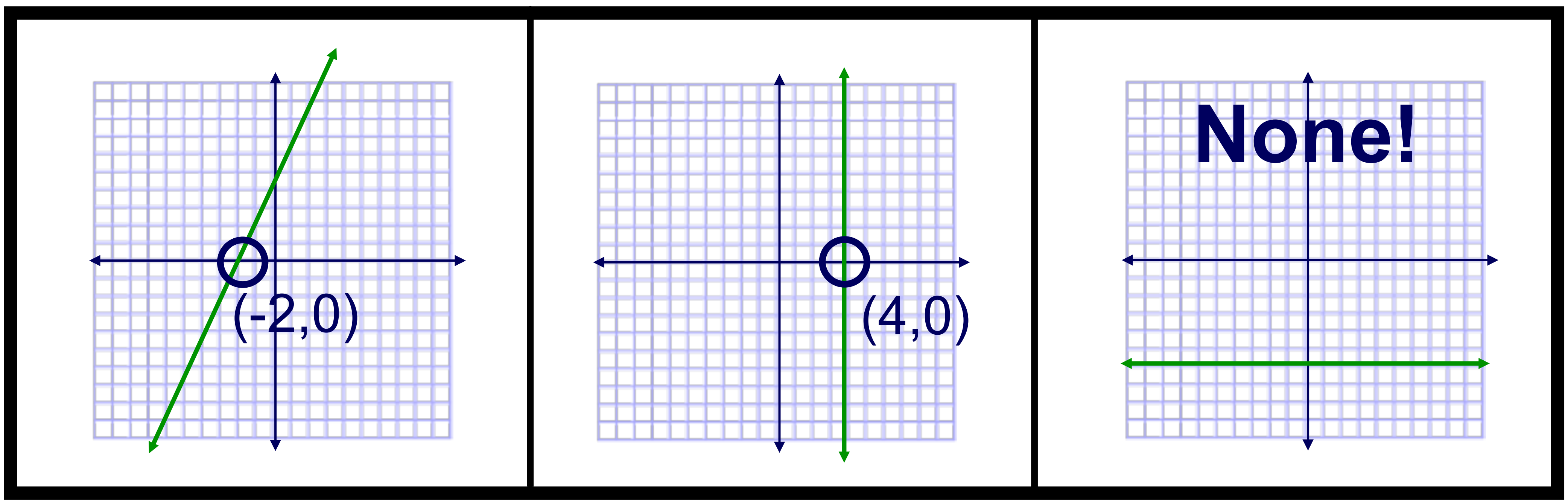
# X-Intercept

Where a line crosses the x-axis.

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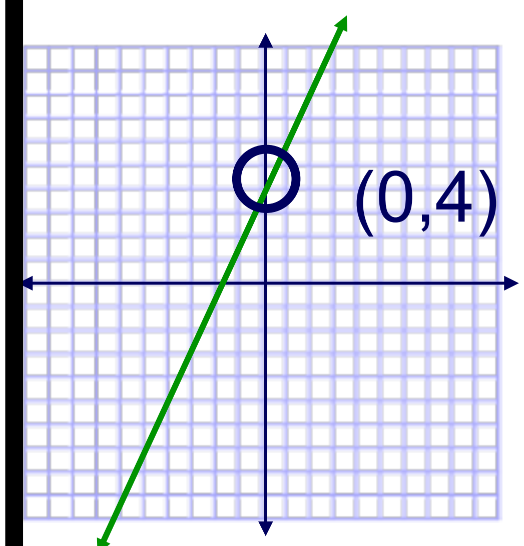
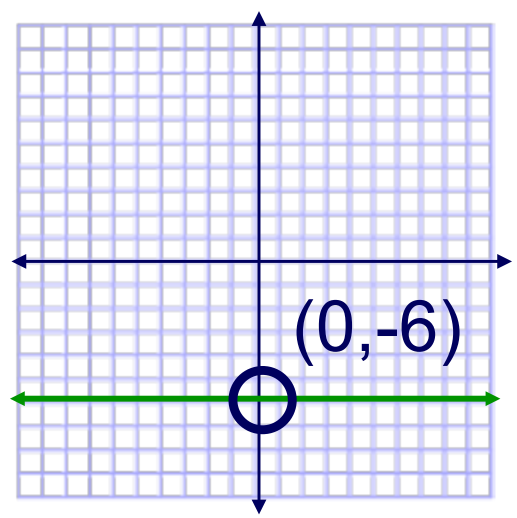
# Y-Intercept

Where a line crosses the y- axis.




$y = mx + b$

**"b" = y-intercept (where it crosses the y-axis)**

(0, b)

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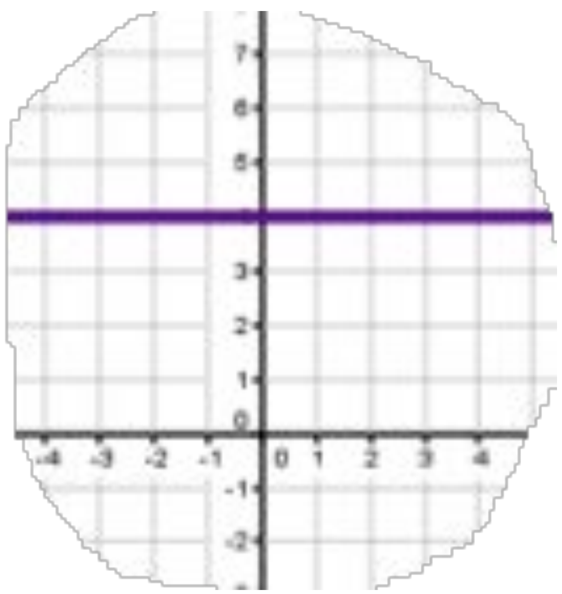
# Zero Slope

When a line does not rise at all as one reads it from left to right on the x-axis.

| | | |
|---|---|---|
|  | $\frac{0}{\text{run}}$ = zero | $y = 3$<br>$y = -2$<br>$y = \frac{1}{5}$ |

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Throughout this unit, the Standards for Mathematical Practice are used.

MP1: Making sense of problems & persevere in solving them.
MP2: Reason abstractly & quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for & make use of structure.
MP8: Look for & express regularity in repeated reasoning.

Additional questions are included on the slides using the "Math Practice" Pull-tabs (e.g. a blank one is shown to the right on this slide) with a reference to the standards used.

If questions already exist on a slide, then the specific MPs that the questions address are listed in the Pull-tab.

Math Practice