Graphing Lines Information Packet:

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Graphing Ordered Pairs

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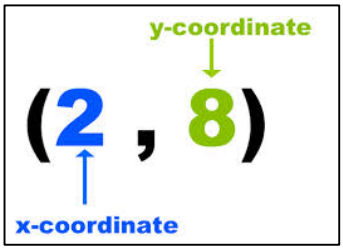
An ordered pair of numbers indicates where a point is on a coordinate plane. When graphing a point, the signs of the numbers indicate which directions to move along the *x*- and *y*-axes, starting from the origin.

Move **right** if positive. Move **left** if negative. — (x, y) — Move **up** if positive. Move **down** if negative.

Tell how to graph each point.

$(3, -5)$ Move 3 right on the *x*-axis and 5 down on the *y*-axis.

$(-4, 2)$ Move 4 left on the *x*-axis and 2 up on the *y*-axis.



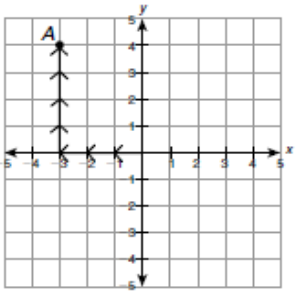
Graph *A*(−3, 4).

Start at the origin (0, 0).

Move 3 spaces to the left.

Move 4 spaces up.

Plot the point.



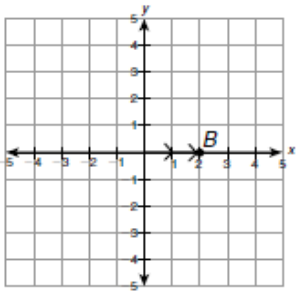
Graph *B*(2, 0).

Start at the origin (0, 0).

Move 2 spaces to the right.

Move 0 spaces up or down.

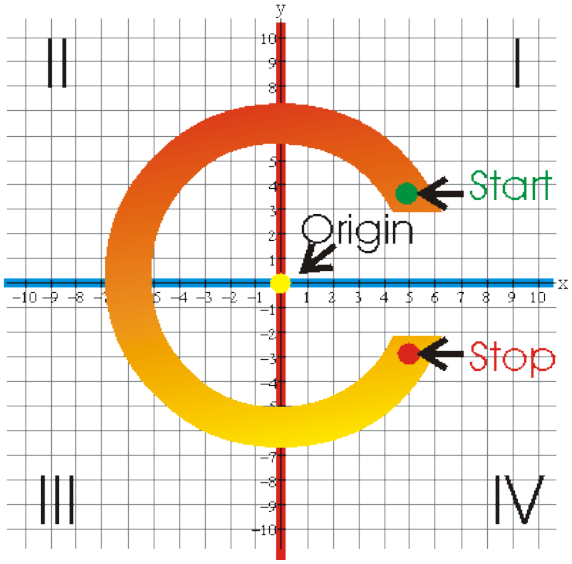
Plot the point.



- **Lattice Point: integer coordinate (where the gridlines intersect)**

Quadrants of The Coordinate Plane

Quadrant 2
all x values are negative
all y values are positive



Quadrant 1
all x values are positive
all y values are positive

Quadrant 3
all x values are negative
all y values are negative

Quadrant 4
all x values are positive
all y values are negative

Return To Start

Slope

Slope: the constant rate of change of the rise (vertical change) to the run (horizontal change).

*Variable is m.

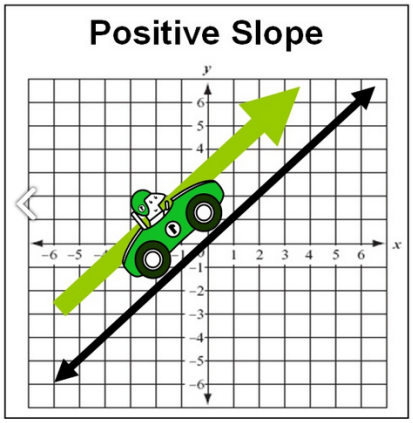
* Put a whole number over 1 to make it into a fraction.

$$4 \sim \frac{4}{1}$$

4 Types of Slope:

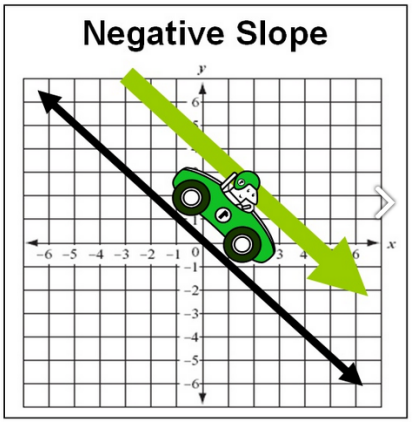
Positive Slope: Rises from left to right

- Examples: $\frac{3}{2}$ *or* 4
- Going up a hill



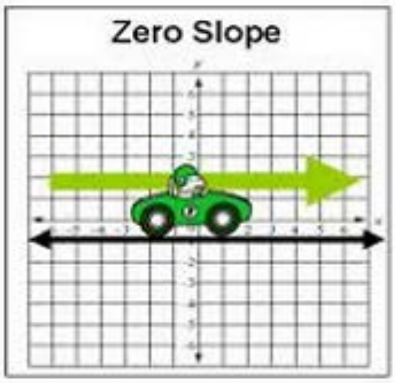
Negative Slope: Falls from left to right

- Examples: $-\frac{1}{2}$ *or* -4
- Going down a hill



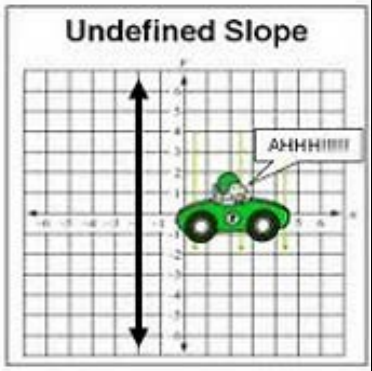
Zero Slope: Horizontal Line

- Y-values are the same
- Going in a straight line (No Vertical Change)



Undefined Slope: Vertical Line

- X-values are the same
- Falling off a cliff (No Horizontal Change)



Slope of a Line

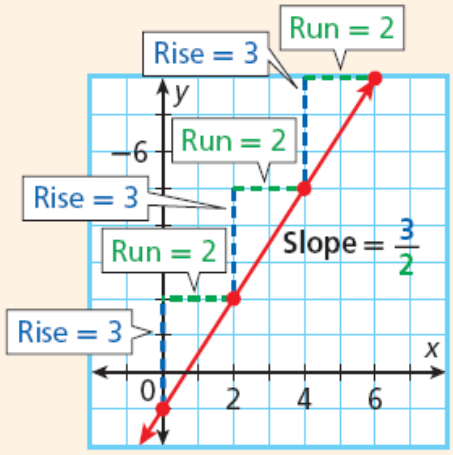
The **rise** is the difference in the *y*-values of two points on a line.

The **run** is the difference in the *x*-values of two points on a line.

The **slope** of a line is the ratio of rise to run for any two points on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

(Remember that *y* is the **dependent variable** and *x* is the **independent variable**.)



Finding Slope: By Graphing

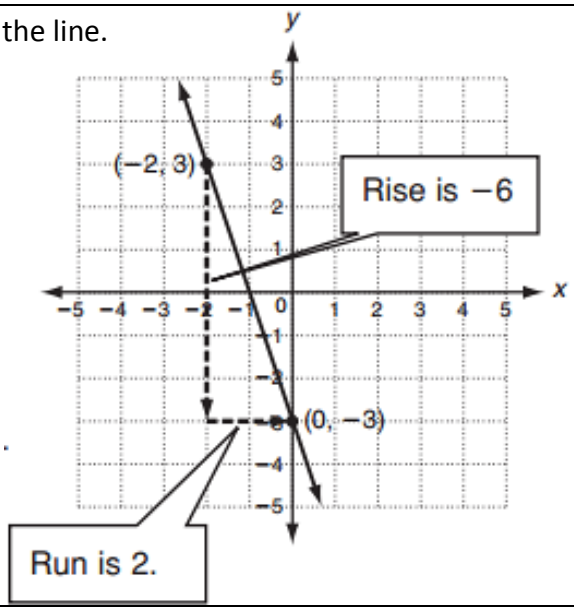
Finding Slope Given a Graph: * Choose any two points on the line.

Step 1: Begin at one of the points and count vertically until you are even with the 2nd point.
*This is the rise.
- If you go down the rise will be negative.
- If you go up the rise will be positive.

Step 2: Count over until you are at the second point.
*This is the run.
- If you go left the run will be negative.
- If you go right the run will be positive.

Step 3: Divide or simplify the fraction to find slope.

$$\text{slope} = \frac{\text{rise}}{\text{sun}} = -\frac{6}{2} = -3$$



Given 2 Points:

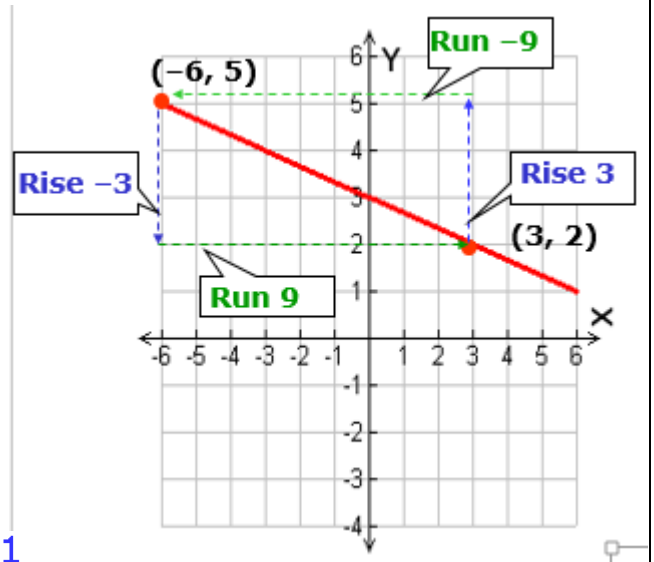
Find the slope of the line that contains (0, –3) and (5, –5).

Step 1: Begin at one point. Count vertically until you are even with the 2nd point.
*This is the rise.
- If you go down the rise will be negative.
- If you go up the rise will be positive.

Step 2: Count horizontally to the 2nd point to find the run.
*This is the run.
- If you go left the run will be negative.
- If you go right the run will be positive.

Step 3: Divide or simplify the fraction to find slope.

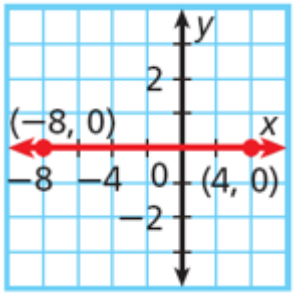
$$\text{Slope} = \frac{-3}{9} = -\frac{1}{3} \quad \text{or} \quad \text{Slope} = \frac{3}{-9} = -\frac{1}{3}$$



(It does not matter which point you start with. The slope is the same.)

Horizontal and Vertical Lines

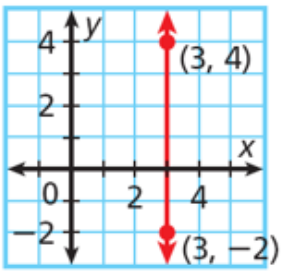
Horizontal Line



$$\frac{\text{rise}}{\text{run}} = \frac{0}{12} = 0$$

The slope is 0.

Vertical Line:



$$\frac{\text{rise}}{\text{run}} = \frac{6}{0}$$

You cannot divide by 0

The slope is undefined.

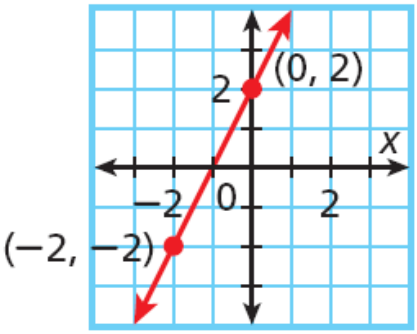
Finding Slope: Formula

Return To Start

Finding Slope Given a Graph:

Step 1: Label Coordinates:

Let (0, 2) be (x_1, y_1) and (–2, –2) be (x_2, y_2).



Step 2: Substitute:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-2}{-2-0} = \frac{-4}{-2} = 2$$

Step 3: Simplify

Given 2 points:

Find the slope of the line that contains **(2, 5)** and **(8, 1).**

Step 1: Label Coordinates:

x_1 y_1 x_2 y_2

Step 2: Substitute:

Step 3: Simplify

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-5}{8-2} = \frac{-4}{6} = -\frac{2}{3}$$

From A Table:

Step 1: Pick 2 points from the table and

Label Coordinates: **(0, 1)** and **(-2, 5).**

x	–2	–1	0	1
y	5	3	1	–1

x_1 y_1 x_2 y_2

Step 2: Substitute:

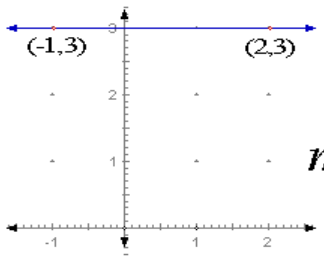
Step 3: Simplify

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{-2-0} = \frac{4}{-2} = -2$$

Horizontal and Vertical Lines

Horizontal Line:

Find the slope of the horizontal line passing through (2,3) and (-1,3).



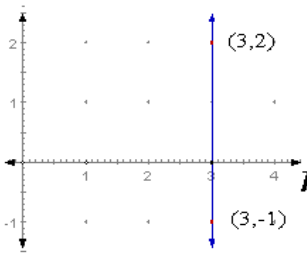
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3-3}{-1-2} = \frac{0}{-3} = 0$$

$$m = 0$$

Vertical Line:

Find the slope of the vertical line passing through (3,2) and (3,-1).



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1-2}{3-3} = \frac{-3}{0} = \text{err}$$

m is undefined

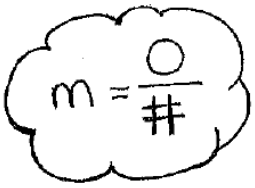
Horizontal and Vertical Lines

Return To Start

Horizontal Lines

Horizontal
O- zero
Y =

- **Equation: y = y-coordinate**
- **Parallel to the x-axis.**
- **Slope is Zero**



Example:

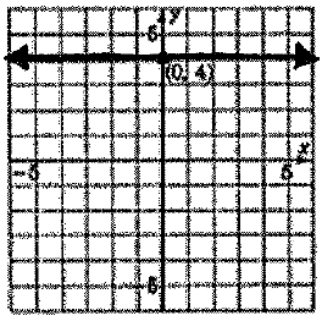
Equation: y = 4

Table:

x	y
-3	4
-2	4
-1	4
0	4

- **All y-values are the same**

Graph:

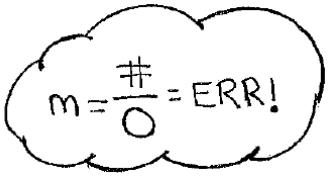


- **Has a y-intercept**

Vertical Lines

Vertical
Undefined
X =

- **Equation is x = x-coordinate**
- **Parallel to y-axis**
- **Slope is undefined**



Example:

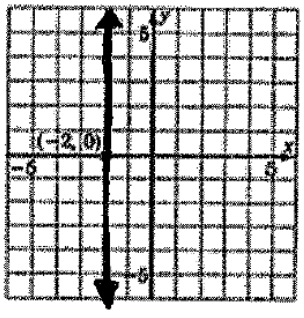
Equation: x = -2

Table:

x	Y
-2	-3
-2	-2
-2	-1

- **All x-values are the same**

Graph:



- **No y-intercept**

Graphing a Linear Equation

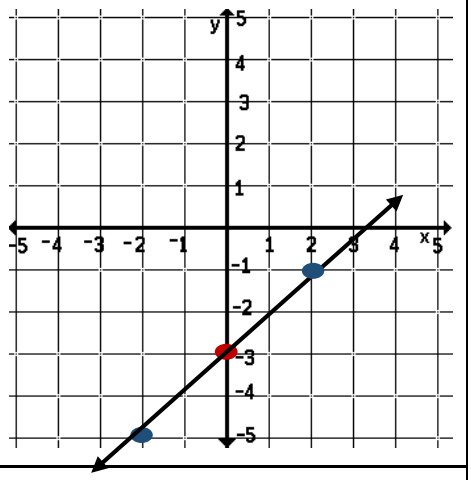
Return To Start

Make A TABLE

Example 1:

Steps	Example
Step 1: Make a t-chart **Step 2**: Pick in 3-5 values for x. *Use (-2, 0, 2) to start unless it is a real life problem. **Step 3**: Substitute each value for x and solve for y. **Step 4**: Record ordered pairs in table. **Step 5**: Graph the points and draw the line.	**2x - 2y = 6** 2 (**-2**) – 2y = 6 -4 – 2y = 6 +4 +4 -2y = 10 -2 -2 y = **-5** 2 (**0**) – 2y = 6 -2y = 6 -2 -2 y = **-3** 2 (**2**) – 2y = 6 4 – 2y = 6 - 4 - 4 -2y = 2 -2 -2 y = **-1**

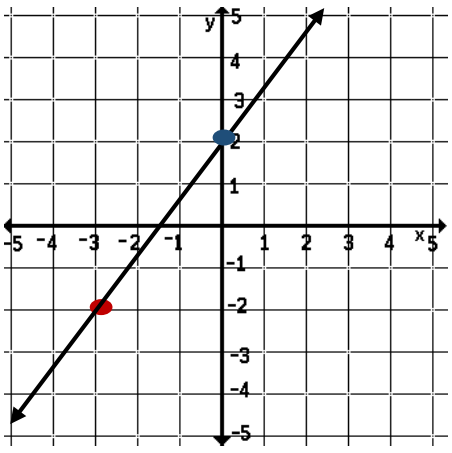
x	y
-2	-5
0	-3
2	-1



Example 2:

Steps	Example
Step 1: Make a t-chart **Step 2**: Pick in 3-5 values for x. ***Use (-2, 0, 2)** to start unless it is a real life problem. * **If slope is a fraction** use the + & – denominator and 0 **Step 3**: Substitute each value for x and solve for y. **Step 4**: Record ordered pairs in table. **Step 5**: Graph the points and draw the line.	**$y = \frac{4}{3}x + 2$** $y = \frac{4}{3}$ (**-3**) + 2 y = **-2** $y = \frac{4}{3}$ (**0**) + 2 y = **2** $y = \frac{4}{3}$ (**3**) + 2 y = **6**

x	y
-3	-2
0	2
3	6



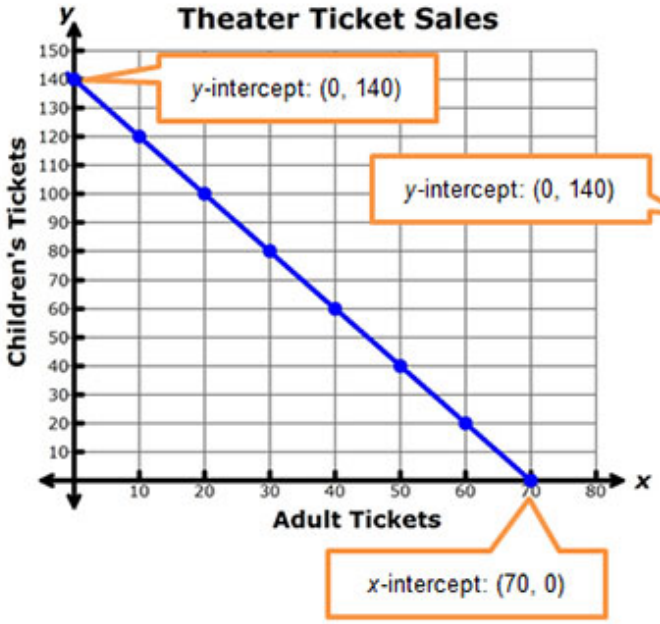
Graphing a Linear Equation

Return To Start

Intercepts:

The x-intercept is where the graph crosses the x-axis. The y-coordinate is always 0.

The y-intercept is where the graph crosses the y-axis. The x-coordinate is always 0.

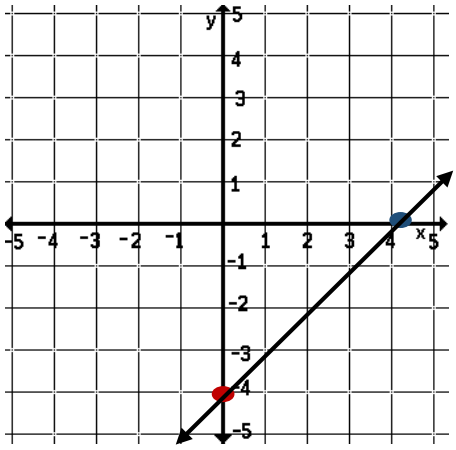


Number of Adult Tickets, x	Number of Children's Tickets, y
0	140
10	120
20	100
30	80
40	60
50	40
60	20
70	0

y-intercept: (0, 140)

x-intercept: (70, 0)

Graphing Lines by Finding the Intercepts:

Steps	Example
Step 1: Find y-intercept • Let x = 0 • Substitute 0 for x; solve for y. • Graph the point on the y-a-xis. **Step 2**: Find x-intercept • Let y = 0 • Substitute 0 for y; solve for x. • Graph the point on the x-a-xis. **Step 3:** Connect the dots.	**2x - 2y = 8** S1) Let x= 0 2 (0) – 2y = 8 – 2y = 8 -2 -2 y = -4 Ordered pair: (0, -4) S2) Let y = 0 2x – 2(0) = 8 2x = 8 2 2 x = 4 Ordered pair: (4, 0) 

Graphing a Linear Equation

Return To Start

Slope Intercept Form:

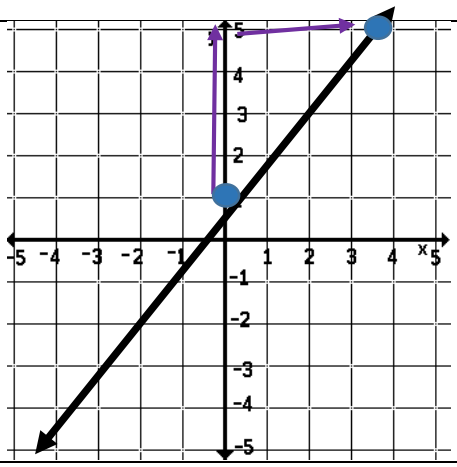
y	=	m	x	+ b
y-coordinate		**Slope:** rate of change (rise/run)	**x-coordinate**	**y-intercept:** point where line crosses the y-axis

Represent an ordered pair on the line.

Graph Using SLOPE and Y-INTERCEPT:

Example 1: Equation in Slope Intercept Form

Steps	Example
Step 1: Identify the y-intercept (b) and plot the point (0, b) **Step 2**: Use the slope (m) to find a second point: $\frac{rise}{run}$ (Put a whole number over 1 to make it a fraction: $2 = (\frac{2}{1})$ **Step 3**: Connect the points	$y = \frac{4}{3}x + 1$ b= 1 $m = \frac{4}{3}$



Example 2: Equation NOT in Slope Intercept Form

Write $2x + 6y = 12$ in slope-intercept form. Then graph the line.

Step 1: Solve for *y*.

$2x + 6y = 12$ — *Subtract 2x from both sides.*

$-2x \quad -2x$

$6y = -2x + 12$

$\frac{6y}{6} = \frac{-2x + 12}{6}$ — *Divide both sides by 6.*

$y = -\frac{1}{3}x + 2$ — *Simplify.*

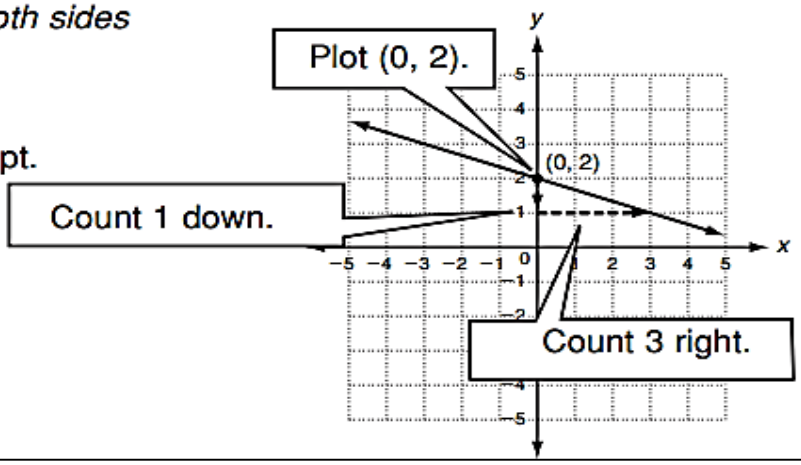
Step 2: Find the slope and *y*-intercept.

slope: $m = -\frac{1}{3} = \frac{-1}{3}$

y-intercept: $b = 2$

Step 3: Graph the line.

- Plot (0, 2).
- Then count 1 **down** (because the rise is **negative**) and 3 **right** (because the run is **positive**) and plot another point.
- Draw a line connecting the points.



Writing Equations of Lines

Return To Start

To write an equation of a line you need the slope and y-intercept.

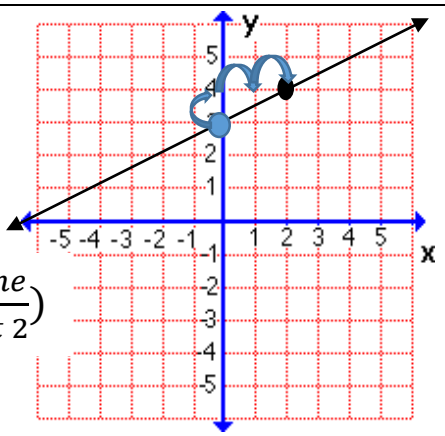
Slope Intercept Form:

y	=	m	x	+ b
y-coordinate		**Slope:** rate of change (rise/run)	**x-coordinate**	**y-intercept:** point where line crosses the y-axis

Represent an ordered pair on the line.

Skeleton Equation: $y = \square x + \square$ **(Fill in the slope and y-intercept)**

Graph:

What to do: (Steps)	Example
1. Find the y-intercept (b) 2. Count the rise over run, or slope (m) 3. Plug them into slope-intercept form: $y=mx+b$	b = 3 m = $\frac{1}{2}$ $(\frac{went\ up\ one}{went\ right\ 2})$ 

Slope and y-intercept:

What to do: (Steps)	Example
1. Plug the slope (m) into slope-intercept form 2. Plug the y-coordinate (b) of the y-intercept into slope-intercept form: $y=mx+b$	Slope is -3, Point is (0, -2) b = -2 m = -3 $y = mx + b$ ↓ ↓ y = -3x -2

Slope of Zero and Point on Line: Horizontal Lines

Return To Start

What to do: (Steps)	Example
*All points on a horizontal line have the same y-coordinate * Equation: y = y-coordinate	Slope is 0, Point is (-2, 4) y = 4

Slope of Undefined and Point on Line: Vertical Lines

What to do: (Steps)	Example
*All points on a vertical line have the same x-coordinate * Equation: x = x-coordinate	Slope is Undefined, Point is (-3, 2) x = -3

Slope and point on a line

What to do: (Steps)	Example
1. Plug the slope (m) into the equation $y=mx+b$ 2. Use the given pt (x, y) to substitute the x- & y-coordinates into the equation $y=mx+b$ 3. Solve equation for b 4. Write equation	Write the equation of line with slope of 1, passing through point (-2, 3) m = 1 $y= m\,x + b$ $3 = 1*-2 + b$ $3 = -2 + b$ $+2 \quad +2$ $5 = b$ Equation: $y = 1x + 5$

Two points on a line

What to do: (Steps)	Example
1. Use the 2 points to calculate the slope of line $m = \frac{y_2 - y_1}{x_2 - x_1}$ 2. Choose 1 of the given points & the calculated slope to substitute into $y= m\,x + b$ 3. Solve for b 4. Write equation	Write an equation through (-4, 4) and (2,-5) $m= \frac{-5-4}{2--4} = \frac{-9}{6} = \frac{-3}{2}$ $-5 = \frac{-3}{2}(2) + b$ $-5 = -3 + b$ $+3 \quad +3$ $-2 = b$ Equation: $y = \frac{-3}{2}x - 2$

Parallel Lines (equation) and point

Return To Start

- Have the **SAME** slope $m_1 = m_2$

Steps:	**Example:**
1. Substitute the slope from original line (3 in this case) into the equation of the line 2. Substitute the given point (1,7) into the x and y values 3. Solve for b 4. Write equation (substitute the value in for b)	Ex: through (1,7) and parallel to y = 3x +5. 1) y = 3x +b 2) 7 = 3(1) +b 3) $7 = 3(1) + b$ $7 = 3 + b$ $-3 \quad -3$ $4 = b$ 4) y = 3x + 4

Perpendicular Lines (equation) and point

$m_1 = -\frac{1}{m_2}$

- slopes that are the **negative reciprocal** of one another

Steps:	**Example:**
1. Id the slope of the given line 2. Use negative (opposite) reciprocal of given slope as m 3. Use given point & substitute coordinates into y=mx+b for x & y 4. Solve for b 5. Write equation	Ex: through (2,1) and perpendicular to y = -3x – 2 1) m_1 = -3 2) $m_2 = \frac{1}{3}$ 3&4) y = m x + b $1 = \frac{1}{3}(2) + b$ $-\frac{2}{3} \quad -\frac{2}{3}$ $\frac{1}{3} = b$ 5) $y = \frac{1}{3}x + \frac{1}{3}$

Writing Equations of Lines from a Word Problem:

Return To Start

Things to do before solving:

1. **Identify the variables in this situation**: independent variable: _____ =_____

   dependent variable: _____ =_____

2. **What is the given information in this problem (find all that apply)?**

   slope _____ y-intercept _____ one point (,) a second point: (,)

3. Find the example that matches the given information.

Given Information: Slope & y-intercept

When a word problem involves a constant rate or speed and a beginning amount, it can be written in slope-intercept form: y = mx + b. To do this, recognize which number will represent *m*, the rate, and which number will represent *b*, the y-intercept.

What to do: (Steps)	Example
1. Identify the variables. 2. Identify the given information: slope _____ y-intercept _____ 3. Plug the slope (m) into slope-intercept form 4. Plug the y-coordinate (b) of the y-intercept into slope-intercept form: $y=mx+b$	**Ex:** Suppose you receive $100 for a graduation present, and you deposit it in a savings account. Then each week thereafter, you add $5 to the account but no interest is earned. 1) independent variable: x= **# of weeks** dependent variable: y= **Total amount** 2) m = 5 (happening each week) b = 100 (initial deposit) $y = mx + b$ ↓ ↓ Equation: $y = 5x + 100$

Return To Start

Given Information: Slope & a point

A word problem will involve a constant rate or speed and gives a relationship at some point in time between each variable.

What to do: (Steps)	Example
1. Identify the variables 2. Identify the given information: slope _____ one point(,) 3. Plug the slope (m) into the equation $y=mx+b$ 4. Use the given pt (x, y) to substitute the x- & y-coordinates into the equation $y=mx+b$ 5. Solve equation for b 6. Write equation	**EX.** Marty is spending money at the average rate of \$3 per day. After 14 days he has \$68 left. The amount left depends on the number of days that have passed. 1) independent variable: x= **# of days** dependent variable: y= **Amount left in account** 2) Slope is -3 (decreasing by 3 each day) Point is (14, 68) 3&4) $y= m\,x + b$ $68 = -3*14 + b$ 5) $68 = -42 + b$ $+42 \quad +42$ $110 = b$ 6) Equation: $y = -3x + 110$

Given Information: Two Points

Sometimes instead of giving a rate, a word problem gives two relationships at different points in time between variables. This kind of problem is giving you two points. You must find the slope and then use one of the points to write an equation.

What to do: (Steps)	Example
1. Identify the variables 2. Identify given information: Point1 (,) Point2(,) 3. Use the 2 points to calculate the slope of line 4. Choose 1 of the given points & the calculated slope to substitute into $y= m\,x + b$ 5. Solve for b 6. Write equation	**EX:** All tickets for a concert are the same price. The ticket agency adds a fixed fee to every order. A person who orders 3 tickets pays \$57. A person who orders 5 tickets pays \$93. 1) independent variable: x= **# of tickets** dependent variable: y= **Total Cost** 2) Point1 (3, 57) Point2 (5, 93) 3) $m=\frac{93-57}{5-3}=\frac{36}{2}=18$ 4&5) $y= m\,x + b$ $57 = 18\ (3) + b$ $57 = 54 + b$ $-54 \quad -54$ $3 = b$ 6) **Equation:** $y = 18x + 3$