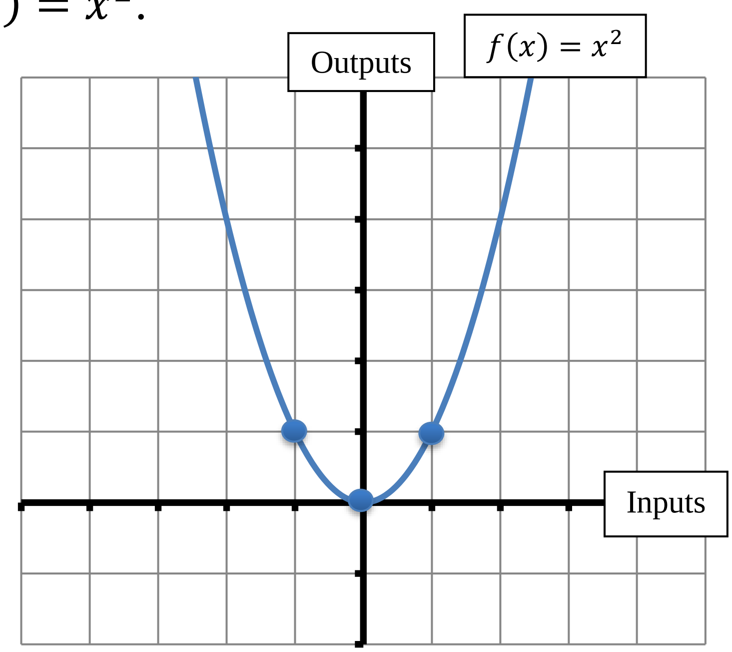
Graphing quadratic functions:

- the only method I will use to graph quadratic functions is transformations
  - remember when using transformations that whatever changes happen **OUT**side the parentheses, do exactly what you see to the **OUT**puts; whatever changes take place **IN**side the parentheses, do the **IN**verse operation to the **IN**puts.
- to graph using transformations, I will use standard form of a quadratic function to transform the parent function $f(x) = x^2$

A parent function is the simplest function of a family of functions. For quadratic functions, the simplest function is $f(x) = x^2$.

Example 1: Graph the quadratic function $g(x) = 2(x-1)^2 - 3$ by transforming the parent function $f(x) = x^2$.

Inputs, outputs, and ordered pairs for the parent function $f(x) = x^2$		
Inputs	**Outputs**	**Ordered Pairs**
x	$f(x) = x^2$	$(x, f(x))$
-1	$f(-1) = 1$	$(-1, 1)$
0	$f(0) = 0$	Vertex $(0, 0)$
1	$f(1) = 1$	$(1, 1)$



The quadratic function g is already in standard form, so we don't need to change it at all to sketch its graph using transformations. I will simply take the three points that are given from the graph of the parent function $f(x) = x^2$, $(-1, 1), (0, 0)$, and $(1, 1)$, and transform them.

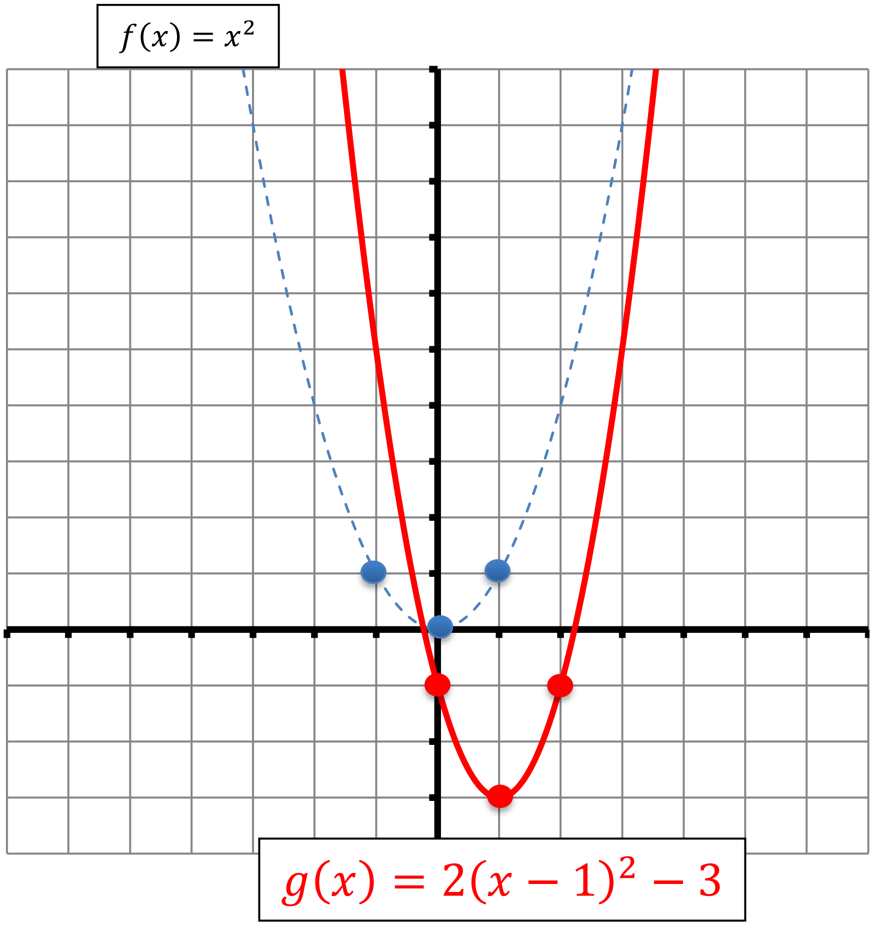
Inside the parentheses of the function $g(x) = 2(x-1)^2 - 3$ we have $x - 1$, which indicates that we will take the inputs of the parent function f and add 1 to them $(\text{inputs} + 1)$. Remember that when changes take place **in**side the parentheses of a function, we do the **in**verse operation to the **in**puts.

Outside the parentheses of the function $g(x) = 2(x-1)^2 - 3$ we have a factor of 2 and a term of -3. This indicates that we will take the outputs of the parent function f, multiply them by 2 first, and then subtract 3 $(2(\text{outputs}) - 3)$. Remember that when changes take place **out**side the parentheses, we do exactly what we see to the **out**puts. Also remember that order of operation says that we multiply/divide first, and add/subtract second.

$$\mathbf{(inputs + 1, 2(outputs) - 3)}$$

Ordered Pairs for $f(x) = x^2$		**Transformations** $(\text{inputs} + 1, 2(\text{outputs}) - 3)$		**Ordered Pairs for $g(x) = 2(x-1)^2 - 3$**
$(-1,1)$	$\rightarrow$	$(-1+1, 2(1)-3)$	$\rightarrow$	$(0,-1)$
Old Vertex $(0,0)$	$\rightarrow$	$(0+1, 2(0)-3)$	$\rightarrow$	New Vertex $(1,-3)$
$(1,1)$	$\rightarrow$	$(1+1, 2(1)-3)$	$\rightarrow$	$(2,-1)$

Transforming the points from the parent function $f(x) = x^2$ to get the new points for the function $g(x) = 2(x-1)^2 - 3$ results in the graph on the following page:



Graphing quadratic functions using transformations is the only method I will cover in-class, but it is not the only option for graphing quadratic functions. The graph of the function g could also have been obtained by making an input/output table. If you plan to use an input/output table, find the vertex first and then choose other inputs that are close to the x-coordinate of the vertex to plug in to the function in order to find ordered pairs.

Keep in mind that we are basically just transforming points, just like we did in Lessons 21 & 22.

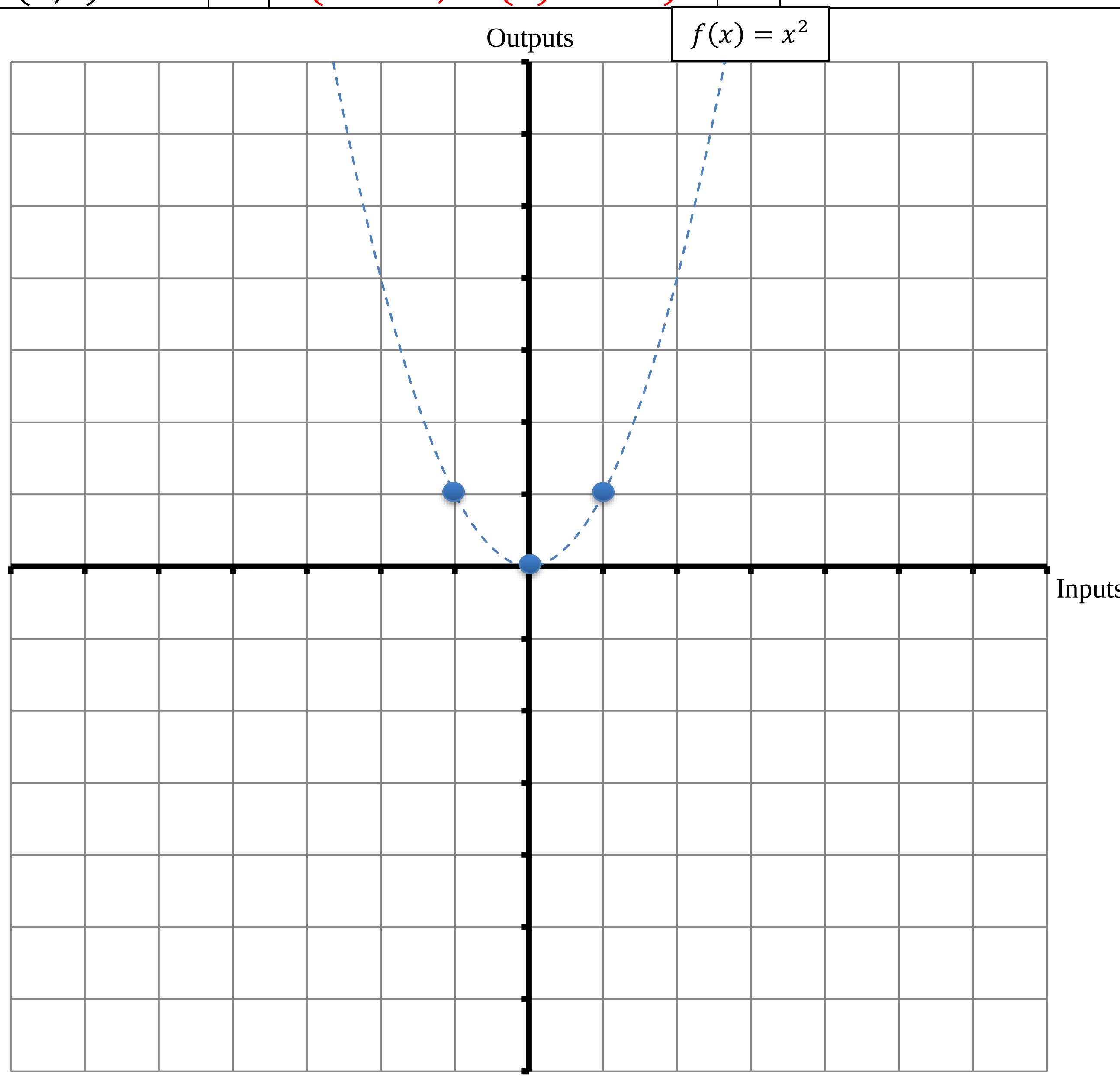
Also, I don't express the new function in terms of the old function like I did in Lesson 21 because I already have parentheses to separate the changes taking place between the inputs and the outputs. If you wanted to express the new function $g(x) = 2(x-1)^2 - 3$ in terms of the original function $f(x) = x^2$, you would say that $g(x) = 2 \cdot f(x-1) - 3$.

Instead of going through another example similar to this on paper, next I will go through a problem like this from LON-CAPA.

First LON-CAPA Problem:

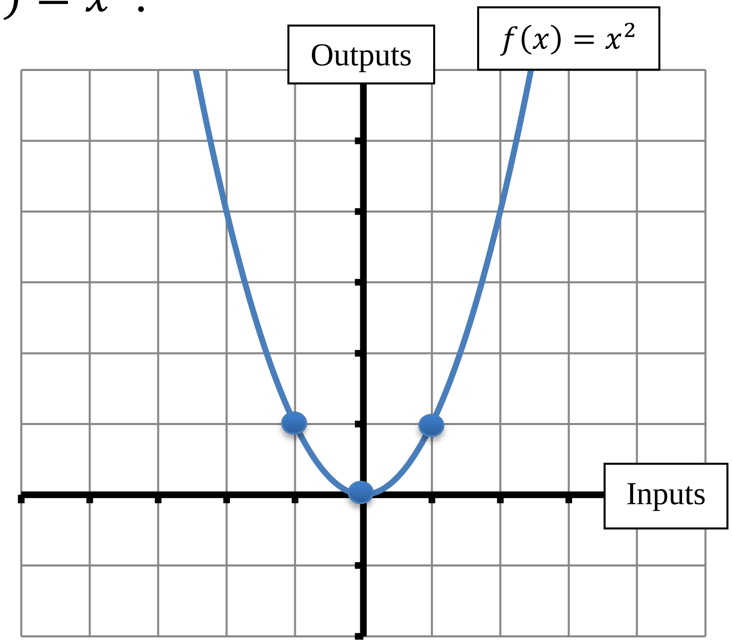
Transformations: (inputs , (outputs))

Ordered Pairs for $f(x) = x^2$		Transformations (inputs , (outputs))		New Ordered Pairs
$(-1,1)$	$\rightarrow$	$(-1$, (1) $)$	$\rightarrow$	
Old Vertex $(0,0)$	$\rightarrow$	$(0$, (0) $)$	$\rightarrow$	New Vertex
$(1,1)$	$\rightarrow$	$(1$, (1) $)$	$\rightarrow$	



Example 2: Graph the quadratic function $j(x) = -x^2 + 6x - 7$ by transforming the parent function $f(x) = x^2$.

Inputs, outputs, and ordered pairs for the parent function $f(x) = x^2$		
Inputs	**Outputs**	**Ordered Pairs**
x	$f(x) = x^2$	$(x, f(x))$
-1	$f(-1) = 1$	$(-1, 1)$
0	$f(0) = 0$	Vertex $(0, 0)$
1	$f(1) = 1$	$(1, 1)$



Since the quadratic function j is in polynomial form, I will convert it to standard form first before transforming the graph of the parent function $f(x) = x^2$. To convert $j(x) = -x^2 + 6x - 7$ to standard form, I will start by finding its vertex.

$$x = \frac{-b}{2a}$$

$$x = \frac{-6}{2(-1)}$$

$$x = \frac{-6}{-2}$$

$$x = 3$$

$$j(3) = -(3)^2 + 6(3) - 7$$

$$j(3) = -9 + 18 - 7$$

$$j(3) = 2$$

Vertex: $(3, 2)$

So the quadratic function $j(x) = -x^2 + 6x - 7$ has a vertex of $(3, 2)$ and a leading coefficient a of -1. Plugging this information into the standard form $f(x) = a(x-h)^2 + k$, I get the following:

$$j(x) = -(x-3)^2 + 2$$

Now that the quadratic function j is in standard form, I will take the points from the parent function $f(x) = x^2$ and transform them using $j(x) = -(x-3)^2 + 2$.

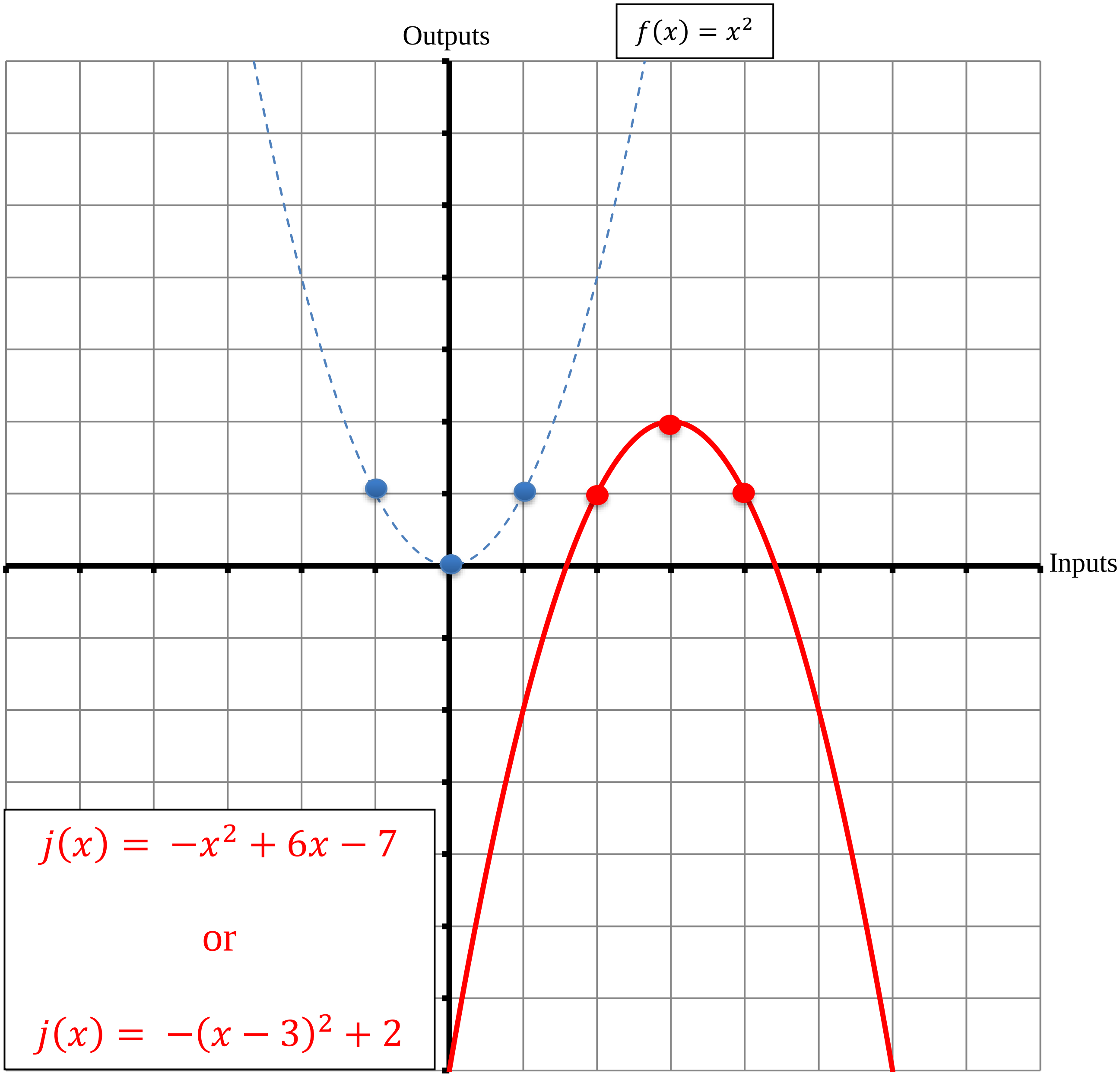
Inside the parentheses of the function $j(x) = -(x-3)^2 + 2$ we have $x - 3$, which indicates that we will take the inputs of the parent function f and add 3 to them (inputs $+ 3$).

Outside the parentheses of the function $j(x) = -(x-3)^2 + 2$ we have -1 times the quantity inside the parentheses, then we have $+2$. This indicates that we will take the outputs of the parent function f, negate them, and then add 2 ($-$(outputs) $+ 2$).

$$\mathbf{(inputs + 3, -(outputs) + 2)}$$

Ordered Pairs for $f(x) = x^2$		**Transformations** (inputs $+ 3, -$(outputs) $+ 2$)		**Ordered Pairs for $j(x) = -(x-3)^2 + 2$**
$(-1,1)$	$\rightarrow$	$(-1+3, -(1)+2)$	$\rightarrow$	$(2, 1)$
Old Vertex $(0,0)$	$\rightarrow$	$(0+3, -(0)+2)$	$\rightarrow$	New Vertex $(3,2)$
$(1,1)$	$\rightarrow$	$(1+3, -(1)+2)$	$\rightarrow$	$(4,1)$

Transforming the points from the parent function $f(x) = x^2$ to get the new points for the function $j(x) = -(x-3)^2 + 2$ results in the graph on the following page:



Students who don't like or don't understand transformations may use other methods such as making an input/output table and/or using intercepts. However making an input/output table may require more work, and not every quadratic function has x-intercepts, so using intercepts may not be a viable option at all.

Next I will go through another problem from LON-CAPA, this time one that is similar to Example 2.

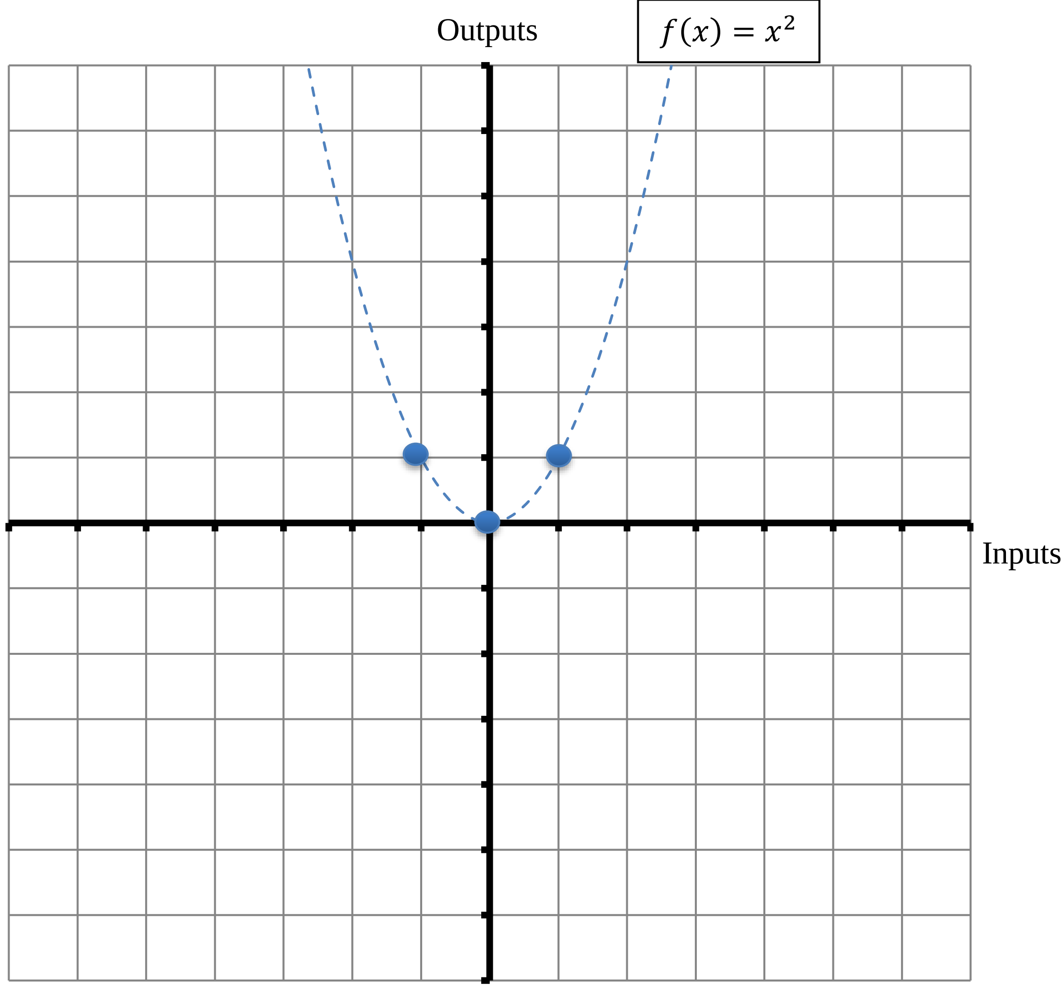
Second LON-CAPA Problem:

Vertex:

Standard form:

Transformations: (inputs , (outputs))

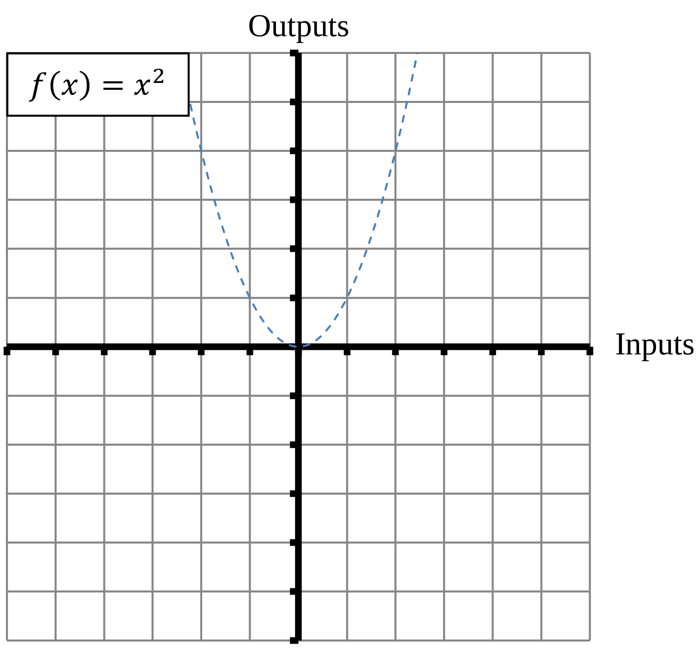
Ordered Pairs for $f(x) = x^2$		**Transformations** (inputs , (outputs))		**New Ordered Pairs**
$(-1,1)$	$\rightarrow$	$(-1 \quad , \quad (1) \quad)$	$\rightarrow$	
Old Vertex $(0,0)$	$\rightarrow$	$(0 \quad , \quad (0) \quad)$	$\rightarrow$	New Vertex
$(1,1)$	$\rightarrow$	$(1 \quad , \quad (1) \quad)$	$\rightarrow$	

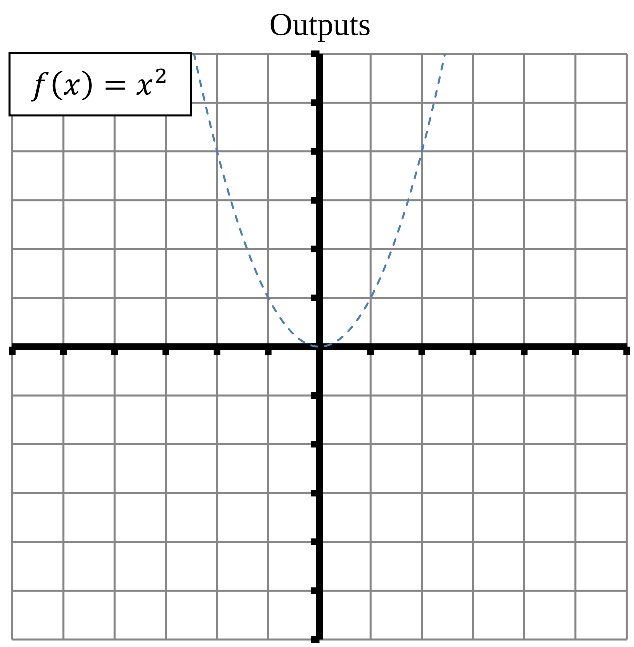


Example 3: Graph each of the following quadratic functions. After graphing, list the domain, the range, the zeros (if any), the positive/negative interval (if any), the increasing/decreasing intervals, and the intercepts.

a. $g(x) = -(x+2)^2 + 4$

b. $h(x) = (x-3)^2 - 2$

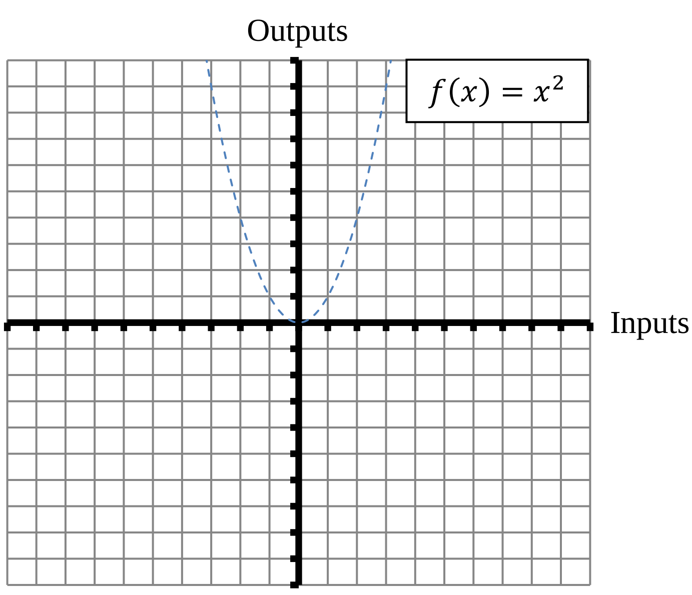


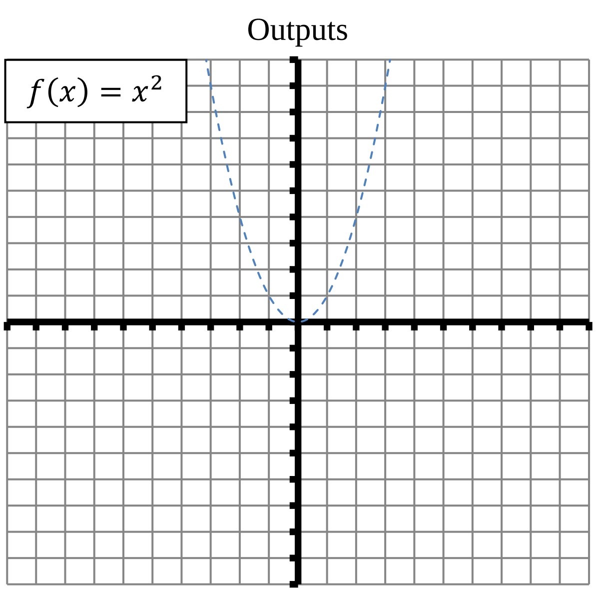


(hint: on these two problems, find the vertex of each quadratic function first, then express each quadratic function in standard form ($f(x) = a(x - h)^2 + k$), and then graph)

c. $j(x) = x^2 + 4x + 9$

d. $k(x) = 2x^2 - 20x + 44$

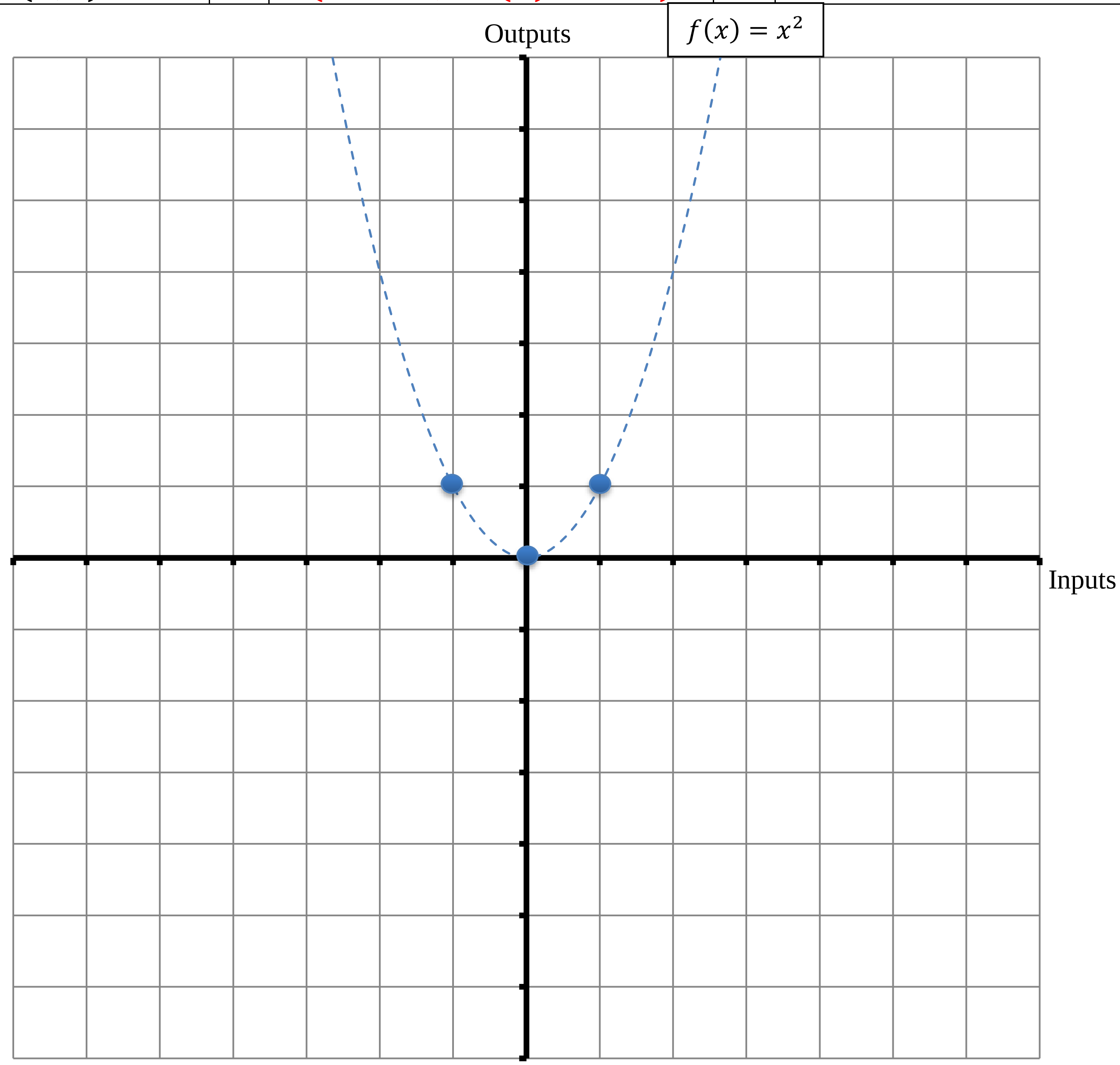




Third LON-CAPA Problem:

Transformations: (inputs , (outputs))

Ordered Pairs for $f(x) = x^2$		Transformations (inputs , (outputs))		New Ordered Pairs
$(-1,1)$	→	(−1 , (1))	→	
Old Vertex $(0,0)$	→	(0 , (0))	→	New Vertex
$(1,1)$	→	(1 , (1))	→	



Answers to Examples:

3a. $V(-2,4)$; $D: (-\infty, \infty)$; $R: (-\infty, 4]$; $g(x) = 0$ when $x = -4, 0$;
$g(x) > 0: (-4, 0)$; $g(x) < 0: (-\infty, -4) \cup (0, \infty)$; $\uparrow: (-\infty, -2)$;
$\downarrow: (-2, \infty)$; $x -$ intercepts: $(-4, 0), (0, 0)$; $y -$ intercept: $(0, 0)$

3b. $V(3,-2)$; $D: (-\infty, \infty)$; $R: [-2, \infty)$; $h(x) = 0$ when $x = 3 \pm \sqrt{2}$;
$h(x) > 0: \left(-\infty, 3 - \sqrt{2}\right) \cup \left(3 - \sqrt{2}, \infty\right)$; $h(x) < 0: \left(3 - \sqrt{2}, 3 + \sqrt{2}\right)$;
$\uparrow: (3, \infty)$; $\downarrow: (-\infty, 3)$; $x -$ intercepts: $\left(3 - \sqrt{2}, 0\right), \left(3 + \sqrt{2}, 0\right)$;
$y -$ intercept: $(0, 7)$

3c. $V(-2,5)$; $D: (-\infty, \infty)$; $R: [5, \infty)$; $j(x) \neq 0$; $j(x) > 0: (-\infty, \infty)$;
$j(x) < 0$: NEVER; $\uparrow: (-2, \infty)$; $\downarrow: (-\infty, -2)$; $x -$ intercepts: NONE;
$y -$ intercept: $(0, 9)$

3d. $V(5,-6)$; $D: (-\infty, \infty)$; $R: [-6, \infty)$; $k(x) = 0$ when $x = 5 \pm \sqrt{3}$;
$k(x) > 0: \left(-\infty, 5 - \sqrt{3}\right) \cup \left(5 - \sqrt{3}, \infty\right)$; $k(x) < 0: \left(5 - \sqrt{3}, 5 + \sqrt{3}\right)$;
$\uparrow: (5, \infty)$; $\downarrow: (-\infty, 5)$; $x -$ intercepts: $\left(5 - \sqrt{3}, 0\right), \left(5 + \sqrt{3}, 0\right)$;
$y -$ intercept: $(0, 44)$