Graphs of Radical Functions

Lesson #4 of
Unit 3: Rational Exponents
and Radical Functions
(Textbook Ch3.5)

Question of the Day

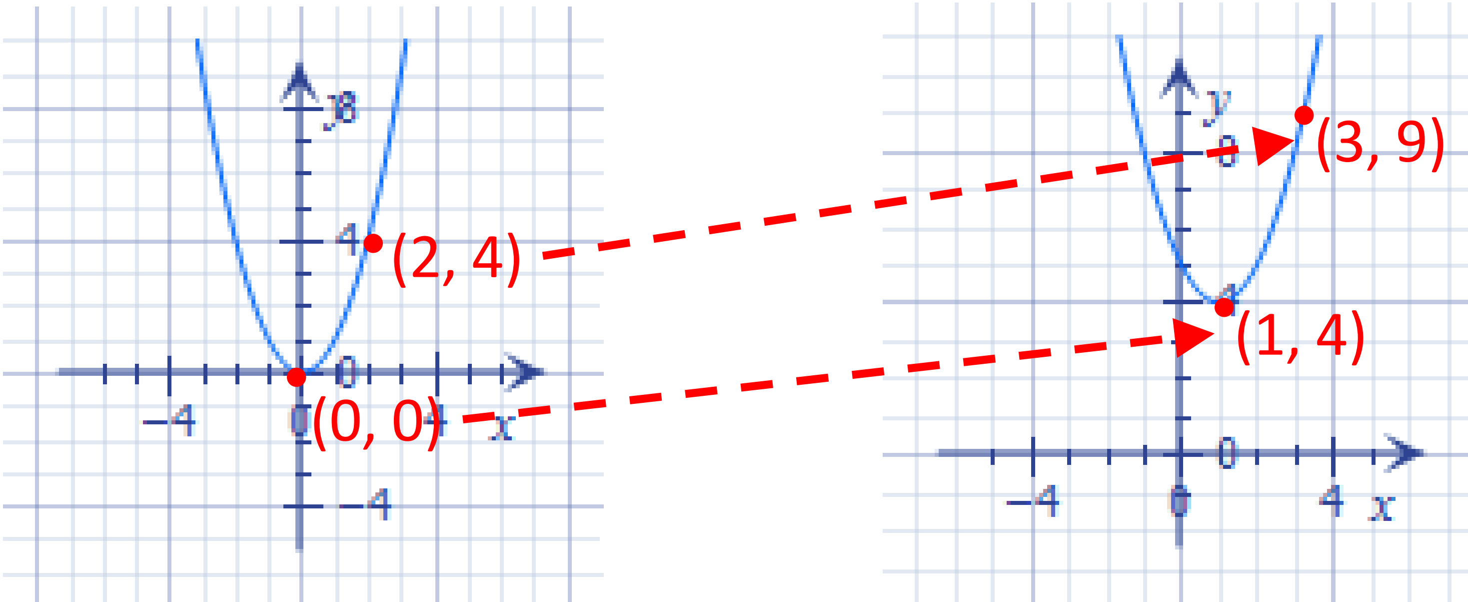
For the polynomial function $f(x) = x^2 - 3$, you can plug-in any real number into *x,* such as 1, -5, ¾, 6.2, and 3.14, to get a real number result.

QUESTION: Is there any real number you <u>cannot</u> plug into the radical function $g(x) = \sqrt{x-3}$ if you want to get real number result?

Translation of Graphs

The vertex form of quadratic functions is $y = a(x-h)^2 + k$ where $a \neq 0$.

$y = x^2$ Vertex: $(0, 0)$ $y = (x-1)^2 + 4$ Vertex: $(1, 4)$

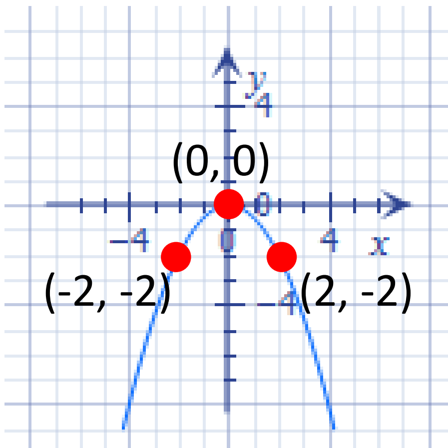


Every point from the graph of $y = x^2$ is moved to new location by the amount of *h* and *k.* All x values are moved horizontally *h* units, and all y values are moved vertically *k* units. This moving, or shifting, is called **translation of graph.**

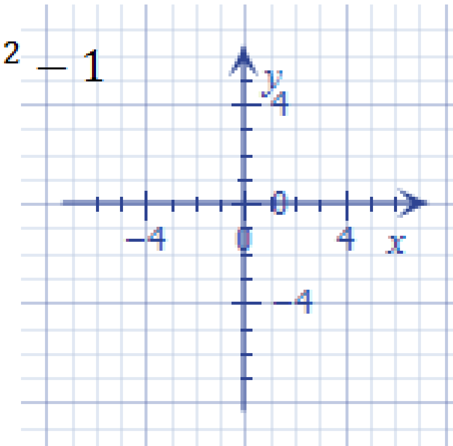
Practice

Below is the graph of $y = -\frac{1}{2}x^2$. Graph each function on the grid by translating given three points.

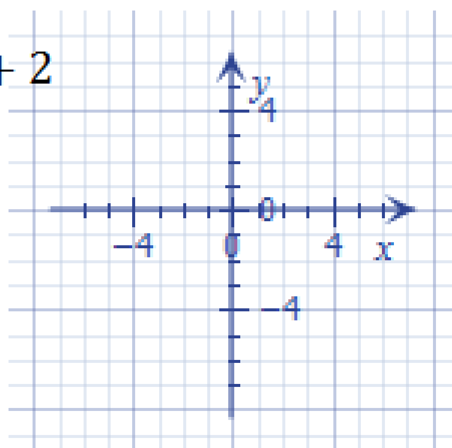
$y = -\frac{1}{2}x^2$



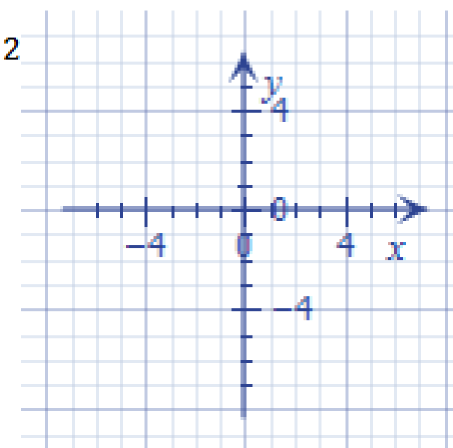
$y = -\frac{1}{2}(x+2)^2 - 1$



$y = -\frac{1}{2}x^2 + 2$



$y = -\frac{1}{2}(x-3)^2$



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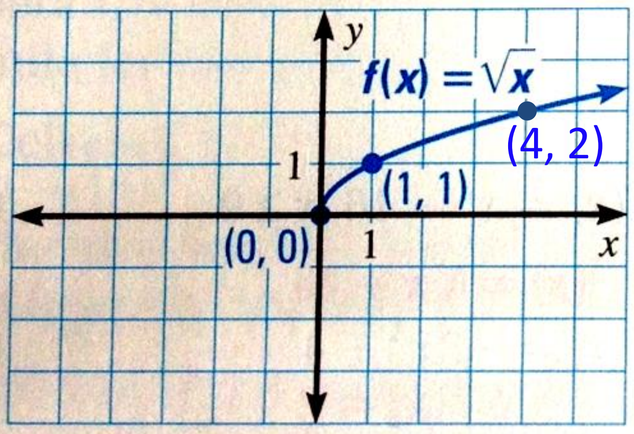
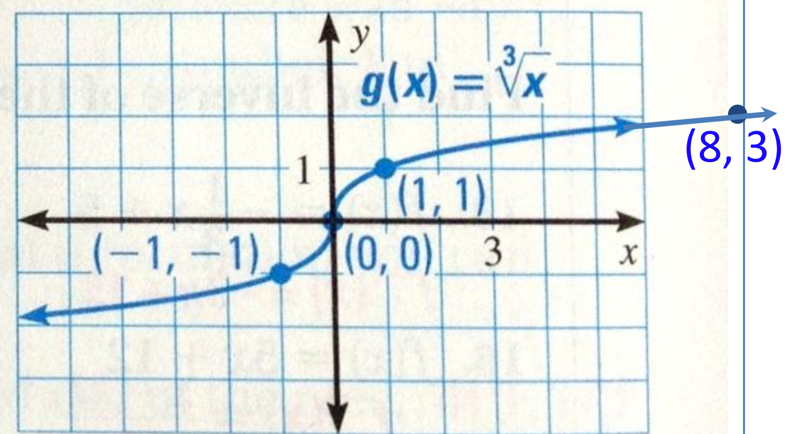
Parent Functions for Square Root and Cube Root Functions

The parent function for the family of square root functions is $f(x) = \sqrt{x}$.



The parent function for the family of cube root functions is $g(x) = \sqrt[3]{x}$.



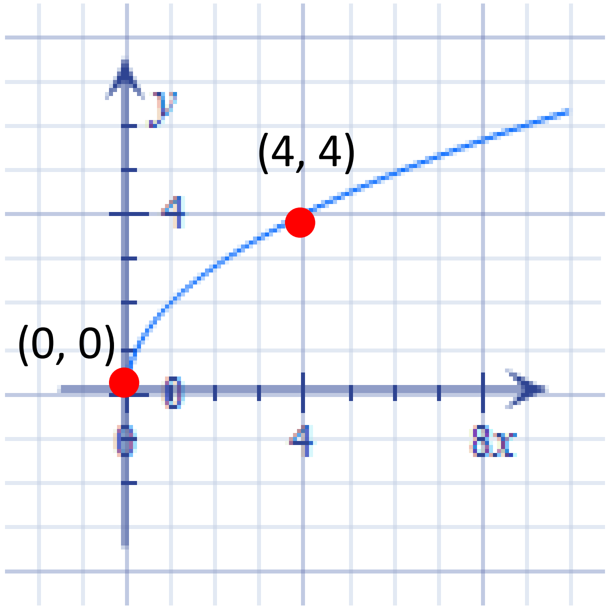
Both $f(x) = \sqrt{x}$ and $g(x) = \sqrt[3]{x}$ go through (0, 0) and (1, 1), but the graph of $f(x) = \sqrt{x}$ does not go through the area where *x* is negative or y is negative. The graph of $g(x) = \sqrt[3]{x}$ go through anywhere regardless of x or y being positive or negative.

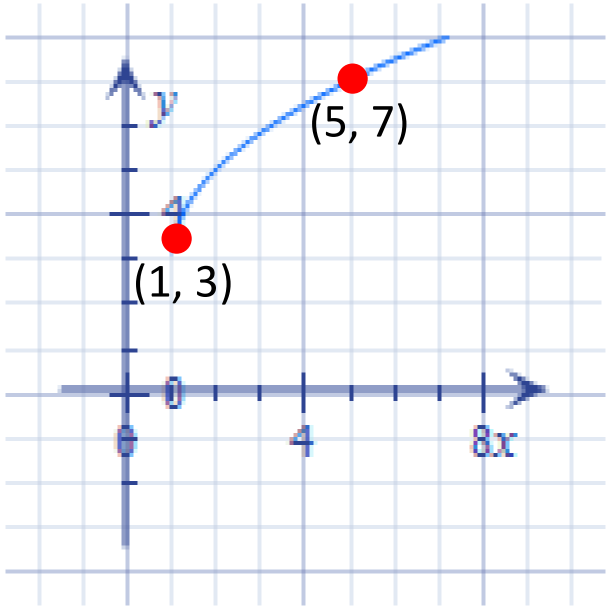
Text p.200

Graphs of Radical Functions

Graphs of radical functions $y = a\sqrt{x-h} + k$ and $y = a\sqrt[3]{x-h} + k$ are horizontally translated *h* units and vertically translated *k* units from $y = a\sqrt{x}$ and $y = a\sqrt[3]{x}$, respectively.

$y = 2\sqrt{x}$ → Translate 1 unit horizontally, Translate 3 units vertically → $y = 2\sqrt{x-1} + 3$

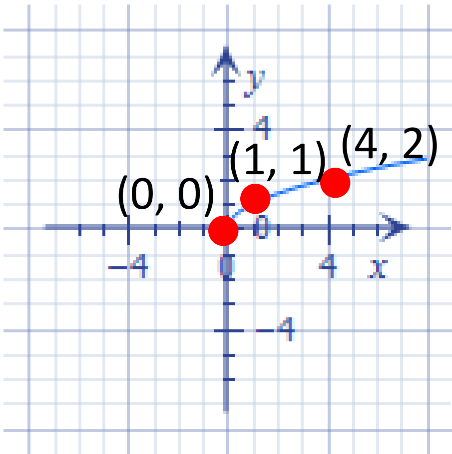




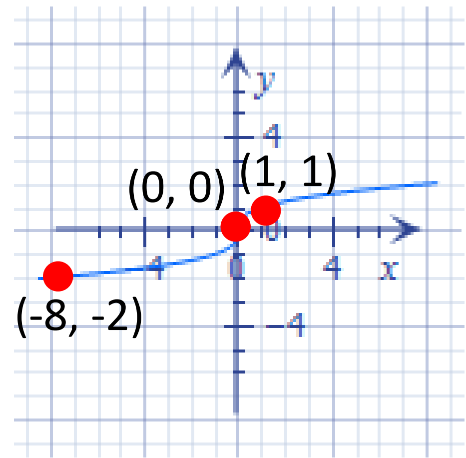
Practice

Graph the function on the grid by translating given three points.

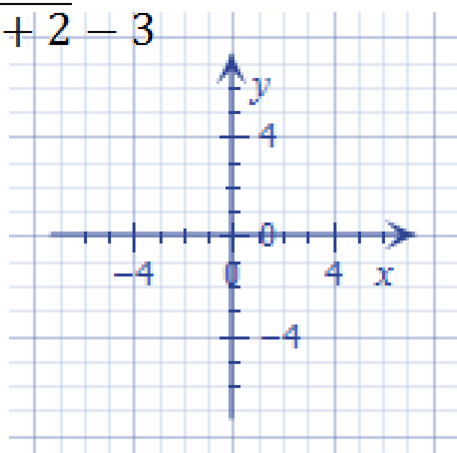
$y = \sqrt{x}$



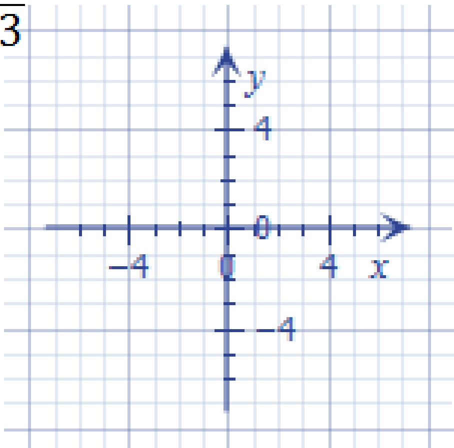
$y = \sqrt[3]{x}$



$y = \sqrt{x+2} - 3$



$y = \sqrt[3]{x-3}$



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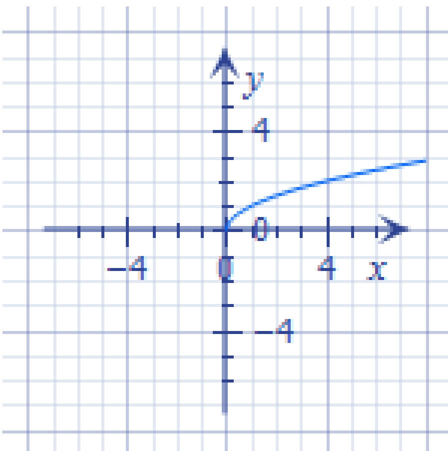
Domain and Range of Radical Functions

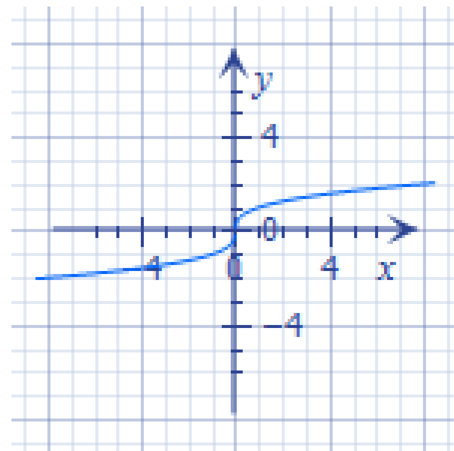
As the graph suggests, $y = \sqrt{x}$ and $y = \sqrt[3]{x}$ have below domain and range.



$y = \sqrt{x}$

Domain: $x \geq 0$

Range: $y \geq 0$



$y = \sqrt[3]{x}$

Domain: All real

Range: All real

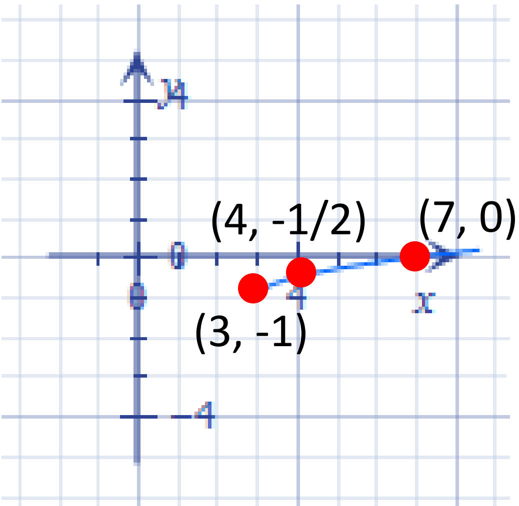
When the square root function is translated, its domain and range changes exactly same as the translation. For example, $y = \sqrt{x}$ is translated horizontally -2 units and vertically -3 units to create the graph of $y = \sqrt{x+2} - 3$ so the domain and range of $y = \sqrt{x+2} - 3$ is $x \geq -2$ and $y \geq 3$, respectively.

The domain and range of cube root function does not change because even if the function is translated, it will still cover all real numbers.

Strategy to Graph Radical Functions

To graph a square root function, for example $f(x) = \frac{1}{2}\sqrt{x-3} - 1$...

1) Find values of *x* that make the inside of the radical equal to 0, 1, and 4.



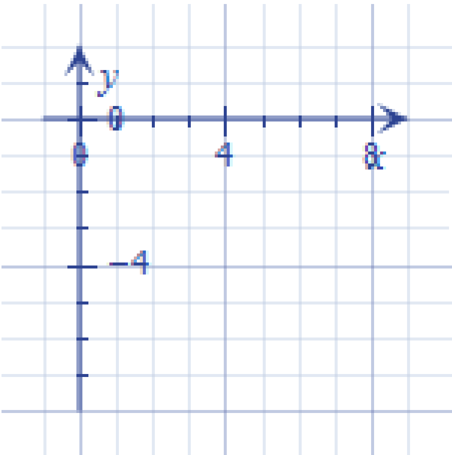
2) By doing that the radical part of the function becomes $\sqrt{0} = 0$, $\sqrt{1} = 1$, and $\sqrt{4} = 2$. For the example function above, these *x* values are 3, 4, and 7, respectively.
3) Calculate remaining part of the function to find corresponding y values. For the example function above, corresponding y values are -1, ½, and 0, respectively.
4) Plot these three (x, y) pair and then smoothly connect them to complete the graph.

To graph a cube root function, for example $y = 2\sqrt[3]{x-4}$, do exactly same as above except finding values of *x* that make the inside of the radical equal to 0, 1, -1, 8, and -8. For the example function, these are 4, 5, 3, 12, -4

Practice

Graph the function, and then state its domain and range.

$y = -4\sqrt{x} + 2$



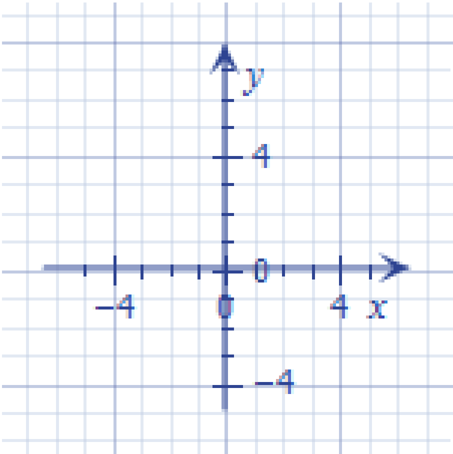
Domain

:

Range

:

$y = 2\sqrt{x+1}$



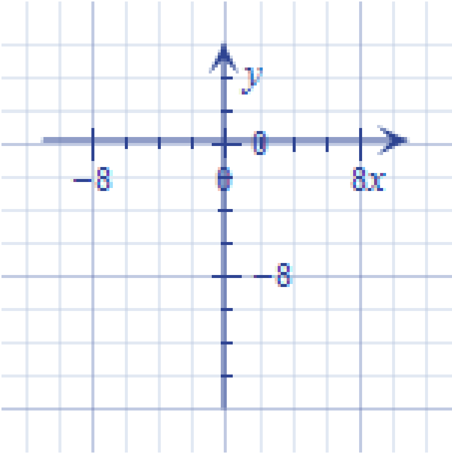
Domain

:

Range

:

$y = \sqrt[3]{x} - 5$



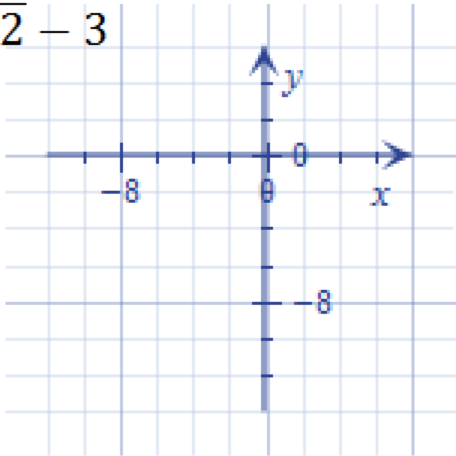
Domain

:

Range

:

$g(x) = -\sqrt[3]{x+2} - 3$



Domain

:

Range

:

Suggested Problems

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1, 2, 4, 6, 7, 11, 12

Suggested Problems

- SOLUTIONS -

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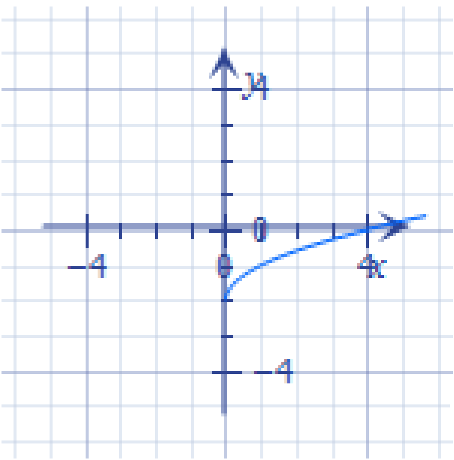
1)



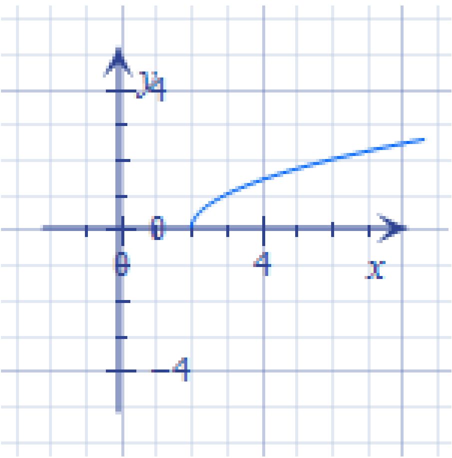
Domain

: x >= 0

Range

: y >= -2

2)



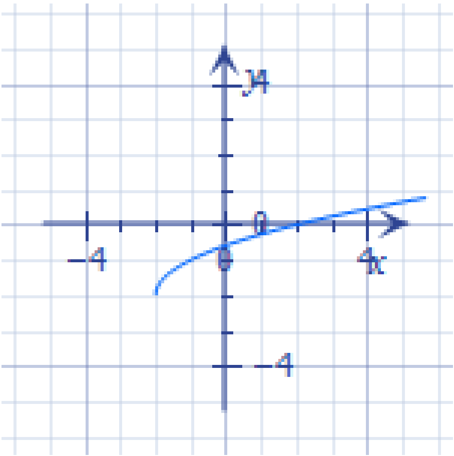
Domain

: x >= 2

Range

: y >= 0

4)



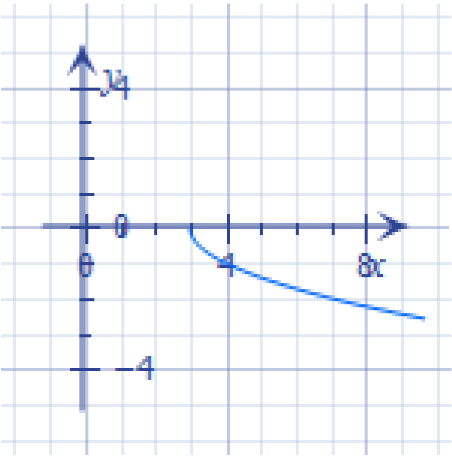
Domain

: x >= -2

Range

: y >= -2

6)



Domain

: x >= 3

Range

: y <= 0

Suggested Problems
- SOLUTIONS -

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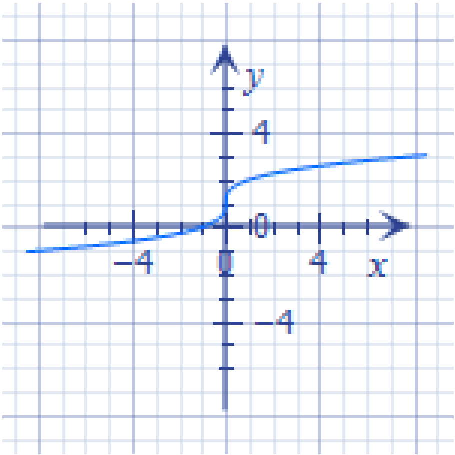
7)



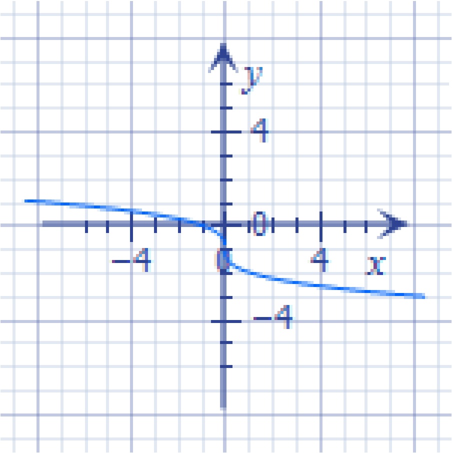
Domain

: All real

Range

: All real

11)



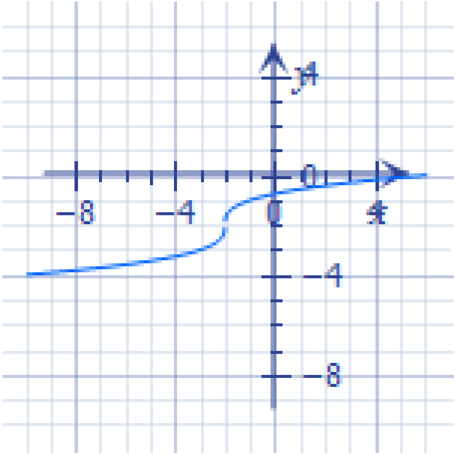
Domain

: All real

Range

: All real

12)



Domain

: All real

Range

: All real