GRAVITATION UNIT H.W. ANS KEY

Question 1.

To launch a spaceship from the earth, an escape velocity of v_{escape} is necessary. For that same spaceship to launch from Saturn, with a radius approximately 10 times that of the Earth, and a mass approximately 100 times that of the Earth, what escape velocity is required?

a. $\dfrac{\sqrt{10}}{v_{escape}}$

b. $\sqrt{10}\ v_{escape}$

c. $10\ v_{escape}$

d. $\dfrac{v_{escape}}{10}$

e. $1000\ v_{escape}$

Answer:

The correct answer is *b*. The relationship between escape velocity, mass, and radius can be derived using conservation of energy:

$$U + K = 0$$

$$\frac{-GMm}{r} = \frac{1}{2}mv^2$$

$$v_{escape} = \sqrt{\frac{2GM}{r}}$$

Now we can use values given in the problem to determine the Saturn escape velocity as a function of v_{escape}:

$$v_{escape}' = \sqrt{\frac{2G(100M_{earth})}{(10r_{earth})}} = \sqrt{10}\ v_{escape}$$

Question 2.

At the surface of a planet with radius R, a mass experiences a gravitational acceleration g. At a height of $3R$ above the surface of the planet, the gravitational acceleration is:

a. $\dfrac{g}{3}$

b. $3g$

c. $\dfrac{g}{9}$

d. $9g$

e. $\dfrac{g}{16}$

Answer:

The correct answer is e. The acceleration due to gravity for a mass M can be determined using Newton's Law of Universal Gravitation:
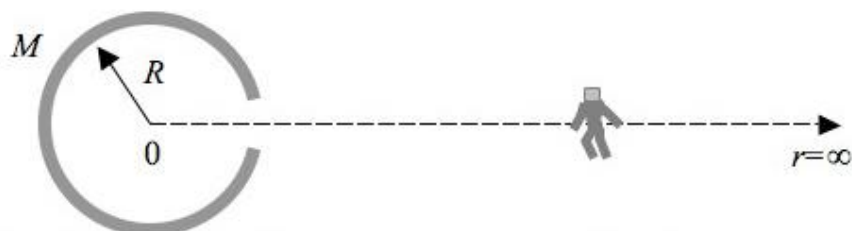
$$F_g = G\frac{Mm}{r^2}$$

$$mg = G\frac{Mm}{r^2}$$

$$g = G\frac{M}{r^2}$$

For the mass at the planet's surface, $g_R = G\dfrac{M}{R^2}$, where r is the distance between the mass's location and the center of the planet. For the mass above the planet's surface, the acceleration is $g_{4R} = G\dfrac{M}{(4R)^2} = G\dfrac{M}{16R^2} = \dfrac{1}{16}g_R$.

Question 3.

A large, massive, satellite is hollow, with all of its mass *m* located at a radius R from its center, as shown above. Which graph best represents the Force of gravity experienced by an astronaut at a distance *r* from the center of the satellite, where *r* goes from 0 to ∞?

a.

b.

c.

d.

e.
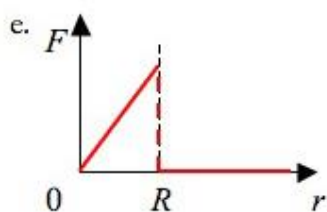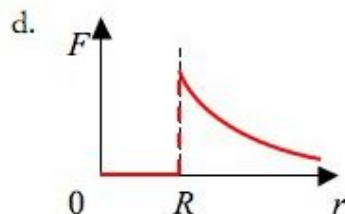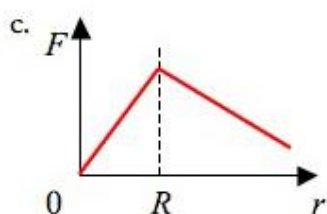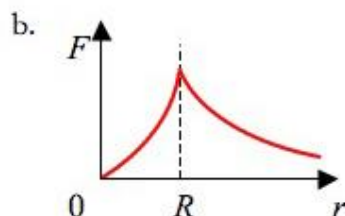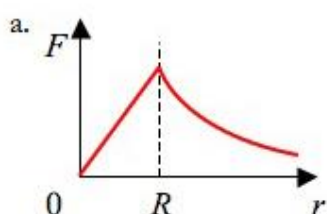
Answer:

The correct answer is *d*. Outside the satellite, its force of gravity acts on the astronaute as an inverse-square law, as if the mass were all located at the center-of-mass. Inside the satellite, the gravitational effects of the distribution of mass add up to produce a net gravitational force of 0 on the astronaut.

Question 4.

A satellite of mass m is in a circular orbit about the earth (mass $= M$) at a height h above the surface, where $h = r$, the radius of the earth. What velocity should this satellite have in order to maintain its orbit?

a. $v = \sqrt{\dfrac{GM}{r}}$

b. $v = \dfrac{GM}{2r}$

c. $v = \sqrt{\dfrac{GM}{2r}}$

d. $v = \sqrt{\dfrac{GMm}{2r}}$

e. $v = \dfrac{\sqrt{GM}}{2r}$

Answer:

The correct answer is c. The centripetal force that keeps the satellite in orbit about the earth is supplied by the force of earth's gravity.
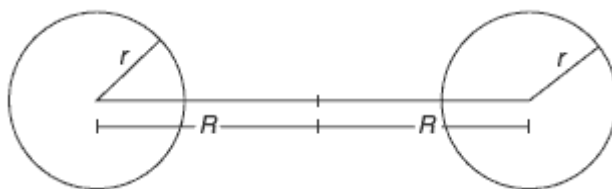
$$F_c = \frac{mv^2}{r}$$

$$F_g = G\frac{mM}{r^2}$$

$$G\frac{mM}{r^2} = \frac{mv^2}{r}$$

$$G\frac{M}{r} = v^2$$

Because the satellite is located one earth radius above the surface,

$$r_{satellite} = h + r = 2r$$

$$v = \sqrt{\frac{GM}{2r}}$$

Two stars, each of mass M, form a binary system. The stars orbit about a point a distance R from the center of each star, as shown in the diagram above. The stars themselves each have radius r.

Question 5. What is the force each star exerts on the other?

(A) $G\dfrac{M^2}{(2r+2R)^2}$

(B) $G\dfrac{M^2}{(R+r)^2}$

(C) $G\dfrac{M^2}{R^2}$
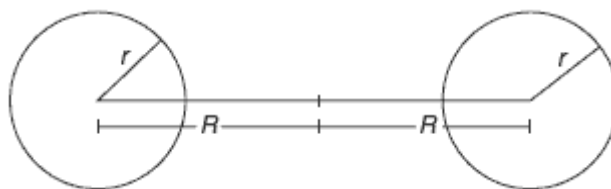
(D) $G\dfrac{M^2}{4R^2}$

(E) $G\dfrac{M^2}{2R^2}$

Answer

D—In Newton's law of gravitation,

$$F = G\frac{m_1 m_2}{r^2},$$

the distance used is the distance between the centers of the planets; here that distance is 2R. But the denominator is squared, so $(2R)^2 = 4R^2$ in the denominator here.

Two stars, each of mass M, form a binary system. The stars orbit about a point a distance R from the center of each star, as shown in the diagram above. The stars themselves each have radius r.

Question 6. In terms of each star's tangential speed v, what is the centripetal acceleration of each star?

(A) $\dfrac{v^2}{2R}$

(B) $\dfrac{v^2}{(r+R)}$

(C) $\dfrac{v^2}{2(r+R)}$

(D) $\dfrac{v^2}{2r}$

(E) $\dfrac{v^2}{R}$

Answer

E—In the centripetal acceleration equation

$$a_c = \frac{v^2}{r},$$

the distance used is the radius of the circular motion. Here, because the planets orbit around a point right in between them, this distance is simply R.

Question 7. A Space Shuttle orbits Earth 300 km above the surface. Why can't the Shuttle orbit 10 km above Earth?

(A) The Space Shuttle cannot go fast enough to maintain such an orbit.

(B) Kepler's Laws forbid an orbit so close to the surface of the Earth.

(C) Because *r* appears in the denominator of Newton's law of gravitation, the force of gravity is much larger closer to the Earth; this force is too strong to allow such an orbit.

(D) The closer orbit would likely crash into a large mountain such as Everest because of its elliptical nature.

(E) Much of the Shuttle's kinetic energy would be dissipated as heat in the atmosphere, degrading the orbit.

Answer

E—A circular orbit is allowed at any distance from a planet, as long as the satellite moves fast enough. At 300 km above the surface Earth's atmosphere is practically nonexistent. At 10 km, though, the atmospheric friction would quickly cause the shuttle to slow down.

Question 8. The orbital speed of a satellite orbiting the earth in a circular orbit at the height of 400 km above the surface of the earth is v_o. If the same satellite is at a distance of 800 km above the surface of the earth and the radius of the earth is 6400 m, the orbital speed of the satellite would be

(A) $2v_o$
(B) v_o
(C) $0.97v_o$
(D) $0.71v_o$
(E) $0.5v_o$

Answer

C—According to Kepler's Third Law, the orbital speed of an object is given by $v = \sqrt{\frac{G \cdot M}{r}}$ where G = 6.67x10⁻¹¹ m³/kg·s², M is the mass of the planet/body around which an object is orbiting, and r is the distance between the bodies' centers of mass.

If at 400 km, the velocity is v_o, the only thing that changes between the two situations is that the distance between the two bodies increases from 400 km to 800 km. Since the distance is increasing, the denominator is getting bigger so the velocity would have to get smaller (inversely proportional). This throws out answers (A) and (B). If the velocity was to be half of what it was before (Ans. E), the orbital distance would have to quadruple (since you take the square root of the distance between the objects). This leaves only answers (C) and (D).

Compare then $v_o = \sqrt{\frac{G \cdot M}{6400}}$ and $v = \sqrt{\frac{G \cdot M}{6800}}$. Simplifying: $v_o = \frac{1}{80}\sqrt{GM}$ and $v = \frac{1}{82.46}\sqrt{GM}$

$82.46x = 80 \rightarrow x = \frac{80}{82.46} \rightarrow x = \frac{80}{82.46} = 0.97$

Free Response 1. A satellite of mass m is moving in a circular orbit with linear speed v, around a planet of mass M, orbiting at a particular distance r from the center of the planet.
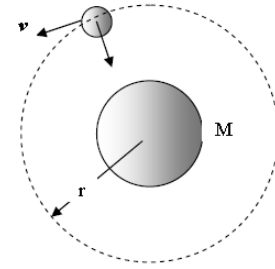
A. Determine the radius of revolution, r of the satellite, in terms of the given quantities and any fundamental constants.

$F_c = F_G$ $$\frac{mv^2}{r} = \frac{GmM}{r^2}$$ $mrv^2 = GmM$ $$r = \frac{GM}{v^2}$$	**1 point** For indicating that the gravitational force plays the role of the centripetal force **1 point** For a correct application of Newton's law of Universal Gravitation **1 point** For a correct application of $$\frac{m_{satellite}v^2}{r}$$ **1 point** For a correct expression for the radius of revolution in terms of the given quantities and the fundamental constant G

B. What is the relationship between the radius of revolution and the mass of the satellite which is undergoing uniform circular motion? Justify your response.

Radius of revolution is independent of the mass of the satellite. Radius of revolution depends upon the mass of the planet the satellite is orbiting, not the mass of the satellite.	**1 point** For indicating that there is no relationship between the radius and the mass of the satellite **1 point** For a proper justification, i.e. the mass of the satellite divides out of both sides of the equation

C. Assume the acceleration due to gravity g at a distance r from the center of the planet of mass M is 9 m/s^2. In terms of the radius of revolution r, what would the speed of the satellite have to be to remain in a circular orbit around this planet at this distance?

$F_w = F_c$ $mg = \dfrac{mv^2}{r}$ $gr = v^2$ $v = \sqrt{gr} = \sqrt{9r} = 3\sqrt{r}$	**1 point** For indicating that the weight force plays the role of the centripetal force **1 point** For indicating that the mass of the satellite divides out of the equation **1 point** For substituting 9 m/s^2 for g **1 point** For the correct expression in terms of the radius of revolution, r