6.1 Greatest Common Factors and Factoring by Grouping

Objectives

1 Factor out the greatest common factor.

2 Factor by grouping.

Greatest Common Factors and Factoring by Grouping

Writing a polynomial as the product of two or more simpler polynomials is called **factoring** the polynomial.

$3x(5x - 2) = 15x^2 - 6x$ **Multiplying**

$15x^2 - 6x = 3x(5x - 2)$ **Factoring**

Factoring "undoes" or reverses, multiplying.

Factor out the greatest common factor.

The first step in factoring a polynomial is to find the ***greatest common factor*** for the terms of the polynomial.

The **greatest common factor** (**GCF**) is the largest term that is a factor of all terms in the polynomial.

CLASSROOM EXAMPLE 1 **Factoring Out the Greatest Common Factor**

Factor out the greatest common factor.

$7k + 28$

Solution:

Since 7 is the GCF; factor 7 from each term.

$= 7 \cdot k + 7 \cdot 4$

$= 7(k + 4)$

Check:

$7(k + 4) = 7k + 28$ (Original polynomial)

CLASSROOM EXAMPLE 1 **Factoring Out the Greatest Common Factor (cont'd)**

Factor out the greatest common factor.

Solution:

$32m + 24$

$= 8 \cdot 4m + 8 \cdot 3$

$= 8(4m + 3)$

$8a - 9$

There is no common factor other than 1.

$5z + 5$

$= 5 \cdot z + 5 \cdot 1$

$= 5(z + 1)$

CLASSROOM EXAMPLE 2 **Factoring Out the Greatest Common Factor**

Factor out the greatest common factor.

$100m^5 - 50m^4 + 25m^3$

Solution:

The numerical part of the GCF is 25.

The variable parts are m^5, m^4, and m^3, use the least exponent that appears on *m.*

The GCF is $25m^3$.

$= 25m^3 \cdot 4m^2 - 25m^3 \cdot 2m + 25m^3 \cdot 1$

$= 25m^3(4m^2 - 2m + 1)$

CLASSROOM EXAMPLE 2 — **Factoring Out the Greatest Common Factor (cont'd)**

Factor out the greatest common factor.

$5m^4x^3 + 15m^5x^6 - 20m^4x^6$

Solution:

The numerical part of the GCF is 5.

The least exponent that occurs on m is m^4.

The least exponent that appears on x is x^3.

The GCF is $5m^4x^3$.

$= 5m^4x^3 \cdot 1 + 5m^4x^3 \cdot 3mx^3 - 5m^4x^3 \cdot 4x^3$

$= 5m^4x^3(1 + 3mx^3 - 4x^3)$

CLASSROOM EXAMPLE 3 — **Factoring Out a Binomial Factor**

Factor out the greatest common factor.

$(a + 2)(a - 3) + (a + 2)(a + 6)$

Solution:

The GCF is the binomial $a + 2$.

$= (a + 2)[(a - 3)+(a + 6)]$
$= (a + 2)(a - 3 + a + 6)$
$= (a + 2)(2a + 3)$

$(y - 1)(y + 3) - (y - 1)(y + 4)$

$= (y - 1)[(y + 3) - (y + 4)]$
$= (y - 1)(y + 3 - y - 4)$
$= (y - 1)(-1)$ or $-y + 1$

CLASSROOM EXAMPLE 3 — **Factoring Out a Binomial Factor (cont'd)**

Factor out the greatest common factor.

$k^2(a + 5b) + m^2(a + 5b)^2$

Solution:

The GCF is the binomial $a + 5b$.

$= k^2(a + 5b) + m^2(a + 5b)^2$

$= (a + 5b)[k^2 + m^2(a + 5b)]$

$= (a + 5b)(k^2 + m^2a + 5m^2b)$

CLASSROOM EXAMPLE 4 — **Factoring Out a Negative Common Factort**

Factor $-6r^2 + 5r$ in two ways.

Solution:

r could be used as the common factor giving

$= r \cdot -6r + r \cdot 5$

$= r(-6r + 5)$

Because of the negative sign, $-r$ could also be used as the common factor.

$= -r(6r) + (-r)(5)$

$= -r(6r - 5)$

Factoring by grouping.

Sometimes ***individual terms*** of a polynomial have a greatest common factor of 1, but it still may be possible to factor the polynomial by using a process called ***factoring by grouping.***

We usually factor by grouping when a polynomial has more than three terms.

CLASSROOM EXAMPLE 5 — **Factoring by Grouping**

Factor.

$6p - 6q + rp - rq$.

Solution:

Group the terms as follows:

Terms with a common factor of p — Terms with a common factor of q.

$(6p + rp) + (-6q - rq)$

Factor $(6p + rp)$ as $p(6 + r)$ and factor $(-6q - rq)$ as $-q(6 + r)$

$= (6p + rp) + (-6q - rq)$
$= p(6 + r) - q(6 + r)$
$= (6 + r)(p - q)$

CLASSROOM EXAMPLE 6 **Factoring by Grouping**

Factor.

$xy - 2y - 4x + 8.$

Solution:

Grouping gives: $xy - 4x - 2y + 8$

$= xy - 4x - 2y + 8$

$= x(y - 4) - 2(y - 4)$

$= (y - 4)(x - 2)$

Factoring by grouping.

Factoring by Grouping

Step 1 **Group terms.** Collect the terms into groups so that each group has a common factor.

Step 2 **Factor within groups.** Factor out the common factor in each group.

Step 3 **Factor the entire polynomial.** If each group now has a common factor, factor it out. If not, try a different grouping.

Always check the factored form by multiplying.

CLASSROOM EXAMPLE 7 **Factoring by Grouping**

Factor.

$kn + mn - k - m$

Solution:

Group the terms:

$= (kn + mn) + (-k - m)$

$= n(k + m) + (-1)(k + m)$

$= (k + m)(n - 1)$

CLASSROOM EXAMPLE 8 **Rearranging Terms before Factoring by Grouping**

Factor.

$10x^2y^2 - 18 + 15y^2 - 12x^2.$

Solution:

Group the terms so that there is a common factor in the first two terms and a common factor in the last two terms.

$= (10x^2y^2 + 15y^2) + (-12x^2 - 18)$

$= 5y^2(2x^2 + 3) - 6(2x^2 + 3)$

$= (2x^2 + 3)(5y^2 - 6)$

6.2 Factoring Trinomials

Objectives

1 Factor trinomials when the coefficient of the quadratic term is 1.

2 Factor trinomials when the coefficient of the quadratic term is not 1.

3 Use an alternative method for factoring trinomials.

4 Factor by substitution.

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Factor trinomials when the coefficient of the quadratic term is 1.

Factoring $x^2 + bx + c$

Step 1 **Find pairs whose product is *c***. Find all pairs of integers whose product is *c*, the third term of the trinomial.

Step 2 **Find pairs whose sum is *b***. Choose the pair whose sum is *b*, the coefficient of the middle term.

If there are no such integers, the polynomials cannot be factored.

A polynomial that cannot be factored with integer coefficient is a **prime polynomial**.

CLASSROOM EXAMPLE 1 **Factoring Trinomials in $x^2 + bx + c$ Form**

Factor the trinomial.

$a^2 + 9a + 20$

Solution:

Step 1 Find pairs of numbers whose product is 20.	***Step 2*** Write sums of those numbers.
20(1)	20 + 1 = 21
–20(–1)	–20 + (–1) = –21
10(2)	10 + 2 = 12
–10(–2)	–10 + (–2) = –12
5(4)	5 + 4 = 9
–5(–4)	–5 + (–4) = –9

CLASSROOM EXAMPLE 1 **Factoring Trinomials in $x^2 + bx + c$ Form (cont'd)**

The coefficient of the middle term is 9, so the required numbers are 5 and 4. The factored form of $a^2 + 9a + 20$ is

$(a + 5)(a + 4)$.

Check $(a + 5)(a + 4) = a^2 + 9a + 20$

CLASSROOM EXAMPLE 1 **Factoring Trinomials in $x^2 + bx + c$ Form (cont'd)**

Factor the trinomial.

$b^2 – 7b + 10$

Solution:

Step 1 Find pairs of numbers whose product is 10.	***Step 2*** Write sums of those numbers.
10(1)	10 + 1 = 11
–10(–1)	–10 + (–1) = –11
5(2)	5 + 2 = 7
–5(–2)	–5 + (–2) = –7

CLASSROOM EXAMPLE 1 **Factoring Trinomials in $x^2 + bx + c$ Form (cont'd)**

The coefficient of the middle term is –7, so the required numbers are –5 and –2. The factored form of $b^2 – 7b + 10$ is

$(b – 5)(b – 2)$.

Check $(b – 5)(b – 2) = b^2 – 7b + 10$

CLASSROOM EXAMPLE 2 — **Recognizing a Prime Polynomial**

Factor $t^2 + 3t – 5$.

Solution:

Look for two expressions whose product is –5 and whose sum is 3. There are no such quantities. Therefore, the trinomial cannot be factored and is ***prime.***

CLASSROOM EXAMPLE 3 — **Factoring a Trinomial in Two Variables**

Factor $p^2 – 5pq + 6q^2$.

Solution:

Look for two expressions whose product is $6q^2$ and whose sum is –5q. The quantities –3q and –2q have the necessary product and sum, so

$$p^2 – 5pq + 6q^2 = (p – 3q)(p – 2q).$$

CLASSROOM EXAMPLE 4 — **Factoring a Trinomial with a Common Factor**

Factor $3a^3 + 12a^2 – 15a$.

Solution:

Start by factoring out the GCF, 3a.

$$= 3a(a^2 + 4a – 5)$$

To factor $a^2 + 4a – 5$, look for two integers whose product is –5 and whose sum is 4. The necessary integers are –1 and 5. Remember to write the common factor 3a as part of the answer.

$$= 3a(a – 1)(a + 5)$$

CAUTION ***When factoring, always look for a common factor first. Remember to write the common factor as part of the answer.***

Objective 2

Factor trinomials when the coefficient of the quadratic term is not 1.

CLASSROOM EXAMPLE 5 — **Factoring a Trinomial in $ax^2 + bx + c$ Form**

Factor $6k^2 – 19k + 10$.

Solution:

The product as is 6 (10) = 60. Look for two integers whose products is 60 and whose sum is –19. The necessary integers are –15 and –4. Write –19k as –15k – 4k and then factor by grouping.

$$= 6k^2 – 15k – 4k + 10$$

$$= (6k^2 – 15k) + (–4k + 10)$$

$$= 3k(2k – 5) –2(2k – 5)$$

$$= (2k – 5)(3k – 2)$$

CLASSROOM EXAMPLE 6 — **Factoring Trinomials in $ax^2 + bx + c$ Form**

Factor each trinomial. Alternative Method

Solution:

$10x^2 + 17x + 3$

By trial and error, the following are factored.

$$= (5x + 1)(2x + 3)$$

$6r^2 + 13r – 5$

$$= (2r + 5)(3r – 1)$$

Use an alternative method for factoring trinomials.

Factoring $ax^2 + bx + c$ (GCF of a, b, c is 1)

Step 1 **Find pairs whose product is *a*.** Write all pairs of integer factors of a, the coefficient of the second-degree term.

Step 2 **Find pairs whose product is *c*.** Write all pairs of integer factors of c, the last term.

Step 3 **Choose inner and outer terms.** Use FOIL and various combinations of the factors from ***Steps 1 and 2*** until the necessary middle term is found.

If no such combinations exist, the trinomial is prime.

CLASSROOM EXAMPLE 7 **Factoring a Trinomial in Two Variables**

Factor $6m^2 + 7mn - 5n^2$.

Solution:

Try some possibilities.

$(6m + n)(m - 5n)$	$= 6m^2 - 29mn - 5n^2$	**No**
$(6m - 5n)(m + n)$	$= 6m^2 + mn - 5n^2$	**No**
$(3m + n)(2m - 5n)$	$= 6m^2 - 13mn - 5n^2$	**No**
$(3m + 5n)(2m - n)$	$= 6m^2 + 7mn - 5n^2$	**Yes**

The correct factoring is

$= (3m + 5n)(2m - n)$.

CLASSROOM EXAMPLE 8 **Factoring a Trinomial in $ax^2 + bx + c$ Form (a < 0)**

Factor $-2p^2 - 5p + 12$.

Solution:

First factor out –1, then proceed.

$= -1(2p^2 + 5p - 12)$

$= -1(p + 4)(2p - 3)$

$= -(p + 4)(2p - 3)$

CLASSROOM EXAMPLE 9 **Factoring a Trinomial with a Common Factor**

Factor $4m^3 + 2m^2 - 6m$.

Solution:

First, factor out the GCF, $2m$.

$= 2m(2m^2 + m - 3)$

Look for two integers whose product is $2(-3) = -6$ and whose sum is 1. The integers are 3 and –2.

$= 2m(2m^2 + 3m - 2m - 3)$

$= 2m[m(2m + 3) - 1(2m + 3)]$

$= 2m[(2m + 3)(m - 1)]$

$= 2m(2m + 3)(m - 1)$

CLASSROOM EXAMPLE 10 **Factoring a Polynomial by Substitution**

Factor $8(z + 5)^2 - 2(z + 5) - 3$.

Solution:

$= 8x^2 - 2x - 3$ **Let $x = z + 5$.**

$= (2x + 1)(4x - 3)$

Now replace x with $z + 5$.

$= [2(z + 5) + 1][4(z + 5) - 3]$

$= (2z + 10 + 1)(4z + 20 - 3)$

$= (2z + 11)(4z + 17)$

CLASSROOM EXAMPLE 11 **Factoring a Trinomial in $ax^4 + bx^2 + c$ Form**

Factor $6r^4 - 13r^2 + 5$.

Solution:

$= 6(r^2)^2 - 13r^2 + 5$

$= 6x^2 - 13x + 5$ **Let $x = r^2$.**

$= (3x - 5)(2x - 1)$ **Factor.**

$= (3r^2 - 5)(2r^2 - 1)$ ***$x = r^2$***

6.3 Special Factoring

Objectives

1. Factor a difference of squares.
2. Factor a perfect square trinomial.
3. Factor a difference of cubes.
4. Factor a sum of cubes.

Factor a difference of squares.

Difference of Squares

$$x^2 - y^2 = (x + y)(x - y)$$

CLASSROOM EXAMPLE 1 **Factoring Differences of Squares**

Factor each polynomial.

$p^2 - 100$

Solution:

$= p^2 - 10^2$

$= (p + 10)(p - 10)$

$2x^2 - 18$

$= 2(x^2 - 9)$ **Factor out the GCF, 2.**

$= 2(x^2 - 3^2)$

$= 2(x + 3)(x - 3)$

$9a^2 - 16b^2$

$= (3a)^2 - (4b)^2$

$= (3a + 4b)(3a - 4b)$

CLASSROOM EXAMPLE 1 **Factoring Differences of Squares (cont'd)**

Factor each polynomial.

$(m + 3)^2 - 49z^2$

Solution:

$= (m + 3)^2 - (7z)^2$

$= (m + 3 + 7z)(m + 3 - 7z)$

$y^4 - 16$

$= (y^2)^2 - 4^2$

$= (y^2 + 4)(y^2 - 4)$

$= (y^2 + 4)(y + 2)(y - 2)$

Objective 2

Factor a prefect square trinomial.

Factor a perfect square trinomial.

Perfect Square Trinomial

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

CLASSROOM EXAMPLE 2 | **Factoring Perfect Square Trinomials**

Factor the polynomial.

$49z^2 - 14z + 1$

Solution:

$= (7z)^2 - 14z + 1^2$

$= (7z - 1)^2$

Check. $2(7z)(-1) = -14z$, which is the middle term.

Thus, $49z^2 - 14z + 1 = (7z - 1)^2$.

CLASSROOM EXAMPLE 2 | **Factoring Perfect Square Trinomials (cont'd)**

Factor the polynomial.

$9a^2 + 48ab + 64b^2$

Solution:

$= (3a)^2 + 48ab + (8b)^2$

$= (3a + 8b)^2$

Check. $2(3a)(8b) = 48ab$, which is the middle term.

Thus, $9a^2 + 48\text{ab} + 64\text{b}^2 = (3a)^2 + 48ab + (8b)^2 = (3a + 8b)^2$

CLASSROOM EXAMPLE 2 | **Factoring Perfect Square Trinomials (cont'd)**

Factor the polynomial.

$x^2 - 2x + 1 - y^2$

Solution:

$= (x^2 - 2x + 1) - y^2$ **Factor by grouping.**

$= (x - 1)^2 - y^2$ **Factor the perfect square trinomial.**

This is the difference of two squares.

$= [(x - 1) + y][(x - 1) - y]$

$= (x - 1 + y)(x - 1 - y)$

Objective 3

Factor a difference of cubes.

Factor a difference of cubes.

Difference of Cubes

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

CLASSROOM EXAMPLE 3 | **Factoring Differences of Cubes**

Factor the polynomial.

$x^3 - 1000$

Solution:

$A^3 - B^3 \quad = (A - B)(A^2 + A \cdot B + B^2)$

$= x^3 - 10^3 \quad = (x - 10)(x^2 + x \cdot 10 + 10^2)$

$= (x - 10)(x^2 + 10x + 100)$

CLASSROOM EXAMPLE 3 | **Factoring Differences of Cubes (cont'd)**

Factor each polynomial.

$8k^3 - y^3$

Solution:

$= (2k)^3 - y^3$

$= (2k - y)[(2k)^2 + 2k(y) + y^2]$

$= (2k - y)(4k^2 + 2ky + y^2)$

$27m^3 - 64n^3$

$= (3m)^3 - (4n)^3$

$= (3m - 4n)[(3m)^2 + 3m(4n) + (4n)^2]$

$= (3m - 4n)(9m^2 + 12mn + 16n^2)$

Objective 4

Factor a sum of cubes.

Factor a sum of cubes.

Sum of Cubes

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

The sign of the second term in the binomial factor of a sum or difference of cubes is ***always the same*** as the sign in the original polynomial. In the trinomial factor, the first and last terms are ***always positive.*** The sign of the middle term is ***the opposite*** *of* the sign of the second term in the binomial factor.

CLASSROOM EXAMPLE 4 | **Factoring Sums of Cubes**

Factor each polynomial.

$8p^3 + 125$

Solution:

$= (2p)^3 + 5^3$

$= (2p + 5)[(2p)^2 - (2p)(5) + 5^2]$

$= (2p + 5)(4p^2 - 10p + 25)$

$64m^3 + 125n^3$

$= (4m)^3 + (5n)^3$

$= (4m + 5n)[(4m)^2 - 4m(5n) + (5n)^2]$

$= (4m + 5n)(16m^2 - 20mn + 25n^2)$

CLASSROOM EXAMPLE 4 | **Factoring Sums of Cubes (cont'd)**

Factor each polynomial.

$2x^3 + 2000$

Solution:

$= 2(x^3 + 1000)$

$= 2(x^3 + 10^3)$

$= 2(x + 10)(x^2 - 10x + 10^2)$

$= 2(x + 10)(x^2 - 10x + 100)$

$(a - 4)^3 + b^3$

$= [(a - 4) + b][(a - 4)^2 - (a - 4)b + b^2$

$= (a - 4 + b)(a^2 - 8a + 16 - ab + 4b + b^2)$

Factor a sum of cubes.

Special Types of Factoring

Difference of Squares	$x^2 - y^2 = (x + y)(x - y)$
Perfect Square Trinomial	$x^2 + 2xy + y^2 = (x + y)^2$
	$x^2 - 2xy + y^2 = (x - y)^2$
Difference of Cubes	$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
Sum of Cubes	$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

6.4 A General Approach to Factoring

Objectives

1. Factor out any common factor.
2. Factor binomials.
3. Factor trinomials.
4. Factor polynomials of more than three terms.

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A General Approach to Factoring

Factoring a Polynomial

Step 1 **Factor out any common factor.**

Step 2 **If the polynomial is a binomial,** check to see if it is the difference of squares, a difference of cubes, or a sum of cubes.

If the polynomial is a trinomial, check to see if it is a perfect square trinomial. If it is not, factor as in **Section 6.2**.

If the polynomial has more than three terms, try to factor by grouping.

Step 3 Check the factored form by multiplying.

CLASSROOM EXAMPLE 1 **Factoring Out a Common Factor**

Factor each polynomial.

$2x^3 + 10x^2 - 4x$

Solution:

The GCF is $2x$.

$= 2x(x^2 + 5x - 2)$

$12m(p - q) - 7n(p - q)$

The GCF is $(p - q)$

$= (p - q)(12m - 7n)$

Objective 2

Factor binomials.

Factor binomials.

Factoring a Binomial

For a **binomial** (two terms), check for the following:

Difference of Squares:

$$x^2 - y^2 = (x - y)(x + y)$$

Difference of Cubes:

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Sum of Cubes:

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

CLASSROOM EXAMPLE 2 **Factoring Binomials**

Factor each binomial, if possible.

$36x^2 - y^2$

Solution:

Difference of squares

$= (6x)^2 - (y)^2$

$= (6x - y)(6x + y)$

$4t^2 + 1$

The binomial is prime. It is the sum of squares.

$125x^3 - 27y^3 =$

$= (5x - 3y)[(5x)^2 + (5x)(3y) + (3y)^2]$

$= (5x - 3y)(25x^2 + 15xy + 9y^2)$

Factor trinomials.

Factoring a Trinomial

For a **trinomial** (three terms), decide whether it is a perfect square trinomial of either of these forms

$x^2 + 2xy + y^2 = (x + y)^2$

or

$x^2 - 2xy + y^2 = (x - y)^2$

If not, use the methods of **Section 6.2.**

CLASSROOM EXAMPLE 3 **Factoring Trinomials**

Factor each trinomial.

$16m^2 + 56m + 49$

Solution:

$= (4m + 7)^2$ **Perfect square trinomial**

$8t^2 - 13t + 5$

Two integer factors whose sum is 8(5) = 40 and whose sum is –13 are –5 and –8.

$= 8t^2 - 5t - 8t + 5$
$= t(8t - 5) - 1(8t - 5)$
$= (8t - 5)(t - 1)$

CLASSROOM EXAMPLE 3 **Factoring Trinomials (cont'd)**

Factor the trinomial.

$6x^2 - 3x - 63$

Solution:

Factor out the GCF of 3.

$= 3(2x^2 - x - 21)$

Two factors whose product is 2(–21) = – 42 and whose sum is –1 are –7 and 6.

$= 3[2x^2 - 7x + 6x - 21]$
$= 3[x(2x - 7) + 3(2x - 7)]$
$= 3(2x - 7)(x + 3)$

Objective 3

Factor polynomials of more than three terms.

CLASSROOM EXAMPLE 4 **Factoring Polynomials with More than Three Terms**

Factor each polynomial.

$p^3 - 2pq^2 + p^2q - 2q^3$

Solution:

$= (p^3 - 2pq^2) + (p^2q - 2q^3)$
$= p(p^2 - 2q^2) + q(p^2 - 2q^2)$
$= (p^2 - 2q^2)(p + q)$

$9x^2 + 24x + 16 - y^2$

$= (9x^2 + 24x + 16) - y^2$
$= (3x + 4)^2 - y^2$
$= [(3x + 4) + y)][(3x + 4) - y)]$
$= (3x + 4 + y)(3x + 4 - y)$

CLASSROOM EXAMPLE 4 **Factoring Polynomials with More than Three Terms (cont'd)**

Factor the polynomial.

$64a^3 + 16a^2 + b^3 - b^2$

Solution:

$= (64a^3 + b^3) + (16a^2 - b^2)$
$= [(4a)^3 + b^3] + [(4a)^2 - b^2]$
$= \{(4a + b)[(4a)^2 - 4ab + b^2]\} + [(4a + b)(4a - b)]$
$= [(4a + b)(16a^2 - 4ab + b^2)] + [(4a + b)(4a - b)]$
$= (4a + b)(16a^2 - 4ab + b^2 + 4a - b)$

6.5 Solving Equations by Factoring

Objectives

1 Learn and use the zero-factor property.

2 Solve applied problems that require the zero-factor property.

3 Solve a formula for a specified variable, where factoring is necessary.

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CLASSROOM EXAMPLE 1 **Using the Zero-Factor Property to Solve an Equation**

Solve $(8x + 3)(2x + 1) = 0$.

Solution:

By the zero-factor property, either

$8x + 3 = 0$ or $2x + 1 = 0$

$$8x + 3 = 0 \quad \text{or} \quad 2x + 1 = 0$$

$$8x = -3 \quad \text{or} \quad 2x = -1$$

$$x = -\frac{3}{8} \quad \text{or} \quad x = -\frac{1}{2}$$

Check the two solutions by substitution into the ***original*** equation.

Learn and use the zero-factor property.

Zero-Factor Property

If two numbers have a product of 0, then at least one of the numbers must be 0.

That is, if $ab = 0$, then either $a = 0$ or $b = 0$.

CAUTION ***The zero-factor property works only for a product equal to 0.***

CLASSROOM EXAMPLE 1 **Using the Zero-Factor Property to Solve an Equation (cont'd)**

Check the solutions.

$(8x + 3)(2x + 1) = 0$.

$$\left((8)\left(-\frac{3}{8}\right)+3\right)\left((2)\left(-\frac{3}{8}\right)+1\right)=0$$

$$(-3+3)\left(-\frac{6}{8}+\frac{8}{8}\right)=0$$

$$0 = 0$$

True

$$\left((8)\left(-\frac{1}{2}\right)+3\right)\left((2)\left(-\frac{1}{2}\right)+1\right)=0$$

$$(-4+3)(0)=0$$

$$0 = 0$$

True

Both solutions check; the solution set is $\left\{-\frac{3}{8}, -\frac{1}{2}\right\}$.

Learn and use the zero-factor property.

Quadratic Equation

An equation that can be written in the form

$$ax^2 + bx + c = 0$$

where *a, b,* and *c* are real numbers, with $a \neq 0$, is a **quadratic equation.** This form is called **standard form**.

Learn and use the zero-factor property.

Solving a Quadratic Equation by Factoring

Step 1 **Write in standard form.** Rewrite the equation if necessary so that one side is 0.

Step 2 **Factor** the polynomial.

Step 3 **Use the zero-factor property.** Set each variable factor equal to 0.

Step 4 **Find the solution(s).** Solve each equation formed in ***Step 3.***

Step 5 **Check** each solution in the ***original*** equation.

CLASSROOM EXAMPLE 2 **Solving Quadratic Equations by Factoring**

Solve the equation.

$3x^2 - x = 4$

Solution:

Step 1 **Standard form.** $3x^2 - x - 4 = 0$

Step 2 **Factor.** $(3x - 4)(x + 1) = 0$

Step 3 **Zero-factor.** $3x - 4 = 0$ or $x + 1 = 0$

Step 4 **Solve.** $3x = 4$, $x = \frac{4}{3}$ | $x = -1$

Step 5 **Check.** $3x^2 - x = 4$

$3\left(\frac{4}{3}\right)^2 - \frac{4}{3} = 4$

$4 = 4$ **True**

$3x^2 - x = 4$

$3(-1)^2 + 1 = 4$

$4 = 4$ **True**

The solution set is $\left\{-1, \frac{4}{3}\right\}$.

CLASSROOM EXAMPLE 2 **Solving Quadratic Equations by Factoring (cont'd)**

Solve the equation.

$16m^2 + 24m + 9 = 0$

Solution:

Step 1 **Standard form.** Already in standard form.

Step 2 **Factor.** $(4m + 3)^2 = 0$

Step 3 **Zero-factor.** $4m + 3 = 0$

Step 4 **Solve.** $4m = -3$, $m = \frac{-3}{4}$

Step 5 **Check.** $16\left(\frac{-3}{4}\right)^2 + 24\left(\frac{-3}{4}\right) + 9 = 0$

$9 - 18 + 9 = 0$

$0 = 0$ **True**

The solution set is $\left\{-\frac{3}{4}\right\}$.

CLASSROOM EXAMPLE 3 **Solving a Quadratic Equation with a Missing Constant Term**

Solve $x^2 + 12x = 0$

Solution:

Factor. $x(x + 12) = 0$

$x = 0$ or $x + 12 = 0$

$x = 0$ or $x = -12$

Check.

$x^2 + 12x = 0$ | $x^2 + 12x = 0$

$0^2 + 12(0) = 0$ | $(-12)^2 + 12(-12) = 0$

True $0 = 0$ | $0 = 0$ **True**

The solution set is $\{-12, 0\}$.

CLASSROOM EXAMPLE 4 **Solving a Quadratic Equation with a Missing Linear Term**

Solve $5x^2 - 80 = 0$

Solution:

$5x^2 - 80 = 5(x^2 - 16)$ **Factor out 5.**

$= 5(x - 4)(x + 4)$ **Factor.**

$x - 4 = 0$ or $x + 4 = 0$ **Solve.**

$x = 4$ or $x = -4$

Check that the solution set is $\{-4, 4\}$.

CLASSROOM EXAMPLE 5 **Solving an Equation That Requires Rewriting**

Solve $(x + 6)(x - 2) = 2 + x - 10$.

Solution:

$x^2 + 4x - 12 = x - 8$ **Multiply.**

$x^2 + 3x - 4 = 0$ **Standard form.**

$(x - 1)(x + 4) = 0$ **Factor.**

$x - 1 = 0$ or $x + 4 = 0$

$x = 1$ or $x = -4$

Check that the solution set is $\{-4, 1\}$.

CLASSROOM EXAMPLE 6 **Solving an Equation of Degree 3**

Solve $3x^3 + x^2 = 4x$

Solution:

$3x^3 + x^2 - 4x = 0$ **Standard form.**

$x(3x^2 + x - 4) = 0$ **Factor out *n*.**

$x(3x + 4)(x - 1) = 0$

$x = 0$ or $3x + 4 = 0$ or $x - 1 = 0$

$x = -\frac{4}{3}$ $x = 1$

Check that the solution set is $\left\{-\frac{4}{3}, 0, 1\right\}$.

CLASSROOM EXAMPLE 7 — **Using a Quadratic Equation in an Application**

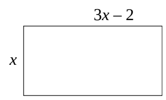
The length of a room is 2 m less than three times the width. The area of the room is 96 m². Find the width of the room.

Solution:

Step 1 **Read** the problem again. There will be one answer.

Step 2 **Assign a variable.**

Let x = the width of the room.
Let $3x - 2$ = the length of the room.
The area is 96.



CLASSROOM EXAMPLE 7 — **Using a Quadratic Equation in an Application (cont'd)**

Step 3 **Write an equation.** The area of a rectangle is the length times the width.

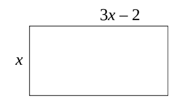
$A = lw$
$96 = (3x - 2)x$

Step 4 **Solve.**

$96 = (3x - 2)x$
$96 = 3x^2 - 2x$
$0 = 3x^2 - 2x - 96$
$0 = (3x + 16)(x - 6)$

$3x + 16 = 0$ or $x - 6 = 0$

$x = -\frac{16}{3}$ $\quad$ $x = 6$



CLASSROOM EXAMPLE 7 — **Using a Quadratic Equation in an Application (cont'd)**

Step 5 **State the answer.**

A distance cannot be negative, so reject $-\frac{16}{3}$ *as a solution.*

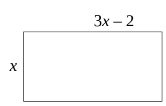
The only possible solution is 6, so the width of the room is 6 m.

Step 6 **Check.**

The length is 2 m less than three times the width, the length would be $3(6) - 2 = 16$.

The area is 6 • 16 = 96, as required.

When application lead to quadratic equations, a solution of the equation may not satisfy the physical requirements of the problem. Reject such solutions as valid answers.



CLASSROOM EXAMPLE 8 — **Using a Quadratic Function in an Application**

If a small rocket is launched vertically upward from ground level with an initial velocity of 128 ft per sec, then its height in feet after t seconds is a function defined by $h(t) = -16t^2 + 128t$ if air resistance is neglected. How long will it take the rocket to reach a height of 256 feet?

Solution:

We let $h(t) = 256$ and solve for t.

$256 = -16t^2 + 128t$
$16t^2 - 128t + 256 = 0$ $\quad$ **Divide by 16.**
$t^2 - 8t + 16 = 0$
$(t - 4)^2 = 0$
$t - 4 = 0$
$t = 4$

It will reach a height of 256 feet in 4 seconds.

Objective 3

Solve a formula for a specified variable, where factoring is necessary.

CLASSROOM EXAMPLE 9 — **Using Factoring to Solve for a Specified Variable**

Solve the formula for W.

$A = 2HW + 2LW + 2LH$

Solution:

$A - 2LH = 2HW + 2LW$

$A - 2LH = W(2H + 2L)$

$\frac{A - 2LH}{2H + 2L} = W$ or $W = \frac{A - 2LH}{2H + 2L}$

We must write the expression so that the specified variable is a factor. Then we can divide by its coefficient in the final step.