

Greece



Math 1700 - Greece

The Origins of (Deductive) Mathematics?

- Greece is often cited as the place where the mathematics, in the modern sense of a logical, deductive system of theorems, was established.
- Why there and not elsewhere?
- Einstein's answer, with respect to the origin of science in general in Greece, was:
 - "The astonishing thing is that these discoveries [the bases of science] were made at all."

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The Origins of Ancient Greece

- What we call ancient Greece might better be called the ancient Aegean Civilizations.



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The Aegean Civilizations

- There have been civilizations in the Aegean area almost as long as there have been in Mesopotamia and Egypt.
- The earliest known in the area was the Minoan Civilization on the island of Crete.
 - Existed from about 3000 – 1450 BCE.
 - Had some kind of written language, never deciphered.
 - Collapsed suddenly for unknown reasons.

The Mycenaean Civilization

- On the Peloponnesus (the southern mainland) another civilization arose and flourished from about 1600-1200 BCE.
- The Mycenaeans adapted the Minoan writing system to their own language, Greek. But it was awkward to use.

Mycenaea

- The peak of the Mycenaean civilization was the reign of Agamemnon, who took his people (the "Greeks") to war against the Trojans.



Agamemnon's Palace

The Trojan War



The Trojan War

- Approx. 1280 – 1180 BCE.
- Mycenaea versus Troy.
- Won by the Greeks, but the war depleted their fighting forces.
- Mycenaea was invaded by Dorians about 1200 BCE, and its culture destroyed.

The Dark Age of Greece

- 1200 – 800 BCE
- The organized Greek civilization was destroyed by the invading Dorians.
- Knowledge of writing was lost.
- People lived in isolated villages.
- What they had in common was spoken Greek and memories of past greatness.

Phoenicia

- Around 1700 BCE, in the Near East, what is now Lebanon, a civilization developed with both Mesopotamian and Egyptian influences.
- The Greeks later called the people from there "Phonicians" – meaning traders in purple.

Phoenician Writing

- Phoenicians developed a style of writing that combined Mesopotamian cuneiform and Egyptian hieratic.
- It had 22 distinct characters, each representing a particular sound (a consonant).

The Phoenician Alphabet



The Phoenician Alphabetic was Phonetic

- Since each character represented a sound, rather than a meaning, the characters could be used to represent words in an entirely different language.
- The Greeks adapted the Phoenician script to their own language and produced an *alphabet*.

The Homeric Age

- 800 – 600 BCE
- The Greek verbal culture could be written down.
- The heroic stories of the Trojan War were written by Homer.
 - *The Iliad, The Odyssey*
- Greek mythology and folk knowledge were recorded by Hesiod.
 - *Theogony, Works and Days*



The Greek Civilization Takes Off

- The first Olympic Games 776 BCE
- The Polis (City-State)
 - Independent governments arose all across the Greek settlements.
 - Experimentation in forms of government:
 - Monarchies, Aristocracies, Dictatorships, Oligarchies, Democracies
 - Independent units, but tied together by a common language, religion, and literature.

Assertion: Deductive Reasoning Began in Ancient Greece

Possible explanations given for the origin of science in Greece:

- **Religion** – The Greek gods were too human-like.
- **Language** – Phonetic alphabet encouraged literacy.
- **Trade** – The Greeks became traders and travellers, bringing home new ideas.
- **Democracy** – Democratic governments, where they existed, encouraged independent thought.
- **Slavery** – Greeks (like many other cultures) had slaves who did the menial work.

The Pre-Socratics

- Thinkers living between about 600 – 450 BCE.
- So named because they (basically) predated Socrates.
- Known only through discussions of their thoughts in later works.
- Some fragments still exist.

Socrates

- Lived in Athens, 470-399 BCE.
- Set the direction of Western philosophical thinking.
- The goal of philosophy – to discover the truth.
- Reasoning, the supreme method.
 - Pursued by asking questions, the dialectical, or "Socratic" method.



Socrates, contd.

- Socrates left no writings at all.
- He is known to us primarily through the works of Plato.
 - It is hard to distinguish Socrates' own thought from Plato's.
- Socrates is an important figure in the development of deductive reasoning, but...
- He had no interest in the natural world, nor in mathematics.

Back to the Pre-Socratics

- Most Pre-Socratics came from the Greek colonies on the eastern side of the Aegean Sea known as Ionia.
 - This is now part of Turkey.

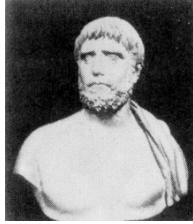


Wondering about Nature

- The importance of the Pre-Socratics is that they appear to be the first people we know of who asked fundamental questions about nature, such as "What is the world made of?"
 - And then they provided reasons to justify their answers.

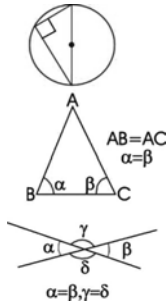
Thales of Miletos

- 625-545 BCE
- Phoenician parents?
- Stories:
 - Predicted solar eclipse of May 28, 585 BCE
 - Falling into a well
 - Olive press
- **Water** is the basic stuff of the world.



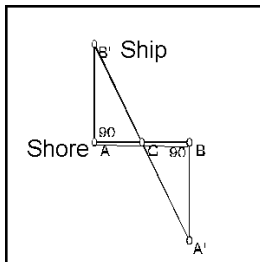
Thales and Mathematics

- Thales is said to have brought Egyptian mathematics to Greeks.
- Examples:
- All triangles constructed on the diameter of a circle are right triangles.
 - The base angles of isosceles triangles are equal.
 - If two straight lines intersect, opposite angles are equal.



Measuring the distance of a ship from shore

- From the desired point on the shore, A, walk off a known distance to point C, at a right angle from the ship and place a marker there.
- Continue walking the same distance again to B.
- At B, turn at a right angle away from the shore and walk until the marker at C and the ship are in a straight line. Call that A'.
- The distance from A' to B is the same as the distance from A to the ship.



Anaximander of Miletos

- 611-547 BCE
- Student of Thales?
- Map of the known world
- **Apeiron** (the Boundless)
 - The basic stuff of the world



Anaximander's Map of the World

Anaximenes of Miletos

- 550-475 BCE
- Student of Anaximander?
- *Air* – the fundamental stuff
- Cosmological view:
 - Crystalline sphere of the fixed stars
 - Earth in centre, planets between



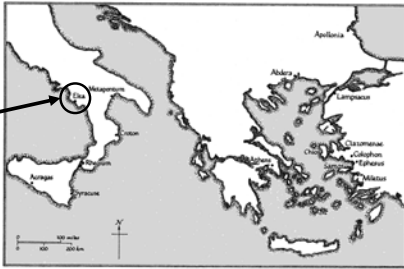
Heraclitus of Ephesus

- Ephesus is 50 km N of Miletos.
- 550?-475? BCE (i.e., about the same as Anaximenes, but uncertain)
- Everything is Flux.
 - Fire fundamental
 - "You can't step in the same river twice."



Elea

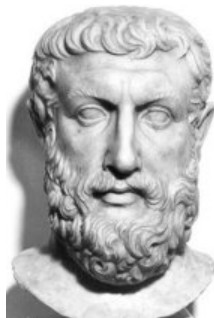
Elea was a Greek colony in southern Italy.



- The minor Pre-Socratic, Xenophanes, fled from Colophon in Ionia to Elea to escape persecution.

Parmenides of Elea

- 510-??
- Student of the exiled Xenophanes
- The goal of philosophy is to attain the truth.
- The path to truth is via reason and logic.
- Reason will distinguish appearance from reality.
 - Nature is comprehensible and logical.



Parmenides and the Law of Contradiction

- Something either *is* or it *is not*.
 - The law of the excluded middle
- Therefore, nothing *is* that *isn't!*
 - It is impossible to be *not being*
 - There is no such thing as *empty* space.
 - *Space* is something and *empty* is nothing.

Parmenides against Heraclitos

- If there is no space that is empty, the universe is everywhere full and occupied.
- Therefore nothing actually changes.
- Therefore motion is impossible.

The Fundamental Problem of Viewpoint

- Focus on the whole – Parmenides
 - Easier to grasp the unity of the world.
 - Difficult to explain processes, events, changes.
- Focus on the parts – Heraclitos
 - Easier to explain changes as rearrangements of the parts.
 - Difficult to make sense of all that is.

The Perils of Logic

- Reasoning with logic inevitably begins with assumed premises, which may or may not be true.
- The reasoning itself may or may not be valid – though this can be checked.
- The truth of conclusions depends on the truth of the premises and the validity of the argument.

Zeno of Elea

- 495-425 BCE
- Student of Parmenides
- Probably moved to Athens later and taught there, making his and Parmenides' views better known.



Zeno's Paradoxes

- Paradox, from the Greek meaning "contrary to opinion."
- Showed that logic can lead to conclusions which defy common sense.
 - Hard to say whether he was attacking common sense beliefs (as seems probable), or demonstrating the dangers of reasoning by logical deduction.

The Stadium

- Consider a stadium—a running track of about 180 meters in ancient Greece.

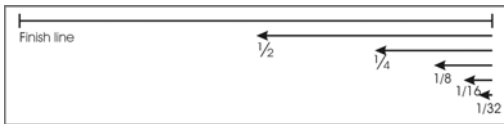


The Stadium



- Will the runner reach the other side of the stadium?

The Stadium Paradox



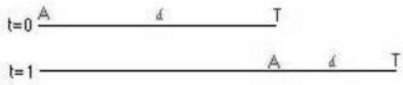
- Before the runner can reach the finish line, the mid-point must be reached.
- Before that, the $\frac{1}{4}$ point. Before that $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$,... and an infinite number of prior events.
- The runner never can leave the starting block.

Achilles and the Tortoise



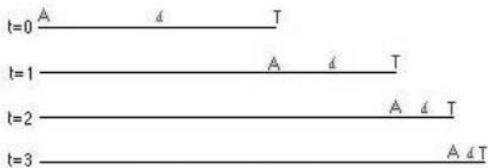
- Achilles, the mythical speedy warrior, is to have a footrace with a tortoise.
- Achilles gives the tortoise a head start.

Achilles and the Tortoise, 2



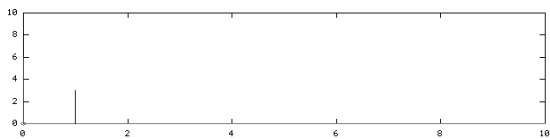
- Call the starting time $t=0$.
- Before Achilles can pass the tortoise, he must reach where the tortoise was at the start.
- Call when Achilles reaches the tortoise's starting position $t=1$
- By then, the tortoise has gone ahead.

Achilles and the Tortoise, 3



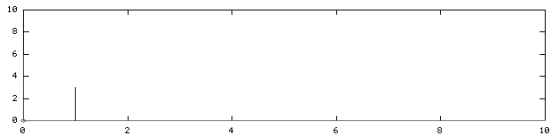
- Now at time $t=1$, Achilles still must reach where the tortoise is before he can pass it.
- Every time Achilles reaches where the tortoise had been, the tortoise is further ahead.
- The tortoise must win the race.

Achilles and the Tortoise, 4



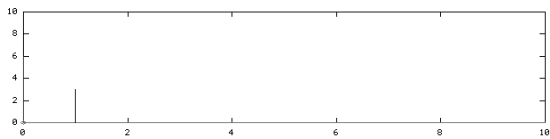
- An animated demonstration of the paradox.

Achilles and the Tortoise, 5



- And again, an animated demonstration of the paradox.

Achilles and the Tortoise, 6



- One more time: An animated demonstration of the paradox.

The Flying Arrow



- Imagine an arrow in flight. Is it moving?
- Motion means moving from place to place.
- At any single moment, the arrow is in a single place, therefore, not moving.

The Flying Arrow, 2



- At every moment of its flight, the arrow is not moving. If it were, it would occupy more space than it does, which is impossible.
- There is no such thing as motion.
