Greeley-Evans School District 6 Geometry 2016-2017

Suggested Big Idea	Unit 1: Basics of Geometry, Congruence, Transformations			
Content Emphasis Cluster	Understand congruence in terms of rigid motions			
Mathematical Practices	MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.			
Common Assessment	End of Unit Assessment			
Graduate Competency	Prepared graduates apply transformation to numbers, shapes, functional representations, and data			
CCSS Priority Standards	**Cross-Content Connections**	**Writing Focus**	**Language/Vocabulary**	**Misconceptions**
G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	Literacy Connections RST.6-8.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics. RST.6-8.5 Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic. RST.6-8.7 Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table). RST.6-8.8 Distinguish among facts, reasoned judgment based on research	Writing Connection WHST.6-8.2 Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate and varied transitions to create cohesion and clarify the relationships	Academic Vocabulary- Transformation Translation Rotation Reflection Congruence Technical Vocabulary- Isometry L.6-8.6 Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression. L.6-8.4 Determine or clarify the meaning of unknown and multiple-meaning words and phrases choosing flexibly from a range of strategies.	G.CO.6 That combinations such as SSA or AAA are also a congruence criterion for triangles. Provide counterexamples for this misconception. That all transformations, including dilations, are rigid motions. Provide counterexamples for this misconception. That any two figures that have the same area represent a rigid transformation. Students should recognize that the areas remain the same, but preservation of side and angle lengths determine that the transformation is rigid. That corresponding vertices do not have to be listed in order; however, it is useful to stress the importance of listing corresponding vertices in the same order so that corresponding sides and angles can be easily identified and that included sides or angles are apparent. G.CO.7 That combinations such as SSA or AAA are also a congruence criterion for triangles. Provide counterexamples for this misconception. That all transformations, including dilations, are rigid motions. Provide counterexamples for this misconception.

	findings, and speculation in a text.	among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented. WHST.6-8.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.		That any two figures that have the same area represent a rigid transformation. Students should recognize that the areas remain the same, but preservation of side and angle lengths determine that the transformation is rigid. That corresponding vertices do not have to be listed in order; however, it is useful to stress the importance of listing corresponding vertices in the same order so that corresponding sides and angles can be easily identified and that included sides or angles are apparent.

Unit 1	Basics of Geometry, Congruence, Transformations	Length of Unit	Est: 22 Days 8/26/16-9/27/16 Unit assessment scanned by 10/6/16
Standards of Mathematical Practices	**MP.2** Reason abstractly and quantitatively **MP.3** Construct viable arguments and critique the reasoning of others. **MP.5** Use appropriate tools strategically. **MP.6** Attend to Precision **MP.7** Look for and make use of structure		
Content Standards (<u>Priority Standards underlined)</u>	**Experiment with transformations in the plane** **G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). **G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. **G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **<u>Understand congruence in terms of rigid motions</u>** **<u>G.CO.6</u>** **<u>Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.</u>** **<u>G.CO.7</u>** **<u>Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.</u>** **Prove geometric theorems** **G.CO.9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.		

	Make geometric constructions **G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. **G.CO.**13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle **C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
Terminology:	
Academic Vocabulary *Cross discipline language*	Transformation, Translation, Rotation, Reflection, Congruence
Technical Vocabulary *Discipline-specific language*	**Isometry** An isometry of the plane is a transformation of the plane that is distance-preserving.
Assessments	**End of Unit Common Assessment on Schoolcity:** • Scanned into School City or students take the assessment online • Should be in addition to individually developed formative assessments

Core Lessons and Notes

Lessons as suggested from Engage NY Module 1 (click links to follow) **Topic A: Basic Constructions (G-CO.A.1, G-CO.D.12, G-CO.D.13)** Lessons 1-2: Construct an Equilateral Triangle Lesson 3: Copy and Bisect an Angle Lesson 4: Construct a Perpendicular Bisector Lesson 5: Points of concurrencies **Topic B: Unknown Angles (G-CO.C.9)** Lesson 6: Solve for Unknown Angles – Angles and lines at a point Lesson 7: Solve for Unknown Angles – Transversals Lesson 8: Solve for Unknown Angles – Angles in a Triangle Lesson 9: Unknown Angle Proofs – Writing Proofs Lesson 10: Unknown Angle Proofs – Proofs with Constructions Lesson 11: Unknown Angle Proofs – Proofs of known Facts **Topic C: Transformations/Rigid Motions (G-CO.A.2, G-CO.A.3, G-CO.A.4, G-CO.A.5, G-CO.B.6, G-CO.B.7, G-CO.D.12)** Lesson 12: Transformations – The Next Level Lesson 13: Rotations Lesson 14: Reflections	• *Lessons listed on the left are OPTIONAL resources from New York Engage. You may choose to use these as a basis or supplement to your own materials.* • When teaching transformations, provide examples with coordinates, but do not limit to coordinates. • Coordinate geometry has been separated into its own units to help students obtain a deeper grasp on the essential concepts presented in these units. • When teaching rigid motions, reiterate the fact that corresponding parts of congruent figures are congruent. • The construction skills learned in lessons 1-5 are key to future lessons in various units. • Engage NY states that lesson 18 is important because it covers the parallel postulate. • After lesson 21 Engage NY provides Assessment Tasks that include grading rubrics along with student work samples.

Lesson 15: Rotations, Reflections, and Symmetry Lesson 16: Translations Lesson 17: Characterize Points on a Perpendicular Lesson 18: Looking More Carefully at Parallel Lines Lesson 19: Construct and Apply a Sequence of Rigid Motions Lesson 20: Applications of Congruence in Terms of Rigid Motions Lesson 21: Correspondence and Transformations Optional Materials and Optional Testing Items from New York Engage	
Materials	Compass, Straightedge, Ruler, MIRA or GeoReflector, Protractor, Graph Paper, Patty Paper, Calculators, Projector, Doc Cam, Promethean Board

Unit 1 - Congruence and Transformations

Topic A – Basic Constructions

The first module of Geometry incorporates and formalizes geometric concepts presented in all the different grade levels up to high school geometry. Topic A brings the relatively unfamiliar concept of construction to life by building upon ideas students are familiar with, such as the constant length of the radius within a circle. While the figures that are being constructed may not be novel, the process of using tools to create the figures is certainly new. Students use construction tools, such as a compass, straightedge, and patty paper to create constructions of varying difficulty, including equilateral triangles, perpendicular bisectors, and angle bisectors. The constructions are embedded in models that require students to make sense of their space and to understand how to find an appropriate solution with their tools. Students will also discover the critical need for precise language when they articulate the steps necessary for each construction. The figures covered throughout the topic provide a bridge to solving, then proving unknown angle problems.

Topic B – Unknown Angles

Constructions segue into Topic B, Unknown Angles, which consists of unknown angle problems and proofs. These exercises consolidate students' prior body of geometric facts and prime students' reasoning abilities as they begin to justify each step for a solution to a problem. Students began the proof writing process in Grade 8 when they developed informal arguments to establish select geometric facts (8.G.5).

Topic C – Transformations/Rigid Motions

Topic C, Transformations, builds on students' intuitive understanding developed in Grade 8. With the help of manipulatives, students observed how reflections, translations, and rotations behave individually and in sequence (8.G.1, 8.G.2). In Grade 10, this experience is formalized by clear definitions (G.CO.4) and more in-depth exploration (G.CO.3, G.CO.5). The concrete establishment of rigid motions also allows proofs of facts formerly accepted to be true (G.CO.9). Similarly, students' Grade 8 concept of congruence transitions from a hands-on understanding (8.G.2) to a precise, formally notated understanding of congruence (G.CO.6). With a solid understanding of how transformations form the basis of congruence, students next examine triangle congruence criteria. Part of this examination includes the use of rigid motions to prove how triangle congruence criteria such as SAS actually work (G.CO.7, G.CO.8).

Resources and Notes by Standard (adapted from [KATM Flip Books])

Standard: G.CO.1

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Connections: G.CO.1-5

Rotations, reflections and translations are developed experimentally in the Grade 8, and this experience should be built upon in high school, giving greater attention to precise definitions and formal reasoning.

Transformations can be studied in terms of functions, where the inputs and outputs are points in the plane, rather than numbers.

Rotations are studied again in the cluster about circles.

Explanations and Examples: G.CO.1

Understand and use definitions of angles, circles, perpendicular lines, parallel lines, and line segments based on the undefined term of a point, a line, the distance along a line, and the length of an arc.

Define angles, circles, perpendicular lines, rays, and line segments precisely using the undefined terms and “if-then” and “if-only-if” statements.

Examples:

- Have students write their own understanding of a given term.
- Give students formal and informal definitions of each term and compare them.
- Develop precise definitions through use of examples and non-examples.

Discuss the importance of having precise definitions.

Instructional Strategies: G.CO.1-5

Review vocabulary associated with transformations (e.g. point, line, segment, angle, circle, polygon, parallelogram, perpendicular, rotation reflection, translation).

Provide both individual and small-group activities, allowing adequate time for students to explore and verify conjectures about transformations and develop precise definitions of rotations, reflections and translations.

Provide real-world examples of rigid motions (e.g. Ferris wheels for rotation; mirrors for reflection; moving vehicles for translation).

Use graph paper, transparencies, tracing paper or dynamic geometry software to obtain images of a given figure under specified transformations.

Instructional Strategies: G.CO.1-5

Provide students with a pre-image and a final, transformed image, and ask them to describe the steps required to generate the final image. Show examples with more than one answer (e.g., a reflection might result in the same image as a translation).

Work backwards to determine a sequence of transformations that will carry (map) one figure onto another of the same size and shape.

Focus attention on the attributes (e.g. distances or angle measures) of a geometric figure that remain constant under various transformations.

Make the transition from transformations as physical motions to functions that take points in the plane as inputs and give other points as outputs. The correspondence between the initial and final points determines the transformation.

Analyze various figures (e.g. regular polygons, folk art designs or product logos) to determine which rotations and reflections carry (map) the figure onto itself. These transformations are the “symmetries” of the figure.

Emphasize the importance of understanding a transformation as the correspondence between initial and final points, rather than the physical motion.

Use a variety of means to represent rigid motions, including physical manipulatives, coordinate methods, and dynamic geometry software.

Common Misconceptions: G.CO.1-5

The terms "mapping" and "under" are used in special ways when studying transformations. A translation is a type of transformation that moves all the points in the object in a straight line in the same direction.

Students should know that not every transformation is a translation.

Students sometimes confuse the terms "transformation" and "translation."

Standard: G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Connections:

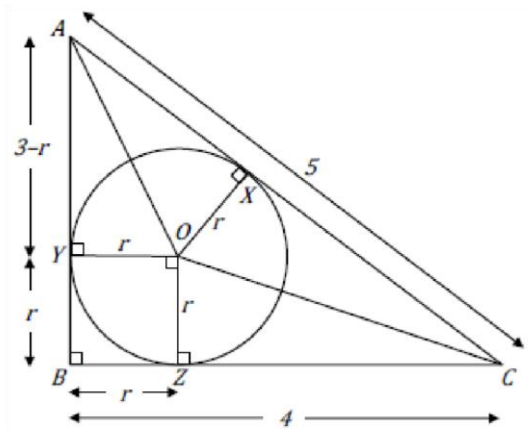
Constructing inscribed and circumscribed circles of a triangle is an application of the formal constructions studied in G.CO.12.

Explanations and Examples: G.C.3

Define the terms inscribed, circumscribed, angle bisector, and perpendicular bisector.
Construct the inscribed circle whose center is the point of intersection of the angle bisectors (the incenter).
Construct the circumscribed circle whose center is the point of intersection of the perpendicular bisectors of each side of the triangle (the circumcenter).
Apply the Arc Addition Postulate to solve for missing arc measures.
Prove that opposite angles in an inscribed quadrilateral are supplementary.
Using definitions, properties, and theorems, prove properties of angles for a quadrilateral inscribed in a circle. Students may use geometric simulation software to make geometric constructions.

Examples:

 The following diagram shows a circle that just touches the sides of a right triangle whose sides are 3 units, 4 units, and 5 units long.
Explain why triangles AOX and AOY are congruent.
What can you say about the measures of the line segments CX and CZ?
Find r, the radius of the circle. Explain your work clearly and show all your calculations.



Standard: G.CO.2

Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Connections:

Rotations, reflections and translations are developed experimentally in the Grade 8, and this experience should be built upon in high school, giving greater attention to precise definitions and formal reasoning.

Transformations can be studied in terms of functions, where the inputs and outputs are points in the plane, rather than numbers.

Rotations are studied again in the cluster about circles.

Explanations and Examples: G.CO.2

In middle school students have worked with translations, reflections, and rotations and informally with dilations. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.

Use various technologies such as transparencies, geometry software, interactive whiteboards, and digital visual presenters to represent and compare rigid and size transformations of figures in a coordinate plane. Comparing transformations that preserve distance and angle to those that do not.

Describe and compare function transformations on a set of points as inputs to produce another set of points as outputs, to include translations and horizontal and vertical stretching.

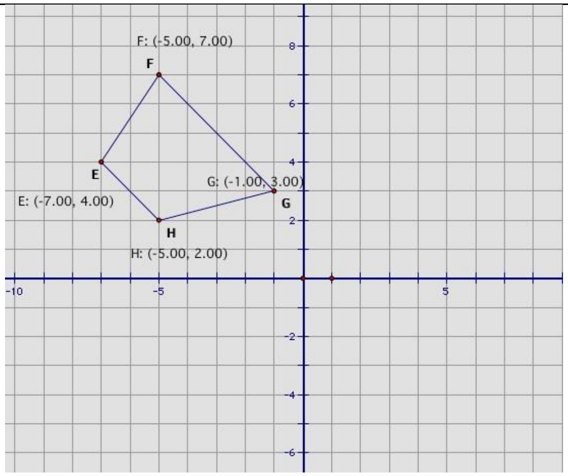
Students may use geometry software and/or manipulatives to model and compare transformations.

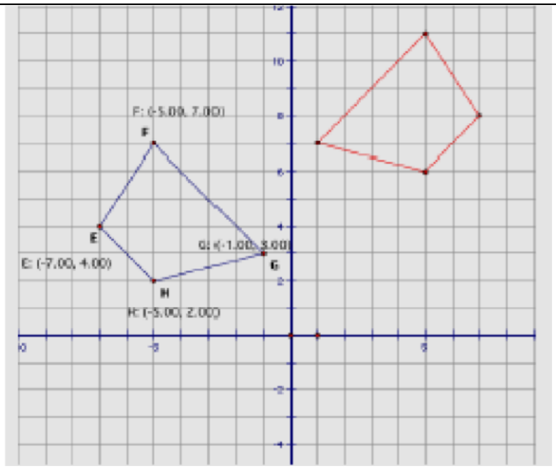
Examples:

- Draw transformations of reflections, rotations, translations, and combinations of these using graph paper, transparencies and/or geometry software.
- Determine the coordinates for the image (output) of a figure when a transformation rule is applied to the preimage (input).
- Distinguish between transformations that are rigid (preserve distance and angle measure-reflections, rotations, translations, or combinations of these) and those that are not (dilations or rigid motions followed by dilations).

The figure below is reflected across the y-axis and then shifted up by 4 units. Draw the transformed figure and label the new coordinates.

What function can be used to describe these transformations in the coordinate plane?





Solution:
(-1x, y)

Instructional Strategies: G.CO.1-5

Provide both individual and small-group activities, allowing adequate time for students to explore and verify conjectures about transformations and develop precise definitions of rotations, reflections and translations.

Provide real-world examples of rigid motions (e.g. Ferris wheels for rotation; mirrors for reflection; moving vehicles for translation).

Use graph paper, transparencies, tracing paper or dynamic geometry software to obtain images of a given figure under specified transformations.

Provide students with a pre-image and a final, transformed image, and ask them to describe the steps required to generate the final image. Show examples with more than one answer (e.g., a reflection might result in the same image as a translation).

Work backwards to determine a sequence of transformations that will carry (map) one figure onto another of the same size and shape.

Focus attention on the attributes (e.g. distances or angle measures) of a geometric figure that remain constant under various transformations.

Make the transition from transformations as physical motions to functions that take points in the plane as inputs and give other points as outputs. The correspondence between the initial and final points determines the transformation.

Analyze various figures (e.g. regular polygons, folk art designs or product logos) to determine which rotations and reflections carry (map) the figure onto itself. These transformations are the “symmetries” of the figure.

Emphasize the importance of understanding a transformation as the correspondence between initial and final points, rather than the physical motion.

Use a variety of means to represent rigid motions, including physical manipulatives, coordinate methods, and dynamic geometry software.

Common Misconceptions: G.CO.1-5

The terms “mapping” and “under” are used in special ways when studying transformations. A translation is a type of transformation that moves all the points in the object in a straight line in the same direction.

Students should know that not every transformation is a translation. Students sometimes confuse the terms "transformation" and "translation."

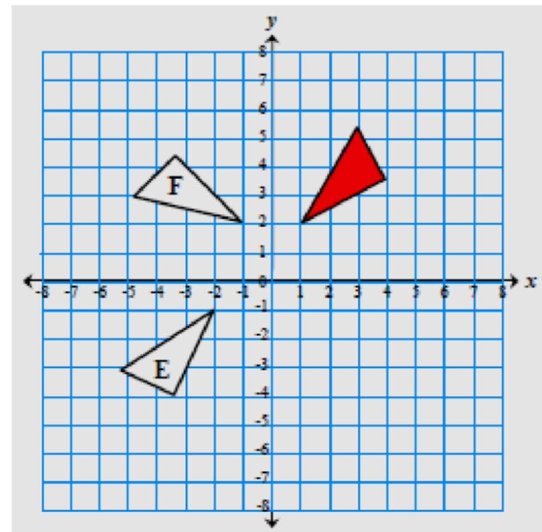
Standard: G.CO.3
Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Explanations and Examples: G.CO.3
Describe and illustrate how a rectangle, parallelogram, and isosceles trapezoid are mapped onto themselves using transformations.

Calculate the number of lines of reflection symmetry and the degree of rotational symmetry of any regular polygon.

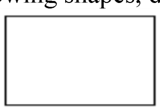
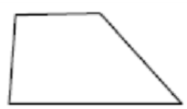
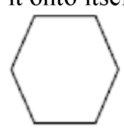
Students may use geometry software and/or manipulatives to model transformations.

Examples:



1. Draw the shaded triangle after:
   a. It has been translated -7 horizontally and $+1$ vertically. Label your answer A.
   b. It has been reflected over the x-axis. Label your answer B.
   c. It has been rotated 90° clockwise around the origin. Label your answer C.
   d. It has been reflected over the line $y = x$. Label your answer D.
2. Describe fully the single transformation that:
   a. Takes the shaded triangle onto the triangle labeled E.
   b. Takes the shaded triangle onto the triangle labeled F.

- For each of the following shapes, describe the rotations and reflections that carry it onto itself.

(a)  (b)  (c) 

Standard: G.CO.4
Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Explanations and Examples: G.CO.4
Using previous comparisons and descriptions of transformations develop and understand the meaning of rotations, reflections, and translations based on angles, circles, perpendicular lines, parallel lines, and line segments.

Students may use geometry software and/or manipulatives to model transformations. Students may observe patterns and develop definitions of rotations, reflections, and translations.

Examples:

- Perform a rotation, reflection, and translation with a given polygon and give a written explanation of how each step meets the definitions of each transformation using correct mathematical terms.
- Construct the reflection definition by connecting any point on the preimage to its corresponding point on the reflected image and describe the line segment's relationship to the line of reflection (e.g., the line of reflection is the perpendicular bisector of the segment).
- Construct the translation definition by connecting any point on the preimage to its corresponding point on the translated image, and connect a second point on the preimage to its corresponding point on the translated image, and describe how the two segments are equal in length, point in the same direction, and are parallel.

Construct the rotation definition by connecting the center of rotation to any point on the preimage and to its corresponding point on the rotated image, and describe the measure of the angle formed and the equal measures of the segments that formed the angle as part of the definition.

Standard: G.CO.5
Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Explanations and Examples: G.CO.5
Transform a geometric figure given a rotation, reflection, or translation using graph paper, tracing paper, or geometric software.

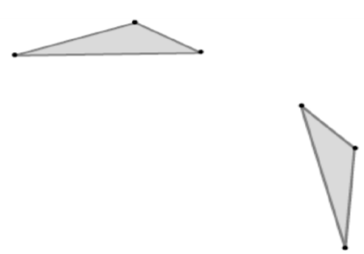
Create sequences of transformations that map a geometric figure on to itself and another geometric figure. Draw a specific transformation when given a geometric figure and a rotation, reflection or translation.

Predict and verify the sequence of transformations (a composition) that will map a figure onto another.

Students may use geometry software and/or manipulatives to model transformations and demonstrate a sequence of transformations that will carry a given figure onto another.

Examples:

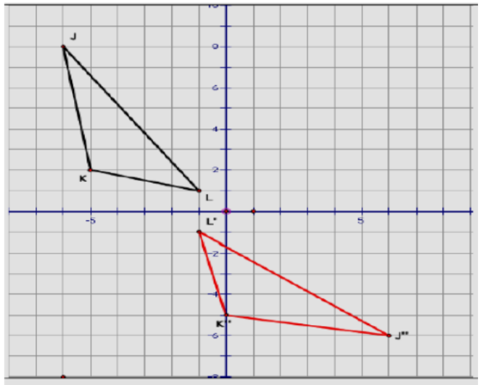
- The triangle in the upper left of the figure below has been reflected across a line into the triangle in the lower right of the figure. Use a straightedge and compass to construct the line across which the triangle was reflected



 For the diagram below, describe the sequence of transformations that was used to carry JKL (Image 1) onto Image 2.

Image 1

Image 2



Standard: G.CO.6
Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

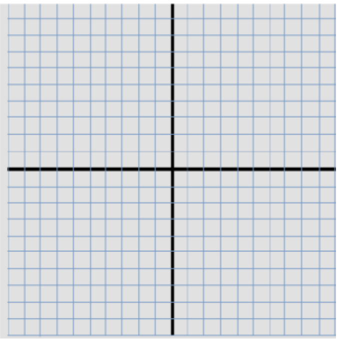
Connections: G.CO.6-8
An understanding of congruence using physical models, transparencies or geometry software is developed in Grade 8, and should be built upon in high school with greater attention to precise definitions, careful statements and proofs of theorems and formal reasoning.

Explanations and Examples: G.CO.6
For standards G.CO.6-8 the focus is for students to understand that rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane.

Knowing that rigid transformations preserve size and shape or distance and angle, use this fact to connect the idea of congruency and develop the definition of congruent. Determine if two figures are congruent by determining if rigid motions will turn one figure into the other.

Examples:

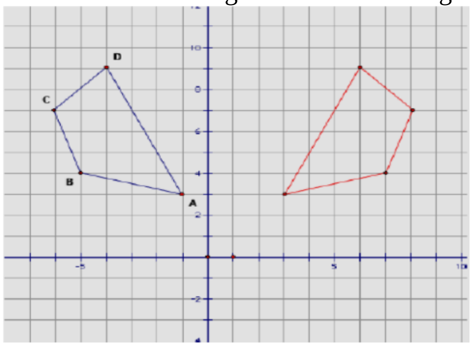


 ΔABC has vertices $A(-1, 0)$, $B(4, 0)$, $C(2, 6)$

a. Draw ΔABC on the coordinate grid provided.
b. Translate ΔABC using the rule $(x, y) \rightarrow (x - 6, y - 5)$ to create $\Delta A'B'C'$.
Record the new coordinate grid (using a different color if possible).
A' _ B' ________________ C' _
c. Rotate $\Delta A'B'C'$ 90°CCW using the rule $(x, y) \rightarrow$ to create $\Delta A''B''C''$.
Record the new coordinates below and add the triangle to your coordinate grid (using a different color if possible).
A'' _ B'' ________________ C'' _

d. Write ONE rule below that would change ΔABC to $\Delta A''B''C''$ in one step.

Determine if the figures below are congruent. If so tell what rigid motions were used.



Instructional Strategies: G.CO.6-8

Develop the relationship between transformations and congruency. Allow adequate time and provide hands-on activities for students to visually and physically explore rigid motions and congruence.

Use graph paper, tracing paper or dynamic geometry software to obtain images of a given figure under specified rigid motions. Note that size and shape are preserved.

Use rigid motions (translations, reflections and rotations) to determine if two figures are congruent. Compare a given triangle and its image to verify that corresponding sides and corresponding angles are congruent.

Work backwards – given two figures that have the same size and shape, find a sequence of rigid motions that will map one onto the other.

Build on previous learning of transformations and congruency to develop a formal criterion for proving the congruency of triangles. Construct pairs of triangles that satisfy the ASA, SAS or SSS congruence criteria, and use rigid motions to verify that they satisfy the definition of congruent figures. Investigate rigid motions and congruence both algebraically (using coordinates) and logically (using proofs).

Common Misconceptions: G.CO.6-8

Some students may believe:

That combinations such as SSA or AAA are also a congruence criterion for triangles. Provide counterexamples for this misconception.
That all transformations, including dilations, are rigid motions. Provide counterexamples for this misconception.
That any two figures that have the same area represent a rigid transformation. Students should recognize that the areas remain the same, but preservation of side and angle lengths determine that the transformation is rigid.
That corresponding vertices do not have to be listed in order; however, it is useful to stress the importance of listing corresponding vertices in the same order so that corresponding sides and angles can be easily identified and that included sides or angles are apparent.

Standard: G.CO.7

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Explanations and Examples: G.CO.7

A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures. Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur.

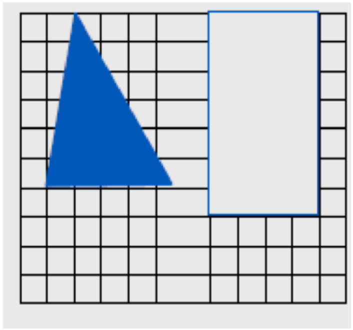
Students identify corresponding sides and corresponding angles of congruent triangles of congruent triangles.

Explain that in a pair of congruent triangles, corresponding sides are congruent (distance is preserved) and corresponding angles are congruent (angles measure is preserved).

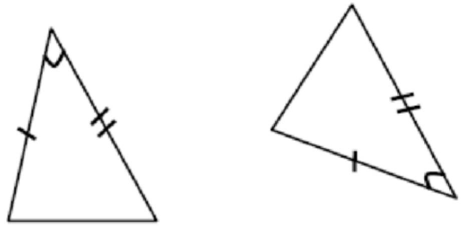
Demonstrate that when distance is preserved (corresponding sides are congruent) and angle measure is preserved (corresponding angles are congruent) the triangles must also be congruent.

Examples:

- How many ways can you construct a triangle congruent to the given triangle inside the rectangle? Demonstrate each.



Are the following triangles congruent? Explain how you know.



Instructional Strategies: G.CO.6-8

Develop the relationship between transformations and congruency. Allow adequate time and provide hands-on activities for students to visually and physically explore rigid motions and congruence.

Use graph paper, tracing paper or dynamic geometry software to obtain images of a given figure under specified rigid motions. Note that size and shape are preserved.

Use rigid motions (translations, reflections and rotations) to determine if two figures are congruent. Compare a given triangle and its image to verify that corresponding sides and corresponding angles are congruent.

Work backwards – given two figures that have the same size and shape, find a sequence of rigid motions that will map one onto the other.

Build on previous learning of transformations and congruency to develop a formal criterion for proving the congruency of triangles. Construct pairs of triangles that satisfy the ASA, SAS or SSS congruence criteria, and use rigid motions to verify that they satisfy the definition of congruent figures. Investigate rigid motions and congruence both algebraically (using coordinates) and logically (using proofs).

Common Misconceptions: G.CO.6-8

Some students may believe:

That combinations such as SSA or AAA are also a congruence criterion for triangles. Provide counterexamples for this misconception.

That all transformations, including dilations, are rigid motions. Provide counterexamples for this misconception.

That any two figures that have the same area represent a rigid transformation. Students should recognize that the areas remain the same, but preservation of side and angle lengths determine that the transformation is rigid.

That corresponding vertices do not have to be listed in order; however, it is useful to stress the importance of listing corresponding vertices in the same order so that corresponding sides and angles can be easily identified and that included sides or angles are apparent.

Standard: G.CO.9

Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

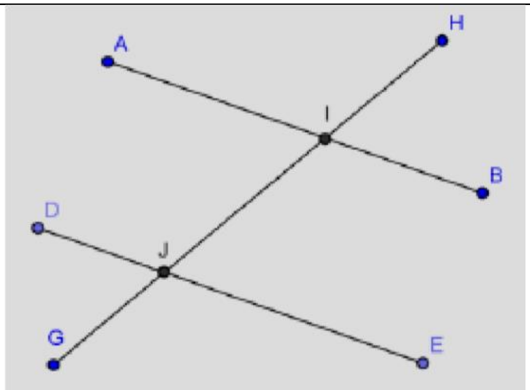
Connections: G.CO.9-11

Properties of lines and angles, triangles and parallelograms were investigated in Grades 7 and 8. In high school, these properties are revisited in a more formal setting, giving greater attention to precise statements of theorems and establishing these theorems by means of formal reasoning.

The theorem about the midline of a triangle can easily be connected to a unit on similarity. The proof of it is usually based on the similarity property that corresponding sides of similar triangles are proportional.

Explanations and Examples: G.CO.9 $\overline{AB}$

 Prove that ∠HIB ≅ ∠DJG, given that $\overline{AB} \cong \overline{DE}$



Instructional Strategies: G.CO.9-11

Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3.

Classroom teachers and mathematics education researchers agree that students have a hard time learning how to do geometric proofs. An article by Battista and Clements (1995) (http://investigations.terc.edu/library/bookpapers/geometryand_proof.cfm) provides information for teachers to help students who struggle learn to do proof. The most significant implication for instructional strategies for proof is stated in their conclusion.

> "Ironically, the most effective path to engendering meaningful use of proof in secondary school geometry is to avoid formal proof for much of students' work. By focusing instead on justifying ideas while helping students build the visual and empirical foundation for higher levels of geometric thought, we can lead students to appreciate the need for formal proof. Only then will we be able to use it meaningfully as a mechanism for justifying ideas."

The article and ideas from Niven (1987) offers a few suggestions about teaching proof in geometry:

- Initial geometric understandings and ideas should be taught "without excessive emphasis on rigor." Develop basic geometric ideas outside an axiomatic framework, and then let the importance of the framework (and the framework itself) emerges from the geometry.
- Geometry is visual and should be taught in ways that leverage this aspect. Sketching, drawing and constructing figures and relationships between geometric objects should be central to any geometric study and certainly to proof. Battista and Clement make a powerful argument that the use of dynamic geometry software can be an important tool for helping students understand proof.
- "Avoid the deadly elaboration of the obvious" (Niven, p. 43). Often textbooks begin the treatment of formal proof with "easy" proofs, which appear to students to need no proof at all. After presenting many opportunities for students to "justify" properties of geometric figures, formal proof activities should begin with non-obvious conjectures.
- Use the history of geometry and real-world applications to help students develop conceptual understandings before they begin to use formal proof.

Proofs in high school geometry should not be restricted to the two-column format. Most proofs at the college level are done in paragraph form, with the writer explaining and defending a conjecture. In many cases, the two-column format can hinder the student from making sense of the geometry by paying too much attention to format rather than mathematical reasoning.

Some of the theorems listed in this cluster (e.g. the ones about alternate interior angles and the angle sum of a triangle) are logically equivalent to the Euclidean parallel postulate, and this should be acknowledged.

Use dynamic geometry software to allow students to make conjectures that can, in turn, be formally proven. For example, students might notice that the base angles of an isosceles triangle always appear to be congruent when manipulating triangles on the computer screen and could then engage in a more formal discussion of why this occurs.

Common Core Standards Appendix A states, "Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words.

Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning" (p. 29). Different methods of proof will appeal to different learning styles in the classroom.

Common Misconceptions: G.CO.9-11

Research over the last four decades suggests that student misconceptions about proof abound:

- even after proving a generalization, students believe that exceptions to the generalization might exist; • one counterexample is not sufficient;
- the converse of a statement is true (parallel lines do not intersect, lines that do not intersect are parallel); and
- a conjecture is true because it worked in all examples that were explored.

Each of these misconceptions needs to be addressed, both by the ways in which formal proof is taught in geometry and how ideas about "justification" are developed throughout a student's mathematical education.

Standard: G.CO.12

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Connections: G.CO.12-13

Drawing geometric shapes with rulers, protractors and technology is developed in Grade 7. In high school, students perform formal geometry constructions using a variety of tools. Students will utilize proofs to justify validity of their constructions.

Explanations and Examples: G.CO.12

For standards G.CO.12-13, the expectation is to build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced with them.

Students may use geometric software to make geometric constructions.

Examples:

- Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.

- Construct the circumcenter of a given triangle.

- Construct the perpendicular bisector of a line segment.

This construction can also be used to construct a 90 degree angle or to find the midpoint of a line.

1. Mark two points on your line, A and B - this construction will give you a straight line which passes exactly half way between these two points and is perpendicular (at right angles) to the line.
2. Open your compasses to a distance more than half way between A and B.
3. With the point of the compass on one of the points, draw circular arcs above and below the line, at P and Q.
4. Keeping the compasses set to exactly the same distance, repeat with the compass point on your other point.
5. Draw a line through the P and Q.
6. PQ is the perpendicular bisector of AB - check that the angles are exactly 90 degrees and that it does indeed halve the distance between A and B.

http://motivate.maths.org/content/accurate-constructions

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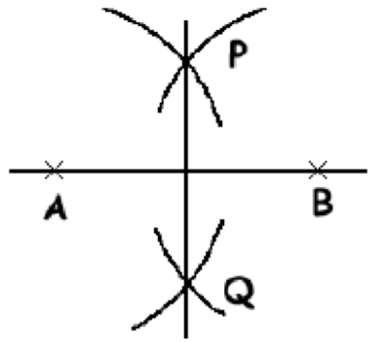


Instructional Strategies: G.CO.12-13

Students should analyze each listed construction in terms of what simpler constructions are involved (e.g., constructing parallel lines can be done with two different constructions of perpendicular lines).
Using congruence theorems, ask students to prove that the constructions are correct.

Provide meaningful problems (e.g. constructing the centroid or the incenter of a triangle) to offer students practice in executing basic constructions.

Challenge students to perform the same construction using a compass and string. Use paper folding to produce a reflection; use bisections to produce reflections.

Ask students to write "how-to" manuals, giving verbal instructions for a particular construction. Offer opportunities for hands-on practice using various construction tools and methods.

Compare dynamic geometry commands to sequences of compass-and-straightedge steps. Prove, using congruence theorems, that the constructions are correct.

Common Misconceptions: G.CO.12-13

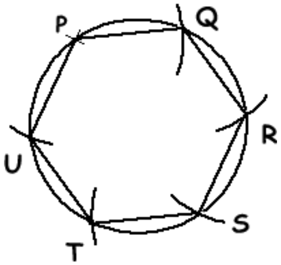
Some students may believe that a construction is the same as a sketch or drawing. Emphasize the need for precision and accuracy when doing constructions. Stress the idea that a compass and straightedge are identical to a protractor and ruler. Explain the difference between measurement and construction.

Standard: G.CO.13

Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Explanations and Examples: G.CO.13

For standards G.CO.12-13, the expectation is to build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced with them.
Students may use geometric software to make geometric constructions.



Examples:

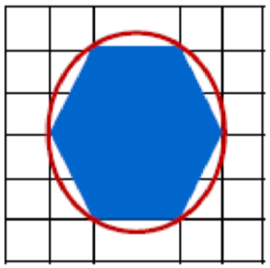
- Construct a regular hexagon inscribed in a circle.

  This construction can also be used to draw a 120° angle.

  Keep your compasses to the same setting throughout this construction.
  Draw a circle.
  Mark a point, P, on the circle.
  Put the point of your compasses on P and draw arcs to cut the circle at Q and U.
  Put the point of your compasses on Q and draw an arc to cut the circle at R.
  Repeat with the point of the compasses at R and S to draw arcs at S and T.
  Join PQRSTU to form a regular hexagon.
  Measure the lengths to check they are all equal, and the angles to check they are all 120 degrees.

  http://motivate.maths.org/content/accurate-constructions

- Find two ways to construct a hexagon inscribed in a circle as shown.



Suggested Big Idea	Unit 2: Triangle/Polygon Congruence and Proof			
Content Emphasis Cluster	Understand congruence in terms of rigid motions			
Mathematical Practices	MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.			
Common Assessment	End of Unit Assessment			
Graduate Competency	Prepared graduates apply transformation to numbers, shapes, functional representations, and data			
CCSS Priority Standards	**Cross-Content Connections**	**Writing Focus**	**Language/Vocabulary**	**Misconceptions**
G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	Literacy Connections RST.6-8.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics. RST.6-8.5 Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic. RST.6-8.7 Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table). RST.6-8.8 Distinguish among facts, reasoned judgment based on research	Writing Connection WHST.6-8.2 Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. f. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. g. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. h. Use appropriate and varied transitions to create cohesion and clarify the relationships	Academic Vocabulary- Congruence L.6-8.6 Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression. L.6-8.4 Determine or clarify the meaning of unknown and multiple-meaning words and phrases choosing flexibly from a range of strategies.	G.CO.7 That combinations such as SSA or AAA are also a congruence criterion for triangles. Provide counterexamples for this misconception. That all transformations, including dilations, are rigid motions. Provide counterexamples for this misconception. That any two figures that have the same area represent a rigid transformation. Students should recognize that the areas remain the same, but preservation of side and angle lengths determine that the transformation is rigid. That corresponding vertices do not have to be listed in order; however, it is useful to stress the importance of listing corresponding vertices in the same order so that corresponding sides and angles can be easily identified and that included sides or angles are apparent.

	findings, and speculation in a text.	among ideas and concepts. i. Use precise language and domain-specific vocabulary to inform about or explain the topic. j. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented. WHST.6-8.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.		

Unit 2	**Triangle Congruence**	**Length of Unit**	**Est: 25 Days** **9/28/16-11/3/16** **Unit assessment scanned by 11/10/16**
Standards of Mathematical Practices	**MP.3** **Construct viable arguments and critique the reasoning of others.** **MP.4** **Model with mathematics.** **MP.5** **Use appropriate tools strategically.** **MP.6** **Attend to precision**		
Content Standards (<u>Priority Standards underlined</u>)	<u>**Understand congruence in terms of rigid motions**</u> <u>**G.CO.7**</u> <u>**Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.**</u> <u>**G.CO.8**</u> <u>**Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.**</u> **Prove geometric theorems** **G.CO.9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. **G.CO.10** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. **G.CO.11** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. **Make geometric constructions** **G.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.		

Terminology:	
Academic Vocabulary *Cross discipline language-*	Congruence
Technical Vocabulary *Discipline-specific language*	
Assessments	End of Unit Common Assessment on Schoolcity: • Scanned into School City or students take the assessment online • Should be in addition to individually developed formative assessments

Core Lessons and Notes

Lessons as suggested from Engage NY Module 1 (click links to follow)

Topic D: Congruence (G-CO.B.7, G-CO.B.8)
- Lesson 22: Congruence Criteria for Triangles—SAS
- Lesson 23: Base Angles of Isosceles Triangles
- Lesson 24: Congruence Criteria for Triangles—ASA and SSS
- Lesson 25: Congruence Criteria for Triangles—AAS and HL
- Lessons 26–27: Triangle Congruency Proofs

Topic E: Proving Properties of Geometric Figures (G-CO.C.9, G-CO.C.10, G-CO.C.11)
- Lesson 28: Properties of Parallelograms
- Lessons 29–30: Special Lines in Triangles

Topic F: Advanced Constructions (G-CO.D.13)
- Lesson 31: Construct a Square and a Nine-Point Circle
- Lesson 32: Construct a Nine-Point Circle.

Topic G: Axiomatic Systems (G-CO.A.1, G-CO.A.2, G-CO.A.3, G-CO.A.4, G-CO.A.5, G-CO.B.6, G-CO.B.7, G-CO.B.8, G-CO.C.9, G-CO.C.10, G-CO.C.11, G-CO.C.12, G-CO.C.13)
- Lessons 33–34: Review of the Assumptions

Optional Materials and Optional Testing Items from New York Engage

- *Lessons listed on the left are OPTIONAL resources from New York Engage. You may choose to use these as a basis or supplement to your own materials.*
- Mention similarity in context that congruent angles (AAA) does not prove congruency.
- Continue to refer to rigid transformations when teaching congruency.
- Topic F - Is for students who have been completely successful with all previous material.
- Topic G – Students review material covered throughout Unit 1 and 2. Students discuss the structure of geometry as an axiomatic system.
- Lesson 33 and 34– Students examine the basic geometric assumptions (axioms) from which all other facts and properties can be derived.
- After lesson 34 Engage NY provides Assessment Tasks that include grading rubrics along with student work samples.

Materials	Compass, Straightedge, Ruler, MIRA or GeoReflector, Geometer's Sketchpad or Geogebra Software, Patty paper, Calculators, Doc Cam, Promethean Board

Unit 2 – Triangle and Polygon Congruence

Topic D – Congruence

In Topic D, Proving Properties of Geometric Figures, students use what they have learned in Topics A through C to prove properties—those that have been accepted as true and those that are new—of parallelograms and triangles (G.CO.10, G.CO.11).

Topic E – Proving Properties of Geometric Figures

The module closes with a return to constructions in Topic E (G.CO.13).

Topic F – Advanced Constructions

Topic F is a review that highlights how geometric assumptions underpin the facts established thereafter.

Topic G – Axiomatic Systems

In Topic G, students review material covered throughout the module. Additionally, students discuss the structure of geometry as an axiomatic system.

Resources and Notes by Standard (adapted from KATM Flip Books)

Standard: G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Explanations and Examples: G.CO.7

A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures. Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur.

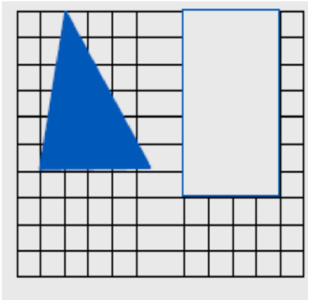
Students identify corresponding sides and corresponding angles of congruent triangles of congruent triangles.

Explain that in a pair of congruent triangles, corresponding sides are congruent (distance is preserved) and corresponding angles are congruent (angles measure is preserved).

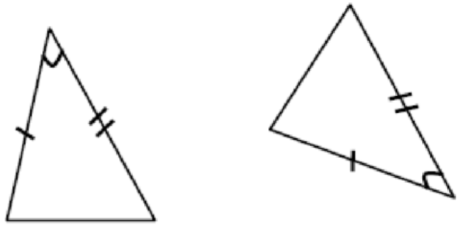
Demonstrate that when distance is preserved (corresponding sides are congruent) and angle measure is preserved (corresponding angles are congruent) the triangles must also be congruent.

Examples:

- How many ways can you construct a triangle congruent to the given triangle inside the rectangle? Demonstrate each.



Are the following triangles congruent? Explain how you know.



Instructional Strategies: G.CO.6-8

Develop the relationship between transformations and congruency. Allow adequate time and provide hands-on activities for students to visually and physically explore rigid motions and congruence.

Use graph paper, tracing paper or dynamic geometry software to obtain images of a given figure under specified rigid motions. Note that size and shape are preserved.

Use rigid motions (translations, reflections and rotations) to determine if two figures are congruent. Compare a given triangle and its image to verify that corresponding sides and corresponding angles are congruent.

Work backwards – given two figures that have the same size and shape, find a sequence of rigid motions that will map one onto the other.

Build on previous learning of transformations and congruency to develop a formal criterion for proving the congruency of triangles. Construct pairs of triangles that satisfy the ASA, SAS or SSS congruence criteria, and use rigid motions to verify that they satisfy the definition of congruent figures. Investigate rigid motions and congruence both algebraically (using coordinates) and logically (using proofs).

Common Misconceptions: G.CO.6-8

Some students may believe:

That combinations such as SSA or AAA are also a congruence criterion for triangles. Provide counterexamples for this misconception.

That all transformations, including dilations, are rigid motions. Provide counterexamples for this misconception.

That any two figures that have the same area represent a rigid transformation. Students should recognize that the areas remain the same, but preservation of side and angle lengths determine that the transformation is rigid.

That corresponding vertices do not have to be listed in order; however, it is useful to stress the importance of listing corresponding vertices in the same order so that corresponding sides and angles can be easily identified and that included sides or angles are apparent.

Standard: G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

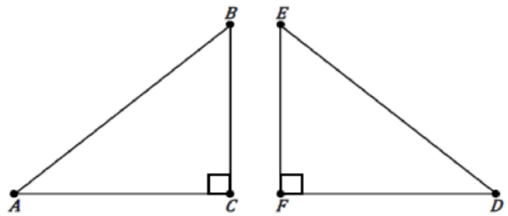
Explanations and Examples: G.CO.8

List the sufficient conditions to prove triangles are congruent.
Map a triangle with one of the sufficient conditions (e.g., SSS) onto the original triangle and show that corresponding sides and corresponding angles are congruent.

Examples:

- Josh is told that two triangles ABC and DEF share two sets of congruent sides and one pair of congruent angles: AB is congruent to DE, BC is congruent to E, and angle C is congruent to angle F.
  He is asked if these two triangles must be congruent.
  Josh draws the two triangles below and says, "They are definitely congruent because they share all three side lengths"!

  - Explain Josh's reasoning using one of the triangle congruence criteria: ASA, SSS, SAS.
  - Give an example of two triangles ABC and DEF, fitting the criteria of this problem, which are not congruent.

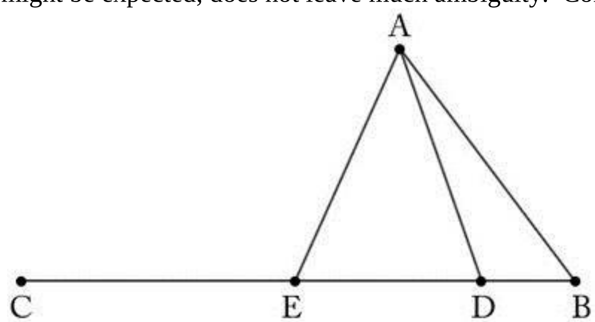


Sample Response:

a. Josh's reasoning is incorrect because he has made the unwarranted assumption that angles C and F are right angles. However, with that additional assumption his statement is correct, since we may apply the Pythagorean theorem to conclude that $|AC|^2 = |AB|^2 - |BC|^2$ and $|DF|^2 = |DE|^2 - |EF|^2$.

Since DE is congruent to AB and EF is congruent to BC by hypothesis we can conclude that AC must be congruent to DF and so, by SSS, triangle ABC is congruent to triangle DEF. Instead of SSS, we could also apply SAS using right angles C and F along with sides AC and BC for triangle ABC and sides DF and EF for triangle DEF.

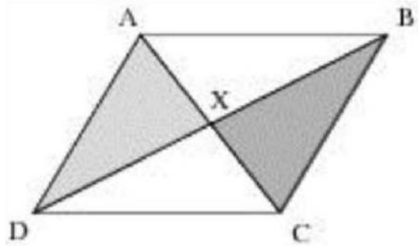
b. The information given amounts to SSA, two congruent sides and a congruent angle which is not the angle determined by the two sets of congruent sides. This is a lot of information and, as might be expected, does not leave much ambiguity. Consider five points A , B , C , D , E as pictured below with isosceles triangle ADE:



Triangles ABD and ABE share angle B and side AB while AD is congruent to AE by construction. The triangles ABD and ABE are definitely not congruent, however, as one of them is properly contained within the other.

The given information heavily restricted this construction but we were still able to find two non congruent triangles sharing two congruent sides and a non-included congruent angle.

- Decide whether there is enough information to prove that the two shaded triangles are congruent. In the figure below, ABCD is a parallelogram.



Solution: The two triangles are congruent by SAS.

We have AX □CX and DX□BX since the diagonals of a parallelogram bisect each other and □AXD □□BXC since they are vertical angles.

Alternatively, we could use and argue via ASA. We have the opposite interior angles □DAX □□BCX and□ADX □□CBX and AD □BC since opposite sides of a parallelogram are congruent.

Instructional Strategies:

Build on previous learning of transformations and congruency to develop a formal criterion for proving the congruency of triangles. Construct pairs of triangles that satisfy the ASA, SAS or SSS congruence criteria, and use rigid motions to verify that they satisfy the definition of congruent figures. Investigate rigid motions and congruence both algebraically (using coordinates) and logically (using proofs).

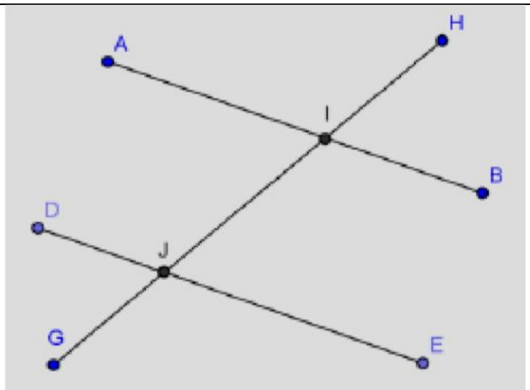
Standard: G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Connections: G.CO.9-11
Properties of lines and angles, triangles and parallelograms were investigated in Grades 7 and 8. In high school, these properties are revisited in a more formal setting, giving greater attention to precise statements of theorems and establishing these theorems by means of formal reasoning.

The theorem about the midline of a triangle can easily be connected to a unit on similarity. The proof of it is usually based on the similarity property that corresponding sides of similar triangles are proportional.

Explanations and Examples: G.CO.9

□ Prove that □HIB □□DJG, given that AB□□ □ DE.□□



Instructional Strategies: G.CO.9-11

Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3.

Classroom teachers and mathematics education researchers agree that students have a hard time learning how to do geometric proofs. An article by Battista and Clements (1995) (http://investigations.terc.edu/library/bookpapers/geometryand_proof.cfm) provides information for teachers to help students who struggle learn to do proof. The most significant implication for instructional strategies for proof is stated in their conclusion.

> "Ironically, the most effective path to engendering meaningful use of proof in secondary school geometry is to avoid formal proof for much of students' work. By focusing instead on justifying ideas while helping students build the visual and empirical foundation for higher levels of geometric thought, we can lead students to appreciate the need for formal proof. Only then will we be able to use it meaningfully as a mechanism for justifying ideas."

The article and ideas from Niven (1987) offers a few suggestions about teaching proof in geometry:

- Initial geometric understandings and ideas should be taught "without excessive emphasis on rigor." Develop basic geometric ideas outside an axiomatic framework, and then let the importance of the framework (and the framework itself) emerges from the geometry.
- Geometry is visual and should be taught in ways that leverage this aspect. Sketching, drawing and constructing figures and relationships between geometric objects should be central to any geometric study and certainly to proof. Battista and Clement make a powerful argument that the use of dynamic geometry software can be an important tool for helping students understand proof.
- "Avoid the deadly elaboration of the obvious" (Niven, p. 43). Often textbooks begin the treatment of formal proof with "easy" proofs, which appear to students to need no proof at all. After presenting many opportunities for students to "justify" properties of geometric figures, formal proof activities should begin with non-obvious conjectures.
- Use the history of geometry and real-world applications to help students develop conceptual understandings before they begin to use formal proof.

Proofs in high school geometry should not be restricted to the two-column format. Most proofs at the college level are done in paragraph form, with the writer explaining and defending a conjecture. In many cases, the two-column format can hinder the student from making sense of the geometry by paying too much attention to format rather than mathematical reasoning.

Some of the theorems listed in this cluster (e.g. the ones about alternate interior angles and the angle sum of a triangle) are logically equivalent to the Euclidean parallel postulate, and this should be acknowledged.

Use dynamic geometry software to allow students to make conjectures that can, in turn, be formally proven. For example, students might notice that the base angles of an isosceles triangle always appear to be congruent when manipulating triangles on the computer screen and could then engage in a more formal discussion of why this occurs.

Common Core Standards Appendix A states, "Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words.

Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning" (p. 29). Different methods of proof will appeal to different learning styles in the classroom.

Common Misconceptions: G.CO.9-11

Research over the last four decades suggests that student misconceptions about proof abound:

- even after proving a generalization, students believe that exceptions to the generalization might exist; • one counterexample is not sufficient;
- the converse of a statement is true (parallel lines do not intersect, lines that do not intersect are parallel); and
- a conjecture is true because it worked in all examples that were explored.

Each of these misconceptions needs to be addressed, both by the ways in which formal proof is taught in geometry and how ideas about "justification" are developed throughout a student's mathematical education.

Standard: G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

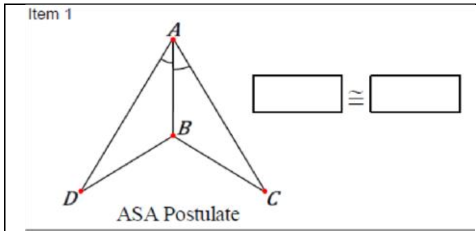
Explanations and Examples: G.CO.10

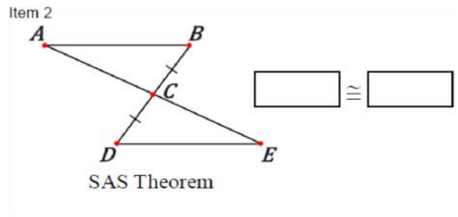
Order statements based on the Law of Syllogism when constructing a proof.
Interpret geometric diagrams by identifying what can and cannot be assumed.
Students may use geometric simulations (computer software or graphing calculator) to explore theorems about triangles.

Examples:

- For items 1 and 2, what additional information is required in order to prove the two triangles are congruent using the provided justification?
  Use the set of choices in the box below. Select a side or angle and place it in the appropriate region. Only one side or angle can be placed in each region.

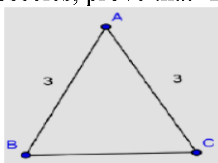
$\overline{AB}$	$\overline{AC}$	$\overline{AD}$	$\overline{BC}$
$\overline{BD}$	$\overline{CD}$	$\overline{CE}$	$\overline{DE}$
$\angle ABC$	$\angle ABD$	$\angle ACB$	$\angle ADB$
$\angle BAC$	$\angle CDE$	$\angle CED$	$\angle DCE$





Key: Item 1 – $\angle ABD \cong \angle ABC$ Item 2 – $AC \cong CE$

Given that □ABC is isosceles, prove that □ABC □□ACB.



Standard: G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

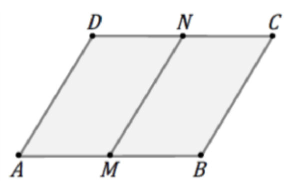
Explanations and Examples: G.CO.11

Students may use geometric simulations (computer software or graphing calculator) to explore theorems about parallelograms.

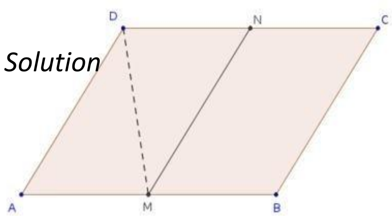
Examples:

□ Suppose that ABCD is a parallelogram, and that M and N are the midpoints of AB and CD, respectively.

Prove that $M = A$, and that the $\overleftrightarrow{M}$ is parallel to $\overleftrightarrow{A}$.



Solution



Solution:

The diagram above consists of the given information, and one additional line segment, MD, which we will use to demonstrate the result. We claim that triangles △AMD and △NDM are congruent by SAS:

We have MD□□ =DM□□ by reflexivity.

We have ∠AMD = ∠NDM since they are opposite interior angles of the transversal MD through parallel lines AB and CD.

We have MA□□ = ND□□, since M and N are midpoints of their respective sides, and opposite sides of parallelograms are congruent MA□□ = 1/2 (A□B□) = 1/2(CD□□) = ND□□

Now since corresponding parts of congruent triangles are congruent, we have DA=NM, as desired. Similarly, we have congruent opposite interior angles ∠DMN ≅ ∠MDA, so MN is parallel to AD.

Standard: G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

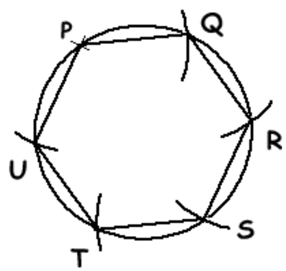
Explanations and Examples: G.CO.13

For standards G.CO.12-13, the expectation is to build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced with them.

Students may use geometric software to make geometric constructions.

Examples:

- Construct a regular hexagon inscribed in a circle.

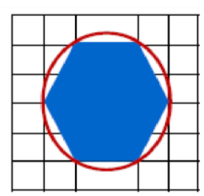


This construction can also be used to draw a 120° angle.

Keep your compasses to the same setting throughout this construction.
Draw a circle.
Mark a point, P, on the circle.
Put the point of your compasses on P and draw arcs to cut the circle at Q and U.
Put the point of your compasses on Q and draw an arc to cut the circle at R.
Repeat with the point of the compasses at R and S to draw arcs at S and T.
Join PQRSTU to form a regular hexagon.
Measure the lengths to check they are all equal, and the angles to check they are all 120 degrees.

http://motivate.maths.org/content/accurate-constructions

- Find two ways to construct a hexagon inscribed in a circle as shown.



Suggested Big Idea	Unit 3: Scale Drawings, Dilations, & Similarity			
Content Emphasis Cluster	Understand similarity in terms of similarity transformations. Apply geometric concepts in modeling situations.			
Mathematical Practices	**MP.3 Construct viable arguments and critique the reasoning of others** **MP.7 Look for and make use of structure**			
Common Assessment	End of Unit Assessment			
Graduate Competency	Prepared graduates use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions			
CCSS Priority Standards	**Cross-Content Connections**	**Writing Focus**	**Language/Vocabulary**	**Misconceptions**
SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. MG.1 Use geometric shapes, their measures, and their properties to describe	Literacy Connections RST.6-8.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics. RST.6-8.5 Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic. RST.6-8.7 Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table). RST.6-8.8 Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.	Writing Connection WHST.6-8.2 Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. k. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. l. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. m. Use appropriate and varied transitions to create cohesion and clarify the relationships	Academic Vocabulary- Composition, Pythagorean theorem, Rigid motions, Scale drawing, Scale factor, Slope Technical Vocabulary- Dilation Sides of a Right Triangle Similar Similarity Transformation L.6-8.6 Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression. L.6-8.4 Determine or clarify the meaning of unknown and multiple-meaning words and phrases choosing	SRT.A.1-3 Some students often do not recognize that congruence is a special case of similarity. Similarity with a scale factor equal to 1 becomes a congruency. Students may not realize that similarities preserve shape, but not size. Angle measures stay the same, but side lengths change by a constant scale factor. Students may incorrectly apply the scale factor. For example students will multiply instead of divide with a scale factor that reduces a figure or divide instead of multiply when enlarging a figure. Some students often do not list the vertices of similar triangles in order. However, the order in which vertices are listed is preferred and especially important for similar triangles so that proportional sides can be correctly identified. MG. 1-3 When students ask to see “useful” mathematics, what they often mean is, “Show me how to use this mathematical concept or skill to solve the homework problems.” Mathematical modeling, on the other hand, involves solving problems in which the path to the solution is not obvious. Geometry may be one of several tools that can be used.

objects (e.g., modeling a tree trunk or a human torso as a cylinder). MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).		among ideas and concepts. n. Use precise language and domain-specific vocabulary to inform about or explain the topic. o. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented. WHST.6-8.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	flexibly from a range of strategies.	

Unit 3	**Scale Drawings, Dilations, & Similarity**	**Length of Unit**	**Est: 24 Days** **11/4/16-12/13/16** **Unit assessment scanned by 12/20/16**
Mathematical Practices	**MP.3** Construct viable arguments and critique the reasoning of others **MP.7** Look for and make use of structure		
Content Standards (<u>Priority Standards Underlined</u>)	<u>**Understand similarity in terms of similarity transformations**</u> <u>**SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor:**</u> <u>**a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.**</u> <u>**b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.**</u> <u>**SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.**</u> <u>**SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.**</u> **Prove theorems involving similarity** **SRT.B.4** Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. **SRT.B.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. <u>**Modeling with geometry**</u> <u>**MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).**</u> <u>**MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).**</u>		
Terminology			
Academic Vocabulary *Cross discipline language*	Composition, Dilation, Pythagorean theorem, Rigid motions, Scale drawing, Scale factor, Slope		

Technical Vocabulary *Discipline-specific language*	**Dilation** For $r > 0$, a *dilation with center C and scale factor r* is a transformation $D_{C,r}$ of the plane defined as follows: 1.For the center C, $D_{C,r}(C) = C$, and 2.For any other point P, $D_{C,r}(P)$ is the point Q on the ray $\overrightarrow{CP}$ so that $CQ = r \cdot CP$. **Sides of a Right Triangle** The *hypotenuse* of a right triangle is the side opposite the right angle; the other two sides of the right triangle are called the *legs*. Let θ be the angle measure of an acute angle of the right triangle. The *opposite side* is the leg opposite that angle. The *adjacent side* is the leg that is contained in one of the two rays of that angle (the hypotenuse is contained in the other ray of the angle). **Similar** Two figures in a plane are *similar* if there exists a similarity transformation taking one figure onto the other figure. A congruence is a similarity with scale factor 1. It can be shown that a similarity with scale factor 1 is a congruence. **Similarity Transformation** A *similarity transformation* (or *similarity*) is a composition of a finite number of dilations or basic rigid motions. The *scale factor* of a similarity transformation is the product of the scale factors of the dilations in the composition; if there are no dilations in the composition, the scale factor is defined to be 1. A similarity is an example of a transformation.
Assessments	**End of Unit Common Assessment on Schoolcity:** • Scanned into School City or students take the assessment online • Should be in addition to individually developed formative assessments

Core Lessons and Notes

Lessons as suggested from Engage NY Module 2 (click links to follow) **Topic A: Scale Drawings (G-SRT.A.1, G-SRT.B.4, G-MG.A.3)** Lesson 1: Scale Drawings Lesson 2: Making Scale Drawings Using the Ratio Method *Lesson 3: Making Scale Drawings Using the Parallel Method Lesson 4: Comparing the Ratio Method with the Parallel Method Lesson 5: Scale Factors **Topic B: Dilations (G-SRT.A.1, G-SRT.B.4)** Lesson 6: Dilations as Transformations of the Plane Lesson 7: How Do Dilations Map Segments? Lesson 8: How Do Dilations Map Lines, Rays, and Circles? Lesson 9: How Do Dilations Map Angles? Lesson 10: Dividing the King's Foot into 12 Equal Pieces Lesson 11: Dilations from Different Centers **Topic C: Similarity and Dilations (G-SRT.A.2, G-SRT.A.3, G-SRT.B.5, G-MG.A.1)** Lesson 12: What Are Similarity Transformations, and Why Do We Need Them? Lesson 13: Properties of Similarity Transformations Lesson 14: Similarity Lesson 15: The Angle-Angle (AA) Criterion for Two Triangles to be Similar Lesson 16: Between-Figure and Within-Figure Ratios	• *Lessons listed on the left are OPTIONAL resources from New York Engage. You may choose to use these as a basis or supplement to your own materials.* • Do examples with and without coordinate grid to demonstrate dilations and the center of dilation • Dilations is taken further than it was in Grade 8 by now exploring why dilations occur the way they do. (particularly in lessons 4 and 5 of Engage NY) • Continue to support and reference transformations as they relate to similarity and congruence • Similarity connects directly to trigonometry, however this is separated into its own unit (Semester 2) in order to guarantee enough time to thoroughly cover trigonometry • **If you choose to do lessons from Engage NY for this unit, it uses all transformations as well as constructions pretty heavily to complete the lessons.** • Lesson 1 utilizes the use of constructing parallel lines to create scale drawings (ratio method). • Lesson 2 starts to introduce the idea of dilations centered about a point not on the figure (parallel method, also called one point perspective in art class) to create scale drawings.

Lesson 17: The Side-Angle-Side (SAS) and Side-Side-Side (SSS) Criteria for Two Triangles to be Similar Lesson 18: Similarity and the Angle Bisector Theorem Lesson 19: Families of Parallel Lines and the Circumference of the Earth *Lesson 20: How Far Away Is the Moon? **Topic D: Applying Similarity to Right Triangles (G-SRT.B.4)** Lesson 21: Special Relationships Within Right Triangles—Dividing into Two Similar Sub-Triangles Lesson 22: Multiplying and Dividing Expressions with Radicals Lesson 23: Adding and Subtracting Expressions with Radicals Lesson 24: Prove the Pythagorean Theorem Using Similarity Optional Materials and Testing Items from New York Engage *Lessons recommended by the Geometry team	• Lesson 6 involves review of all transformations using constructions. • Lesson 9 is dilating figures to show that angles are preserved while size changes. • Lesson 15 worksheet has good homework problems for proportions between similar figures. • Lesson 16 dives into real world applications of proportionality. (Measure the height of the flag pole by measuring shadows) • Lesson 19 refers back to parallel lines cut by a transversal when discussing similar figures. • Lesson 21 introduces an altitude in right triangles to create similar figures as well as the Pythagorean theorem.

Materials	Compass and straightedge

Unit 3 Scale Factor, Dilations, Similarity

Topic A – Scale Drawings

Students embark on Topic A with a brief review of scale drawings and scale factor, which they last studied in Grades 7 and 8. In Lesson 1, students recall the properties of a well-scaled drawing and practice creating scale drawings using basic construction techniques. Lessons 2 and 3 explore systematic techniques for creating scale drawings. With the ratio method, students dilate key points of a figure according to the scale factor to produce a scale drawing (G-SRT.A.1). Note that exercises within Lesson 2 where students apply the ratio method to solve design problems relate to the modeling standard G-MG.A.3. With the parallel method, students construct sides parallel to corresponding sides of the original figure to create a scale drawing.

Topic B - Dilations

Topic B is an in depth study of the properties of dilations. Though students applied dilations in Topic A, their use in the ratio and parallel methods was to establish relationships that were consequences of applying a dilation, not directly about the dilation itself. In Topic B, students explore observed properties of dilations (Grade 8, Module 3) and reason why these properties are true. This reasoning is possible because of what students have studied regarding scale drawings and the triangle side splitter and dilation theorems. With these theorems, it is possible to establish why dilations map segments to segments, lines to lines, etc. Some of the arguments involve an examination of several sub-cases; it is in these instances of thorough examination that students must truly make sense of problems and persevere in solving them (MP.1).

Topic C – Similarity and Dilations

With an understanding of dilations, students are now ready to study similarity in Topic C. This is an appropriate moment to pause and reflect on the change in how the study of similarity is studied in this curriculum versus traditional geometry curricula. It is not uncommon to open to a similarity unit in a traditional textbook and read about polygons, chiefly triangles, which are of the same shape but different size. Some may emphasize the proportional relationship between corresponding sides early in the unit. The point is that similarity is an instance in grade school mathematics where the information has traditionally been packaged into a distilled version of the bigger picture. The unpackaged view requires a more methodical journey to arrive at the concept of similarity, including the use of transformations. It is in Topic C, after a foundation of scale drawings and dilations, that we can discuss similarity. Students are introduced to the concept of a similarity transformation in Lesson 12, which they learn is needed to identify figures as being similar. Just as with rigid motions and congruence, the lesson intentionally presents curvilinear examples to emphasize that the use of similarity transformations allows us to compare both rectilinear and curvilinear figures. Next, in Lesson 13, students apply similarity transformations to figures by construction. This is the only lesson where students actually perform similarity transformations. The goals are to simply be able to apply a similarity as well as observe how the properties of the individual transformations that compose each similarity hold throughout construction. In Lesson 14, students observe the reflexive, symmetric, and transitive properties of similarity. The scope of figures used in Lessons 15 through 18 narrows to triangles. In these lessons, students discover and prove the AA, SSS, and SAS similarity criteria. Students use these criteria and length relationships between similar figures and within figures to solve for unknown lengths in triangles (G-SRT.A.3, G-SRT.B.5). Note that when students solve problems in Lesson 16 they are using geometric shapes, their measures and properties to describe situations, e.g., similar triangles, is work related to the modeling standard G-MG.A.1. Lessons 19 and 20 are modeling lessons (GMG.A.1) that lead students through the reasoning the ancient Greeks used to determine the circumference of the earth (Lesson 19) and the distance from the earth to the moon (Lesson 20).

Topic D – Applying Similarity to Right Triangles

In Topic D, students use their understanding of similarity and focus on right triangles as a lead up to trigonometry. In Lesson 21, students use the AA criterion to show how an altitude drawn from the vertex of the right angle of a right triangle to the hypotenuse creates two right triangles similar to the original right triangle. Students examine how the ratios within the three similar right triangles can be used to find unknown side lengths. Work with lengths in right triangles lends itself to expressions with radicals. In Lessons 22 and 23 students learn to rationalize fractions with radical expressions in the denominator and also to simplify, add, and subtract radical expressions. In the final lesson of Topic D, students use the relationships created by an altitude to the hypotenuse of a right triangle to prove the Pythagorean theorem.

Resources and Notes by Standard (adapted from KATM Flip Books)

Standard: G.SRT.1
Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Connections: G.SRT.1-3
Dilations and similarity, including the AA criterion, are investigated in Grade 8, and these experiences should be built upon in high school with greater attention to precise definitions, careful statements and proofs of theorems and formal reasoning.

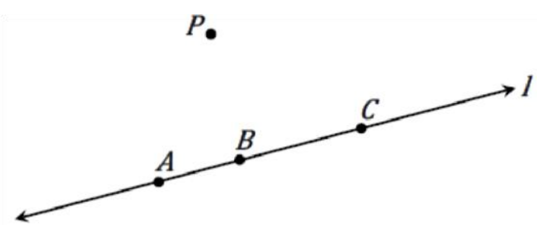
Explanations and Examples: G.SRT.1
Students should understand that a dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Perform a dilation with a given center and scale factor on a figure in the coordinate plane. Verify that when a side passes through the center of dilation, the side and its image lie on the same line. Verify that corresponding sides of the preimage and images are parallel. Verify that a side length of the image is equal to the scale factor multiplied by the corresponding side length of the preimage.

Students may use geometric simulation software to model transformations. Students may observe patterns and verify experimentally the properties of dilations.

Examples:

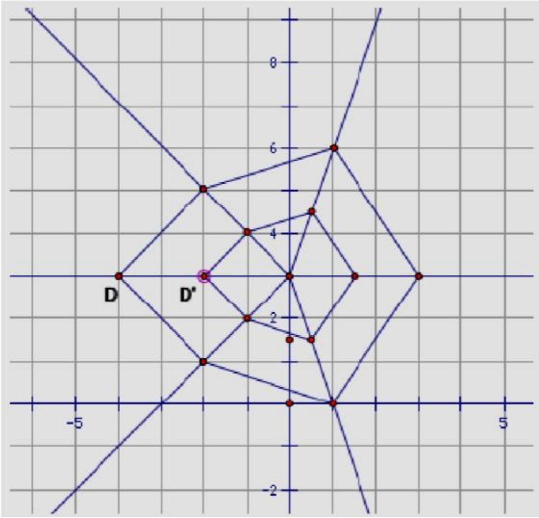
- Suppose we apply a dilation by a factor of 2, centered at the point P to the figure below.



a. In the picture, locate the images A’, B’, and C’ of the points A, B, C under this dilation.
b. Based on you picture in part a., what do you think happens to the line l when we perform the dilation?
c. Based on your picture in part a., what appears to be the relationship between the distance A’B’ and the distance AB?
d. Can you prove your observations in part c?

- Draw a polygon. Pick a point and construct a dilation of the polygon with that point as the center. Identify the scale factor that you used.

Example Response:



Instructional Strategies: G.SRT.1-3

- Allow adequate time and hands-on activities for students to explore dilations visually and physically.
- Use graph paper and rulers or dynamic geometry software to obtain images of a given figure under dilations having specified centers and scale factors. Carefully observe the images of lines passing through the center of dilation and those not passing through the center, respectively. A line segment passing through the center of dilation will simply be shortened or elongated but will lie on the same line, while the dilation of a line segment that does not pass through the center will be parallel to the original segment (this is intended as a clarification of Standard 1a).
- Illustrate two-dimensional dilations using scale drawings and photocopies.
- Measure the corresponding angles and sides of the original figure and its image to verify that the corresponding angles are congruent and the corresponding sides are proportional (i.e. stretched or shrunk by the same scale factor). Investigate the SAS and SSS criteria for similar triangles.
- Use graph paper and rulers or dynamic geometry software to obtain the image of a given figure under a combination of a dilation followed by a sequence of rigid motions (or rigid motions followed by dilation).
- Work backwards – given two similar figures that are related by dilation, determine the center of dilation and scale factor. Given two similar figures that are related by a dilation followed by a sequence of rigid motions, determine the parameters of the dilation and rigid motions that will map one onto the other.
- Using the theorem that the angle sum of a triangle is 180°, verify that the AA criterion is equivalent to the AAA criterion. Given two triangles for which AA holds, use rigid motions to map a vertex of one triangle onto the corresponding vertex of the other in such a way that their corresponding sides are in line. Then show that dilation will complete the mapping of one triangle onto the other.
- Students may be interested in scale models or experiences with blueprints and scale drawings
- (perhaps in a work related situation) to illustrate similarity.

Common Misconceptions: G.SRT.1-3

- Some students often do not recognize that congruence is a special case of similarity. Similarity with a scale factor equal to 1 becomes a congruency.
- Students may not realize that similarities preserve shape, but not size. Angle measures stay the same, but side lengths change by a constant scale factor.
- Students may incorrectly apply the scale factor. For example students will multiply instead of divide with a scale factor that reduces a figure or divide instead of multiply when enlarging a figure.
- Some students often do not list the vertices of similar triangles in order. However, the order in which vertices are listed is preferred and especially important for similar triangles so that proportional sides can be correctly identified.

Standard: G.SRT.2

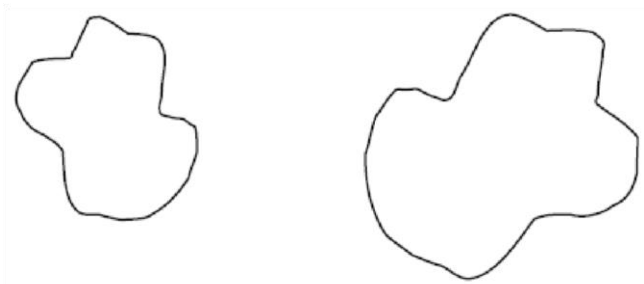
Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Explanations and Examples: G.SRT.2

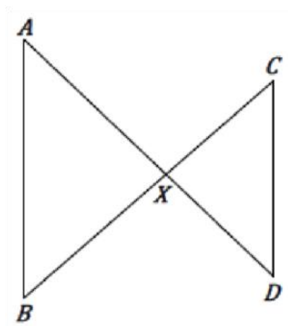
- Use the idea of dilation transformations to develop the definition of similarity. Understand that a similarity transformation is a rigid motion followed by a dilation.
- Demonstrate that in a pair of similar triangles, corresponding angles are congruent (angle measure is preserved) and corresponding sides are proportional.
- Determine that two figures are similar by verifying that angle measure is preserved and corresponding sides are proportional.
- Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.

Examples:

- Are these two figures similar? Explain why or why not.



- In the picture below, line segments AD and BC intersect at X. Line segments AB and CD are drawn, forming two triangles AXB and CXD.



- In each part a-d below, some additional assumptions about the picture are given. In each problem, determine whether the given assumptions are enough to prove that the two triangles are similar, and if so, what the correct correspondence of vertices is. If the two triangles must be similar, prove this result by describing a sequence of similarity transformations that maps one variable to the other. If not explain why not.

a. The lengths of AX and AD satisfy the equation 2AX = 3XD.

b. The lengths AX, BX, CX, and DX satisfy the equation $\frac{AX}{BX} = \frac{DX}{CX}$.

c. Lines AB and CD are parallel.

d. ∠XAB is congruent to angle ∠XCD .

Standard: G.SRT.3

Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Explanations and Examples: G.SRT.3

- Show and explain that when two angle measures are known (AA), the third angle measure is also know (Third Angle Theorem).
- Identify and explain that AA similarity is a sufficient condition for two triangles to be similar.

Examples:

- Are all right triangles similar to one another? How do you know?
- What is the least amount of information needed to prove two triangles are similar? How do you know?
- Using a ruler and a protractor, prove AA similarity.

Instructional Strategies: See G.SRT.1

- Using the theorem that the angle sum of a triangle is 180°, verify that the AA criterion is equivalent to the AAA criterion.
- Given two triangles for which AA holds, use rigid motions to map a vertex of one triangle onto the corresponding vertex of the other in such a way that their corresponding sides are in line. Then show that dilation will complete the mapping of one triangle onto the other.

Standard: G.SRT.4

Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

Connections: G.SRT.4-5

- The Pythagorean theorem and its converse are proved and applied in Grade 8. In high school, another proof, based on similar triangles, is presented.
- The alternate interior angle theorem and its converse, as well as properties of parallelograms, are established informally in Grade 8 and proved formally in high school.

Explanations and Examples: G.SRT.4

- Use AA, SAS, SSS similarity theorems to prove triangles are similar.
- Use triangle similarity to prove other theorems about triangles o Prove a line parallel to one side of a triangle divides the other two proportionally, and it's converse o Prove the Pythagorean Theorem using triangle similarity.

- Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.

Examples:

- Prove that if two triangles are similar, then the ratio of corresponding attitudes is equal to the ratio of corresponding sides.
- How does the Pythagorean Theorem support the case for triangle similarity?
  - View the video below and create a visual proving the Pythagorean Theorem using similarity.
    http://www.youtube.com/watch?v=LrS5_l-gk94
- To prove the Pythagorean Theorem using triangle similarity:
  We can cut a right triangle into two parts by dropping a perpendicular onto the hypotenuse. Since these triangles and the original one have the same angles, all three are similar.
  http://www.math.ubc.ca/~cass/euclid/java/html/pythagorassimilarity.html

Instructional Strategies: G.SRT.4-5

- Review triangle congruence criteria and similarity criteria, if it has already been established.
- Review the angle sum theorem for triangles, the alternate interior angle theorem and its converse, and properties of parallelograms. Visualize it using dynamic geometry software.
- Using SAS and the alternate interior angle theorem, prove that a line segment joining midpoints of two sides of a triangle is parallel to and half the length of the third side. Apply this theorem to a line segment that cuts two sides of a triangle proportionally.
- Generalize this theorem to prove that the figure formed by joining consecutive midpoints of sides of an arbitrary quadrilateral is a parallelogram. (This result is known as the Midpoint Quadrilateral Theorem or Varignon's Theorem.)
- Use cardboard cutouts to illustrate that the altitude to the hypotenuse divides a right triangle into two triangles that are similar to the original triangle. Then use AA to prove this theorem. Then, use this result to establish the Pythagorean relationship among the sides of a right triangle (a2 + b2 = c2) and thus obtain an algebraic proof of the Pythagorean Theorem.
- Prove that the altitude to the hypotenuse of a right triangle is the geometric mean of the two segments into which its foot divides the hypotenuse.
- Prove the converse of the Pythagorean Theorem, using the theorem itself as one step in the proof. Some students might engage in an exploration of Pythagorean Triples (e.g., 3-4-5, 5-12-13, etc.), which provides an algebraic extension and an opportunity to explore patterns.

Common Misconceptions: G.SRT.4-5

- Some students may confuse the alternate interior angle theorem and its converse as well as the Pythagorean theorem and its converse.

Standard: G.SRT.5

Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

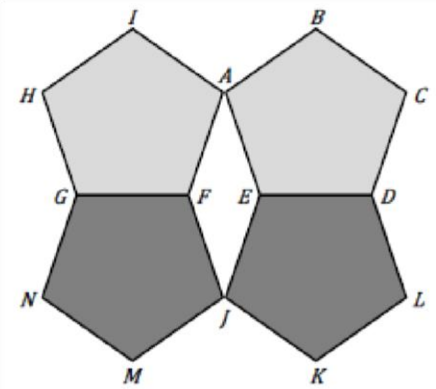
Explanations and Examples: G.SRT.5

- Similarity postulates include SSS, SAS, and AA.
- Congruence postulates include SSS, SAS, ASA, AAS, and H-L.
- Apply triangle congruence and triangle similarity to solve problem situations (e.g., indirect measurement, missing sides/angle measures, side splitting).

- Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.

Examples:

- This diagram is made up of four regular pentagons that are all the same size.
  1. Find the measure of angle AEJ
  2. Find the measure of angle BJF
  3. Find the measure of angle KJM



Standard: G.MG.1

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). (★)

Connections: G.MG.1-3
Modeling activities are a good way to show connections among various branches of mathematics.

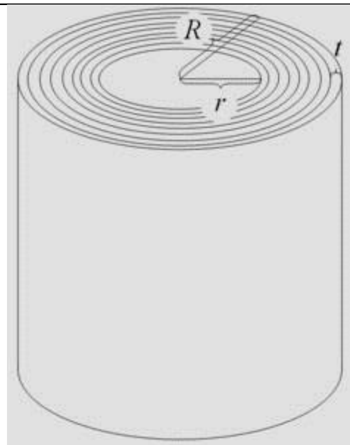
Explanations and Examples: G.MG.1

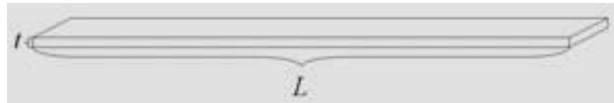
- Focus on situations that require relating two- and three- dimensional objects.
- Estimate measures (circumference, area, perimeter, volume) of real-world objects using comparable geometric shapes or three-dimensional objects.
- Apply the properties of geometric figures to comparable real-world objects (e.g., The spokes of a wheel of a bicycle are equal lengths because they represent the radii of a circle).
- Students may us simulation software and modeling software to explore which model best describes a set of data or situation.

Examples:

- How can you model objects in your classroom as geometric shapes?
- Picture a roll of toilet paper; assume that the paper in the roll is very tightly rolled. Assuming that the paper in the roll is very thin, find a relationship between the thickness of the paper, the inner and outer radii of the roll, and the length of the paper in the roll.
- Express your answer as an algebraic formula involving the four listed variables.
- The purpose of this task is to engage students in geometric modeling, and in particular to deduce algebraic relationships between variables stemming from geometric constraints. The modeling process is a challenging one, and will likely elicit a variety of attempts from the students. Teachers should expect to spend time guiding students away from overly complicated models. Similarly, the task presents one solution, but alternatives abound: For example, students could imagine slicing the roll along a radius, unraveling the cross-section into a sequence of trapezoids whose area can be computed.

Solution:





- We begin by labeling the variables, for which the above diagrams may be useful. Let t denote the thickness of the paper, let r denote the inner radius, let R denote the outer radius and let L denote the length of the paper, all measured in inches. We now consider the area A, measured in square inches, of the annular cross-section displayed at the top of the first image, consisting of concentric circles. Namely, we see that this area can be expressed in two ways: First, since this area is the area of the circle of radius R minus the area of the circle of radius r, we learn that
  $A = \pi(R^2 - r^2)$.
- Second, if the paper were unrolled, laid on a (very long) table and viewed from the side, we would see a very long thin rectangle. When the paper is rolled up, this rectangle is distorted, but -- assuming r is large in comparison to t -- the area of the distorted rectangle is nearly identical to that of the flat one. As in the second figure, the formula for the area of a rectangle now gives $A = t \cdot L$.
- Comparing the two formulas for A, we find that the four variables are related by: $t \cdot L = \pi(R^2 - r^2)$.

Instructional Strategies: G.MG.1-3

- Genuine mathematical modeling typically involves more than one conceptual category. For example, modeling a herd of wild animals may involve geometry, measurement, proportional reasoning, estimation, probability and statistics, functions, and algebra. It would be somewhat misleading to try to teach a unit with the title of "modeling with geometry." Instead, these standards can be woven into other content clusters.
- A challenge for teaching modeling is finding problems that are interesting and relevant to high school students and, at the same time, solvable with the mathematical tools at the students' disposal. The resources listed below are a beginning for addressing this difficulty.

Common Misconceptions: G.MG.1-3

- When students ask to see "useful" mathematics, what they often mean is, "Show me how to use this mathematical concept or skill to solve the homework problems." Mathematical modeling, on the other hand, involves solving problems in which the path to the solution is not obvious. Geometry may be one of several tools that can be used.

Standard: G.MG.3

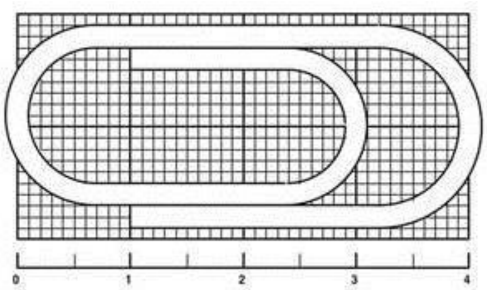
Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). (★)

Explanations and Examples: G.MG.3

- Create a visual representation of a design problem and solve using a geometric model (graph, equation, table, formula).
- Interpret the results and make conclusions based on the geometric model.
- Students may us simulation software and modeling software to explore which model best describes a set of data or situation.

Examples:

- Given one geometric solid, design a different geometric solid that will hold the same amount of substance (e.g., a cone to a prism).
- This paper clip is just over 4 cm long.



- How many paper clips like this may be made from a straight piece of wire 10 meters long?

- In this task, a typographic grid system serves as the background for a standard paper clip. A metric measurement scale is drawn across the bottom of the grid and the paper clip extends in both directions slightly beyond the grid. Students are given the approximate length of the paper clip and determine the number of like paper clips made from a given length of wire. Extending the paper clip beyond the grid provides an opportunity to include an estimation component in the problem. In the interest of open-ended problem solving, no scaffolding or additional questions are posed in this task. The paper clip modeled in this problem is an actual large standard paper clip.

Suggested Big Idea	**Unit 4 Trigonometry**			
Content Emphasis Cluster	Define trigonometric ratios and solve problems involving right triangles.			
Mathematical Practices	**MP.1 Make sense of problems and persevere in solving them.** **MP.2 Reason abstractly and quantitatively.**. **MP.3 Construct viable arguments and critique the reasoning of others.** **MP.4 Model with mathematics.** **MP.5 Use appropriate tools strategically.** **MP.6 Attend to precision.** **MP.7 Look for and make use of structure.**			
Common Assessment	End of Unit Assessment			
Graduate Competency	Prepared graduates use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions			
CCSS Priority Standards	**Cross-Content Connections**	**Writing Focus**	**Language/Vocabulary**	**Misconceptions**
SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles. SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*	Literacy Connections RST.6-8.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics. RST.6-8.5 Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic. RST.6-8.7 Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g.,	Writing Connection WHST.6-8.2 Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. p.Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. q.Develop the topic with relevant, well-chosen facts, definitions, concrete details,	Academic Vocabulary- Composition, Dilation, Pythagorean theorem, Rigid Motions, Scale Drawing, Scale Factor, Slope Technical Vocabulary- Cosine Sides of a Right Triangle Sine Tangent L.6-8.6 Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.	SRT.C.6-8 Some students believe that right triangles must be oriented a particular way. Some students do not realize that opposite and adjacent sides need to be identified with reference to a particular acute angle in a right triangle. Some students believe that the trigonometric ratios defined in this cluster apply to all triangles, but they are only defined for acute angles in right triangles.

	in a flowchart, diagram, model, graph, or table). RST.6-8.8 Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.	quotations, or other information and examples. r. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. s. Use precise language and domain-specific vocabulary to inform about or explain the topic. t. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented. WHST.6-8.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	L.6-8.4 Determine or clarify the meaning of unknown and multiple-meaning words and phrases choosing flexibly from a range of strategies.	

Greeley-Evans School District 6 Geometry 2016-2017

Unit 4	**Trigonometry**	**Length of Unit**	**Est: 21 Days** **1/4/17-2/3/17** **Unit assessment scanned by 2/13/17**
Standards of Mathematical Practices	**MP.1** Make sense of problems and persevere in solving them. **MP.2** Reason abstractly and quantitatively.. **MP.3** Construct viable arguments and critique the reasoning of others. **MP.4** Model with mathematics. **MP.5** Use appropriate tools strategically. **MP.6** Attend to precision. **MP.7** Look for and make use of structure.		
Content Standards (<u>Priority Standards are underlined)</u>	**<u>Define trigonometric ratios and solve problems involving right triangles</u>** **<u>SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.</u>** **<u>SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.</u>** **<u>SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*</u>** **Apply trigonometry to general triangles** **SRT.D.9** Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **SRT.D.10** Prove the Laws of Sines and Cosines and use them to solve problems. **SRT.D.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). **Translate between the geometric description and the equation for a conic section** **GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean theorem; complete the square to find the center and radius of a circle given by an equation.		

Terminology	
Academic Vocabulary *Cross discipline language-*	Composition, Dilation, Pythagorean theorem, Rigid Motions, Scale Drawing, Scale Factor, Slope

Technical Vocabulary *Discipline-specific language*	**Cosine** (Let θ be the angle measure of an acute angle of the right triangle. The cosine of θ of a right triangle is the value of the ratio of the length of the adjacent side (denoted adj) to the length of the hypotenuse (denoted hyp). As a formula, cos $\theta = adj/hyp$.) **Sides of a Right Triangle** (The hypotenuse of a right triangle is the side opposite the right angle; the other two sides of the right triangle are called the legs. Let θ be the angle measure of an acute angle of the right triangle. The opposite side is the leg opposite that angle. The adjacent side is the leg that is contained in one of the two rays of that angle (the hypotenuse is contained in the other ray of the angle).) **Sine** (Let θ be the angle measure of an acute angle of the right triangle. The sine of θ of a right triangle is the value of the ratio of the length of the opposite side (denoted opp) to the length of the hypotenuse (denoted hyp). As a formula, sin $\theta = opp/hyp$.) **Tangent (**Let θ be the angle measure of an acute angle of the right triangle. The tangent of θ of a right triangle is the value of the ratio of the length of the opposite side (denoted opp) to the length of the adjacent side (denoted adj). As a formula, tan $\theta = opp/adj$.)
Assessments	**End of Unit Common Assessment on Schoolcity:** • Scanned into School City or students take the assessment online • Should be in addition to individually developed formative assessments

Core Lessons and Notes

Lessons as suggested from Engage NY Module 2 (click links to follow)

Topic A: Trigonometry (G-SRT.C.6, G-SRT.C.7, G-SRT.C.8, G-SRT.D.9, G-SRT.D.10, G-SRT.D.11)

- *Lesson 25: Incredibly Useful Ratios
- Lesson 26: The Definition of Sine, Cosine, and Tangent
- Lesson 27: Sine and Cosine of Complementary Angles and Special Angles
- *Lesson 28: Solving Problems using Sine and Cosine
- Lesson 29: Applying Tangents
- Lesson 30: Trigonometry and the Pythagorean Theorem
- Lesson 31: Using Trigonometry to Determine Area
- *Lesson 32: Using Trigonometry to Find Side Lengths of an Acute Triangle
- Lesson 33: Applying the Laws of Sines and Cosines
- Lesson 34: Unknown Angles

Lessons as suggested from Engage NY Module 5 (click links to follow)

Topic B: Equations for Circles and Their Tangents (G-GPE.A.1, G-GPE.B.4)

- *Lesson 17: Writing the Equation for a Circle
- *Lesson 18: Recognizing Equations of Circles
- Lesson 19: Equations for Tangent Lines to Circles

Optional Materials and Testing Items from New York Engage

*Lessons recommended by the Geometry Team

- *Lessons listed on the left are OPTIONAL resources from New York Engage. You may choose to use these as a basis or supplement to your own materials.*
- This unit builds on similarity from the last semester—emphasize how identical angles in right triangles will always give sides that are proportional
- Using notation such as (sin x) instead of function notation sin(x) may help student realize that the trigonometric function with angle is simply a placeholder for a ratio
- Lesson 25 introduces the idea of what opposite, adjacent, and hypotenuse are depending on angles (and it does well) as well introducing the idea of Opp/Adj; Opp/Hyp. Great table for students to do.
- Lesson 27 covers topics that are represented in the Unit Assessment.
- Throughout the lessons, the charts that students are to fill out will help students gain more of an understanding of the ratios as well as make the students fluent using the calculator.
- Lesson 30 explores the relationship between the trig ratios and Pythagorean theorem. Also introduces a couple of trig identities.

- Lesson 31 brings back the altitude in a triangles to help solve for the area of a triangles through Trigonometry. The worksheet has problems that involve non right triangles.
- Lesson 32 introduces the explanation of law of sines and cosines.

Module 5

- Lesson 17 recalls on knowledge of the distance formula and Pythagorean theorem to discuss where the equation of a circle comes from.
- Lesson 18 requires students to be able to factor trinomials and complete the square in order to get to the equation of a circle.

Materials	Calculator, Straight Edge

Unit 4 Trigonometry

Topic A - Trigonometry

Students begin the study of trigonometry in the final topic of the module. The emphasis in the module on side length relationships within similar triangles (Topic C) and the specific emphasis on right triangles (Topic D) help set the foundation for trigonometry. Lesson 25 is a last highlight of the side length ratios within and between right triangles. Students are guided to the idea that the values of the ratios depend solely on a given acute angle in the right triangle before the basic trigonometric ratios are explicitly defined in Lesson 26 (G.SRT.C.6). After practice with ratios labeled as shorter leg: hypotenuse (Lesson 21) and opp: hyp (Lesson 25), students are introduced to the trigonometric ratios sine, cosine, and tangent (G-SRT.C.6) in Lesson 26. Students examine the relationship between sine and cosine in Lesson 27, discovering that the sine and cosine of complementary angles are equal (G-SRT.C.7). They are also introduced to the common sine and cosine values of angle measures frequently seen in trigonometry. Students apply the trigonometric ratios to solve for unknown lengths in Lessons 28 and 29; students also learn about the relationship between tangent and slope in Lesson 29 (G-SRT.C.8). In Lesson 30, students use the Pythagorean theorem to prove the identity sin2 θ + cos2 θ = 1 and also show why tan θ = sin θ cos . In Lessons 31–33, students study the application of trigonometry to determine area and solve for unknown lengths using the laws of sines and cosines (G-SRT.9, G-SRT.10, G-SRT.11). Finally, in Lesson 34 students learn how to determine the unknown measure of an angle of a right triangle. Students are introduced to the trigonometric functions arcsin, arccos, and arctan. These inverse functions are taught formally in Grade 11. For now, students should understand the meaning of and how to use arcsin, arccos, and arctan to determine unknown measures of angles.

Topic B – Equations for Circles and their Tangents

Topic B consists of three lessons focusing on MP.7. Students see the structure in the different forms of equations of a circle and lines tangent to circles. In Lesson 17, students deduce the equation for a circle in center-radius form using what they know about the Pythagorean theorem and the distance between two points on the coordinate plane (G-GPE.A.1). Students first understand that a circle whose center is at the origin of the coordinate plane is given by $xx2 + yy2 = rr2$, where rr is the radius. Using their knowledge of translation, students derive the general formula for a circle as $(xx - aa)2 + (yy - bb)2 = rr2$, where rr is the radius of the circle, and (aa, bb) is the center of the circle. In Lesson 18, students use their algebraic skills of factoring and completing the square to transform equations into center-radius form. Students prove that $xx2 + yy2 + AAAA + BBBB + CC = 0$ is the equation of a circle and find the formula for the center and radius of this circle (G-GPE.A.4). Students know how to recognize the equation of a circle once the equation format is in center-radius form. In Lesson 19, students again use algebraic skills to write the equations of lines, specifically lines tangent to a circle, using information about slope and/or points on the line. Recalling students' understanding of tangent lines from Lesson 11 and combining that with the equations of circles from Lessons 17 and 18, students determine the equation of tangent lines to a circle from points outside of the circle.

Resources and Notes by Standard (adapted from KATM Flip Books)

Standard: G.SRT.6

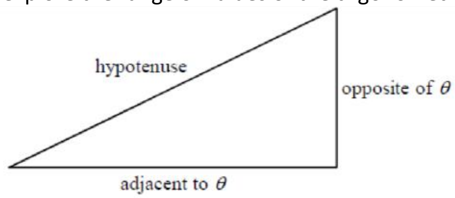
Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Connections: G.SRT.6-8

Trigonometry is not introduced until high school. Right triangle trigonometry (a geometry topic) has implications when studying algebra and functions. For example, trigonometric ratios are functions of the size of an angle, the trigonometric functions can be revisited after radian measure has been studied, and the Pythagorean theorem can be used to show that (sin A)2 + (cos A)2 = 1.

Explanations and Examples: G.SRT.6

Students may use applets to explore the range of values of the trigonometric ratios as Θ ranges from 0 to 90 degrees. Use the characteristics of similar figures to justify trigonometric ratios.



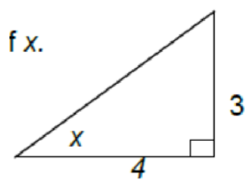
$$\text{sine of } \theta = \sin\theta = \frac{opposite}{hypotenuse} \qquad \text{cosecant of } \theta = \csc\theta = \frac{hypotenuse}{opposite}$$

$$\text{cosine of } \theta = \cos\theta = \frac{adjacent}{hypotenuse} \qquad \text{secant of } \theta = \sec\theta = \frac{hypotenuse}{adjacent}$$

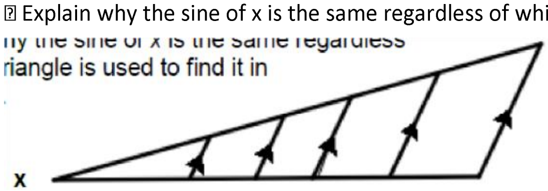
$$\text{tangent of } \theta = \tan\theta\ \frac{opposite}{adjacent} \qquad \text{cotangent of } \theta = \cot\theta = \frac{adjacent}{opposite}$$

Examples:

- Find the sine, cosine, and tangent of x.



- Explain why the sine of x is the same regardless of which triangle is used to find it in the figure below.



Instructional Strategies: G.SRT.6-8

Review vocabulary (opposite and adjacent sides, legs, hypotenuse and complementary angles) associated with right triangles.

Make cutouts or drawings of right triangles or manipulate them on a computer screen using dynamic geometry software and ask students to measure side lengths and compute side ratios. Observe that when triangles satisfy the AA criterion, corresponding side ratios are equal. Side ratios are given standard names, such as sine, cosine and tangent. Allow adequate time for students to discover trigonometric relationships and progress from concrete to abstract understanding of the trigonometric ratios.

Show students how to use the trigonometric function keys on a calculator. Also, show how to find the measure of an acute angle if the value of its trigonometric function is known.

Investigate sines and cosines of complementary angles, and guide students to discover that they are equal to one another. Point out to students that the “co” in cosine refers to the “sine of the complement.”

Observe that, as the size of the acute angle increases, sines and tangents increase while cosines decrease.
Stress trigonometric terminology by the history of the word “sine” and the connection between the term “tangent” in trigonometry and tangents to circles.

Have students make their own diagrams showing a right triangle with labels showing the trigonometric ratios.
Although students like mnemonics such as SOHCAHTOA, these are not a substitute for conceptual understanding. Some students may investigate the reciprocals of sine, cosine, and tangent to discover the other three trigonometric functions.

Use the Pythagorean theorem to obtain exact trigonometric ratios for 30°, 45°, and 60° angles.
Use cooperative learning in small groups for discovery activities and outdoor measurement projects.

Have students work on applied problems and project, such as measuring the height of the school building or a flagpole, using clinometers and the trigonometric functions.

Common Misconceptions: G.SRT.6-8

Some students believe that right triangles must be oriented a particular way.

Some students do not realize that opposite and adjacent sides need to be identified with reference to a particular acute angle in a right triangle.

Some students believe that the trigonometric ratios defined in this cluster apply to all triangles, but they are only defined for acute angles in right triangles.

Standard: G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

Explanations and Examples: G.SRT.7

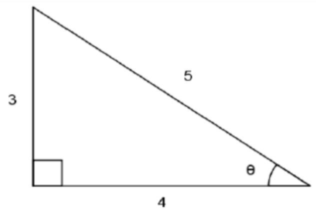
Calculate sine and cosine ratios for acute angles in a right triangle when given two side lengths.
Use a diagram of a right triangle to explain that for a pair of complimentary angles A and B, the sine of angle A is equal to the cosine of angle B and the cosine of angle A is equal to the sine of angle B.

Geometric simulation software applets and graphing calculators can be used to explore the relationship between sine and cosine.

Examples:

- What is the relationship between cosine and sine in relation to complementary angles?
  - Construct a table demonstrating the relationship between sine and cosine of complementary angles.
- Find the second acute angle of a right triangle given that the first acute angles has a measure of 39°.
- Complete the following statement: If sin 30 = 1/2, then the cos______ = 1/2.

- Find the sine and cosine of angle Θ in the triangle below. What do you notice?



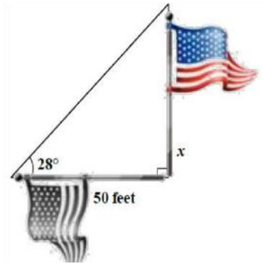
Standard: G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (★)

Explanations and Examples: G.SRT.8

Use angle measures to estimate side lengths (e.g., The side across from a 33 angle will be shorter than the side across from a 57 angle).
Use side lengths to estimate angle measures (e.g., The angle opposite of a 10 cm side will be larger than the angle across from a 9 cm side).
Draw right triangles that describe real world problems and label the sides and angles with their given measures. Solve application problems involving right triangles, including angle of elevation and depression, navigation, and surveying.
Students may use graphing calculators or programs, tables, spreadsheets, or computer algebra systems to solve right triangle problems.

Examples:

- Find the height of a flagpole to the nearest tenth if the angle of elevation of the sun is 28° and the shadow of the flagpole is 50 feet.



- A teenager whose eyes are 5 feet above ground level is looking into a mirror on the ground and can see the top of a building that is 30 feet away from the teenager. The angle of elevation from the center of the mirror to the top of the building is 75°. How tall is the building? How far away from the teenager's feet is the mirror?

- While traveling across flat land, you see a mountain directly in front of you. The angle of elevation to the peak is 3.5°. After driving 14 miles closer to the mountain, the angle of elevation is 9°24'36".

Explain how you would set up the problem, and find the approximate height of the mountain.

Standard: G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Connections: G.GPE.1-2

In Grade 8 the Pythagorean theorem was applied to find the distance between two particular points. In high school, the application is generalized to obtain formulas related to conic sections.

Quadratic functions and the method of completing the square are studied in the domain of interpreting functions.

The methods are applied here to transform a quadratic equation representing a conic section into standard form.

Instructional Strategies: G.GPE.1

Review the definition of a circle as a set of points whose distance from a fixed point is constant.

Review the algebraic method of completing the square and demonstrate it geometrically.

Illustrate conic sections geometrically as cross sections of a cone.

Use the Pythagorean theorem to derive the distance formula. Then, use the distance formula to derive the equation of a circle with a given center and radius, beginning with the case where the center is the origin.

Starting with any quadratic equation in two variables (x and y) in which the coefficients of the quadratic terms are equal, complete the squares in both x and y and obtain the equation of a circle in standard form.

Given two points, find the equation of the circle passing through one of the points and having the other as its center.

Import images of circle from fields from Google Earth into a coordinate grid system and find their equations.

Common Misconceptions: G.GPE.1-2

Because new vocabulary is being introduced in this cluster, remembering the names of the conic sections can be problematic for some students.

The Euclidean distance formula involves squared, subscripted variables whose differences are added. The notation and multiplicity of steps can be a serious stumbling block for some students.

The method of completing the square is a multi-step process that takes time to assimilate. A geometric demonstration of completing the square can be helpful in promoting conceptual understanding.

Suggested Big Idea	Unit 5 Area and Volume			
Content Emphasis Cluster	Explain volume formulas and use them to solve problems. Visualize relationships between two-dimensional and three-dimensional objects.			
Mathematical Practices	**MP.1 Make sense of problems and persevere in solving them.** **MP.2 Reason abstractly and quantitatively.** **MP.3 Construct viable arguments and critique the reasoning of others.** **MP.4 Model with mathematics.** **MP.6 Attend to precision.**			
Common Assessment	End of Unit Assessment			
Graduate Competency	Prepared graduates make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics			
CCSS Priority Standards	**Cross-Content Connections**	**Writing Focus**	**Language/Vocabulary**	**Misconceptions**
GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.* Visualize relationships between two-dimensional and three-dimensional objects GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* MG.3 Apply geometric methods to solve design problems (e.g., designing an object	Literacy Connections RST.6-8.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics. RST.6-8.5 Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic. RST.6-8.7 Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table). RST.6-8.8	Writing Connection WHST.6-8.2 Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. u. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. v. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples.	Academic Vocabulary- Surface Area, volume, Area, circumference, circle, cylinder, pyramid, cone, lateral area, prism, sphere, composite figure, density, dimension, net, cross sections, polyhedron, slant height, Disk Technical Vocabulary- Cavalieri's Principle, Cone, General Cylinder, Inscribed Polygon, Intersection, Rectangular Pyramid, Right Rectangular Prism, Solid Sphere or Ball, Sphere, Subset, Tangent to a Circle, Union L.6-8.6 Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary	GMD.1-3 An informal survey of students from elementary school through college showed the number pi to be the mathematical idea about which more students were curious than any other. There are at least three facets to this curiosity: the symbol π itself, the number 3.14159…, and the formula for the area of a circle. All of these facets can be addressed here, at least briefly. Many students want to think of infinity as a number. Avoid this by talking about a quantity that becomes larger and larger with no upper bound. The inclusion of the coefficient 1/3 in the formulas for the volume of a pyramid or cone and 4/3 in the formula for the volume of a sphere remains a mystery for many students. In high school, students should attain a conceptual understanding of where these coefficients come from. Concrete demonstrations, such as pouring water from one shape into another should be followed by more formal reasoning. MG.1-3 When students ask to see "useful" mathematics, what they often mean is, "Show me how to use this mathematical concept or skill to solve the homework

or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).* GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).	Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.	w. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. x. Use precise language and domain-specific vocabulary to inform about or explain the topic. y. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented. WHST.6-8.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	knowledge when considering a word or phrase important to comprehension or expression. L.6-8.4 Determine or clarify the meaning of unknown and multiple-meaning words and phrases choosing flexibly from a range of strategies.	problems." Mathematical modeling, on the other hand, involves solving problems in which the path to the solution is not obvious. Geometry may be one of several tools that can be used. GPE.4 Students may claim that a vertical line has infinite slopes. This suggests that infinity is a number. Since applying the slope formula to a vertical line leads to division by zero, we say that the slope of a vertical line is undefined. Also, the slope of a horizontal line is 0. Students often say that the slope of vertical and/or horizontal lines is "no slope," which is incorrect.

Unit 5	**Area and Volume**	**Length of Unit**	**Est 27 Days** **2/7/17-3/17/17** **Unit assessment scanned by 3/31/17**
Standards of Mathematical Practices	**MP.1** Make sense of problems and persevere in solving them. **MP.2** Reason abstractly and quantitatively. **MP.3** Construct viable arguments and critique the reasoning of others. **MP.4** Model with mathematics. **MP.6** Attend to precision.		
Content Standards (<u>Priority Standards are Underlined</u>)	**<u>Explain volume formulas and use them to solve problems</u>** **<u>GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.</u>** **<u>GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*</u>** **<u>Visualize relationships between two-dimensional and three-dimensional objects</u>** **<u>GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.</u>** **Apply geometric concepts in modeling situations** **<u>MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*</u>** **<u>MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*</u>** **<u>MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*</u>** **C.2** Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. **C.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. **C.4** (+) Construct a tangent line from a point outside a given circle to the circle.		

Critical Language:	
Academic Vocabulary *Cross discipline language*	Surface Area, volume, Area, circumference, circle, cylinder, pyramid, cone, lateral area, prism, sphere, composite figure, density, dimension, net, cross sections, polyhedron, slant height, Disk,
Technical Vocabulary *Discipline-specific language*	**Cavalieri's Principle** (Given two solids that are included between two parallel planes, if every plane parallel to the two planes intersects both solids in cross-sections of equal area, then the volumes of the two solids are equal.) **Cone** (Let BB be a region in a plane EE, and VV be a point not in EE. The cone with base BB and vertex VV is the union of all segments $VVVV$ for all points PP in BB. If the base is a polygonal region, then the cone is usually called a pyramid.) **General Cylinder** (Let EE and EE' be two parallel planes, let BB be a region in the plane EE, and let LL be a line which intersects EE and EE' but not BB. At each point PP of BB, consider the segment $PPPP'$ parallel to LL, joining PP to a point PP' of the plane EE'. The union of all these segments is called a cylinder with base BB.) **Inscribed Polygon** (A polygon is inscribed in a circle if all of the vertices of the polygon lie on the circle.) **Intersection** (The intersection of AA and BB is the set of all objects that are elements of AA and also elements of BB. The intersection is denoted $AA \cap BB$.) **Rectangular Pyramid** (Given a rectangular region BB in a plane EE, and a point VV not in EE, the rectangular pyramid with base BB and vertex VV is the union of all segments $VVVV$ for points PP in BB.) **Right Rectangular Prism** (Let EE and EE' be two parallel planes. Let BB be a rectangular region in the plane EE. At each point PP of BB, consider the segment $PPPP'$ perpendicular to EE, joining PP to a point PP' of the plane EE'. The union of all these segments is called a right rectangular prism.) **Solid Sphere or Ball** (Given a point CC in the three-dimensional space and a number $rr > 0$, the solid sphere (or ball) with center CC and radius rr is the set of all points in space whose distance from point CC is less than or equal to rr.) **Sphere** (Given a point CC in the three-dimensional space and a number $rr > 0$, the sphere with center CC and radius rr is the set of all points in space that are distance rr from the point CC.) **Subset** (A set AA is a subset of a set BB if every element of AA is also an element of BB.)

	Tangent to a Circle (A tangent line to a circle is a line that intersects a circle in one and only one point.) **Union** (The union of AA and BB is the set of all objects that are either elements of AA or of BB or of both. The union is denoted $AA \cup BB$.) **Arc Length** (The length of an arc is the circular distance around the arc.) **Central Angle** (A central angle of a circle is an angle whose vertex is the center of a circle.) **Chord** (Given a circle C, let P and Q be points on C. Then PQ is called a chord of CC.) **Cyclic Quadrilateral** (A quadrilateral inscribed in a circle is called a cyclic quadrilateral.) Inscribed Angle (An inscribed angle is an angle whose vertex is on a circle, and each side of the angle intersects the circle in another point.) **Inscribed Polygon** (A polygon is inscribed in a circle if all vertices of the polygon lie on the circle.) **Secant Line** (A secant line to a circle is a line that intersects a circle in exactly two points.) **Sector** (Let A be an arc of a circle. The sector of a circle with arc A is the union of all radii of the circle that have an endpoint in arc A. The arc A is called the arc of the sector, and the length of any radius of the circle is called the radius of the sector.) **Tangent Line** (A tangent line to a circle is a line in the same plane that intersects the circle in one and only one point. This point is called the point of tangency.)
Assessment	**End of Unit Common Assessment on Schoolcity:** • Scanned into School City or students take the assessment online • Should be in addition to individually developed formative assessments

Core Lessons and Notes

Lessons as suggested from Engage NY Module 3 (click links to follow)

Topic A: Area (G-GMD.A.1)

- *Lesson 1: What is Area?
- Lesson 2: Properties of Area
- *Lesson 3: The Scaling Principle for Area
- Lesson 4: Proving the Area of a Disk

Lessons as suggested from Engage NY Module 5 (click links to follow)

Topic B: Central and Inscribed Angles (G-C.A.2, G-C.A.3)

- Lesson 1: Thales' Theorem
- Lesson 2: Circles, Chords, Diameters, and Their Relationships
- Lesson 3: Rectangles Inscribed in Circles
- Lesson 4: Experiments with Inscribed Angles
- *Lesson 5: Inscribed Angle Theorem and Its Applications
- Lesson 6: Unknown Angle Problems with Inscribed Angles in Circles

Topic C: Arcs and Sectors (G-C.A.1, G-C.A.2, G-C.B.5)

- Lesson 7: The Angle Measure of an Arc
- *Lesson 8: Arcs and Chords
- Lesson 9: Arc Length and Areas of Sectors
- Lesson 10: Unknown Length and Area Problems

Topic D: Secants and Tangents (G-C.A.2, G-C.A.3)

- Lesson 11: Properties of Tangents
- Lesson 12: Tangent Segments
- Lesson 13: The Inscribed Angle Alternate—A Tangent Angle
- Lesson 14: Secant Lines; Secant Lines That Meet Inside a Circle
- Lesson 15: Secant Angle Theorem, Exterior Case
- Lesson 16: Similar Triangles in Circle-Secant (or Circle-Secant-Tangent) Diagrams

Lessons as suggested from Engage NY Module 3 (click links to follow)

Topic E: Volume (G-GMD.A.1, G-GMD.A.3, G-GMD.B.4, G-MG.A.1, G-MG.A.2, G-MG.A.3)

- Lesson 5: Three-Dimensional Space
- Lesson 6: General Prisms and cylinders and Their Cross Sections
- Lesson 7: General Pyramids and Cones and Their Cross sections
- Lesson 8: Definition and Properties of Volume
- Lesson 9: Scaling Principle for Volumes
- *Lesson 10: The Volume of Prisms and Cylinders and Cavalieri's Principle
- Lesson 11: The Volume Formula of a Pyramid and Cone
- Lesson 12: The Volume Formula of a Sphere
- *Lesson 13: How do 3D Printers Work?

Module 3

- *Lessons listed on the left are OPTIONAL resources from New York Engage. You may choose to use these as a basis or supplement to your own materials.*
- Lesson 1 engages students on a conceptual understanding of area applied to a unit coordinate grid, then broadens the concept to area of irregular shapes.
- Lesson 2 uses sets (unions and intersections) to help explain finding area of shapes that overlap.
- Lesson 3 provides an exploratory basis for the effect scale factor has on area. This is repeated in lesson 9 for volume.
- Lesson 4 develops the area of a circle using inscribed and circumscribed polygons with continually more sides.

Module 5

- Lesson 2 reviews constructions. (If you create a perpendicular bisector of a chord in a circle, the perpendicular bisector will always go through the center of the circle)
- Lesson 3 – students need to know properties of rectangles.
- Lesson 9 has a short, to the point, homework that covers arc length and area of a sector.
- Lesson 11 requires the use of constructions.
- Lesson 13 recalls the information from inscribed angles and their intercepted arcs.

Module 3

- Lesson 7 uses similarity to discuss the cross sections found in cones/pyramids at different heights.
- Lesson 10 incorporate Cavalieri's principle to demonstrate how the volume of different shaped solids may actually be the same due to the cross sections
- Lessons 10-12 provide numerous real-world example problems involving volume
- Watch the 3-D printer videos in Lesson 13!
- Be sure to explain what density is in context of volume

Optional Materials and Testing Items from New York Engage * Lessons recommended by the geometry team	• Show answer in terms of pi, or calculations using approximations of pi (3.14 or 22/7)

Materials	3D models, deck of cards, stack of coins, images of "sliced" figures such as a loaf of bread, Patty Paper, Calculators, Projector, Doc Cam, Promethean Board

Unit 5 – Area and Volume

Topic A: Area

The topic of area must be revisited in order to have a conversation about figures in three dimensions; we first have the necessary discussion around area. Area is introduced in Grade 3, but only figures that are easy to "fill" with units are considered. In Grade 5, the need to use parts of unit squares for some figures is understood, and is applied in Grade 6. In Grade 7, students realize that a figure may have an area even if, like the disk, it cannot be decomposed into a finite number of unit squares, but (appropriately) this is treated at the intuitive level. As the grades progress, the link between area as a measurable geometric quantity and area represented by numbers for calculations is elaborated. The culmination is a universal method for measuring areas, even when they are not finite unions of unit squares or simple parts of unit squares. Mathematicians refer to this as Jordan measure. While this may seem like an intimidating idea to introduce at this level, it can be thought of simply as the well-known properties of area that students are already familiar with. This is not only an intuitively acceptable approach to area, but it is completely rigorous at the university level. The concept forms an important bridge to calculus, as Jordan measure is the idea employed in defining the Riemann integral.

In this topic, Lesson 1 shows how finding the area of a curved figure can be approximated by rectangles and triangles. By refining the size of the rectangles and triangles, the approximation of the area becomes closer to the actual area. Students experience a similar process of approximation in Grade 8 (Module 7, Lesson 14) in order to estimate $\pi\pi$. The informal limit argument prepares students for the development of volume formulas for cylinders and cones and foreshadows ideas that students will formally explore in calculus. This process of approximation is important to developing the volume formula of cylinders and cones. In Lesson 2, students study the basic properties of area using set notation; in Topic B they will see how the properties are analogous to those of volume. In Lesson 3, students study the scaling principle, which states that if a planar region is scaled by factors a and b in two perpendicular directions, then its area is multiplied by a factor of $a \times b$. Again, we study this in two dimensions to set the stage for three dimensions when we scale solids. Finally, in Lesson 4, students develop the formula for the area of a disk, and just as in Lesson 1, incorporate an approximation process. Students approximate the area of the disk, or circle, by inscribing a polygon within the circle, and consider how the area of the polygonal region changes as the number of sides increases and the polygon looks more and more like the disk it is inscribed within.

Topic B – Central and Inscribed Angles

The module begins with students exploring Thales' theorem in Lesson 1. The first exercise is a paper pushing discovery exercise where students push angles of triangles and trapezoids through a segment of fixed length to discover arcs of a circle (G-C.A.2). Students revisit the terms diameter and radius and are introduced to the terms central angle and inscribed angle. Lesson 2 begins the study of inscribed angles, which is the focus of this module. In this lesson, students study the relationships between circles and their diameters and between circles and their chords. Through the use of proofs (G-C.A.2), students realize that the perpendicular bisector of a chord contains the center. They also realize that the diameter is the longest chord, and congruent chords are equidistant from the center. Lesson 3 continues the study of inscribed angles by having students use a compass and straight edge to inscribe a rectangle in a circle (G-C.A.3). They then study the similarities of circles and the properties of other polygons that allow certain polygons to be inscribed in circles. Inscribed angles are compared to central angles in the same arcs in Lesson 4 with students using trapezoids and a paper pushing exercise similar to that of Lesson 1 to understand the difference between a major and minor arc. Students then explore inscribed and central angles and use repeated patterns (MP.7) to realize that the measure of a central angle is double the angle inscribed in the same arc. In Lesson 5, students are introduced to the inscribed angle theorem, but they are only introduced to inscribed angles that are not obtuse. Students prove that inscribed angles are half the angle measure of the arcs they subtend. Lesson 6 completes the study of the inscribed angle theorem as students use their knowledge of chords, radii, diameters, central angles, and inscribed angles to persevere in solving a variety of unknown angle problems (MP.1). Throughout this module, students perform activities and constructions to enhance their understanding of the concepts studied. Through the use of proofs (MP.3) and concepts previously studied, students arrive at new theorems and definitions.

Topic C – Arcs and Sectors

In Topic B, students continue studying the relationships between chords, diameters, and angles and extend that work to arcs, arc length, and areas of sectors. Students use prior knowledge of the structure of inscribed and central angles together with repeated reasoning to develop an understanding of circles, secant lines, and tangent lines (MP.7). In Lesson 7, students revisit the inscribed angle theorem, this time stating it in terms of inscribed arcs (G-C.A.2). This concept is extended to studying similar arcs, which leads students to understand that all circles are similar (G-C.A.1). Students then look at the relationships between chords and subtended arcs and prove that congruent chords lie in congruent arcs. They also prove that arcs between parallel lines are congruent using transformations (G-C.A.2). Lessons 9 and 10 switch the focus from angles to arc length and areas of sectors. Students combine previously learned formulas for area and circumference of circles with concepts learned in this module to determine arc length, areas of sectors, and similar triangles (G-C.B.5). In Lesson 9, students are introduced to radians as the ratio of arc length to the radius of a circle. Lesson 10 reinforces these concepts with problems involving unknown length and area. Topic B requires that students use and apply prior knowledge to see the structure in new applications and to see the repeated patterns in these problems in order to arrive at theorems relating chords, arcs, angles, secant lines, and tangent lines to circles (MP.7). For example, students know that an inscribed angle has a measure of half the central angle intercepting the same arc. When they discover that the measure of a central angle is equal to the angle measure of its intersected arc, they conclude that the measure of an inscribed angle is half the angle of its intercepted arc. Students then conclude that congruent arcs have congruent chords and that arcs between parallel chords are congruent.

Topic D – Secants and Tangents

Topic C focuses on secant and tangent lines intersecting circles, the relationships of angles formed, and segment lengths. In Lesson 11, students study properties of tangent lines and construct tangents to a circle through a point outside the circle and through points on the circle (G-C.A.4). Students prove that at the point of tangency, the tangent line and radius meet at a right angle. Lesson 12 continues the study of tangent lines proving segments tangent to a circle from a point outside the circle are congruent. In Lesson 13, students inscribe a circle in an angle and a circle in a triangle with constructions (G-C.A.3) leading to the study of inscribed angles with one ray being part of the tangent line (G-C.A.2). Students solve a variety of missing angle problems using theorems introduced in Lessons 11–13 (MP.1). The study of secant lines begins in Lesson 14 as students study two secant lines that intersect inside a circle. Students prove that an angle whose vertex is inside a circle is equal in measure to half the sum of arcs intercepted by it and its vertical angle. Lesson 15 extends this study to secant lines that intersect outside of a circle. Students understand that an angle whose vertex is outside of a circle is equal in measure to half the difference of the degree measure of its larger and smaller intercepted arcs. This concept is extended as the secant rays rotate to form tangent rays, and that relationship is developed. Topic C and the study of secant lines concludes in Lesson 16 as students discover the relationships between segment lengths of secant lines intersecting inside and outside of a circle. Students find similar triangles and use proportional sides to develop this relationship (G-SRT.B.5). Topic C highlights MP.1 as students persevere in solving missing angle and missing length problems; it also highlights MP.6 as students extend known relationships to limiting cases.

Topic E: Volume

With a reference to area established in Topic A, students study volume in Topic B. In Grade 8, volume is treated independent of the subtle problems that arise when we attempt to measure the volume of figures other than rectangular solids. From an advanced mathematical perspective, area and volume are conceptually very close in that Jordan measure provides a good foundation, but there are profound differences between area and volume that show up mathematically only when we consider the problem of cutting bodies along planes and reassembling them. Two bodies of the same volume might not be “equi-decomposable” in this sense. This, of course, is much more advanced an idea than anything in the curriculum, but it is one of the mathematical reasons Cavalieri's principle is indispensable. In contrasting Grade 8 with Module 3, the role of this principle is a prominent difference. More generally, understanding and predicting the shapes of cross-sections of three-dimensional figures—though it was done in Grade 7—is a complex skill that needs a lot of work to fully develop. We return to that with a level of sophistication that was absent in Grade 7.

In Lesson 5, students study the basic properties of two-dimensional and three-dimensional space, noting how ideas shift between the dimensions. For example, in two-dimensional space, two lines perpendicular to the same line are parallel, but in three-dimensional space we consider how two planes perpendicular to the same line are parallel. In Lesson 6, students learn that general cylinders are the parent category for prisms, circular cylinders, right cylinders, and oblique cylinders (MP.6). Students also study why the cross-section of a cylinder is congruent to its base (G-GMD.B.4). In Lesson 7, students study the explicit definition of a cone and learn what distinguishes pyramids from general cones. Students also see how dilations explain why a cross-section taken parallel to the base of a cone is similar to the base (G-GMD.B.4, MP.7). Lesson 8 demonstrates the properties of volume, which are analogous to the properties of area (seen in Lesson 2). Students reason why the volume of any right triangular prism has the same volume formula as that of a right triangular prism with a right triangle as a base. This leads to the generalization of the volume formula for any right cylinder (G-GMD.A.1, G-GMD.A.3). In Lesson 9, students examine the scaling principle for volume (they have seen the parallel situation regarding area in Lesson 3) and see that a solid scaled by factors a, b, and c in three perpendicular directions will result in a volume multiplied by a factor of a, b, c. In Lesson 10, students learn Cavalieri’s principle, which describes the relationship between cross-sections of two solids and their respective volumes. If two solids are included between two parallel planes, and cross-sections taken parallel to the bases are of equal area at every level, then the volumes of the solids must be equal. Cavalieri’s principle is used to reason why the volume formula of any cylinder is area of base × height (G-GMD.A.1). Lesson 11 focuses on the derivation of the volume formulas for cones, and Lesson 12 focuses on the derivation of the volume formula for spheres, which depends partly on the volume formula of a cone (G-GMD.A.1). Lesson 13 is a look at 3D printers and ultimately how the technology is linked to Cavalieri’s principle.

Module 3 is a natural place to see geometric concepts in modeling situations. Modeling-based problems are found throughout Topic B and include the modeling of real-world objects, the application of density, the occurrence of physical constraints, and issues regarding cost and profit (G-MG.A.1, G-MG.A.2, G-MG.A.3).

Resources and Notes by Standard (adapted from KATM Flip Books)

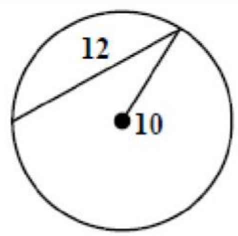
Standard: G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

Explanations and Examples: G.C.2

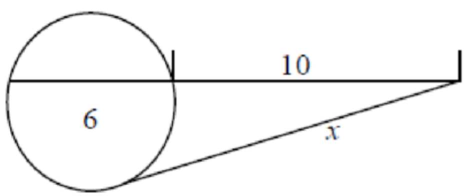
Identify central angles, inscribed angles, circumscribed angles, diameters, radii, chords, and tangents.
Describe the relationship between a central angle and the arc it intercepts.
Describe the relationship between an inscribed angle and the arc it intercepts.
Describe the relationship between a circumscribed angle and the arcs it intercepts.
Recognize that an inscribed angle whose sides intersect the endpoints of the diameter of a circle is a right angle.
Recognize that the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Examples:

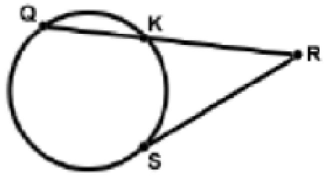
 Given the circle below with radius of 10 and chord length of 12, find the distance from the chord to the center of the circle.



Find the unknown length in the picture below.



Solution:
The theorem for a secant segment and a tangent segment that share an endpoint not on the circle states that for the picture below secant segment QR and the tangent segment SR share and endpoint R, not on the circle. Then the length of SR squared is equal to the product of the lengths of QR and KR.



$x2 = 16 \cdot 10$

S So for the example above $x2 = 160$

$$x = \sqrt{160} = 4\sqrt{10} \approx 12.6$$

How does the angle between a tangent to a circle and the line connecting the point of tangency and the center of the circle change as you move the tangent point?

Standard: G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Connections:

Formulas for area and circumference of a circle were developed in Grade 7. In this cluster the formulas are generalized to fractional parts of a circle and will prepare students for the study of trigonometry.

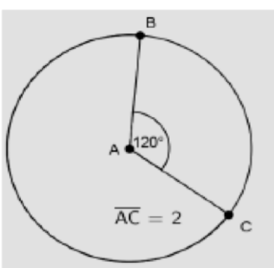
Explanations and Examples: G.C.5

Emphasize the similarity of circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry at this point.

Students can use geometric simulation software to explore angle and radian measure and derive the formula for the area of a sector.

Examples:

 Find the area of the sectors below. What general formula can you develop based on this information?



The amusement park has discovered that the brace that provides stability to the Ferris wheel has been damaged and needs work. The arc length of steel reinforcement that must be replaces is between the two seats shown below. If the sector area is 28.25 ft2 and the radius is 12 feet, what is the length of steel that must be replaced? Describe the steps you used to find your answer.

If the amusement park owners wanted to decorate each sector of this Ferris wheel with a different color of fabric, how much of each color fabric would they need to purchase?
The area to be covered is described by an arc length of 5.9 feet. The circle has a radius of 15 feet. Describe the steps you used to find your answer.

Instructional Strategies: G.C.5 1

Begin by calculating lengths of arcs that are simple fractional parts of a circle (e.g. /6), and do this for circles of various radii so that students discover a proportionality relationship.

Provide plenty of practice in assigning radian measure to angles that are simple fractional parts of a straight angle. Stress the definition of radian by considering a central angle whose intercepted arc has its length equal to the radius, making the constant of proportionality 1. Students who are having difficulty understanding radians may benefit from constructing cardboard sectors whose angles are one radian. Use a ruler and string to approximate such an angle.

Compute areas of sectors by first considering them as fractional parts of a circle. Then, using proportionality, derive a formula for their area in terms of radius and central angle. Do this for angles that are measured both in degrees and radians and note that the formula is much simpler when the angels are measured in radians.
Derive formulas that relate degrees and radians.

Introduce arc measures that are equal to the) measures of the intercepted central angles in degrees or radians.
Emphasize appropriate use of terms, such as, angle, arc, radian, degree, and sector.

Common Misconceptions: G.C.5

Sectors and segments are often used interchangeably in everyday conversation. Care should be taken to distinguish these two geometric concepts.
The formulas for converting radians to degrees and vice versa are easily confused. Knowing that the degree measure of given angle is always a number larger than the radian measure can help students use the correct unit.

Standard: G.GMD.1

Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

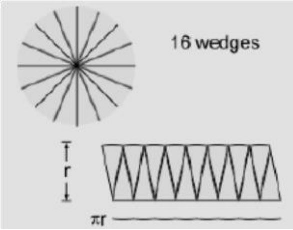
Connections: G.GMD1; G.GMD.3

In Grade 8, students were required to know and use the formulas for volumes of cylinders, cones, and spheres.
In Grade 7 students informally derived the formula for the area of a circle from the circumference.
In this cluster those formulas are derived by a combination of concrete demonstrations and formal reasoning.

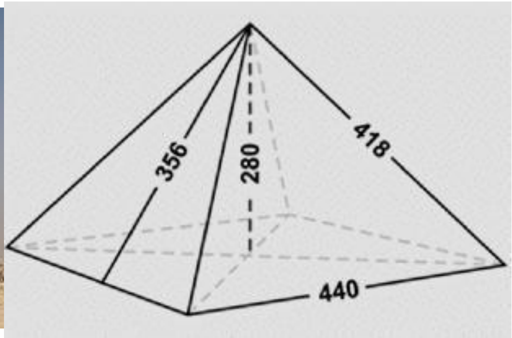
Explanations and Examples: G.GMD.1

Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when on figure in the plane results from another by applying similarity transformation with scale factor k; its area is k2 times the area of the first. Similarly, volumes of solid figure scale k3 under a similarity transformation with scale factor k.
Explain the formulas for the circumference of a circle and the area of circle by determining the meaning of each term or factor. Explain the formulas for the volume of a cylinder, pyramid and cone by determining the meaning of each term or factor.
Understand Cavalieri's principle - if two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

Examples:
Use the diagram to give an informal argument for the formula for finding the area of a circle. (This concept was introduced in Grade 7).



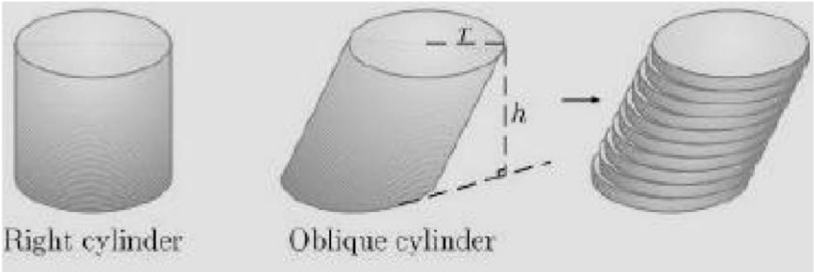
- Justify using the given measurements to find the volume of the Great Pyramid of Giza. (Dimensions shown on the diagram below are in royal cubits).



440 ≈ 755.7 feet
418 ≈ 718 feet
356 ≈ 611.4 feet
280 ≈ 480.9 feet

- Explain why the volume of a cylinder is V = πr2h.

- Prove that the right cylinder and the oblique cylinder have the same volume.



Instructional Strategies: G.GMD.1 & G.GMD.3

Revisit formulas C = πd and C = 2πr. Observe that the circumference is a little more than three times the diameter of the circle. Briefly discuss the history of this number and attempts to compute its value.
Review alternative ways to derive the formula for the area of the circle A = πr2. For example,

Cut a cardboard circular disk into 6 congruent sectors and rearrange the pieces to form a shape that looks like a parallelogram with two scalloped edges. Repeat the process with 12 sectors and note how the edges of the parallelogram look "straighter." Discuss what would happen in the case as the number of sectors becomes infinitely large. Then calculate the area of a parallelogram with base ½ C and altitude r to derive the formula A = πr2 .

Wind a piece of string or rope to form a circular disk and cut it along a radial line. Stack the pieces to form a triangular shape with base C and altitude r. Again discuss what would happen if the string became thinner and thinner so that the number of pieces in the stack became infinitely large. Then calculate the area of the triangle to derive the formula A = πr2.

Introduce Cavalieri's principle using a concrete model, such as a deck of cards. Use Cavalieri's principle with cross sections of cylinders, pyramids, and cones to justify their volume formulas.

For pyramids and cones, the factor 1/3 will need some explanation. An informal demonstration can be done using a volume relationship set of plastic shapes that permit one to pour liquid or sand from one shape into another. Another way to do this for pyramids is with Geoblocks. The set includes three pyramids with equal bases and altitudes that will stack to form a cube. An algebraic approach involves the formula for the sum of squares (12 + 22 +…+ n2).
After the coefficient 1/3 has been justified for the formula of the volume of the pyramid (A = 1/3Bh), one can argue that it must also apply to the formula of the volume of the cone by considering a cone to be a pyramid that has a base with infinitely many sides.

The formulas for volumes of cylinders, pyramids, cones and spheres can be applied to a wide variety of problems such as finding the capacity of a pipeline; comparing the amount of food in cans of various shapes; comparing capacities of cylindrical, conical and spherical storage tanks; using pyramids and cones in architecture; etc. Use a combination of concrete models and formal reasoning to develop conceptual understanding of the volume formulas.

Common Misconceptions: G.GMD.1 & G.GMD.3

An informal survey of students from elementary school through college showed the number pi to be the mathematical idea about which more students were curious than any other. There are at least three facets to this curiosity: the symbol π itself, the number 3.14159…, and the formula for the area of a circle. All of these facets can be addressed here, at least briefly.

Many students want to think of infinity as a number. Avoid this by talking about a quantity that becomes larger and larger with no upper bound.

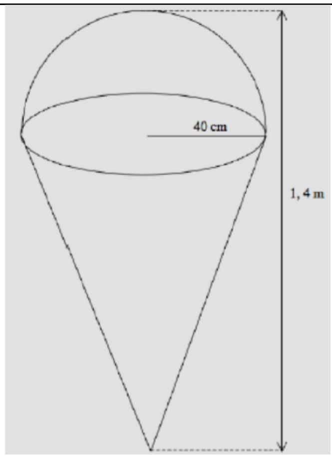
The inclusion of the coefficient 1/3 in the formulas for the volume of a pyramid or cone and 4/3 in the formula for the volume of a sphere remains a mystery for many students. In high school, students should attain a conceptual understanding of where these coefficients come from. Concrete demonstrations, such as pouring water from one shape into another should be followed by more formal reasoning.

Standard: G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (★)

Explanations and Examples: G.GMD.3

Missing measures can include but are not limited to slant height, altitude, height, diagonal of a prism, edge length, and radius.
Find the volume of cylinders, pyramids, cones and spheres in contextual problems.
Examples:

Determine the volume of the figure below.



- Find the volume of a cylindrical oatmeal box.

- Given a three-dimensional object, compute the effect on volume of doubling or tripling one or more dimension(s). For example, how is the volume of a cone affected by doubling the height?

Standard: G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Connections:

Slices of rectangular prisms and pyramids were explored in Grade 7. In high school, the concept is extended to a wider class of solids.

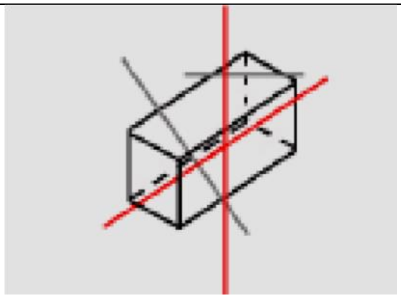
Students who eventually take calculus will learn how to compute volumes of solids of revolution by a method involving cross-sectional disks.

Explanations and Examples: G.GMD.4

Given a three-dimensional object, identify the shape made when the object is cut into cross-sections.
When rotating a two-dimensional figure, such as a square, know the three-dimensional figure that is generated, such as a cylinder. Understand that a cross section of a solid is an intersection of a plane (two-dimensional and a solid (three-dimensional).
Students may use geometric simulation software to model figures and create cross sectional views.

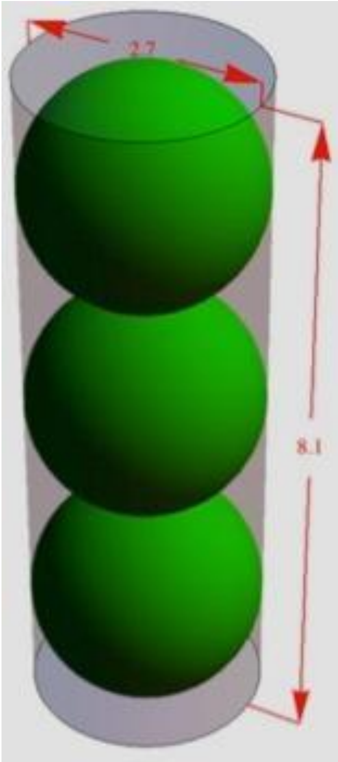
Examples:

Identify the shape of the vertical, horizontal, and other cross sections of a rectangular prism.



Identify the shape of the vertical, horizontal, and other cross sections of a rectangular prism.

The official diameter of a tennis ball, as defined by the International Tennis Federation, is at least 2.575 inches and at most 2.700 inches. Tennis balls are sold in cylindrical containers that contain three balls each. To model the container and the balls in it, we will assume that the balls are 2.7 inches in diameter and that the container is a cylinder the interior of which measures 2.7 inches in diameter and $3 \times 2.7 = 8.1$ inches high.



a. Lying on its side, the container passes through an X-ray scanner in an airport. If the material of the container is opaque to X-rays, what outline will appear? With what dimensions?

b. If the material of the container is partially opaque to X-rays and the material of the balls is completely opaque to X-rays, what will the outline look like (still assuming the can is lying on its side)?

c. The central axis of the container is a line that passes through the centers of the top and bottom. If one cuts the container and balls by a plane passing through the central axis, what does the intersection of the plane with the container and balls look like? (The intersection is also called a cross section. Imagine putting the cut surface on an ink pad and then stamping a piece of paper. The stamped image is a picture of the intersection.)

d. If the can is cut by a plane parallel to the central axis, but at a distance of 1 inch from the axis, what will the intersection of this plane with the container and balls look like?

e. If the can is cut by a plane parallel to one end of the can—a horizontal plane—what are the possible appearances of the intersections?

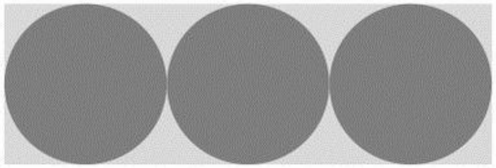
f. A cross-section by a horizontal plane at a height of 1.35 + w inches from the bottom is made, with $0 < w < 1.35$ (so the bottom ball is cut). What is the area of the portion of the cross section inside the container but outside the tennis ball?

g. Suppose the can is cut by a plane parallel to the central axis but at a distance of w inches from the axis ($0 < w < 1.35$). What fractional part of the cross section of the container is inside of a tennis ball?

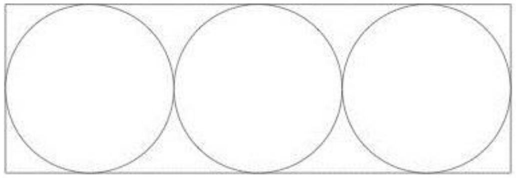
Solution: (This task connects with domain G.MG)

a. The shadow is a rectangle measuring 2.7 inches by 8.1 inches.

b. The shadow is a light rectangle (2.7 × 8.1 inches) with three disks inside. It looks like a traffic light:



c. The image is similar to the previous one, but now only the outlines are seen:



Instructional Strategies: G.GMD.4

Review vocabulary for names of solids (e.g., right prism, cylinder, cone, sphere, etc.).

Slice various solids to illustrate their cross sections. For example, cross sections of a cube can be triangles, quadrilaterals or hexagons. Rubber bands may also be stretched around a solid to show a cross section.

Cut a half-inch slit in the end of a drinking straw, and insert a cardboard cutout shape. Rotate the straw and observe the three-dimensional solid of revolution generated by the two-dimensional cutout.

Java applets on some web sites can also be used to illustrate cross sections or solids of revolution.

Encourage students to create three-dimensional models to be sliced and cardboard cutouts to be rotated. Students can also make three-dimensional models out of modeling clay and slice through them with a plastic knife.

Common Misconceptions: G.GMD.4

Some cross sections are more difficult to visualize than others. For example, it is often easier to visualize a rectangular cross section of a cube than a hexagonal cross section.

Generating solids of revolution involves motion and is difficult to visualize by merely looking at drawings.

Standard: G.MG.1

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). (★)

Connections: G.MG.1-3

Modeling activities are a good way to show connections among various branches of mathematics.

Explanations and Examples: G.MG.1

Focus on situations that require relating two- and three- dimensional objects.
Estimate measures (circumference, area, perimeter, volume) of real-world objects using comparable geometric shapes or three-dimensional objects.
Apply the properties of geometric figures to comparable real-world objects (e.g., The spokes of a wheel of a bicycle are equal lengths because they represent the radii of a circle).

Students may us simulation software and modeling software to explore which model best describes a set of data or situation.

Examples:

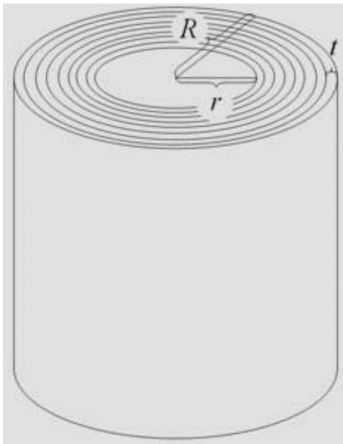
How can you model objects in your classroom as geometric shapes?

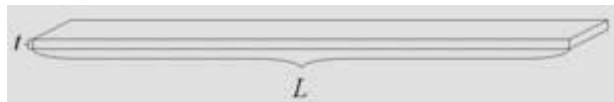
Picture a roll of toilet paper; assume that the paper in the roll is very tightly rolled. Assuming that the paper in the roll is very thin, find a relationship between the thickness of the paper, the inner and outer radii of the roll, and the length of the paper in the roll.

Express your answer as an algebraic formula involving the four listed variables.

The purpose of this task is to engage students in geometric modeling, and in particular to deduce algebraic relationships between variables stemming from geometric constraints. The modeling process is a challenging one, and will likely elicit a variety of attempts from the students. Teachers should expect to spend time guiding students away from overly complicated models. Similarly, the task presents one solution, but alternatives abound: For example, students could imagine slicing the roll along a radius, unraveling the cross-section into a sequence of trapezoids whose area can be computed.

Solution:





We begin by labeling the variables, for which the above diagrams may be useful. Let t denote the thickness of the paper, let r denote the inner radius, let R denote the outer radius and let L denote the length of the paper, all measured in inches. We now consider the area A, measured in square inches, of the annular cross-section displayed at the top of the first image, consisting of concentric circles. Namely, we see that this area can be expressed in two ways: First, since this area is the area of the circle of radius R minus the area of the circle of radius r, we learn that $A = \pi(R2 - r2)$.

Second, if the paper were unrolled, laid on a (very long) table and viewed from the side, we would see a very long thin rectangle. When the paper is rolled up, this rectangle is distorted, but -- assuming r is large in comparison to t -- the area of the distorted rectangle is nearly identical to that of the flat one. As in the second figure, the formula for the area of a rectangle now gives $A = t \cdot L$.

Comparing the two formulas for A, we find that the four variables are related by: $t \cdot L = \pi(R2 - r2)$.

Instructional Strategies: G.MG.1-3

- Genuine mathematical modeling typically involves more than one conceptual category. For example, modeling a herd of wild animals may involve geometry, measurement, proportional reasoning, estimation, probability and statistics, functions, and algebra. It would be somewhat misleading to try to teach a unit with the title of “modeling with geometry.” Instead, these standards can be woven into other content clusters.

A challenge for teaching modeling is finding problems that are interesting and relevant to high school students and, at the same time, solvable with the mathematical tools at the students’ disposal. The resources listed below are a beginning for addressing this difficulty.

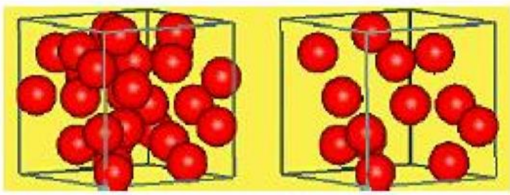
Common Misconceptions: G.MG.1-3

- When students ask to see “useful” mathematics, what they often mean is, “Show me how to use this mathematical concept or skill to solve the homework problems.” Mathematical modeling, on the other hand, involves solving problems in which the path to the solution is not obvious. Geometry may be one of several tools that can be used.

Standard: G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). (★)

Explanations and Examples: G.MG.1

- Decide whether it is best to calculate or estimate the area or volume of a geometric figure and perform the calculation or estimation.
- Students may us simulation software and modeling software to explore which model best describes a set of data or situation.
- Examples:
- Wichita, Kansas has 344,234 people within 165.9 square miles. What is Wichita's population density?
- Consider the two boxes below. Each box has the same volume. If each ball has the same mass, which box would weight more? Why



Block I
Mass = 79.4 grams
Volume = 29.8 cubic cm

Block II
Mass = 25.4 grams
Volume = 29.8 cubic cm

- A King Size waterbed has the following dimensions 72 in. X 84 in. X 9.5in. It takes 240.7 gallons of water to fill it which would weigh 2071 pounds. What is the weight of a cubic foot of water?

Standard: G.MG.3

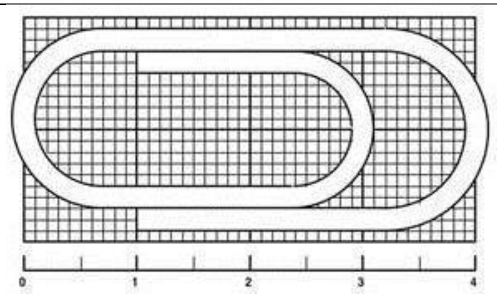
Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). (★)

Explanations and Examples: G.MG.3

- Create a visual representation of a design problem and solve using a geometric model (graph, equation, table, formula).
- Interpret the results and make conclusions based on the geometric model.
- Students may us simulation software and modeling software to explore which model best describes a set of data or situation.

Examples:

- Given one geometric solid, design a different geometric solid that will hold the same amount of substance (e.g., a cone to a prism).
- This paper clip is just over 4 cm long.



- How many paper clips like this may be made from a straight piece of wire 10 meters long?

In this task, a typographic grid system serves as the background for a standard paper clip. A metric measurement scale is drawn across the bottom of the grid and the paper clip extends in both directions slightly beyond the grid. Students are given the approximate length of the paper clip and determine the number of like paper clips made from a given length of wire. Extending the paper clip beyond the grid provides an opportunity to include an estimation component in the problem. In the interest of open-ended problem solving, no scaffolding or additional questions are posed in this task. The paper clip modeled in this problem is an actual large standard paper clip.

Suggested Big Idea	**Unit 6 Coordinate Plane Geometry**			
Content Emphasis Cluster	**Use coordinates to prove simple geometric theorems algebraically**			
Mathematical Practices	**MP.1 Make sense of problems and persevere in solving them.** **MP.2 Reason abstractly and quantitatively.** **MP.4 Model with mathematics.** **MP.7 Look for and make use of structure.** **MP.8 Look for and express regularity in repeated reasoning.**			
Common Assessment	End of Unit Assessment			
Graduate Competency	Prepared graduates make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics			
CCSS Priority Standards	**Cross-Content Connections**	**Writing Focus**	**Language/Vocabulary**	**Misconceptions**
GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2). GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio. GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula	Literacy Connections RST.6-8.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics. RST.6-8.5 Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic. RST.6-8.7 Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table). RST.6-8.8	Writing Connection WHST.6-8.2 Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. z. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. aa. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples.	Academic Vocabulary- Slope, Parallel, Perpendicular, Distance, Bisect, Directed Line Segment Technical Vocabulary- Normal Segment to a Line L.6-8.6 Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression. L.6-8.4 Determine or clarify the meaning of unknown and multiple-meaning words and phrases choosing	GPE.B.4-7 Students may claim that a vertical line has infinite slopes. This suggests that infinity is a number. Since applying the slope formula to a vertical line leads to division by zero, we say that the slope of a vertical line is undefined. Also, the slope of a horizontal line is 0. Students often say that the slope of vertical and/or horizontal lines is “no slope,” which is incorrect.

	Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.	bb. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. cc. Use precise language and domain-specific vocabulary to inform about or explain the topic. dd. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented. WHST.6-8.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	flexibly from a range of strategies.	

Unit 6	Rectangular and Triangular Regions Parallel and Perpendicular Lines	Length of Unit	Est: 24 Days (total for unit 6 and 7) 3/27/17-continue to unit 7
Standards of Mathematical Practices	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning		
Content Standards (Priority Standards underlined)	Use coordinates to prove simple geometric theorems algebraically GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2). GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio. GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.		

Terminology:	
Academic Vocabulary *Cross discipline language-*	Slope, Parallel, Perpendicular, Distance, Bisect, Directed Line Segment
Technical Vocabulary *Discipline-specific language*	**Normal Segment to a Line** A line segment with one endpoint on a line and perpendicular to the line is called a normal segment to the line.
Assessments	**End of Unit Common Assessment on Schoolcity:** • Scanned into School City or students take the assessment online • Should be in addition to individually developed formative assessments

Core Lessons and Notes

Lessons as suggested from Engage NY Module 4 (click links to follow)

Topic A: Rectangular and Triangular Regions Defined by Inequalities (G-GPE.B.7)

- *Lesson 1: Searching a Region in the Plane
- Lesson 2: Finding Systems of Inequalities That Describe Triangular and Rectangular Regions
- Lesson 3: Lines That Pass Through Regions
- Lesson 4: Designing a Search Robot to Find a Beacon

Topic B: Perpendicular and Parallel Lines in the Cartesian Plane (G-GPE.B.4, G-GPE.B.5)

- *Lesson 5: Criterion for Perpendicularity
- Lesson 6: Segments That Meet at Right Angles
- Lesson 7: Equations for Lines Using Normal Segments
- Lesson 8: Parallel and Perpendicular Lines.

Optional Materials and Testing Items from New York Engage

* Lessons recommended by the geometry team

- *Lessons listed on the left are OPTIONAL resources from New York Engage. You may choose to use these as a basis or supplement to your own materials.*
- ***District unit assessment comes after unit 7***
- In this unit students begin to use coordinates to formalize a method to solve geometry or algebra problems
- Lesson 1 uses robots to motivate the need for knowledge to work in the coordinate plane
- Lesson 2 deals with inequalities used to enclose a shape – review of graphing lines is necessary for success
- Lessons 1-4 use y=mx+b, Lessons 5 onward also uses ax+by=c
- Lessons 5-7 set up testing perpendicular lines in a way that would generalize to testing vectors in higher level math courses

Materials	Graph Paper, Graphing Calculator, Wolfram Alpha Software, Geometer's Sketchpad Software

Unit 6 Parallel and Perpendicular Lines

Topic A – Rectangular and Triangular Regions Defined by Inequalities

The module opens with a modeling challenge (G-MG.A.1, G-MG.A.3) that re-occurs throughout the lessons. Students use coordinate geometry to program the motion of a robot bound in a polygonal region (a room) of the plane. MP.4 is highlighted throughout this module as students transition from the verbal tasks to determining how to use coordinate geometry, algebra, and graphical thinking to complete the task. The modeling task varies in each lesson as students define regions, constrain motion along segments, rotate motion, and move through a real-world task of programming a robot. While this robot moves at a constant speed and its motion is very basic, it allows students to see the usefulness of the concepts taught in this module and put them in context. In Lesson 1 students use the distance formula and previous knowledge of angles to program a robot to search a plane. Students impose a coordinate system and describe the movement of the robot in terms of line segments and points. In Lesson 2, students graph inequalities and discover that a rectangular or triangular region (G-GPE.B.7) in the plane can be defined by a system of algebraic inequalities (A-REI.D.12). In Lesson 3, students study lines that cut through these previously described regions. Students are given two points in the plane and a region and determine whether a line through those points meets the region. If it does, they describe the intersection as a segment and name the endpoints. Topic A ends with Lesson 4, where students return to programming the robot while constraining motion along line segments within the region (G-GPE.B.7, A-REI.C.6) and rotating a segment 90° clockwise or counterclockwise about an endpoint (G-MG.A.1, G-MG.A.3). Revisiting A-REI.C.6 (solving systems of linear equations in two variables) and A-REI.D.12 (graphing linear inequalities in two variables and the solution sets of a system of linear inequalities) shows the coherence between algebra and geometry

Topic B – Perpendicular and Parallel Lines in the Cartesian Plane

The challenge of programming robot motion along segments parallel or perpendicular to a given segment leads to an analysis of slopes of parallel and perpendicular lines and the need to prove results about these quantities (G-GPE.B.5). MP.3 is highlighted in this topic as students engage in proving criteria and then extending that knowledge to reason about lines and segments. This work highlights the role of the converse of the Pythagorean theorem in the identification of perpendicular directions of motion (G-GPE.B.4). In Lesson 5, students explain the connection between the Pythagorean theorem and the criterion for perpendicularity studied in Module 2 (G-GPE.B.4). Lesson 6 extends that study by generalizing the criterion for perpendicularity to any two segments and applying this criterion to determine if segments are perpendicular. In Lesson 7, students learn a new format for a line, $aa1xx + aa2yy = cc$, and recognize the segment from (0, 0) to $(aa1, aa2)$ as normal with a slope of $- aa2\ aa1$. Lesson 8 concludes Topic B when students recognize parallel and perpendicular lines from their slopes and create equations for parallel and perpendicular lines. The criterion for parallel and perpendicular lines and the work from this topic with the distance formula will be extended in the last two topics of this module as students use these foundations to determine perimeter and area of polygonal regions in the coordinate plane defined by systems of inequalities. Additionally, students will study the proportionality of segments formed by diagonals of polygons.

Resources and Notes by Standard (adapted from KATM Flip Books)

Standard: G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

Connections: G.GPE.4-7

Rates of change and graphs of linear equations were studied in Grade 8 and generalized in the Functions and Geometry Conceptual Categories in high school. Therefore, an alternative way to define the slope of a line is to call it the tangent of an angle of inclination of the line.

Explanations and Examples: G.GPE.4

Represent the vertices of a figure in the coordinate plane using variables.

Use coordinates to prove or disprove a claim about a figure.

For example: use slope to determine if sides are parallel, intersecting, or perpendicular; use the distance formula to determine if sides are congruent or to decide if appoint is inside a circle, outside a circle, or on the circle; use the midpoint formula or the distance formula to decide if a side has been bisected.

Students may use geometric simulation software to model figures and prove simple geometric theorems.

Examples:

Use slope and distance formula to verify the polygon formed by connecting the points (-3, -2), (5, 3), (9, 9), (1, 4) is a parallelogram.

Prove or disprove that triangle ABC with coordinates A (-1, 2), B (1, 5), C (-2, 7) is an isosceles right triangle.

Take a picture or find a picture which includes a polygon. Overlay the picture on a coordinate plane (manually or electronically). Determine the coordinates of the vertices. Classify the polygon. Use the coordinates to justify the classification.

Instructional Strategies: G.GPE.4-7

Review the concept of slope as the rate of change of the y-coordinate with respect to the x-coordinate for a point moving along a line, and derive the slope formula.

Use similar triangles to show that every nonvertical line has a constant slope.

Review the point-slope, slope-intercept and standard forms for equations of lines.

Investigate pairs of lines that are known to be parallel or perpendicular to each other and discover that their slopes are either equal or have a product of –1, respectively.

Pay special attention to the slope of a line and its applications in analyzing properties of lines.

Allow adequate time for students to become familiar with slopes and equations of lines and methods of computing them.

- Use slopes and the Euclidean distance formula to solve problems about figures in the coordinate plane such as:
- Given three points, are they vertices of an isosceles, equilateral, or right triangle?
- Given four points, are they vertices of a parallelogram, a rectangle, a rhombus, or a square?
- Given the equation of a circle and a point, does the point lie outside, inside, or on the circle?

- Given the equation of a circle and a point on it, find an equation of the line tangent to the circle at that point.
- Given a line and a point not on it, find an equation of the line through the point that is parallel to the given line.
- Given a line and a point not on it, find an equation of the line through the point that is perpendicular to the given line.
- Given the equations of two non-parallel lines, find their point of intersection.

Given two points, use the distance formula to find the coordinates of the point halfway between them. Generalize this for two arbitrary points to derive the midpoint formula.

Use linear interpolation to generalize the midpoint formula and find the point that partitions a line segment in any specified ratio.

Given the vertices of a triangle or a parallelogram, find the equation of a line containing the altitude to a specified base and the point of intersection of the altitude and the base. Use the distance formula to find the length of that altitude and base, and then compute the area of the figure.

Common Misconceptions: G.GPE.4-7

Students may claim that a vertical line has infinite slopes. This suggests that infinity is a number. Since applying the slope formula to a vertical line leads to division by zero, we say that the slope of a vertical line is undefined. Also, the slope of a horizontal line is 0. Students often say that the slope of vertical and/or horizontal lines is "no slope," which is incorrect.

Standard: G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*

Connections: G.GPE.4-7

Rates of change and graphs of linear equations were studied in Grade 8 and generalized in the Functions and Geometry Conceptual Categories in high school. Therefore, an alternative way to define the slope of a line is to call it the tangent of an angle of inclination of the line.

Explanations and Examples: G.GPE.4

Represent the vertices of a figure in the coordinate plane using variables.
Use coordinates to prove or disprove a claim about a figure.
For example: use slope to determine if sides are parallel, intersecting, or perpendicular; use the distance formula to determine if sides are congruent or to decide if appoint is inside a circle, outside a circle, or on the circle; use the midpoint formula or the distance formula to decide if a side has been bisected.
Students may use geometric simulation software to model figures and prove simple geometric theorems.

Examples:

Use slope and distance formula to verify the polygon formed by connecting the points (-3, -2), (5, 3), (9, 9), (1, 4) is a parallelogram.

Prove or disprove that triangle ABC with coordinates A (-1, 2), B (1, 5), C (-2, 7) is an isosceles right triangle.

Take a picture or find a picture which includes a polygon. Overlay the picture on a coordinate plane (manually or electronically). Determine the coordinates of the vertices. Classify the polygon. Use the coordinates to justify the classification.

Instructional Strategies: G.GPE.4-7

Review the concept of slope as the rate of change of the y-coordinate with respect to the x-coordinate for a point moving along a line, and derive the slope formula.

Use similar triangles to show that every nonvertical line has a constant slope.

Review the point-slope, slope-intercept and standard forms for equations of lines.

Investigate pairs of lines that are known to be parallel or perpendicular to each other and discover that their slopes are either equal or have a product of –1, respectively. Pay special attention to the slope of a line and its applications in analyzing properties of lines.

Allow adequate time for students to become familiar with slopes and equations of lines and methods of computing them.

Use slopes and the Euclidean distance formula to solve problems about figures in the coordinate plane such as:

- Given three points, are they vertices of an isosceles, equilateral, or right triangle?
- Given four points, are they vertices of a parallelogram, a rectangle, a rhombus, or a square?
- Given the equation of a circle and a point, does the point lie outside, inside, or on the circle?
- Given the equation of a circle and a point on it, find an equation of the line tangent to the circle at that point.
- Given a line and a point not on it, find an equation of the line through the point that is parallel to the given line.
- Given a line and a point not on it, find an equation of the line through the point that is perpendicular to the given line.
- Given the equations of two non-parallel lines, find their point of intersection.

Given two points, use the distance formula to find the coordinates of the point halfway between them. Generalize this for two arbitrary points to derive the midpoint formula.

Use linear interpolation to generalize the midpoint formula and find the point that partitions a line segment in any specified ratio.

Given the vertices of a triangle or a parallelogram, find the equation of a line containing the altitude to a specified base and the point of intersection of the altitude and the base. Use the distance formula to find the length of that altitude and base, and then compute the area of the figure.

Common Misconceptions: G.GPE.4-7

Students may claim that a vertical line has infinite slopes. This suggests that infinity is a number. Since applying the slope formula to a vertical line leads to division by zero, we say that the slope of a vertical line is undefined. Also, the slope of a horizontal line is 0. Students often say that the slope of vertical and/or horizontal lines is “no slope,” which is incorrect.

Standard: G.GPE.5

Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Explanations and Examples: G.GPE.5

Relate work on parallel lines to standard A.REI.5 involving systems of equations having no solution or infinitely many solutions.

Lines can be horizontal, vertical or neither.

Prove that the slopes of parallel lines are equal.

Prove that the product of the slopes of perpendicular lines is -1.

Write the equation of a line parallel or perpendicular to a given line, passing through a given point.

Students may use a variety of different methods to construct a parallel or perpendicular line to a given line and calculate the slopes to compare relationships.

Examples:

Find the equation of a line perpendicular to 3x + 5y + 15 through the point (-3. 2).

Find an equation of a line perpendicular to y = 3x - 4 that passes through (3, 4).

Verify that the distance between two parallel lines is constant. Justify your answer.

Instructional Strategies:

Allow students to explore and make conjectures about relationships between lines and segments using a variety of methods.

Discuss the role of algebra in providing a precise means of representing a visual image.

Standard: G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Explanations and Examples: G.GPE.6

Students may use geometric simulation software to model figures or line segments.

Examples:

- Given *A*(3, 2) and *B*(6, 11),
  - Find the point that divides the line segment AB two-thirds of the way from *A* to *B*.

    Solution:
    The point two-thirds of the way from *A* to *B* has an *x*-coordinate two-thirds of the way from 3 to 6 and a *y*-coordinate two-thirds of the way from 2 to 11. So (5, 8) is the point that is two-thirds from point *A* to *B*.

  - Find the midpoint of the line segment *AB*.

- For the line segment whose endpoints are (0, 0) and (4, 3), find the point that partitions the segment into a ratio of 3 to 2.

  Solution:
  $x = \dfrac{(2 \bullet 0) + (3 \bullet 4)}{(3 + 2)} = \dfrac{12}{5}$ $\quad y = \dfrac{(2 \bullet 0) + (3 \bullet 3)}{(3 + 2)} = \dfrac{9}{5}$, so the point is $(\dfrac{12}{5}, \dfrac{9}{5})$

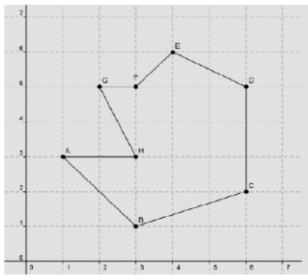
Standard: G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. (★)

Explanations and Examples: G.GPE.7

This standard provides practice with the distance formula and its connection with the Pythagorean Theorem. Use the coordinates of the vertices of a polygon graphed in the coordinate plane and use the distance formula to compute the perimeter.
Use the coordinates of the vertices of triangles and rectangles graphed in the coordinate plane to compute the area. Students may use geometric simulation software to model figures.

Examples:

- Find the perimeter and area of a rectangle with vertices at C (-1, 1), D (3, 4), E (6, 0), F (2, -3). Round your answer to the nearest hundredth when necessary.
- Find the area and perimeter for the figure below.



- Calculate the area of triangle ABC with altitude CD, given A (-4, -2), B (8, 7), C (1, 8) and D (4, 4).

Instructional Strategies:

Graph polygons using coordinates. Explore perimeter and area of a variety of polygons, including convex, concave, and irregularly shaped polygons.

Given a triangle, use slopes to verify that the length and height are perpendicular. Find the area.

Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth. Select a shape (yard, parking lot, school, etc.). Use the tool menu to overlay a coordinate grid. Use coordinates to find the perimeter and area of the shape selected. Determine the scale factor of the picture as related to the actual real-life view. Then find the actual perimeter and area.

Suggested Big Idea	**Unit 7 Coordinate Plane Geometry**			
Content Emphasis Cluster	**Use coordinates to prove simple geometric theorems algebraically**			
Mathematical Practices	**MP.1 Make sense of problems and persevere in solving them.** **MP.2 Reason abstractly and quantitatively.** **MP.4 Model with mathematics.** **MP.7 Look for and make use of structure.** **MP.8 Look for and express regularity in repeated reasoning.**			
Common Assessment	End of Unit Assessment			
Graduate Competency	Prepared graduates make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics			
CCSS Priority Standards	**Cross-Content Connections**	**Writing Focus**	**Language/Vocabulary**	**Misconceptions**
GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio. GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula	Literacy Connections RST.6-8.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics. RST.6-8.5 Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic. RST.6-8.7 Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table). RST.6-8.8 Distinguish among facts, reasoned judgment based	Writing Connection WHST.6-8.2 Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. ee. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. ff. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. gg. Use appropriate and varied transitions to create cohesion and	Academic Vocabulary-Slope, Parallel, Perpendicular, Bisect Technical Vocabulary-Normal Segment to a Line L.6-8.6 Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression. L.6-8.4 Determine or clarify the meaning of unknown and multiple-meaning words and phrases choosing flexibly from a range of strategies.	GPE.B.4-7 Students may claim that a vertical line has infinite slopes. This suggests that infinity is a number. Since applying the slope formula to a vertical line leads to division by zero, we say that the slope of a vertical line is undefined. Also, the slope of a horizontal line is 0. Students often say that the slope of vertical and/or horizontal lines is "no slope," which is incorrect.

	on research findings, and speculation in a text.	clarify the relationships among ideas and concepts. hh. Use precise language and domain-specific vocabulary to inform about or explain the topic. ii. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented. WHST.6-8.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.		

Unit 7	Perimeter and Area of Polygons	Length of Unit	Est: 24 days (total for unit 6 and 7) Continued from unit 6 - 4/27/17 Unit assessment scanned by 5/8/17
Standards of Mathematical Practices	**MP.1** Make sense of problems and persevere in solving them. **MP.2** Reason abstractly and quantitatively. **MP.4** Model with mathematics. **MP.7** Look for and make use of structure. **MP.8** Look for and express regularity in repeated reasoning.		
Content Standards (Priority Standards underlined)	**<u>Use coordinates to prove simple geometric theorems algebraically</u>** GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2). **<u>GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.</u>** **<u>GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula</u>** **<u>Use coordinates to prove simple geometric theorems algebraically</u>** **<u>GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).</u>**		

Terminology:	
Academic Vocabulary *Cross discipline language-*	Slope, Parallel, Perpendicular, Distance, Bisect
Technical Vocabulary *Discipline-specific language*	**Normal Segment to a Line** (A line segment with one endpoint on a line and perpendicular to the line is called a normal segment to the line.)
Assessments	**End of Unit Common Assessment on Schoolcity:** • Scanned into School City or students take the assessment online • Should be in addition to individually developed formative assessments

Core Lessons and Notes

Lessons as suggested from Engage NY Module 4 (click links to follow)

Topic C: Perimeters and Areas of Polygonal Regions in the Cartesian Plane (G-GPE.B.7)

- *Lesson 9: Perimeter and Area of Triangles in the Cartesian Plane
- Lesson 10: Perimeter and Area of Polygonal Regions in the Cartesian Plane
- Lesson 11: Perimeters and Areas of Polygonal Regions Defined by Systems of Inequalities

Topic D: Partitioning and Extending Segments and Parameterization of Lines (G-GPE.B.4, G-GPE.B.6)

- *Lesson 12: Dividing Segments Proportionately
- Lesson 13: Analytic Proofs of Theorems Previously Proved by Synthetic Means
- Lesson 14: Motion Along a Line—Search Robots Again (Optional)
- Lesson 15: The Distance from a Point to a Line

Optional Materials and Testing Items from New York Engage

* Lessons recommended by the geometry team

- *Lessons listed on the left are OPTIONAL resources from New York Engage. You may choose to use these as a basis or supplement to your own materials.*
- ***District unit assessment from unit 6 and 7 comes after this unit***
- Lessons 9-10 construct a generalized method for finding the area of figures in the plane. It turns out that this method is a simplified form of Green's theorem, a calculus method for finding area.
- Lesson 11 combines unit 6 and the previous two lessons
- Lesson 12 is about dividing line segments – understanding ratios is essential. This may take several lessons to fully convey
- Lesson 13 proves previously discussed theorems analytically (in the coordinate plane)
- Lesson 14 is an optional extension to line equations that builds the idea of parametric equations – separate equations to govern the change in the x and y values

Materials	Graph Paper, Graphing Calculator, Wolfram Alpha Software, Geometer's Sketchpad Software

Unit 7 Perimeter and Area of Polygons

Topic C – Perimeters and Areas of Polygonal Regions in the Cartesian Plane

Lesson 9 begins Topic C with students finding the perimeter of triangular regions using the distance formula and deriving the formula for the area of a triangle with vertices (0, 0),$(x1,\ y1)$, $(x2,\ y2)$ as $A = 1/2\ |x1y2 - x2y1|$ (G-GPE.B.7). Students are introduced to the "shoelace" formula for area and understand that this formula is useful because only the coordinates of the vertices of a triangle are needed. In Lesson 10, students extend the "shoelace" formula to quadrilaterals, showing that the traditional formulas are verified with general cases of the "shoelace" formula and even extend this work to other polygons (pentagons and hexagons). Students compare the traditional formula for area and area by decomposition of figures and see that the "shoelace" formula is much more efficient in some cases. This work with the "shoelace" formula is the high school Geometry version of Green's theorem and subtly exposes students to elementary ideas of vector and integral calculus. Lesson 11 concludes this work as the regions are described by a system of inequalities. Students sketch the regions, determine points of intersection (vertices), and use the distance formula to calculate perimeter and the "shoelace" formula to determine area of these regions. Students return to the real-world application of programming a robot and extend this work to robots not just confined to straight line motion but also motion bound by regions described by inequalities and defined areas.

Topic D – Partitioning and Extending Segments and Parameterization of Lines

Topic D concludes the work of Module 4. In Lesson 12, students find midpoints of segments and points that divide segments into 3 or more equal and proportional parts. Students will also find locations on a directed line segment between two given points that partition the segment in given ratios (G-GPE.B.6). Lesson 13 requires students to show that if B' and C' cut line segment AB and line segmentAC proportionately, then the intersection of line segment BC and line segment CC' lies on the median of ABC from vertex A and connects this work to proving classical results in geometry (G-GPE.B.4). For instance, the diagonals of a parallelogram bisect one another and the medians of a triangle meet at the point 2/3 of the way from the vertex for each. Lesson 14 is an optional lesson that allows students to explore parametric equations and compare them with more familiar linear equations (G-GPE.B.6, G-MG.A.1). Parametric equations make both the x- and y- variables in an equation dependent on a third variable, usually time; for example, $f(t) = (t, 2t - 1)$ represents a function, f, with both x- and y-coordinates dependent on the independent variable, tt (time). In this lesson, parametric equations model the robot's horizontal and vertical motion over a period of time, $P = (20 + t/2\ (100), 10 + t/2\ (40))$. Introducing parametric equations in the Geometry course prepares students for higher level courses and also represents an opportunity to show coherence between functions, algebra, and coordinate geometry. Students extend their knowledge of parallel and perpendicular lines to lines given parametrically. Students complete the work of this module in Lesson 15 by deriving and applying the distance formula (G-GPE.B.4) and with the challenge of locating the point along a line closest to a given point, again given as a robot challenge.

Resources and Notes by Standard (adapted from KATM Flip Books)

Standard: G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

Connections: G.GPE.4-7

Rates of change and graphs of linear equations were studied in Grade 8 and generalized in the Functions and Geometry Conceptual Categories in high school. Therefore, an alternative way to define the slope of a line is to call it the tangent of an angle of inclination of the line.

Explanations and Examples: G.GPE.4

Represent the vertices of a figure in the coordinate plane using variables.
Use coordinates to prove or disprove a claim about a figure.
For example: use slope to determine if sides are parallel, intersecting, or perpendicular; use the distance formula to determine if sides are congruent or to decide if appoint is inside a circle, outside a circle, or on the circle; use the midpoint formula or the distance formula to decide if a side has been bisected.
Students may use geometric simulation software to model figures and prove simple geometric theorems.

Examples:

Use slope and distance formula to verify the polygon formed by connecting the points (-3, -2), (5, 3), (9, 9), (1, 4) is a parallelogram.

Prove or disprove that triangle ABC with coordinates A (-1, 2), B (1, 5), C (-2, 7) is an isosceles right triangle.

Take a picture or find a picture which includes a polygon. Overlay the picture on a coordinate plane (manually or electronically). Determine the coordinates of the vertices. Classify the polygon. Use the coordinates to justify the classification.

Instructional Strategies: G.GPE.4-7

Review the concept of slope as the rate of change of the y-coordinate with respect to the x-coordinate for a point moving along a line, and derive the slope formula.

Use similar triangles to show that every nonvertical line has a constant slope.

Review the point-slope, slope-intercept and standard forms for equations of lines.

Investigate pairs of lines that are known to be parallel or perpendicular to each other and discover that their slopes are either equal or have a product of –1, respectively.
Pay special attention to the slope of a line and its applications in analyzing properties of lines.

Allow adequate time for students to become familiar with slopes and equations of lines and methods of computing them.

Use slopes and the Euclidean distance formula to solve problems about figures in the coordinate plane such as:

- Given three points, are they vertices of an isosceles, equilateral, or right triangle?
- Given four points, are they vertices of a parallelogram, a rectangle, a rhombus, or a square?
- Given the equation of a circle and a point, does the point lie outside, inside, or on the circle?
- Given the equation of a circle and a point on it, find an equation of the line tangent to the circle at that point.
- Given a line and a point not on it, find an equation of the line through the point that is parallel to the given line.
- Given a line and a point not on it, find an equation of the line through the point that is perpendicular to the given line.
- Given the equations of two non-parallel lines, find their point of intersection.

Given two points, use the distance formula to find the coordinates of the point halfway between them. Generalize this for two arbitrary points to derive the midpoint formula.

Use linear interpolation to generalize the midpoint formula and find the point that partitions a line segment in any specified ratio.

Given the vertices of a triangle or a parallelogram, find the equation of a line containing the altitude to a specified base and the point of intersection of the altitude and the base. Use the distance formula to find the length of that altitude and base, and then compute the area of the figure.

Common Misconceptions: G.GPE.4-7

Students may claim that a vertical line has infinite slopes. This suggests that infinity is a number. Since applying the slope formula to a vertical line leads to division by zero, we say that the slope of a vertical line is undefined. Also, the slope of a horizontal line is 0. Students often say that the slope of vertical and/or horizontal lines is "no slope," which is incorrect.

Standard: G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Explanations and Examples: G.GPE.6

Students may use geometric simulation software to model figures or line segments.

Examples:

Given A(3, 2) and B(6, 11), o Find the point that divides the line segment AB two-thirds of the way from A to B.

Solution:

The point two-thirds of the way from A to B has an x-coordinate two-thirds of the way from 3 to 6 and a y-coordinate two-thirds of the way from 2 to 11. So (5, 8) is the point that is two-thirds from point A to B.

o Find the midpoint of the line segment AB.

For the line segment whose endpoints are (0, 0) and (4, 3), find the point that partitions the segment into a ratio of 3 to 2.

Solution:

$$x = \frac{(2 \bullet 0) + (3 \bullet 4)}{(3 + 2)} = \frac{12}{5} \qquad y = \frac{(2 \bullet 0) + (3 \bullet 3)}{(3 + 2)} = \frac{9}{5}\text{, so the point is } \left(\frac{12}{5}, \frac{9}{5}\right)$$

Standard: G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. (★)

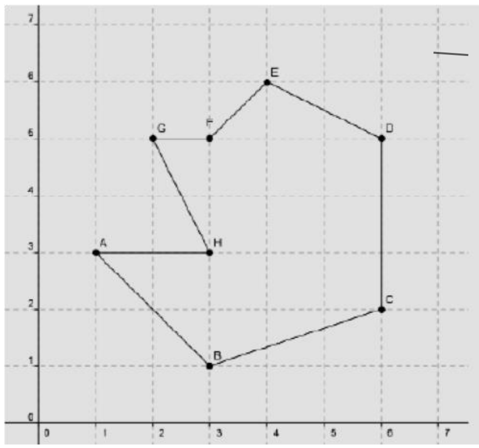
Explanations and Examples: G.GPE.7

This standard provides practice with the distance formula and its connection with the Pythagorean Theorem. Use the coordinates of the vertices of a polygon graphed in the coordinate plane and use the distance formula to compute the perimeter.
Use the coordinates of the vertices of triangles and rectangles graphed in the coordinate plane to compute the area. Students may use geometric simulation software to model figures.

Examples:

- Find the perimeter and area of a rectangle with vertices at C (-1, 1), D (3, 4), E (6, 0), F (2, -3). Round your answer to the nearest hundredth when necessary.

- Find the area and perimeter for the figure below.



- Calculate the area of triangle ABC with altitude CD, given A (-4, -2), B (8, 7), C (1, 8) and D (4, 4).

Instructional Strategies:

Graph polygons using coordinates. Explore perimeter and area of a variety of polygons, including convex, concave, and irregularly shaped polygons.

Given a triangle, use slopes to verify that the length and height are perpendicular. Find the area.

Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth. Select a shape (yard, parking lot, school, etc.). Use the tool menu to overlay a coordinate grid. Use coordinates to find the perimeter and area of the shape selected. Determine the scale factor of the picture as related to the actual real-life view. Then find the actual perimeter and area.

Suggested Big Idea	**Unit 8 Conics**			
Content Emphasis Cluster	**Use coordinates to prove simple geometric theorems algebraically** **Apply geometric concepts in modeling situations**			
Mathematical Practices	**MP.1 Make sense of problems and persevere in solving them.** **MP.3 Construct viable arguments and critique the reasoning of others.** **MP.4 Model with mathematics.** **MP.5 Use appropriate tools strategically.** **MP.6 Attend to precision.** **MP.7 Look for and make use of structure.** **MP.8 Look for and express regularity in repeated reasoning.**			
Common Assessment	End of Unit Assessment			
Graduate Competency	Prepared graduates make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics Prepared graduates use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions			
CCSS Priority Standards	**Cross-Content Connections**	**Writing Focus**	**Language/Vocabulary**	**Misconceptions**
MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	Literacy Connections RST.6-8.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics. RST.6-8.5 Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic. RST.6-8.7 Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g.,	Writing Connection WHST.6-8.2 Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. jj. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. kk. Develop the topic with relevant, well-chosen facts,	Academic Vocabulary- Circle, Diameter, Radius Technical Vocabulary- Ellipse, hyperbola, parabola, vertex, conics. L.6-8.6 Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression. L.6-8.4 Determine or clarify the meaning of unknown and multiple-meaning	MG.1 When students ask to see “useful” mathematics, what they often mean is, “Show me how to use this mathematical concept or skill to solve the homework problems.” Mathematical modeling, on the other hand, involves solving problems in which the path to the solution is not obvious. Geometry may be one of several tools that can be used.

	in a flowchart, diagram, model, graph, or table). RST.6-8.8 Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.	definitions, concrete details, quotations, or other information and examples. ll. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. mm. Use precise language and domain-specific vocabulary to inform about or explain the topic. nn. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented. WHST.6-8.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	words and phrases choosing flexibly from a range of strategies.	

Unit 8	**Conics**	**Length of Unit**	**Est: 13 Days** **5/1/17-5/17/17** **Unit assessment scanned by 5/24/17**
Standards of Mathematical Practices	**MP.1** Make sense of problems and persevere in solving them. **MP.3** Construct viable arguments and critique the reasoning of others. **MP.4** Model with mathematics. **MP.5** Use appropriate tools strategically. **MP.6** Attend to precision. **MP.7** Look for and make use of structure. **MP.8** Look for and express regularity in repeated reasoning.		
Content Standards (Priority Standards Underlined	**Translate between the geometric description and the equation for a conic section** <u>**Apply geometric concepts in modeling situations**</u> <u>**MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).</u> **Translate between the geometric description and the equation for a conic section** GPE.A.2 Derive the equation of a parabola, given a focus and a directrix. GPE.A.3 Derive the equation of ellipses and hyperbolas given the foci, using the fact that the sum or differences of distances from the foci is constant.		
Terminology			
Academic Vocabulary *Cross discipline language-*	Circle, Diameter, Radius		
Technical Vocabulary *Discipline-specific language*	**Parabola:** (G-GPE.A.2) A parabola with directrix L and focus F is the set of all points in the plane that are equidistant from the point F and line L. **Axis of symmetry of a parabola**: (G-GPE.A.2) The axis of symmetry of a parabola given by a focus point and a directrix is the perpendicular line to the directrix that passes through the focus. **Vertex of a parabola:** (G-GPE.A.2) The vertex of a parabola is the point where the axis of symmetry intersects the parabola.		
Assessments	**End of Unit Common Assessment on Schoolcity:** • Scanned into School City or students take the assessment online • Should be in addition to individually developed formative assessments		

Core Lessons and Notes	
Lessons as suggested from Engage NY ALGEBRA 2 Module 1 (click links to follow) **Topic A:** Lesson 33: Parabolas https://www.khanacademy.org/math/algebra/quadratics/graphing-quadratic-functions/e/graphing_parabolas_1 **Topic B:** https://www.khanacademy.org/math/precalculus/conics-precalc/center-and-radii-of-an-ellipse/v/conic-sections-intro-to-ellipses **Topic C:** https://www.khanacademy.org/math/precalculus/conics-precalc/intro-to-hyperbolas/v/conic-sections-intro-to-hyperbolas Optional Materials and Mid-Unit Testing Items from New York Engage and End of Unit Testing Items *Lessons recommended by the Geometry Team	• *Lessons listed on the left are OPTIONAL resources from New York Engage. You may choose to use these as a basis or supplement to your own materials.* • *Khan academy lessons are also included on the left as a resource.*
Materials	Compass and straight edge, Geometer's Sketchpad or Geogebra software, white and colored paper, markers

Unit 8 Conics

Topic A – Parabola

Students will have familiarity with the family of functions of quadratics (parabolas). The students in algebra 2 will be expected to model the locus of points at equal distance between a point (focus) and a line (directrix). They construct a parabola and understand this geometric definition of the curve. They use algebraic techniques to derive the analytic equation of the parabola. In geometry, the objective for students will have a basic understanding of the structure of the equation of a parabola in both standard and vertex form and graph it on a coordinate plane. Application problems (such as maximization of a rectangular pig pen) will help link Parabolas and Geometry concepts.

Topic B – Ellipse

Students will have a prior vocabulary of "oval" from elementary school. In geometry, the objective for students will be to recognize the equation is an ellipse, identify and describe translations from the equation, write equations from a graph, and graph from an equation.

Topic C – Hyperbola

Students will have no prior knowledge of hyperbolas. In geometry, the objective for students will be to recognize the equation is a hyperbola, identify and describe translations from the equation, write equations from a graph, and graph from an equation.

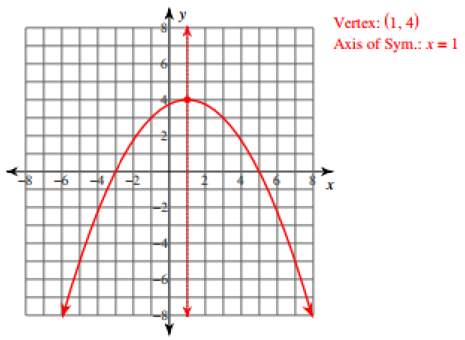
Resources and Notes by Standard (adapted from KATM Flip Books)

Standard: G.GPE.2. Translate between the geometric description and the equation for a conic section.

Explanations and Examples: G.GPE.2

Identify the vertex and axis of symmetry of each. Then sketch the graph.

$$f(x) = -\frac{1}{4}(x-1)^2 + 4$$

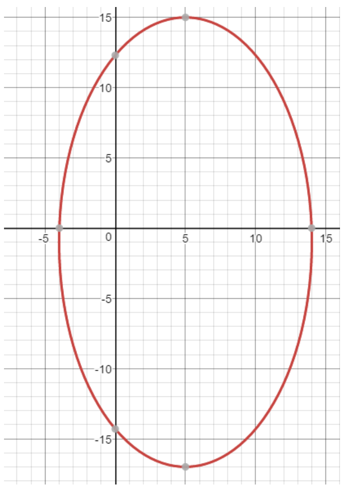


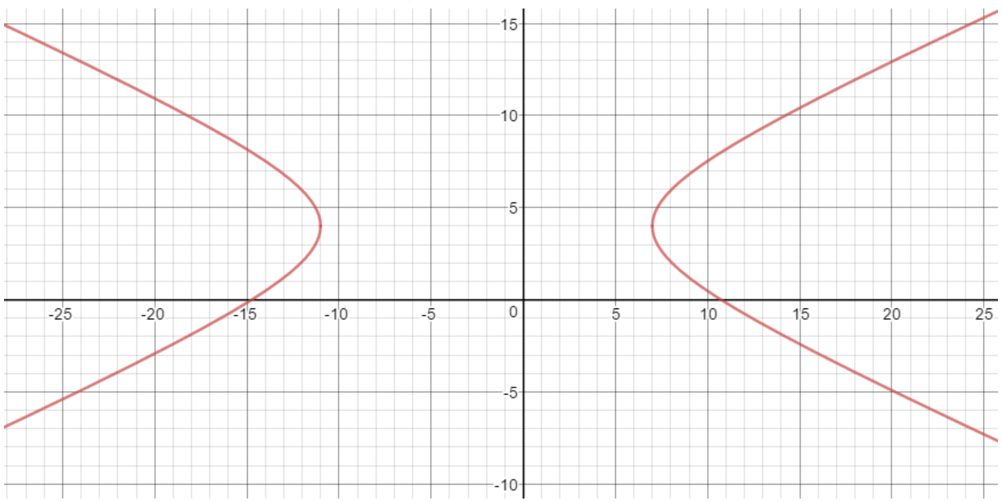
Graph the ellipse:

$$\frac{(x-5)^2}{9} + \frac{(y+1)^2}{16} = 1,$$

Graph the hyperbola:

$$\left(\frac{(x+2)}{9}\right)2 - \left(\frac{(y-4)}{4}\right)2 = 1$$





Common Misconceptions: G.GPE.4-7 Students may confuse the equations for a hyperbola and an ellipse. Focus on the operation connecting the two fractions.

Standard: G.MG.1

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). (★)

Connections: G.MG.1-3

Modeling activities are a good way to show connections among various branches of mathematics.

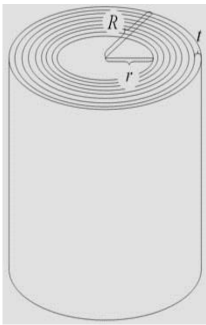
Explanations and Examples: G.MG.1

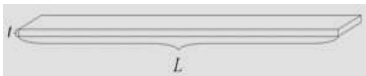
- Focus on situations that require relating two- and three- dimensional objects.
- Estimate measures (circumference, area, perimeter, volume) of real-world objects using comparable geometric shapes or three-dimensional objects.
- Apply the properties of geometric figures to comparable real-world objects (e.g., The spokes of a wheel of a bicycle are equal lengths because they represent the radii of a circle).
- Students may us simulation software and modeling software to explore which model best describes a set of data or situation.

Examples:

- How can you model objects in your classroom as geometric shapes?
- Picture a roll of toilet paper; assume that the paper in the roll is very tightly rolled. Assuming that the paper in the roll is very thin, find a relationship between the thickness of the paper, the inner and outer radii of the roll, and the length of the paper in the roll.
- Express your answer as an algebraic formula involving the four listed variables.
- The purpose of this task is to engage students in geometric modeling, and in particular to deduce algebraic relationships between variables stemming from geometric constraints. The modeling process is a challenging one, and will likely elicit a variety of attempts from the students. Teachers should expect to spend time guiding students away from overly complicated models. Similarly, the task presents one solution, but alternatives abound: For example, students could imagine slicing the roll along a radius, unraveling the cross-section into a sequence of trapezoids whose area can be computed.

Solution:





- We begin by labeling the variables, for which the above diagrams may be useful. Let t denote the thickness of the paper, let r denote the inner radius, let R denote the outer radius and let L denote the length of the paper, all measured in inches. We now consider the area A, measured in square inches, of the annular cross-section displayed at the top of the first image, consisting of concentric circles. Namely, we see that this area can be expressed in two ways: First, since this area is the area of the circle of radius R minus the area of the circle of radius r, we learn that $A = \pi(R2 - r2)$.

- Second, if the paper were unrolled, laid on a (very long) table and viewed from the side, we would see a very long thin rectangle. When the paper is rolled up, this rectangle is distorted, but -- assuming r is large in comparison to t -- the area of the distorted rectangle is nearly identical to that of the flat one. As in the second figure, the formula for the area of a rectangle now gives $A = t \cdot L$.
- Comparing the two formulas for A, we find that the four variables are related by: $t \cdot L = \pi(R2 - r2)$.

Instructional Strategies: G.MG.1-3

- Genuine mathematical modeling typically involves more than one conceptual category. For example, modeling a herd of wild animals may involve geometry, measurement, proportional reasoning, estimation, probability and statistics, functions, and algebra. It would be somewhat misleading to try to teach a unit with the title of "modeling with geometry." Instead, these standards can be woven into other content clusters.
- A challenge for teaching modeling is finding problems that are interesting and relevant to high school students and, at the same time, solvable with the mathematical tools at the students' disposal. The resources listed below are a beginning for addressing this difficulty.

Common Misconceptions: G.MG.1-3

- When students ask to see "useful" mathematics, what they often mean is, "Show me how to use this mathematical concept or skill to solve the homework problems." Mathematical modeling, on the other hand, involves solving problems in which the path to the solution is not obvious. Geometry may be one of several tools that can be used.