

GRILLAGE ANALOGY METHOD

1.0 SCOPE

This document is intended as a Design Manual for the application of Grillage Analogy Method for the Bridge Deck Analysis.

The theoretical principles has been expounded in the book of Edmund C. Hambly, "Bridge Deck Behaviour", Chapman and Hall, 1976, First Edition E & FN Spon, Second Edition, 1991, (Ref. N° 1)

The Structural modeling for the Bridge Deck behaviour as an equivalent grillage consists of a grid of longitudinal and transverse beams, following the arrangement of the main beams, diaphragms and the deck slab

These beams are bar elements, with unidirectional behaviour whose properties will be conveniently modified, to represent the continuous bidirectional element of the actual deck (Note 1)

For the deck slab a proper number of bar elements should be assigned to model the continuity of the longitudinal stresses.

We would then have, mainly, 3 types of bar elements:

- a. Slab section
- b. Beam and slab section
- c. Box sections

In this way, the equivalent grillage will be composed, essentially with these 3 types of elements

In the modeling expounded in Ref. N° 1, it has been considered the Bending Moment M_x (MF33, for the SAP), Shear Force S_x (FC22) and the Torsional Moment T_x (MT), (Note 2), which are the principal effects in the grillage for the more important loading cases (gravitational) but require some refinements to satisfy the force equilibrium and displacement compatibility equations in certain other cases.

These special aspects will be treated in section 7.0 of final remarks

For the application of this manual, it has been used the SAP 2000 software (CSI Computer and Structures, Inc)

2.0 PLAN GEOMETRY OF GRILLAGE

We have 3 types of plane meshes:

1. Rectangular or orthogonal decks, where diaphragm beams are perpendicular to the main beams and the deck slab is rectangular
2. Skew decks, where diaphragm beams at the supports line, are skew to the main beams and the deck slab is a parallelogram

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Support line diaphragms would be unavoidable skew, so that in the case of important torsional moment should occur, the section of the diaphragm should be reduced or eliminate the continuity with the deck slab.

Interior diaphragms should be, preferably, orthogonal with the main beams, because in this way, we get the best lateral distribution of eccentric loads and produce the lesser torsional moments.

Also unavoidably, slab modeling will produce triangular and trapezoidal slab elements. In such cases a discretionary criteria should prevail, to determine the equivalent width of the slab elements

3. Curved deck, when main beams are curved in plan and the diaphragms on the support line could be perpendicular or skew with respect to the main beams.

In the curved decks, diaphragm in the support line should be, preferably radial to the curvature of the deck, so as to reduce torsional moments in the diaphragm. Also as in the case of skew decks, should torsional moments of importance occur, diaphragm section should be reduced or eliminate the deck slab continuity.

Also unavoidably, slab modeling will produce triangular and trapezoidal slab elements. In such cases a discretionary criteria should prevail to determine the equivalent width of the slab.

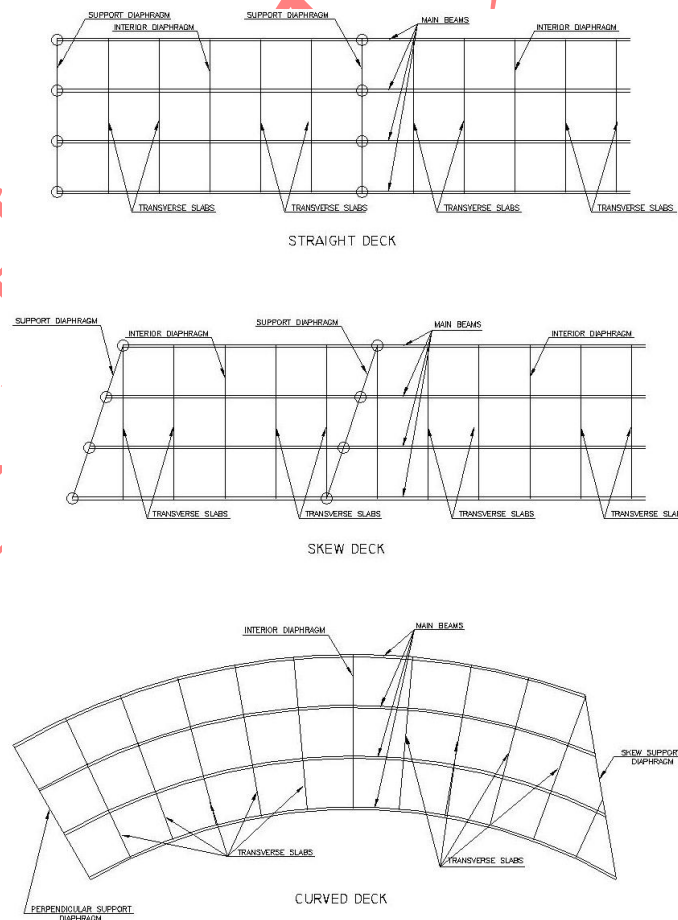


Fig. N° 1: Deck types

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3.0 GEOMETRY IN ELEVATION OF THE ELEMENTS

For beams with significant varying depth, it should be taken into account the curved shape of the centroidal line, so as to consider the effect of arching for this type of beams

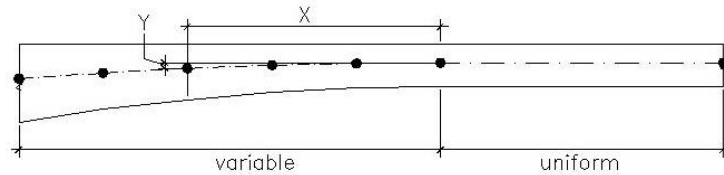


Fig. N° 2: Beams with varying depth

4.0 READJUSTMENT OF THE GRILLAGE GEOMETRY

In modeling with SAP, the element axis coincide with the centroidal axis of the beams, so when partitioning the transverse section of the deck, asymmetrical sections displaces from the actual position.

Also, transverse slabs element, will lie in a different vertical position to the connection with the longitudinal beam, as well to the diaphragm beam.

In Ref. N° 1, use is made to a refinement of the grillage model called "downstand grillage", inserting short and very stiff elements with 0 mass (called rigid arm), to become into space grillage.

In SAP with a command "insertion point", automatically introduces these elements, to move the element from one position to another

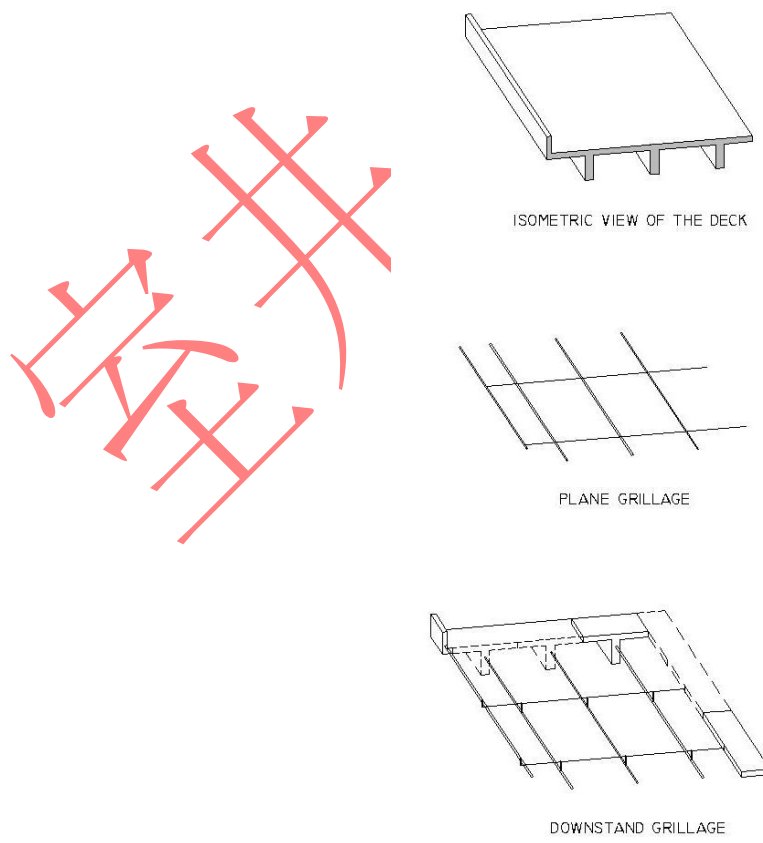


Fig. N° 3: Grillage models (Ref. N° 1)

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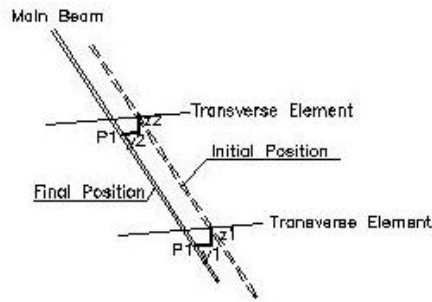


Fig. N° 4: Readjustment of the element geometry

With the SAP option of extruded view, readjustment of the elements geometry could be displayed.

5.0 ACTING FORCES AND MOMENTS ON THE SECTIONS

In the following tables the different actions acting on the element sections are illustrated.

It has already been said that the basic model of Ref. N° 1, only the effects of the Bending Moments MF33, Torsional Moments MT and Shearing Forces FC22 are considered.

In the model to be used here, the total of six degrees of freedom of the bar element are considered.

From these tables, we can check that for orthogonal sections, the Axial Force FA in the main beam interacts with the shearing force FC33 of the transverse element.

Bending Moments MF33 in the main beam interacts with the torsional moment MT of the transverse element

Bending Moment MF22 in the main beam interacts with the bending moment MF22 of the transverse element.

Shearing Force FC22 in the main beam interacts with the shearing Force FC22 of the transverse element.

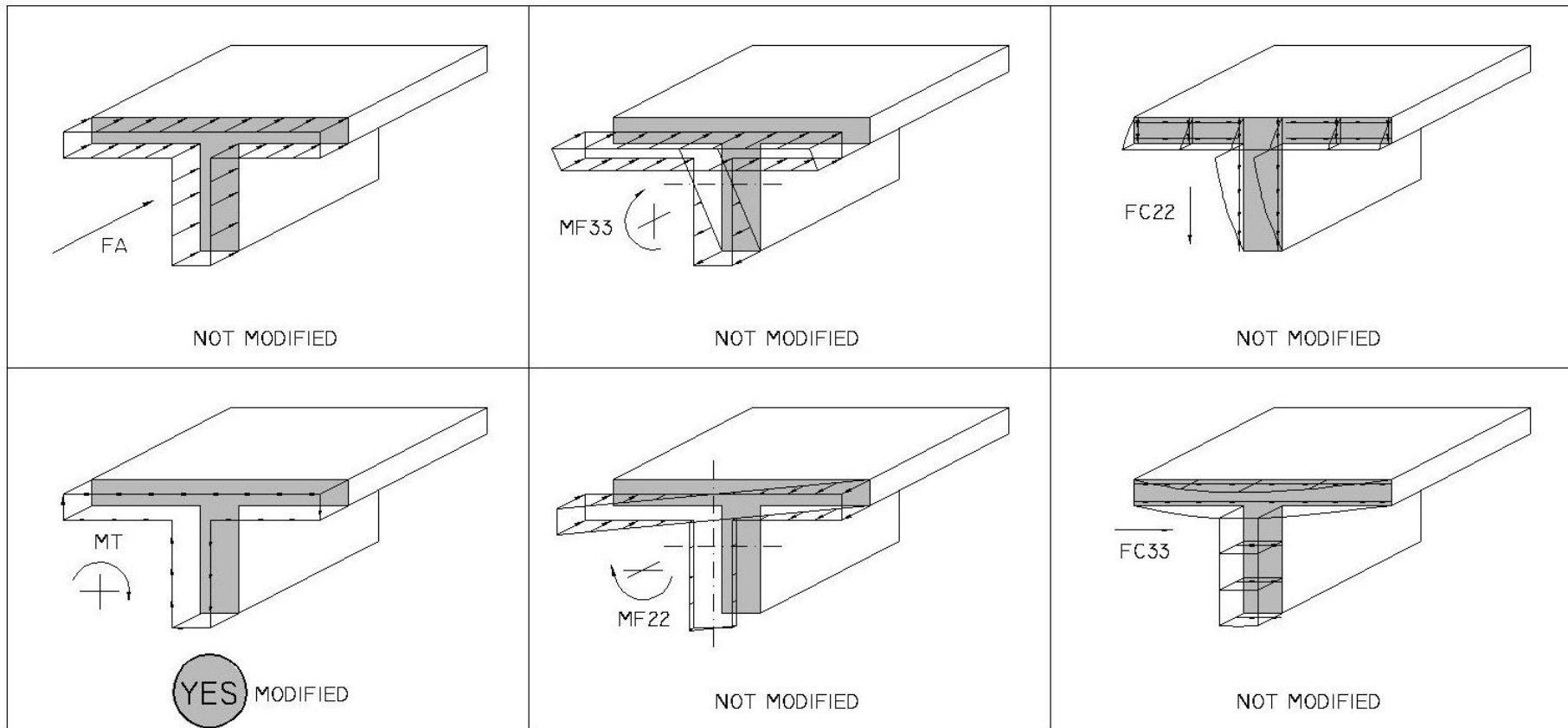
Shearing Force FC33 in the main beam interacts with the axial force FA of the transverse element

Torsional Moment MT in the main beam interacts with the bending moment MF33 of the transverse element

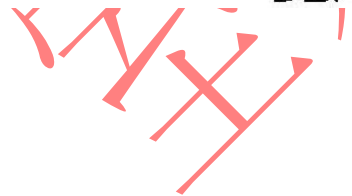
Distribution of stresses due to the action on the section of the element are shown and it is indicated if it is modified or not for the equivalent grillage model

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LONGITUDINAL ELEMENTS MAIN BEAMS

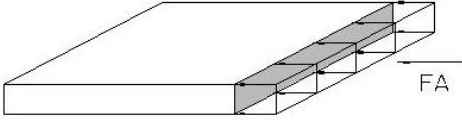
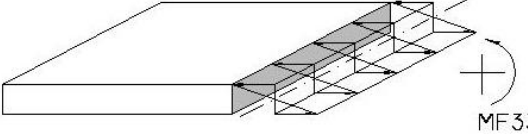
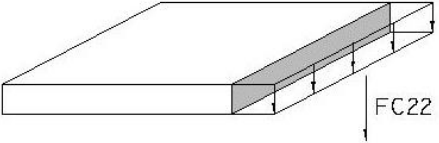
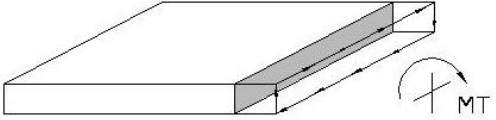
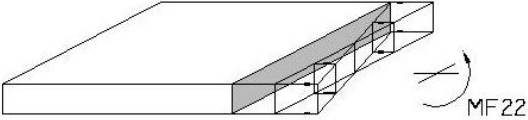
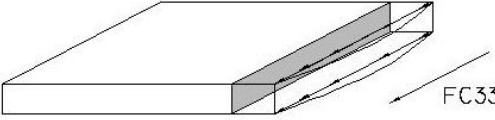


BEAM AND SLAB DECK (1/3)

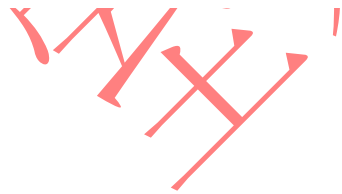


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TRANSVERSE ELEMENTS TRANSVERSE SLABS

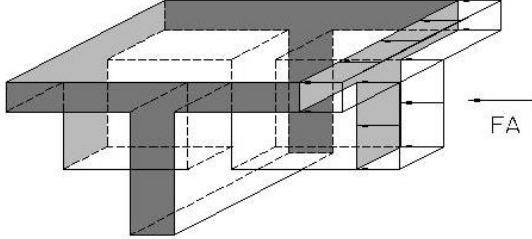
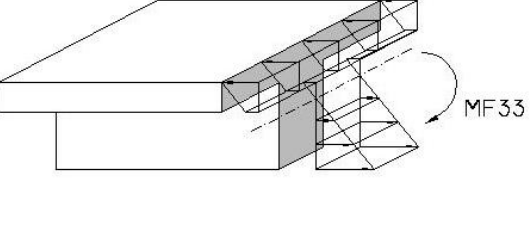
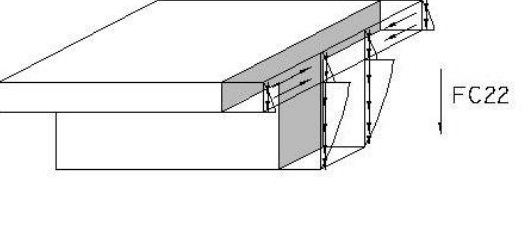
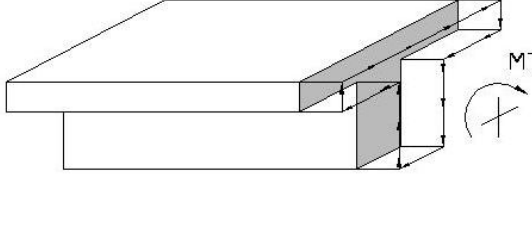
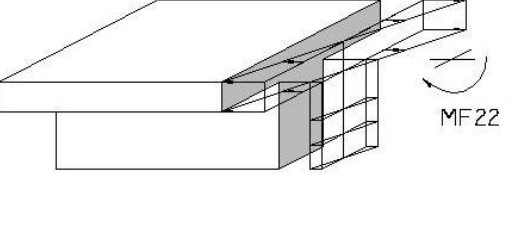
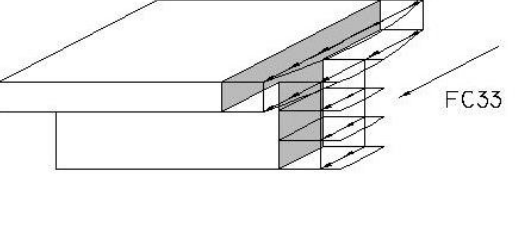
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BEAM AND SLAB DECK (2/3)

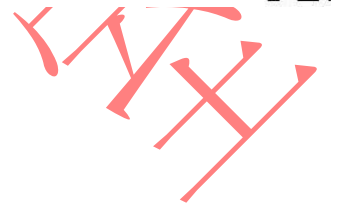


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TRANSVERSE ELEMENTS DIAPHRAGM BEAMS

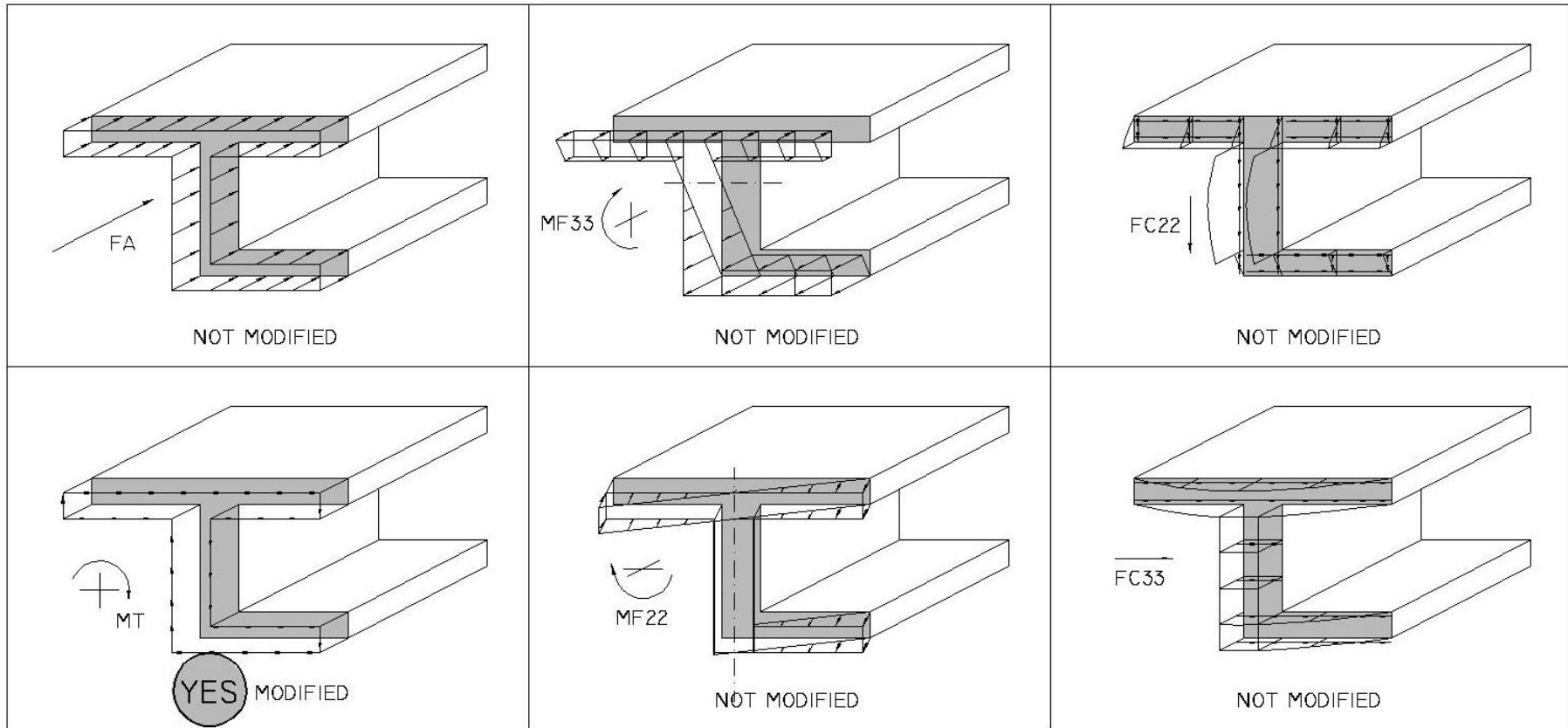
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BEAM AND SLAB DECK (3/3)

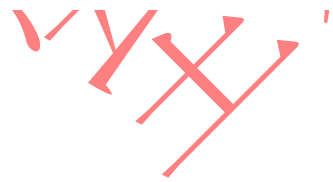


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LONGITUDINAL ELEMENTS MAIN BEAMS

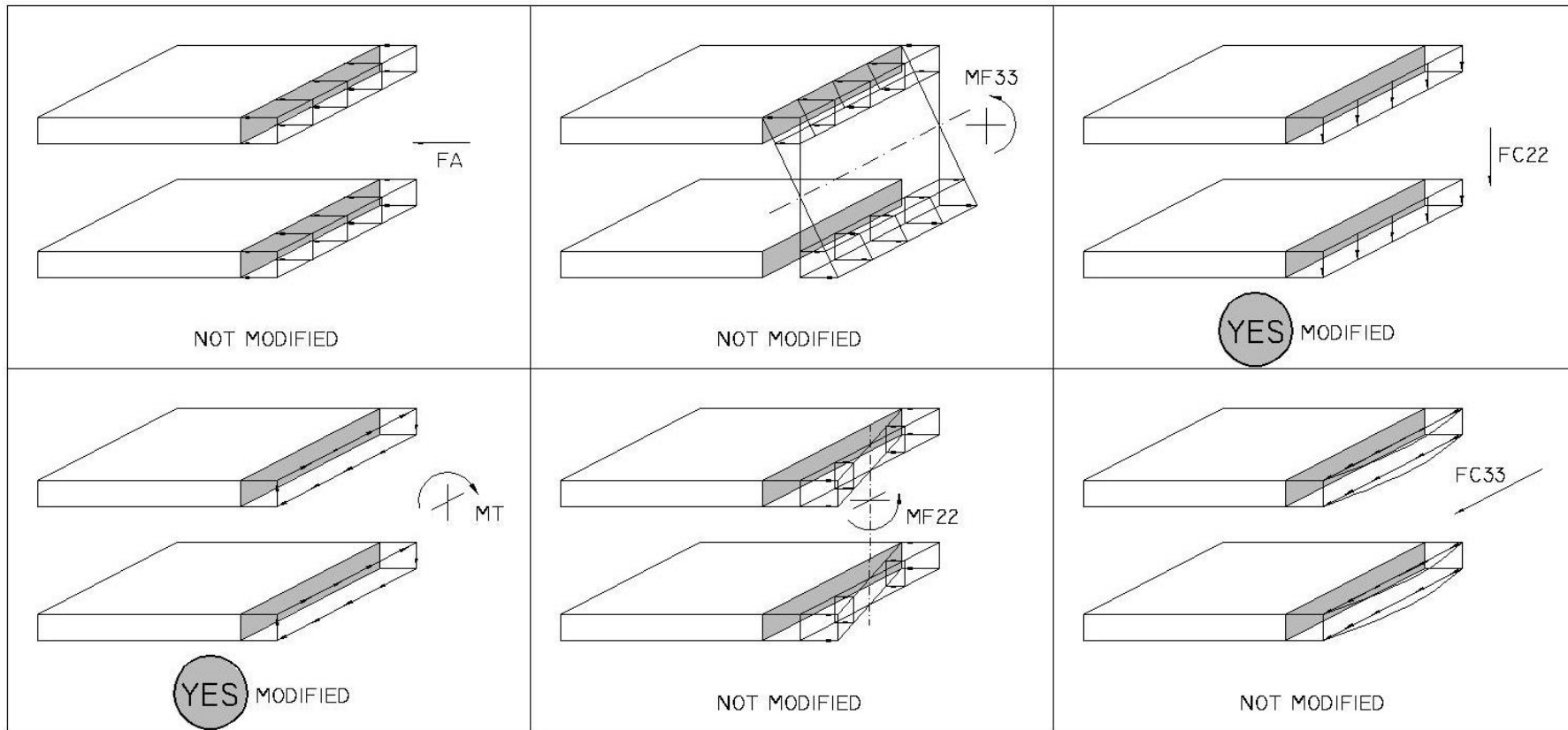


BOX BEAM DECK (1/3)



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TRANSVERSE ELEMENTS TRANSVERSE SLABS

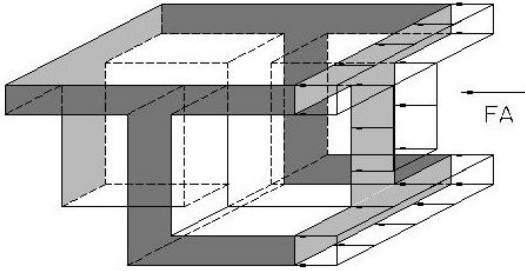
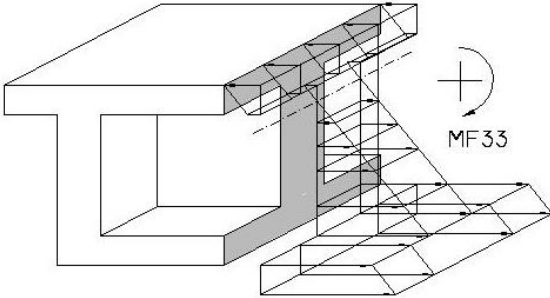
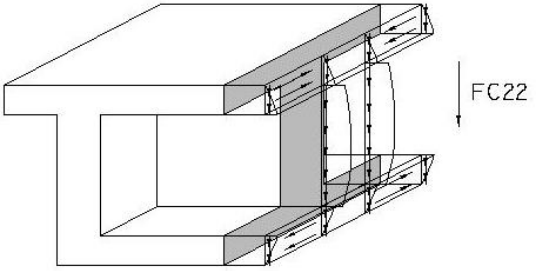
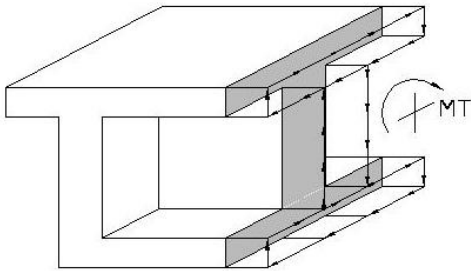
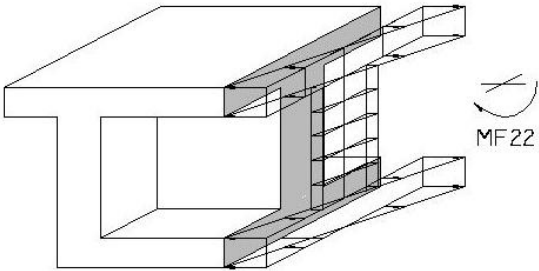
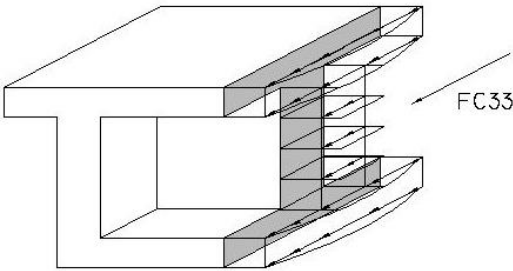


BOX BEAM DECK (2/3)

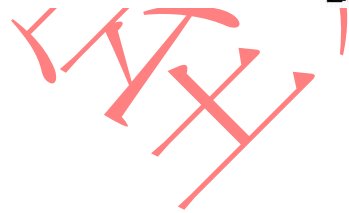


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TRANSVERSE ELEMENTS DIAPHRAGM BEAMS

 <p>NOT MODIFIED</p>	 <p>NOT MODIFIED</p>	 <p>YES MODIFIED</p>
 <p>YES MODIFIED</p>	 <p>NOT MODIFIED</p>	 <p>NOT MODIFIED</p>

BOX BEAM DECK (3/3)



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6.0 MODIFICATION OF THE SECTION PROPERTIES

Weight of Sections

In the transverse slabs it is modified to zero, because this weight have already been considered in the main beam

Also, in the diaphragm beams, the weight of the fraction of slab which have already been considered in the main beams, should be reduced

Torsional Inertia

In the beam and slab decks, contribution of the slab should be reduced to a half.

For the diaphragm beams, contribution of the torsional inertia of the diaphragm should be included

In the box beam deck, torsional inertia of the portion of the box beam in the section should be calculated and reduced to a half of its value.

Shearing area of the transverse slabs and diaphragm beams

In the first place, distortion w_s should be found, due to a distorting load s , with the formula given in the next tables or solving the structural problem of the frame (cross section model) or a beam subjected to a distorting load s .

With the value w_s , it is found the equivalent area AS_2 of the transverse cross section

Next it is shown the tables with the Modification Factors formulas, to be introduced in the sections data of the SAP file.

7.0 FINAL REMARKS

In relation to the basic model of the Ref. N° 1, we will be referring to points which the same Ref. N° 1, gives as especial aspects which should merit a especial treatment

Longitudinal Axial Forces F_A

In the first place, it is required to model prestressing forces, see Ref. N° 1, Sect. 11.6.

Also, the temperature effects, plastic flow and shrinkage shortening of concrete, produce axial forces, see Ref. N° 1, Sect. 11.2 to 11.5.

Due to eccentric loadings, will result in a transverse deflection of the deck, activating shear forces FC_{33} in the transverse slabs, which in turn creates axial forces in the main beams, see Ref. N° 1, Sect. 4.10

Transverse Axial Forces F_A

For transverse loads such as wind, earth quake and when transverse prestressing is applied.

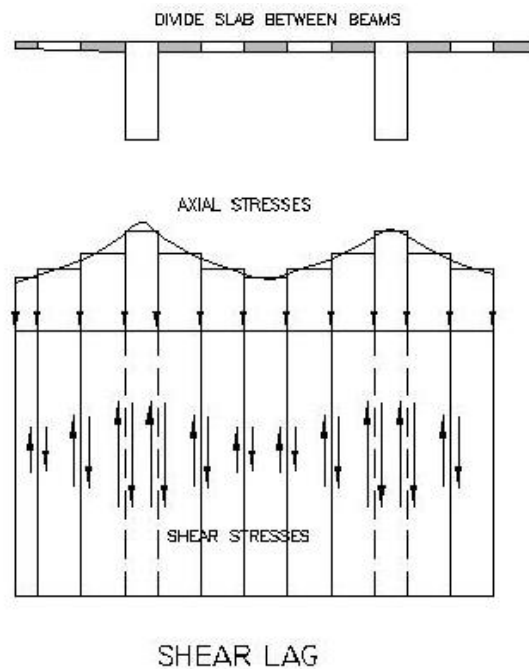
In skew and curved decks, axial forces in the transverse elements will occur.

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Transverse Shear Forces FC33 and transverse Bending Moments MF22

Transverse deflections of the deck will produce warping of the longitudinal beams, which will generate shear force FC33 and bending moment MF22 in plane of the transverse slab, see Ref. N° 1, Sect. 7.5

It could be modelled the shear lag effect, occurring in very large spaced slabs between beams, introducing a number of slabs in between the beams, to get a stepwise mean value of the axial force, due to bending moments MF33 in the deck slab (see chapter 8, Ref. N° 1)



8.0 BIBLIOGRAPHY

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9.0 NOTES

NOTE 1

Recent publications are using new designations to distinguish structures and types of structural elements

STRUCTURES

Dim	Designation	Usual Designation
1	One dimensional	Beams, Columns and Cables
2	Two dimensional	Plane Truss, Plane Frames, Plane Grillages
3	Three dimensional	Space Truss, Space Frames, Space Grillages, Blocks, Three dimensional Solids.

ELEMENTS

Dim	Designation	Usual Designation
0	Point Element	Supports, Concrete Hinges, Steel Connections
1	Line Element	Bar Element, Beam Element, Column Element, Cable Element
2	Surface Element	Membrane Finite Element, Plate Finite Element, Shell Finite Element
3	Volume Element	Solid Finite Element

See Reference N°2 and Reference N°4

NOTE 2

Essentially, the problem is finding the concentrated load distribution between the deck elements.

First researches for the analysis of bridge decks dates back to the 40' decade, with works like J. Melan, "Die genaue Berechnung von Trägerrosten"

During the 50' decade, diverse methods of calculation based on grillage analogy (Leonhardt and Homberg) or an equivalent plate (Guyon-Massonnet) were developed, whose final results were obtained by means of surface Influence diagrams.

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Working with these diagrams were extremely cumbersome and also prone to errors from one hand and on the other was its limited scope of validity (only for rectangular simply supported decks).

It should be remembered that up to the beginnings of the 60' decade, the common calculation tool was the slide rule.

A great technological step was done with the advent of the computer (main Frame) and the development of the matrix methods in Structures in the 60' decade.

In this way you could count with generic methods to solve the basic problem of the bridge deck as a grillage for different configurations and support conditions.

This first approach was still deficient in modeling the equivalent grillage and was limited to beam and slab bridge deck, neglecting the torsional stiffness of the slab.

In the second half of the 60', appears the Finite Element Method, as a powerful tool to deal with the study of continuous medium problems, such as slabs and solids, examining the behaviour of the elements to stress and strain level

Also, in this decade, a number of box beams analysis were developed.

NOTE 3

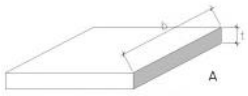
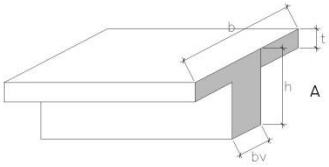
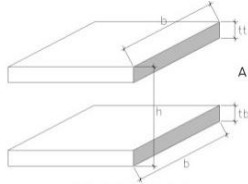
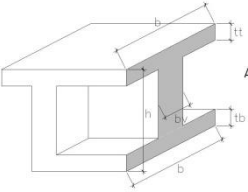
In the book by Ing. J. Manterola a comprehensive examination of the state of art (year 2006) has been made on the analysis of bridge deck behaviour, using finite element and grillage analogy. Acknowledging significant progress been made in the implementation of finite element method, there are still diverse aspects to hamper for the practical use of the finite element method as an every day tool in the design office, limiting for the time being to the research investigation of very specific matters.

Among aspects which should be undertaken, would be the orientation of the Standards for the elements Design, which are notionally using the properties of the sections (areas, inertia) and the applied actions (axial, forces, shearing forces and bending moments). This will require an important adaptation of the design standards

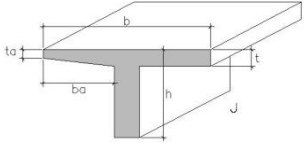
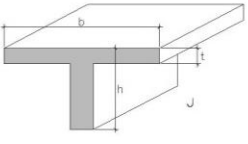
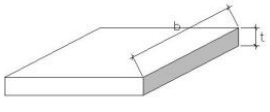
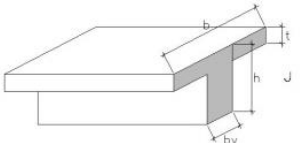
Finally, it is included a number of bridge decks types, with a comparative study between the finite element method and the grillage analogy method.

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MODIFICATION FACTORS FOR WEIGHT AND MASSES

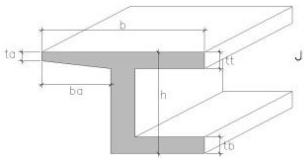
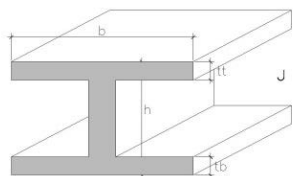
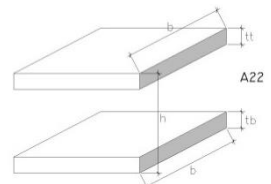
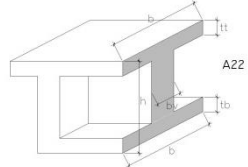
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 <p>TRANSVERSE SLAB BEAM AND SLAB DECK</p>	$A1 = b \times t$	$A2 = 0.0$	$FM = 0.0$
 <p>DIAPHRAGM BEAM BEAM AND SLAB DECK</p>	$A1 = b \times t + (h - t) \times bv$	Diaph Int $A2 = (h - t) \times bv$ Diaph Ext $A2 = \frac{b}{2} \times t + (h - t) \times bv$	$FM = \frac{A2}{A1}$
 <p>TRANSVERSE SLAB BOX BEAM DECK</p>	$A1 = b \times (tt + tb)$	$A2 = 0.0$	$FM = 0.0$
 <p>DIAPHRAGM BOX BEAM DECK</p>	$A1 = b \times (tt + tb) + (h - tt - tb) \times bv$	Diaph Int $A2 = (h - tt - tb) \times bv$ Diaph Ext $A2 = \frac{b}{2} \times (tt + tb) + (h - tt - tb) \times bv$	$FM = \frac{A2}{A1}$

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MODIFICATION FACTORS FOR TORSIONAL INERTIA

SECTION	THEORETICAL PROPERTY (as per SAP) (1)	EQUIVALENT PROPERTY (2)	FM=(2)/(1)
 <p>MAIN BEAM EXT. BEAM AND SLAB DECK</p>	$J1 = \sum \frac{1}{3} b \times t^3 = \frac{1}{3} (b - ba) \times t^3 + \frac{1}{3} ba \times \left(\frac{t + ta}{2}\right)^3 + \frac{1}{3} (h - t) \times bv^3$	 $J2 = \frac{1}{6} (b - ba) \times t^3 + \frac{1}{6} ba \times \left(\frac{t + ta}{2}\right)^3 + \frac{1}{3} (h - t) \times bv^3$ 	$FM = \frac{J2}{J1}$
 <p>MAIN BEAM INT. BEAM AND SLAB DECK</p>	$J1 = \frac{1}{3} b \times t^3 + \frac{1}{3} (h - t) \times bv^3$	 $J2 = \frac{1}{6} b \times t^3 + \frac{1}{3} (h - t) \times bv^3$ 	$FM = \frac{J2}{J1}$
 <p>TRANSVERSE SLAB BEAM AND SLAB DECK</p>	$J1 = \frac{1}{3} b \times t^3$	$J2 = \frac{1}{6} b \times t^3$	$FM = 0.5$
 <p>DIAPHRAGM BEAM BEAM AND SLAB DECK</p>	$J1 = \frac{1}{3} b \times t^3 + \frac{1}{3} (h - t) \times bv^3$	$J2 = \frac{1}{6} b \times t^3 + \frac{1}{3} (h - t) \times bv^3$	$FM = \frac{J2}{J1}$

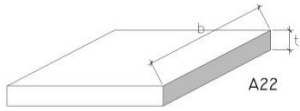
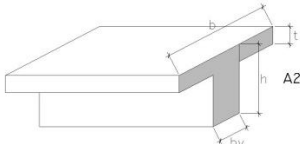
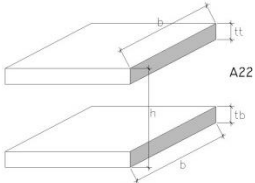
GRILLAGE ANALOGY METHOD

MODIFICATION FACTORS FOR TORSIONAL INERTIA

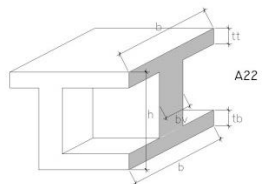
SECTION	THEORETICAL PROPERTY (as per SAP) (1)	EQUIVALENT PROPERTY (2)	FM=(2)/(1)
 <p>MAIN BEAM EXT BOX BEAM DECK</p>	$J1 = \sum \frac{1}{3} b \times t^3 = \frac{1}{3} ba \times \left(\frac{t+ta}{2}\right)^3 + \frac{1}{3} (b-ba) \times (tt^3 + tb^3) + \frac{1}{3} (h-tt-tb) \times bv^3$	$J2 = \frac{1}{6} ba \times \left(\frac{t+ta}{2}\right)^3 + \frac{2 \times H^2 \times tt \times tb}{(tt+tb)} B$ <p>Being $B = b - ba - \frac{bv}{2}$ $H = h - \left(\frac{tt+tb}{2}\right)$</p> <p><i>Torsional Inertia of 1/2 cell of box beam</i></p>	$FM = \frac{J2}{J1}$
 <p>MAIN BEAM INT. BOX BEAM DECK</p>	$J1 = \frac{1}{3} b \times (tt^3 + tb^3) + \frac{1}{3} (h-tt-tb) \times bv^3$	$J2 = \frac{2 \times H^2 \times tt \times tb}{(tt+tb)} b$ <p>Being $H = h - \left(\frac{tt+tb}{2}\right)$</p> <p><i>Torsional Inertia of a cell of box beam</i></p>	$FM = \frac{J2}{J1}$
 <p>TRANSVERSE SLAB BOX BEAM DECK</p>	$J1 = \frac{1}{3} b \times (tt^3 + tb^3)$	$J2 = \frac{2 \times H^2 \times tt \times tb}{(tt+tb)} b$ <p>Being $H = h - \left(\frac{tt+tb}{2}\right)$</p> <p><i>Torsional Inertia of a cell of box beam</i></p>	$FM = \frac{J2}{J1}$
 <p>DIAPHRAGM BOX BEAM DECK</p>	$J1 = \frac{1}{3} b \times (tt^3 + tb^3) + \frac{1}{3} (h-tt-tb) \times bv^3$	$J2 = \frac{2 \times H^2 \times tt \times tb}{(tt+tb)} b$ <p>Being $H = h - \left(\frac{tt+tb}{2}\right)$</p> <p><i>Torsional Inertia of a cell of box beam</i></p>	$FM = \frac{J2}{J1}$

GRILLAGE ANALOGY METHOD

MODIFICATION FACTORS FOR DISTORSIONAL INERTIA

SECTION	THEORETICAL PROPERTY (as per SAP) (1)	EQUIVALENT PROPERTY (2)	FM=(2)/(1)
		$as = \frac{s \times l}{G \times ws}$ $AS2 = as \times b$ <p>Being l, spacing between main beams ws, deflection due to distortion s, Distorsional Force b, section width G, Shear Modulus</p>	
 TRANSVERSE SLAB BEAM AND SLAB DECK	$AS2_1 = \frac{5}{6} b \times t$	$ws = \frac{s \times l^3}{E \times t^3}$ <p>Being E, Young Modulus</p>	$FM = \frac{AS2_2}{AS2_1}$
 DIAPHRAGM BEAM BEAM AND SLAB DECK	$AS2_1 \sim bv \times h$	$ws = \frac{s \times l^3}{12 \times E \times I_{33}}$ <p>Being I_{33}, Moment of Inertia E, Young Modulus</p>	$FM = \frac{AS2_2}{AS2_1}$
 TRANSVERSE SLAB BOX BEAM DECK	$AS2_1 = b \times (tt + tb)$	$ws = \frac{s \times l^2}{(tt^3 + tb^3)} \left[\frac{bv^3 \times l + (tt^3 + tb^3) \times H}{bv^3 \times E} \right]$ <p>Being $H = h - (tt + tb)/2$ E, Young Modulus</p>	$FM = \frac{AS2_2}{AS2_1}$

GRILLAGE ANALOGY METHOD

 <p style="text-align: center;">DIAPHRAGM BOX BEAM DECK</p>	$AS2_1 \sim bv \times h$	$ws = \frac{s \times l^3}{12 \times E \times I_{33}}$ <p style="text-align: center;"><i>Being I_{33}, Moment of Inertia</i> <i>E, Young Modulus</i></p>	$FM = \frac{AS2_2}{AS2_1}$
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GRILLAGE ANALOGY METHOD

EXAMPLE N°1: BEAM AND SLAB DECKS

General Layout

Beam and slab Bridge Deck, rectangular deck, 13.00m span and 9.60m wide

Beams are 1.00m depth and 0.30m width, spaced at 2.00m c/c

Slab is 0.175m thickness and end diaphragms are 0.80m depth and 0.20m width

Deck is simply supported at both ends

Equivalent grillage are made up of 20 nodes and 31 members

Longitudinal beams are of VTABI, VATB2 Y VATB3 sections, and the diaphragms are of VD1 section

Deck has been split in 3 LOSA1 section of 4.00m width

The four supports are at one end fixed and the other end could move longitudinally.

See Fig. N° 1

GRILLAGE GEOMETRY ADJUSTMENT

		Centroidal Coordinates			Corrections (Insertion point)		
Section		X	Y	Z	X	Y	Z
Main Beam	VTAB1	0.0000	-0.1245	-0.0959	0.0000	-0.1245	0.0237
	VTAB2	0.0000	0.1245	-0.0959	0.0000	0.1245	0.0237
	VTAB3	0.0000	0.0000	-0.1196	0.0000	0.0000	0.0000
Transversal	LOSA1	0.0000	0.0000	0.0875	0.0000	0.0000	0.2071
	VD1	0.0000	0.0000	-0.0792	0.0000	0.0000	0.0404

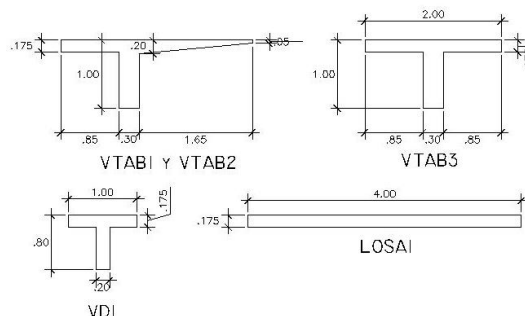


FIG. N° 2

GRILLAGE ANALOGY METHOD

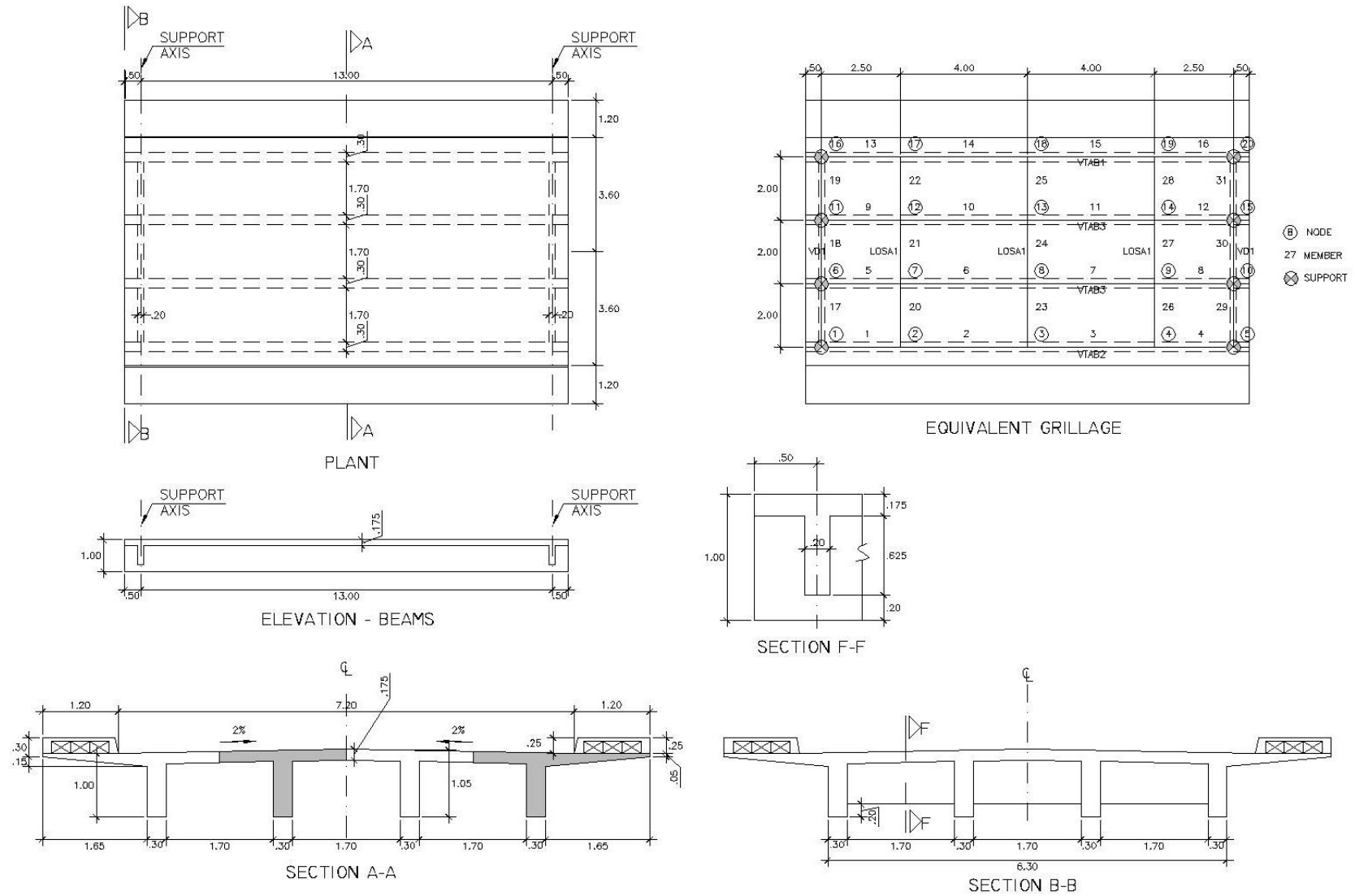


FIG. N° I

GRILLAGE ANALOGY METHOD

PROPERTY MODIFYING FACTORS

LONGITUDINAL BEAMS

VTAB3
WEIGHT 1

BEAM	SLAB	SUM
0.00743	0.00357	0.01100
0.00743	0.00179	0.00921
	FM=	0.83756

TORSION

VTAB1 VTAB2
WEIGHT 1

BEAM	SLAB	VOLADO	SUM
0.00743	0.00205	0.00107	0.01055
0.00743	0.00103	0.00054	0.00899
		FM=	0.85177

TORSION

TRANSVERSE BEAMS

LOSA1
SLAB
WEIGHT 0

SLAB	SUM
0.00715	0.00715
0.00357	0.00357
FM=	0.50000

TORSION

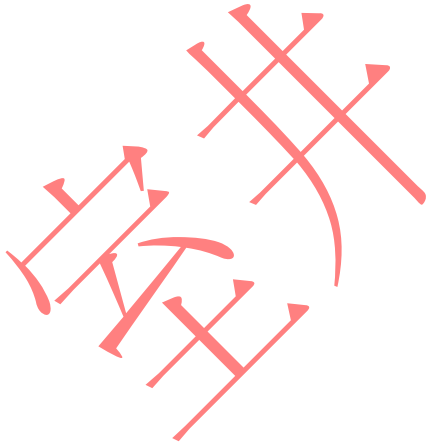
VD1

BEAM	SLAB	SUMA
0.12500	0.17500	0.30000
0.12500	0.08750	0.21250
	FM=	0.70833

WEIGHT

BEAM	SLAB	SUM
0.00167	0.00179	0.00345
0.00167	0.00089	0.00256
	FM=	0.74133

TORSION



GRILLAGE ANALOGY METHOD

DISTORSIONAL INERTIA MODIFYING FACTORS

SLAB DISTORSION	DIAPHRAGM DISTORSION
<p>Formula</p> <p>E= 2534563.5</p> <p>G= 1056068.1</p> <p>s= 10.0</p> <p>t= 0.175</p> <p>l= 2.000</p> <p>t³= 0.005359</p> <p>l²= 4.000</p> <p>ws= 0.00589</p> <p>as=Sl/Gxws= 0.00322</p> <p>b= 4.000</p> <p>AS2= 0.01286</p>	<p>Formula</p> <p>E= 2534563.5</p> <p>G= 1056068.1</p> <p>s= 10.0</p> <p>t= 0.175</p> <p>l= 2.000</p> <p>h= 1.000</p> <p>bv= 0.200</p> <p>I₃₃= 0.0162</p> <p>l²= 4.000</p> <p>ws= 0.00016</p> <p>as=Sl/Gxws= 0.11664</p> <p>b= 1.000</p> <p>AS2= 0.11664</p>
<p>SAP Model</p> <p>ws= 0.00602 from SAP</p> <p>as=Sl/Gxws= 0.00315</p> <p>b= 4.000</p> <p>AS2= 0.01258</p>	<p>SAP Model</p> <p>ws= 0.00019 from SAP</p> <p>as=Sl/Gxws= 0.09967</p> <p>b= 1.000</p> <p>AS2= 0.09967</p>
<p>AS2= 0.58333</p> <p>FM= 0.02205 SAP</p> <p>FM= 0.02157 Formula</p>	<p>AS2= 0.20000</p> <p>FM= 0.58320 SAP</p> <p>FM= 0.49837 Formula</p>

0.51030 from SAP

AS2 de LOSA1

0.32394 from SAP

AS2 de VD1

GRILLAGE ANALOGY METHOD

Applied Loads

Dead Weight

Interior Beam

Asphalt $0.05 \times 2.00 \times 2.2 = 0.22 \text{ T/m}$

Exterior Beam

Asphalt	0.05 × 1.60 × 2.1	= .176 T/m	+0.200
Sidewalk	0.15 × .25 × 2.4	= .090	-1.725
	0.125 × .25 × 2.4	= .075	-0.6625
	0.05 × .90 × 2.4	= .108	-1.200
Voided brick	$0.10 \times 3 \times \frac{100}{30}$	= .100	-1.200
Railings		= .100	-1.650
		<u>.649 T/m</u>	

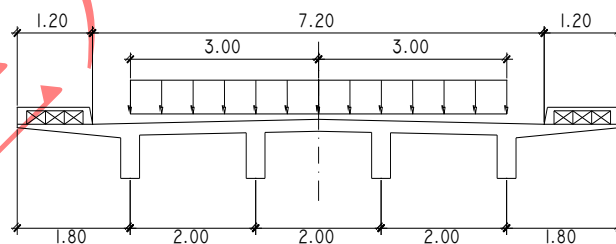
$$M = .176 \times .200 - (.090 \times 1.725 + .075 \times .6625 + .108 \times 1.200 + .100 \times 1.20 + 0.100 \times 1.65)$$

$$M = 0.0352 - 0.6195 = -0.5843 \text{ Tm/m}$$

Vehicular Live Load

Lane Live Loading

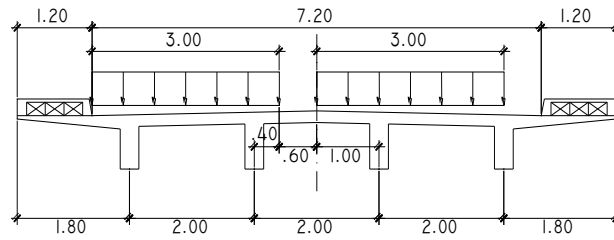
Concentric Loads



1.00w	2.00w	2.00w	1.00w	= 6.00w
0.323	0.647	0.647	0.323	= 1.94

GRILLAGE ANALOGY METHOD

Eccentric Loads



$1.00w$	$1.00w$	$1.00w$	$1.00w$	
$\frac{.60w \times 2.30}{2.00}$	$\frac{.60w \times 0.30}{2.00}$	$\frac{.40w \times 1.80}{2.00}$	$\frac{.40w \times .20}{2.00}$	
	$1.00w \times .50$	$1.00w \times 1.50$		
	$\frac{\quad}{2.00}$	$\frac{\quad}{2.00}$		
$1.69w$	$1.52w$	$1.79w$	$1.00w$	$= 6.00w$
$.546$	$.491$	$.579$	$.323$	$= 1.939 \sim 1.94$

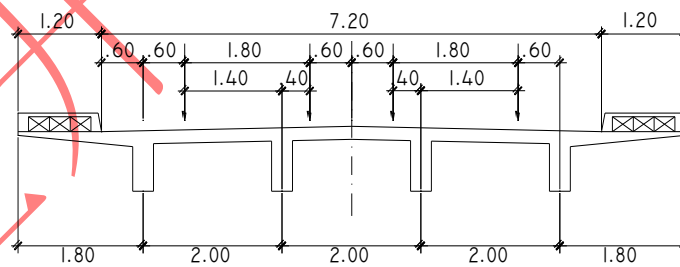
Truck and Tandem Live Loading

Traffic lanes are along each main beam

Wheel Concentrated Loads to be distributed simply between adjacent beams, applying Saint Venant principle

It should be remembered that with this model we are analysing the main beams and not the deck slab

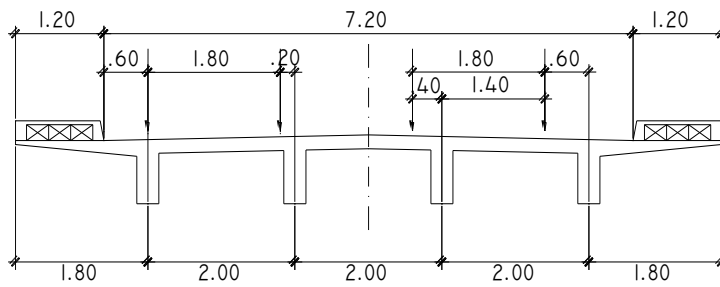
Concentric Loads



$\frac{1.40P}{2.00}$	$\frac{.60P}{2.00}$	$\frac{.60P}{2.00}$	$\frac{1.40P}{2.00}$	
	$\frac{1.60P}{2.00}$	$\frac{0.40P}{2.00}$		
	$\frac{.40P}{2.00}$	$\frac{1.60P}{2.00}$		
$.70P$	$1.30P$	$1.30P$	$.70P$	$= 4P$

GRILLAGE ANALOGY METHOD

Eccentric Loads



$$1.00P$$

$$0.00P$$

$$\frac{0.20P}{2.00}$$

$$\frac{1.80P}{2.00}$$

$$\frac{.40P}{2.00}$$

$$\frac{1.60P}{2.00}$$

$$\frac{.60P}{2.00}$$

$$\frac{1.40P}{2.00}$$

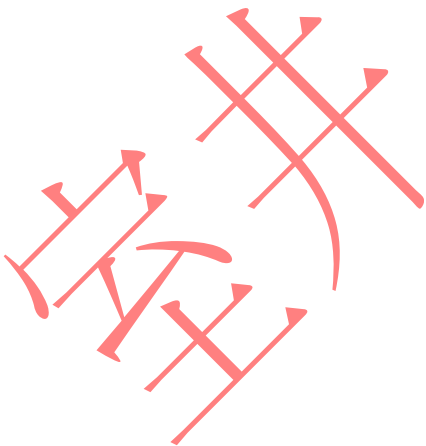
$$1.10P$$

$$1.10P$$

$$1.10P$$

$$.70P$$

$$=4.00P$$



GRILLAGE ANALOGY METHOD

EXAMPLE N°2: BOX BEAM DECK

1. GENERAL LAYOUT

The Bridge is a continuous box beam deck of 3 spans, 27.00m, 36.00m and 27.00m in length, of variable depth from 1.20m to 2.20m, with parabolic haunches, over the intermediate supports.

We have diaphragms at the supports and at mid spans

2. ADJUSTMENTS AT THE NODE LOCATIONS

1. MAIN BEAMS (VERTICAL)

	Exterior	Interior	Ordinate Z
VL VIGA 1	- 0.312	-0.317	-0.314
VL VIGA 2	-0.329	-0.334	-0.332
VL VIGA 3	-0.383	-0.387	-0.385
VL VIGA 4	-0.473	-0.476	-0.475
VL VIGA 5	-0.601	-0.603	-0.602
VL VIGA 6	-0.768	-0.768	-0.768

1. TRANSVERSE BEAMS (VERTICAL)

DIAF 1	VL = -0.314	D1 = -0.375	$\Delta = -0.061$
DIAF 2	VL = -0.314	D2 = -0.362	$\Delta = -0.048$
DIAF 3	VL = -0.768	D3 = -0.875	$\Delta = -0.107$
LOSA 1	VL = -0.314	L1 = -0.339	$\Delta = -0.025$
LOSA 2	VL = -0.314	L2 = -0.339	$\Delta = -0.025$
LOSA 3	VL = -0.332	L3 = -0.365	$\Delta = -0.033$
LOSA 4	VL = -0.430	L4 = -0.449	$\Delta = -0.019$
LOSA 5	VL = -0.602	L5 = -0.622	$\Delta = -0.020$
LOSA 6	VL = -0.602	L6 = -0.614	$\Delta = -0.012$
LOSA 7	VL = -0.332	L7 = -0.365	$\Delta = -0.033$
LOSA 8	VL = -0.430	L8 = -0.449	$\Delta = -0.019$

1. MAIN BEAMS (TRANSVERSALLY)

	Exterior		
VL VIGA 1	+ 0.072	prom = 1+2	VAR 12
VL VIGA 2	+ 0.071	prom = 2+3	VAR 23
VL VIGA 3	+ 0.068		VAR 34 (I)
VL VIGA 4	+ 0.064		VAR 34 (J)
VL VIGA 5	+ 0.059	prom = 4+5	VAR 45
VL VIGA 6	+ 0.054	prom = 5+6	VAR 56

GRILLAGE ANALOGY METHOD

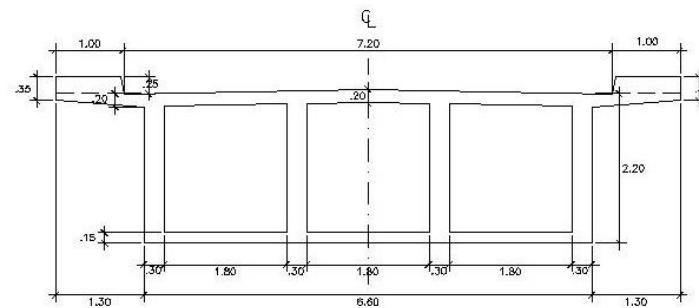
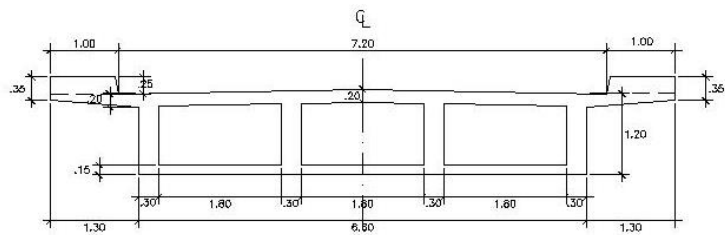
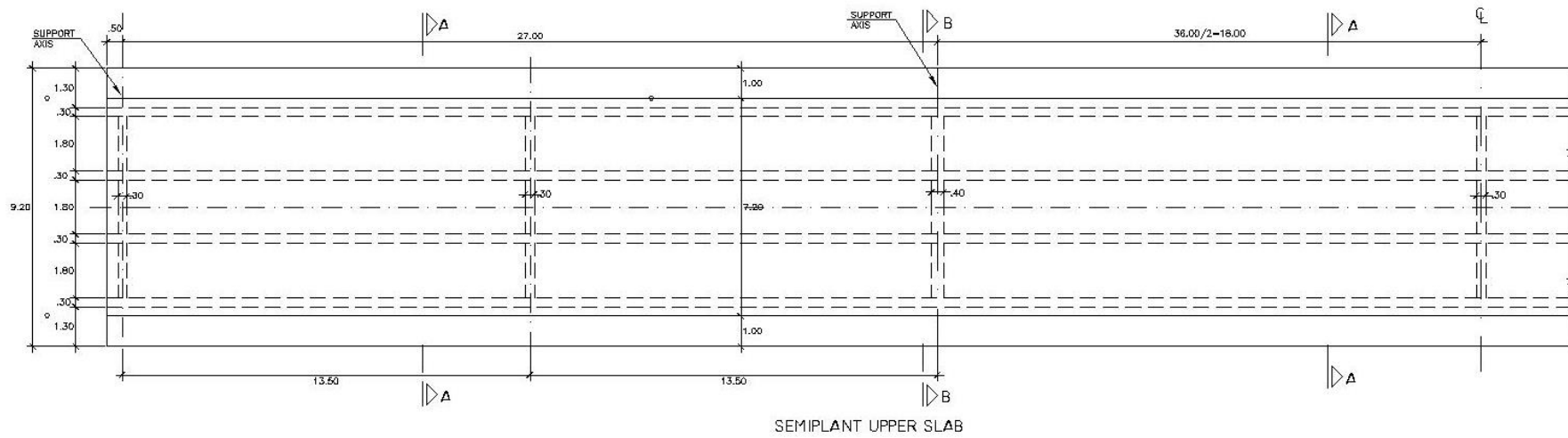
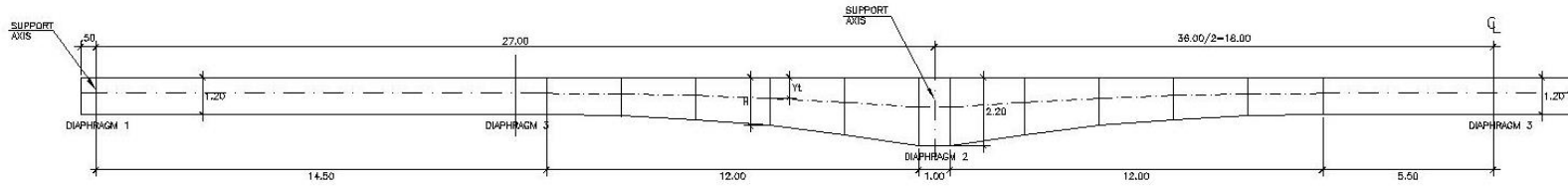


FIG. N° I

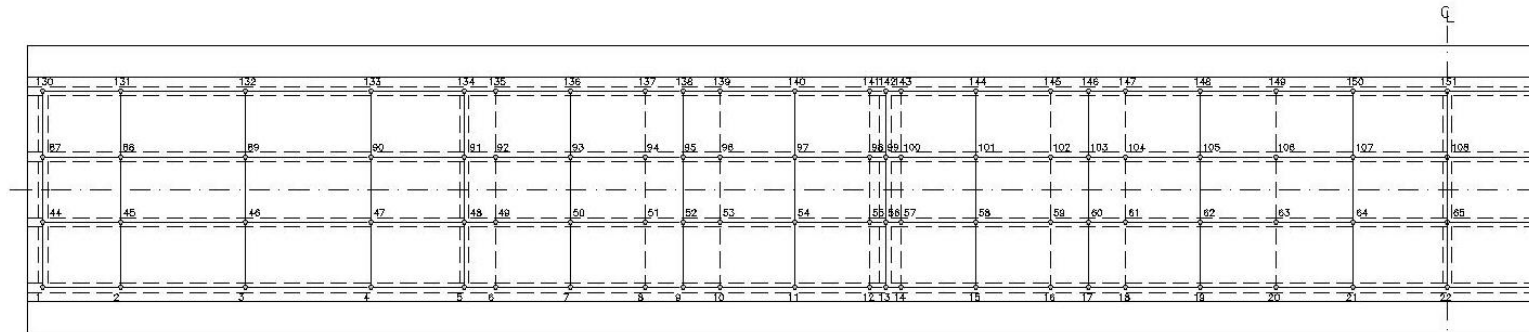


GRILLAGE ANALOGY METHOD

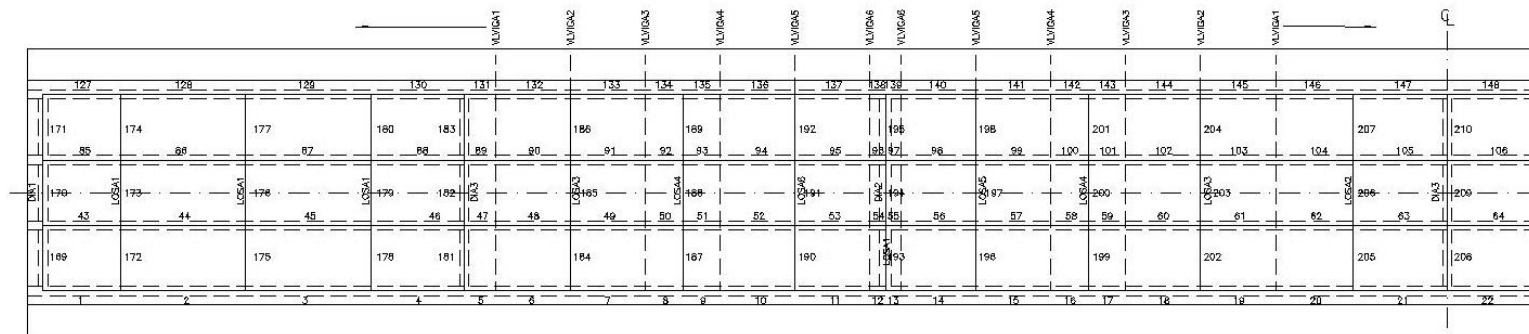


DISTANCIAS (M)	0.000	14.500	16.900	19.300	21.700	24.100	26.500	27.900	29.900	32.300	34.700	37.100	39.500	45.000
H (M)	1.200	1.200	1.240	1.360	1.560	1.840	2.200	2.200	1.840	1.560	1.360	1.240	1.200	1.200
yt (M)	0.517	0.517	0.534	0.587	0.676	0.803	0.968	0.968	0.803	0.676	0.587	0.534	0.517	0.517

GEOMETRY OF THE BEAM ELEVATION



SEMIPLANT - NODES NUMBERING



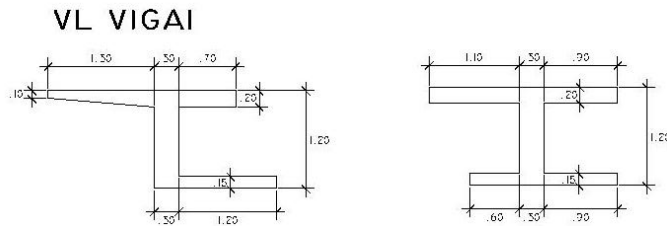
SEMIPLANT - ELEMENT NUMBERING

FIG. N° 2

GRILLAGE ANALOGY METHOD

3. CROSS SECTIONS PROPERTIES

VL VIGA 1



$$A = 0.875 \text{ m}^2$$

$$I = 0.1754 \text{ m}^4$$

$$Y_t = 0.512 \text{ m}$$

$$Y_b = 0.688 \text{ m}$$

$$J = 0.0154 \text{ m}^4$$

$$A = 0.985 \text{ m}^2$$

$$I = 0.2002 \text{ m}^4$$

$$Y_t = 0.517 \text{ m}$$

$$Y_b = 0.683 \text{ m}$$

$$J = 0.0177 \text{ m}^4$$

Torsional Inertia adjustment:

$$c = \frac{2h^2 d' d''}{(d' + d'')} = \frac{2 \times 1.025^2 \times 0.20 \times 0.15}{(0.20 + 0.15)} = 0.1801$$

$$h = 1.20 - 0.10 - 0.075 = 1.025 \text{ m}$$

In the exterior beams $c \times b = 0.1801 \times 0.90 = 0.1621$

$$J = 0.0154$$

$$J/J = 0.1621/0.0154 = 10.526$$

In the interior beams $c \times b = 0.1801 \times 1.80 = 0.3242$

$$J = 0.0177$$

$$J/J = 0.3242/0.0177 = 18.32$$

VL VIGA 2

Similar to VL VIGA1, only that the beam depth being 1.24 m instead of 1.20 m

$$A = 0.887 \text{ m}^2$$

$$I = 0.2916 \text{ m}^4$$

$$Y_t = 0.529 \text{ m}$$

$$Y_b = 0.711 \text{ m}$$

$$J = 0.0158 \text{ m}^4$$

$$A = 0.997 \text{ m}^2$$

$$I = 0.2171 \text{ m}^4$$

$$Y_t = 0.534 \text{ m}$$

$$Y_b = 0.706 \text{ m}$$

$$J = 0.018 \text{ m}^4$$

Torsional Inertia adjustment:

$$h = 1.24 - 0.10 - 0.075 = 1.065 \text{ m}$$

GRILLAGE ANALOGY METHOD

$$c = \frac{2 \times 1.065^2 \times 0.20 \times 0.15}{(0.20 + 0.15)} = 0.1944$$

In the exterior beams $c \times b = 0.1944 \times 0.90 = 0.1750$

$$J = 0.0158$$

$$J/J = 0.1750/0.0158 = 11.076$$

In the interior beams $c \times b = 0.1944 \times 1.80 = 0.3500$

$$J = 0.018$$

$$J/J = 0.3500/0.018 = 19.444$$

VL VIGA 3

Similar to VL VIGA1, only that the beam depth being 1.36 m instead of 1.20 m

$$A = 0.923 \text{ m}^2$$

$$I = 0.2391 \text{ m}^4$$

$$Y_t = 0.583 \text{ m}$$

$$Y_b = 0.777 \text{ m}$$

$$J = 0.0168 \text{ m}^4$$

$$A = 1.033 \text{ m}^2$$

$$I = 0.2727 \text{ m}^4$$

$$Y_t = 0.587 \text{ m}$$

$$Y_b = 0.773 \text{ m}$$

$$J = 0.0191 \text{ m}^4$$

Torsional Inertia adjustment:

Unit Torsion in the slabs, as a box beam:

$$h = 1.36 - 0.10 - 0.075 = 1.185 \text{ m}$$

$$c = \frac{2 \times 1.185^2 \times 0.20 \times 0.15}{(0.20 + 0.15)} = 0.24072$$

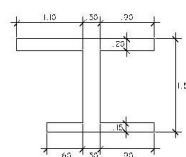
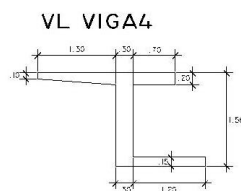
In the exterior beams $J = 0.24072 \times 0.90 = 0.2167$

$$J/J = 0.2167/0.0168 = 12.896$$

In the interior beams $J = 0.24072 \times 1.80 = 0.4333$

$$J/J = 0.4333/0.0191 = 22.686$$

VL VIGA 4



GRILLAGE ANALOGY METHOD

$A = 0.983\text{m}^2$	$A = 1.093\text{m}^2$
$I = 0.3353\text{ m}^4$	$I = 0.3817\text{ m}^4$
$Y_t = 0.6729\text{ m}$	$Y_t = 0.6763\text{ m}$
$Y_b = 0.8871\text{ m}$	$Y_b = 0.8837\text{ m}$
$J = 0.0187\text{ m}^4$	$J = 0.0209\text{ m}^4$

Torsional Inertia adjustment:

Unit Torsion in the slabs, as a box beam:

$$c = \frac{2h^2d'd''}{(d' + d'')} = \frac{2 \times (1.56 - 0.10 - 0.75)^2 \times 0.20 \times 0.15}{(0.20 + 0.15)} = 0.32884\text{ m}^4/\text{m}$$

In the exterior beams $c \times b = 0.32884 \times 0.90 = 0.2960$

$$J = 0.0187$$

$$J/J = 0.2960/0.0187 = 15.829$$

In the interior beams $c \times b = 0.32884 \times 1.80 = 0.5919$

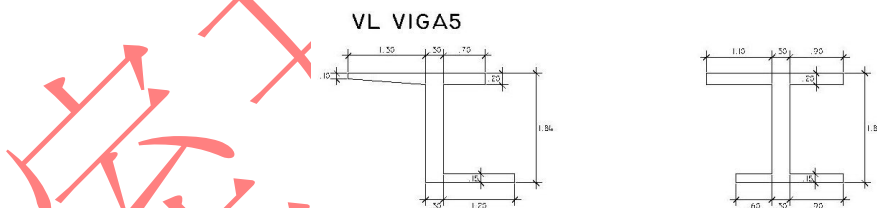
$$J = 0.0209$$

$$J/J = 0.5919/0.0209 = 28.321$$

Torsional Inertia of the beam $J = 3.9553\text{m}^2 \rightarrow 1/2 = 1.9777$

Sum $2 \times (0.2960 + 0.5919) = 1.7758 \sim 1.9777$

VL VIGA 5



$A = 1.067\text{m}^2$	$A = 1.177\text{m}^2$
$I = 0.5029\text{ m}^4$	$I = 0.571\text{ m}^4$
$Y_t = 0.801\text{ m}$	$Y_t = 0.8029\text{ m}$
$Y_b = 1.039\text{ m}$	$Y_b = 1.0371\text{ m}$
$J = 0.0212\text{ m}^4$	$J = 0.0235\text{m}^4$

Torsional Inertia adjustment:

Unit Torsion in the slabs, as a box beam:

GRILLAGE ANALOGY METHOD

$$c = \frac{2h^2d'd''}{(d' + d'')} = \frac{2 \times (1.84 - 0.10 - 0.75)^2 \times 0.20 \times 0.15}{(0.20 + 0.15)} = 0.47524$$

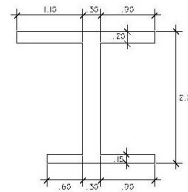
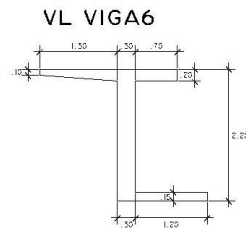
In the exterior beams $J = 0.47524 \times 0.90 = 0.427716 \text{ m}^4$

$$J/J = 0.4277/0.0212 = 20.175$$

In the interior beams $J = 0.47524 \times 1.80 = 0.8554 \text{ m}^4$

$$J/J = 0.8554/0.0235 = 36.400$$

VL VIGA 6



$$A = 1.175 \text{ m}^2$$

$$I = 0.7799 \text{ m}^4$$

$$Y_t = 0.9682 \text{ m}$$

$$Y_b = 1.2318 \text{ m}$$

$$J = 0.0245 \text{ m}^4$$

$$A = 1.285 \text{ m}^2$$

$$I = 0.882 \text{ m}^4$$

$$Y_t = 0.9682 \text{ m}$$

$$Y_b = 1.2318 \text{ m}$$

$$J = 0.0268 \text{ m}^4$$

Torsional Inertia adjustment:

Unit Torsion in the slabs, as a box beam:

$$c = \frac{2h^2d'd''}{(d' + d'')} = \frac{2 \times (2.20 - 0.10 - 0.75)^2 \times 0.20 \times 0.15}{(0.20 + 0.15)} = 0.7029$$

In the exterior beams $J = 0.702964 \times 0.90 = 0.6327 \text{ m}^4$

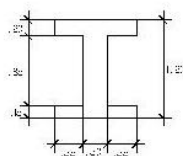
$$J/J = 0.6327/0.0245 = 25.824$$

In the interior beams $J = 0.702964 \times 1.80 = 1.2653 \text{ m}^4$

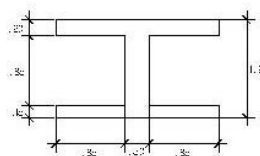
$$J/J = 1.2653/0.0268 = 47.213$$

TRANSVERSE BEAMS

Diaphragm DIAF1



Diaphragm DIAF2



GRILLAGE ANALOGY METHOD

$A = 0.605\text{m}^2$	$A = 0.955\text{m}^2$
$I = 0.1074\text{ m}^4$	$I = 0.1987\text{ m}^4$
$Y_t = 0.575\text{ m}$	$Y_t = 0.562\text{ m}$
$Y_b = 0.625\text{ m}$	$Y_b = 10.638\text{m}$
$J = 0.013\text{ m}^4$	$J = 0.017\text{m}^4$

Torsional Inertia adjustment:

Unit Torsion in the slabs, as a box beam:

DIAF1 $h = 1.20 - 0.10 - 0.075 = 1.025\text{ m}$

$$c = \frac{2 \times 1.025^2 \times 0.20 \times 0.15}{(0.20 + 0.15)} = 0.1801$$

$$J = 0.1801 \times 0.90 + \frac{1}{3} 1.20 \times 0.30^3 = 0.1261 + 0.0108 = 0.1369\text{ m}^4$$

$$J = 0.013$$

$$J/J = 0.1369/0.013 = 10.53$$

Weight $w = 0.30 \times 1.20 = 0.36$

$$A = 0.35 \times 0.35 + 0.30 \times 1.20 = 0.4825$$

$$A/A = 0.4825/0.605 = 0.798$$

DIAF2 $b = 2.00 - 0.30 = 1.70$

$$J = 0.1801 \times 1.70 + \frac{1}{3} 1.20 \times 0.30^3 = 0.3062 + 0.108 = 0.3170$$

$$J = 0.017$$

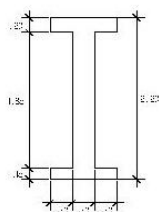
$$J/J = 0.3170/0.017 = 18.65$$

$$A = 0.30 \times 0.85 = 0.255$$

$$A/A = 0.255/0.955 = 0.267$$

DIAPHRAGM DIAF3

Diaphragm DIAF3



GRILLAGE ANALOGY METHOD

$$A = 1.09\text{m}^2$$

$$I = 0.5694 \text{ m}^4$$

$$Y_t = 1.0745 \text{ m}$$

$$Y_b = 1.1255 \text{ m}$$

$$J = 0.0458 \text{ m}^4$$

Torsional Inertia adjustment:

Unit Torsion in the slabs, as a box beam:

$$c = \frac{2h^2 d' d''}{(d' + d'')} = \frac{2 \times (2.20 - 0.10 - 0.75)^2 \times 0.20 \times 0.15}{(0.20 + 0.15)} = 0.7029$$

$$J = \frac{1}{3} 2.20 \times 0.40^3 + 0.702964 \times 0.60 = 0.046933 + 0.4217$$

$$J = 0.468711$$

$$J/J = 0.468711/0.0458 = 10.2339$$

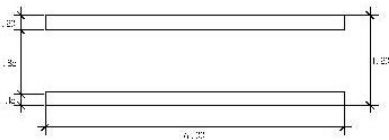
Weight reduction

$$A = 0.40 \times 1.85 = 0.74 \text{ m}^2$$

$$A/A = 0.74/1.09 = 0.679$$

LOSA 1

$$L = 4.00\text{m}$$



$$c \times b = 0.1801 \times 4.00 = 0.7204$$

$$J = \frac{1}{3} 4 \times 0.20^3 + \frac{1}{3} 4 \times 0.15^3 = 0.01517$$

$$J/J = 0.7204/0.1517 = 47.5$$

Weight $w = 0.0$

LOSA 2

$$L = 5.00\text{m}$$

$$c \times b = 0.1801 \times 5.00 = 0.9005$$

$$J = \frac{1}{3} 5 \times 0.20^3 + \frac{1}{3} 5 \times 0.15^3 = 0.01896$$

$$J/J = 0.9005/0.1896 = 47.5$$

GRILLAGE ANALOGY METHOD

Weight $w = 0.0$

LOSA 3 $b = 4.8 \text{ m}$ $h_{prom} = 1.26 \text{ m}$ $h = 1.26 - 0.10 - 0.075 = 1.085 \text{ m}$

$$c = \frac{2 \times 1.085^2 \times 0.20 \times 0.15}{(0.20 + 0.15)} = 0.20181$$

$$c \times b = 0.20181 \times 4.80 = 0.969$$

$$J = 0.0182$$

$$J/J = 0.969/0.0182 = 53.24$$

Weight $w = 0.0$ equal as LOSA 7

LOSA 4 $b = 2.40 \text{ m}$ $h_{prom} = 1.255 \text{ m}$ $h = 1.255 - 0.10 - 0.075 = 1.08 \text{ m}$

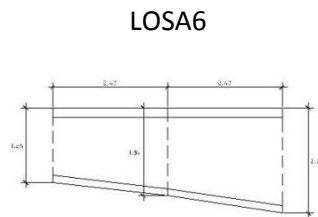
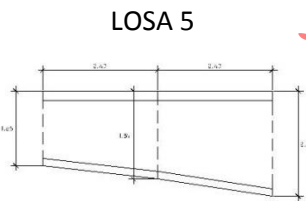
$$c = \frac{2 \times 1.08^2 \times 0.20 \times 0.15}{(0.20 + 0.15)} = 0.2000$$

$$J = 0.20 \times 2.4 = 0.48$$

$$J = 0.008853$$

$$J/J = 0.48/0.008853 = 54.22$$

Weight $w = 0.0$ equal as LOSA 8



~~$$A = 1.68 \text{ m}^2$$~~

~~$$I = 1.1974 \text{ m}^4$$~~

~~$$Y_t = 0.8221 \text{ m}$$~~

~~$$Y_b =$$~~

~~$$J = 0.0173 \text{ m}^4$$~~

$$A = 0.1668 \text{ m}^2$$

$$I = 1.1823 \text{ m}^4$$

$$Y_t = 0.8141 \text{ m}$$

$$Y_b =$$

$$J = 0.0186 \text{ m}^4$$

Mean value h $h_{mean} = \frac{1}{2} \left[\frac{1}{2} (1.56 + 1.84) + \frac{1}{2} (1.84 + 2.20) \right] = 1.86 \text{ m}$

$$c = \frac{2h^2 d' d''}{(d' + d'')} = \frac{2 \times (1.86 - 0.10 - 0.75)^2 \times 0.20 \times 0.15}{(0.20 + 0.15)} = 0.486$$

$$J = 0.486724 \times 4.80 = 2.3363 \text{ m}^4$$

For LOSA 5 $J/J = 2.3363/0.0173 = 135.045$

For LOSA 6 $J/J = 2.3363/0.0186 = 125.606$

GRILLAGE ANALOGY METHOD

DISTORSIONAL INERTIA MODIFICATION FACTORS

DISTORSION VLVIGA1	DISTORSION VLVIGA2	DISTORSION VLVIGA3
<p>Hambly Formula</p> <p style="text-align: right;">2534563.</p> <p>E= 5</p> <p style="text-align: right;">1056068.</p> <p>G= 1</p> <p>s= 1.0</p> <p>tt= 0.200</p> <p>tb= 0.150</p> <p>l= 2.100</p> <p>h= 1.200</p> <p>H= 1.025</p> <p>bv= 0.300</p> <p>tt³+tb³= 0.011375</p> <p>bv³= 0.027</p> <p>l²= 4.410</p> <p>ws= 0.00039</p> <p>as=Sl/Gxws</p> <p>= 0.00513</p> <p>b= 3.250</p> <p>AS2= 0.01669</p>	<p>Hambly Formula</p> <p style="text-align: right;">2534563.</p> <p>E= 5</p> <p style="text-align: right;">1056068.</p> <p>G= 1</p> <p>s= 1.0</p> <p>tt= 0.200</p> <p>tb= 0.150</p> <p>l= 2.100</p> <p>h= 1.240</p> <p>H= 1.065</p> <p>bv= 0.300</p> <p>tt³+tb³= 0.011375</p> <p>bv³= 0.027</p> <p>l²= 4.410</p> <p>ws= 0.00039</p> <p>as=Sl/Gxws</p> <p>= 0.00510</p> <p>b= 3.250</p> <p>AS2= 0.01658</p>	<p>Hambly Formula</p> <p style="text-align: right;">2534563.</p> <p>E= 5</p> <p style="text-align: right;">1056068.</p> <p>G= 1</p> <p>s= 1.0</p> <p>tt= 0.200</p> <p>tb= 0.150</p> <p>l= 2.100</p> <p>h= 1.360</p> <p>H= 1.185</p> <p>bv= 0.300</p> <p>tt³+tb³= 0.011375</p> <p>bv³= 0.027</p> <p>l²= 4.410</p> <p>ws= 0.00040</p> <p>as=Sl/Gxws</p> <p>= 0.00500</p> <p>b= 3.250</p> <p>AS2= 0.01625</p>
<p>SAP Model</p> <p>ws= 0.00038 From SAP</p> <p>as=Sl/Gxws</p> <p>= 0.00523</p> <p>b= 3.250</p> <p>AS2= 0.01701</p>	<p>SAP Model</p> <p>ws= 0.00038 From SAP</p> <p>as=Sl/Gxws</p> <p>= 0.00523</p> <p>b= 3.250</p> <p>AS2= 0.01701</p>	<p>SAP Model</p> <p>ws= 0.00039 From SAP</p> <p>as=Sl/Gxws</p> <p>= 0.00510</p> <p>b= 3.250</p> <p>AS2= 0.01657</p>
<p>AS2= 1.1375</p> <p>FM= 0.01467 SAP</p> <p>FM= 0.01495 Form.</p>	<p>AS2= 1.1375</p> <p>FM= 0.01457 SAP</p> <p>FM= 0.01495 Form.</p>	<p>AS2= 1.1375</p> <p>FM= 0.01429 SAP</p> <p>FM= 0.01457 Form.</p>

AS2= 1.1375 from
SAP

AS2 in LOSA1

AS2= 1.1375 from
SAP

AS2 in LOSA2

AS2= 1.1375 from
SAP

AS2 in LOSA3

GRILLAGE ANALOGY METHOD

DISTORSIONAL INERTIA MODIFICATION FACTORS

DISTORSION VLVIGA4	DISTORSION VLVIGA5	DISTORSION VLVIGA6
<p>Hambly Formula</p> <p style="text-align: right;">2534563.</p> <p>E= 5</p> <p style="text-align: right;">1056068.</p> <p>G= 1</p> <p>s= 1.0</p> <p>tt= 0.200</p> <p>tb= 0.150</p> <p>l= 2.100</p> <p>h= 1.560</p> <p>H= 1.385</p> <p>bv= 0.300</p> <p>tt³+tb³= 0.011375</p> <p>bv³= 0.027</p> <p>l²= 4.410</p> <p>ws= 0.00041</p> <p>as=Sl/Gxws</p> <p>= 0.00484</p> <p>b= 3.250</p> <p>AS2= 0.01574</p>	<p>Hambly Formula</p> <p style="text-align: right;">2534563.</p> <p>E= 5</p> <p style="text-align: right;">1056068.</p> <p>G= 1</p> <p>s= 1.0</p> <p>tt= 0.200</p> <p>tb= 0.150</p> <p>l= 2.100</p> <p>h= 1.840</p> <p>H= 1.665</p> <p>bv= 0.300</p> <p>tt³+tb³= 0.011375</p> <p>bv³= 0.027</p> <p>l²= 4.410</p> <p>ws= 0.00043</p> <p>as=Sl/Gxws</p> <p>= 0.00464</p> <p>b= 3.250</p> <p>AS2= 0.01508</p>	<p>Hambly Formula</p> <p style="text-align: right;">2534563.</p> <p>E= 5</p> <p style="text-align: right;">1056068.</p> <p>G= 1</p> <p>s= 1.0</p> <p>tt= 0.200</p> <p>tb= 0.150</p> <p>l= 2.100</p> <p>h= 2.200</p> <p>H= 2.025</p> <p>bv= 0.300</p> <p>tt³+tb³= 0.011375</p> <p>bv³= 0.027</p> <p>l²= 4.410</p> <p>ws= 0.00045</p> <p>as=Sl/Gxws</p> <p>= 0.00440</p> <p>b= 3.250</p> <p>AS2= 0.01431</p>
<p>SAP Model</p> <p>ws= 0.00039 from SAP</p> <p>as=Sl/Gxws</p> <p>= 0.00510</p> <p>b= 3.250</p> <p>AS2= 0.01657</p>	<p>SAP Model</p> <p>ws= 0.00040 from SAP</p> <p>as=Sl/Gxws</p> <p>= 0.00497</p> <p>b= 3.250</p> <p>AS2= 0.01616</p>	<p>SAP Model</p> <p>ws= 0.00042 from SAP</p> <p>as=Sl/Gxws</p> <p>= 0.00473</p> <p>b= 3.250</p> <p>AS2= 0.01539</p>
<p>AS2= 1.1375</p> <p>FM= 0.01384 SAP</p> <p>FM= 0.01457 form</p>	<p>AS2= 1.1375</p> <p>FM= 0.01326 SAP</p> <p>FM= 0.01420 form</p>	<p>AS2= 1.1375</p> <p>FM= 0.01258 SAP</p> <p>FM= 0.01353 form</p>
<p>AS2= 1.1375 from SAP</p> <p>AS2 in LOSA4</p>	<p>AS2= 1.1375 from SAP</p> <p>AS2 in LOSA5</p>	<p>AS2= 1.1375 from SAP</p> <p>AS2 in LOSA6</p>

GRILLAGE ANALOGY METHOD

APPLIED LOADS

1. Self Weight

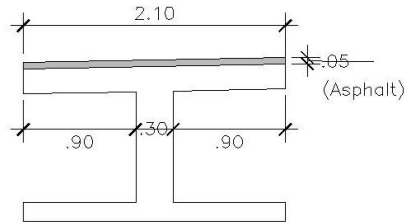
Automatically calculated by the program

Reinforced concrete density, $\gamma = 2.4 \text{ T/m}^3$

2. Dead Weight

Asphalt Weight and railing

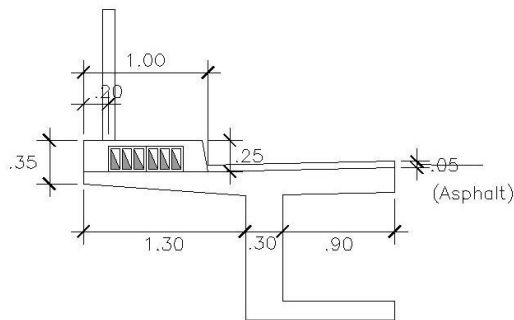
Interior Beams



INTERIOR BEAM

Asphalt weight: $2.10 \times 0.05 \times 2.20 \text{ T/m}^3 = 0.231 \text{ T/m}$

Exterior beams



EXTERIOR BEAM

Asphalt weight: $1.50 \times 0.05 \times 2.20 = 0.165 \text{ T/m}$

Railing: 0.100 T/m

0.265 T/m

Moments $0.165 \times (1.05 - 0.75) = 0.0495$

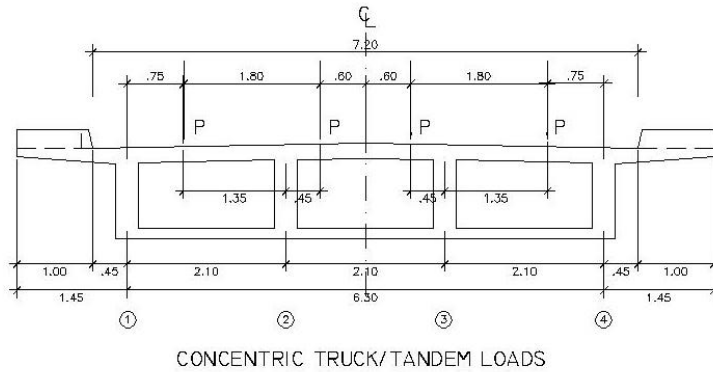
$-0.100 \times (1.45 - 0.20) = -0.125$

$m = -0.0755 \text{ Tm/m}$

GRILLAGE ANALOGY METHOD

3. Vehicular Loads

a. Trucks and Tandem concentric (SC y ST)



$$\text{Beam 1} \quad \frac{1.35P}{2.10} = 0.643P$$

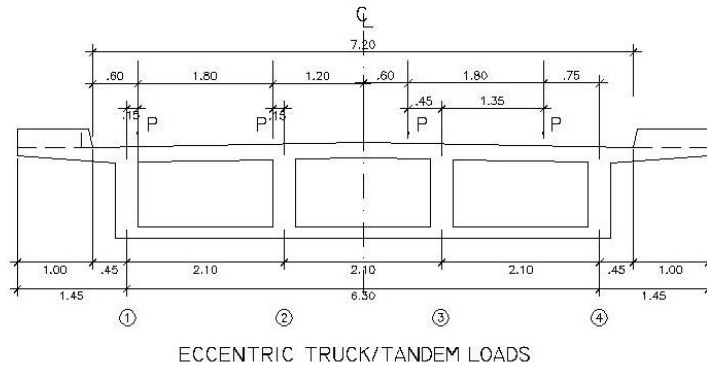
$$\text{Beam 2} \quad \frac{0.75P}{2.10} + \frac{1.65P}{2.10} + \frac{0.45P}{2.10} = \frac{2.85P}{2.10} = 1.357P$$

$$\text{Beam 3} \quad \frac{0.45P}{2.10} + \frac{1.65P}{2.10} + \frac{0.75P}{2.10} = \frac{2.85P}{2.10} = 1.357P$$

$$\text{Beam 4} \quad \frac{1.35P}{2.10} = 0.643P$$

$$\Sigma = 4.000P$$

b. Trucks and Tandem eccentric (SC y ST)



$$\text{Beam 1} \quad \frac{1.95P}{2.10} + \frac{0.15P}{2.10} = 1.000P$$

$$\text{Beam 2} \quad \frac{1.95P}{2.10} + \frac{0.15P}{2.10} + \frac{0.45P}{2.10} = 1.214P$$

$$\text{Beam 3} \quad \frac{1.65P}{2.10} + \frac{0.75P}{2.10} = 1.143P$$

$$\text{Beam 4} \quad \frac{1.35P}{2.10} = 0.643P$$

$$\Sigma = 4.000P$$

Truck Rear Wheel $14.78/2 = 7.39 \text{ T/rueda}$

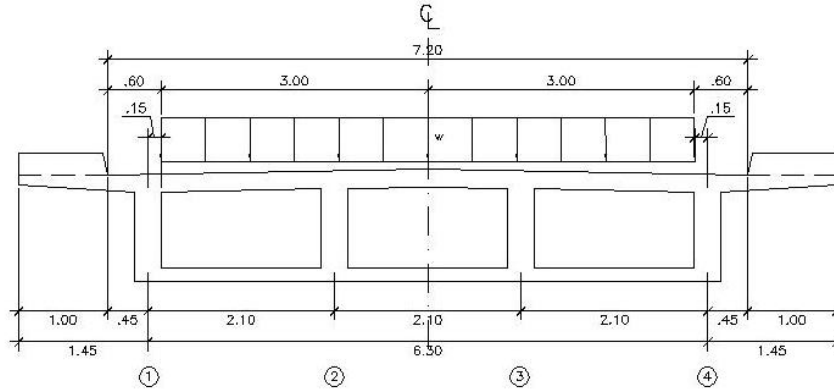
GRILLAGE ANALOGY METHOD

$$7.39 \times 1.00 \times 1.33 = 9.83T$$

Tandem 11.2T each axis $11.2/2 = 5.6 T/\text{rueda}$

$$5.6 \times 1.00 \times 1.33 = 7.4 ST$$

c. Lane Load Concentric



$$\text{Beam 1} \quad \frac{1.95\omega \times 0.975}{2.10} = 0.905\omega$$

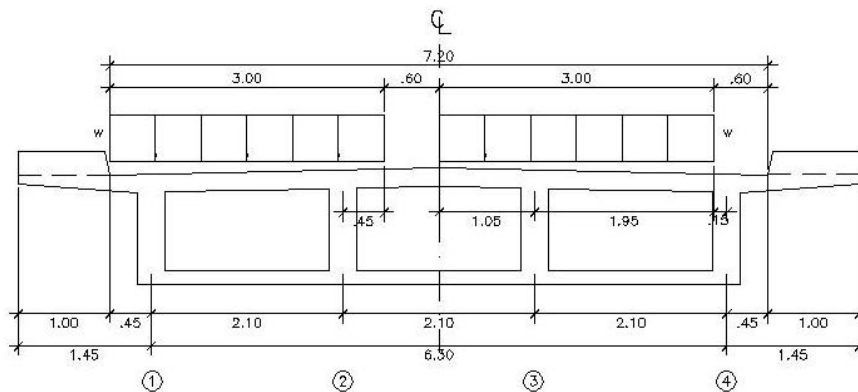
$$\text{Beam 2} \quad \frac{1.95\omega \times 1.125}{2.10} + 1.05\omega = 1.045\omega + 1.05\omega = 2.095\omega$$

$$\text{Beam 3} \quad 1.05\omega + \frac{1.95\omega \times 1.125}{2.10} = 2.095\omega$$

$$\text{Beam 4} \quad \frac{1.95\omega \times 0.975}{2.10} = 0.905\omega$$

$$\Sigma = 6.00\omega$$

d. Lane Load Eccentric



$$\text{Beam 1} \quad \frac{0.45\omega \times 2.325}{2.10} + \frac{2.10\omega \times 1.05}{2.10} = 1.548\omega$$

$$\text{Beam 2} \quad \frac{0.45\omega \times 0.225}{2.10} + \frac{2.10\omega \times 1.05}{2.10} + \frac{0.45\omega \times 1.875}{2.10} + \frac{1.05\omega \times 0.525}{2.10} = 1.666\omega$$

GRILLAGE ANALOGY METHOD

$$\text{Beam 3} \quad \frac{0.45\omega \times 0.225}{2.10} + \frac{1.05\omega \times 1.575}{2.10} + \frac{1.95\omega \times 1.125}{2.10} = 1.880\omega$$

$$\text{Beam 4} \quad \frac{1.95\omega \times 0.975}{2.10} = 0.905\omega$$

$$\Sigma = 5.999\omega$$

$$3.0\omega = 0.97 \text{ T/m} \rightarrow \omega = 0.323 \text{ T/m}^2$$

$$0.905\omega = 0.292 \text{ T/m}$$

$$2.095\omega = 0.677 \text{ T/m}$$

$$1.548\omega = 0.500 \text{ T/m}$$

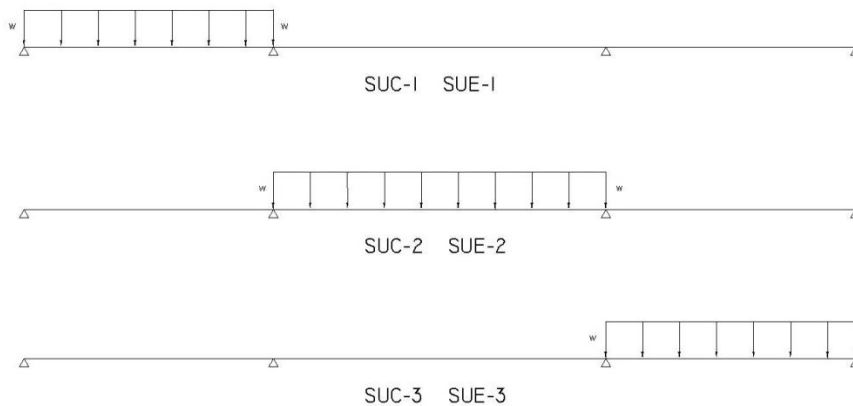
$$1.666\omega = 0.538 \text{ T/m}$$

$$1.880\omega = 0.607 \text{ T/m}$$

$$0.905\omega = 0.292 \text{ T/m}$$

SURCHARGE LOADS COMBINATION

Lane Loads



The same as concentric or eccentric lane loads

Truck and Tandem Loads

Trucks and Tandem Loads travels along the whole length of the lanes, in concentric and eccentric position.

Case of Load SCC SCE Truck AASHTO

Case of Load STC STE Tandem

Lane Loads Cases Combination

$$\text{SUC 123} = \text{SUC1} + \text{SUC2} + \text{SUC3}$$

$$\text{SUC 12} = \text{SUC1} + \text{SUC2}$$

$$\text{SUC 23} = \text{SUC2} + \text{SUC3}$$

$$\text{SUC 13} = \text{SUC1} + \text{SUC3}$$

GRILLAGE ANALOGY METHOD

SUC envelope of concentric lane load cases

Equally SUE envelope of eccentric lane load cases

S/C simultaneous lane and S/C concentrated loads

SC1 = SUC + SCC truck concentric

SC2 = SUC + STC tandem concentric

SC envelope SC1 and SC2

SE1 = SUE + SCE truck eccentric

SE2 = SUE + STE tandem eccentric

SE envelope SE1 and SE2

SMax envelope SC and SE

