Chapter 4 (Part 1)

Growth and Non-Renewable Resources

References:

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Questions in the Context of Non-Renewable Resource Use

Timing: Hotelling 1931, Solow 1974, Stiglitz 1974

- How should resource extraction be optimally allocated over time?
- How are resources extracted over time in a market economy?

Sustainability: Hartwick 1977

- What are the conditions for the use of exhaustible resources to be compatible with sustainable development?
- Role of capital accumulation and technological change.

Long-run growth: Smulders 1999, van den Bergh/de Mooij 1999

- Can economies grow in the long run when resources are essential inputs?
- When is long-run growth feasible and optimal?

Technological Development: Di Maria and Valente 2008

- What is the direction of technological change in a resource-constrained economy?

4.1 Resource Extraction: Fundamental Issues

Resource Extraction by a Competitive Firm

- Determination of resource extraction and price path of a non-renewable resource on a competitive market.

  → partial equilibrium model of the resource extraction sector

- Resource extracting firms own a given and known initial stock of exhaustible resources that they extract over time (assume infinite time horizon)

  natural resource stock: $S(t)$

  initial stock available to firm: $S(0)$

  extraction at each point in time: $R(t)$

  equation of motion for the resource stock: $\dot{S}(t) = -R(t)$

- Intertemporal resource constraint → integrating over time gives stock remaining at each point in time:

$$S(\bar{t}) = S(0) - \int_0^{\bar{t}} R(t)dt$$

- Property rights are perfect → firms face no risk of expropriation
- Further assumptions:
  - no extraction costs
  - perfect competition

    → firms take resource price $p_R(t)$ as exogenous

    → perfect information about future prices
  - an exogenously given interest rate represents the return to alternative investments
- Firms maximize the present value of profits from resource extraction:

$$\max_R \int_0^\infty \pi(t)e^{-rt}dt = \int_0^\infty p_R(t)R(t)e^{-rt}dt$$

subject to

$$\dot{S} = -R$$

$$S(0) = S_0$$

Cost Empirics

Estimates for *all-in costs of producing oil* from various types of hydrocarbons in different parts of the world:

Oilfields/source	Estimated Production Costs ($ 2008)
Mideast/N.Africa oilfields	6 - 28
Other conventional oilfields	6 - 39
CO2 enhanced oil recovery	30 - 80
Deep/ultra-deep-water oilfields	32 - 65
Enhanced oil recovery	32 - 82
Arctic oilfields	32 - 100
Heavy oil/bitumen	32 - 68
Oil shales	52 - 113
Gas to liquids	38 - 113
Coal to liquids	60 - 113

Source: International Energy Agency (2008)

Derivation of Profit-Maximizing Price Path

- Hamiltonian: $H(p, R, \lambda) = p_R \, R \, e^{-rt} - \lambda \, R$

- FOCs: $H_R = 0$: $p_R e^{-rt} = \lambda$ (1)

  $H_S = -\dot{\lambda}$: $0 = \dot{\lambda}$ (2)

  TVC: $\lim_{t \to \infty} S(t) \, \lambda(t) = 0$ (3)

- *Intuition*
  - λ = present (shadow) value of leaving resources in the ground
  - (1) amount extracted is optimal when the present value of the market price p_R is equal to the shadow price
  - (2) in the optimum, the present value of a resource left in the ground must be constant over time

    → present value of profits cannot be increased by reallocation of extraction

From FOC for R: $\boldsymbol{g_{p_R} = r}$ **(Hotelling rule)**

No-arbitrage condition:

- Resource owner can choose between extracting the resource now or in the future

  1. If he decides to extract a unit of the resource, he can invest the profits on the capital market and receive interest on this asset.

  2. If he decides not to leave the resource in the ground for future extraction, this can only be optimal if the additional return from the increase in the resource price compensates for the foregone interest from investing on the capital market.

  → in equilibrium each type of asset in an economy has to earn the same rate of return

Intertemporal efficiency condition:

- The discounted marginal value of extracted resources must be constant over time as $p_R(t)e^{-rt} = p(0)$ for all t.

- From TVC: $\lim_{t\to\infty} S(t)\,\lambda(t) = \lim_{t\to\infty} S(t)\,p_R(t)e^{-rt} = \lim_{t\to\infty} S(t)\,p_R(0) = 0$

  → Given that the initial price of the resource is positive, it is optimal to exhaust the entire resource stock.

$$S_\infty = 0 \quad \Leftrightarrow \quad S_0 = \int_0^\infty R(t)dt$$

Resource Price Level

- To determine the profit maximizing level of the resource price, the Hotelling rule is necessary but not sufficient.

- Components required for the derivation of the price level:

  - Hotelling price path $p_R(t) = p_{R_0} e^{rt}$ → to be determined: p_0
  - initial resource stock S_0
  - resource demand $R(p_R(t))$, $R_{p_R} < 0$

  Example: $R(t) = p_R(t)^{-\gamma}$, $\gamma > 0$ (isoelastic demand function)

→ initial resource price: $S_0 = \int_0^\infty R(t)dt = \int_0^\infty p_R(t)^{-\gamma} dt = p_{R0}^{-\gamma} \int_0^\infty e^{-\gamma r t} dt = \frac{p_{R_0}^{-\gamma}}{\gamma r}$

$$\Leftrightarrow \quad p_{R_0} = (\gamma r S_0)^{-\frac{1}{\gamma}} \quad \rightarrow \quad p_R(t) = (\gamma r S_0)^{-\frac{1}{\gamma}} e^{rt}$$

Resource Extraction Path

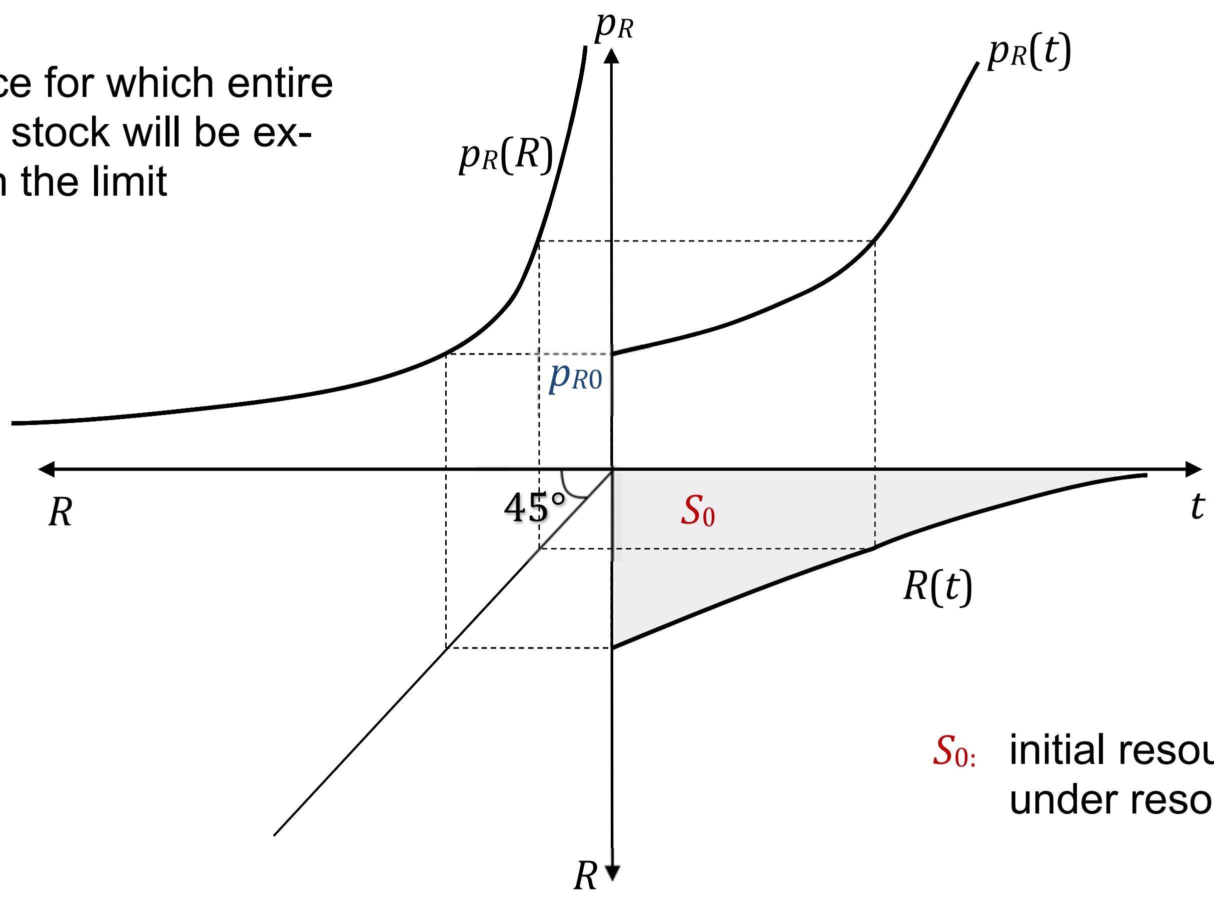
From the resource price and its development over time, the level and development of resource extraction follow immediately:

- initial extraction level: $R_0 = {p_R}_0^{-\gamma} = \gamma r S_0$
- development of resource extraction: $R(t) = p_R(t)^{-\gamma}$ $\Longrightarrow$ $g_R = -\gamma g_{p_R} = -\gamma r$

→ extraction decreases monotonously towards zero if demand is isoelastic: $R(t) = \gamma r S_0 e^{-\gamma r t}$

Graphical Representation: Hotelling Path and Resource Extraction Path

p_{R0}: initial price for which entire resource stock will be extracted in the limit



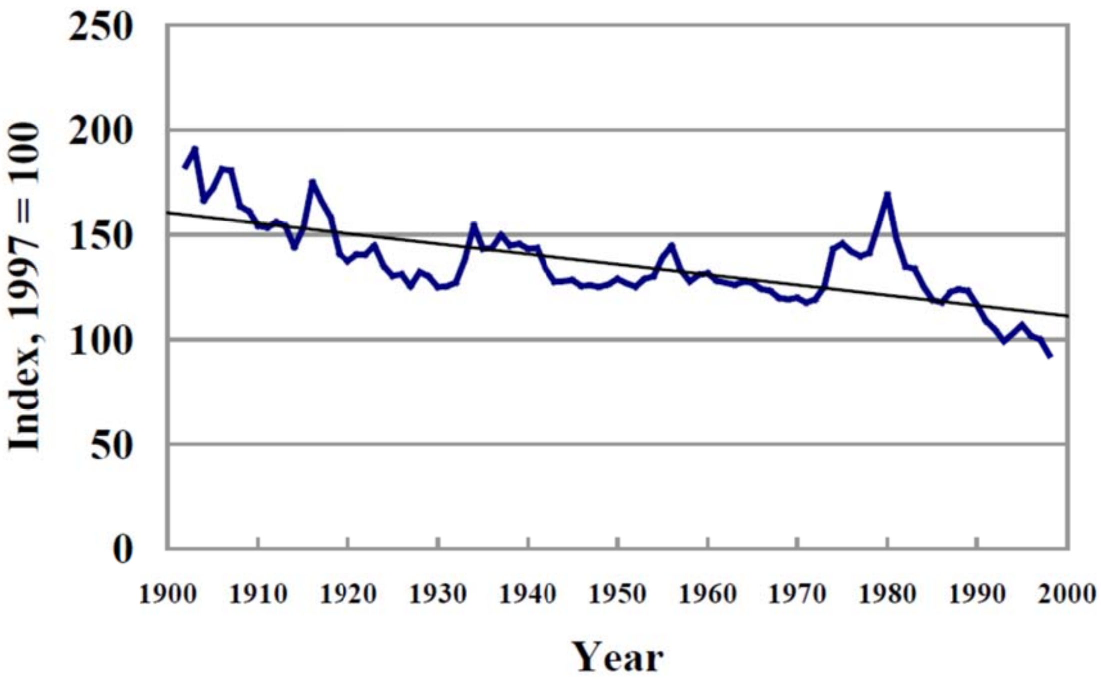
S_0: initial resource stock = area under resource extraction path

Excercise: Derive the profit maximizing growth rate of the price in the presence of extraction costs, i.e. $\pi(t) = p_R(t)R(t) - c(R(t))$. Give intuition for your result.

Empirical Observations

U.S. mine production composite price index

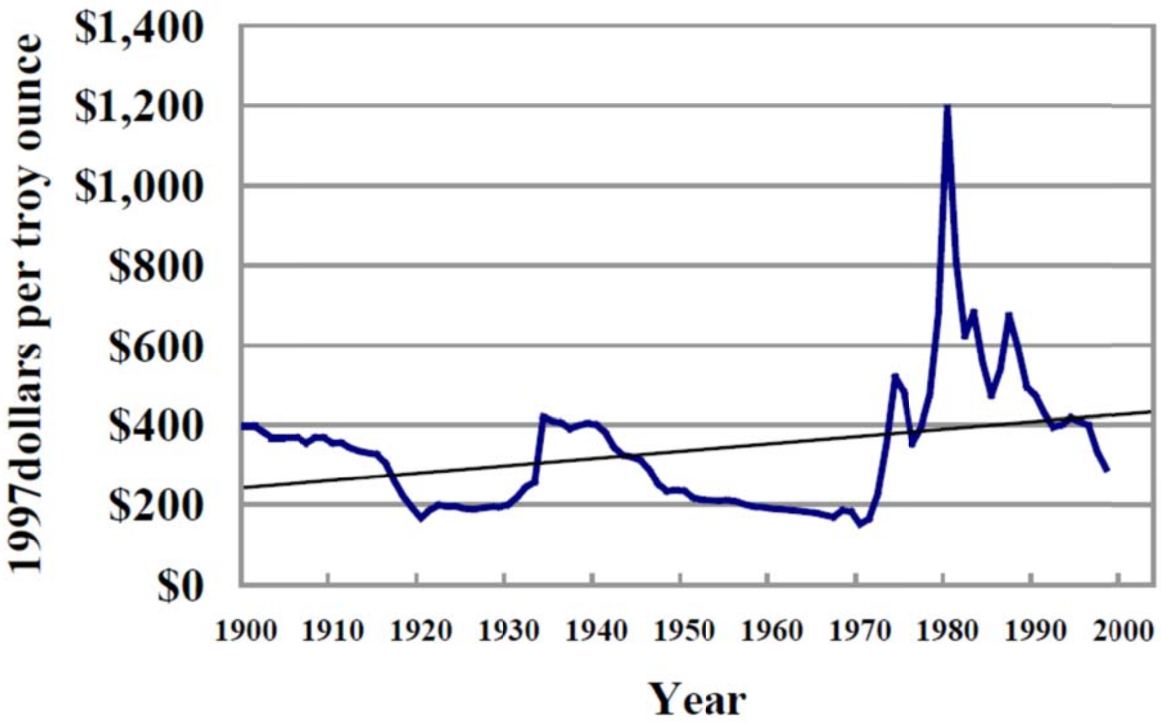
(5 metal commodities: copper, gold, iron ore, lead, zinc, and 7 industrial mineral commodities: cement, clay, crushed stone, lime, phosphate rock, salt, sand and gravel).



Source: USGS (2000)

Gold

(price in constant 1997 dollars)



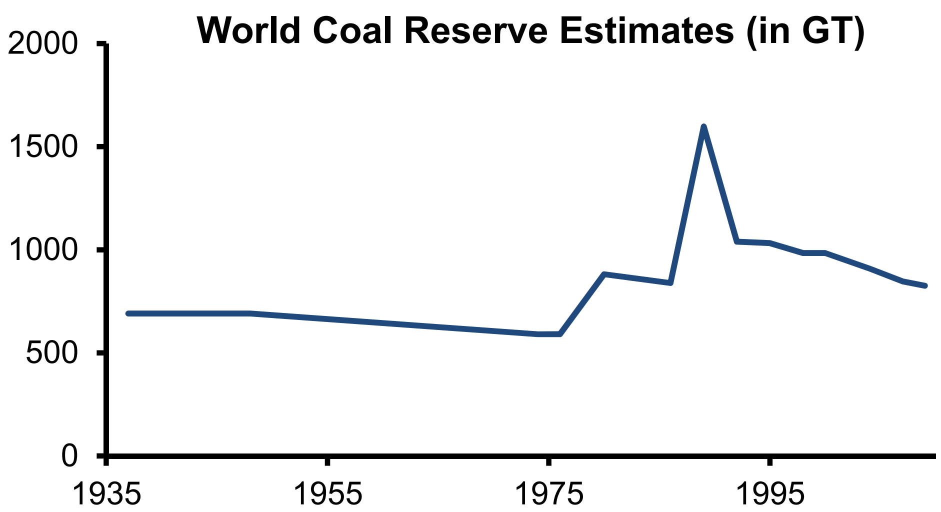
Source: USGS (2000)

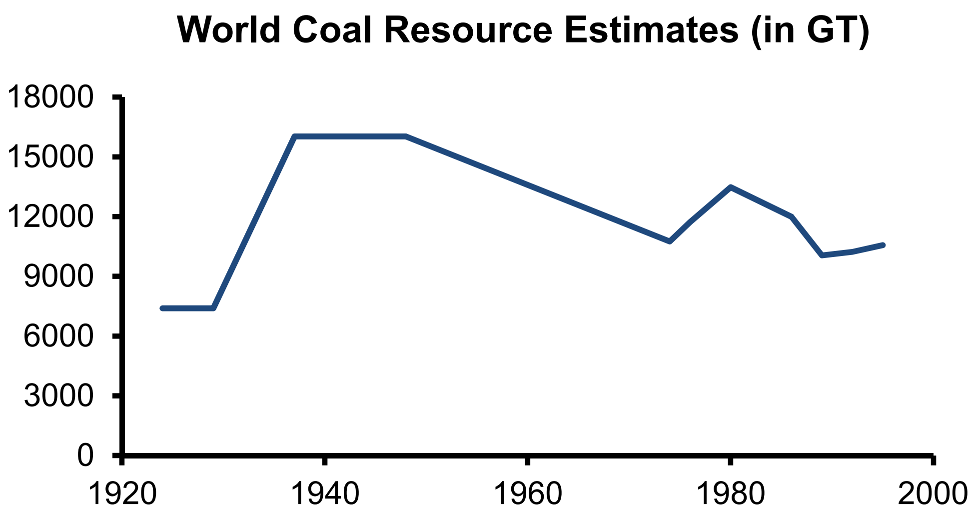
→ Empirically it is hard to find a resource with perpetually growing prices.

Why don't we observe the Hotelling rule?

- new resources were detected or became economically depletable
- short time horizons (of incumbents in office)
- missing or hardly enforceable property rights, risk of expropriation
- technological progress (backstop technologies, costs of extraction)

Empirics of New Discoveries





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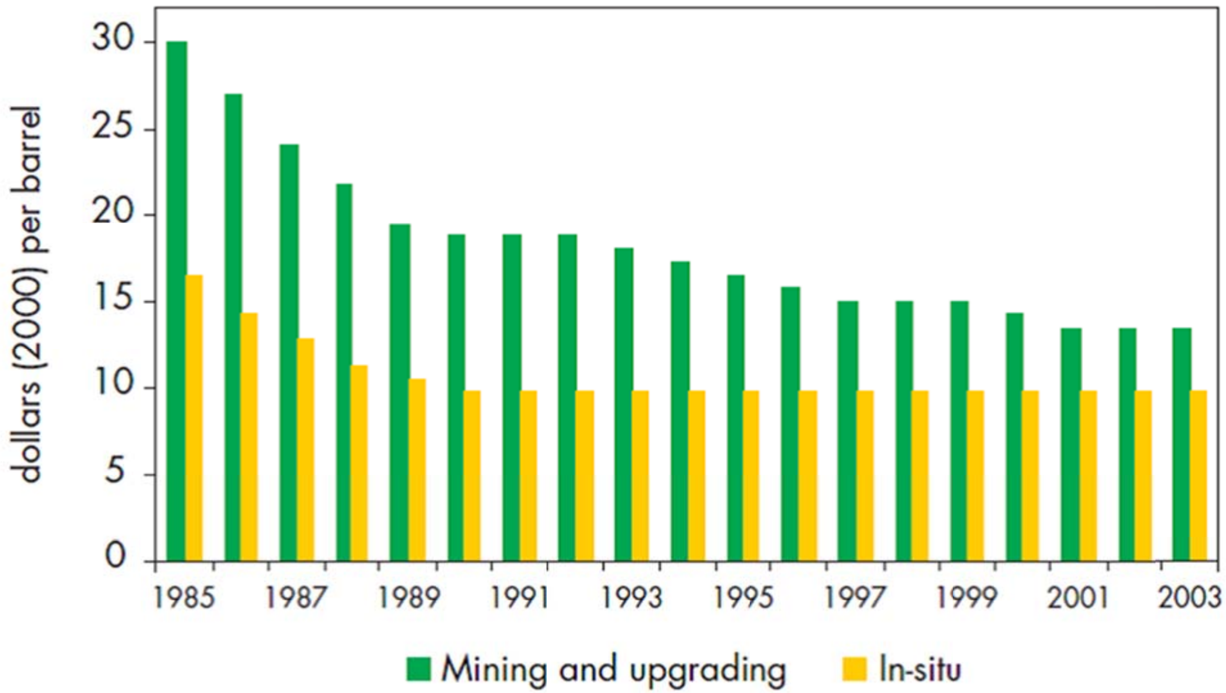
World Energy Council (1924-1960) Statistical Yearbooks
World Energy Council (1962-2009) Surveys of Energy Resources

Empirics of Extraction Cost Development

The case of Canadian Tar Sands



Source: IEA (2004)

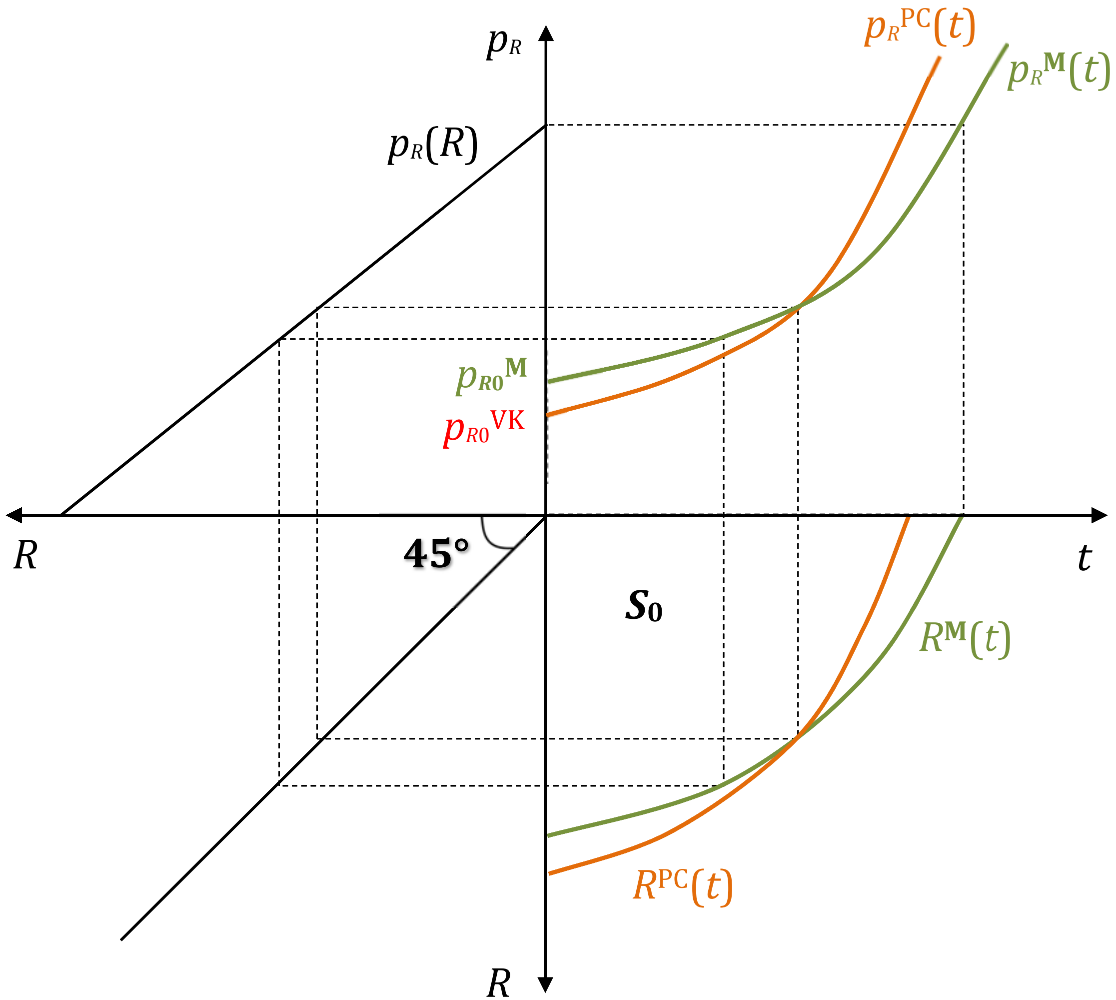
Resource Monopoly

- Perfect competition (PC): - market share of individual firm small. Individual firm takes resource price as given
  → marginal profits rise with constant rate $g_{p_R} = r$
- Monopoly (M): - monopolist chooses profit maximizing price-output combination
  - marginal profits depend on reaction of demand on rising prices (i.e. on the price elasticity of demand)

Example*:* linear demand function (price elasticity rises with increasing prices)

→ optimal for the monopolist to postpone resource extraction (i.e. to lower extraction and charge higher prices today)

→ „monopolist as friend of environmentalist"



Note:

Whether resource extraction is slower in the monopoly case depends crucially on the demand function.

(e.g. isoelastic demand:

no change in price path compared to perfect competition ($g_{p^R} = r$) due to the constant price elasticity of demand)

Profit Maximization of the Monopolist

$$\max_R \int_0^\infty p_R(R(t))R(t)e^{-rt}dt \qquad \text{s.t.} \qquad \dot{S} = -R,\ S(0) = S_0$$

$\boldsymbol{H_S = -\dot{\lambda}}$: $\quad \dot{\lambda} = 0$

$\boldsymbol{H_R = 0}$: $\quad \left(\frac{\partial p_R}{\partial R}R + p_R\right)e^{-rt} = \lambda \iff \left(\frac{\partial p_R}{\partial R}\frac{R}{p_R} + 1\right)p_R e^{-rt} = \lambda \iff \left(1 + \frac{1}{\epsilon}\right)p_R e^{-rt} = \lambda$

with ϵ = price elasticity of demand < 0

Taking the time derivative gives: $\left[\left(1 + \dot{\frac{1}{\epsilon}}\right)p_R + \left(1 + \frac{1}{\epsilon}\right)\dot{p_R}\right]e^{-rt} - r\left(1 + \frac{1}{\epsilon}\right)p_R e^{-rt} = \dot{\lambda} = 0$

$$r = \frac{\left(1+\dot{\frac{1}{\epsilon}}\right)p_R + \left(1+\frac{1}{\epsilon}\right)\dot{p_R}}{\left(1+\frac{1}{\epsilon}\right)p_R} \qquad \iff \qquad r = g_{p_R} + g_{\left(1+\frac{1}{\epsilon}\right)}$$

→ isoelastic demand: $\quad g_{\left(1+\frac{1}{\epsilon}\right)} = 0 \quad \rightarrow \quad g_{p_R} = r$

→ linear demand: $\quad g_{\left(1+\frac{1}{\epsilon}\right)} > 0 \quad \rightarrow \quad g_{p_R} < r$

Green Paradox

→ Seemingly paradox reaction of the extraction of fossil energy resources on increasingly strict climate policy

- example: tax on the sale of exhaustible resource (tax rate: τ)

→ price which resource owners receive: $p_R^\tau = p_R\,(1-\tau)$

→ profit maximizing price path: $g_{p_R^\tau} = r \quad \Leftrightarrow \quad \boldsymbol{g_{p_R} = r + g_\tau \frac{\tau}{1-\tau}}$

Tax scenarios:

1. constant tax rate ($g_\tau = 0$): $g_{p_R} = r$ → no effect on price path
2. decreasing tax rate ($g_\tau < 0$): $g_{p_R} < r$ → price rises slower
3. increasing tax rate ($g_\tau > 0$): $g_{p_R} > r$ → price rises faster (adjustment of optimal time path:

→ short term: price lower and extraction higher

→ long term: price higher and extraction lower)

Green Paradox

Intuition

Constant tax rate

→ only effect of taxation: price that resource owner receives is lower due to taxation

→ redistribution of rents from resource owner to policy maker

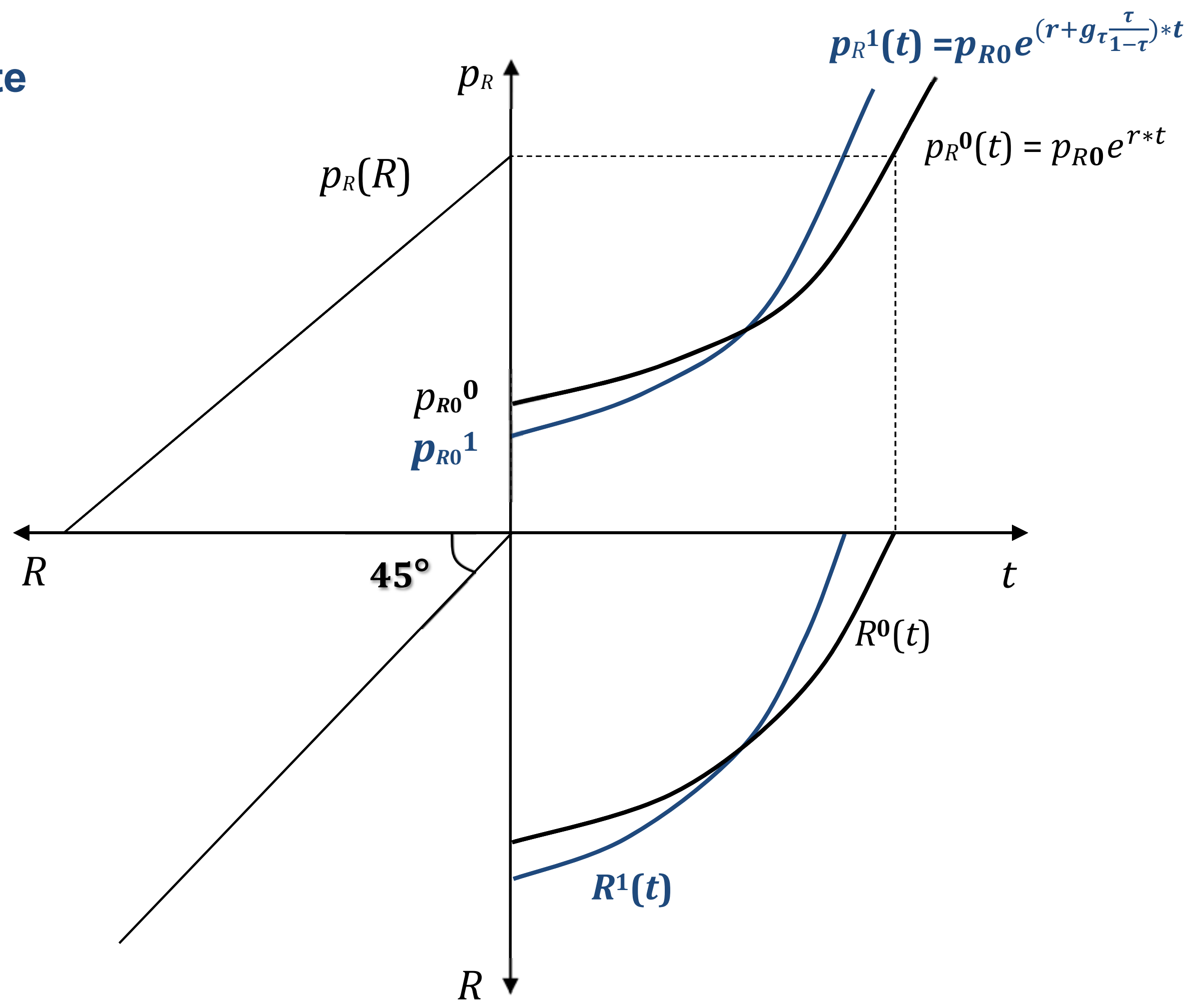
Increasing tax rate

→ resource owner knows that the present value of future marginal profits decreases over time due to increasingly strict taxation

→ optimal to sell more of the resource today when marginal profits are higher

Green Paradox

Increasing tax rate



4.2 Socially Optimal Allocation of Extraction

- How should the extraction of a non-renewable resource be socially optimally allocated over time?
- Answer depends crucially on choice of the objective function

Types of objective functions

1. $\max_C \int_0^\infty U(C)e^{-\rho t}dt$ → maximize present value of utility from consumption
   → see section 4.2.1
2. $\max_C \int_0^\infty U(C,S)e^{-\rho t}dt$ → present value maximization when the natural resource has a positive effect on utility (Krautkrämer 1985)
   → see section 4.2.2
3. $\max_C \ \inf\{u(C)\}$ → maximize utility of worst off generation (Rawls 1971)
4. $\max_C \alpha \int_0^\infty U(C)e^{-\rho t}dt + (1-\alpha)\lim_{t\to\infty} u(C)$ → maximize weighted average of present value and utility of the last generation (Chichilnisky 1997)

Resources as Consumption Goods: The Cake Eating Problem

- Consider a simple economy in which extracted exhaustible resources are directly used for consumption $(\dot{S} = -C)$

  → allocation of a cake of a given size between generations

- Independent of choice of objective function: Consumption of the resource will decrease at some point due to limited availability

4.2.1 Resources as Inputs to Production: The Solow-Stiglitz Efficiency Condition

- Derivation of the *welfare optimal* extraction path

- Assumption: output that is produced from capital and exhaustible resources can be used for consumption and capital accumulation

- Optimization problem of a social planner ($L = 1$):

$$\max_{C,R} \int_0^\infty \frac{C^{1-\sigma}}{1-\sigma} e^{-\rho t} dt$$

subject to

$$\dot{K} = F(K,R) - C \qquad F_K > 0, F_{KK} < 0, F_R > 0, F_{RR} < 0$$

$$\dot{S} = -R$$

$$S(0) = S_0, \qquad K(0) = K_0$$

→ two control variables (C, R) and two state variables (K, S)

- Hamiltonian: $H = \frac{C^{1-\sigma}}{1-\sigma} e^{-\rho t} + \mu(F(K,R) - C) - \lambda R$

- FOCs:
  - $H_C = 0$: $C^{-\sigma} e^{-\rho t} = \mu$
  - $H_R = 0$: $\mu F_R = \lambda$
  - $H_K = -\dot{\mu}$: $\dot{\mu} = -\mu F_K$
  - $H_S = -\dot{\lambda}$: $\dot{\lambda} = 0$

- From FOCs for *C* and *K*: $g_C = \frac{1}{\sigma}(F_K - \rho)$

- From FOCs for *R, S* and *K*: $\boldsymbol{g_{F_R} = F_K}$ → **Solow-Stiglitz Efficiency Condition (SSEC)**

→ *Intuition*: along the optimal path, the marginal productivity of capital has to be equal to the growth rate of the marginal productivity of the resource

→ *Analogy to private optimum on competitive markets*: $g_{F_R} = g_{p_R}$ and $F_K = r$

→ private optimum is also pareto-optimal

4.2.2 The SSEC with Preferences for Resources

$$\max_{C,R} \int_0^\infty U(C,S)e^{-\rho t}dt, \qquad U_C > 0, U_{CC} < 0, U_S > 0, U_{SS} < 0$$

subject to

$$\dot{K} = F(K,R) - C$$

$$\dot{S} = -R$$

$$S(0) = S_0, \qquad K(0) = K_0$$

- Hamiltonian: $H = U(C,S)e^{-\rho t} + \mu(F(K,R) - C) - \lambda R$

- FOCs:
  $H_C = 0$: $\quad U_C e^{-\rho t} = \mu$

  $H_R = 0$: $\quad \mu F_R = \lambda$

  $H_K = -\dot{\mu}$: $\quad \dot{\mu} = -\mu F_K \quad \rightarrow \quad g_\mu = -F_K$

  $H_S = -\dot{\lambda}$: $\quad \dot{\lambda} = -U_S e^{-\rho t}$

- Derivation of modified Keynes-Ramsey rule (from FOCs for C and K)

$\dot{\mu} = (U_{CC}\dot{C} + U_{CS}\dot{S})e^{-\rho t} - \rho U_C e^{-\rho t}$ $\quad\rightarrow\quad g_\mu = \frac{U_{CC}C}{U_C} g_C + \frac{U_{CS}}{U_C}\dot{S} - \rho$

$\rightarrow\quad g_C = -\frac{U_C}{U_{CC}C}\left(F_K + \frac{U_{CS}}{U_C}\dot{S} - \rho\right)$

Intuition:

$U_{CS} = \frac{\partial U_C}{\partial S} > 0$: the marginal utility of consumption decreases when the stock of natural resources shrinks

$\rightarrow\quad \frac{U_{CS}}{U_C}\dot{S} < 0$ for $\dot{S} < 0$:

faster resource extraction lowers the growth rate as it affects the utility negatively and thereby raises the costs of growth

$U_{CS} = \frac{\partial U_C}{\partial S} < 0$: the marginal utility of consumption increases for a decrease in S

$\rightarrow\quad \frac{U_{CS}}{U_C}\dot{S} > 0$ for $\dot{S} < 0$:

faster resource extraction raises the growth rate as consumption is valued more when natural resources become scarcer

- Derivation of modified SSEC (from FOCs for *S, R* and *K*)

$$\mu F_R = \lambda \quad \rightarrow \quad g_\mu + g_{F_R} = g_\lambda$$

$$\dot{\lambda} = -U_S e^{-\rho t} \quad \rightarrow \quad g_\lambda = -\frac{U_S}{U_C}\frac{1}{F_R}$$

$$\Rightarrow \quad F_K = g_{F_R} + \frac{U_S}{U_C}\frac{1}{F_R}$$

Intuition:

Standard SSEC: If resources have no amenity value, the loss of the output that could have been produced if an additional unit of the resource had been extracted and invested, has to be compensated by the increase in the value of the resource.

Modified SSEC: If resources have an amenity value, the compensation in equilibrium is lower as the non-extracted resources also increase utility.

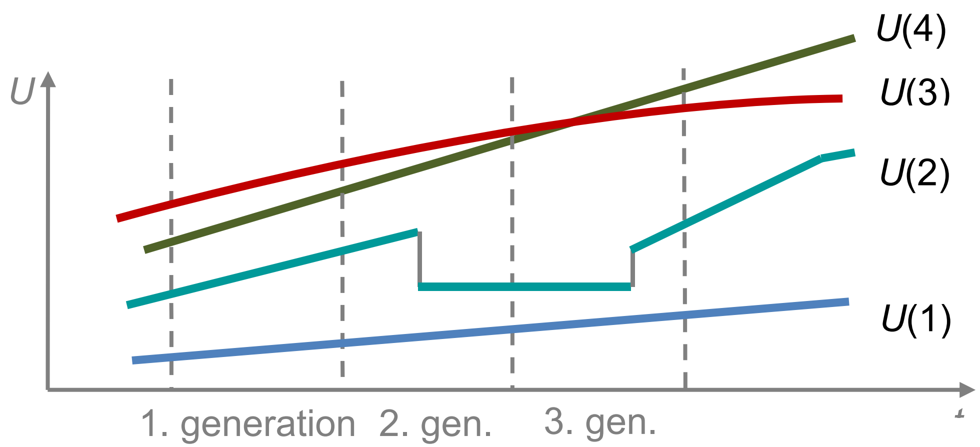
4.2 Sustainability and Non-Renewable Resources

Economic Interpretation of Sustainability

Pezzey (1997): *„I see little point in expanding the collection of fifty sustainability definitions which I made in 1989, to the five thousand definitions that one could readily find today“*

Frequently employed criterion: $\boldsymbol{\dot{U} = \frac{dU}{dt} \geq 0}$ → non-decreasing utility over time

(Alternatively, if utility depends solely on consumption: $\dot{C} = \frac{dC}{dt} \geq 0$)



Sustainable paths: $U(1), U(3), U(4)$

Non-sustainable path: $U(2)$

→ Recall the *Cake Eating Problem*: Non-decreasing level of utility/consumption with $C > 0$ cannot be maintained over time

→ not sustainable

→ Under which conditions is sustainability possible given that exhaustible resources are employed in production?

→ Under which conditions is not only a constant but a rising level of utility possible?

→ Obviously crucial: Potential to substitute out of the diminishing input of exhaustible resources

4.2.1 The Role of the Substitution Elasticity

- The elasticity of substitution describes the possibility to substitute the input of one factor of production by another.

- Assume that output is produced from resources and capital ($F(R,K)$). Then the elasticity of substitution between the two inputs is given by:

$$\varepsilon = \frac{d\left(\frac{K}{R}\right)/\frac{K}{R}}{d\left(\frac{p_R}{p_K}\right)/\frac{p_R}{p_K}} = \frac{d\left(\frac{K}{R}\right)/\frac{K}{R}}{d\left(\frac{F_R}{F_K}\right)/\frac{F_R}{F_K}}$$

$\rightarrow$ relative change of the input ratio of two production factors given a change in relative factor prices.

- Consider the Constant-Elasticity-of-Substitution (CES) production function

$$\boldsymbol{Y = F(R,K)) = (\alpha K^{\frac{\varepsilon-1}{\varepsilon}} + \beta R^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}}$$

where ε denotes the elasticity of substitution between capital and resources.

Exercise: Derivation of Elasticity of Substitution for CES-function

$$Y = (\alpha K^{\frac{\varepsilon-1}{\varepsilon}} + \beta R^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}$$

- $F_R = (\alpha K^{\frac{\varepsilon-1}{\varepsilon}} + \beta R^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}-1} \beta R^{\frac{\varepsilon-1}{\varepsilon}-1}$
- $F_K = (\alpha K^{\frac{\varepsilon-1}{\varepsilon}} + \beta R^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}-1} \alpha K^{\frac{\varepsilon-1}{\varepsilon}-1}$

$$\frac{F_R}{F_K} = \frac{\beta R^{\frac{\varepsilon-1}{\varepsilon}-1}}{\alpha K^{\frac{\varepsilon-1}{\varepsilon}-1}} = \frac{\beta}{\alpha}\left(\frac{K}{R}\right)^{\frac{1}{\varepsilon}}$$

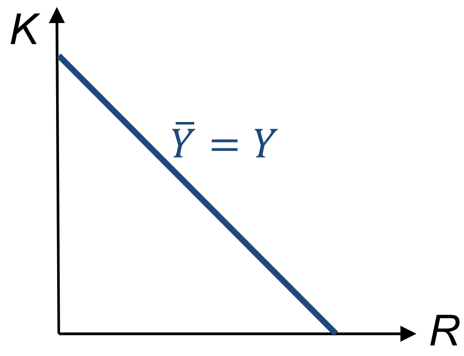
- $\frac{d\frac{F_R}{F_K}}{d\frac{K}{R}} = \frac{1}{\varepsilon}\frac{\beta}{\alpha}\left(\frac{K}{R}\right)^{\frac{1}{\varepsilon}-1}$
- Elasticity of substitution: $\frac{d\left(\frac{K}{R}\right)/\frac{K}{R}}{d\left(\frac{F_R}{F_K}\right)/\frac{F_R}{F_K}} = \left(\frac{d\frac{F_R}{F_K}}{d\frac{K}{R}}\right)^{-1} \frac{\frac{F_R}{F_K}}{\frac{R}{K}} = \varepsilon \frac{\alpha}{\beta}\left(\frac{K}{R}\right)^{-\frac{1}{\varepsilon}+1} \frac{\frac{\beta}{\alpha}\left(\frac{K}{R}\right)^{\frac{1}{\varepsilon}}}{\frac{K}{R}} = \varepsilon$

Examples

ε > 1:

$$\lim_{R\to 0}\left(\alpha K^{\frac{\varepsilon-1}{\varepsilon}}+\beta R^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}=\alpha^{\frac{\varepsilon}{\varepsilon-1}}K$$

→ production possible without the input of resources (*non-essential resource*)



ε = ∞:

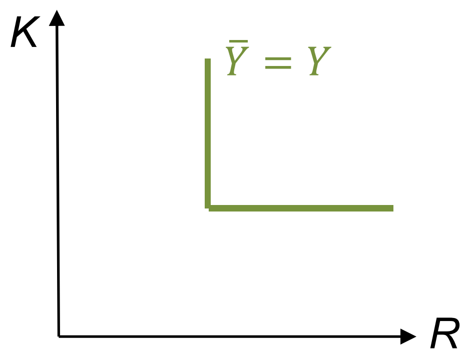
$$Y=\alpha K+\beta R$$

ε < 1:

$$\lim_{R\to 0}\frac{Y}{R}=\lim_{R\to 0}\left(\alpha\left(\frac{K}{R}\right)^{\frac{\varepsilon-1}{\varepsilon}}+\beta\right)^{\frac{\varepsilon}{\varepsilon-1}}=\beta^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\lim_{R\to 0} R\left(\frac{Y}{R}\right)=0$$

→ capital accumulation cannot compensate for the decreasing resource input (*essential resource*)
→ doomsday



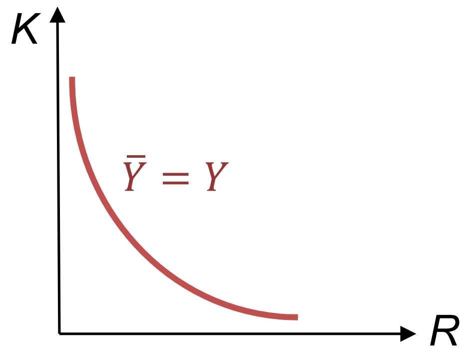
ε = 0:

$$Y=\min\{\alpha K,\beta R\}$$

ε = 1:

$$\lim_{\varepsilon\to 1}\left(\alpha K^{\frac{\varepsilon-1}{\varepsilon}}+\beta R^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}=K^{\alpha}R^{\beta}$$

$$\lim_{R\to 0}\left(\frac{Y}{R}\right)=\lim_{R\to 0}K^{\alpha}R^{\beta-1}=\infty$$



→ capital and resources are both essential for production, but the input of the resource going to zero does not imply that production also goes to zero

→ prerequisite: sufficient accumulation of capital

→ Obviously, the decreasing input of resources has to be compensated by the accumulation of capital.

→ Question: How much has to be invested in the capital stock to keep output at least constant over time?

4.4.2 The Hartwick Rule

The Cobb-Douglas Case

- $Y = K^{\alpha} R^{1-\alpha}$ $\quad\rightarrow\quad$ $g_Y = \alpha g_K + (1-\alpha) g_R = 0$

  $\rightarrow\quad g_K = -\frac{1-\alpha}{\alpha} g_R$

- from the Solow-Stiglitz condition: $F_K = \frac{\dot{F_R}}{F_R} \Leftrightarrow \alpha \frac{Y}{K} = g_{(1-\alpha)\frac{Y}{R}} = g_Y - g_R = -g_R$

- $g_K = \frac{1-\alpha}{\alpha} \alpha \frac{Y}{K} = (1-\alpha) \frac{Y}{K}$ $\quad\rightarrow\quad$ $\dot{K} = (1-\alpha) Y$ $\quad\rightarrow\quad$ *if households save enough for this rule to hold, income is constant over time*

Sustainability in the Solow model with exhaustible resources

$\dot{K} = sY \quad\rightarrow\quad s = (1-\alpha)$

sustainability possible if savings rate is equal to income share of resource

Sustainability in the Ramsey model with exhaustible resources

- Keynes-Ramsey rule: $g_C = \frac{1}{\sigma}(F_K - \rho)$

- Hotelling rule: $\frac{\dot{F_R}}{F_R} = F_K \quad \Leftrightarrow \quad \left(\frac{\dot{K}}{R}\right) = \left(\frac{K}{R}\right)^{\alpha}$ as $F_K = \alpha \left(\frac{K}{R}\right)^{\alpha-1}$

$$F_R = (1-\alpha)\left(\frac{K}{R}\right)^{\alpha}$$

$$\dot{F_R} = \alpha(1-\alpha)\left(\frac{K}{R}\right)^{\alpha-1}\left(\frac{\dot{K}}{R}\right)$$

$$\rightarrow \lim_{R\to 0} K/R = \infty$$

$$\rightarrow \lim_{R\to 0} F_K = 0$$

$$\rightarrow \lim_{R\to 0} g_C = \frac{1}{\sigma}(0-\rho) < 0$$

→ as the marginal product of capital decreases with the falling input of resources, the incentives to save decrease

→ not enough capital accumulation to keep output from decreasing (due to preference for present consumption)

→ sustainable development not optimal

Important Assumptions

- no technological progress
- no population growth
- resources are non-renewable
- elasticity of substitution = 1

changes in assumptions
→ modifications of Hartwick rule

1. Example: Technological progress $Y = F(K, AR) = K^{\alpha}(AR)^{1-\alpha}$ with $g_A > 0$

Solow model → $s = (1-\alpha)\frac{Y}{K} - \frac{1-\alpha}{\alpha} g_A$ → less than the resource income has to be saved for production to be non-decreasing

Ramsey model → $g_C = \frac{1}{\sigma}\left(\alpha \left(\frac{AR}{K}\right)^{1-\alpha} - \rho\right)$ → technological progress can prevent the decrease of the marginal product of capital

→ Stiglitz (1974): If the rate of resource-augmenting technical progress is sufficiently high, consumption is sustained in the long run. Resource use declines, but the productivity loss is compensated by means of innovations.

Empirics: Resource- and energy-saving technological progress

- During the period of 1900 to the 1960s, the quantity of coal required to generate a kwh electricity fell from nearly seven pounds to less than one pound.

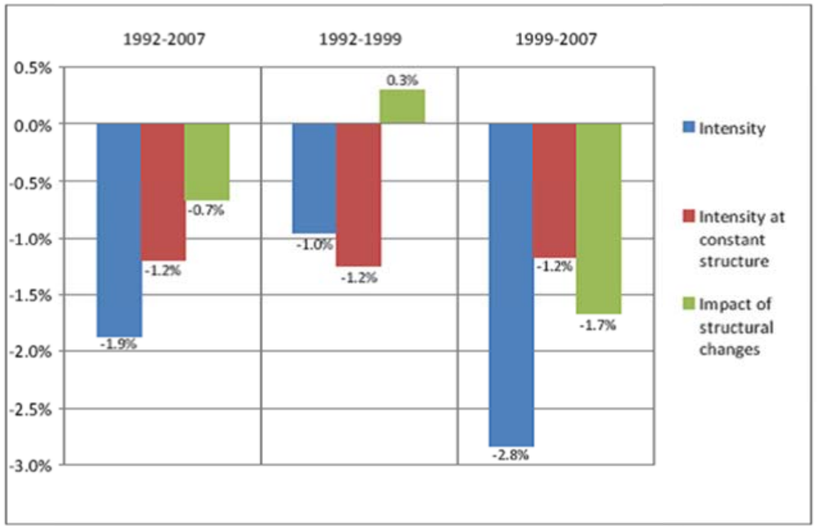
- Energy consumption in tonnes of oil equivalent per million constant 2000 international $:

OECD/IEA (2008)

1990	2000	2005
279.6	204.3	171.7

- Energy intensity in the German manufac-turing industry (Fraunhofer 2009)

Figure 3-6: Energy efficiency and structural change effects in manufacturing industry



Source: Calculations ODYSSEE database

2. Example: Substitution Elasticity $\neq 1$: CES-Production Function $Y = (\alpha K^{\frac{\varepsilon-1}{\varepsilon}} + \beta R^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}$

- Differentiating output wrt time gives $\dot{Y} = (\cdot)^{\frac{\varepsilon}{\varepsilon-1}-1}(\alpha K^{\frac{\varepsilon-1}{\varepsilon}-1}\dot{K} + \beta R^{\frac{\varepsilon-1}{\varepsilon}-1}\dot{R})$

$$\Leftrightarrow \quad \frac{\dot{Y}}{Y} = Y^{\frac{1-\varepsilon}{\varepsilon}}\left(\alpha K^{\frac{\varepsilon-1}{\varepsilon}}\frac{\dot{K}}{K} + \beta R^{\frac{\varepsilon-1}{\varepsilon}}\frac{\dot{R}}{R}\right)$$

$$\Leftrightarrow \quad \frac{\dot{Y}}{Y} = \left(\alpha\left(\frac{K}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\frac{\dot{K}}{K} + \beta\left(\frac{R}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\frac{\dot{R}}{R}\right) \qquad \text{(a)}$$

- Solow-Stiglitz efficiency condition: $\frac{\dot{F_R}}{F_R} = F_K$

$$F_K = \alpha\left(\frac{K}{Y}\right)^{-\frac{1}{\varepsilon}}$$

$$F_R = \beta\left(\frac{R}{Y}\right)^{-\frac{1}{\varepsilon}} \quad \rightarrow \quad \frac{\dot{F_R}}{F_R} = -\frac{1}{\varepsilon}\left(\frac{\dot{R}}{R} - \frac{\dot{Y}}{Y}\right) = -\frac{1}{\varepsilon}\left(\left(1 - \beta\left(\frac{R}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)\frac{\dot{R}}{R} - \alpha\left(\frac{K}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\frac{\dot{K}}{K}\right)$$

such that $\frac{\dot{F_R}}{F_R} = F_K \quad \Leftrightarrow \quad \alpha\left(\frac{K}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\varepsilon\frac{Y}{K}-\frac{\dot{K}}{K}\right) = -\left(1-\beta\left(\frac{R}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)\frac{\dot{R}}{R}$

$$\Leftrightarrow \quad -\alpha\left(\frac{K}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\frac{\left(\varepsilon\frac{Y}{K}-\frac{\dot{K}}{K}\right)}{\left(1-\beta\left(\frac{R}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)} = \frac{\dot{R}}{R}$$

Inserting into (a) gives: $\frac{\dot{Y}}{Y} = \alpha\left(\frac{K}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\frac{\dot{K}}{K}\left(1+\frac{\beta\left(\frac{R}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}}{1-\beta\left(\frac{R}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}}\right)+\alpha\,\beta\,\varepsilon\left(\frac{\left(\frac{R}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{K}{Y}\right)^{-\frac{1}{\varepsilon}}}{1-\beta\left(\frac{R}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}}\right)$

Therefore, for development to be sustainable it has to hold that

$$\frac{\dot{Y}}{Y} \geq 0 \quad \Leftrightarrow \quad \alpha\left(\frac{K}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\frac{\dot{K}}{K} \geq -\alpha\,\beta\,\varepsilon\left(\left(\frac{R}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{K}{Y}\right)^{-\frac{1}{\varepsilon}}\right)$$

$$\Leftrightarrow \quad \frac{\dot{K}}{K} \geq -\beta\,\varepsilon\left(\frac{R}{Y}\right)^{\frac{\varepsilon-1}{\varepsilon}}\frac{Y}{K} = -\beta\,\varepsilon\left(\frac{R}{Y}\right)^{-\frac{1}{\varepsilon}}\frac{R}{K}$$

$\Leftrightarrow$ $\boldsymbol{\dot{K} \geq -\varepsilon F_R R}$ (for $\varepsilon = 1$, we are back at $\dot{K} \geq -F_R R$)