

Guía 1: FIS140

March 27, 2016

Bibliografía: Sears & Zemansky's University Physics, 13th Edition.

- Ondas mecánicas: 15.1, 15.3, 15.7, 15.23, 15.49
- Ondas electromagnéticas: 32.1, 32.4, 32.7, 32.17
- Naturaleza y propagación de la luz: 33.1, 33.4, 33.11, 33.19, 33.26, 33.28, 33.29
- Óptica geométrica: 34.2, 34.7, 34.18, 34.39, 34.58
- Interferencia: 35.1, 35.4, 35.11, 35.35, 35.30
- Difracción: 36.1, 36.4, 36.14, 36.41, 36.39

Waves and their properties: A wave is any disturbance that propagates from one region to another. A mechanical wave travels within some material called the medium. The wave speed v depends on the type of wave and the properties of the medium.

In a periodic wave, the motion of each point of the medium is periodic with frequency f and period T . The wavelength λ is the distance over which the wave pattern repeats, and the amplitude A is the maximum displacement of a particle in the medium. The product of λ and f equals the wave speed. A sinusoidal wave is a special periodic wave in which each point moves in simple harmonic motion. (See Example 15.1.)

$$v = \lambda f \quad (15.1)$$

Wave functions and wave dynamics: The wave function $y(x, t)$ describes the displacements of individual particles in the medium. Equations (15.3), (15.4), and (15.7) give the wave equation for a sinusoidal wave traveling in the $+x$ -direction. If the wave is moving in the $-x$ -direction, the minus signs in the cosine functions are replaced by plus signs. (See Example 15.2.)

The wave function obeys a partial differential equation called the wave equation, Eq. (15.12).

The speed of transverse waves on a string depends on the tension F and mass per unit length μ . (See Example 15.3.)

$$\begin{aligned} y(x, t) &= A \cos \left[\omega \left(\frac{x}{v} - t \right) \right] \\ &= A \cos 2\pi f \left(\frac{x}{v} - t \right) \end{aligned} \quad (15.3)$$

$$y(x, t) = A \cos 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \quad (15.4)$$

$$y(x, t) = A \cos(kx - \omega t) \quad (15.7)$$

$$\text{where } k = 2\pi/\lambda \text{ and } \omega = 2\pi f = vk$$

$$\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x, t)}{\partial t^2} \quad (15.12)$$

$$v = \sqrt{\frac{F}{\mu}} \quad (\text{waves on a string}) \quad (15.13)$$

Wave power: Wave motion conveys energy from one region to another. For a sinusoidal mechanical wave, the average power P_{av} is proportional to the square of the wave amplitude and the square of the frequency. For waves that spread out in three dimensions, the wave intensity I is inversely proportional to the square of the distance from the source. (See Examples 15.4 and 15.5.)

$$P_{\text{av}} = \frac{1}{2} \sqrt{\mu F} \omega^2 A^2 \quad (15.25)$$

(average power, sinusoidal wave)

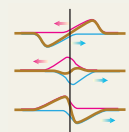
$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad (15.26)$$

(inverse-square law for intensity)

Wave superposition: A wave reflects when it reaches a boundary of its medium. At any point where two or more waves overlap, the total displacement is the sum of the displacements of the individual waves (principle of superposition).

$$y(x, t) = y_1(x, t) + y_2(x, t) \quad (15.27)$$

(principle of superposition)



Standing waves on a string: When a sinusoidal wave is reflected from a fixed or free end of a stretched string, the incident and reflected waves combine to form a standing sinusoidal wave with nodes and antinodes. Adjacent nodes are spaced a distance $\lambda/2$ apart, as are adjacent antinodes. (See Example 15.6.)

When both ends of a string with length L are held fixed, standing waves can occur only when L is an integer multiple of $\lambda/2$. Each frequency with its associated vibration pattern is called a normal mode. (See Examples 15.7 and 15.8.)

$$y(x, t) = (A_{\text{SW}} \sin kx) \sin \omega t \quad (15.28)$$

(standing wave on a string, fixed end at $x = 0$)

$$f_n = n \frac{v}{2L} = n f_1 \quad (n = 1, 2, 3, \dots) \quad (15.33)$$

$$f_1 = \frac{1}{2L} \sqrt{\frac{F}{\mu}} \quad (15.35)$$

(string fixed at both ends)

along the string and passes the discontinuity in μ . Which of the following wave properties will be the same on both sides of the discontinuity, and which ones will change? speed of the wave; frequency; wavelength. Explain the physical reasoning behind each of your answers.

Q15.15 A long rope with mass m is suspended from the ceiling and hangs vertically. A wave pulse is produced at the lower end of the rope, and the pulse travels up the rope. Does the speed of the wave pulse change as it moves up the rope, and if so, does it increase or decrease?

Q15.16 In a transverse wave on a string, the motion of the string is perpendicular to the length of the string. How, then, is it possible for energy to move along the length of the string?

Q15.17 Both wave intensity and gravitation obey inverse-square laws. Do they do so for the same reason? Discuss the reason for each of these inverse-square laws as well as you can.

Q15.18 Energy can be transferred along a string by wave motion. However, in a standing wave on a string, no energy can ever be transferred past a node. Why not?

Q15.19 Can a standing wave be produced on a string by superposing two waves traveling in opposite directions with the same frequency but different amplitudes? Why or why not? Can a standing wave be produced by superposing two waves traveling in opposite directions with different frequencies but the same amplitude? Why or why not?

Q15.20 If you stretch a rubber band and pluck it, you hear a (somewhat) musical tone. How does the frequency of this tone change as you stretch the rubber band further? (Try it!) Does this agree with Eq. (15.35) for a string fixed at both ends? Explain.

Q15.21 A musical interval of an *octave* corresponds to a factor of 2 in frequency. By what factor must the tension in a guitar or violin string be increased to raise its pitch one octave? To raise it two octaves? Explain your reasoning. Is there any danger in attempting these changes in pitch?

Q15.22 By touching a string lightly at its center while bowing, a violinist can produce a note exactly one octave above the note to which the string is tuned—that is, a note with exactly twice the frequency. Why is this possible?

Q15.23 As we discussed in Section 15.1, water waves are a combination of longitudinal and transverse waves. Defend the following statement: “When water waves hit a vertical wall, the wall is a node of the longitudinal displacement but an antinode of the transverse displacement.”

Q15.24 Violins are short instruments, while cellos and basses are long. In terms of the frequency of the waves they produce, explain why this is so.

Q15.25 What is the purpose of the frets on a guitar? In terms of the frequency of the vibration of the strings, explain their use.

EXERCISES

Section 15.2 Periodic Waves

15.1 • The speed of sound in air at 20°C is 344 m/s. (a) What is the wavelength of a sound wave with a frequency of 784 Hz, corresponding to the note G₅ on a piano, and how many milliseconds does each vibration take? (b) What is the wavelength of a sound wave one octave higher than the note in part (a)?

15.2 • **BIO Audible Sound.** Provided the amplitude is sufficiently great, the human ear can respond to longitudinal waves over a range of frequencies from about 20.0 Hz to about 20.0 kHz. (a) If you were to mark the beginning of each complete wave pattern with a red dot for the long-wavelength sound and a blue dot

for the short-wavelength sound, how far apart would the red dots be, and how far apart would the blue dots be? (b) In reality would adjacent dots in each set be far enough apart for you to easily measure their separation with a meter stick? (c) Suppose you repeated part (a) in water, where sound travels at 1480 m/s. How far apart would the dots be in each set? Could you readily measure their separation with a meter stick?

15.3 • **Tsunami!** On December 26, 2004, a great earthquake occurred off the coast of Sumatra and triggered immense waves (tsunami) that killed some 200,000 people. Satellites observing these waves from space measured 800 km from one wave crest to the next and a period between waves of 1.0 hour. What was the speed of these waves in m/s and in km/h? Does your answer help you understand why the waves caused such devastation?

15.4 • **BIO Ultrasound Imaging.** Sound having frequencies above the range of human hearing (about 20,000 Hz) is called *ultrasound*. Waves above this frequency can be used to penetrate the body and to produce images by reflecting from surfaces. In a typical ultrasound scan, the waves travel through body tissue with a speed of 1500 m/s. For a good, detailed image, the wavelength should be no more than 1.0 mm. What frequency sound is required for a good scan?

15.5 • **BIO** (a) **Audible wavelengths.** The range of audible frequencies is from about 20 Hz to 20,000 Hz. What is the range of the wavelengths of audible sound in air? (b) **Visible light.** The range of visible light extends from 400 nm to 700 nm. What is the range of visible frequencies of light? (c) **Brain surgery.** Surgeons can remove brain tumors by using a cavitron ultrasonic surgical aspirator, which produces sound waves of frequency 23 kHz. What is the wavelength of these waves in air? (d) **Sound in the body.** What would be the wavelength of the sound in part (c) in bodily fluids in which the speed of sound is 1480 m/s but the frequency is unchanged?

15.6 •• A fisherman notices that his boat is moving up and down periodically, owing to waves on the surface of the water. It takes 2.5 s for the boat to travel from its highest point to its lowest, a total distance of 0.62 m. The fisherman sees that the wave crests are spaced 6.0 m apart. (a) How fast are the waves traveling? (b) What is the amplitude of each wave? (c) If the total vertical distance traveled by the boat were 0.30 m but the other data remained the same, how would the answers to parts (a) and (b) be affected?

Section 15.3 Mathematical Description of a Wave

15.7 • Transverse waves on a string have wave speed 8.00 m/s, amplitude 0.0700 m, and wavelength 0.320 m. The waves travel in the $-x$ -direction, and at $t = 0$ the $x = 0$ end of the string has its maximum upward displacement. (a) Find the frequency, period, and wave number of these waves. (b) Write a wave function describing the wave. (c) Find the transverse displacement of a particle at $x = 0.360$ m at time $t = 0.150$ s. (d) How much time must elapse from the instant in part (c) until the particle at $x = 0.360$ m next has maximum upward displacement?

15.8 • A certain transverse wave is described by

$$y(x, t) = (6.50 \text{ mm}) \cos 2\pi \left(\frac{x}{28.0 \text{ cm}} - \frac{t}{0.0360 \text{ s}} \right)$$

Determine the wave's (a) amplitude; (b) wavelength; (c) frequency; (d) speed of propagation; (e) direction of propagation.

15.9 • **CALC** Which of the following wave functions satisfies the wave equation, Eq. (15.12)? (a) $y(x, t) = A \cos(kx + \omega t)$; (b) $y(x, t) = A \sin(kx + \omega t)$; (c) $y(x, t) = A(\cos kx + \cos \omega t)$. (d) For the wave of part (b), write the equations for the transverse velocity and transverse acceleration of a particle at point x .

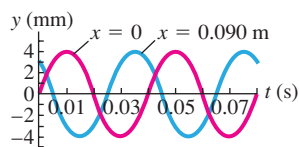
15.10 • A water wave traveling in a straight line on a lake is described by the equation

$$y(x, t) = (3.75 \text{ cm}) \cos(0.450 \text{ cm}^{-1} x + 5.40 \text{ s}^{-1} t)$$

where y is the displacement perpendicular to the undisturbed surface of the lake. (a) How much time does it take for one complete wave pattern to go past a fisherman in a boat at anchor, and what horizontal distance does the wave crest travel in that time? (b) What are the wave number and the number of waves per second that pass the fisherman? (c) How fast does a wave crest travel past the fisherman, and what is the maximum speed of his cork floater as the wave causes it to bob up and down?

15.11 • A sinusoidal wave is propagating along a stretched string that lies along the x -axis. The displacement of the string as a function of time is graphed in Fig. E15.11 for particles at $x = 0$ and at $x = 0.0900 \text{ m}$. (a) What is the amplitude of the wave?

Figure E15.11



(b) What is the period of the wave? (c) You are told that the two points $x = 0$ and $x = 0.0900 \text{ m}$ are within one wavelength of each other. If the wave is moving in the $+x$ -direction, determine the wavelength and the wave speed. (d) If instead the wave is moving in the $-x$ -direction, determine the wavelength and the wave speed. (e) Would it be possible to determine definitively the wavelength in parts (c) and (d) if you were not told that the two points were within one wavelength of each other? Why or why not?

15.12 • **CALC** Speed of Propagation vs. Particle Speed.

(a) Show that Eq. (15.3) may be written as

$$y(x, t) = A \cos\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

(b) Use $y(x, t)$ to find an expression for the transverse velocity v_y of a particle in the string on which the wave travels. (c) Find the maximum speed of a particle of the string. Under what circumstances is this equal to the propagation speed v ? Less than v ? Greater than v ?

15.13 •• A transverse wave on a string has amplitude 0.300 cm , wavelength 12.0 cm , and speed 6.00 cm/s . It is represented by $y(x, t)$ as given in Exercise 15.12. (a) At time $t = 0$, compute y at 1.5-cm intervals of x (that is, at $x = 0, x = 1.5 \text{ cm}, x = 3.0 \text{ cm}$, and so on) from $x = 0$ to $x = 12.0 \text{ cm}$. Graph the results. This is the shape of the string at time $t = 0$. (b) Repeat the calculations for the same values of x at times $t = 0.400 \text{ s}$ and $t = 0.800 \text{ s}$. Graph the shape of the string at these instants. In what direction is the wave traveling?

15.14 • A wave on a string is described by $y(x, t) = A \cos(kx - \omega t)$. (a) Graph y , v_y , and a_y as functions of x for time $t = 0$. (b) Consider the following points on the string: (i) $x = 0$; (ii) $x = \pi/4k$; (iii) $x = \pi/2k$; (iv) $x = 3\pi/4k$; (v) $x = \pi/k$; (vi) $x = 5\pi/4k$; (vii) $x = 3\pi/2k$; (viii) $x = 7\pi/4k$. For a particle at each of these points at $t = 0$, describe in words whether the particle is moving and in what direction, and whether the particle is speeding up, slowing down, or instantaneously not accelerating.

Section 15.4 Speed of a Transverse Wave

15.15 • One end of a horizontal rope is attached to a prong of an electrically driven tuning fork that vibrates the rope transversely at 120 Hz . The other end passes over a pulley and supports a 1.50-kg mass. The linear mass density of the rope is 0.0550 kg/m .

(a) What is the speed of a transverse wave on the rope? (b) What is the wavelength? (c) How would your answers to parts (a) and (b) change if the mass were increased to 3.00 kg ?

15.16 • With what tension must a rope with length 2.50 m and mass 0.120 kg be stretched for transverse waves of frequency 40.0 Hz to have a wavelength of 0.750 m ?

15.17 •• The upper end of a 3.80-m -long steel wire is fastened to the ceiling, and a 54.0-kg object is suspended from the lower end of the wire. You observe that it takes a transverse pulse 0.0492 s to travel from the bottom to the top of the wire. What is the mass of the wire?

15.18 •• A 1.50-m string of weight 0.0125 N is tied to the ceiling at its upper end, and the lower end supports a weight W . Neglect the very small variation in tension along the length of the string that is produced by the weight of the string. When you pluck the string slightly, the waves traveling up the string obey the equation

$$y(x, t) = (8.50 \text{ mm}) \cos(172 \text{ m}^{-1} x - 4830 \text{ s}^{-1} t)$$

Assume that the tension of the string is constant and equal to W . (a) How much time does it take a pulse to travel the full length of the string? (b) What is the weight W ? (c) How many wavelengths are on the string at any instant of time? (d) What is the equation for waves traveling *down* the string?

15.19 • A thin, 75.0-cm wire has a mass of 16.5 g . One end is tied to a nail, and the other end is attached to a screw that can be adjusted to vary the tension in the wire. (a) To what tension (in newtons) must you adjust the screw so that a transverse wave of wavelength 3.33 cm makes 875 vibrations per second? (b) How fast would this wave travel?

15.20 • **Weighty Rope.** If in Example 15.3 (Section 15.4) we do *not* neglect the weight of the rope, what is the wave speed (a) at the bottom of the rope; (b) at the middle of the rope; (c) at the top of the rope?

15.21 • A simple harmonic oscillator at the point $x = 0$ generates a wave on a rope. The oscillator operates at a frequency of 40.0 Hz and with an amplitude of 3.00 cm . The rope has a linear mass density of 50.0 g/m and is stretched with a tension of 5.00 N . (a) Determine the speed of the wave. (b) Find the wavelength. (c) Write the wave function $y(x, t)$ for the wave. Assume that the oscillator has its maximum upward displacement at time $t = 0$. (d) Find the maximum transverse acceleration of points on the rope. (e) In the discussion of transverse waves in this chapter, the force of gravity was ignored. Is that a reasonable approximation for this wave? Explain.

Section 15.5 Energy in Wave Motion

15.22 •• A piano wire with mass 3.00 g and length 80.0 cm is stretched with a tension of 25.0 N . A wave with frequency 120.0 Hz and amplitude 1.6 mm travels along the wire. (a) Calculate the average power carried by the wave. (b) What happens to the average power if the wave amplitude is halved?

15.23 • A horizontal wire is stretched with a tension of 94.0 N , and the speed of transverse waves for the wire is 492 m/s . What must the amplitude of a traveling wave of frequency 69.0 Hz be in order for the average power carried by the wave to be 0.365 W ?

15.24 •• A light wire is tightly stretched with tension F . Transverse traveling waves of amplitude A and wavelength λ_1 carry average power $P_{\text{av},1} = 0.400 \text{ W}$. If the wavelength of the waves is doubled, so $\lambda_2 = 2\lambda_1$, while the tension F and amplitude A are not altered, what then is the average power $P_{\text{av},2}$ carried by the waves?

15.25 •• A jet plane at takeoff can produce sound of intensity 10.0 W/m^2 at 30.0 m away. But you prefer the tranquil sound of

the wave equation, Eq. (15.12), for $v = \omega/k$. (b) Explain why the relationship $v = \omega/k$ for *traveling* waves also applies to *standing* waves.

15.39 • CALC Let $y_1(x, t) = A \cos(k_1x - \omega_1t)$ and $y_2(x, t) = A \cos(k_2x - \omega_2t)$ be two solutions to the wave equation, Eq. (15.12), for the same v . Show that $y(x, t) = y_1(x, t) + y_2(x, t)$ is also a solution to the wave equation.

15.40 • A 1.50-m-long rope is stretched between two supports with a tension that makes the speed of transverse waves 48.0 m/s. What are the wavelength and frequency of (a) the fundamental; (b) the second overtone; (c) the fourth harmonic?

15.41 • A wire with mass 40.0 g is stretched so that its ends are tied down at points 80.0 cm apart. The wire vibrates in its fundamental mode with frequency 60.0 Hz and with an amplitude at the antinodes of 0.300 cm. (a) What is the speed of propagation of transverse waves in the wire? (b) Compute the tension in the wire. (c) Find the maximum transverse velocity and acceleration of particles in the wire.

15.42 • A piano tuner stretches a steel piano wire with a tension of 800 N. The steel wire is 0.400 m long and has a mass of 3.00 g. (a) What is the frequency of its fundamental mode of vibration? (b) What is the number of the highest harmonic that could be heard by a person who is capable of hearing frequencies up to 10,000 Hz?

15.43 • CALC A thin, taut string tied at both ends and oscillating in its third harmonic has its shape described by the equation $y(x, t) = (5.60 \text{ cm}) \sin[(0.0340 \text{ rad/cm})x] \sin[(50.0 \text{ rad/s})t]$, where the origin is at the left end of the string, the x -axis is along the string, and the y -axis is perpendicular to the string. (a) Draw a sketch that shows the standing-wave pattern. (b) Find the amplitude of the two traveling waves that make up this standing wave. (c) What is the length of the string? (d) Find the wavelength, frequency, period, and speed of the traveling waves. (e) Find the maximum transverse speed of a point on the string. (f) What would be the equation $y(x, t)$ for this string if it were vibrating in its eighth harmonic?

15.44 • The wave function of a standing wave is $y(x, t) = 4.44 \text{ mm} \sin[(32.5 \text{ rad/m})x] \sin[(754 \text{ rad/s})t]$. For the two traveling waves that make up this standing wave, find the (a) amplitude; (b) wavelength; (c) frequency; (d) wave speed; (e) wave functions. (f) From the information given, can you determine which harmonic this is? Explain.

15.45 •• Consider again the rope and traveling wave of Exercise 15.28. Assume that the ends of the rope are held fixed and that this traveling wave and the reflected wave are traveling in the opposite direction. (a) What is the wave function $y(x, t)$ for the standing wave that is produced? (b) In which harmonic is the standing wave oscillating? (c) What is the frequency of the fundamental oscillation?

15.46 •• One string of a certain musical instrument is 75.0 cm long and has a mass of 8.75 g. It is being played in a room where the speed of sound is 344 m/s. (a) To what tension must you adjust the string so that, when vibrating in its second overtone, it produces sound of wavelength 0.765 m? (Assume that the breaking stress of the wire is very large and isn't exceeded.) (b) What frequency sound does this string produce in its fundamental mode of vibration?

15.47 • The portion of the string of a certain musical instrument between the bridge and upper end of the finger board (that part of the string that is free to vibrate) is 60.0 cm long, and this length of the string has mass 2.00 g. The string sounds an A_4 note (440 Hz) when played. (a) Where must the player put a finger (what distance x from the bridge) to play a D_5 note (587 Hz)? (See Fig. E15.47.)

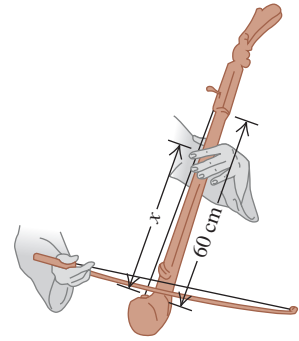
For both the A_4 and D_5 notes, the string vibrates in its fundamental mode. (b) Without retuning, is it possible to play a G_4 note (392 Hz) on this string? Why or why not?

15.48 •• (a) A horizontal string tied at both ends is vibrating in its fundamental mode. The traveling waves have speed v , frequency f , amplitude A , and wavelength λ . Calculate the maximum transverse velocity and maximum transverse acceleration of points located at (i) $x = \lambda/2$, (ii) $x = \lambda/4$, and (iii) $x = \lambda/8$ from the left-hand end of the string. (b) At each of the points in part (a), what is the amplitude of the motion? (c) At each of the points in part (a), how much time does it take the string to go from its largest upward displacement to its largest downward displacement?

15.49 • Guitar String. One of the 63.5-cm-long strings of an ordinary guitar is tuned to produce the note B_3 (frequency 245 Hz) when vibrating in its fundamental mode. (a) Find the speed of transverse waves on this string. (b) If the tension in this string is increased by 1.0%, what will be the new fundamental frequency of the string? (c) If the speed of sound in the surrounding air is 344 m/s, find the frequency and wavelength of the sound wave produced in the air by the vibration of the B_3 string. How do these compare to the frequency and wavelength of the standing wave on the string?

15.50 • Waves on a Stick. A flexible stick 2.0 m long is not fixed in any way and is free to vibrate. Make clear drawings of this stick vibrating in its first three harmonics, and then use your drawings to find the wavelengths of each of these harmonics. (*Hint:* Should the ends be nodes or antinodes?)

Figure E15.47



PROBLEMS

15.51 • CALC A transverse sine wave with an amplitude of 2.50 mm and a wavelength of 1.80 m travels from left to right along a long, horizontal, stretched string with a speed of 36.0 m/s. Take the origin at the left end of the undisturbed string. At time $t = 0$ the left end of the string has its maximum upward displacement. (a) What are the frequency, angular frequency, and wave number of the wave? (b) What is the function $y(x, t)$ that describes the wave? (c) What is $y(t)$ for a particle at the left end of the string? (d) What is $y(t)$ for a particle 1.35 m to the right of the origin? (e) What is the maximum magnitude of transverse velocity of any particle of the string? (f) Find the transverse displacement and the transverse velocity of a particle 1.35 m to the right of the origin at time $t = 0.0625$ s.

15.52 • A transverse wave on a rope is given by

$$y(x, t) = (0.750 \text{ cm}) \cos \pi[(0.400 \text{ cm}^{-1})x + (250 \text{ s}^{-1})t]$$

(a) Find the amplitude, period, frequency, wavelength, and speed of propagation. (b) Sketch the shape of the rope at these values of t : 0, 0.0005 s, 0.0010 s. (c) Is the wave traveling in the $+x$ - or $-x$ -direction? (d) The mass per unit length of the rope is 0.0500 kg/m. Find the tension. (e) Find the average power of this wave.

15.53 •• Three pieces of string, each of length L , are joined together end to end, to make a combined string of length $3L$. The first piece of string has mass per unit length μ_1 , the second piece

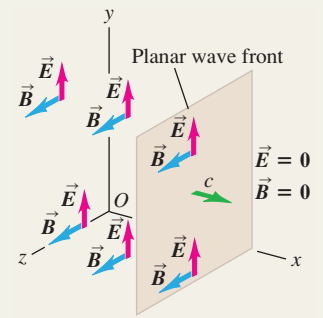
Maxwell's equations and electromagnetic waves:

Maxwell's equations predict the existence of electromagnetic waves that propagate in vacuum at the speed of light c . The electromagnetic spectrum covers frequencies from at least 1 to 10^{24} Hz and a correspondingly broad range of wavelengths. Visible light, with wavelengths from 380 to 750 nm, is only a very small part of this spectrum. In a plane wave, \vec{E} and \vec{B} are uniform over any plane perpendicular to the propagation direction. Faraday's law and Ampere's law both give relationships between the magnitudes of \vec{E} and \vec{B} ; requiring both of these relationships to be satisfied gives an expression for c in terms of ϵ_0 and μ_0 . Electromagnetic waves are transverse; the \vec{E} and \vec{B} fields are perpendicular to the direction of propagation and to each other. The direction of propagation is the direction of $\vec{E} \times \vec{B}$.

$$E = cB \tag{32.4}$$

$$B = \epsilon_0 \mu_0 c E \tag{32.8}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{32.9}$$

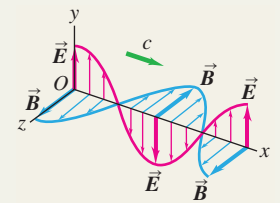


Sinusoidal electromagnetic waves: Equations (32.17) and (32.18) describe a sinusoidal plane electromagnetic wave traveling in vacuum in the $+x$ -direction. If the wave is propagating in the $-x$ -direction, replace $kx - \omega t$ by $kx + \omega t$. (See Example 32.1.)

$$\vec{E}(x, t) = \hat{j} E_{\max} \cos(kx - \omega t) \tag{32.17}$$

$$\vec{B}(x, t) = \hat{k} B_{\max} \cos(kx - \omega t) \tag{32.18}$$

$$E_{\max} = c B_{\max} \tag{32.18}$$



Electromagnetic waves in matter: When an electromagnetic wave travels through a dielectric, the wave speed v is less than the speed of light in vacuum c . (See Example 32.2.)

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{KK_m}} \frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{c}{\sqrt{KK_m}} \tag{32.21}$$

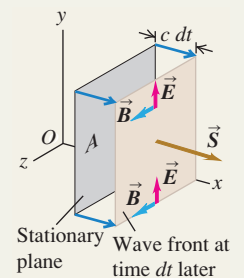
Energy and momentum in electromagnetic waves: The energy flow rate (power per unit area) in an electromagnetic wave in vacuum is given by the Poynting vector \vec{S} . The magnitude of the time-averaged value of the Poynting vector is called the intensity I of the wave. Electromagnetic waves also carry momentum. When an electromagnetic wave strikes a surface, it exerts a radiation pressure p_{rad} . If the surface is perpendicular to the wave propagation direction and is totally absorbing, $p_{\text{rad}} = I/c$; if the surface is a perfect reflector, $p_{\text{rad}} = 2I/c$. (See Examples 32.3–32.5.)

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \tag{32.28}$$

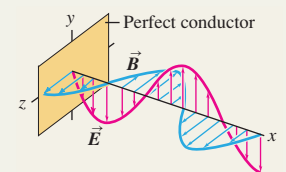
$$I = S_{\text{av}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} E_{\max}^2 = \frac{1}{2} \epsilon_0 c E_{\max}^2 \tag{32.29}$$

$$\frac{1}{A} \frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c} \tag{32.31}$$

(flow rate of electromagnetic momentum)



Standing electromagnetic waves: If a perfect reflecting surface is placed at $x = 0$, the incident and reflected waves form a standing wave. Nodal planes for \vec{E} occur at $kx = 0, \pi, 2\pi, \dots$, and nodal planes for \vec{B} at $kx = \pi/2, 3\pi/2, 5\pi/2, \dots$. At each point, the sinusoidal variations of \vec{E} and \vec{B} with time are 90° out of phase. (See Examples 32.6 and 32.7.)



EXERCISES

Section 32.2 Plane Electromagnetic Waves and the Speed of Light

32.1 • (a) How much time does it take light to travel from the moon to the earth, a distance of 384,000 km? (b) Light from the star Sirius takes 8.61 years to reach the earth. What is the distance from earth to Sirius in kilometers?

32.2 • Consider each of the electric- and magnetic-field orientations given next. In each case, what is the direction of propagation of the wave? (a) \vec{E} in the $+x$ -direction, \vec{B} in the $+y$ -direction; (b) \vec{E} in the $-y$ -direction, \vec{B} in the $+x$ -direction; (c) \vec{E} in the $+z$ -direction, \vec{B} in the $-x$ -direction; (d) \vec{E} in the $+y$ -direction, \vec{B} in the $-z$ -direction.

32.3 • A sinusoidal electromagnetic wave is propagating in vacuum in the $+z$ -direction. If at a particular instant and at a certain point in space the electric field is in the $+x$ -direction and has magnitude 4.00 V/m, what are the magnitude and direction of the magnetic field of the wave at this same point in space and instant in time?

32.4 • Consider each of the following electric- and magnetic-field orientations. In each case, what is the direction of propagation of the wave? (a) $\vec{E} = E\hat{i}$, $\vec{B} = -B\hat{j}$; (b) $\vec{E} = E\hat{j}$, $\vec{B} = B\hat{i}$; (c) $\vec{E} = -E\hat{k}$, $\vec{B} = -B\hat{i}$; (d) $\vec{E} = E\hat{i}$, $\vec{B} = -B\hat{k}$.

Section 32.3 Sinusoidal Electromagnetic Waves

32.5 • **BIO Medical X rays.** Medical x rays are taken with electromagnetic waves having a wavelength of around 0.10 nm. What are the frequency, period, and wave number of such waves?

32.6 • **BIO Ultraviolet Radiation.** There are two categories of ultraviolet light. Ultraviolet A (UVA) has a wavelength ranging from 320 nm to 400 nm. It is not harmful to the skin and is necessary for the production of vitamin D. UVB, with a wavelength between 280 nm and 320 nm, is much more dangerous because it causes skin cancer. (a) Find the frequency ranges of UVA and UVB. (b) What are the ranges of the wave numbers for UVA and UVB?

32.7 • A sinusoidal electromagnetic wave having a magnetic field of amplitude 1.25 μ T and a wavelength of 432 nm is traveling in the $+x$ -direction through empty space. (a) What is the frequency of this wave? (b) What is the amplitude of the associated electric field? (c) Write the equations for the electric and magnetic fields as functions of x and t in the form of Eqs. (32.17).

32.8 • An electromagnetic wave of wavelength 435 nm is traveling in vacuum in the $-z$ -direction. The electric field has amplitude 2.70×10^{-3} V/m and is parallel to the x -axis. What are (a) the frequency and (b) the magnetic-field amplitude? (c) Write the vector equations for $\vec{E}(z, t)$ and $\vec{B}(z, t)$.

32.9 • Consider electromagnetic waves propagating in air. (a) Determine the frequency of a wave with a wavelength of (i) 5.0 km, (ii) 5.0 μ m, (iii) 5.0 nm. (b) What is the wavelength (in meters and nanometers) of (i) gamma rays of frequency 6.50×10^{21} Hz and (ii) an AM station radio wave of frequency 590 kHz?

32.10 • The electric field of a sinusoidal electromagnetic wave obeys the equation $E = (375 \text{ V/m}) \cos[(1.99 \times 10^7 \text{ rad/m})x + (5.97 \times 10^{15} \text{ rad/s})t]$. (a) What are the amplitudes of the electric and magnetic fields of this wave? (b) What are the frequency, wavelength, and period of the wave? Is this light visible to humans? (c) What is the speed of the wave?

32.11 • An electromagnetic wave has an electric field given by $\vec{E}(y, t) = (3.10 \times 10^5 \text{ V/m})\hat{k} \cos[ky - (12.65 \times 10^{12} \text{ rad/s})t]$. (a) In which direction is the wave traveling? (b) What is the wavelength of the wave? (c) Write the vector equation for $\vec{B}(y, t)$.

32.12 • An electromagnetic wave has a magnetic field given by $\vec{B}(x, t) = -(8.25 \times 10^{-9} \text{ T})\hat{j} \cos[(1.38 \times 10^4 \text{ rad/m})x + \omega t]$. (a) In which direction is the wave traveling? (b) What is the frequency f of the wave? (c) Write the vector equation for $\vec{E}(x, t)$.

32.13 • Radio station WCCO in Minneapolis broadcasts at a frequency of 830 kHz. At a point some distance from the transmitter, the magnetic-field amplitude of the electromagnetic wave from WCCO is 4.82×10^{-11} T. Calculate (a) the wavelength; (b) the wave number; (c) the angular frequency; (d) the electric-field amplitude.

32.14 • An electromagnetic wave with frequency 65.0 Hz travels in an insulating magnetic material that has dielectric constant 3.64 and relative permeability 5.18 at this frequency. The electric field has amplitude 7.20×10^{-3} V/m. (a) What is the speed of propagation of the wave? (b) What is the wavelength of the wave? (c) What is the amplitude of the magnetic field?

32.15 • An electromagnetic wave with frequency 5.70×10^{14} Hz propagates with a speed of 2.17×10^8 m/s in a certain piece of glass. Find (a) the wavelength of the wave in the glass; (b) the wavelength of a wave of the same frequency propagating in air; (c) the index of refraction n of the glass for an electromagnetic wave with this frequency; (d) the dielectric constant for glass at this frequency, assuming that the relative permeability is unity.

Section 32.4 Energy and Momentum in Electromagnetic Waves

32.16 • **BIO High-Energy Cancer Treatment.** Scientists are working on a new technique to kill cancer cells by zapping them with ultrahigh-energy (in the range of 10^{12} W) pulses of light that last for an extremely short time (a few nanoseconds). These short pulses scramble the interior of a cell without causing it to explode, as long pulses would do. We can model a typical such cell as a disk 5.0 μ m in diameter, with the pulse lasting for 4.0 ns with an average power of 2.0×10^{12} W. We shall assume that the energy is spread uniformly over the faces of 100 cells for each pulse. (a) How much energy is given to the cell during this pulse? (b) What is the intensity (in W/m^2) delivered to the cell? (c) What are the maximum values of the electric and magnetic fields in the pulse?

32.17 • **Fields from a Light Bulb.** We can reasonably model a 75-W incandescent light bulb as a sphere 6.0 cm in diameter. Typically, only about 5% of the energy goes to visible light; the rest goes largely to nonvisible infrared radiation. (a) What is the visible-light intensity (in W/m^2) at the surface of the bulb? (b) What are the amplitudes of the electric and magnetic fields at this surface, for a sinusoidal wave with this intensity?

32.18 • A sinusoidal electromagnetic wave from a radio station passes perpendicularly through an open window that has area 0.500 m^2 . At the window, the electric field of the wave has rms value 0.0200 V/m. How much energy does this wave carry through the window during a 30.0-s commercial?

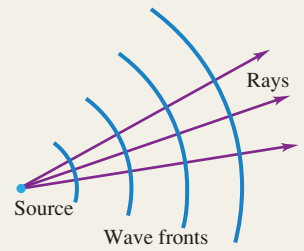
32.19 • **Testing a Space Radio Transmitter.** You are a NASA mission specialist on your first flight aboard the space shuttle. Thanks to your extensive training in physics, you have been assigned to evaluate the performance of a new radio transmitter on board the International Space Station (ISS). Perched on the shuttle's movable arm, you aim a sensitive detector at the ISS, which is 2.5 km away. You find that the electric-field amplitude of the radio waves coming from the ISS transmitter is 0.090 V/m and that the frequency of the waves is 244 MHz. Find the following: (a) the intensity of the radio wave at your location; (b) the magnetic-field amplitude of the wave at your location; (c) the total power output of the ISS radio transmitter. (d) What assumptions, if any, did you make in your calculations?

Light and its properties: Light is an electromagnetic wave. When emitted or absorbed, it also shows particle properties. It is emitted by accelerated electric charges.

$$n = \frac{c}{v} \quad (33.1)$$

A wave front is a surface of constant phase; wave fronts move with a speed equal to the propagation speed of the wave. A ray is a line along the direction of propagation, perpendicular to the wave fronts.

$$\lambda = \frac{\lambda_0}{n} \quad (33.5)$$



When light is transmitted from one material to another, the frequency of the light is unchanged, but the wavelength and wave speed can change. The index of refraction n of a material is the ratio of the speed of light in vacuum c to the speed v in the material. If λ_0 is the wavelength in vacuum, the same wave has a shorter wavelength λ in a medium with index of refraction n . (See Example 33.2.)

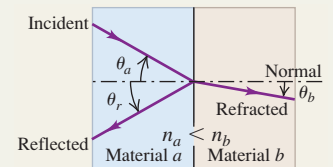
Reflection and refraction: At a smooth interface between two optical materials, the incident, reflected, and refracted rays and the normal to the interface all lie in a single plane called the plane of incidence. The law of reflection states that the angles of incidence and reflection are equal. The law of refraction relates the angles of incidence and refraction to the indexes of refraction of the materials. (See Examples 33.1 and 33.3.)

$$\theta_r = \theta_a \quad (33.2)$$

(law of reflection)

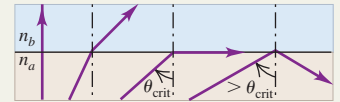
$$n_a \sin \theta_a = n_b \sin \theta_b \quad (33.4)$$

(law of refraction)



Total internal reflection: When a ray travels in a material of greater index of refraction n_a toward a material of smaller index n_b , total internal reflection occurs at the interface when the angle of incidence exceeds a critical angle θ_{crit} . (See Example 33.4.)

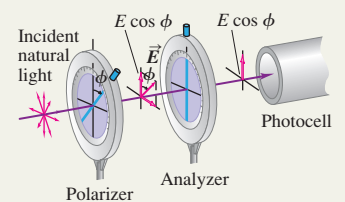
$$\sin \theta_{\text{crit}} = \frac{n_b}{n_a} \quad (33.6)$$



Polarization of light: The direction of polarization of a linearly polarized electromagnetic wave is the direction of the \vec{E} field. A polarizing filter passes waves that are linearly polarized along its polarizing axis and blocks waves polarized perpendicularly to that axis. When polarized light of intensity I_{max} is incident on a polarizing filter used as an analyzer, the intensity I of the light transmitted through the analyzer depends on the angle ϕ between the polarization direction of the incident light and the polarizing axis of the analyzer. (See Example 33.5.)

$$I = I_{\text{max}} \cos^2 \phi \quad (33.7)$$

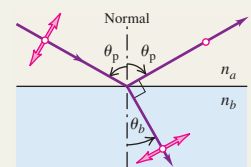
(Malus's law)



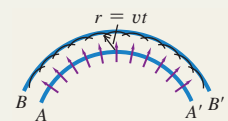
Polarization by reflection: When unpolarized light strikes an interface between two materials, Brewster's law states that the reflected light is completely polarized perpendicular to the plane of incidence (parallel to the interface) if the angle of incidence equals the polarizing angle θ_p . (See Example 33.6.)

$$\tan \theta_p = \frac{n_b}{n_a} \quad (33.8)$$

(Brewster's law)



Huygens's principle: Huygens's principle states that if the position of a wave front at one instant is known, then the position of the front at a later time can be constructed by imagining the front as a source of secondary wavelets. Huygens's principle can be used to derive the laws of reflection and refraction.



Q33.18 For the old “rabbit-ear” style TV antennas, it’s possible to alter the quality of reception considerably simply by changing the orientation of the antenna. Why?

Q33.19 In Fig. 33.32, since the light that is scattered out of the incident beam is polarized, why is the transmitted beam not also partially polarized?

Q33.20 You are sunbathing in the late afternoon when the sun is relatively low in the western sky. You are lying flat on your back, looking straight up through Polaroid sunglasses. To minimize the amount of sky light reaching your eyes, how should you lie: with your feet pointing north, east, south, west, or in some other direction? Explain your reasoning.

Q33.21 Light scattered from blue sky is strongly polarized because of the nature of the scattering process described in Section 33.6. But light scattered from white clouds is usually *not* polarized. Why not?

Q33.22 Atmospheric haze is due to water droplets or smoke particles (“smog”). Such haze reduces visibility by scattering light, so that the light from distant objects becomes randomized and images become indistinct. Explain why visibility through haze can be improved by wearing red-tinted sunglasses, which filter out blue light.

Q33.23 The explanation given in Section 33.6 for the color of the setting sun should apply equally well to the *rising* sun, since sunlight travels the same distance through the atmosphere to reach your eyes at either sunrise or sunset. Typically, however, sunsets are redder than sunrises. Why? (*Hint:* Particles of all kinds in the atmosphere contribute to scattering.)

Q33.24 Huygens’s principle also applies to sound waves. During the day, the temperature of the atmosphere decreases with increasing altitude above the ground. But at night, when the ground cools, there is a layer of air just above the surface in which the temperature *increases* with altitude. Use this to explain why sound waves from distant sources can be heard more clearly at night than in the daytime. (*Hint:* The speed of sound increases with increasing temperature. Use the ideas displayed in Fig. 33.37 for light.)

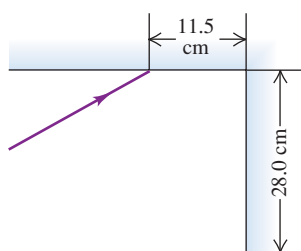
Q33.25 Can water waves be reflected and refracted? Give examples. Does Huygens’s principle apply to water waves? Explain.

EXERCISES

Section 33.2 Reflection and Refraction

33.1 • Two plane mirrors intersect at right angles. A laser beam strikes the first of them at a point 11.5 cm from their point of intersection, as shown in Fig. E33.1 For what angle of incidence at the first mirror will this ray strike the midpoint of the second mirror (which is 28.0 cm long) after reflecting from the first mirror?

Figure E33.1



33.2 • **BIO Light Inside the Eye.** The vitreous humor, a transparent, gelatinous fluid that fills most of the eyeball, has an index of refraction of 1.34. Visible light ranges in wavelength from 380 nm (violet) to 750 nm (red), as measured in air. This light travels through the vitreous humor and strikes the rods and cones at the surface of the retina. What are the ranges of (a) the wavelength, (b) the frequency, and (c) the speed of the light just as it approaches the retina within the vitreous humor?

33.3 • A beam of light has a wavelength of 650 nm in vacuum. (a) What is the speed of this light in a liquid whose index of refraction at this wavelength is 1.47? (b) What is the wavelength of these waves in the liquid?

33.4 • Light with a frequency of 5.80×10^{14} Hz travels in a block of glass that has an index of refraction of 1.52. What is the wavelength of the light (a) in vacuum and (b) in the glass?

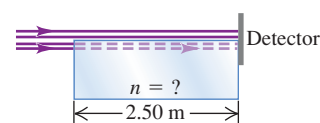
33.5 • A light beam travels at 1.94×10^8 m/s in quartz. The wavelength of the light in quartz is 355 nm. (a) What is the index of refraction of quartz at this wavelength? (b) If this same light travels through air, what is its wavelength there?

33.6 •• Light of a certain frequency has a wavelength of 438 nm in water. What is the wavelength of this light in benzene?

33.7 •• A parallel beam of light in air makes an angle of 47.5° with the surface of a glass plate having a refractive index of 1.66. (a) What is the angle between the reflected part of the beam and the surface of the glass? (b) What is the angle between the refracted beam and the surface of the glass?

33.8 •• A laser beam shines along the surface of a block of transparent material (see Fig. E33.8.). Half of the beam goes straight to a detector, while the other half travels through the block and then hits the detector. The time delay between the arrival of the two light beams at the detector is 6.25 ns. What is the index of refraction of this material?

Figure E33.8

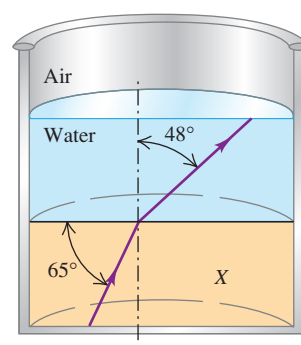


33.9 • Light traveling in air is incident on the surface of a block of plastic at an angle of 62.7° to the normal and is bent so that it makes a 48.1° angle with the normal in the plastic. Find the speed of light in the plastic.

33.10 • (a) A tank containing methanol has walls 2.50 cm thick made of glass of refractive index 1.550. Light from the outside air strikes the glass at a 41.3° angle with the normal to the glass. Find the angle the light makes with the normal in the methanol. (b) The tank is emptied and refilled with an unknown liquid. If light incident at the same angle as in part (a) enters the liquid in the tank at an angle of 20.2° from the normal, what is the refractive index of the unknown liquid?

33.11 •• As shown in Fig. E33.11, a layer of water covers a slab of material X in a beaker. A ray of light traveling upward following the path indicated. Using the information on the figure, find (a) the index of refraction of material X and (b) the angle the light makes with the normal in the *air*.

Figure E33.11



33.12 •• A horizontal, parallel-sided plate of glass having a refractive index of 1.52 is in contact with the surface of

water in a tank. A ray coming from above in air makes an angle of incidence of 35.0° with the normal to the top surface of the glass. (a) What angle does the ray refracted into the water make with the normal to the surface? (b) What is the dependence of this angle on the refractive index of the glass?

33.13 •• In a material having an index of refraction n , a light ray has frequency f , wavelength λ , and speed v . What are the frequency, wavelength, and speed of this light (a) in vacuum and

(b) in a material having refractive index n' ? In each case, express your answers in terms of *only* f , λ , v , n , and n' .

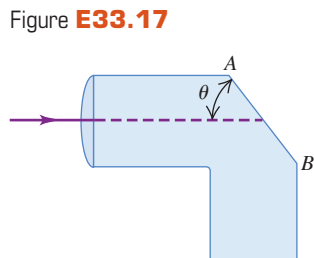
33.14 • A ray of light traveling in water is incident on an interface with a flat piece of glass. The wavelength of the light in the water is 726 nm and its wavelength in the glass is 544 nm. If the ray in water makes an angle of 42.0° with respect to the normal to the interface, what angle does the refracted ray in the glass make with respect to the normal?

33.15 • A ray of light is incident on a plane surface separating two sheets of glass with refractive indexes 1.70 and 1.58. The angle of incidence is 62.0° , and the ray originates in the glass with $n = 1.70$. Compute the angle of refraction.

Section 33.3 Total Internal Reflection

33.16 • A flat piece of glass covers the top of a vertical cylinder that is completely filled with water. If a ray of light traveling in the glass is incident on the interface with the water at an angle of $\theta_a = 36.2^\circ$, the ray refracted into the water makes an angle of 49.8° with the normal to the interface. What is the smallest value of the incident angle θ_a for which none of the ray refracts into the water?

33.17 • **Light Pipe.** Light enters a solid pipe made of plastic having an index of refraction of 1.60. The light travels parallel to the upper part of the pipe (Fig. E33.17). You want to cut the face AB so that all the light will reflect back into the pipe after it first strikes that face. (a) What is the largest that θ can be if the pipe is in air? (b) If the pipe is immersed in water of refractive index 1.33, what is the largest that θ can be?



33.18 • A beam of light is traveling inside a solid glass cube having index of refraction 1.53. It strikes the surface of the cube from the inside. (a) If the cube is in air, at what minimum angle with the normal inside the glass will this light *not* enter the air at this surface? (b) What would be the minimum angle in part (a) if the cube were immersed in water?

33.19 •• The critical angle for total internal reflection at a liquid–air interface is 42.5° . (a) If a ray of light traveling in the liquid has an angle of incidence at the interface of 35.0° , what angle does the refracted ray in the air make with the normal? (b) If a ray of light traveling in air has an angle of incidence at the interface of 35.0° , what angle does the refracted ray in the liquid make with the normal?

33.20 • At the very end of Wagner’s series of operas *Ring of the Nibelung*, Brünnhilde takes the golden ring from the finger of the dead Siegfried and throws it into the Rhine, where it sinks to the bottom of the river. Assuming that the ring is small enough compared to the depth of the river to be treated as a point and that the Rhine is 10.0 m deep where the ring goes in, what is the area of the largest circle at the surface of the water over which light from the ring could escape from the water?

33.21 • A ray of light is traveling in a glass cube that is totally immersed in water. You find that if the ray is incident on the glass–water interface at an angle to the normal larger than 48.7° , no light is refracted into the water. What is the refractive index of the glass?

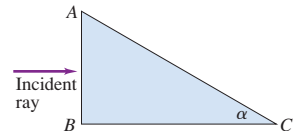
33.22 • Light is incident along the normal on face AB of a glass prism of refractive index 1.52, as shown in Fig. E33.22. Find the

largest value the angle α can have without any light refracted out of the prism at face AC if (a) the prism is immersed in air and (b) the prism is immersed in water.

33.23 • A piece of glass with a flat surface is at the bottom of a tank of water. If a ray of light traveling in the glass is incident on the interface with the water at an angle with respect to the normal that is greater than 62.0° , no light is refracted into the water. For smaller angles of incidence, part of the ray is refracted into the water. If the light has wavelength 408 nm in the glass, what is the wavelength of the light in the water?

33.24 •• We define the index of refraction of a material for sound waves to be the ratio of the speed of sound in air to the speed of sound in the material. Snell’s law then applies to the refraction of sound waves. The speed of a sound wave is 344 m/s in air and 1320 m/s in water. (a) Which medium has the higher index of refraction for sound? (b) What is the critical angle for a sound wave incident on the surface between the air and water? (c) For total internal reflection to occur, must the sound wave be traveling in the air or in the water? (d) Use your results to explain why it is possible to hear people on the opposite shore of a river or small lake extremely clearly.

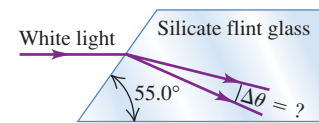
Figure E33.22



Section 33.4 Dispersion

33.25 •• A narrow beam of white light strikes one face of a slab of silicate flint glass. The light is traveling parallel to the two adjoining faces, as shown in Fig. E33.25. For the transmitted light inside the glass, through what angle $\Delta\theta$ is the portion of the visible spectrum between 400 nm and 700 nm dispersed? (Consult the graph in Fig. 33.18.)

Figure E33.25



33.26 • A beam of light strikes a sheet of glass at an angle of 57.0° with the normal in air. You observe that red light makes an angle of 38.1° with the normal in the glass, while violet light makes a 36.7° angle. (a) What are the indexes of refraction of this glass for these colors of light? (b) What are the speeds of red and violet light in the glass?

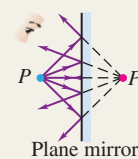
Section 33.5 Polarization

33.27 • Unpolarized light with intensity I_0 is incident on two polarizing filters. The axis of the first filter makes an angle of 60.0° with the vertical, and the axis of the second filter is horizontal. What is the intensity of the light after it has passed through the second filter?

33.28 •• (a) At what angle above the horizontal is the sun if sunlight reflected from the surface of a calm lake is completely polarized? (b) What is the plane of the electric-field vector in the reflected light?

33.29 •• A beam of unpolarized light of intensity I_0 passes through a series of ideal polarizing filters with their polarizing directions turned to various angles as shown in Fig. E33.29. (a) What is the light intensity (in terms of I_0) at points A, B, and C? (b) If we remove the middle filter, what will be the light intensity at point C?

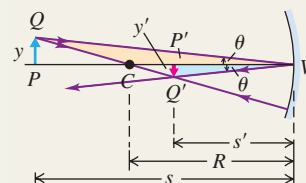
Reflection or refraction at a plane surface: When rays diverge from an object point P and are reflected or refracted, the directions of the outgoing rays are the same as though they had diverged from a point P' called the image point. If they actually converge at P' and diverge again beyond it, P' is a real image of P ; if they only appear to have diverged from P' , it is a virtual image. Images can be either erect or inverted.



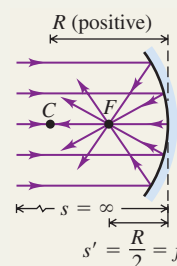
Lateral magnification: The lateral magnification m in any reflecting or refracting situation is defined as the ratio of image height y' to object height y . When m is positive, the image is erect; when m is negative, the image is inverted.

$$m = \frac{y'}{y}$$

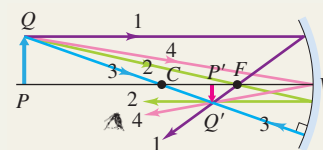
(34.2)



Focal point and focal length: The focal point of a mirror is the point where parallel rays converge after reflection from a concave mirror, or the point from which they appear to diverge after reflection from a convex mirror. Rays diverging from the focal point of a concave mirror are parallel after reflection; rays converging toward the focal point of a convex mirror are parallel after reflection. The distance from the focal point to the vertex is called the focal length, denoted as f . The focal points of a lens are defined similarly.



Relating object and image distances: The formulas for object distance s and image distance s' for plane and spherical mirrors and single refracting surfaces are summarized in the table. The equation for a plane surface can be obtained from the corresponding equation for a spherical surface by setting $R = \infty$. (See Examples 34.1–34.7.)



	Plane Mirror	Spherical Mirror	Plane Refracting Surface	Spherical Refracting Surface
Object and image distances	$\frac{1}{s} + \frac{1}{s'} = 0$	$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} = \frac{1}{f}$	$\frac{n_a}{s} + \frac{n_b}{s'} = 0$	$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$
Lateral magnification	$m = -\frac{s'}{s} = 1$	$m = -\frac{s'}{s}$	$m = -\frac{n_a s'}{n_b s} = 1$	$m = -\frac{n_a s'}{n_b s}$

Object–image relationships derived in this chapter are valid only for rays close to and nearly parallel to the optic axis; these are called paraxial rays. Nonparaxial rays do not converge precisely to an image point. This effect is called spherical aberration.

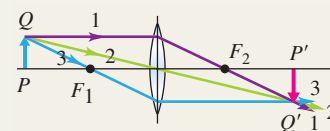
Thin lenses: The object–image relationship, given by Eq. (34.16), is the same for a thin lens as for a spherical mirror. Equation (34.19), the lensmaker’s equation, relates the focal length of a lens to its index of refraction and the radii of curvature of its surfaces. (See Examples 34.8–34.11.)

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

(34.16)

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

(34.19)



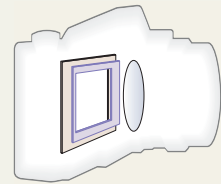
Sign rules: The following sign rules are used with all plane and spherical reflecting and refracting surfaces.

- $s > 0$ when the object is on the incoming side of the surface (a real object); $s < 0$ otherwise.

- $s' > 0$ when the image is on the outgoing side of the surface (a real image); $s' < 0$ otherwise.
- $R > 0$ when the center of curvature is on the outgoing side of the surface; $R < 0$ otherwise.
- $m > 0$ when the image is erect; $m < 0$ when inverted.

Cameras: A camera forms a real, inverted, reduced image of the object being photographed on a light-sensitive surface. The amount of light striking this surface is controlled by the shutter speed and the aperture. The intensity of this light is inversely proportional to the square of the f -number of the lens. (See Example 34.12.)

$$\begin{aligned} f\text{-number} &= \frac{\text{Focal length}}{\text{Aperture diameter}} \\ &= \frac{f}{D} \end{aligned} \quad (34.20)$$



Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q34.1 A spherical mirror is cut in half horizontally. Will an image be formed by the bottom half of the mirror? If so, where will the image be formed?

Q34.2 For the situation shown in Fig. 34.3, is the image distance s' positive or negative? Is the image real or virtual? Explain your answers.

Q34.3 The laws of optics also apply to electromagnetic waves invisible to the eye. A satellite TV dish is used to detect radio waves coming from orbiting satellites. Why is a curved reflecting surface (a “dish”) used? The dish is always concave, never convex; why? The actual radio receiver is placed on an arm and suspended in front of the dish. How far in front of the dish should it be placed?

Q34.4 Explain why the focal length of a *plane* mirror is infinite, and explain what it means for the focal point to be at infinity.

Q34.5 If a spherical mirror is immersed in water, does its focal length change? Explain.

Q34.6 For what range of object positions does a concave spherical mirror form a real image? What about a convex spherical mirror?

Q34.7 When a room has mirrors on two opposite walls, an infinite series of reflections can be seen. Discuss this phenomenon in terms of images. Why do the distant images appear fainter?

Q34.8 For a spherical mirror, if $s = f$, then $s' = \infty$, and the lateral magnification m is infinite. Does this make sense? If so, what does it mean?

Q34.9 You may have noticed a small convex mirror next to your bank’s ATM. Why is this mirror convex, as opposed to flat or concave? What considerations determine its radius of curvature?

Q34.10 A student claims that she can start a fire on a sunny day using just the sun’s rays and a concave mirror. How is this done? Is the concept of image relevant? Can she do the same thing with a convex mirror? Explain.

Q34.11 A person looks at his reflection in the concave side of a shiny spoon. Is it right side up or inverted? Does it matter how far his face is from the spoon? What if he looks in the convex side? (Try this yourself!)

Q34.12 In Example 34.4 (Section 34.2), there appears to be an ambiguity for the case $s = 10$ cm as to whether s' is $+\infty$ or $-\infty$ and whether the image is erect or inverted. How is this resolved? Or is it?

Q34.13 Suppose that in the situation of Example 34.7 of Section 34.3 (see Fig. 34.26) a vertical arrow 2.00 m tall is painted on the side of the pool beneath the water line. According to the calculations in the example, this arrow would appear to the person shown in Fig. 34.26 to be 1.50 m long. But the discussion following Eq. (34.13) states that the magnification for a plane refracting surface is $m = 1$, which suggests that the arrow would appear to the person to be 2.00 m long. How can you resolve this apparent contradiction?

Q34.14 The bottom of the passenger-side mirror on your car notes, “Objects in mirror are closer than they appear.” Is this true? Why?

Q34.15 How could you very quickly make an approximate measurement of the focal length of a converging lens? Could the same method be applied if you wished to use a diverging lens? Explain.

Q34.16 The focal length of a simple lens depends on the color (wavelength) of light passing through it. Why? Is it possible for a lens to have a positive focal length for some colors and negative for others? Explain.

Q34.17 When a converging lens is immersed in water, does its focal length increase or decrease in comparison with the value in air? Explain.

Q34.18 A spherical air bubble in water can function as a lens. Is it a converging or diverging lens? How is its focal length related to its radius?

Q34.19 Can an image formed by one reflecting or refracting surface serve as an object for a second reflection or refraction? Does it matter whether the first image is real or virtual? Explain.

Q34.20 If a piece of photographic film is placed at the location of a real image, the film will record the image. Can this be done with a virtual image? How might one record a virtual image?

Q34.21 According to the discussion in Section 34.2, light rays are reversible. Are the formulas in the table in this chapter’s Summary still valid if object and image are interchanged? What does reversibility imply with respect to the *forms* of the various formulas?

Q34.22 You’ve entered a survival contest that will include building a crude telescope. You are given a large box of lenses. Which two lenses do you pick? How do you quickly identify them?

Q34.23 **BIO** You can’t see clearly underwater with the naked eye, but you *can* if you wear a face mask or goggles (with air between your eyes and the mask or goggles). Why is there a difference? Could you instead wear eyeglasses (with water between your eyes and the eyeglasses) in order to see underwater? If so, should the lenses be converging or diverging? Explain.

Q34.24 You take a lens and mask it so that light can pass through only the bottom half of the lens. How does the image formed by the masked lens compare to the image formed before masking?

EXERCISES

Section 34.1 Reflection and Refraction at a Plane Surface

34.1 • A candle 4.85 cm tall is 39.2 cm to the left of a plane mirror. Where is the image formed by the mirror, and what is the height of this image?

34.2 • The image of a tree just covers the length of a plane mirror 4.00 cm tall when the mirror is held 35.0 cm from the eye. The tree is 28.0 m from the mirror. What is its height?

34.3 • A pencil that is 9.0 cm long is held perpendicular to the surface of a plane mirror with the tip of the pencil lead 12.0 cm from the mirror surface and the end of the eraser 21.0 cm from the mirror surface. What is the length of the image of the pencil that is formed by the mirror? Which end of the image is closer to the mirror surface: the tip of the lead or the end of the eraser?

Section 34.2 Reflection at a Spherical Surface

34.4 • A concave mirror has a radius of curvature of 34.0 cm. (a) What is its focal length? (b) If the mirror is immersed in water (refractive index 1.33), what is its focal length?

34.5 • An object 0.600 cm tall is placed 16.5 cm to the left of the vertex of a concave spherical mirror having a radius of curvature of 22.0 cm. (a) Draw a principal-ray diagram showing the formation of the image. (b) Determine the position, size, orientation, and nature (real or virtual) of the image.

34.6 • Repeat Exercise 34.5 for the case in which the mirror is convex.

34.7 •• The diameter of Mars is 6794 km, and its minimum distance from the earth is 5.58×10^7 km. When Mars is at this distance, find the diameter of the image of Mars formed by a spherical, concave telescope mirror with a focal length of 1.75 m.

34.8 •• An object is 24.0 cm from the center of a silvered spherical glass Christmas tree ornament 6.00 cm in diameter. What are the position and magnification of its image?

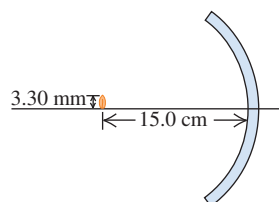
34.9 • A coin is placed next to the convex side of a thin spherical glass shell having a radius of curvature of 18.0 cm. Reflection from the surface of the shell forms an image of the 1.5-cm-tall coin that is 6.00 cm behind the glass shell. Where is the coin located? Determine the size, orientation, and nature (real or virtual) of the image.

34.10 • You hold a spherical salad bowl 90 cm in front of your face with the bottom of the bowl facing you. The salad bowl is made of polished metal with a 35-cm radius of curvature. (a) Where is the image of your 2.0-cm-tall nose located? (b) What are the image's size, orientation, and nature (real or virtual)?

34.11 • (a) Show that Eq. (34.6) can be written as $s' = sf/(s - f)$ and hence the lateral magnification given by Eq. (34.7) can be expressed as $m = f/(f - s)$. (b) A concave spherical mirror has focal length $f = +14.0$ cm. What is the nonzero distance of the object from the mirror vertex if the image has the same height as the object? In this case, is the image erect or inverted? (c) A convex spherical mirror has $f = -8.00$ cm. What is the nonzero distance of the object from the mirror vertex if the height of the image is one-half the height of the object?

34.12 • The thin glass shell shown in Fig. E34.12 has a spherical shape with a radius of curvature of 12.0 cm, and both of its surfaces can act as mirrors. A seed 3.30 mm high is placed 15.0 cm from the center of the mirror along the optic axis, as shown in the figure. (a) Calculate the location and height of the image of this seed. (b) Suppose now that the shell is reversed. Find the location and height of the seed's image.

Figure E34.12



34.13 • **Dental Mirror.** A dentist uses a curved mirror to view teeth on the upper side of the mouth. Suppose she wants an erect image with a magnification of 2.00 when the mirror is 1.25 cm from a tooth. (Treat this problem as though the object and image lie along a straight line.) (a) What kind of mirror (concave or convex) is needed? Use a ray diagram to decide, without performing any calculations. (b) What must be the focal length and radius of curvature of this mirror? (c) Draw a principal-ray diagram to check your answer in part (b).

34.14 • A spherical, concave shaving mirror has a radius of curvature of 32.0 cm. (a) What is the magnification of a person's face when it is 12.0 cm to the left of the vertex of the mirror? (b) Where is the image? Is the image real or virtual? (c) Draw a principal-ray diagram showing the formation of the image.

Section 34.3 Refraction at a Spherical Surface

34.15 • A speck of dirt is embedded 3.50 cm below the surface of a sheet of ice ($n = 1.309$). What is its apparent depth when viewed at normal incidence?

34.16 •• A tank whose bottom is a mirror is filled with water to a depth of 20.0 cm. A small fish floats motionless 7.0 cm under the surface of the water. (a) What is the apparent depth of the fish when viewed at normal incidence? (b) What is the apparent depth of the image of the fish when viewed at normal incidence?

34.17 • A person swimming 0.80 m below the surface of the water in a swimming pool looks at the diving board that is directly overhead and sees the image of the board that is formed by refraction at the surface of the water. This image is a height of 5.20 m above the swimmer. What is the actual height of the diving board above the surface of the water?

34.18 • A person is lying on a diving board 3.00 m above the surface of the water in a swimming pool. The person looks at a penny that is on the bottom of the pool directly below her. The penny appears to the person to be a distance of 8.00 m from her. What is the depth of the water at this point?

34.19 •• **A Spherical Fish Bowl.** A small tropical fish is at the center of a water-filled, spherical fish bowl 28.0 cm in diameter. (a) Find the apparent position and magnification of the fish to an observer outside the bowl. The effect of the thin walls of the bowl may be ignored. (b) A friend advised the owner of the bowl to keep it out of direct sunlight to avoid blinding the fish, which might swim into the focal point of the parallel rays from the sun. Is the focal point actually within the bowl?

34.20 • The left end of a long glass rod 6.00 cm in diameter has a convex hemispherical surface 3.00 cm in radius. The refractive index of the glass is 1.60. Determine the position of the image if an object is placed in air on the axis of the rod at the following distances to the left of the vertex of the curved end: (a) infinitely far, (b) 12.0 cm; (c) 2.00 cm.

34.21 •• The glass rod of Exercise 34.20 is immersed in oil ($n = 1.45$). An object placed to the left of the rod on the rod's axis is to be imaged 1.20 m inside the rod. How far from the left end of the rod must the object be located to form the image?

34.22 •• The left end of a long glass rod 8.00 cm in diameter, with an index of refraction of 1.60, is ground and polished to a convex hemispherical surface with a radius of 4.00 cm. An object in the form of an arrow 1.50 mm tall, at right angles to the axis of the rod, is located on the axis 24.0 cm to the left of the vertex of the convex surface. Find the position and height of the image of the arrow formed by paraxial rays incident on the convex surface. Is the image erect or inverted?

34.23 •• Repeat Exercise 34.22 for the case in which the end of the rod is ground to a *concave* hemispherical surface with radius 4.00 cm.

34.24 •• The glass rod of Exercise 34.23 is immersed in a liquid. An object 14.0 cm from the vertex of the left end of the rod and on its axis is imaged at a point 9.00 cm from the vertex inside the liquid. What is the index of refraction of the liquid?

Section 34.4 Thin Lenses

34.25 • An insect 3.75 mm tall is placed 22.5 cm to the left of a thin planoconvex lens. The left surface of this lens is flat, the right surface has a radius of curvature of magnitude 13.0 cm, and the index of refraction of the lens material is 1.70. (a) Calculate the location and size of the image this lens forms of the insect. Is it real or virtual? Erect or inverted? (b) Repeat part (a) if the lens is reversed.

34.26 • A lens forms an image of an object. The object is 16.0 cm from the lens. The image is 12.0 cm from the lens on the same side as the object. (a) What is the focal length of the lens? Is the lens converging or diverging? (b) If the object is 8.50 mm tall, how tall is the image? Is it erect or inverted? (c) Draw a principal-ray diagram.

34.27 • A converging meniscus lens (see Fig. 34.32a) with a refractive index of 1.52 has spherical surfaces whose radii are 7.00 cm and 4.00 cm. What is the position of the image if an object is placed 24.0 cm to the left of the lens? What is the magnification?

34.28 • A converging lens with a focal length of 90.0 cm forms an image of a 3.20-cm-tall real object that is to the left of the lens. The image is 4.50 cm tall and inverted. Where are the object and image located in relation to the lens? Is the image real or virtual?

34.29 •• A converging lens forms an image of an 8.00-mm-tall real object. The image is 12.0 cm to the left of the lens, 3.40 cm tall, and erect. What is the focal length of the lens? Where is the object located?

34.30 • A photographic slide is to the left of a lens. The lens projects an image of the slide onto a wall 6.00 m to the right of the slide. The image is 80.0 times the size of the slide. (a) How far is the slide from the lens? (b) Is the image erect or inverted? (c) What is the focal length of the lens? (d) Is the lens converging or diverging?

34.31 •• A double-convex thin lens has surfaces with equal radii of curvature of magnitude 2.50 cm. Looking through this lens, you observe that it forms an image of a very distant tree at a distance of 1.87 cm from the lens. What is the index of refraction of the lens?

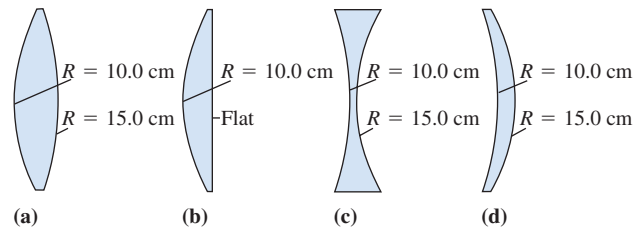
34.32 • **BIO The Lens of the Eye.** The crystalline lens of the human eye is a double-convex lens made of material having an index of refraction of 1.44 (although this varies). Its focal length in air is about 8.0 mm, which also varies. We shall assume that the radii of curvature of its two surfaces have the same magnitude. (a) Find the radii of curvature of this lens. (b) If an object 16 cm tall is placed 30.0 cm from the eye lens, where would the lens focus it and how tall would the image be? Is this image real or virtual? Is it erect or inverted? (*Note:* The results obtained here are not strictly accurate because the lens is embedded in fluids having refractive indexes different from that of air.)

34.33 • **BIO The Cornea As a Simple Lens.** The cornea behaves as a thin lens of focal length approximately 1.8 cm, although this varies a bit. The material of which it is made has an index of refraction of 1.38, and its front surface is convex, with a radius of curvature of 5.0 mm. (a) If this focal length is in air, what is the radius of curvature of the back side of the cornea? (b) The closest distance at which a typical person can focus on an object (called the near point) is about 25 cm, although this varies considerably with age. Where would the cornea focus the image of an 8.0-mm-tall object at the near point? (c) What is the height of the image in part (b)? Is this image real or virtual? Is it erect or inverted? (*Note:* The results obtained here are not strictly accurate because, on one side, the cornea has a fluid with a refractive index different from that of air.)

34.34 • A converging lens with a focal length of 7.00 cm forms an image of a 4.00-mm-tall real object that is to the left of the lens. The image is 1.30 cm tall and erect. Where are the object and image located? Is the image real or virtual?

34.35 • For each thin lens shown in Fig. E34.35, calculate the location of the image of an object that is 18.0 cm to the left of the lens. The lens material has a refractive index of 1.50, and the radii of curvature shown are only the magnitudes.

Figure E34.35



34.36 • A converging lens with a focal length of 12.0 cm forms a virtual image 8.00 mm tall, 17.0 cm to the right of the lens. Determine the position and size of the object. Is the image erect or inverted? Are the object and image on the same side or opposite sides of the lens? Draw a principal-ray diagram for this situation.

34.37 • Repeat Exercise 34.36 for the case in which the lens is diverging, with a focal length of -48.0 cm.

34.38 • An object is 16.0 cm to the left of a lens. The lens forms an image 36.0 cm to the right of the lens. (a) What is the focal length of the lens? Is the lens converging or diverging? (b) If the object is 8.00 mm tall, how tall is the image? Is it erect or inverted? (c) Draw a principal-ray diagram.

34.39 •• **Combination of Lenses I.** A 1.20-cm-tall object is 50.0 cm to the left of a converging lens of focal length 40.0 cm. A second converging lens, this one having a focal length of 60.0 cm, is located 300.0 cm to the right of the first lens along the same optic axis. (a) Find the location and height of the image (call it I_1) formed by the lens with a focal length of 40.0 cm. (b) I_1 is now the object for the second lens. Find the location and height of the image produced by the second lens. This is the final image produced by the combination of lenses.

34.40 •• **Combination of Lenses II.** Repeat Problem 34.39 using the same lenses except for the following changes: (a) The second lens is a *diverging* lens having a focal length of magnitude 60.0 cm. (b) The first lens is a *diverging* lens having a focal length of magnitude 40.0 cm. (c) Both lenses are diverging lenses having focal lengths of the same *magnitudes* as in Problem 34.39.

34.41 •• **Combination of Lenses III.** Two thin lenses with a focal length of magnitude 12.0 cm, the first diverging and the second converging, are located 9.00 cm apart. An object 2.50 mm tall is placed 20.0 cm to the left of the first (diverging) lens. (a) How far from this first lens is the final image formed? (b) Is the final image real or virtual? (c) What is the height of the final image? Is it erect or inverted? (*Hint:* See the preceding two problems.)

Section 34.5 Cameras

34.42 • You wish to project the image of a slide on a screen 9.00 m from the lens of a slide projector. (a) If the slide is placed 15.0 cm from the lens, what focal length lens is required? (b) If the dimensions of the picture on a 35-mm color slide are 24 mm \times 36 mm, what is the minimum size of the projector screen required to accommodate the image?

34.43 •• A camera lens has a focal length of 200 mm. How far from the lens should the subject for the photo be if the lens is 20.4 cm from the film?

34.44 • When a camera is focused, the lens is moved away from or toward the film. If you take a picture of your friend, who is standing 3.90 m from the lens, using a camera with a lens with a 85-mm focal length, how far from the film is the lens? Will the whole image of your friend, who is 175 cm tall, fit on film that is 24 \times 36 mm?

34.45 • Figure 34.41 shows photographs of the same scene taken with the same camera with lenses of different focal length. If the

object is 200 m from the lens, what is the magnitude of the lateral magnification for a lens of focal length (a) 28 mm; (b) 105 mm; (c) 300 mm?

34.46 • A photographer takes a photograph of a Boeing 747 airliner (length 70.7 m) when it is flying directly overhead at an altitude of 9.50 km. The lens has a focal length of 5.00 m. How long is the image of the airliner on the film?

34.47 • Choosing a Camera Lens. The picture size on ordinary 35-mm camera film is $24\text{ mm} \times 36\text{ mm}$. Focal lengths of lenses available for 35-mm cameras typically include 28, 35, 50 (the “normal” lens), 85, 100, 135, 200, and 300 mm, among others. Which of these lenses should be used to photograph the following objects, assuming that the object is to fill most of the picture area? (a) a building 240 m tall and 160 m wide at a distance of 600 m, and (b) a mobile home 9.6 m in length at a distance of 40.0 m.

34.48 • Zoom Lens. Consider the simple model of the zoom lens shown in Fig. 34.43a. The converging lens has focal length $f_1 = 12\text{ cm}$, and the diverging lens has focal length $f_2 = -12\text{ cm}$. The lenses are separated by 4 cm as shown in Fig. 34.43a. (a) For a distant object, where is the image of the converging lens? (b) The image of the converging lens serves as the object for the diverging lens. What is the object distance for the diverging lens? (c) Where is the final image? Compare your answer to Fig. 34.43a. (d) Repeat parts (a), (b), and (c) for the situation shown in Fig. 34.43b, in which the lenses are separated by 8 cm.

34.49 •• A camera lens has a focal length of 180.0 mm and an aperture diameter of 16.36 mm. (a) What is the f -number of the lens? (b) If the correct exposure of a certain scene is $\frac{1}{30}\text{ s}$ at $f/11$, what is the correct exposure at $f/2.8$?

34.50 •• Recall that the intensity of light reaching film in a camera is proportional to the effective area of the lens. Camera A has a lens with an aperture diameter of 8.00 mm. It photographs an object using the correct exposure time of $\frac{1}{30}\text{ s}$. What exposure time should be used with camera B in photographing the same object with the same film if this camera has a lens with an aperture diameter of 23.1 mm?

34.51 • Photography. A 35-mm camera has a standard lens with focal length 50 mm and can focus on objects between 45 cm and infinity. (a) Is the lens for such a camera a concave or a convex lens? (b) The camera is focused by rotating the lens, which moves it on the camera body and changes its distance from the film. In what range of distances between the lens and the film plane must the lens move to focus properly over the 45 cm to infinity range?

Section 34.6 The Eye

34.52 •• BIO Curvature of the Cornea. In a simplified model of the human eye, the aqueous and vitreous humors and the lens all have a refractive index of 1.40, and all the refraction occurs at the cornea, whose vertex is 2.60 cm from the retina. What should be the radius of curvature of the cornea such that the image of an object 40.0 cm from the cornea’s vertex is focused on the retina?

34.53 •• BIO (a) Where is the near point of an eye for which a contact lens with a power of +2.75 diopters is prescribed? (b) Where is the far point of an eye for which a contact lens with a power of -1.30 diopters is prescribed for distant vision?

34.54 • BIO Contact Lenses. Contact lenses are placed right on the eyeball, so the distance from the eye to an object (or image) is the same as the distance from the lens to that object (or image). A certain person can see distant objects well, but his near point is 45.0 cm from his eyes instead of the usual 25.0 cm. (a) Is this person nearsighted or farsighted? (b) What type of lens (converging or diverging) is needed to correct his vision? (c) If the correcting

lenses will be contact lenses, what focal length lens is needed and what is its power in diopters?

34.55 •• BIO Ordinary Glasses. Ordinary glasses are worn in front of the eye and usually 2.0 cm in front of the eyeball. Suppose that the person in Problem 34.54 prefers ordinary glasses to contact lenses. What focal length lenses are needed to correct his vision, and what is their power in diopters?

34.56 • BIO A person can see clearly up close but cannot focus on objects beyond 75.0 cm. She opts for contact lenses to correct her vision. (a) Is she nearsighted or farsighted? (b) What type of lens (converging or diverging) is needed to correct her vision? (c) What focal length contact lens is needed, and what is its power in diopters?

34.57 •• BIO If the person in Problem 34.56 chooses ordinary glasses over contact lenses, what power lens (in diopters) does she need to correct her vision if the lenses are 2.0 cm in front of the eye?

Section 34.7 The Magnifier

34.58 •• A thin lens with a focal length of 6.00 cm is used as a simple magnifier. (a) What angular magnification is obtainable with the lens if the object is at the focal point? (b) When an object is examined through the lens, how close can it be brought to the lens? Assume that the image viewed by the eye is at the near point, 25.0 cm from the eye, and that the lens is very close to the eye.

34.59 • The focal length of a simple magnifier is 8.00 cm. Assume the magnifier is a thin lens placed very close to the eye. (a) How far in front of the magnifier should an object be placed if the image is formed at the observer’s near point, 25.0 cm in front of her eye? (b) If the object is 1.00 mm high, what is the height of its image formed by the magnifier?

34.60 • You want to view an insect 2.00 mm in length through a magnifier. If the insect is to be at the focal point of the magnifier, what focal length will give the image of the insect an angular size of 0.025 radian?

Section 34.8 Microscopes and Telescopes

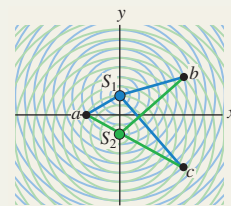
34.61 • A certain microscope is provided with objectives that have focal lengths of 16 mm, 4 mm, and 1.9 mm and with eyepieces that have angular magnifications of $5\times$ and $10\times$. Each objective forms an image 120 mm beyond its second focal point. Determine (a) the largest overall angular magnification obtainable and (b) the smallest overall angular magnification obtainable.

34.62 •• Resolution of a Microscope. The image formed by a microscope objective with a focal length of 5.00 mm is 160 mm from its second focal point. The eyepiece has a focal length of 26.0 mm. (a) What is the angular magnification of the microscope? (b) The unaided eye can distinguish two points at its near point as separate if they are about 0.10 mm apart. What is the minimum separation between two points that can be observed (or resolved) through this microscope?

34.63 •• The focal length of the eyepiece of a certain microscope is 18.0 mm. The focal length of the objective is 8.00 mm. The distance between objective and eyepiece is 19.7 cm. The final image formed by the eyepiece is at infinity. Treat all lenses as thin. (a) What is the distance from the objective to the object being viewed? (b) What is the magnitude of the linear magnification produced by the objective? (c) What is the overall angular magnification of the microscope?

34.64 •• The eyepiece of a refracting telescope (see Fig. 34.53) has a focal length of 9.00 cm. The distance between objective and eyepiece is 1.80 m, and the final image is at infinity. What is the angular magnification of the telescope?

Interference and coherent sources: Monochromatic light is light with a single frequency. Coherence is a definite, unchanging phase relationship between two waves. The overlap of waves from two coherent sources of monochromatic light forms an interference pattern. The principle of superposition states that the total wave disturbance at any point is the sum of the disturbances from the separate waves.



Two-source interference of light: When two sources are in phase, constructive interference occurs where the difference in path length from the two sources is zero or an integer number of wavelengths; destructive interference occurs where the path difference is a half-integer number of wavelengths. If two sources separated by a distance d are both very far from a point P , and the line from the sources to P makes an angle θ with the line perpendicular to the line of the sources, then the condition for constructive interference at P is Eq. (35.4). The condition for destructive interference is Eq. (35.5). When θ is very small, the position y_m of the m th bright fringe on a screen located a distance R from the sources is given by Eq. (35.6). (See Examples 35.1 and 35.2.)

$$d \sin \theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \dots)$$

(constructive interference) (35.4)

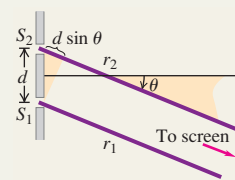
$$d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$$

($m = 0, \pm 1, \pm 2, \dots$) (35.5)

(destructive interference)

$$y_m = R \frac{m\lambda}{d}$$

(bright fringes) (35.6)



Intensity in interference patterns: When two sinusoidal waves with equal amplitude E and phase difference ϕ are superimposed, the resultant amplitude E_P and intensity I are given by Eqs. (35.7) and (35.10), respectively. If the two sources emit in phase, the phase difference ϕ at a point P (located a distance r_1 from source 1 and a distance r_2 from source 2) is directly proportional to the difference in path length $r_2 - r_1$. (See Example 35.3.)

$$E_P = 2E \left| \cos \frac{\phi}{2} \right|$$

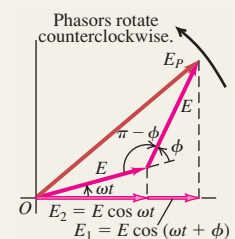
(35.7)

$$I = I_0 \cos^2 \frac{\phi}{2}$$

(35.10)

$$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = k(r_2 - r_1)$$

(35.11)



Interference in thin films: When light is reflected from both sides of a thin film of thickness t and no phase shift occurs at either surface, constructive interference between the reflected waves occurs when $2t$ is equal to an integral number of wavelengths. If a half-cycle phase shift occurs at one surface, this is the condition for destructive interference. A half-cycle phase shift occurs during reflection whenever the index of refraction in the second material is greater than that in the first. (See Examples 35.4–35.7.)

$$2t = m\lambda \quad (m = 0, 1, 2, \dots)$$

(constructive reflection from thin film, no relative phase shift) (35.17a)

$$2t = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \dots)$$

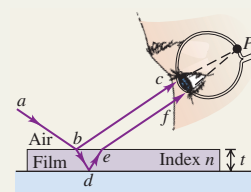
(destructive reflection from thin film, no relative phase shift) (35.17b)

$$2t = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \dots)$$

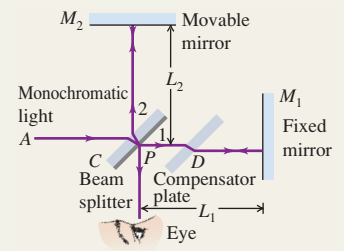
(constructive reflection from thin film, half-cycle relative phase shift) (35.18a)

$$2t = m\lambda \quad (m = 0, 1, 2, \dots)$$

(destructive reflection from thin film, half-cycle relative phase shift) (35.18b)



Michelson interferometer: The Michelson interferometer uses a monochromatic light source and can be used for high-precision measurements of wavelengths. Its original purpose was to detect motion of the earth relative to a hypothetical ether, the supposed medium for electromagnetic waves. The ether has never been detected, and the concept has been abandoned; the speed of light is the same relative to all observers. This is part of the foundation of the special theory of relativity.



BRIDGING PROBLEM

Modifying a Two-Slit Experiment

An oil tanker spills a large amount of oil ($n = 1.45$) into the sea ($n = 1.33$). (a) If you look down onto the oil spill from overhead, what predominant wavelength of light do you see at a point where the oil is 380 nm thick? What color is the light? (*Hint:* See Table 32.1.) (b) In the water under the slick, what visible wavelength (as measured in air) is predominant in the transmitted light at the same place in the slick as in part (a)?

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IDENTIFY and SET UP

- The oil layer acts as a thin film, so we must consider interference between light that reflects from the top and bottom surfaces of the oil. If a wavelength is prominent in the *transmitted* light, there is destructive interference for that wavelength in the *reflected* light.

- Choose the appropriate interference equations that relate the thickness of the oil film and the wavelength of light. Take account of the indexes of refraction of the air, oil, and water.

EXECUTE

- For part (a), find the wavelengths for which there is constructive interference as seen from above the oil film. Which of these are in the visible spectrum?
- For part (b), find the visible wavelength for which there is destructive interference as seen from above the film. (This will ensure that there is substantial transmitted light at the wavelength.)

EVALUATE

- If a diver below the water's surface shines a light up at the bottom of the oil film, at what wavelengths would there be constructive interference in the light that reflects back downward?

Problems

For instructor-assigned homework, go to www.masteringphysics.com



•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

- Q35.1** A two-slit interference experiment is set up, and the fringes are displayed on a screen. Then the whole apparatus is immersed in the nearest swimming pool. How does the fringe pattern change?
- Q35.2** Could an experiment similar to Young's two-slit experiment be performed with sound? How might this be carried out? Does it matter that sound waves are longitudinal and electromagnetic waves are transverse? Explain.
- Q35.3** Monochromatic coherent light passing through two thin slits is viewed on a distant screen. Are the bright fringes equally spaced on the screen? If so, why? If not, which ones are closest to being equally spaced?
- Q35.4** In a two-slit interference pattern on a distant screen, are the bright fringes midway between the dark fringes? Is this ever a good approximation?
- Q35.5** Would the headlights of a distant car form a two-source interference pattern? If so, how might it be observed? If not, why not?

Q35.6 The two sources S_1 and S_2 shown in Fig. 35.3 emit waves of the same wavelength λ and are in phase with each other. Suppose S_1 is a weaker source, so that the waves emitted by S_1 have half the amplitude of the waves emitted by S_2 . How would this affect the positions of the antinodal lines and nodal lines? Would there be total reinforcement at points on the antinodal curves? Would there be total cancellation at points on the nodal curves? Explain your answers.

Q35.7 Could the Young two-slit interference experiment be performed with gamma rays? If not, why not? If so, discuss differences in the experimental design compared to the experiment with visible light.

Q35.8 Coherent red light illuminates two narrow slits that are 25 cm apart. Will a two-slit interference pattern be observed when the light from the slits falls on a screen? Explain.

Q35.9 Coherent light with wavelength λ falls on two narrow slits separated by a distance d . If d is less than some minimum value,

no dark fringes are observed. Explain. In terms of λ , what is this minimum value of d ?

Q35.10 A fellow student, who values memorizing equations above understanding them, combines Eqs. (35.4) and (35.13) to “prove” that ϕ can *only* equal $2\pi m$. How would you explain to this student that ϕ can have values other than $2\pi m$?

Q35.11 If the monochromatic light shown in Fig. 35.5a were replaced by white light, would a two-slit interference pattern be seen on the screen? Explain.

Q35.12 In using the superposition principle to calculate intensities in interference patterns, could you add the intensities of the waves instead of their amplitudes? Explain.

Q35.13 A glass windowpane with a thin film of water on it reflects less than when it is perfectly dry. Why?

Q35.14 A *very* thin soap film ($n = 1.33$), whose thickness is much less than a wavelength of visible light, looks black; it appears to reflect no light at all. Why? By contrast, an equally thin layer of soapy water ($n = 1.33$) on glass ($n = 1.50$) appears quite shiny. Why is there a difference?

Q35.15 Interference can occur in thin films. Why is it important that the films be *thin*? Why don't you get these effects with a relatively *thick* film? Where should you put the dividing line between “thin” and “thick”? Explain your reasoning.

Q35.16 If we shine white light on an air wedge like that shown in Fig. 35.12, the colors that are weak in the light *reflected* from any point along the wedge are strong in the light *transmitted* through the wedge. Explain why this should be so.

Q35.17 Monochromatic light is directed at normal incidence on a thin film. There is destructive interference for the reflected light, so the intensity of the reflected light is very low. What happened to the energy of the incident light?

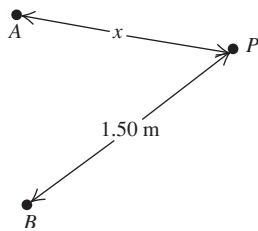
Q35.18 When a thin oil film spreads out on a puddle of water, the thinnest part of the film looks dark in the resulting interference pattern. What does this tell you about the relative magnitudes of the refractive indexes of oil and water?

EXERCISES

Section 35.1 Interference and Coherent Sources

35.1 • Two small stereo speakers A and B that are 1.40 m apart are sending out sound of wavelength 34 cm in all directions and all in phase. A person at point P starts out equidistant from both speakers and walks so that he is always 1.50 m from speaker B (Fig. E35.1). For what values of x will the sound this person hears be (a) maximally reinforced, (b) cancelled? Limit your solution to the cases where $x \leq 1.50$ m.

Figure E35.1



35.2 • • Two speakers that are 15.0 m apart produce in-phase sound waves of frequency 250.0 Hz in a room where the speed of sound is 340.0 m/s. A woman starts out at the midpoint between the two speakers. The room's walls and ceiling are covered with absorbers to eliminate reflections, and she listens with only one ear for best precision. (a) What does she hear: constructive or destructive interference? Why? (b) She now walks slowly toward one of the speakers. How far from the center must she walk before she first hears the sound reach a minimum intensity? (c) How far from the center must she walk before she first hears the sound maximally enhanced?

35.3 • • Two identical audio speakers connected to the same amplifier produce in-phase sound waves with a single frequency that can be varied between 300 and 600 Hz. The speed of sound is 340 m/s. You find that where you are standing, you hear minimum-intensity sound. (a) Explain why you hear minimum-intensity sound. (b) If one of the speakers is moved 39.8 cm toward you, the sound you hear has maximum intensity. What is the frequency of the sound? (c) How much closer to you from the position in part (b) must the speaker be moved to the next position where you hear maximum intensity?

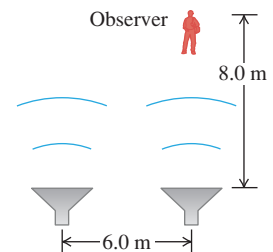
35.4 • **Radio Interference.** Two radio antennas A and B radiate in phase. Antenna B is 120 m to the right of antenna A . Consider point Q along the extension of the line connecting the antennas, a horizontal distance of 40 m to the right of antenna B . The frequency, and hence the wavelength, of the emitted waves can be varied. (a) What is the longest wavelength for which there will be destructive interference at point Q ? (b) What is the longest wavelength for which there will be constructive interference at point Q ?

35.5 • • A radio transmitting station operating at a frequency of 120 MHz has two identical antennas that radiate in phase. Antenna B is 9.00 m to the right of antenna A . Consider point P between the antennas and along the line connecting them, a horizontal distance x to the right of antenna A . For what values of x will constructive interference occur at point P ?

35.6 • Two light sources can be adjusted to emit monochromatic light of any visible wavelength. The two sources are coherent, $2.04 \mu\text{m}$ apart, and in line with an observer, so that one source is $2.04 \mu\text{m}$ farther from the observer than the other. (a) For what visible wavelengths (380 to 750 nm) will the observer see the brightest light, owing to constructive interference? (b) How would your answers to part (a) be affected if the two sources were not in line with the observer, but were still arranged so that one source is $2.04 \mu\text{m}$ farther away from the observer than the other? (c) For what visible wavelengths will there be *destructive* interference at the location of the observer?

35.7 • Two speakers, emitting identical sound waves of wavelength 2.0 m in phase with each other, and an observer are located as shown in Fig. E35.7. (a) At the observer's location, what is the path difference for waves from the two speakers? (b) Will the sound waves interfere constructively or destructively at the observer's location—or something in between constructive and destructive? (c) Suppose the observer now increases her distance from the closest speaker to 17.0 m, staying directly in front of the same speaker as initially. Answer the questions of parts (a) and (b) for this new situation.

Figure E35.7



35.8 • • Figure 35.3 shows the wave pattern produced by two identical, coherent sources emitting waves with wavelength λ and separated by a distance $d = 4\lambda$. (a) Explain why the positive y -axis above S_1 constitutes an antinodal curve with $m = +4$ and why the negative y -axis below S_2 constitutes an antinodal curve with $m = -4$. (b) Draw the wave pattern produced when the separation between the sources is reduced to 3λ . In your drawing, sketch all antinodal curves—that is, the curves on which $r_2 - r_1 = m\lambda$. Label each curve by its value of m . (c) In general, what determines the maximum (most positive) and minimum (most negative) values of the integer m that labels the antinodal lines? (d) Suppose the separation between the sources is increased

to $7\frac{1}{2}\lambda$. How many antinodal curves will there be? To what values of m do they correspond? Explain your reasoning. (You should not have to make a drawing to answer these questions.)

Section 35.2 Two-Source Interference of Light

35.9 • Young's experiment is performed with light from excited helium atoms ($\lambda = 502$ nm). Fringes are measured carefully on a screen 1.20 m away from the double slit, and the center of the 20th fringe (not counting the central bright fringe) is found to be 10.6 mm from the center of the central bright fringe. What is the separation of the two slits?

35.10 •• Coherent light with wavelength 450 nm falls on a double slit. On a screen 1.80 m away, the distance between dark fringes is 4.20 mm. What is the separation of the slits?

35.11 •• Two slits spaced 0.450 mm apart are placed 75.0 cm from a screen. What is the distance between the second and third dark lines of the interference pattern on the screen when the slits are illuminated with coherent light with a wavelength of 500 nm?

35.12 •• If the entire apparatus of Exercise 35.11 (slits, screen, and space in between) is immersed in water, what then is the distance between the second and third dark lines?

35.13 •• Two thin parallel slits that are 0.0116 mm apart are illuminated by a laser beam of wavelength 585 nm. (a) On a very large distant screen, what is the *total* number of bright fringes (those indicating complete constructive interference), including the central fringe and those on both sides of it? Solve this problem without calculating all the angles! (*Hint:* What is the largest that $\sin\theta$ can be? What does this tell you is the largest value of m ?) (b) At what angle, relative to the original direction of the beam, will the fringe that is most distant from the central bright fringe occur?

35.14 • Coherent light with wavelength 400 nm passes through two very narrow slits that are separated by 0.200 mm, and the interference pattern is observed on a screen 4.00 m from the slits. (a) What is the width (in mm) of the central interference maximum? (b) What is the width of the first-order bright fringe?

35.15 •• Two very narrow slits are spaced 1.80 μm apart and are placed 35.0 cm from a screen. What is the distance between the first and second dark lines of the interference pattern when the slits are illuminated with coherent light with $\lambda = 550$ nm? (*Hint:* The angle θ in Eq. (35.5) is *not* small.)

35.16 •• Coherent light that contains two wavelengths, 660 nm (red) and 470 nm (blue), passes through two narrow slits separated by 0.300 mm, and the interference pattern is observed on a screen 5.00 m from the slits. What is the distance on the screen between the first-order bright fringes for the two wavelengths?

35.17 •• Coherent light with wavelength 600 nm passes through two very narrow slits and the interference pattern is observed on a screen 3.00 m from the slits. The first-order bright fringe is at 4.84 mm from the center of the central bright fringe. For what wavelength of light will the first-order dark fringe be observed at this same point on the screen?

35.18 •• Coherent light of frequency 6.32×10^{14} Hz passes through two thin slits and falls on a screen 85.0 cm away. You observe that the third bright fringe occurs at ± 3.11 cm on either side of the central bright fringe. (a) How far apart are the two slits? (b) At what distance from the central bright fringe will the third dark fringe occur?

Section 35.3 Intensity in Interference Patterns

35.19 •• In a two-slit interference pattern, the intensity at the peak of the central maximum is I_0 . (a) At a point in the pattern

where the phase difference between the waves from the two slits is 60.0° , what is the intensity? (b) What is the path difference for 480-nm light from the two slits at a point where the phase angle is 60.0° ?

35.20 • Coherent sources A and B emit electromagnetic waves with wavelength 2.00 cm. Point P is 4.86 m from A and 5.24 m from B . What is the phase difference at P between these two waves?

35.21 • Coherent light with wavelength 500 nm passes through narrow slits separated by 0.340 mm. At a distance from the slits large compared to their separation, what is the phase difference (in radians) in the light from the two slits at an angle of 23.0° from the centerline?

35.22 • Two slits spaced 0.260 mm apart are placed 0.700 m from a screen and illuminated by coherent light with a wavelength of 660 nm. The intensity at the center of the central maximum ($\theta = 0^\circ$) is I_0 . (a) What is the distance on the screen from the center of the central maximum to the first minimum? (b) What is the distance on the screen from the center of the central maximum to the point where the intensity has fallen to $I_0/2$?

35.23 •• Points A and B are 56.0 m apart along an east-west line. At each of these points, a radio transmitter is emitting a 12.5-MHz signal horizontally. These transmitters are in phase with each other and emit their beams uniformly in a horizontal plane. A receiver is taken 0.500 km north of the AB line and initially placed at point C , directly opposite the midpoint of AB . The receiver can be moved only along an east-west direction but, due to its limited sensitivity, it must always remain within a range so that the intensity of the signal it receives from the transmitter is no less than $\frac{1}{4}$ of its maximum value. How far from point C (along an east-west line) can the receiver be moved and always be able to pick up the signal?

35.24 • Consider two antennas separated by 9.00 m that radiate in phase at 120 MHz, as described in Exercise 35.5. A receiver placed 150 m from both antennas measures an intensity I_0 . The receiver is moved so that it is 1.8 m closer to one antenna than to the other. (a) What is the phase difference ϕ between the two radio waves produced by this path difference? (b) In terms of I_0 , what is the intensity measured by the receiver at its new position?

Section 35.4 Interference in Thin Films

35.25 • What is the thinnest film of a coating with $n = 1.42$ on glass ($n = 1.52$) for which destructive interference of the red component (650 nm) of an incident white light beam in air can take place by reflection?

35.26 •• **Nonglare Glass.** When viewing a piece of art that is behind glass, one often is affected by the light that is reflected off the front of the glass (called *glare*), which can make it difficult to see the art clearly. One solution is to coat the outer surface of the glass with a film to cancel part of the glare. (a) If the glass has a refractive index of 1.62 and you use TiO_2 , which has an index of refraction of 2.62, as the coating, what is the minimum film thickness that will cancel light of wavelength 505 nm? (b) If this coating is too thin to stand up to wear, what other thickness would also work? Find only the three thinnest ones.

35.27 •• Two rectangular pieces of plane glass are laid one upon the other on a table. A thin strip of paper is placed between them at one edge so that a very thin wedge of air is formed. The plates are illuminated at normal incidence by 546-nm light from a mercury-vapor lamp. Interference fringes are formed, with 15.0 fringes per centimeter. Find the angle of the wedge.

35.28 •• A plate of glass 9.00 cm long is placed in contact with a second plate and is held at a small angle with it by a metal strip

0.0800 mm thick placed under one end. The space between the plates is filled with air. The glass is illuminated from above with light having a wavelength in air of 656 nm. How many interference fringes are observed per centimeter in the reflected light?

35.29 •• A uniform film of TiO_2 , 1036 nm thick and having index of refraction 2.62, is spread uniformly over the surface of crown glass of refractive index 1.52. Light of wavelength 520.0 nm falls at normal incidence onto the film from air. You want to increase the thickness of this film so that the reflected light cancels. (a) What is the *minimum* thickness of TiO_2 that you must *add* so the reflected light cancels as desired? (b) After you make the adjustment in part (a), what is the path difference between the light reflected off the top of the film and the light that cancels it after traveling through the film? Express your answer in (i) nanometers and (ii) wavelengths of the light in the TiO_2 film.

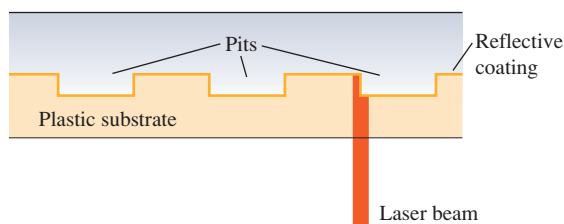
35.30 • A plastic film with index of refraction 1.85 is put on the surface of a car window to increase the reflectivity and thus to keep the interior of the car cooler. The window glass has index of refraction 1.52. (a) What minimum thickness is required if light with wavelength 550 nm in air reflected from the two sides of the film is to interfere constructively? (b) It is found to be difficult to manufacture and install coatings as thin as calculated in part (a). What is the next greatest thickness for which there will also be constructive interference?

35.31 • The walls of a soap bubble have about the same index of refraction as that of plain water, $n = 1.33$. There is air both inside and outside the bubble. (a) What wavelength (in air) of visible light is most strongly reflected from a point on a soap bubble where its wall is 290 nm thick? To what color does this correspond (see Fig. 32.4 and Table 32.1)? (b) Repeat part (a) for a wall thickness of 340 nm.

35.32 •• Light with wavelength 648 nm in air is incident perpendicularly from air on a film $8.76 \mu\text{m}$ thick and with refractive index 1.35. Part of the light is reflected from the first surface of the film, and part enters the film and is reflected back at the second surface, where the film is again in contact with air. (a) How many waves are contained along the path of this second part of the light in its round trip through the film? (b) What is the phase difference between these two parts of the light as they leave the film?

35.33 •• Compact Disc Player. A compact disc (CD) is read from the bottom by a semiconductor laser with wavelength 790 nm passing through a plastic substrate of refractive index 1.8. When the beam encounters a pit, part of the beam is reflected from the pit and part from the flat region between the pits, so these two beams interfere with each other (Fig. E35.33). What must the minimum pit depth be so that the part of the beam reflected from a pit cancels the part of the beam reflected from the flat region? (It is this cancellation that allows the player to recognize the beginning and end of a pit.)

Figure E35.33



35.34 • What is the thinnest soap film (excluding the case of zero thickness) that appears black when illuminated with light with

wavelength 480 nm? The index of refraction of the film is 1.33, and there is air on both sides of the film.

Section 35.5 The Michelson Interferometer

35.35 • How far must the mirror M_2 (see Fig. 35.19) of the Michelson interferometer be moved so that 1800 fringes of He-Ne laser light ($\lambda = 633 \text{ nm}$) move across a line in the field of view?

35.36 • Jan first uses a Michelson interferometer with the 606-nm light from a krypton-86 lamp. He displaces the movable mirror away from him, counting 818 fringes moving across a line in his field of view. Then Linda replaces the krypton lamp with filtered 502-nm light from a helium lamp and displaces the movable mirror toward her. She also counts 818 fringes, but they move across the line in her field of view opposite to the direction they moved for Jan. Assume that both Jan and Linda counted to 818 correctly. (a) What distance did each person move the mirror? (b) What is the resultant displacement of the mirror?

PROBLEMS

35.37 ••• The radius of curvature of the convex surface of a planoconvex lens is 68.4 cm. The lens is placed convex side down on a perfectly flat glass plate that is illuminated from above with red light having a wavelength of 580 nm. Find the diameter of the second bright ring in the interference pattern.

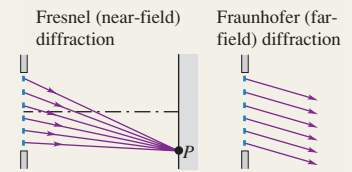
35.38 •• Newton's rings can be seen when a planoconvex lens is placed on a flat glass surface. For a particular lens with an index of refraction of $n = 1.50$ and a glass plate with an index of $n = 1.80$, the diameter of the third bright ring is 0.720 mm. If water ($n = 1.33$) now fills the space between the lens and the plate, what is the new diameter of this ring?

35.39 • BIO Coating Eyeglass Lenses. Eyeglass lenses can be coated on the *inner* surfaces to reduce the reflection of stray light to the eye. If the lenses are medium flint glass of refractive index 1.62 and the coating is fluorite of refractive index 1.432, (a) what minimum thickness of film is needed on the lenses to cancel light of wavelength 550 nm reflected toward the eye at normal incidence? (b) Will any other wavelengths of visible light be cancelled or enhanced in the reflected light?

35.40 •• BIO Sensitive Eyes. After an eye examination, you put some eyedrops on your sensitive eyes. The cornea (the front part of the eye) has an index of refraction of 1.38, while the eyedrops have a refractive index of 1.45. After you put in the drops, your friends notice that your eyes look red, because red light of wavelength 600 nm has been reinforced in the reflected light. (a) What is the minimum thickness of the film of eyedrops on your cornea? (b) Will any other wavelengths of visible light be reinforced in the reflected light? Will any be cancelled? (c) Suppose you had contact lenses, so that the eyedrops went on them instead of on your corneas. If the refractive index of the lens material is 1.50 and the layer of eyedrops has the same thickness as in part (a), what wavelengths of visible light will be reinforced? What wavelengths will be cancelled?

35.41 •• Two flat plates of glass with parallel faces are on a table, one plate on the other. Each plate is 11.0 cm long and has a refractive index of 1.55. A very thin sheet of metal foil is inserted under the end of the upper plate to raise it slightly at that end, in a manner similar to that discussed in Example 35.4. When you view the glass plates from above with reflected white light, you observe that, at 1.15 mm from the line where the sheets are in contact, the violet light of wavelength 400.0 nm is enhanced in this reflected light, but no visible light is enhanced closer to the line of contact.

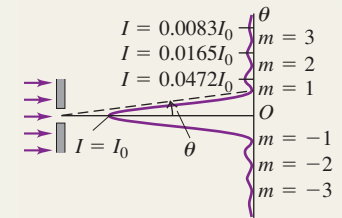
Fresnel and Fraunhofer diffraction: Diffraction occurs when light passes through an aperture or around an edge. When the source and the observer are so far away from the obstructing surface that the outgoing rays can be considered parallel, it is called Fraunhofer diffraction. When the source or the observer is relatively close to the obstructing surface, it is Fresnel diffraction.



Single-slit diffraction: Monochromatic light sent through a narrow slit of width a produces a diffraction pattern on a distant screen. Equation (36.2) gives the condition for destructive interference (a dark fringe) at a point P in the pattern at angle θ . Equation (36.7) gives the intensity in the pattern as a function of θ . (See Examples 36.1–36.3.)

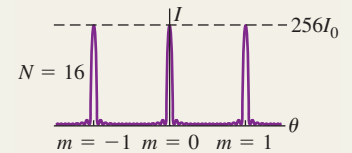
$$\sin \theta = \frac{m\lambda}{a} \quad (m = \pm 1, \pm 2, \dots) \quad (36.2)$$

$$I = I_0 \left\{ \frac{\sin[\pi a(\sin \theta)/\lambda]}{\pi a(\sin \theta)/\lambda} \right\}^2 \quad (36.7)$$



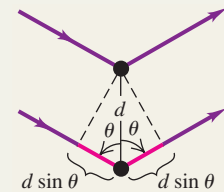
Diffraction gratings: A diffraction grating consists of a large number of thin parallel slits, spaced a distance d apart. The condition for maximum intensity in the interference pattern is the same as for the two-source pattern, but the maxima for the grating are very sharp and narrow. (See Example 36.4.)

$$d \sin \theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \dots) \quad (36.13)$$



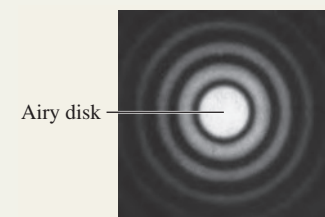
X-ray diffraction: A crystal serves as a three-dimensional diffraction grating for x rays with wavelengths of the same order of magnitude as the spacing between atoms in the crystal. For a set of crystal planes spaced a distance d apart, constructive interference occurs when the angles of incidence and scattering (measured from the crystal planes) are equal and when the Bragg condition [Eq. (36.16)] is satisfied. (See Example 36.5.)

$$2d \sin \theta = m\lambda \quad (m = 1, 2, 3, \dots) \quad (36.16)$$



Circular apertures and resolving power: The diffraction pattern from a circular aperture of diameter D consists of a central bright spot, called the Airy disk, and a series of concentric dark and bright rings. Equation (36.17) gives the angular radius θ_1 of the first dark ring, equal to the angular size of the Airy disk. Diffraction sets the ultimate limit on resolution (image sharpness) of optical instruments. According to Rayleigh's criterion, two point objects are just barely resolved when their angular separation θ is given by Eq. (36.17). (See Example 36.6.)

$$\sin \theta_1 = 1.22 \frac{\lambda}{D} \quad (36.17)$$



Q36.13 At the end of Section 36.4, the following statements were made about an array of N slits. Explain, using phasor diagrams, why each statement is true. (a) A minimum occurs whenever ϕ is an integral multiple of $2\pi/N$, except when ϕ is an integral multiple of 2π (which gives a principal maximum). (b) There are $(N - 1)$ minima between each pair of principal maxima.

Q36.14 Could x-ray diffraction effects with crystals be observed by using visible light instead of x rays? Why or why not?

Q36.15 Why is a diffraction grating better than a two-slit setup for measuring wavelengths of light?

Q36.16 One sometimes sees rows of evenly spaced radio antenna towers. A student remarked that these act like diffraction gratings. What did she mean? Why would one want them to act like a diffraction grating?

Q36.17 If a hologram is made using 600-nm light and then viewed with 500-nm light, how will the images look compared to those observed when viewed with 600-nm light? Explain.

Q36.18 A hologram is made using 600-nm light and then viewed by using white light from an incandescent bulb. What will be seen? Explain.

Q36.19 Ordinary photographic film reverses black and white, in the sense that the most brightly illuminated areas become blackest upon development (hence the term *negative*). Suppose a hologram negative is viewed directly, without making a positive transparency. How will the resulting images differ from those obtained with the positive? Explain.

EXERCISES

Section 36.2 Diffraction from a Single Slit

36.1 • Monochromatic light from a distant source is incident on a slit 0.750 mm wide. On a screen 2.00 m away, the distance from the central maximum of the diffraction pattern to the first minimum is measured to be 1.35 mm. Calculate the wavelength of the light.

36.2 • Parallel rays of green mercury light with a wavelength of 546 nm pass through a slit covering a lens with a focal length of 60.0 cm. In the focal plane of the lens the distance from the central maximum to the first minimum is 10.2 mm. What is the width of the slit?

36.3 • Light of wavelength 585 nm falls on a slit 0.0666 mm wide. (a) On a very large and distant screen, how many *totally* dark fringes (indicating complete cancellation) will there be, including both sides of the central bright spot? Solve this problem *without* calculating all the angles! (*Hint*: What is the largest that $\sin\theta$ can be? What does this tell you is the largest that m can be?) (b) At what angle will the dark fringe that is most distant from the central bright fringe occur?

36.4 • Light of wavelength 633 nm from a distant source is incident on a slit 0.750 mm wide, and the resulting diffraction pattern is observed on a screen 3.50 m away. What is the distance between the two dark fringes on either side of the central bright fringe?

36.5 • Diffraction occurs for all types of waves, including sound waves. High-frequency sound from a distant source with wavelength 9.00 cm passes through a slit 12.0 cm wide. A microphone is placed 8.00 m directly in front of the center of the slit, corresponding to point O in Fig. 36.5a. The microphone is then moved in a direction perpendicular to the line from the center of the slit to point O . At what distances from O will the intensity detected by the microphone be zero?

36.6 • **CP** **Tsunami!** On December 26, 2004, a violent earthquake of magnitude 9.1 occurred off the coast of Sumatra. This

quake triggered a huge tsunami (similar to a tidal wave) that killed more than 150,000 people. Scientists observing the wave on the open ocean measured the time between crests to be 1.0 h and the speed of the wave to be 800 km/h. Computer models of the evolution of this enormous wave showed that it bent around the continents and spread to all the oceans of the earth. When the wave reached the gaps between continents, it diffracted between them as through a slit. (a) What was the wavelength of this tsunami? (b) The distance between the southern tip of Africa and northern Antarctica is about 4500 km, while the distance between the southern end of Australia and Antarctica is about 3700 km. As an approximation, we can model this wave's behavior by using Fraunhofer diffraction. Find the smallest angle away from the central maximum for which the waves would cancel after going through each of these continental gaps.

36.7 • **CP** A series of parallel linear water wave fronts are traveling directly toward the shore at 15.0 cm/s on an otherwise placid lake. A long concrete barrier that runs parallel to the shore at a distance of 3.20 m away has a hole in it. You count the wave crests and observe that 75.0 of them pass by each minute, and you also observe that no waves reach the shore at ± 61.3 cm from the point directly opposite the hole, but waves do reach the shore everywhere within this distance. (a) How wide is the hole in the barrier? (b) At what other angles do you find no waves hitting the shore?

36.8 • Monochromatic electromagnetic radiation with wavelength λ from a distant source passes through a slit. The diffraction pattern is observed on a screen 2.50 m from the slit. If the width of the central maximum is 6.00 mm, what is the slit width a if the wavelength is (a) 500 nm (visible light); (b) 50.0 μm (infrared radiation); (c) 0.500 nm (x rays)?

36.9 • **Doorway Diffraction.** Sound of frequency 1250 Hz leaves a room through a 1.00-m-wide doorway (see Exercise 36.5). At which angles relative to the centerline perpendicular to the doorway will someone outside the room hear no sound? Use 344 m/s for the speed of sound in air and assume that the source and listener are both far enough from the doorway for Fraunhofer diffraction to apply. You can ignore effects of reflections.

36.10 • **CP** Light waves, for which the electric field is given by $E_y(x, t) = E_{\text{max}} \sin[(1.20 \times 10^7 \text{ m}^{-1})x - \omega t]$, pass through a slit and produce the first dark bands at $\pm 28.6^\circ$ from the center of the diffraction pattern. (a) What is the frequency of this light? (b) How wide is the slit? (c) At which angles will other dark bands occur?

36.11 • Parallel rays of light with wavelength 620 nm pass through a slit covering a lens with a focal length of 40.0 cm. The diffraction pattern is observed in the focal plane of the lens, and the distance from the center of the central maximum to the first minimum is 36.5 cm. What is the width of the slit? (*Note*: The angle that locates the first minimum is *not* small.)

36.12 • Red light of wavelength 633 nm from a helium–neon laser passes through a slit 0.350 mm wide. The diffraction pattern is observed on a screen 3.00 m away. Define the width of a bright fringe as the distance between the minima on either side. (a) What is the width of the central bright fringe? (b) What is the width of the first bright fringe on either side of the central one?

Section 36.3 Intensity in the Single-Slit Pattern

36.13 • Monochromatic light of wavelength 580 nm passes through a single slit and the diffraction pattern is observed on a screen. Both the source and screen are far enough from the slit for Fraunhofer diffraction to apply. (a) If the first diffraction minima are at $\pm 90.0^\circ$, so the central maximum completely fills the screen,

what is the width of the slit? (b) For the width of the slit as calculated in part (a), what is the ratio of the intensity at $\theta = 45.0^\circ$ to the intensity at $\theta = 0^\circ$?

36.14 •• Monochromatic light of wavelength $\lambda = 620$ nm from a distant source passes through a slit 0.450 mm wide. The diffraction pattern is observed on a screen 3.00 m from the slit. In terms of the intensity I_0 at the peak of the central maximum, what is the intensity of the light at the screen the following distances from the center of the central maximum: (a) 1.00 mm; (b) 3.00 mm; (c) 5.00 mm?

36.15 •• A slit 0.240 mm wide is illuminated by parallel light rays of wavelength 540 nm. The diffraction pattern is observed on a screen that is 3.00 m from the slit. The intensity at the center of the central maximum ($\theta = 0^\circ$) is 6.00×10^{-6} W/m². (a) What is the distance on the screen from the center of the central maximum to the first minimum? (b) What is the intensity at a point on the screen midway between the center of the central maximum and the first minimum?

36.16 • Monochromatic light of wavelength 486 nm from a distant source passes through a slit that is 0.0290 mm wide. In the resulting diffraction pattern, the intensity at the center of the central maximum ($\theta = 0^\circ$) is 4.00×10^{-5} W/m². What is the intensity at a point on the screen that corresponds to $\theta = 1.20^\circ$.

36.17 •• A single-slit diffraction pattern is formed by monochromatic electromagnetic radiation from a distant source passing through a slit 0.105 mm wide. At the point in the pattern 3.25° from the center of the central maximum, the total phase difference between wavelets from the top and bottom of the slit is 56.0 rad. (a) What is the wavelength of the radiation? (b) What is the intensity at this point, if the intensity at the center of the central maximum is I_0 ?

36.18 • Consider a single-slit diffraction experiment in which the amplitude of the wave at point O in Fig. 36.5a is E_0 . For each of the following cases, draw a phasor diagram like that in Fig. 36.8c and determine *graphically* the amplitude of the wave at the point in question. (*Hint:* Use Eq. (36.6) to determine the value of β for each case.) Compute the intensity and compare to Eq. (36.5). (a) $\sin\theta = \lambda/2a$; (b) $\sin\theta = \lambda/a$; (c) $\sin\theta = 3\lambda/2a$.

36.19 •• Public Radio station KXPR-FM in Sacramento broadcasts at 88.9 MHz. The radio waves pass between two tall skyscrapers that are 15.0 m apart along their closest walls. (a) At what horizontal angles, relative to the original direction of the waves, will a distant antenna not receive any signal from this station? (b) If the maximum intensity is 3.50 W/m² at the antenna, what is the intensity at $\pm 5.00^\circ$ from the center of the central maximum at the distant antenna?

Section 36.4 Multiple Slits

36.20 •• **Diffraction and Interference Combined.** Consider the interference pattern produced by two parallel slits of width a and separation d , in which $d = 3a$. The slits are illuminated by normally incident light of wavelength λ . (a) First we ignore diffraction effects due to the slit width. At what angles θ from the central maximum will the next four maxima in the two-slit interference pattern occur? Your answer will be in terms of d and λ . (b) Now we include the effects of diffraction. If the intensity at $\theta = 0$ is I_0 , what is the intensity at each of the angles in part (a)? (c) Which double-slit interference maxima are missing in the pattern? (d) Compare your results to those illustrated in Fig. 36.12c. In what ways is your result different?

36.21 • **Number of Fringes in a Diffraction Maximum.** In Fig. 36.12c the central diffraction maximum contains exactly seven

interference fringes, and in this case $d/a = 4$. (a) What must the ratio d/a be if the central maximum contains exactly five fringes? (b) In the case considered in part (a), how many fringes are contained within the first diffraction maximum on one side of the central maximum?

36.22 • An interference pattern is produced by eight parallel and equally spaced, narrow slits. There is an interference minimum when the phase difference ϕ between light from adjacent slits is $\pi/4$. The phasor diagram is given in Fig. 36.14b. For which pairs of slits is there totally destructive interference?

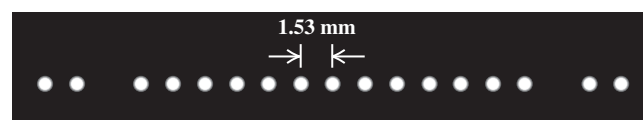
36.23 •• An interference pattern is produced by light of wavelength 580 nm from a distant source incident on two identical parallel slits separated by a distance (between centers) of 0.530 mm. (a) If the slits are very narrow, what would be the angular positions of the first-order and second-order, two-slit, interference maxima? (b) Let the slits have width 0.320 mm. In terms of the intensity I_0 at the center of the central maximum, what is the intensity at each of the angular positions in part (a)?

36.24 • Parallel rays of monochromatic light with wavelength 568 nm illuminate two identical slits and produce an interference pattern on a screen that is 75.0 cm from the slits. The centers of the slits are 0.640 mm apart and the width of each slit is 0.434 mm. If the intensity at the center of the central maximum is 5.00×10^{-4} W/m², what is the intensity at a point on the screen that is 0.900 mm from the center of the central maximum?

36.25 •• An interference pattern is produced by four parallel and equally spaced, narrow slits. By drawing appropriate phasor diagrams, show that there is an interference minimum when the phase difference ϕ from adjacent slits is (a) $\pi/2$; (b) π ; (c) $3\pi/2$. In each case, for which pairs of slits is there totally destructive interference?

36.26 •• A diffraction experiment involving two thin parallel slits yields the pattern of closely spaced bright and dark fringes shown in Fig. E36.26. Only the central portion of the pattern is shown in the figure. The bright spots are equally spaced at 1.53 mm center to center (except for the missing spots) on a screen 2.50 m from the slits. The light source was a He-Ne laser producing a wavelength of 632.8 nm. (a) How far apart are the two slits? (b) How wide is each one?

Figure E36.26



36.27 •• Laser light of wavelength 500.0 nm illuminates two identical slits, producing an interference pattern on a screen 90.0 cm from the slits. The bright bands are 1.00 cm apart, and the third bright bands on either side of the central maximum are missing in the pattern. Find the width and the separation of the two slits.

Section 36.5 The Diffraction Grating

36.28 •• Monochromatic light is at normal incidence on a plane transmission grating. The first-order maximum in the interference pattern is at an angle of 8.94° . What is the angular position of the fourth-order maximum?

36.29 • If a diffraction grating produces its third-order bright band at an angle of 78.4° for light of wavelength 681 nm, find (a) the number of slits per centimeter for the grating and (b) the angular location of the first-order and second-order bright bands. (c) Will there be a fourth-order bright band? Explain.

36.30 • If a diffraction grating produces a third-order bright spot for red light (of wavelength 700 nm) at 65.0° from the central maximum, at what angle will the second-order bright spot be for violet light (of wavelength 400 nm)?

36.31 • Visible light passes through a diffraction grating that has 900 slits/cm, and the interference pattern is observed on a screen that is 2.50 m from the grating. (a) Is the angular position of the first-order spectrum small enough for $\sin\theta \approx \theta$ to be a good approximation? (b) In the first-order spectrum, the maxima for two different wavelengths are separated on the screen by 3.00 mm. What is the difference in these wavelengths?

36.32 • The wavelength range of the visible spectrum is approximately 380–750 nm. White light falls at normal incidence on a diffraction grating that has 350 slits/mm. Find the angular width of the visible spectrum in (a) the first order and (b) the third order. (Note: An advantage of working in higher orders is the greater angular spread and better resolution. A disadvantage is the overlapping of different orders, as shown in Example 36.4.)

36.33 • When laser light of wavelength 632.8 nm passes through a diffraction grating, the first bright spots occur at $\pm 17.8^\circ$ from the central maximum. (a) What is the line density (in lines/cm) of this grating? (b) How many additional bright spots are there beyond the first bright spots, and at what angles do they occur?

36.34 • (a) What is the wavelength of light that is deviated in the first order through an angle of 13.5° by a transmission grating having 5000 slits/cm? (b) What is the second-order deviation of this wavelength? Assume normal incidence.

36.35 • Plane monochromatic waves with wavelength 520 nm are incident normally on a plane transmission grating having 350 slits/mm. Find the angles of deviation in the first, second, and third orders.

36.36 • **Identifying Isotopes by Spectra.** Different isotopes of the same element emit light at slightly different wavelengths. A wavelength in the emission spectrum of a hydrogen atom is 656.45 nm; for deuterium, the corresponding wavelength is 656.27 nm. (a) What minimum number of slits is required to resolve these two wavelengths in second order? (b) If the grating has 500.00 slits/mm, find the angles and angular separation of these two wavelengths in the second order.

36.37 • A typical laboratory diffraction grating has 5.00×10^3 lines/cm, and these lines are contained in a 3.50-cm width of grating. (a) What is the chromatic resolving power of such a grating in the first order? (b) Could this grating resolve the lines of the sodium doublet (see Section 36.5) in the first order? (c) While doing spectral analysis of a star, you are using this grating in the *second* order to resolve spectral lines that are very close to the 587.8002-nm spectral line of iron. (i) For wavelengths longer than the iron line, what is the shortest wavelength you could distinguish from the iron line? (ii) For wavelengths shorter than the iron line, what is the longest wavelength you could distinguish from the iron line? (iii) What is the range of wavelengths you could *not* distinguish from the iron line?

36.38 • The light from an iron arc includes many different wavelengths. Two of these are at $\lambda = 587.9782$ nm and $\lambda = 587.8002$ nm. You wish to resolve these spectral lines in first order using a grating 1.20 cm in length. What minimum number of slits per centimeter must the grating have?

Section 36.6 X-Ray Diffraction

36.39 • X rays of wavelength 0.0850 nm are scattered from the atoms of a crystal. The second-order maximum in the Bragg reflection occurs when the angle θ in Fig. 36.22 is 21.5° . What is the spacing between adjacent atomic planes in the crystal?

36.40 • If the planes of a crystal are 3.50 \AA ($1 \text{ \AA} = 10^{-10} \text{ m} = 1 \text{ \AA}$ ngstrom unit) apart, (a) what wavelength of electromagnetic waves is needed so that the first strong interference maximum in the Bragg reflection occurs when the waves strike the planes at an angle of 15.0° , and in what part of the electromagnetic spectrum do these waves lie? (See Fig. 32.4.) (b) At what other angles will strong interference maxima occur?

36.41 • Monochromatic x rays are incident on a crystal for which the spacing of the atomic planes is 0.440 nm. The first-order maximum in the Bragg reflection occurs when the incident and reflected x rays make an angle of 39.4° with the crystal planes. What is the wavelength of the x rays?

Section 36.7 Circular Apertures and Resolving Power

36.42 • **BIO** If you can read the bottom row of your doctor's eye chart, your eye has a resolving power of 1 arcminute, equal to $\frac{1}{60}$ degree. If this resolving power is diffraction limited, to what effective diameter of your eye's optical system does this correspond? Use Rayleigh's criterion and assume $\lambda = 550$ nm.

36.43 • Two satellites at an altitude of 1200 km are separated by 28 km. If they broadcast 3.6-cm microwaves, what minimum receiving-dish diameter is needed to resolve (by Rayleigh's criterion) the two transmissions?

36.44 •• The VLBA (Very Long Baseline Array) uses a number of individual radio telescopes to make one unit having an equivalent diameter of about 8000 km. When this radio telescope is focusing radio waves of wavelength 2.0 cm, what would have to be the diameter of the mirror of a visible-light telescope focusing light of wavelength 550 nm so that the visible-light telescope has the same resolution as the radio telescope?

36.45 •• Monochromatic light with wavelength 620 nm passes through a circular aperture with diameter $7.4 \mu\text{m}$. The resulting diffraction pattern is observed on a screen that is 4.5 m from the aperture. What is the diameter of the Airy disk on the screen?

36.46 • **Photography.** A wildlife photographer uses a moderate telephoto lens of focal length 135 mm and maximum aperture $f/4.00$ to photograph a bear that is 11.5 m away. Assume the wavelength is 550 nm. (a) What is the width of the smallest feature on the bear that this lens can resolve if it is opened to its maximum aperture? (b) If, to gain depth of field, the photographer stops the lens down to $f/22.0$, what would be the width of the smallest resolvable feature on the bear?

36.47 • **Observing Jupiter.** You are asked to design a space telescope for earth orbit. When Jupiter is 5.93×10^8 km away (its closest approach to the earth), the telescope is to resolve, by Rayleigh's criterion, features on Jupiter that are 250 km apart. What minimum-diameter mirror is required? Assume a wavelength of 500 nm.

36.48 • A converging lens 7.20 cm in diameter has a focal length of 300 mm. If the resolution is diffraction limited, how far away can an object be if points on it 4.00 mm apart are to be resolved (according to Rayleigh's criterion)? Use $\lambda = 550$ nm.

36.49 •• **Hubble Versus Arecibo.** The Hubble Space Telescope has an aperture of 2.4 m and focuses visible light (380–750 nm). The Arecibo radio telescope in Puerto Rico is 305 m (1000 ft) in diameter (it is built in a mountain valley) and focuses radio waves of wavelength 75 cm. (a) Under optimal viewing conditions, what is the smallest crater that each of these telescopes could resolve on our moon? (b) If the Hubble Space Telescope were to be converted to surveillance use, what is the highest orbit above the surface of the earth it could have and still be able to resolve the license plate (not the letters, just the plate) of a car on the ground? Assume optimal viewing conditions, so that the resolution is diffraction limited.