Guide to Maths

GCSE (9-1) Design and Technology

Pearson Edexcel Level 1/Level 2 GCSE (9-1) in Design and Technology (1DT0)

GCSE Guide to Maths for Design and Technology

Contents

Introduction

This guide to maths for design and technology outlines the content that students will have covered in their maths lessons throughout KS3 and KS4. You can use this guide to help you understand how different areas are approached in maths, and therefore support your teaching of mathematical content in design and technology lessons.

The content is split into distinct mathematical concepts. Each chapter takes you through the terminology used in that area, as well as examples taken from Pearson maths textbooks to show you the methods students should be familiar with when solving mathematical problems.

Sections which are highlighted green have a particular connection or reference to design and technology and may include examples.

Sections which are highlighted blue refer to student inconsistencies and common errors.

1a Recognise and use expressions in decimal and standard form

Rounding

Demand

All KS3 students will have learned to round to the nearest whole number and 1, 2 or 3 decimal places. They should be able to cope with rounding to more decimal places as an extension of rounding to 3 decimal places.

Significant figures

At GCSE only Higher tier students learn about upper and lower bounds and percentage error.

For percentage error they answer questions such as: Given a percentage error of ± 10%, what is the largest/smallest possible value?

Answering questions such as 'What is the percentage error?' for a given value is not covered in the GCSE maths specification.

Approach

Look at the digit *after* the last one you want to keep. Round up if this digit is 5 or more; round down if it is 4 or less.

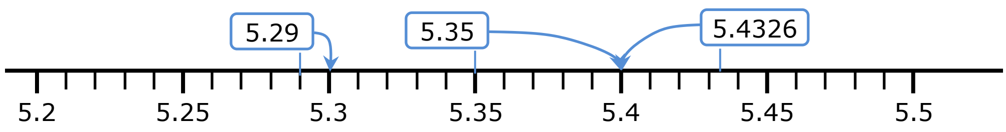
Rounding to 1 decimal place (1 d.p.)

5.4|326
less than 5
round down
5.4 (1 d.p.)

5.2|91
5 or more
round up
5.3 (1 d.p.)

5.3|5
5 or more
round up
5.4 (1 d.p.)

On a number line, round to the nearest value with 1 decimal place:



Rounding to 2 decimal places (2 d.p.)

35.42|9
5 or more
round up
35.43 (2 d.p.)

126.37|2
less than 5
round down
126.37 (2 d.p.)

Rounding to 3 decimal places (3 d.p.)

0.053|21
less than 5
round down
0.053 (3 d.p.)

11.291|5
5 or more
round up
11.292 (3 d.p.)

Rounding to significant figures

Small numbers

1st significant figure
= 4 ten thousandths

0.000 483

Large numbers

1st significant figure
= 5 hundred million

518 376 000

Round to 2 significant figures (2 s.f.)

0.000 48|3
less than 5
round down
0.000 48

51|8 376 000
5 or more
round up
520 000 000
Add zeroes so the 5 is still in the 'hundred million' position

Upper and lower bounds calculations

Find the upper and lower bounds of the given values, before doing the calculation.

Terminology

- The number of **decimal places** is the number of digits after the decimal point. So, 10.5219 has 4 decimal places and 10 has no decimal places.
- In any number the first **significant figure** is the one with the highest place value. It is the first non-zero digit counting from the left.

Inconsistencies: Zero is counted as a significant figure if it is between two other non-zero significant figures. Other zeros are place holders – if you took them out, the place value of the other digits would change.

place holders

0 . 0 0 5 0 7 3

1st 2nd 3rd 4th

significant figures

place holders

3 9 0 4 1 2 0 0 0

1st 2nd 3rd 4th 5th 6th

significant figures

- To round a number to a given number of significant figures or decimal places, look at the digit after the last one you need. Round up if the digit is 5 or more, and round down if the digit is 4 or less.

Inconsistency: Rounding numbers reduces accuracy. Your results cannot be more accurate than your starting values. In design and technology, your answer should not have more significant values than the numbers in the calculation. In maths, students are told not to give more decimal places in the answer than in the calculation. They are also told to consider if their answers are practical. For example, can you measure 4.321 cm to that level of accuracy? In design and technology, where workshop measuring equipment might be more accurate, the answer to this may be 'yes'.

Writing numbers in standard form

Demand

All KS3 students learn to write numbers in index form and use the index laws for multiplication and division.

All GCSE students learn the positive and negative powers of 10. Foundation students often find the negative and zero powers difficult to understand or remember, as they are the only negative and zero powers they use. Higher students use negative and zero indices with a range of numbers so are likely to have a better understanding.

All GCSE students learn to read and write very small and very large numbers in standard form.

Approach

Calculating powers of 10

Follow a pattern:

$$10^1 = 10$$
$$10^2 = 10 \times 10 = \square$$
$$10^3 = 10 \times 10 \times 10 = \square$$
$$10^4 = 10 \times 10 \times 10 \times 10 = 10\,000$$
$$10^5 = \ldots$$
$$10^6 = \ldots$$

$$10^3 = 1000$$
$$10^2 = 100$$
$$10^1 = 10$$
$$10^{\square} = 1$$
$$10^{-1} = \frac{1}{10} = 0.1$$
$$10^{\square} = \frac{1}{100} = \frac{1}{10^2} = 0.01$$
$$10^{\square} = \square = \square = \square$$

Writing large numbers in standard form

The following examples of writing standard form are taken from the *Edexcel GCSE (9-1) Mathematics Foundation* student book.

Example 4

Write 4000 in standard form.

$4000 = 4 \times 1000$ — Write the number as a number between 1 and 10 multiplied by a power of 10.

$= 4 \times 10^3$ — Write the power of 10 using indices.

Example 5

Write 45 600 in standard form.

$45\,600 = 4.56 \times 10^4$ — 4.56 lies between 1 and 10.
Multiply by the power of 10 needed to give the original number.
4 5 6 0 0

Writing small numbers in standard form

Example 6

Write 0.000 05 in standard form.

$0.000\,05 = 5 \times 0.000\,01$ — Write the number as a number between 1 and 9 multiplied by a power of 10.

$= 5 \times 10^{-5}$

Key point 7

To write a small number in standard form:

- Place the decimal point after the first non-zero digit.
- How many places has this moved the digit? This is the negative power of 10.

Example 7

Write 0.003 52 in standard form.

$0.003\,52 = 3.52 \times 10^{-3}$ — 3.52 lies between 1 and 10.
Multiply by the power of 10 needed to give the original number.
0 . 0 0 3 5 2

Calculating with numbers in standard form

Multiplication and division

Example 3

Work out $(5 \times 10^3) \times (7 \times 10^6)$

$5 \times 7 \times 10^3 \times 10^6$ — Rewrite the multiplication grouping the numbers and the powers.

35×10^9 — Simplify using multiplication and the index law $x^m \times x^n = x^{m+n}$.
This is not in standard form because 35 is not between 1 and 10.

$35 = 3.5 \times 10^1$ — Write 35 in standard form.

$35 \times 10^9 = 3.5 \times 10^1 \times 10^9 = 3.5 \times 10^{10}$ — Work out the final answer.

Work out $\dfrac{2.4 \times 10^5}{3 \times 10^2}$

Divide 2.4 by 3.
Use the index law $x^m \div x^n = x^{m-n}$ to divide 10^5 by 10^2.

$= 0.8 \times 10^3$

$= 8 \times 10^2$

Addition and subtraction

Write numbers in decimal form before adding and subtracting.

Write the answer in standard form.

Work out $3.6 \times 10^2 + 4.1 \times 10^{-2}$

$= 360 + 0.041$

$= 360.041$

$= 3.600\,41 \times 10^2$

Work out $2.5 \times 10^6 - 4 \times 10^4$

$$\begin{array}{r} 2\,500\,000 \\ -\underline{\quad 40\,000} \\ 2\,460\,000 \end{array}$$

2.46×10^6

Terminology

- Any number can be raised to a power or index. The power or index tells you how many times the number is multiplied by itself. $3^4 = 3 \times 3 \times 3 \times 3$
- Read 3^4 as '3 to the power 4'.
- Some calculators have a power or index key. In maths, students are not instructed about the key presses needed to work out indices, as calculators vary. Instead they are told to find out how to input numbers raised to a power for their own calculator.
- Any number raised to the power zero = 1.
- The index laws are:
    - to multiply powers of the same number, add the indices –
    - to divide powers of the same number, subtract the indices.
- Some of the powers of 10 are:

10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^0	10^1	10^2	10^3	10^4
0.0001 or $\frac{1}{10\,000}$	0.001 or $\frac{1}{1000}$	0.01 or $\frac{1}{100}$	0.1 or $\frac{1}{10}$	1	10	100	1000	10 000

- Design and technology sometimes deal with very small or very large numbers in standard from. For example, electronic current is sometimes measured in milliamps. One milliamp is 1/1000th of an amp and is often written as 1 mA. Similarly, resistance is often measured in kilo ohms or mega ohms. One kilo ohm is a 1000 ohms and is written as 1 KΩ or $1 \times 10^3\,\Omega$ and 1 mega ohm is 1 000 000 ohms and is written as 1 MΩ or $1 \times 10^6\,\Omega$.
- Standard form is a way of writing very large or very small numbers as a number between 1 and 10 multiplied by a power of 10:
  $A \times 10^n$ where A is between 1 and 10 and n is the power of 10.

Inconsistency: When writing numbers in standard form, do not talk about 'moving the decimal point'. The position of the decimal point remains fixed. Multiplying by a power of 10 moves digits places to the left and dividing by a power of 10 moves digits places to the right.

- On some calculators you can enter numbers in standard form, or answers may be given in standard form. Students need to know how to enter and read numbers in standard form for their type of calculator.

1b Use ratios, fractions and percentages

Fractions and decimals

Demand

All KS3 students learn how to add, subtract, multiply and divide decimals and find a fraction of a quantity.

They also learn how to convert fractions to decimals and vice versa, and to use and interpret recurring decimal notation.

Lower ability KS3 students will not learn to find the reciprocal of a fraction. All students will learn this in GCSE maths.

Approach

Convert a fraction to a decimal

Divide the top number by the bottom number.

For example: $\frac{3}{8} = 0.375$

$\frac{12}{50} = 0.24$

Convert a decimal to a fraction

Worked example

Write 0.32 as a fraction in its simplest form.

$$0.32 = \frac{32}{100} \qquad \frac{32}{100} \overset{\div 4}{\underset{\div 4}{=}} \frac{8}{25}$$

...	H	T	U	•	$\frac{1}{10}$	$\frac{1}{100}$	...
			0	•	3	2	

0.32 is the same as $\frac{32}{100}$

Source: *KS3 Maths Progress*

Calculate a fraction of a quantity

Example 3

Work out $\frac{3}{5}$ of 40.

In mathematics, 'of' means multiply.

$\frac{1}{5}$ of $40 = \frac{1}{5} \times 40 = 40 \div 5 = 8$

$\times 3$ $\times 3$

$\frac{3}{5}$ of $40 = 3 \times 8 = 24$

Multiply by 3 to find $\frac{3}{5}$

Source: *Edexcel GCSE (9-1) Mathematics*

Terminology

- Write fractions on two lines, i.e. $\frac{1}{1000}$ not 1/1000 on one line.
- Avoid 'cancelling' – instead, 'write fractions in their simplest form'.
- In a fraction, the horizontal line means 'divide'. So $\frac{3}{5}$ means 3 ÷ 5. Understanding this helps students remember how to convert fractions to decimals.
- A 'dot' over a decimal value shows the number recurs, e.g. $0.\dot{6}$ means 0.666 66…
- A dot over two decimal values shows the numbers between the dots recur, e.g. $0.\dot{1}\dot{5}$ means 0.151 515… and $0.\dot{2}47\,5\dot{1}$ means 0.247 514 751…

Percentages

Demand

In KS3, students in lower maths sets will only have written one number as a percentage of another, where the larger number is a multiple or factor of 100, so they may find this difficult. They would not be expected to do this calculation in maths without a calculator.

Percentage change

Foundation level students learn this in Year 11.

Approach

Convert a percentage to a fraction

Worked example

Write 20% as a fraction.

÷10 ÷2

$$20\% = \frac{20}{100} = \frac{2}{10} = \frac{1}{5}$$

÷10 ÷2

First write 20% as a fraction of 100.

Then simplify the fraction by dividing the numerator and denominator by the same number. Keep doing this until the fraction is in its simplest form.

Source: *KS3 Maths Progress*

Convert a percentage to a decimal

Worked example

Write 35% as a decimal.

$$35\% = \frac{35}{100} = 0.35$$

Write 35% as a fraction out of 100. Then divide 35 by 100 to write it as a decimal.

Source: *KS3 Maths Progress*

Convert a fraction to a percentage

Convert the fraction to a decimal, then convert the decimal to a percentage.

For example: $\frac{34}{80} = 0.425 = 42.5\%$

Students can input $\frac{34}{80}$ as a fraction into a scientific calculator and press = (or the S-D button on some calculators) to get the equivalent decimal.

Write one number as a percentage of another

Write as a fraction, then convert to a percentage.

For example: in a class of 28 students, 13 are boys. What percentage are boys?

$\frac{13}{28}$ = 0.464 2… = 46.4% (1 d.p.)

Calculating a percentage of an amount

50% is the same as $\frac{1}{2}$, so to find 50% divide by 2.

10% is the same as $\frac{1}{10}$, so to find 10% divide by 10.

To calculate 30% mentally, find 10% and multiply by 3.

To calculate 5% mentally, find 10% and halve it.

Calculating percentages using a calculator

Input the percentage as a fraction

For example: to calculate 30% of 20 m, input $\frac{30}{100}$ × 20 and press = to get 6 m.

Input the percentage using a decimal multiplier

65% = 0.65

To calculate 65% of 80, input 0.65 × 80 and press = to get 52.

Percentage increase/decrease

Work out the increase and add it on/subtract it

Examples

To increase 45 by 20%:

20% of 45 = 9

45 + 9 = 54

To decrease 220 by 5%:

5% of 220 = 11

220 – 11 = 209

Using a multiplier

Examples

To increase 45 by 20%:

After the increase you will have 100% + 20% = 120% = 1.2

1.2 × 45 = 54

To decrease 220 by 5%:

After the decrease you will have 100% – 5% = 95%

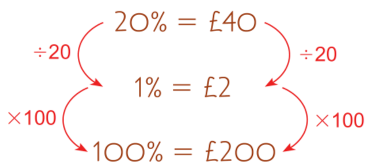
0.95 × 220 = 209

Finding the original amount

Using arrow diagrams

Worked example

20% of an amount is £40.
Work out the original amount.



Source: *KS3 Maths Progress*

Using function machines

Example

When studying ecological footprints, for example, consider the area of a woodland that has been reduced by 15% because of deforestation.

The final area is 320 km².

Calculate the original area.

To calculate the area after 15% decrease, you would multiply by 0.85:

$$\text{original} \rightarrow \boxed{\times\ 0.85} \rightarrow 320\ \text{km}^2$$

$$376.5\ \text{km}^2 \leftarrow \boxed{\div\ 0.85} \leftarrow 320\ \text{km}^2$$

Percentage change

Percentage change $= \frac{\text{actual change}}{\text{original amount}} \times 100$

Example

When studying sources, generation and storage of energy, renewable energy provided 8% of the country's needs in 2010.

In 2014, renewable energy was providing 12% of the country's needs.

The actual increase in energy generation is 12% – 8% = 4%.

The fractional increase is $\frac{\text{actual increase}}{\text{original supply}} = \frac{4}{8}$

$\frac{4}{8}$ as decimal is 0.5

Percentage increase is $0.5 \times 100 = 50\%$

Terminology

- Percent means 'out of 100'. A percentage is a fraction with a denominator of 100.
- You can calculate percentages of amounts, e.g. 20% of £500.
- You can write one number as a percentage of another, e.g. write $\frac{7}{50}$ as a percentage.

Converting between fractions, decimals and percentages

Demand

All KS3 students will learn how to convert between fractions, decimals and percentages.

Approach

Percentage to fraction to decimal

$40\% = \frac{40}{100} = 0.4$

Decimal to percentage

Multiply by 100:

0.3 = 30%

0.02 = 2%

Percentage to decimal

Divide by 100:

62% = 0.62

7.5% = 0.075

Simple fractions to percentages

Multiply or divide both numbers to get a fraction with a denominator of 100:

$$\times 5$$

$$\frac{3}{20} = \frac{15}{100} = 15\%$$

$$\times 5$$

$$\div 2$$

$$\frac{62}{200} = \frac{31}{100} = 31\%$$

$$\div 2$$

Convert a fraction to a percentage

Convert the fraction to a decimal, then convert the decimal to a percentage.

For example:

$\frac{34}{80}$ = 0.425 = 42.5%

Students can input $\frac{34}{80}$ as a fraction into a scientific calculator and press = (or the S-D button on some calculators) to get the equivalent decimal.

Terminology

- When converting decimals to percentages or vice versa, do not say 'move the decimal point two places'. Instead, say 'multiply by 100' or 'divide by 100' as appropriate.

  For example:

  0.52 = 52%

  3% = 0.03

Ratios

Demand

Students learn to simplify ratios and write them in the form 1 : *n* or *n* : 1 in KS3.

Students learn to relate ratios to fractions in KS3 but many continue to make errors with this type of calculation.

Approach

Simplifying ratios

A ratio in its simplest form only contains whole number values.

Divide all the numbers in the ratio by the highest common factor:

$$\begin{array}{ccc} & 2 : 6 & \\ \div 2 & & \div 2 \\ & 1 : 3 & \end{array}$$

$$\begin{array}{ccc} & 6 : 15 & \\ \div 3 & & \div 3 \\ & 2 : 5 & \end{array}$$

The following ratio is not in its simplest form, because the two numbers both still have a common factor, 2:

$$\begin{array}{ccc} & 8 : 20 & \\ \div 2 & & \div 2 \\ & 4 : 10 & \end{array}$$

Writing in the form 1 : *n* (sometimes called a unit ratio)

Divide both numbers by the first number in the ratio:

$$\begin{array}{ccc} & 5 : 7 & \\ \div 5 & & \div 5 \\ & 1 : 1.4 & \end{array}$$

Writing in the form *n* : 1

Divide both numbers by the second number in the ratio:

$$\begin{array}{ccc} & 20 : 12 & \\ \div 12 & & \div 12 \\ & 1.67 : 1 & \end{array}$$

Comparing ratios

Write both ratios in the form 1 : *n* or *n* : 1.

Example

When studying industry and unemployment, in county A there are 20 people who are unemployed for every 120 people who are economically active.

In county B there are 15 people who are unemployed for every 85 people who are economically active.

Which county has more unemployment?

County A	County B
20 : 120	15 : 85
÷20 ÷20	÷15 ÷15
1 : 6	1 : 5.7

County B has a lower ratio, so as a proportion there are more unemployed people in county B.

Ratio and proportion

When studying sources of energy, a manufacturing company uses two different forms of energy A and B, in the ratio 2 : 3. What fraction of the energy is:

a) A – gas

b) B – electricity?

Draw a bar model to illustrate the energy forms:

A	**A**	**B**	**B**	**B**

$\frac{2}{5}$ is A gas and $\frac{3}{5}$ is B electricity.

Terminology

'Write the ratio of *A* to *B*' means write *A* : *B*. If you want students to write the ratio as $\frac{A}{B}$ you need to say 'write the ratio as $\frac{A}{B}$'.

To simplify a ratio, divide all the numbers in the ratio by their highest common factor.

To compare ratios, write them in the form 1 : *n* or *n* : 1. This is sometimes called a unit ratio.

A **ratio** compares two quantities. Ratios translate into a statements such as 'for every 3 black there are 2 red'.

A **proportion** compares a part with a whole. A proportion can be given as a fraction or a percentage.

Common error

Students look at 2 : 3 and think the fraction is $\frac{2}{3}$.

1c Calculate surface area and volume

Converting units of surface area and volume

Demand

All students meet the prefixes for metric units in GCSE maths. The only ones they are likely to use frequently in maths are kg, km, cm, ml, mm.

All students learn to convert between metric units of area and volume in GCSE maths.

All students learn to convert speeds in m/s to km/h and vice versa in GCSE maths.

Students are not expected to know metric/imperial conversions or imperial-to-imperial conversions, such as lbs to stones or feet to yards.

Approach

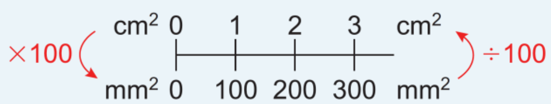
Area conversions

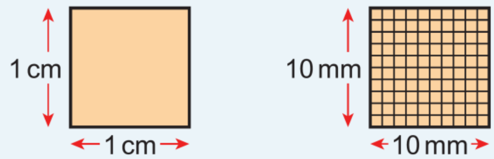
Students may not remember area conversion factors, but will learn in GCSE maths how to work them out, as follows.

You can use a double number line to convert between area measures.

Key point 4

These two squares have the same area.
To convert from cm^2 to mm^2, multiply by 100.
To convert from mm^2 to cm^2, divide by 100.





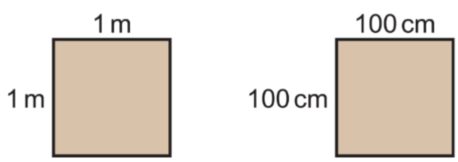
Area = 1 cm × 1 cm
= $1\,cm^2$

Area = 10 mm × 10 mm
= $100\,mm^2$

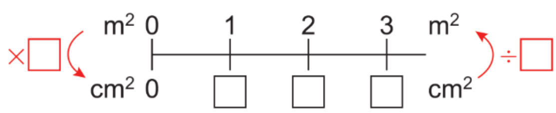
Source: *Edexcel GCSE (9-1) Mathematics*

This question shows students how to convert m^2 to cm^2 and vice versa.

Use these diagrams to help you work out the number of cm^2 in $1\,m^2$.



Copy and complete the double number line.

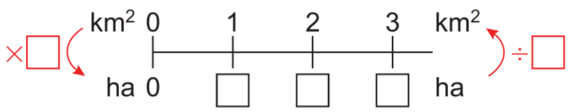


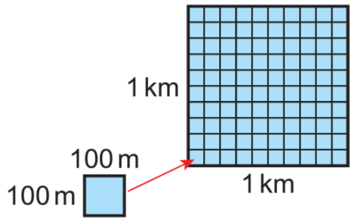
The diagram shows 1 km^2 divided into 100 m squares.

a What is the area of each 100 m square?

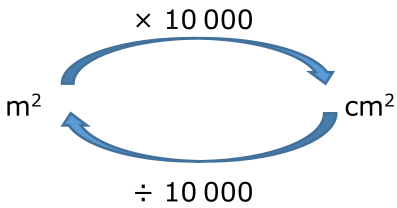
b How many hectares are there in 1 km^2?

c Copy and complete the double number line.





You can use arrow diagrams to help convert between area measures.



Terminology

Inconsistency: Maths, science and design and technology uses cm, mm, kg, etc. These are abbreviations, not symbols.

- Use the abbreviations min (not m) for minutes.
- For compound units use m/s, km/h, g/cm^3, rather than ms^{-1} etc.
- In maths books, where 'per' is used, there will also be a literacy hint, e.g. '8 g/cm^2 means 8 grams in every cm^2'.
- Area is measured in squared units: mm^2, cm^2, m^2.
- Volume is measured in cubed units: mm^3, cm^3, m^3.
- Capacity is measured in litres and ml. Students do not use dl in maths. Some students may use cl.

1 cm^3 = 1 ml. This might be relevant in design and technology when designing packages to hold fluids such as drinks containers or perfume bottles.

- Some students meet the prefixes for metric units in KS3 maths, e.g. M stands for Mega and means 10^6.
- It is often easier to convert measures to the units required before doing a calculation, than converting the answer into the units required.
- If students are required to convert between metric and imperial units, they should be given the conversion factor. In GCSE maths they are not expected to know metric/imperial equivalents.

2a Presentation of data, diagrams, bar charts and histograms

Demand

All students learn the difference between discrete and continuous data in KS3. They also come across categorical data.

Terminology

Data is either qualitative (descriptive) or quantitative (numerical), as well as either discrete (only certain values according to context) or continuous (any value).

- Qualitative data is information that describes something in more subjective terms, e.g. what people think about a garment.
- Quantitative data is data that has a numerical value and can be measured precisely, e.g. the size of a component part.
- Discrete data can only take certain values, e.g. whole numbers, or shoe sizes.
- Continuous data is measured and can take any value, e.g. length, time.
- Categorical data is where there is no numerical value but data can still be sorted into groups, e.g. preferences from an interview.

Bar charts

Demand

All KS3 students learn to draw and interpret bar charts for discrete and continuous data. Students may need help with interpreting scales on axes given in large numbers like thousands or millions (i.e. 2.2 thousand = 2200).

Approach

- Bar charts can show qualitative or quantitative, discrete or continuous data.
- One axis is usually labelled 'Number of …' or shows frequency.
- Frequency is usually shown on the vertical axis (but can be on the horizontal axis with the bars in the chart shown horizontally).
- Bars should be equal in width.
- For discrete and qualitative data there are gaps between the bars.
- A bar-line graph, for discrete and qualitative data, uses lines instead of bars. It can be used to save time drawing the bars.
- For continuous data there are no gaps between the bars.
- In questions on interpreting proportions from bar charts, ask for 'the fraction of students' or 'the percentage of students', not 'the proportion of students'.

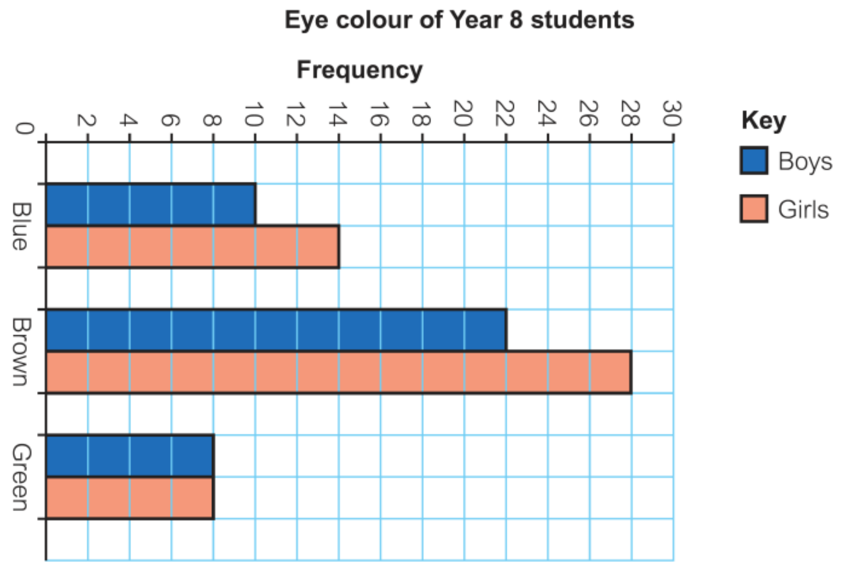


Figure 1 Horizontal bar chart (discrete, qualitative data)

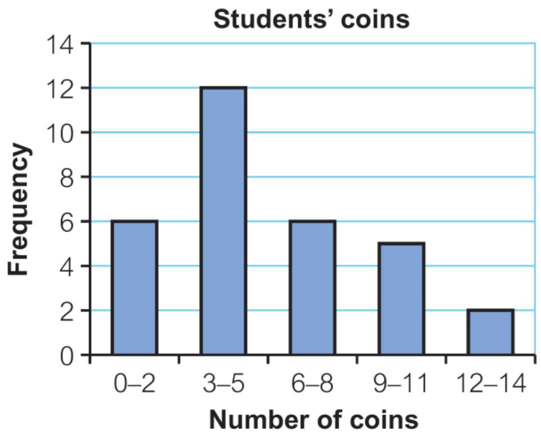


Figure 2 Bar chart (discrete, quantitative data) with gaps between the bars

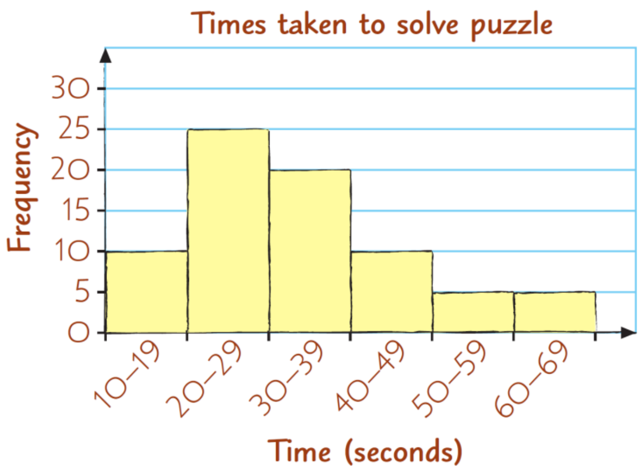


Figure 3 Bar chart (continuous, quantitative data) with no gaps between the bars

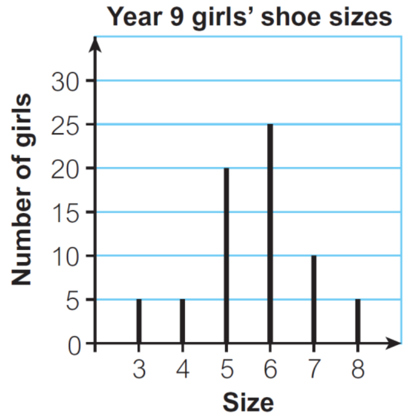


Figure 4 Bar-line chart (discrete, quantitative data)

Frequency tables

Demand

All KS3 students learn to draw and interpret frequency tables for discrete and continuous data.

Approach

- A frequency table shows the number of items, or the frequency of each data value or group.
- The data can be grouped. For discrete data, use groups such as 0–5, 6–10, etc. For continuous data use groups such as $0 \leq t < 10$, $10 \leq t < 20$. The groups must not overlap.
- In maths, students learn that it is best to group numerical data into a maximum of 6 groups. If students need to group data differently, tell them how many groups of equal width they need.

Shoe size	Frequency
3	3
4	5
5	7
6	10
7	10
8	6
9	1

Figure 5 Frequency table (ungrouped discrete, quantitative data)

Design and technology test mark	0–10	11–20	21–30	31–40	41–50
Frequency	4	13	17	19	7

Figure 6 Frequency table (grouped discrete, quantitative data)

Distance (d metres)	Frequency
$10 \le d < 20$	2
$20 \le d < 30$	6
$30 \le d < 40$	15
$40 \le d < 50$	20
$50 \le d < 60$	4

Figure 7 Frequency table (grouped continuous data)

Frequency diagrams

Demand

All KS3 students learn to draw and interpret bar charts for discrete and continuous data.

Lower ability maths students (KS3 and KS4) may need help with interpreting scales on axes given in large numbers like thousands or millions (i.e. 2.2 thousand = 2200).

Approach

Inconsistency: In KS3 maths, the name 'frequency diagram' is not used.

- Frequency diagram is another name for a bar chart where the vertical axis is labelled 'Frequency'.
- Frequency diagrams can be used to show discrete or continuous data.

For example, these two bar charts could also be called frequency diagrams:

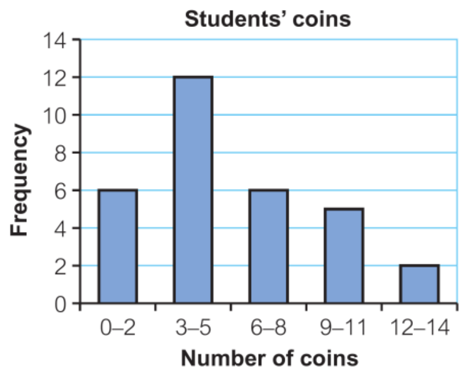


Figure 8 Frequency diagram (discrete, quantitative data)

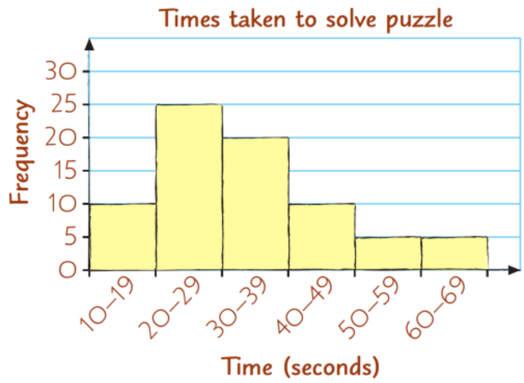


Figure 9 Frequency diagram (continuous, quantitative data)

Comparative bar charts

Demand

All KS3 students learn to draw and interpret comparative bar charts.

Approach

- Comparative bar charts compare two or more sets of data.
- Use different coloured bars for each set of data.
- A key is needed to show what each colour bar represents.

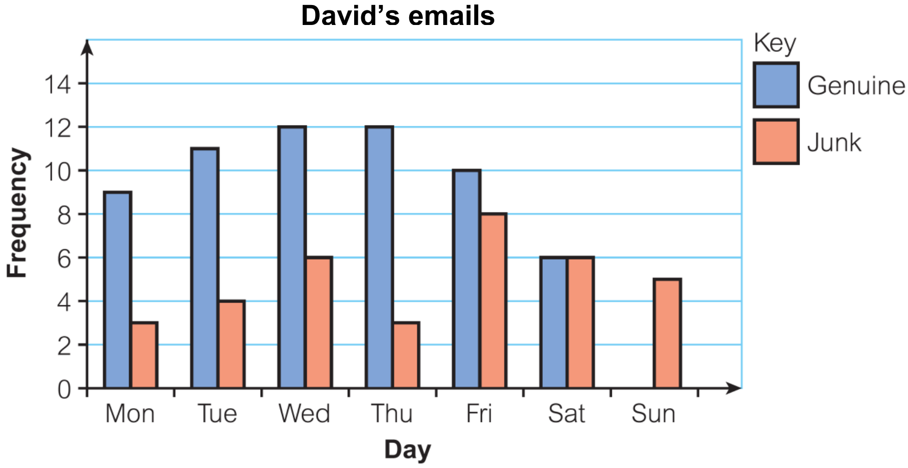


Figure 10 Comparative bar chart (discrete, qualitative data)

Compound bar charts

Demand

All KS3 students learn to draw and interpret compound bar charts.

Approach

- Compound bar charts combine different sets of data in one bar.
- A key is needed to show what each colour section represents.
- In questions on interpreting proportions in compound bar charts, ask for 'the fraction' or 'the percentage of chemistry students getting A*', not 'the proportion of students'.

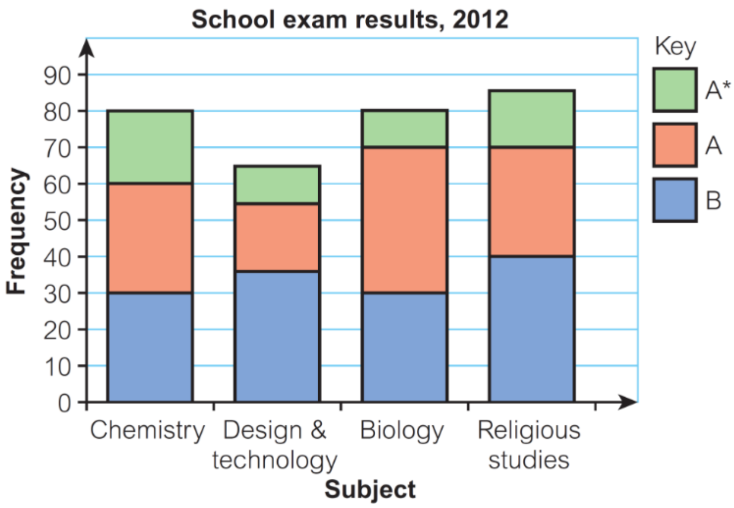


Figure 11 Compound bar chart (discrete, quantitative data)

Histograms

Demand

The bar charts students draw in KS3 for grouped continuous data could be called histograms, although students do not meet histograms until KS4

Only Higher tier GCSE students study histograms with unequal width bars/groups, where frequency density is plotted on the vertical axis.

Approach

- Histograms can be drawn for grouped continuous data where groups/bars are of equal width.
- o gaps between the bars.
- If groups/bars are of unequal width, the vertical axis is labelled frequency density, which is calculated as:

$$\text{Frequency density} = \frac{\text{number in group}}{\text{group width}}$$

- The area of the bar is proportional to the number of items it represents (frequency).

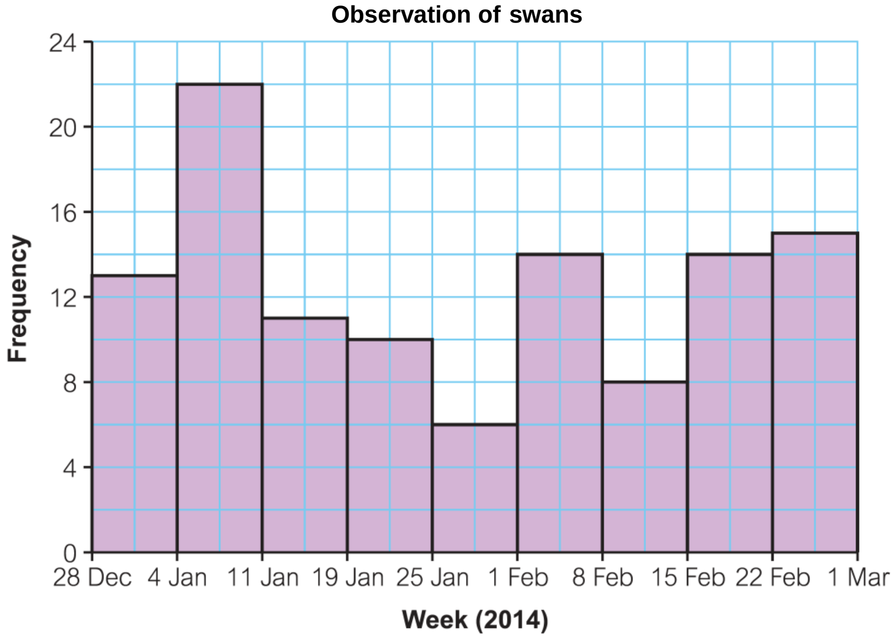


Figure 12 Histogram with equal width bars/groups

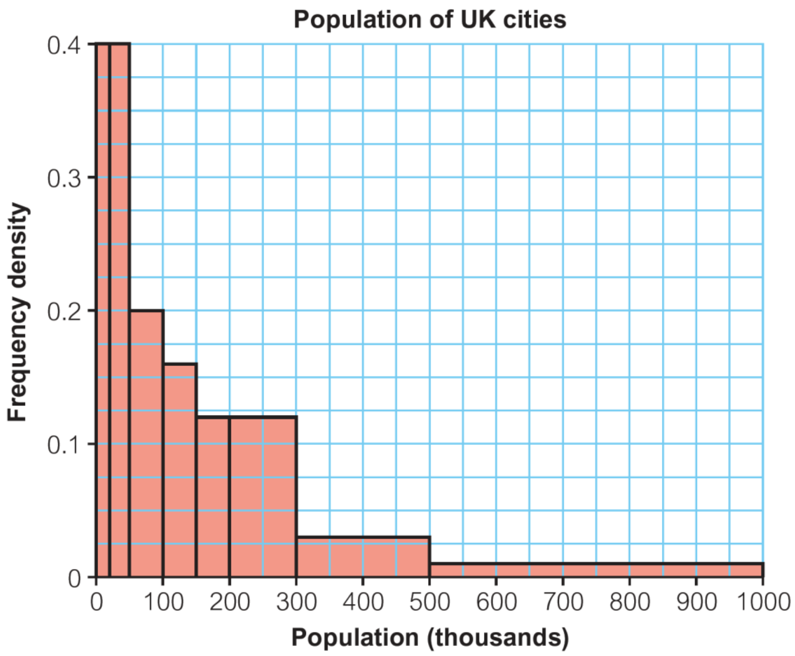


Figure 13 Histogram with unequal width bars/groups, used when the data is grouped into classes of unequal width

Drawing a histogram

Key point 12

In a **histogram** the area of the bar represents the frequency. The height of each bar is the frequency density.

$$\text{Frequency density} = \frac{\text{frequency}}{\text{class width}}$$

Example 4

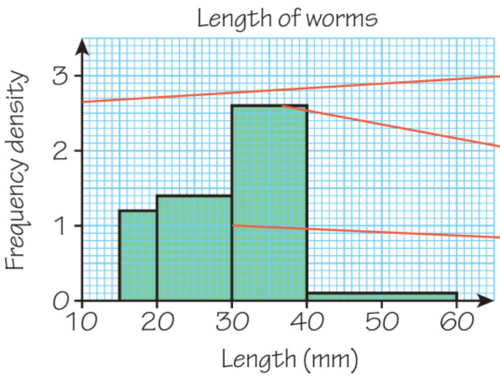
The lengths of 48 worms are recorded in this table.

Length, x (mm)	$15 < x \leqslant 20$	$20 < x \leqslant 30$	$30 < x \leqslant 40$	$40 < x \leqslant 60$
Frequency	6	14	26	2

Draw a histogram to display this data.

$6 \div 5 = 1.2,\ 14 \div 10 = 1.4,\ 26 \div 10 = 2.6,\ 2 \div 20 = 0.1$

Work out the frequency density for each class



Label the y-axis 'Frequency density'.

The height of each bar is the frequency density for each class.

Draw the bars with no gaps between them.

Source: *Edexcel GCSE (9-1) Mathematics Higher* student book

Pie charts

Demand

All KS3 students should learn how to construct a simple pie chart.

Lower ability maths students (KS3 and KS4) will probably struggle with working out the angles as they will not have learned how to calculate percentages or fractions that are not straight forward numbers.

All KS4 students should know how to draw and interpret pie charts.

Terminology

- A pie chart is a circle divided into **sectors**. (A 'slice' of a pie is a sector, not a **segment**.)
- The angle of each sector is proportional to the number of items in that category.
- A pie chart shows proportions of a set of data, e.g. fraction or percentage of waste recycled.
- A key is needed if labels do not fit on the chart.

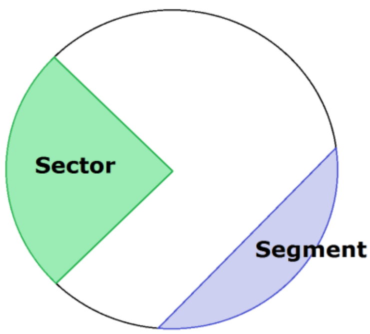


Figure 14 Pie chart terminology

Approach

Drawing a pie chart

Example 5

The table shows the match results of a football team.
Draw a pie chart to represent the data.

Result	Won	Drawn	Lost
Frequency	28	12	20

Total number of games = 28 + 12 + 20 = 60

÷60 60 games : 360° ÷60

1 game : 6°

The total number of games is the total frequency.

1 game = 360 ÷ 60 = 6°

Work out the angle for one game.

Won: $28 \times 6° = 168°$

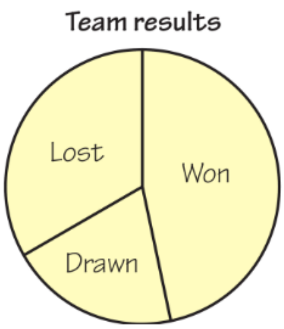
Drawn: $12 \times 6° = 72°$

Lost: $20 \times 6° = 120°$

Work out the angle for each result.

Check: 168 + 72 + 120 = 360

Check that your angles total 360°.



Draw the pie chart. Give it a title and label each section, or make a key.

Source: *Edexcel GCSE (9-1) Mathematics Foundation* student book

In questions asking students to interpret a pie chart, ask for the 'fraction' or 'percentage' who learn design and technology, not the 'proportion' who learn design and technology.

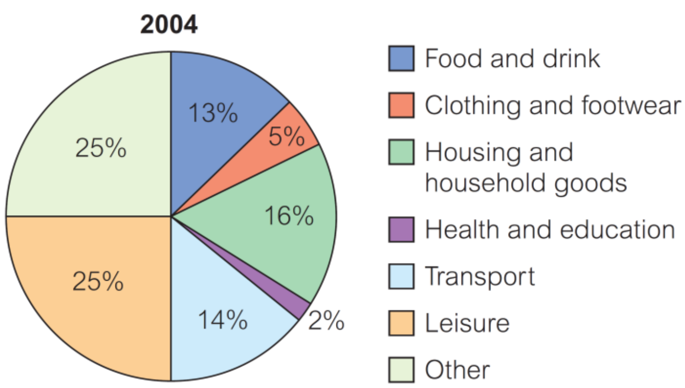


Figure 15 Pie chart showing average household spending (percentages) in 2004

Selecting charts for displaying data

Demand

Choosing a suitable graph or chart to draw to display data is quite a high level skill in maths.

Lower ability maths students (KS3 and KS4) may need some guidance on the type of chart to draw for given sets of data. They may also need help with choosing suitable scales for axes.

Design and technology students should think carefully about the number of categories that are used so that their graphs make sense. They should also consider why alternative methods might need to be used sometimes.

3a Plot, draw and interpret appropriate graphs

Scatter graphs, correlation and lines of best fit

Terminology

Inconsistency: In maths the terms scatter diagrams or scatter plots are not used to describe scatter graphs. In design and technology we may, unknowingly, mix up the terminology.

- A scatter graph plots two sets of data on the same graph to see if there is a relationship or correlation between them.
- Scatter graphs can show positive, negative or no correlation.

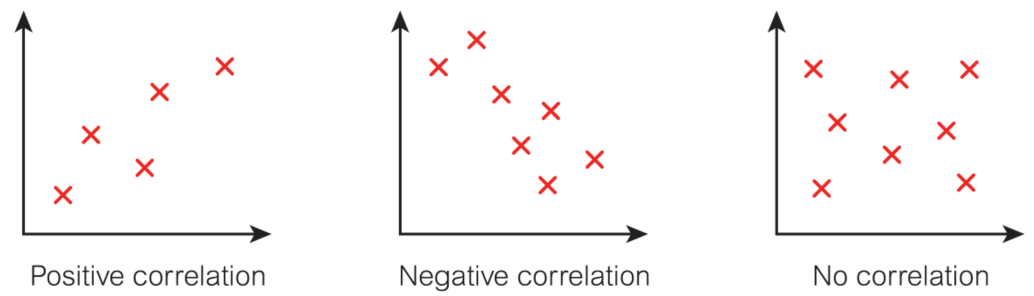


Figure 16 Correlation in scatter graphs

- Correlation is when two sets of data are linked. For example, when one value increases as the other decreases, or when one value decreases as the other increases.
- In maths, points on scatter graphs are plotted with crosses.
- The line of best fit follows the shape of the data and has roughly the same number of crosses above and below the line. There may also be crosses on the line.

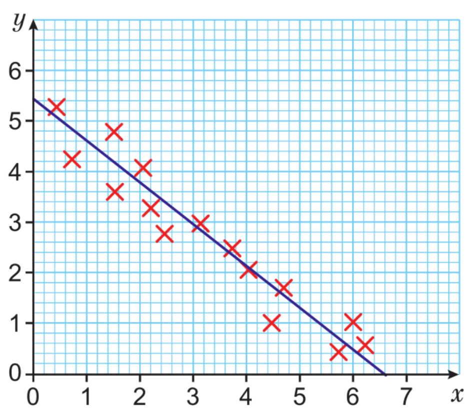


Figure 17 Strong correlation

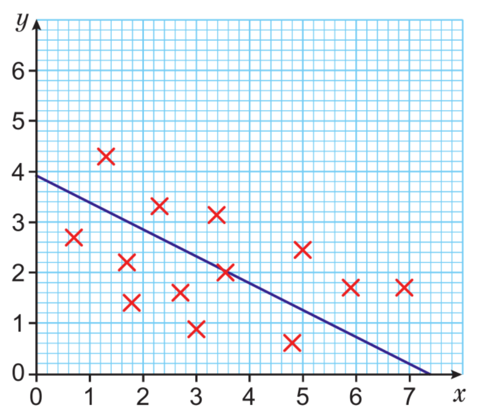


Figure 18 Weak correlation

Inconsistency: When interpreting a scatter graph in maths, an acceptable answer is 'shows positive correlation' unless the question explicitly asks for this to be explained in context.

Use this language in design and technology, but also always comment on the meaning or relevance of the relationship you have described.

- The line of best fit shows a relationship between two sets of data.
- When the points on a scatter graph are on, or close to, a straight line:
  - there is strong correlation between the variables
  - there could be a linear relationship between the variables, e.g. $y = mx$ or $y = mx + c$. The equation of the line of best fit describes this relationship.
- When the points on a scatter graph are not close to a straight line there may be another relationship between the variables.

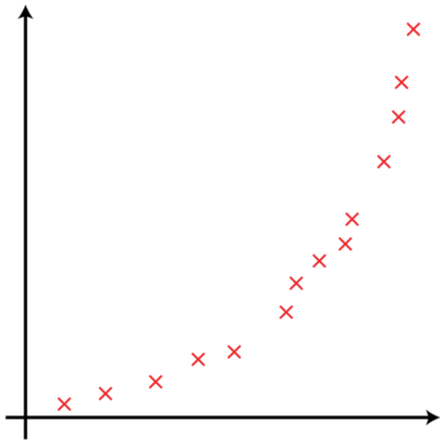


Figure 19 Scatter graph showing a non-linear relationship

- Correlation does not imply causation. Sometimes there may be another factor that affects both variables, or there may be no connection between them at all. For example, there is a negative correlation between number of pirates and mean global temperature, but it is unlikely that one causes the other!

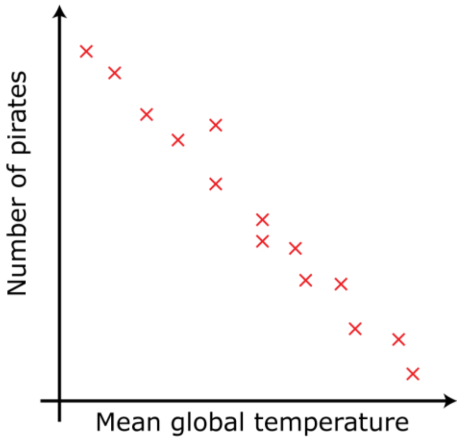


Figure 20 Scatter graph showing a non-causal **negative correlation**

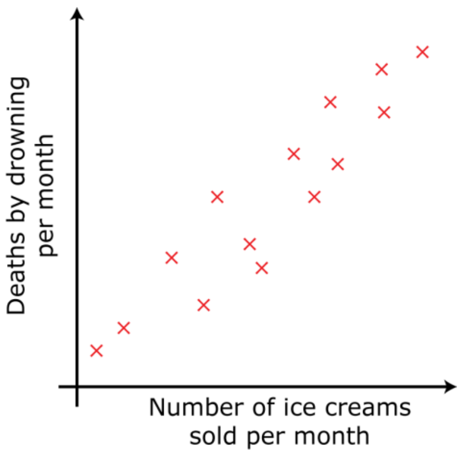


Figure 21 Scatter graph showing non-causal, positive correlation

- There is positive correlation between number of ice-creams sold and death by drowning, but it is unlikely that one causes the other (i.e. directly causal). A more likely explanation is a third factor – temperature. On hot days more people buy ice creams and more people swim, leading to increased numbers of drownings.

Many areas of design and technology do not show clear causal linkages. Factors beyond the relationship shown in the graph must be considered.

Approach

Drawing scatter graphs

Lower ability maths students would not be expected to know which variables to put on which axis for a scatter graph. They may need help with deciding which is the independent variable and a reminder that this goes on the horizontal axis.

Drawing a line of best fit

Place your ruler on the graph, on its edge. Move the ruler until it is following the shape of the data, with roughly the same number of points above and below it. Ignore any points on the line.

Common error

Students often try to make their lines of best fit go through (0, 0). A line of best fit does not necessarily pass through the origin. It should stop at the first or last plot point or cross.

Interpreting scatter graphs

Demand

All KS3 students should have learned to interpret scatter graphs. They will have learned about correlation and causation in KS3 maths.

Lower ability students at GCSE would not be expected to know which variables to put on which axis for a scatter graph.

Students are not expected to find equation of a curve of best fit in GCSE maths.

At GCSE Higher tier students would be expected to state that there is a possible non-linear relationship if the points on a scatter graph closely follow a smooth curve.

Drawing line graphs

Demand

Choosing a suitable graph or chart to draw to display data is quite a high-level skill in maths.

In design and technology the independent variable is often time.

Lower ability students in KS3 and Foundation GCSE maths students may need some guidance on the type of chart to draw for given sets of data.

Lower ability students at KS3 would not be expected to know which variables to put on which axis for a scatter graph. They may need help with deciding which is the independent variable and a reminder that this goes on the horizontal axis.

Lower ability maths students may also need help with choosing suitable scales for axes.

Approach

- In maths, students draw graphs on squared or graph paper.
- They plot points with crosses (×).
- They join points with straight lines or a smooth curve – the question needs to tell them which to use.
- All graphs should have labels on the axes and a title.
- When more than one data set is shown, the lines could be different to show this clearly, e.g. one solid and one dashed line. The graph will need a key to explain solid/dashes. Alternatively, colour-code the lines according to category.

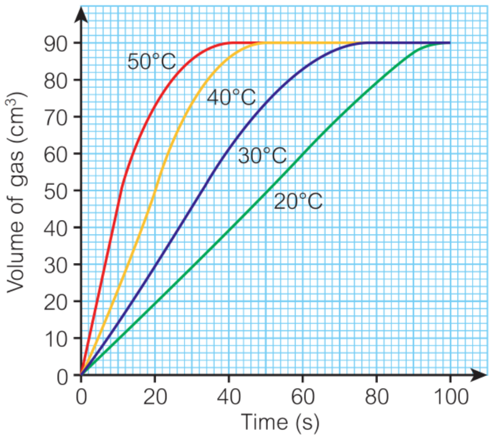


Figure 22 Line graph showing more than one data set

Terminology

- In maths, a line graph that shows how a variable changes over time (i.e. with time on the horizontal axis) is often called a time-series graph.
- Line graphs can show trends in data. The trend is the general direction of change seen in the graph, ignoring individual ups and downs.

In design and technology, graphs showing trends are common. For example, the environmental impact of sourcing materials, manufacturing and transportation can be described using climate change graphs, trend lines in surface temperatures or population predictions.

The following website has climate related graphs (from the IPCC) which show clear trends: www.carbonbrief.org/ipcc-six-graphs-that-explain-how-the-climate-is-changing.

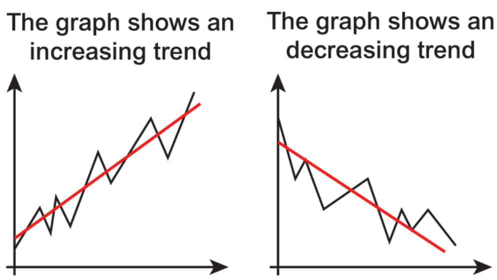


Figure 23 Examples of graphs showing increasing and decreasing trends

Interpreting line graphs

Demand

All KS3 students should learn to interpret line graphs.

Lower ability maths students (KS3 and KS4) may need help with interpreting scales on axes given in large numbers like thousands or millions (i.e. 2.2 thousand = 2200).

At KS4 all students will have limited experience of interpreting real-life graphs that dip below zero.

Only higher ability maths students are likely to have seen graphs with two different vertical scales to read from.

Approach

Students interpret real-life graphs in maths, in a variety of contexts.

They may be less familiar with:

- graphs showing negative values, such as the one below

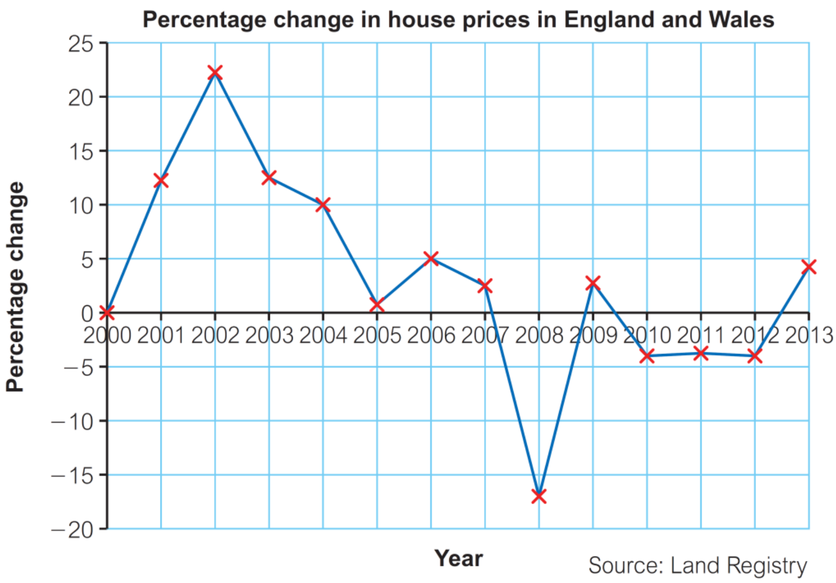


Figure 24 Graph showing negative values

- two types of graph on one set of axes
- two different vertical axes for the same graph.

3b Translate information between graphical and numeric form

Demand

In Year 9, the majority of maths students should know that a straight line graph shows two variables that are in direct proportion. Only top sets maths students will have met graphs showing inverse proportion.

In KS4 all students should learn that the origin is the point (0, 0).

In KS4 all students will meet graphs showing inverse proportion.

Terminology

- A straight line graph through the origin (0, 0) shows that the two variables are in **direct proportion**.
  When one variable doubles, so does the other. When one halves, so does the other.
- The relationship is of the form $y = mx$, where m is the gradient of the graph.

Common error

Use the term 'direct proportion' never just 'proportion.

- A straight line graph not through the origin shows a **linear relationship**.
- The relationship is of the form $y = mx + c$, where m is the gradient of the graph and c is the y-intercept (where the graph crosses the y-axis).

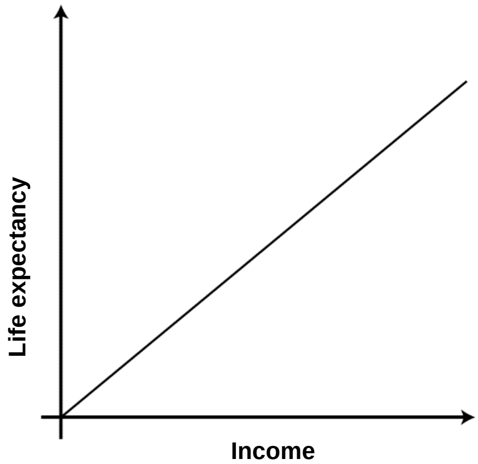
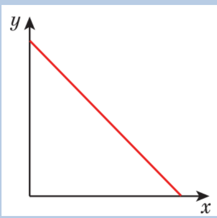


Figure 25 Graph showing two variables in direct proportion to each other

Common error

A straight line graph with negative gradient shows a linear relationship – not inverse proportion. This is a fairly common misconception.



4a Use angular measures in degrees

Demand

All KS3 students learn to measure angles in degrees and to calculate missing angles in simple diagrams.

Terminology

- Angles around a point add up to 360°. Angles on a straight line add up to 180°.
- A right angle is 90°.
- Perpendicular lines meet at 90°.
- Students can use a circular protractor to measure angles greater than 180°.

- Angles less than 90° are acute; angles between 90° and 180° are obtuse; angles greater than 180° are reflex.

4b Visualise and represent 2-D and 3-D forms, including two-dimensional representations of 3-D objects

Demand

All KS3 students learn about simple nets.

Terminology

- 2-D representations of 3-D shapes include 3-D sketches, accurate 3-D drawings on isometric paper, nets, plans and elevations.
- A net is the 2-D shape that can be folded up to make a 3-D shape.

The plan view is the view from above an object. The side elevation is the view from one side and the front elevation is the view from the front. Orthographic projection is a drawing method that combines all three views.

4c Calculate areas of triangles and rectangles, surface areas and volumes of cubes

Demand

All KS3 students learn to calculate the area of a rectangle and triangle.

All KS3 students calculate surface area and volume of cubes and cuboids.

Approach

Calculate the area of a rectangle

Area of a rectangle = length × width

$A = l \times w$ or $A = lw$

This formula works for all triangles, not just right-angled ones.

Area of triangle = $\frac{1}{2}$ × base × height

$A = \frac{1}{2}bh$

h is the perpendicular height.

In a right-angled triangle, the two sides that meet at the right angle are the base and the height.

Worked example

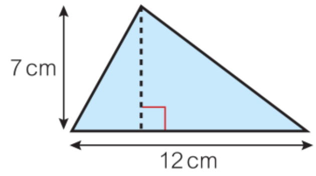
Work out the area of this triangle.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 12 \times 7$

$= 42\ \text{cm}^2$

Write the formula, then substitute the numbers into the formula.



Key point

Area of a triangle = $\frac{1}{2}$ × **base** length × **perpendicular height** which can be written as $A = \frac{1}{2}bh$.
The height measurement must be perpendicular (at 90°) to the base.

Source: *KS3 Maths Progress*

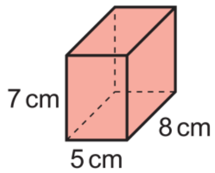
Calculate surface area by drawing the net

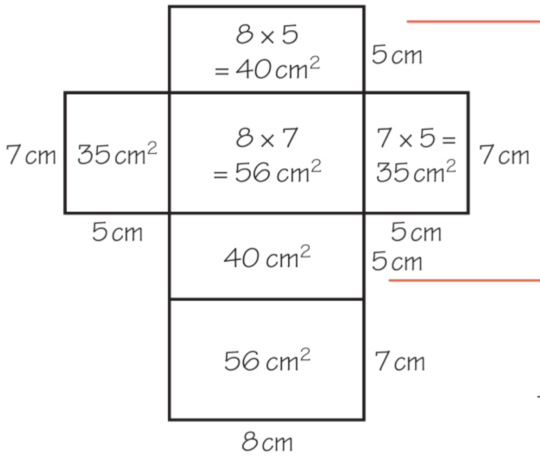
Some 3D shapes like cubes and prisms can be opened out and folded flat. The unfolded flat shape is known as a net.

When calculating the surface area of a net it is a good idea to sketch out the whole net and then every individual surface area can be calculated before adding them all together.

Example 4

Work out the surface area of this cuboid.





Sketch the net.

Label the lengths.

Work out the area of each face.

Total surface area $= 40 + 56 + 40 + 56 + 35 + 35$
$= 262\ \text{cm}^2$

Source: *Edexcel GCSE (9-1) Mathematics*

Terminology

- Use the mathematical terms rectangle and rectangular (instead of oblong) and rhombus (instead of diamond).
- Use the mathematical term cuboid (instead of box-like).
- Specify the shape of an object when you ask students to calculate the area. If it is a rectangle, say so clearly.

Common error

Not stating the shape of an object reinforces a common misconception that the area of any shape is length × width. For example, 'Estimate the number of trees in an area 60 m long and 10 m wide' is not accurate enough. Tell them it is a rectangular area.

- The perimeter of a 2-D shape is the distance all around the outside.
- The area of a 2-D shape is the amount of space inside the shape

Area is measured in squared units, mm^2, cm^2, m^2. Shifting between scales and areas is important in design and technology.

- You can estimate the area of an irregular shape by drawing around it on centimetre-squared paper and counting the squares.
- If a shape is close to a rectangle, you can estimate the area by approximating it to a rectangle.
- The surface area of a 3-D shape is the total area of all the surfaces added together.
- In maths 'area' is used for 2-D shapes (e.g. a rectangular section of piece of fabric) and 'surface area' is used for 3-D shapes (because it is the area of all the surfaces added together).
- To calculate the surface area of a cuboid, find the areas of all the faces and add them together.

Inconsistency: In maths, students are not given a formula for the surface area of a cuboid.

- The volume of a 3-D shape is the amount of space it takes up. It is measured in cubed units, mm^3, cm^3, m^3.
- Capacity is the amount of liquid a 3-D solid can hold. It is measured in ml or litres.