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Guiding Principles for the Design and Development of *Everyday Mathematics*

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The principles that informed the original design and the continuing iterative development of *Everyday Mathematics* are founded in the learning sciences, authoritative recommendations, and the authors' experience and judgment. They underlie an instructional approach that maximizes student learning while keeping teachers' work manageable.

2.1 Foundational Principles

The foundational principles that guide *Everyday Mathematics*' development address what children know when they come to school, how they learn best, what they should learn, and the role of problem solving and assessment in the curriculum.

2.1.1 What Children Bring to School: Prior Knowledge and Disposition

A body of research, including research by the original developers of *Everyday Mathematics*, Max and Jean Bell, has established new understandings about the informal mathematical knowledge of young children. These understandings continue to inform the development of the curriculum.

- Children construct mathematical understandings and problem-solving strategies by building on their own knowledge and experiences. Most children begin school knowing a great deal about numbers, measurement, and geometry. Children have abundant common sense and knowledge of their everyday worlds and come to school with inquisitive dispositions and

positive attitudes. Curricula can aim higher than in the past by building on these early dispositions, attitudes, and knowledge.

- Children’s social environments, including peers, parents, teachers, and other adults, are also critically important to their development. Teachers’ work involves connecting children’s experiences with the discipline of mathematics. The curriculum should foster strong home-school connections. Children learn best when home and school work in partnership.
- Children should expect mathematics to make sense. They should learn that things are true in mathematics not simply because the teacher says so but because mathematics is a coherent network of ideas that fit together. Mathematics should be woven into daily classroom routines so that it becomes a habitual way of making sense of the world.
- Hard work is more important than talent for success in mathematics.

2.1.2 How and What Children Should Learn

Everyday Mathematics incorporates findings from more than 50 years of research in mathematics education. This edition integrates established findings with more recent research about how best to teach specific content.

How Children Learn Best

- The elementary school curriculum should help children progress from intuitions and concrete operations to abstractions and symbolic manipulations, while at the same time building new intuitions that will mature in middle school and beyond.
- The development of skills and concepts works best if spread over relatively long time periods, sometimes over months or years, with early introduction, multiple exposures with increasing sophistication, and spaced practice. Concepts, skills, and applications should be interwoven over time.
- All children can and should learn mathematics. All children can master the Common State Standards for Mathematics (CCSS-M) and become proficient with the practice standards, if they receive high-quality instruction and work hard.
- Children should be doing the thinking in mathematics class. Whole-class discussions, small-group explorations, individual and group practice, problem-solving activities, and guided instruction all have a place in a balanced curriculum. Varied modes of instruction with rich, high cognitive demand content facilitate many opportunities for differentiation.
- Children should have a voice in mathematics class; they should explain, compare, and discuss problems and solutions. A classroom with lots of “math talk” helps make children’s thinking visible, which informs teacher decision making during a lesson. Reasoning, conjecture, and proof are closely connected to communication and discourse. The beginnings of mathematical proof are in children’s explanations and justifications of their ideas.

What Children Should Learn

- The Common Core provides a framework that guides curriculum development, but coherent instructional materials are needed to translate standards into implementable classroom reality. In such instructional

materials, the CCSS-M Standards for Mathematical Practice should be embedded in every lesson.

- Fluency with basic facts is essential for building number sense, estimation skills, and flexibility in problem solving. In early work, the curriculum should emphasize strategies. Later, it should include practical routines to help build fact fluency. Games can be especially useful for building fluency.
- Instruction should connect children’s common sense with formal mathematics. By connecting children’s everyday experience with mathematical concepts and procedures, teachers can help children learn to use the mathematics they are learning. Realistic applications and mathematical modeling should be at the core of school mathematics.
- Conceptual understanding supports learning of both content and procedures. Understanding includes knowing how to carry out a procedure, why that procedure works, how that procedure can be used to solve problems, how mathematical ideas can be represented in various ways, and how concepts, procedures, and representations are connected.

Learning Through and About Problem Solving

- Children can and should devise their own methods for solving problems. If children are to learn how to solve problems, it is not enough to drill them on procedures.
- Cognitive demand should be maintained at a high level. Children should learn that not every problem can be solved in three minutes or less. Some problems should be novel, not routine, and should take longer, sometimes much longer.
- Children should learn and come to expect that there are multiple strategies for solving problems. They should make sense of others’ strategies and solutions, which deepens their understanding of the content and practices addressed in the problem. Discussions should compare strategies in terms of ease of use, understandability, and efficiency.

For more information on problem solving and the Standards for Mathematical Practices, see Section 1.2 *Standards for Mathematical Practice*.

2.1.3 Assessment in a Comprehensive, Standards-Based Curriculum

- Assessment should be ongoing, should reflect the types of activities in which students are engaged, and should use data from a variety of sources. Assessment tasks should themselves be valuable learning experiences. Most importantly, assessment should provide actionable information that teachers can use to make decisions about instruction.
- The curriculum should prepare students to perform well on high-quality assessments, including those from PARCC and SBAC. The best preparation for such tests is a proven, research-based curriculum that is aligned with the content of those tests and is coherent within and across grades to allow for continuous, cumulative learning.

2.2 Design of *Everyday Mathematics*

The work of curriculum developers is to translate general principles into practical tools that teachers can use. The *Everyday Mathematics* authors used the principles outlined above to develop a comprehensive and cohesive educational experience for students and their teachers.

2.2.1 The Spiral: How *Everyday Mathematics* Distributes Learning

In a spiral curriculum, learning is spread over time rather than concentrated in shorter periods. Content is revisited repeatedly over months and across grades. The “spacing effect”—the learning boost from distributing rather than massing learning and practice—has been verified by many researchers for decades: “Space learning over time” is the first research-based recommendation in a practice guide from the U. S. Department of Education’s Institute of Educational Sciences (Pashler et al., 2007). In a recent literature review, Son and Simon write, “On the whole, both in the laboratory and the classroom, both in adults and in children, and in the cognitive and motor learning domains, spacing leads to better performance than massing” (2012).

The spiraling of instruction and practice has been a defining characteristic of *Everyday Mathematics* since its inception and has proven effective: EM students outscore comparable non-EM students on assessments of long-term learning, such as end-of-year standardized tests. Spiraling leads to better long-term mastery of facts, skills, and concepts.

The Spiral in *Everyday Mathematics*

Using the design principles that have made previous editions successful, this edition of *Everyday Mathematics* (EM) has been rebuilt from the ground up to support the Common Core State Standards for Mathematics (CCSS-M). In particular, the content defined by the standards was carefully sequenced and incorporated into the spiral to distribute both instruction and practice of concepts and skills to maximize long-term learning.

Each grade’s Common Core content standards were unpacked into 45 to 80 *Everyday Mathematics* Goals for Mathematical Content (GMC). The standards and the corresponding GMCs are listed in the back of the *Teacher’s Lesson Guide* for each grade. See excerpt below.

For information on the CCSS content standards and *Everyday Mathematics*’ GMCs, see Section 1.1.5 *Unpacking the Content Standards*.

Common Core State Standards



Standards for Mathematical Content

Domain Operations and Algebraic Thinking 1.OA

Everyday Mathematics Goals for Mathematical Content

Cluster Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

GMC Solving number stories by adding and subtracting.

GMC Model parts-and-total, change, and comparison situations.

the first standard under this Domain

program goals for finer-grained tracking of student progress

Excerpt from the table listing the Common Core State Standards and corresponding GMCs, Grade 1 *Teacher’s Lesson Guide*

The Common Core standards and corresponding GMCs were carefully integrated into the unit structure of the curriculum in three phases: Introductory Instruction, Developing Instruction, and Concluding Instruction. For each GMC within a content standard, Introductory Instruction begins through exploration or focus activities in one or more lessons in a unit. Initial practice also begins in this unit. Developing Instruction continues in succeeding units that include additional activities that teach the concept or skill and provide continuing practice and applications. This phase may include “pauses” during which little or no instruction or practice related to the concept or skill takes place. In the final phase, Concluding Instruction, work narrowly focused on the concept or skill is completed, as is most practice. Instruction during this phase involves practice for long-term retention, applications in more complex problem situations, occasional review, and generalization and transfer.

Spiral Transparency

A major goal of *Everyday Mathematics 4* is to make the spiral more transparent to teachers and district leaders. Teachers will be able to see where content is introduced, developed, and concluded within a grade; where it is assessed; and when students are expected to master curricular goals and standards. This information will help teachers know “when to worry” and when (and how) to intervene with students who struggle to meet expectations for a given curricular goal or standard. Teachers will also know when “watchful waiting” is appropriate and intervention may be premature.

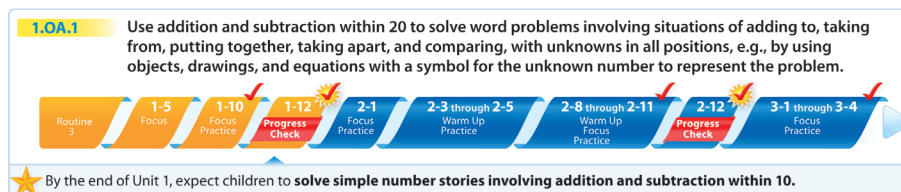
Three features of *Everyday Mathematics 4* are particularly useful for making the spiral more transparent: the Spiral Snapshot, the Spiral Trace, and the Spiral Tracker.

The **Spiral Snapshot** is a lesson-level feature that sketches previous and future experiences with one of the content goals that is a focus of the lesson. The Spiral Snapshot is intended to help teachers connect each lesson’s content to previous and upcoming lessons.



Spiral Snapshot, Grade 2 Lesson 2-11

Every Unit Opener includes a **Spiral Trace** that outlines instructional trajectories for key standards in the unit, highlighting focus activities, practice, and assessment opportunities for each standard and describing the degree of mastery—as measured against the entire standard—that is expected at that point in the year.



Information on CCSS 1.OA.1 from Grade 1 Unit 1 Spiral Trace in the Unit Organizer

For more information on assessment, see Section 9.3 *Assessment Opportunities*.

For more information on assessment and tracking tools, see Section 9.4 *Assessment Tools*.

The most extensive guide to the spiral is the digital **Spiral Tracker**, accessible from every lesson Overview, which provides the user with exhaustive tracking for every standard, including every activity and assessment linked to that standard. For each standard, the tracker moves through the curriculum to show which *Everyday Mathematics* goals are addressed in Warm Up, Focus, and Practice activities, as well as assessments. Once in the Tracker, users can launch specific activities to preview the lesson, scroll through the entire curriculum to scan the complete trajectory for that standard, select other content standards to track, and review the GMC statements that apply.

[Go Online](#) to the Spiral Tracker.

The standard being tracked

Additional standards to track

Click on the blue bead to bring up that lesson.

Scroll through the curriculum to scan all the instances of warm-up, focus instruction, and follow-up practice for each standard.

For more information on *Everyday Mathematics* lessons, see Section 3 *Everyday Mathematics in Kindergarten*.

For more information on *Everyday Mathematics* lessons, see Section 4 *Everyday Mathematics in Grades 1–6*.

2.2.2 *Everyday Mathematics* Instructional Design

One goal of the instructional design is to make the curriculum manageable for teachers. The curriculum should also be educative for teachers as well as for students. It should help teachers better understand the mathematics they are teaching and how students best learn that mathematics. Features in the program utilize the design principles described above and integrate rigorous mathematics with effective strategies for high-quality mathematics instruction.

Lesson Structure and Features

- The **Lesson Opener** includes information on content, standards, and assessment, including the Spiral Snapshot discussed in Section 2.2.1. In the digital Teacher Center, information beyond the Materials and Before You Begin notes is accessible through the buttons on the right. (See *top of the next page*.)

The screenshot shows the 'EverydayMathematics' website interface. At the top, there's a navigation bar with 'Unit 6' and 'Lesson 6-10' selected. The main heading is 'Exploration Exploring Arrays, Length, and Shapes'. The page is divided into several sections: 'Materials' (listing MATH MASTERS, Handbook, and ACTIVITY CARDS), 'Additional Materials' (listing various supplies like state, scissors, eraser, etc.), 'Overview' (describing the activity), 'Before You Begin' (providing instructions for the optional Readiness activity), and 'Vocabulary' (listing 'geoboard' and 'rectangular array'). On the right side, there is a vertical navigation menu with icons for Overview, Activities, Differentiation, Presentation, Standards, Notes, and Resources. At the bottom of the page, there are links for 'Print Worksheets', 'Add a Note', and 'Upload Files'.

Access background and overview information from the Lesson Overview screen, including information about CCSS Standards and Differentiation Options

- **Mental Math and Fluency** exercises develop fluency with basic facts and other skills that need to be automatic. Starting lessons with these exercises establishes a brisk pace that helps engage students. These daily exercises distribute learning over time.
- **Daily Routines** in the primary grades integrate mathematics and real-world applications into classroom life. The routines offer students opportunities to use mathematics as they contribute to the classroom organization. They also provide daily reinforcement and application of important mathematical concepts and skills.
- The **Math Message** asks students to solve a problem they have not already been shown how to solve. Math Messages thus provide daily opportunities to engage in problem solving and the mathematical practices. Teachers should allow students to solve the Math Message problems without first being shown how.
- The **Math Message Follow-Up** is an opportunity for students to share and discuss how they used what they know to solve the Math Message problem. Connecting students' own thinking with more formal mathematics is a key objective of the Math Message Follow-Up. The Math Message and Math Message Follow-up serve as an entry point to the focus activities of the lesson.

For more information, see Section 7 *Daily Routines*.

Lesson 6-6 : Focus : Math Message

Children solve an addition problem mentally and show why their strategies worked.

Materials slate

Uploaded Resources Add a Note Upload Files

Activity Resources Enter a Note here...

Math Message

Use a mental strategy to solve $26 + 74$.
Use words, numbers, or pictures to show why your strategy works.

Activity Kit Writing Tools

Math Message
in Teacher Center and
in Student Learning
Center for Grade 2,
Lesson 6-6

- The other activities in the **Focus** part of the lesson serve a variety of purposes. Some are exploratory activities that introduce new content; others are routine exercises that consolidate recent learning; some are challenging problem-solving tasks that stretch students' thinking; and others are games or other activities that build procedural fluency, conceptual understanding, or students' abilities to use what they know to solve problems.
- The **Assessment Check-In** is a daily opportunity to gather data that will help teachers evaluate student performance on specific standards and goals that are the focus of the lesson. Assessment Check-Ins provide actionable data that teachers can use to make instructional decisions.
- A **Summarize** question or statement at the end of the focus activities is an opportunity for teachers and students to step back and draw key themes and understandings from the lesson.
- The **Practice** portion of the lesson provides ongoing distributed practice of skills, concepts, and applications from past lessons and units through additional activities and games. The Practice portion includes a daily set of **Math Boxes**, a format for distributed practice. Since some Math Box problems may not have been the focus of recent instruction, students may have to work hard to recall or reconstruct how to solve them. Math Boxes should be completed with minimal help from the teacher.
- **Home Links** allow students to practice school mathematics at home and help family members understand the school mathematics program.

Lesson Features that Facilitate Differentiation. An important objective of this edition of *Everyday Mathematics* is to make differentiation achievable for all teachers who use the program. To that end, the *Everyday Mathematics* authors examined every new and revised lesson through the lens of the diverse learning needs of students in a typical classroom. **Differentiation Options** in every lesson help teachers meet all students' needs, providing readiness opportunities for students who need additional background in order to access the core lesson, extra practice activities, and enrichment activities. Specific support for English language learners is included at point of use in the lessons and online.

For more information, see Section 10 *Differentiating Instruction with Everyday Mathematics*.

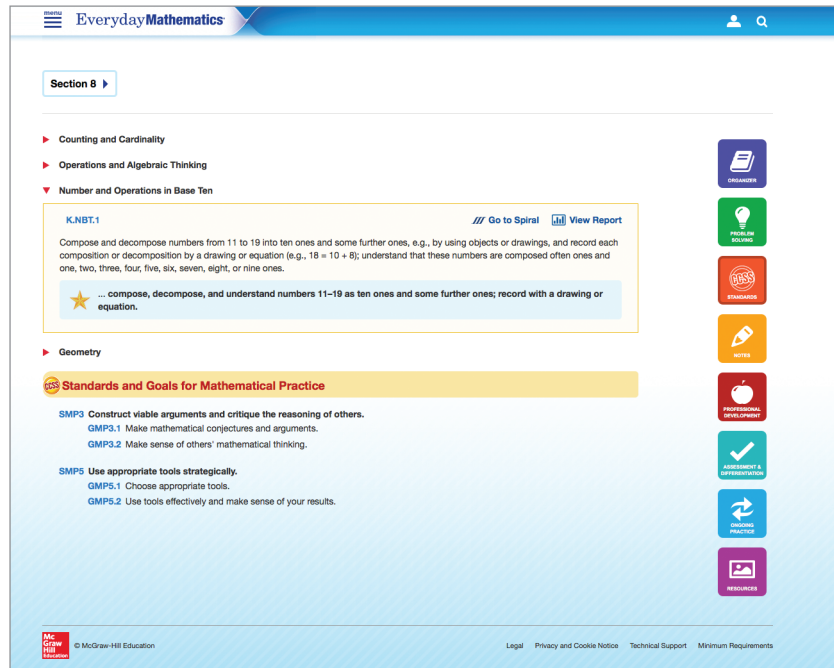
Differentiation Options in Teacher Center and *Teacher's Lesson Guide*, Grade 2, Lesson 6-6

Lesson Features that Support Development of Academic Language. Mastery of content and processes in a mathematics class is dependent upon students' abilities to communicate effectively. Nearly all instruction and assessment requires facility with language. Students need language skills so they can share their thinking publicly in class discussions and so teachers can assess their understanding of mathematical concepts. The Common Core State Standards for both Mathematics and English Language Arts cite the importance of developing and using language in the content areas. As such, the development of students' academic language has become an important element of instruction in all content areas, including mathematics. Many lessons include **Academic Language Development** notes that provide suggestions for promoting language development for all students or highlight important aspects of the language in the lesson that can help foster better understanding of the mathematical ideas.

For more information, see Section 5.1.2 *Academic Language Development*.

Unit Structure and Features

- The **Unit Organizer** includes information on the content and practice standards highlighted in the unit, assessment and differentiation opportunities, and ongoing practice through games. It includes the Spiral Trace described in Section 2.2.1.



Spiral Trace in Teacher Center, Kindergarten, Section 8 Organizer

- Each grade includes 8 or 9 units and each unit includes approximately 10 to 15 lessons which introduce and develop content through varied activities and practice opportunities. In addition to regular lessons, there are three special types of lessons:
 - One lesson in each unit in Grades 1–3 is an **Explorations** lesson with open-ended activities that students complete with minimal teacher guidance. These activities provide initial exposure to content that is developed more fully in later lessons.
 - One lesson in each unit in Grades K–6 is a two-day **Open Response and Reengagement** lesson. On Day 1 students solve a challenging problem that involves more than one possible strategy or solution. On Day 2 students reengage in the problem by examining and discussing other students' solutions to deepen their understanding of the mathematics content and practices in the problem.
 - The final lesson in each unit in Grades 1–6 is a two-day **Progress Check** lesson. Day 1 provides opportunities to assess and track students' performance on the content and practice standards that were the focus of the unit. Day 2 provides an additional assessment opportunity through an Open Response problem (in odd-numbered units) or an opportunity to assess content and practices from prior units in a Cumulative Assessment (in even-numbered units).

For more information, see Section 4.3.1 *Explorations*.

For more information, see Section 6 *Open Response and Reengagement*.

For more information, see Section 9.3.2 *Progress Check Lessons*.