GY403 Structural Geology

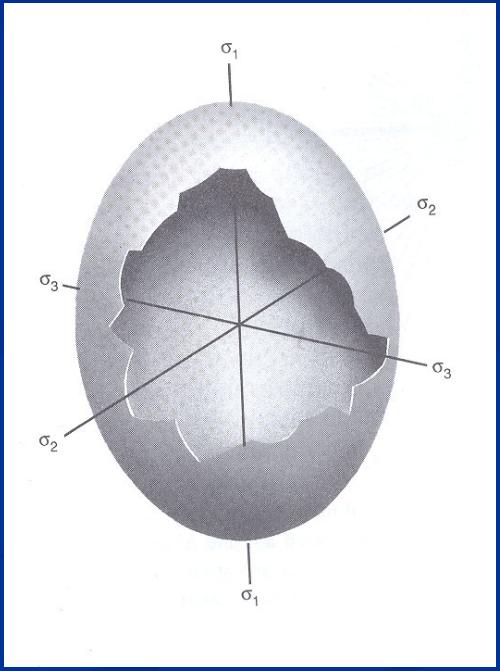
Lecture 7: Dynamic Analysis

Dynamic Analysis Goals

- Determine the magnitude and orientation of forces that produce rigid and non-rigid body strain.
- Determine the mechanical factors in earth materials that favor/disfavor deformation
- Relate forces to tectonic evolution of deformed terranes

Stress

- Stress: force applied to an area (i.e. p.s.i. in tire inflation specifications).
- Stress Ellipsoid: the magnitude of stress in any direction relative to a point in a rock mass can be conceptualized as a stress ellipsoid. The larger the size of the ellipsoid, the higher the stress on the rock.
- Lithostatic stress: the stress on a rock mass due to the overlying column of rock (stress ellipsoid is a sphere).
- Directed stress (Differential stress): produced when plate motion produces a maximum (σ_1) and minimum (σ_3) compressive stress direction (stress ellipsoid: $\sigma_1 > \sigma_2 > \sigma_3$).

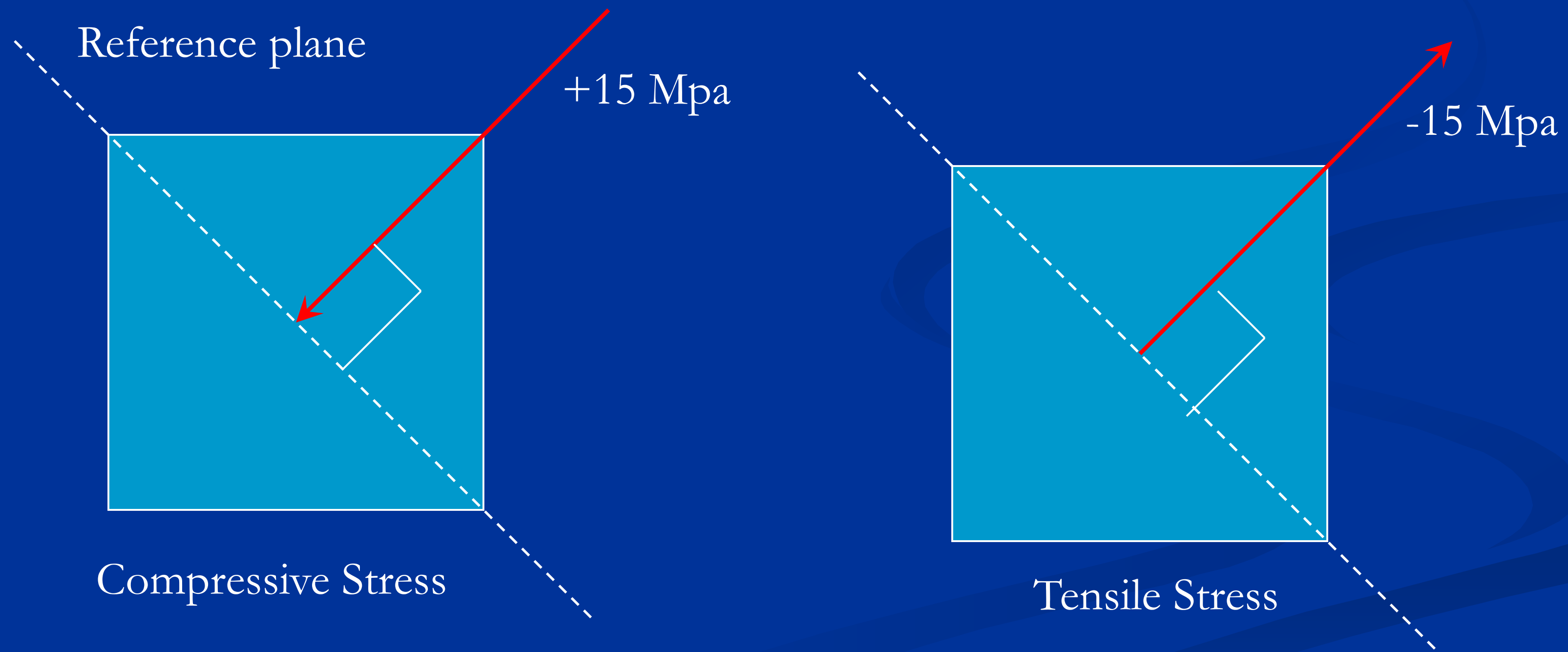


Stress Conventions

- Stress is not a vector, instead it is considered a 2nd order tensor. This means that you cannot add 2 stress tensors “head-to-tail” as you can with 2 force vectors (1st order tensors)
- Compressive (Normal) Stress: The stress tensor acting perpendicular to an imaginary plane passing through a rock mass. It is considered positive if it would cause shortening of material along axis of stress tensor. If normal stress is negative it would potentially cause a stretching along this direction. The symbol σ is used to represent normal stress.

Normal Stress

- Normal Stress: stress tensor acts perpendicular to a reference plane passing through the rock mass.

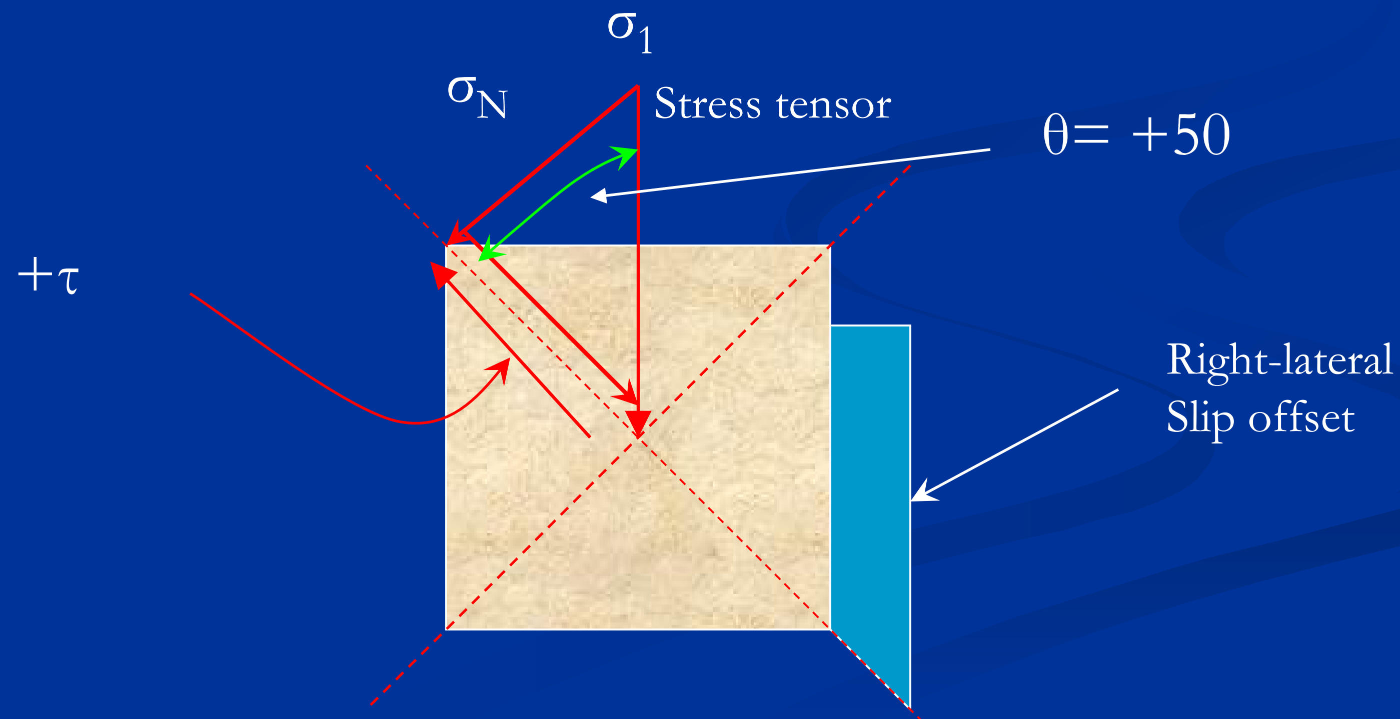


Stress Conventions cont.

- Shear stress (τ): produced when differential stress field exists (i.e. stress ellipsoid)
  - If shear would cause right-lateral offset in a rock it is positive. The shear plane thus produced has a positive angle θ to σ_1.
  - If shear would cause a left-lateral offset in a rock it is negative, and the shear plane would have a negative θ angle relative to σ_1.

Shear Stress (τ)

- Shear Stress (τ): Positive shear stress is by convention right-lateral (dextral), negative shear stress is left-lateral (sinistral).

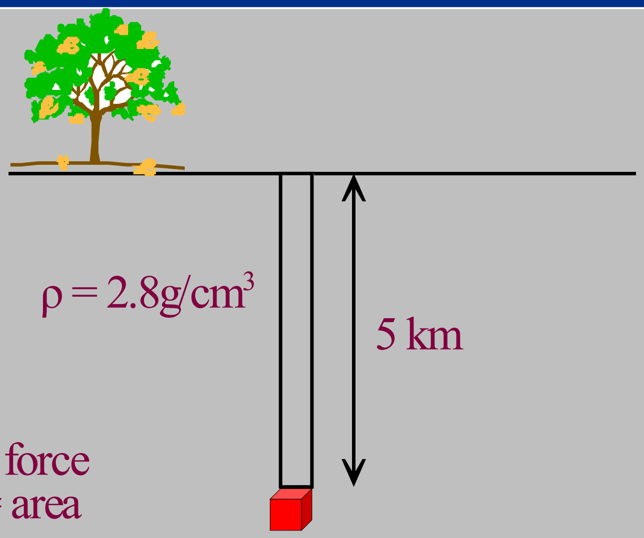


Resolution of Stress via Vector Addition

- Stress is a 2^{nd} order tensor therefore it cannot be directly resolved by simple vector addition
- If the stress tensors can be converted to forces vectors (1^{st} order tensors) then the overall stress can be evaluated by vector addition
- Lithostatic stress: all of the principle stress directions have equal values – the stress ellipsoid is a sphere.
- Lithostatic stress is a result of overburden pressure.

Calculation of Lithostatic Stress

- Use a specified density and depth to calculate lithostatic stress value.



F = force
A = area
m = mass
a = acceleration
F = ma

stress (σ) = $\dfrac{F}{A}$

Given a depth of burial of 5 km, density of 2.8, calculate the lithostatic stress on a one cm^3 volume.

$a = 980\ cm/sec^2$

$F = ma$

$F = V\rho a$ (where ρ = density, V = volume)

$F = (\rho)(h)(b)a$
(where h = height of column; b = area of base)

$F = (2.8 g/cm^3)(5.0x10^5 cm)(1.0 cm^2)(980 cm/sec^2)$

$F = [1,372,000,000\ (g)(cm)]/sec^2$
$= 1,372,000,000$ dynes

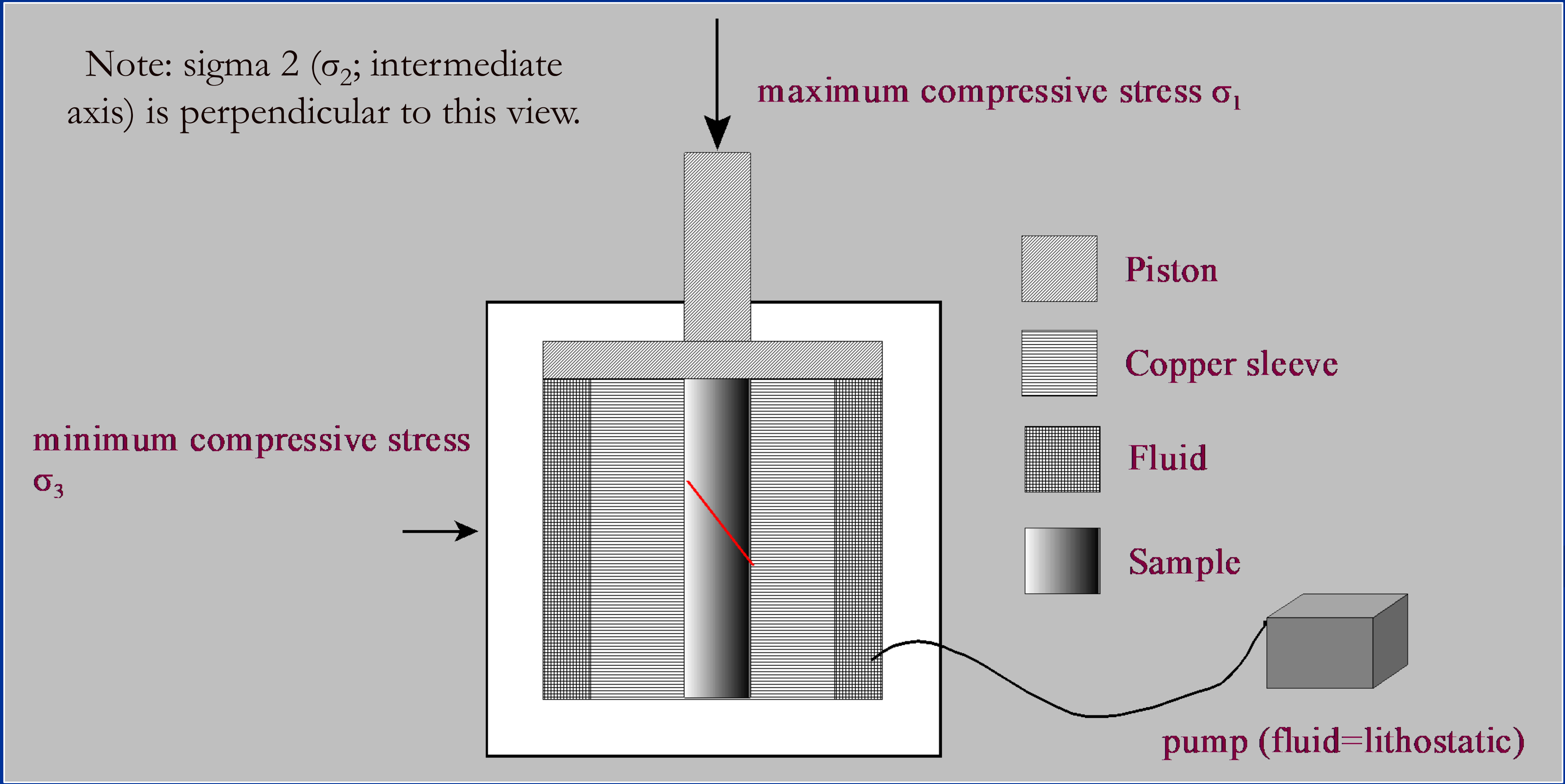
$\sigma = (1,372,000,000 \text{ dynes}) / cm^2 *$
$(1.0 \text{ bar})/(1,000,000 \text{ dynes}/cm^2)$

$\sigma = 1,372$ bar $= 1.372$ kbar

Pressure gradient:
(5.0 km)/(1.372 kbar) = (3.64 km)/(kbar)

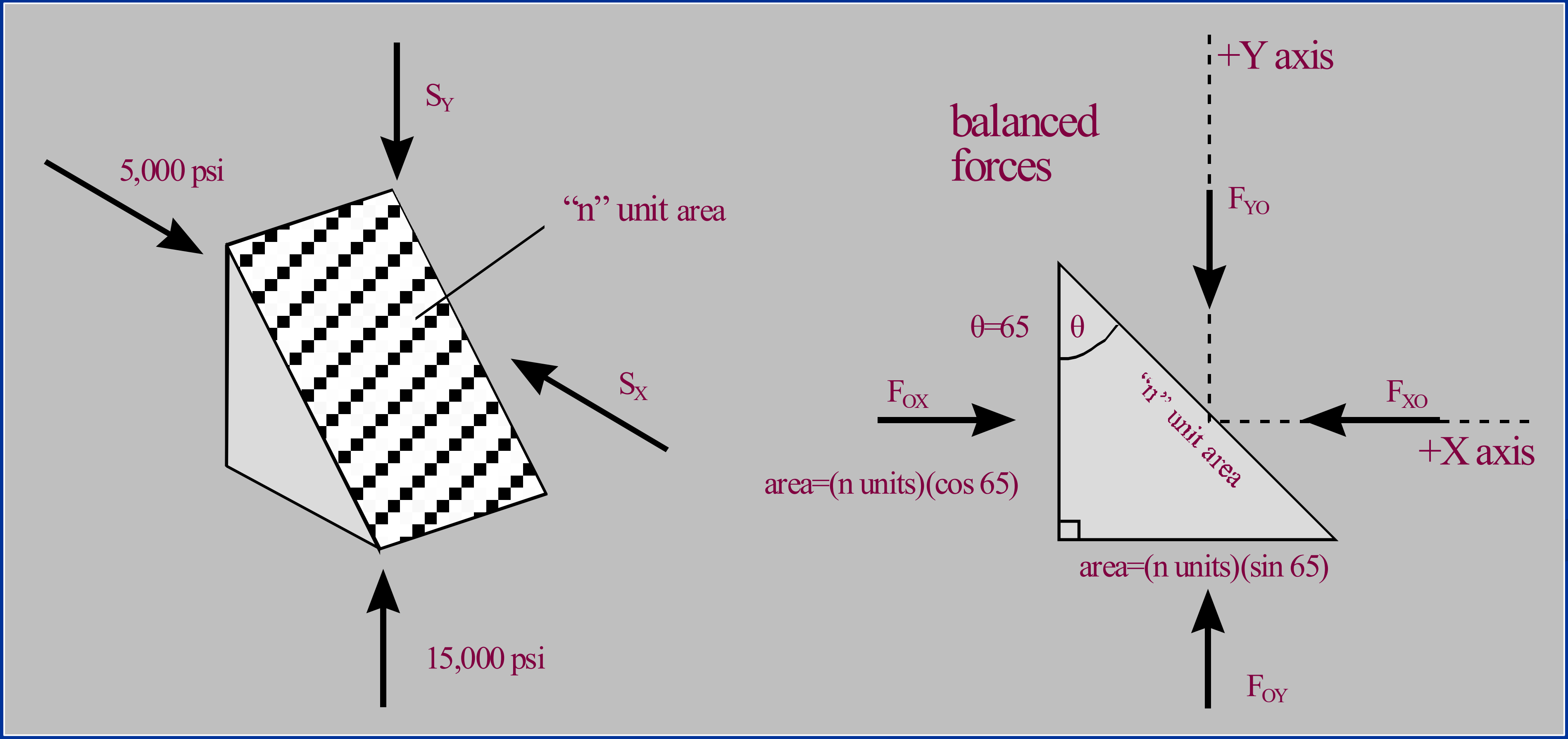
The Triaxial Stress Apparatus

- Otherwise known as the "Bomb"!



Calculation of Stress using Resolution of Forces

- Triaxial stress apparatus example.



Resolution of Forces

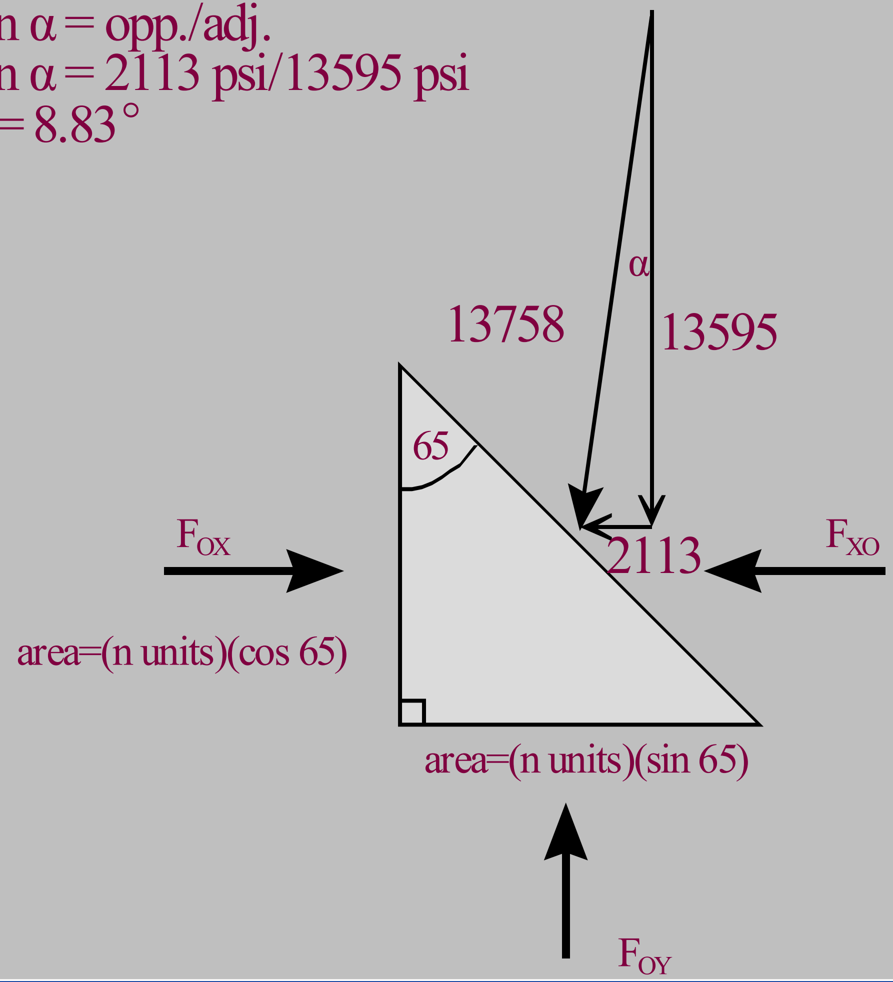
- Using the balanced forces assumption = no significant acceleration.

$\sigma = F/A$
$F = \sigma A$
$F_{xo} = F_{ox}$ (balanced forces)
$F_{xo} = S_x A = S_x (n\ units)^2$
$F_{ox} = (5000psi)(\cos 65)(n\ units)^2$
$S_x (n\ units)^2 = (5000psi)(\cos 65)(n\ units)^2$
$S_x = 2113\ psi$

$S_y = (15000\ psi)(\sin 65)$
$S_y = 13595\ psi$

$\sigma^2 = S_x^{\ 2} + S_y^{\ 2}$
$\sigma^2 = (2113psi)^2 + (13595\ psi)^2$
$\sigma = 13758\ psi$

$\tan \alpha = opp./adj.$
$\tan \alpha = 2113\ psi/13595\ psi$
$\alpha = 8.83°$



Resolving Stress Tensor

- Components of the stress tensor parallel and perpendicular to the potential shear plane.

$\cos(16.17^\circ) = (\sigma_N)/(13758\text{ psi})$

$\sigma_N = 13214\text{ psi}$

$\sin(16.17^\circ) = \tau/(13758\text{ psi})$

$\tau = +3831\text{ psi}$

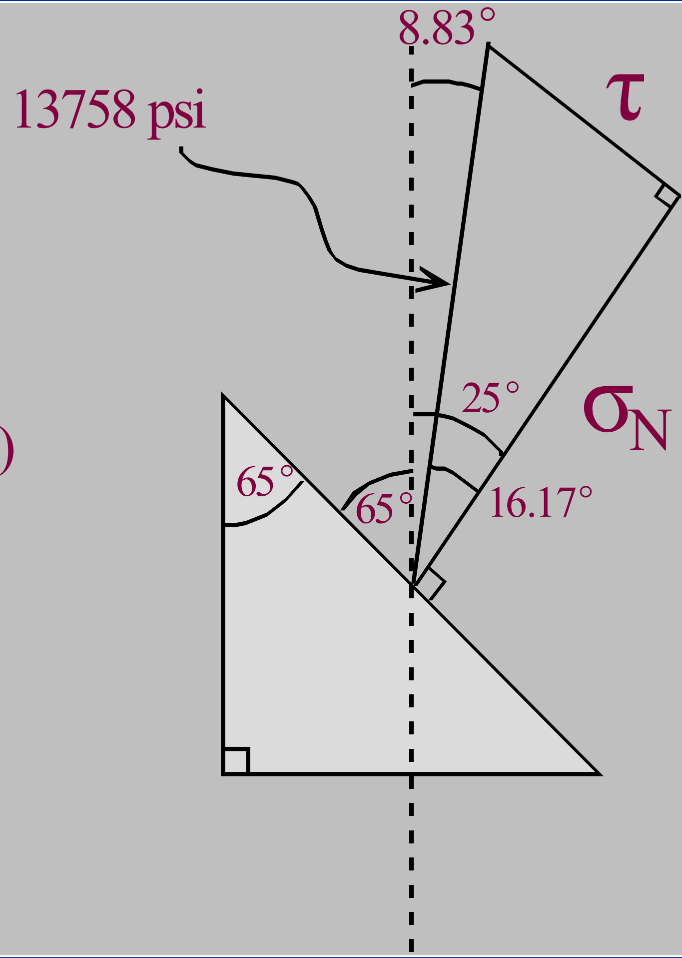
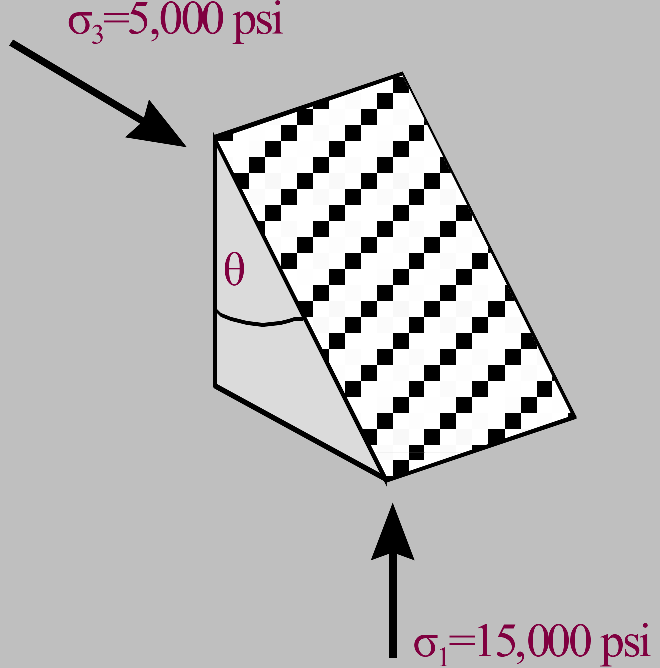


Table of Normal(σ) and Shear(τ) Stress

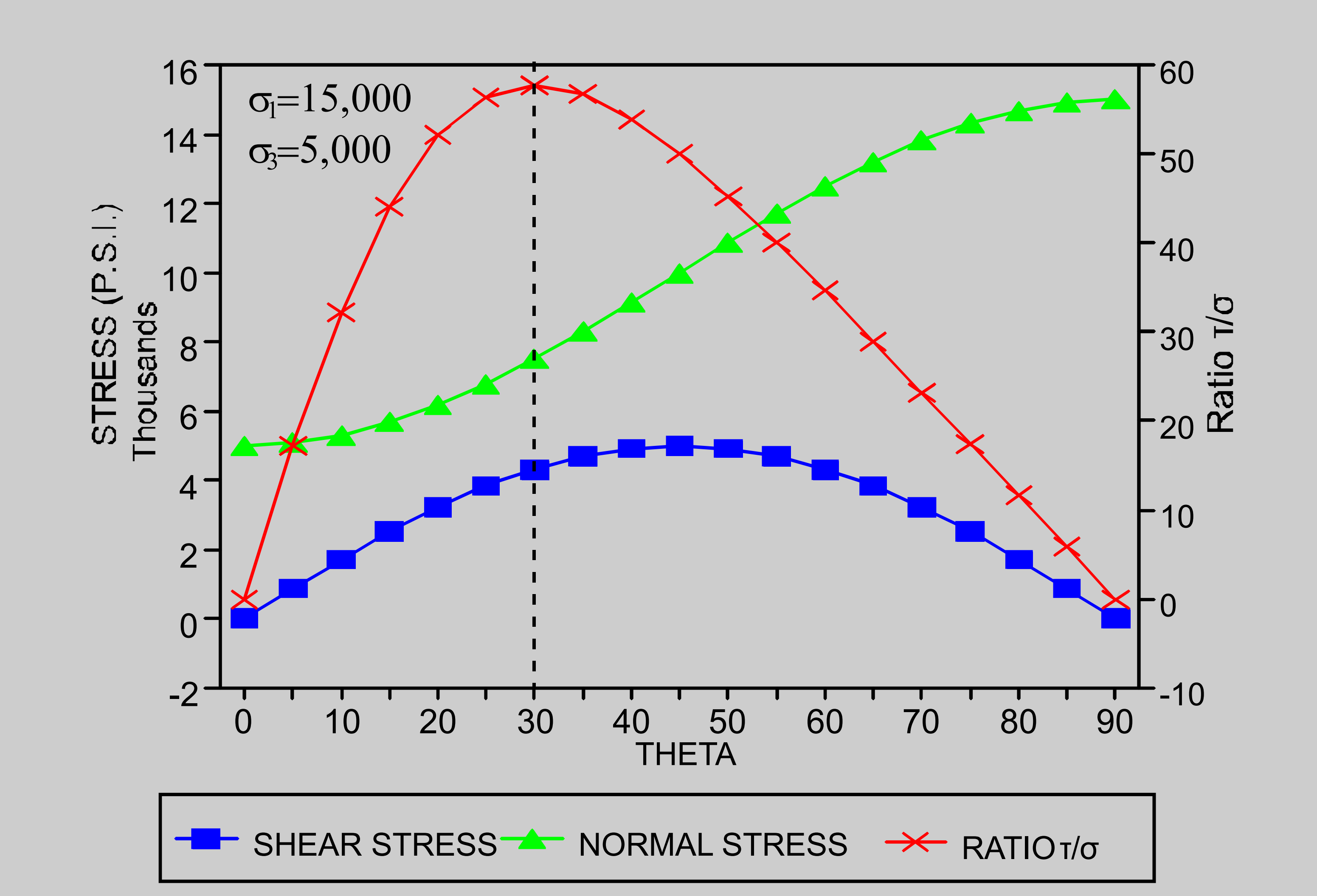
- As a function of theta (θ) angle from potential shear plane.

θ(angle with σ_1)	Normal Stress(σ)	Shear Stress(τ)	Ratio (τ/σ)
0	5000	0	0.0
5	5075	868	17.1
10	5301	1710	32.3
15	5669	2500	44.1
20	6169	3214	52.1
25	6786	3830	56.4
30	7500	4330	57.7
35	8289	4698	56.7
40	9131	4924	53.9
45	10000	5000	50.0
50	10868	4924	45.3
55	11710	4698	40.1
60	12500	4330	34.6
65	13213	3830	29.0
70	13830	3214	23.2
75	14330	2500	17.4
80	14698	1710	11.6
85	14924	868	5.8
90	15000	0	-0.0



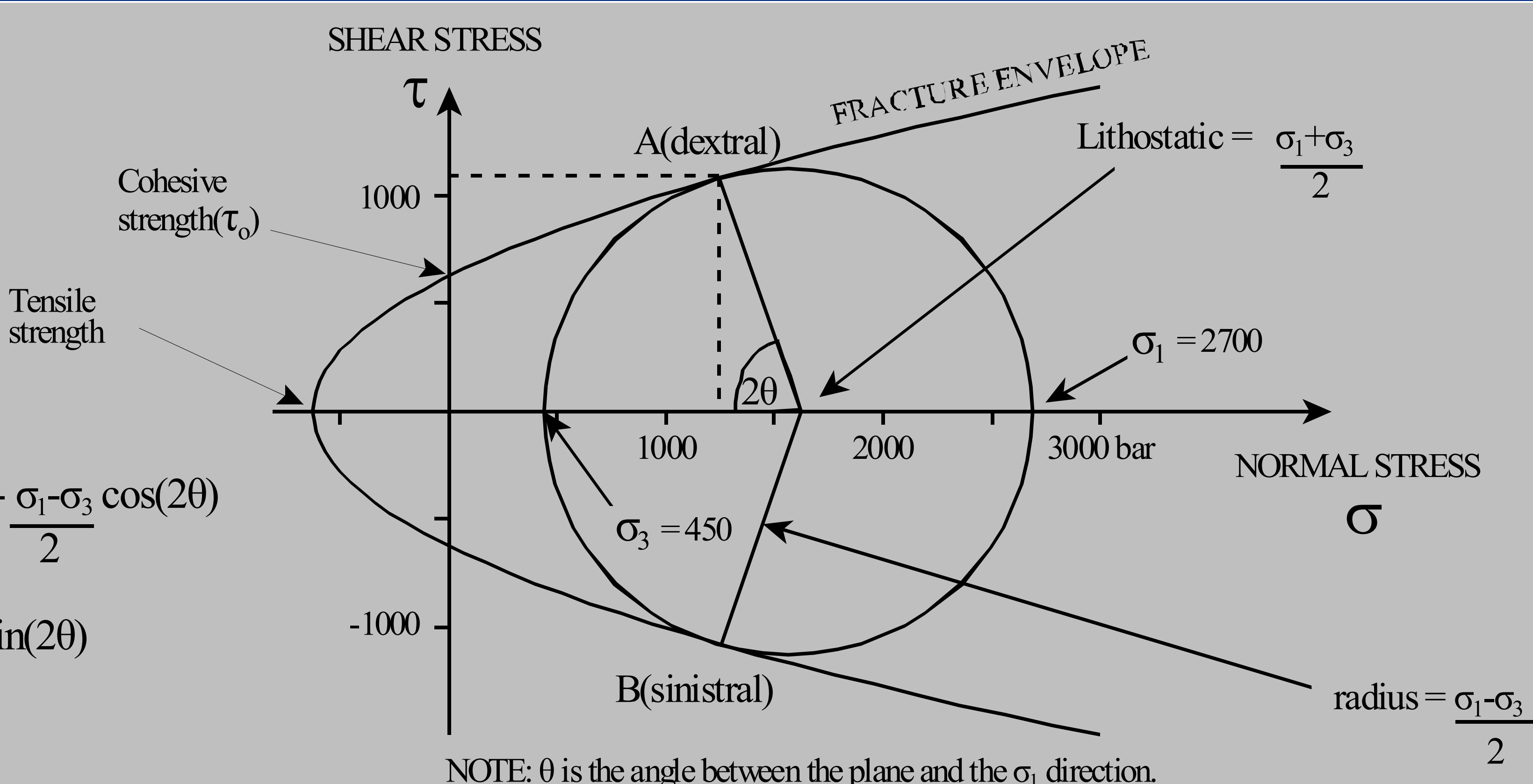
Normal and Shear Stress Values

- As a function of theta angle.



Mohr Circle for Stress

- General equations for σ and τ.

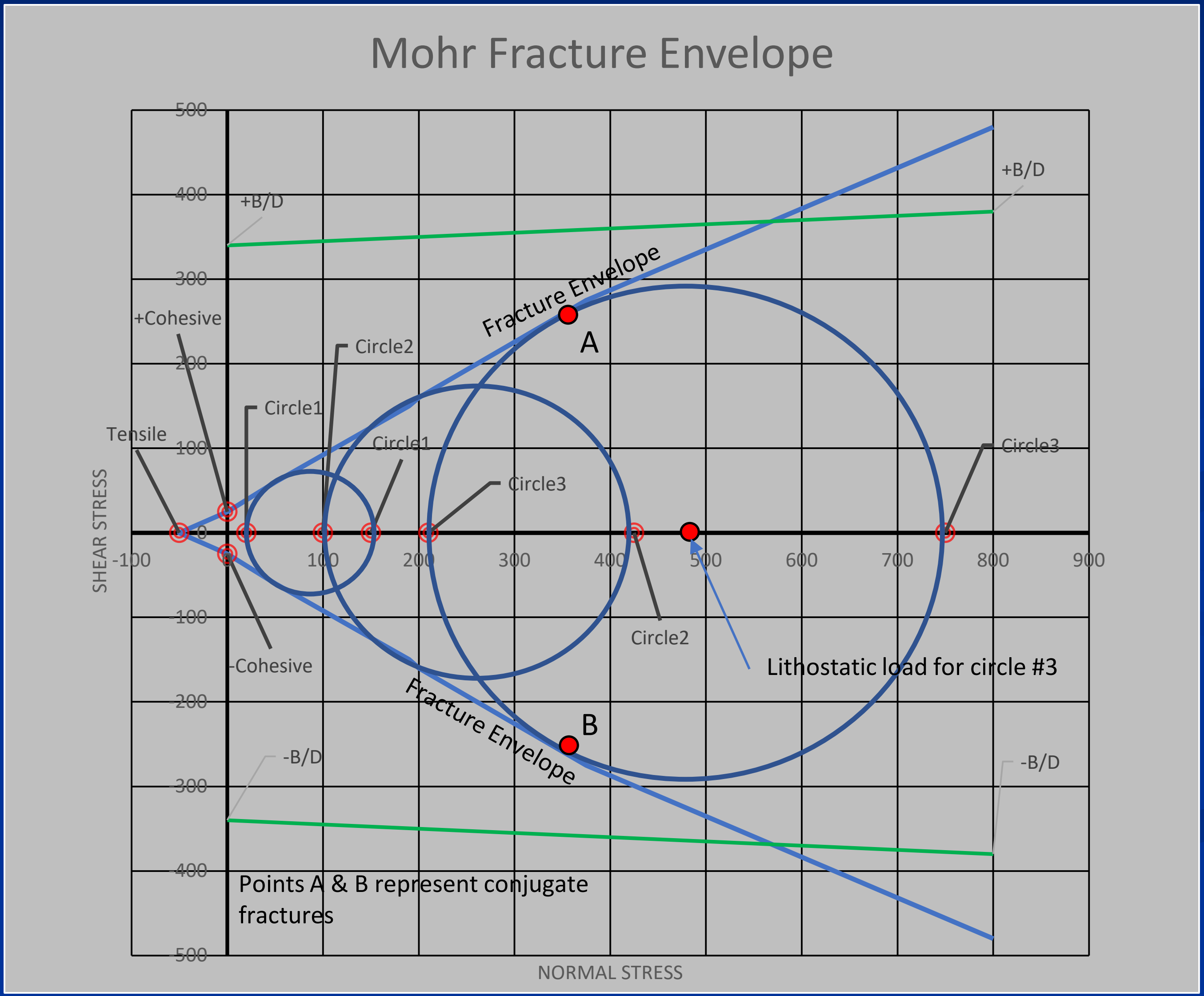


$$\sigma = \frac{\sigma_1+\sigma_3}{2} - \frac{\sigma_1-\sigma_3}{2}\cos(2\theta)$$

$$\tau = \frac{\sigma_1-\sigma_3}{2}\sin(2\theta)$$

Mohr Circle & Fracture Envelope

- Fracture envelope displays the stress state requirements for fracture formation



Fracture Envelope Properties

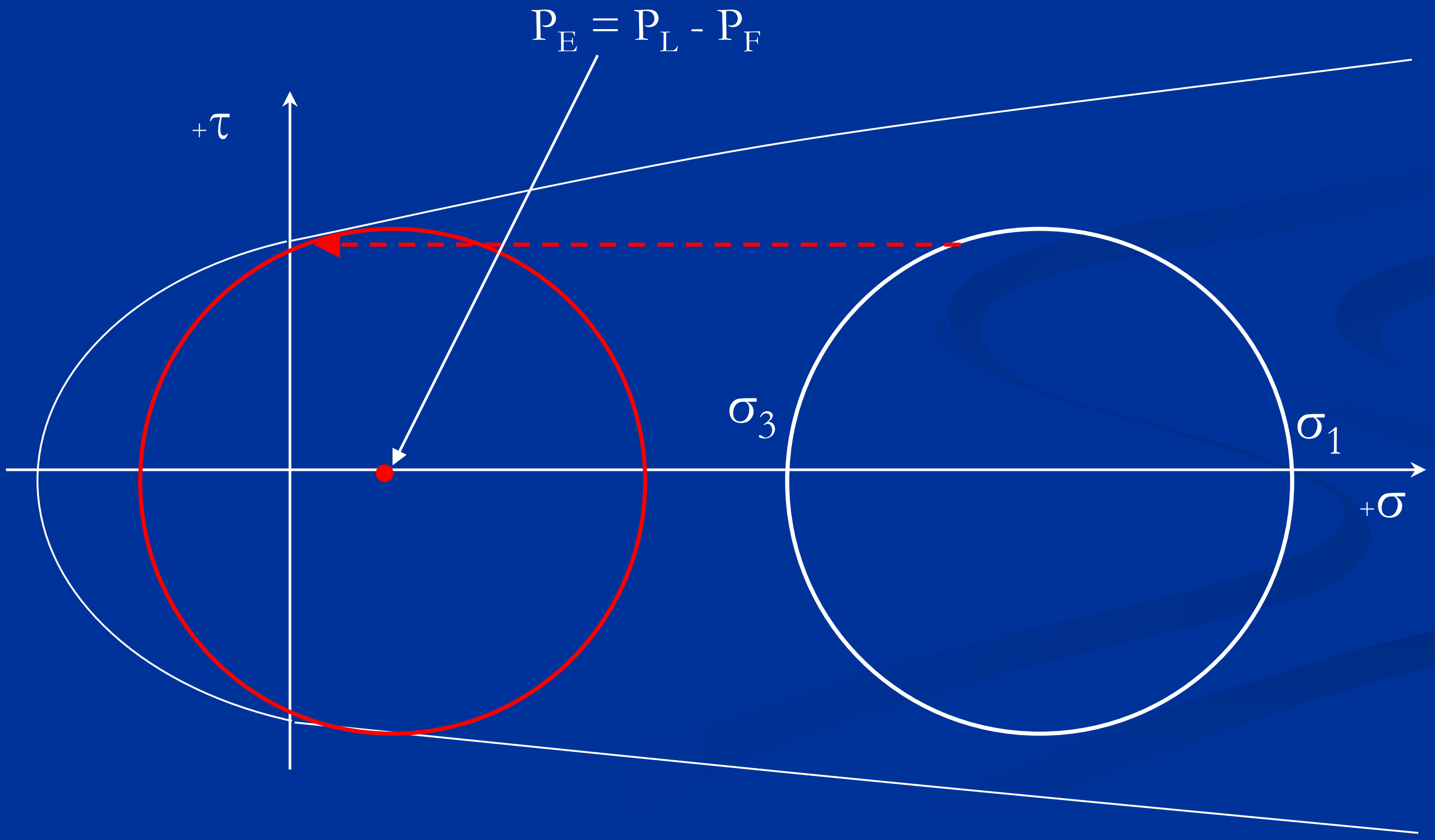
- Increasing lithostatic σ requires greater differential σ to produce fractures
- Because of the parabolic shape of the fracture envelope conjugate fractures will tend to form at 30° to σ_1.
- Fracture envelope only predicts fracture failure via brittle behavior- ductile deformation is not addressed by the envelope

Mohr Circle & Fluid Over-pressure

- If Fluid pressure approaches that of the $\sigma 1$ the Mohr circle is shifted left toward the origin making fracture much more likely:
- $P_E = P_L - P_F$ (Effective Pressure = Lithostatic Pressure – Fluid Pressure)
- Petroleum companies exploit this property by "Fracing" the reservoir after traditional production declines (fracturing dramatically increases porosity & permeability)

Mohr Fracture Envelope & Fluid Over-Pressure

- Fluid over-pressure shifts the Mohr circle toward the origin greatly increasing chances for fracture formation

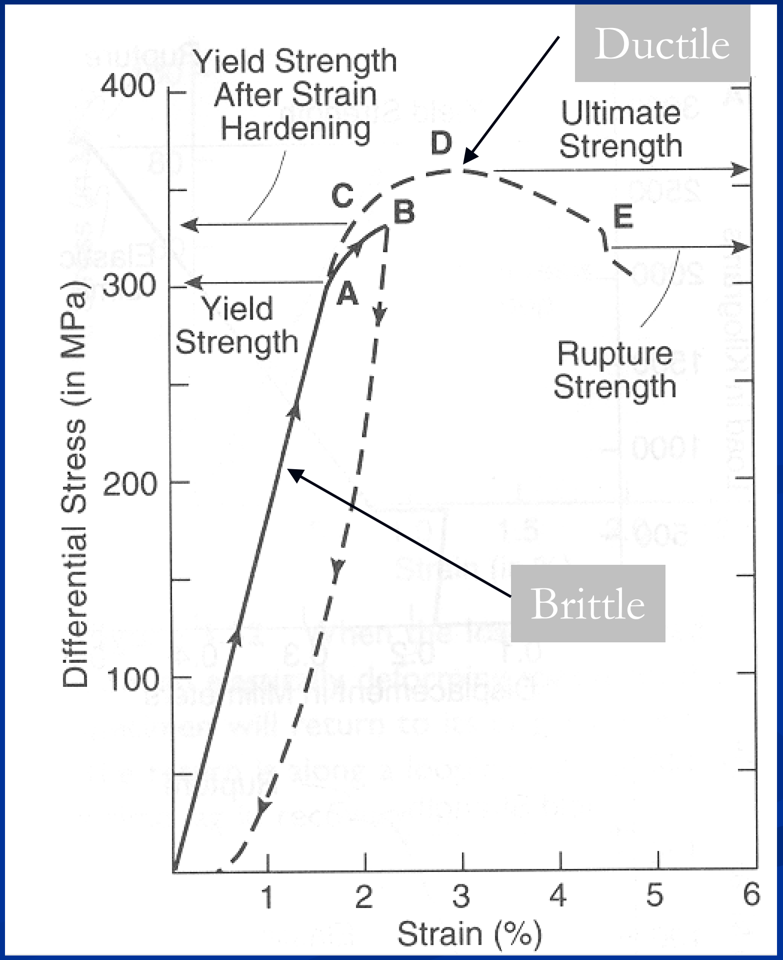


Rock Mechanical Properties

- Triaxial stress apparatus is used to test the mechanical strength of various rocks under a variety of different conditions (Lithostatic load, Temp, Fluid pressure, Strain Rate, etc.)
- Several "Ideal" mechanical behaviors are useful in understanding rock mechanics:
  - Elastic: stress produces strain up to the yield strength at which point the rock fractures, however, releasing stress before the yield strength allows the rock to recover all strain ("Rubber Band")
  - Plastic: any amount of applied stress will produce permanent strain ("Silly Putty")

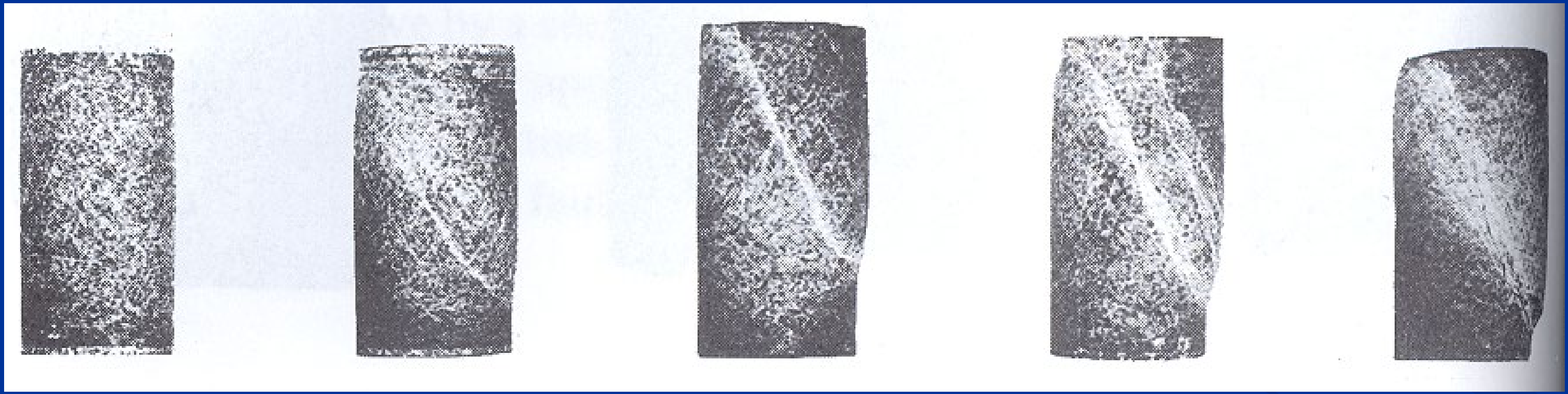
Stress v. Strain Graphs

- Graphical plots of rock mechanical behavior with strain (ε) on the X axis and differential stress ($\sigma = \sigma1 - \sigma3$) on the Y axis
- Rock mechanics dominated by elastic component is "Brittle"
- A significant plastic flow component is termed "Ductile"



Examples of Brittle v. Ductile Tests

- Same rock (limestone) deformed to 15% ε under various Lithostatic and Temperature conditions



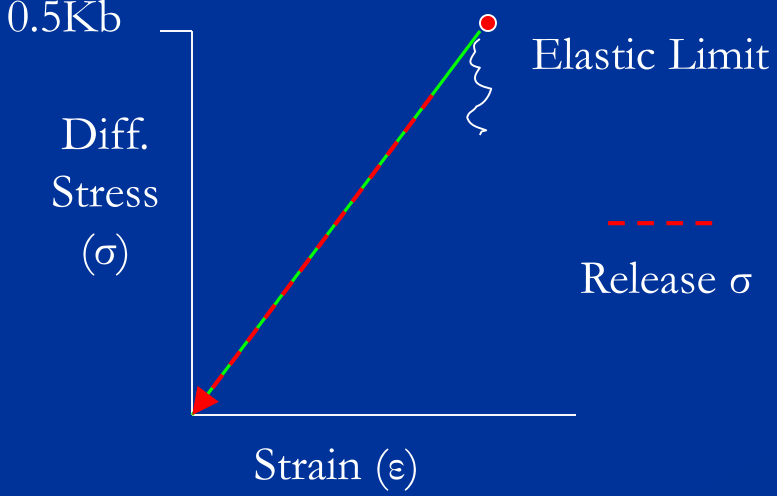
Original High confining Low confining High confining Low confining

Lithostatic Load (Confining)

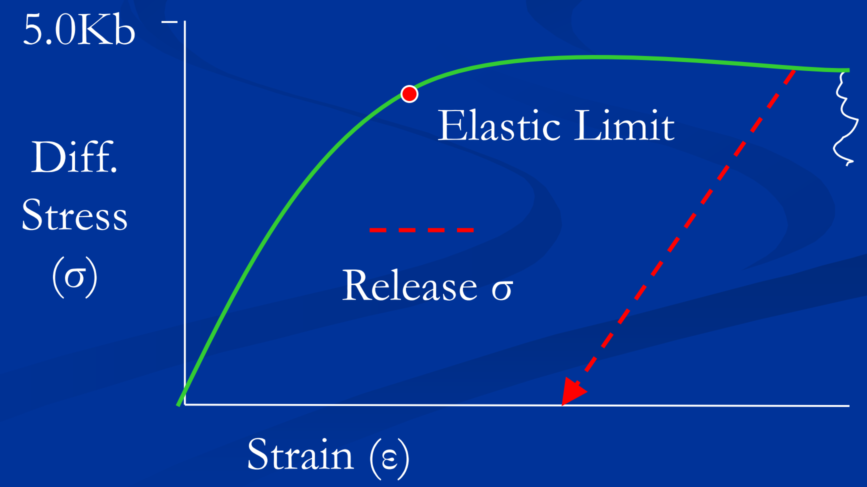
- Mechanical Effects of Increasing Lithostatic Load (i.e. Depth of Burial):
  - > Lithostatic = > Rock Strength
  - > Lithostatic = > Ductility

Elastic Limit

Low Lithostatic

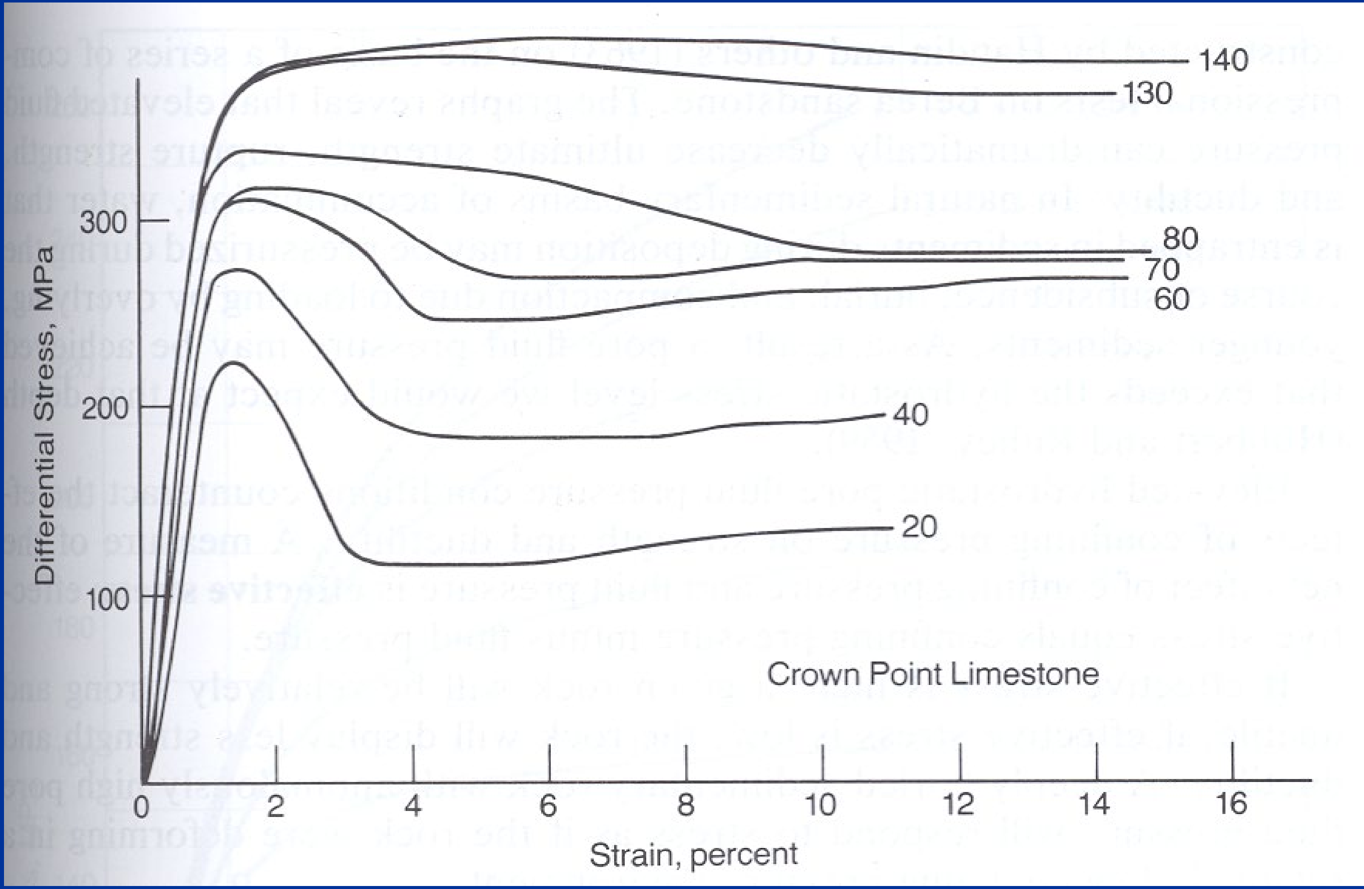


High Lithostatic



Lithostatic Load cont.

- Actual test with limestone at various levels of lithostatic stress

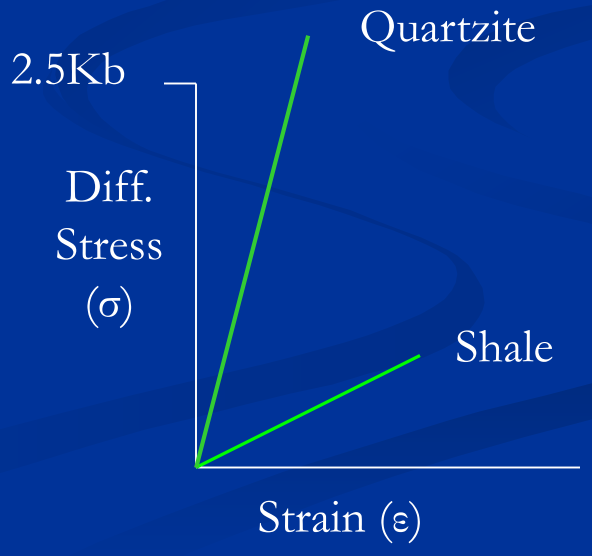


Lithology

- The type of Lithology exerts a strong influence on rock strength

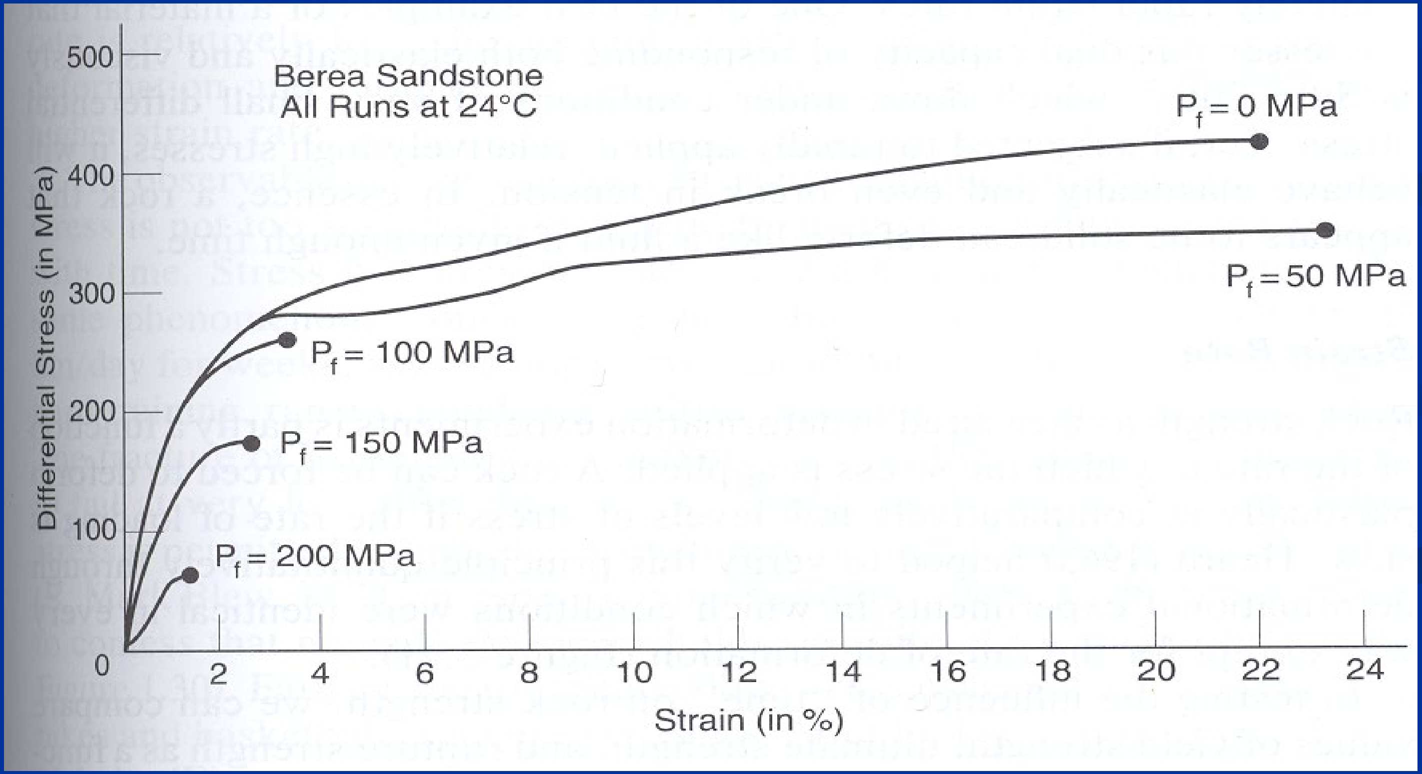
Rock Strength	Lithology
300 MPa (strongest)	Quartzite
280 MPa	Granite
250 MPa	Basalt
213 MPa	Limestone
167 MPa	Schist
140 MPa	Marble
120 MPa	Shale
45 MPa	Anhydrite
22 MPa	Salt

NOTE: a strong anisotropy such as foliation may cause a dramatic weakening of the rock parallel to the fabric



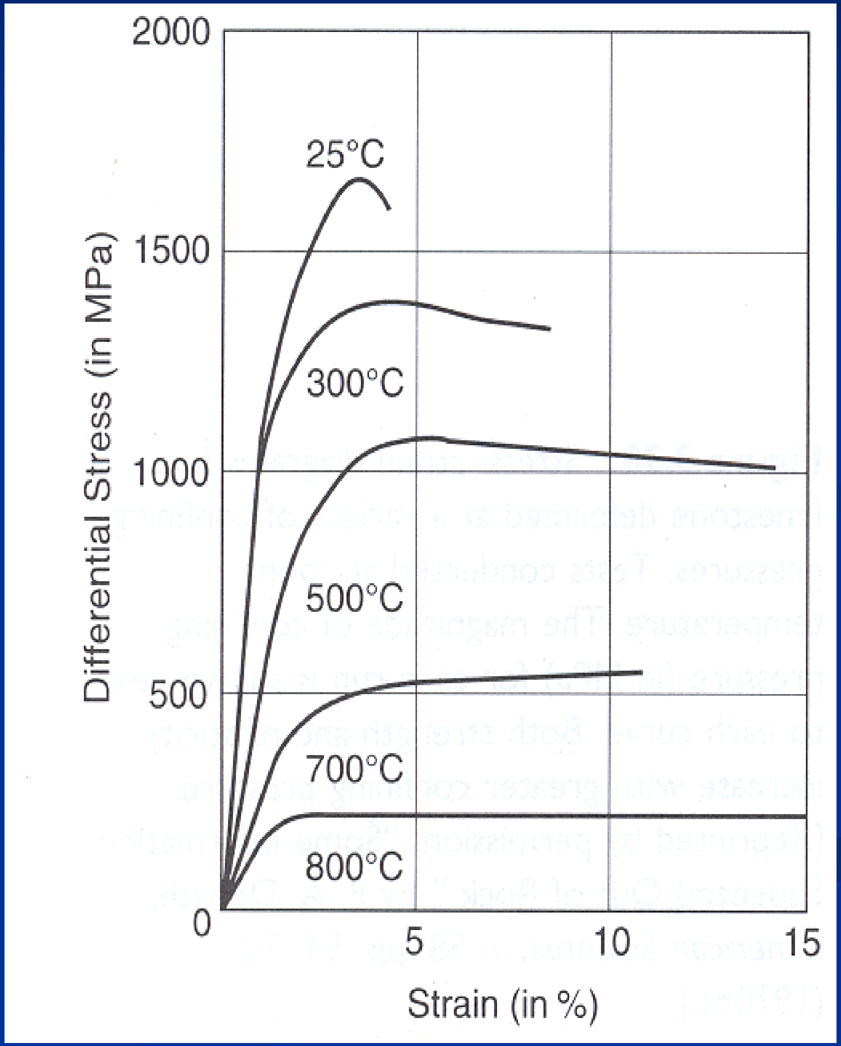
Pore Fluid Pressure

- Increasing pore fluid pressure counteracts increasing lithostatic:
  - Rock is weakened
  - Rock is likely to deform by brittle failure



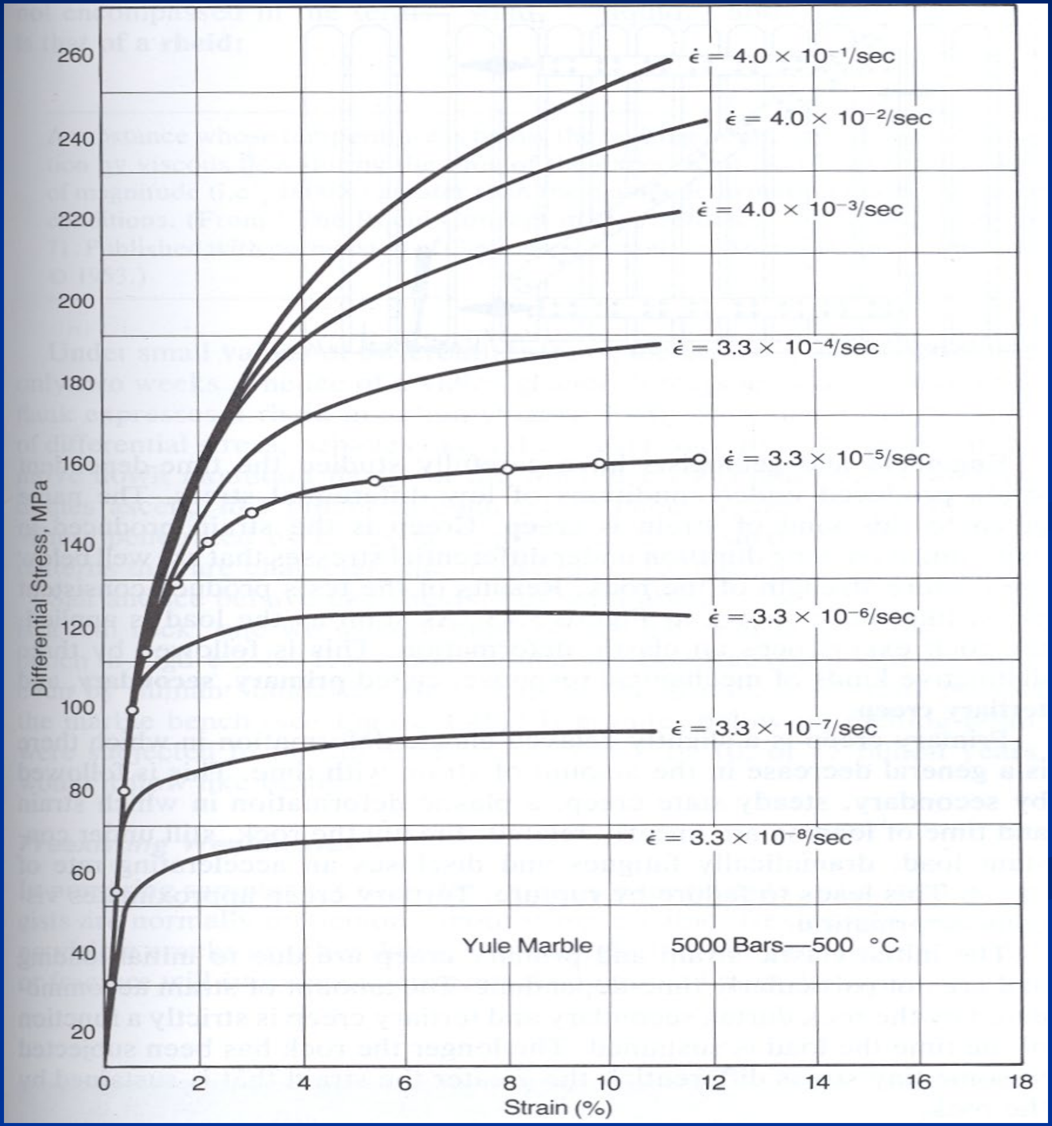
Temperature

- Increasing T will decrease the rock strength and favor ductile behavior (test is on basalt at various T°C)



Strain Rate

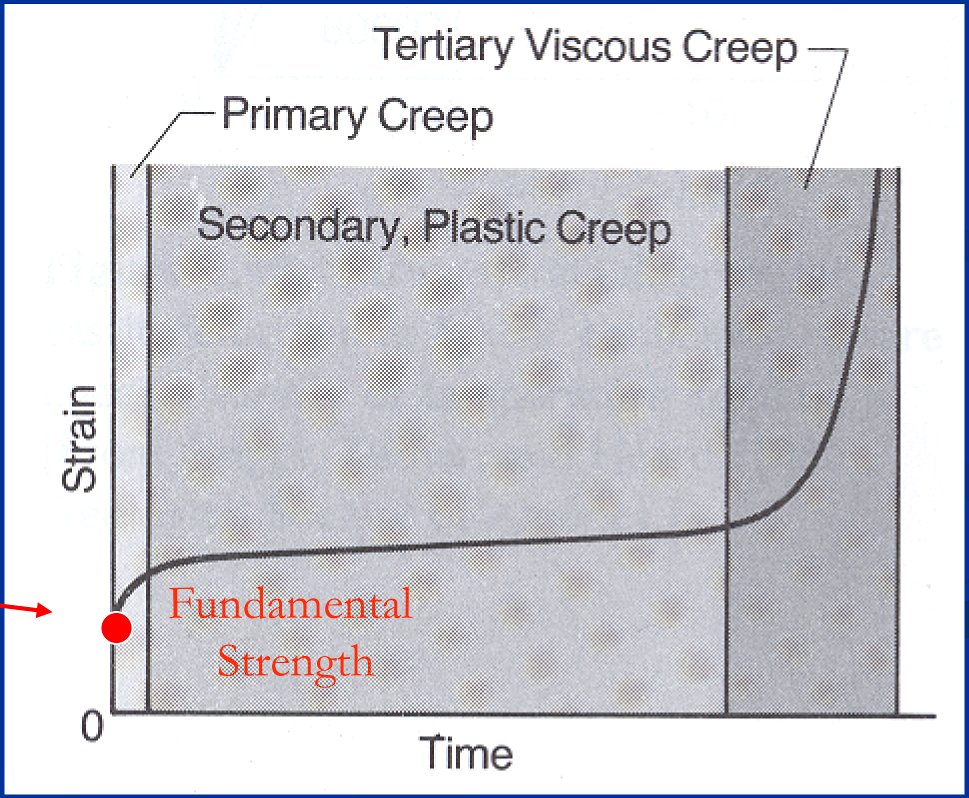
- Increasing the strain rate will increase the rock strength and favor brittle behavior



Time Factor

- Given enough time any solid material will "flow" below the elastic limit- this is termed "Mechanical Creep"

Differential Stress < Elastic Limit



Rheidity

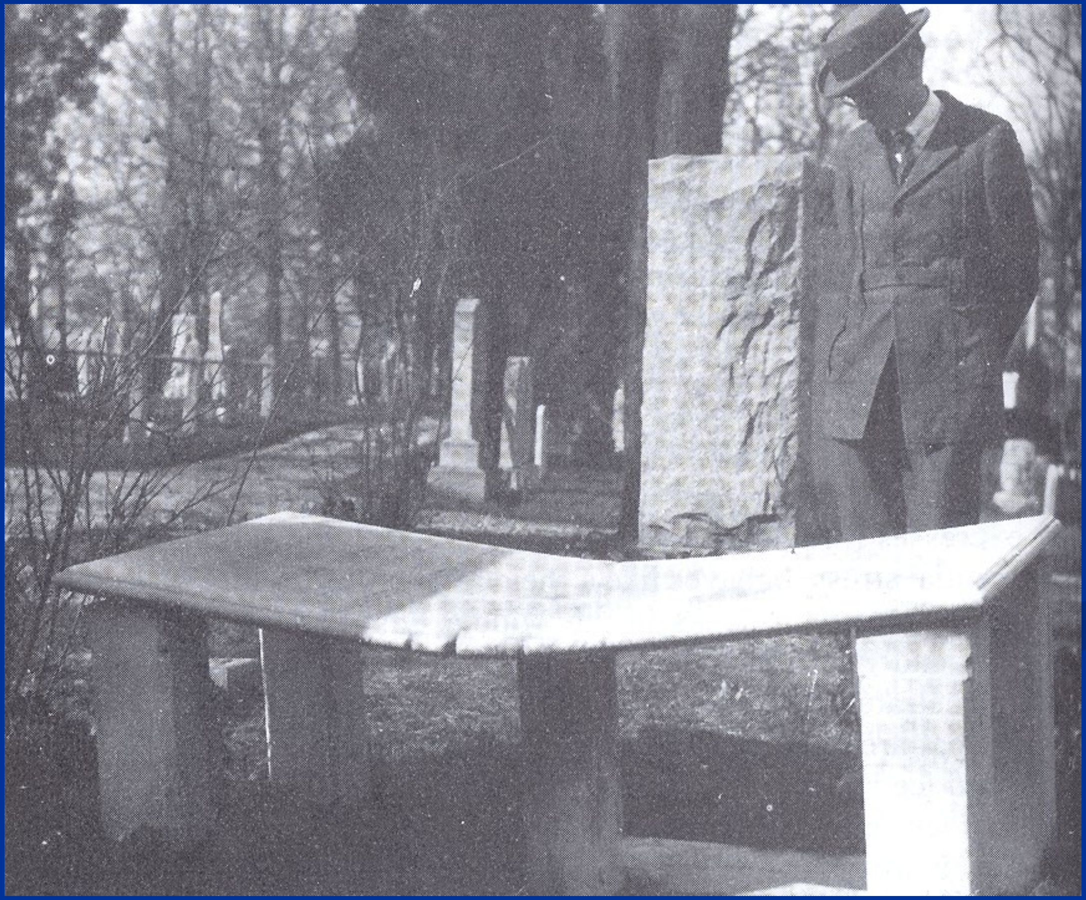
- Definition (Carey, 1953):

not encompassed in the terms "solid," "liquid," and "gas." The state is that of a **rheid:**

A substance whose temperature is below the melting point, and whose deformation by viscous flow during the time of the experiment is at least three orders of magnitude (i.e., 1000×) greater than the elastic deformation under the given conditions. (From "The Rheid Concept in Geotectonics" by S. W. Carey, p. 71. Published with permission of Geological Society of Australia, Inc., copyright © 1953.)

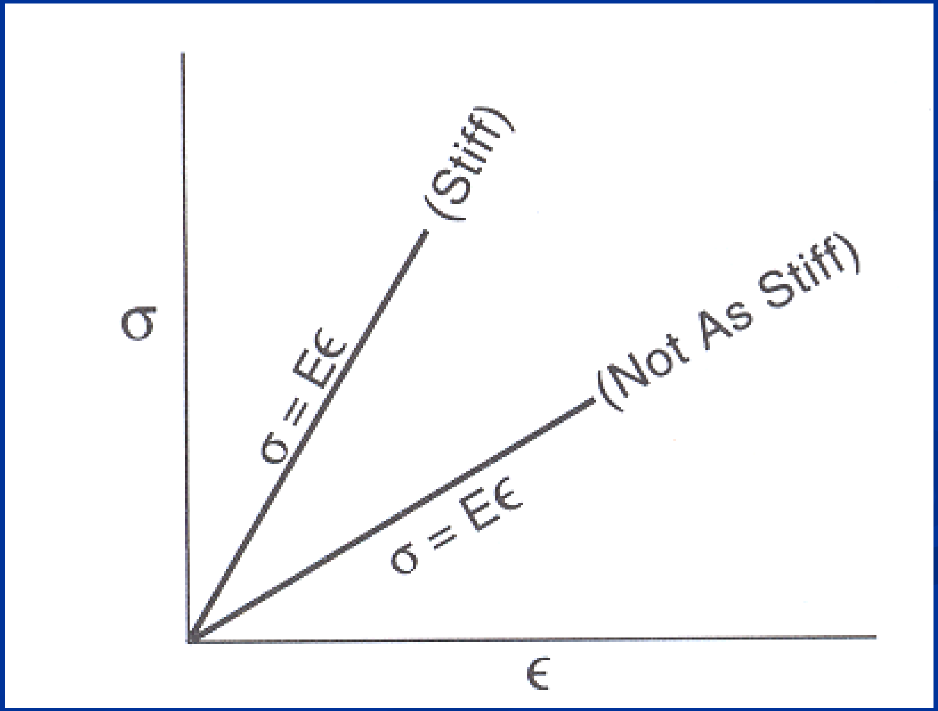
Rheidity Examples

- Marble Bench: (see photo)
- Continental Crust: has suffered no appreciable creep since Archean



Young's Modulus

- Young's Modulus (E): relationship between σ and ε (i.e. the slope on a stress v. strain graph



Bulk and Shear Modulus

- Bulk Modulus (K): measures resistance to Δ volume (dilation)
- Shear Modulus (G): measures resistance to shear (τ)

$$K = \text{bulk modulus} = \frac{\Delta_{\text{hydrostatic stress}}}{\Delta_{\text{dilation}}}$$

$$G = \text{shear modulus} = \frac{\sigma_s}{\gamma}$$

Poisson's Ratio

- Poisson's Ratio (ν "nu"): ration of lateral E to longitudinal E

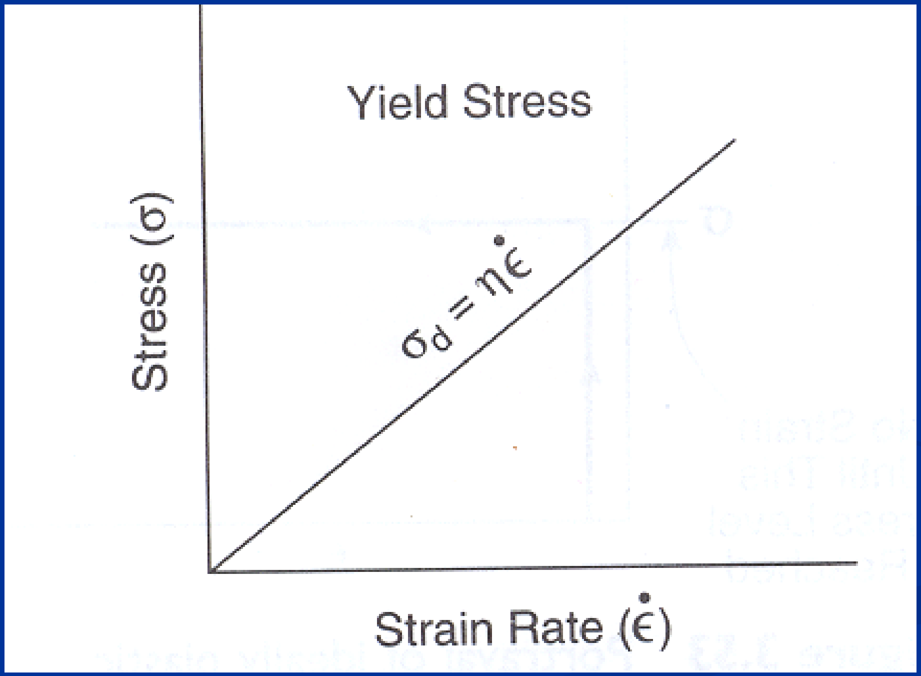
TABLE 3.5
Typical values of Poisson's ratio (ν)

Rock Type	
Limestone, fine grained	0.25
Aplite	0.20
Limestone, porous	0.18
Limestone, oolitic	0.18
Limestone, chalcedonic	0.18
Limestone, medium grained	0.17
Limestone, stylolitic	0.11
Granite	0.11
Shale, quartzose	0.08
Graywacke, coarse grained	0.05
Diorite	0.05
Granite, altered	0.04
Graywacke, fine grained	0.04
Shale, calcareous	0.02
Schist, biotite	0.01

$$\nu = \frac{e_{\text{lat}}}{e_{\text{long}}}$$

Viscosity

- Viscosity (η "eta"): resistance of a fluid to flow



Exam 2: Dynamic Analysis Summary

- Be able to solve strain equations for S, λ, γ, Ψ, α
- Be able to discuss the difference between homogenous and inhomogeneous strain- give geological examples
- Know how to calculate lithostatic stress given depth and density
- Know how to solve a resolution of stress by vector addition problem
- Know the general equations for σ and τ for the Mohr Circle, and know the relationship between terms in the equation and circle geometry
- Be able to interpret the Mohr Circle/Mohr Fracture envelope diagram
- Know the definitions and positions on a σ versus ε graph of Yield strength, Ultimate Strength, and Rupture Strength
- Be able to discuss the effects of the following on a σ versus ε graph:
    - Lithostatic load
    - Temperature
    - Strain rate
    - Pore Fluid pressure
    - Lithology
- Be able to discuss the Rheid concept and describe examples; know where Fundamental Strength is located
- Be able to discuss Young's Modulus, Bulk Modulus, Shear Modulus, Poisson's Ratio, and Viscosity