Dynamic Analysis of Unbalance in Rotating Machinery

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Abstract—**Rotating machines can develop inertia effects and this inertia effects in rotating structures are usually caused by gyroscopic moment introduced by the precise motions of the vibrating rotor as it spins. As spin velocity increases, the gyroscopic moment acting on the rotor becomes critical. It can be analyzed to improve the design and decrease the possibility of failure. This paper focuses on dynamic unbalance detection in rotating machinery by incorporating whirl, gyroscopic effects, mass effects and the effect of unbalance using the Finite Element Method (FEM) .**

Keywords—*Gyroscopic effects, Rotating machine unbalance, Rotatory Inertia, Coriolis Force.*

I. INTRODUCTION

The study of the mechanical signature or the vibration spectrum of a rotating machine allows to identify operating problems even before they become dangerous [4].Any deviation of the signature from its usual pattern provides a symptoms of a problem that is developing and allows the required counter measures to be taken in time. The objective is to study the rotor dynamic aspects of rotors and develop an understanding of critical speed, the effects of mass in a simple model and Rotor dynamics analysis using FEM by incorporating whirl, gyroscopic effects and the effect of unbalance.

Modal analysis using damped Eigen value solver is carried out in order to predict the dynamic behavior of the rotating system [11]. In modal analysis, the natural frequencies are evaluated in order to decide the critical speeds and mode shapes of the rotor. The process is repeated for different rotational speeds. Study of a simple rotating shaft (Jeffcott rotor) with a) centrally mounted disc, b) multiple discs, c) varying the mass, d) for varying distance in order to find the gyroscopic effects and mass effects in the system. To understand the gyroscopic effects a system with four degree of freedom is considered .i.e. two lateral and two rotational motion.

A. Theory

The theory of dynamics acts as a tool in understanding the physics of a rotating system and the theory of vibration serves as a powerful instrument to mathematically quantify the behavior of a rotor. Stability problems are the reason why rotor dynamics are important [3]. Any instability in the rotor can easily lead to disaster and end up to be very expensive. Oscillations can shorten the lifetime of the machine. Oscillations can also make the environment around the machine intolerable with heavy vibrations and high sound so it is advisable to avoid or reduce oscillations.

1) Gyroscopic effects.

The perpendicular rotation or precession motion is applied to the spinning rotor about its spin axis, a reaction moment appears and this effect is described as gyroscopic effect [5]. The interaction of the angular momentum of the rotating rotor and the wobbling motion introduce gyroscopic effect. The gyroscopic effect is only observed for modes which includes an angular (wobbling) component. The gyroscopic effect implies a change in the stiffness of the system and hence its frequency. A forward whirling increases the stiffness of a system while backward whirling decreases the stiffness of a system. A frequency map, is a graphically way to illustrate the influence of the gyroscopic effect on natural frequencies.

2) Critical Speed.

When the excitation frequency equal to natural frequency. The critical speed can be identified with both a modal analysis and a forced response analysis. Critical speed should always be avoided in any machines parts because otherwise significant vibrations may occur. These vibrations are the result of a resonant condition [8].

3) Campbell Diagram

Campbell diagram is the graphical representation of the system frequency Vs excitation frequency as a function of rotational speed. The diagram has the rotational speed of the rotor on the x-axis and the mode frequencies on the y-axis. The modes are plotted for different rotational speeds. The frequencies are not constant over the rotational speed range because most of the modes will be increased or decreased with higher rotational speed. Frequencies of two circular whirling motion, one occurring in the same direction of the spin motion known as Forward whirling, and one in the opposite direction is known as reverse whirling or Backward whirling. Torsional and

longitudinal modes are constant over the rotational speed range because they are not affected by the gyroscopic effect, the bearings or the stator. Forces in rotating machine may occur due to misalignments, a bended rotor, or due to a certain imbalance in the rotating disk. Machines are designed on the basis of manufacturing tolerances; there will always be small imbalance. An imbalance causes lateral forces, if the frequency of the force is equal the rotor speed the force is said to be synchronous. This produces a straight line of positive slope in the Campbell diagram, and is referred to as the one-times spin speed and labeled as 1X. The critical speeds are where the excitation line crosses any of the mode line. Campbell diagram shown in figure 1.
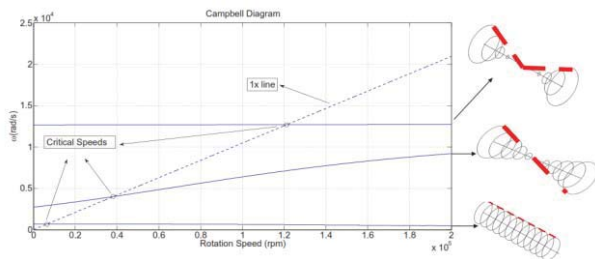

Fig.1 An example for Campbell diagram for a rotor system

4) Rotor Unbalance

Unbalance is the most common rotor system malfunction. Its primary symptom is 1X vibration, which, when excessive, can lead to fatigue of machine components. In extreme cases, it can cause wear in bearings or internal rubs that can damage seals and degrade machine performance. Unbalance can produce high rotor and casing vibration, and it can produce vibration in foundation and piping systems. 1X vibration can also contribute to stress cycling in rotors, which can lead to eventual fatigue failure [6]. There are two types of rotating unbalance:

A. Static Unbalance

All the unbalanced masses lie in a single plane resulting in an unbalance of single a radial force. Static unbalance can be detected by placing the shaft between two horizontal rails and allowing the shaft to naturally roll to the position at which the unbalance is below the shaft axis. Static unbalance is shown in figure.2.

B. Dynamic Unbalance

This is when the unbalanced masses lie in more than one plane. The static test will only detect the resultant force. The unbalance has to be detected by rotating the shaft and measuring the unbalance. The machine for carrying out this detection are called 'balancing machines' and consist of spring mounted bearings that support the shaft. By obtaining the amplitude and relative phase it is possible to calculate the unbalance and correct for it. This is a problem of two degrees of freedom as transitional and angular motions take place.

Classification of unbalances for a short rigid rotor is shown in figure .2.
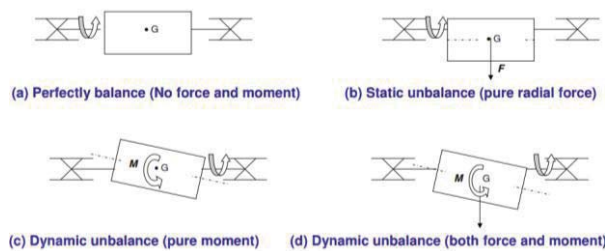

Fig.2 . Classification of unbalances for a short rigid rotor
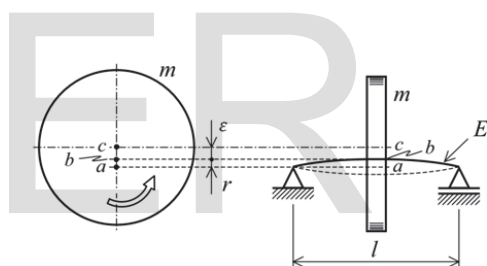
II. MATHEMATICAL MODEL

A. Equations of motion

The motion of the disc is described with four degree of freedom .Two lateral and two rotational motion

$\{U\}=\{x\ y\ \theta\ \psi\}$,where U is the nodal vector of displacements and rotations.

1) Equation Of Motion -Lateral Modes

The disc has a mass m and the centre of gravity offset from the shaft centre is defined an eccentricity ε as shown in figure 3. It is assumed that the damping of the shaft is zero

Where,

a   Intersection with the line through the points of support

b   Intersection with the axis of rotation

c   Centre of the rotating mass

The distance bc is indicated as ε and the distance ab as r, where ε is the eccentricity and r is the deflection of the beam. The equation of motion for the mass center is derived from Newton's second law

$$m\frac{d^2}{dt^2}(r+\varepsilon)=-kr \qquad (1)$$
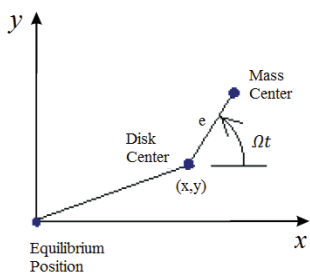
Where k is the bending stiffness of the shaft

Fig.4 The position of the mass center expressed in the x-y coordinate system

The equation of motion in x-y coordinate system, is given by,

$$m\frac{d^2}{dt^2}\left(x + \varepsilon \cos(\Omega t)\right) = -k_x x$$

$$m\frac{d^2}{dt^2}\left(x + \varepsilon \sin(\Omega t)\right) = -k_y y \tag{2}$$

Equation becomes,

$$m\ddot{x} + k_1 x = m\varepsilon\Omega^2 \cos(\Omega t)$$

$$m\ddot{y} + k_1 y = m\varepsilon\Omega^2 \sin(\Omega t) \tag{3}$$

2) Equation Of Motion- Angular Momentum

$$I_d\ddot{\theta} + I_p\Omega\dot{\Psi} + k_3\theta = 0$$

$$I_d\ddot{\Psi} - I_p\Omega\dot{\theta} + k_3\Psi = 0 \tag{4}$$

Where,

I_d -Diametric mass moment of inertia,

I_p - Polar moment of inertia,

3) The Stiffness Matrix

$$\begin{pmatrix} k_{ll} & 0 & k_{al} & 0 \\ k_{al} & 0 & k_{aa} & 0 \\ 0 & k_{ll} & 0 & k_{al} \\ 0 & k_{al} & 0 & k_{aa} \end{pmatrix} \tag{5}$$

4) Equation Of Motion For The Complete System

$$\begin{pmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_d & 0 \\ 0 & 0 & 0 & I_d \end{pmatrix}\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ \ddot{\Psi} \end{pmatrix}$$

$$-\Omega\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p \\ 0 & 0 & -I_p & 0 \end{pmatrix}\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\Psi} \end{pmatrix}$$

$$+\begin{pmatrix} k_{ll} & 0 & k_{al} & 0 \\ k_{al} & 0 & k_{aa} & 0 \\ 0 & k_{ll} & 0 & k_{al} \\ 0 & k_{al} & 0 & k_{aa} \end{pmatrix}\begin{pmatrix} x \\ y \\ \theta \\ \Psi \end{pmatrix} = 0 \tag{6}$$

By solving,

$$\omega_{1,2} = \sqrt{\frac{2k}{m}}$$

$$\omega_{3,4} = \left[\frac{I_p}{2I_d}\omega \pm \sqrt{\frac{kl^2}{2I_d} + \left(\frac{I_p}{2I_d}\omega\right)^2}\right] \tag{7}$$

Where, ω_n is the Natural frequency of the rotor

III. FINITE ELEMENT ANALYSIS

A. Case1:Modal Analysis Without Spin

This analysis is done without any rotation, the gyroscopic effects does not take place. Basic characteristic of a rotating machinery can be determined by introducing b a simple rotor model (Jeffcott rotor) shown in figure 5. The model consists of a single rigid disc centrally mounted on a uniform flexible mass less shaft supported by two identical rigid bearings at each side. The mass of the disc is 110 Kg, Specifications of the rotor system is given in the table 1.
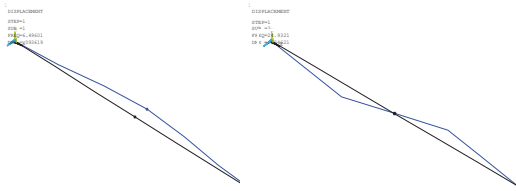

Fig.5 simple rotor model

TABLE.1.

Description of the system	
Disc: steel	*Shaft :steel*
Inner Diameter: 0.03 m	Length : 2 m
Outer diameter: 0.6 m	Diameter : 0.03 m
Thickness: 0.05 m	Modulus (E): 2.10^{11} N/m^2
Density: 7800 kg/m^3	Density: 7800 kg/m^3

The eigen frequencies obtained from the analysis is tabulated in the table 2.

TABLE 2.

Natural Frequencies (Hz)	
Ansys	*Analytical (Matlab)*
6.4960	6.5091
24.932	24.9373
137.41	138.5571

Fig

It is observed that the Ansys results are very much comparable with the analytical results obtained by Matlab. The rotor is not rotating and the modes are a pure planar motion in the z-x plane [9]. In the first mode the centrally mounted disc translate vertically in z direction .In the second mode the vertical translation is zero and the movement is pure wobbling mode of the shaft. First mode is cylindrical mode and the second mode is conical mode.

B. Case2:Analysis of Rotating Shaft

In this analysis, several set of damped Eigen frequency analysis is performed on the rotor model between speed range 0 to 1000 rpm. The rotational velocity is increased ins steps of 100 rpm. Damped modal analysis is performed and validated by comparing the result against corresponding analytical solution.

TABLE 3.

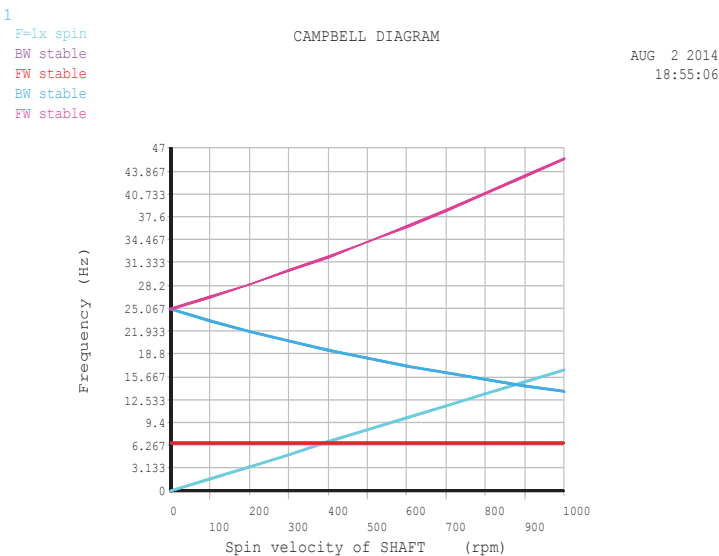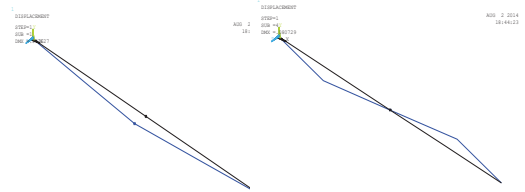| Eigen Frequencies and Critical speeds			
Natural Frequencies(Hz)		Critical speed ,RPM (slope of excitation line = 1)	
Ansys	Matlab	Critical speed	Whirl

Fig.7 Campbell diagram

Fig.8 . First and second mode shape

Since the model undergoes several rotation speed gyroscopic effects take place. The critical speeds are calculated from the Campbell diagram at the intersection of excitation line with modal frequency lines. From the Campbell diagram we can see first mode is independent of the rotational speed and does separate. This is because the disc does not have any wobbling. In the backward whirling the natural frequency is decreasing as the rotational speed increasing, In forward whirling and the natural frequency is increasing as the rotational speed is increasing. This behavior is due to the influence of the gyroscopic effect which is caused by the wobbling of the disc. The intersections of the various branches of the Campbell diagram with the y axis are the natural frequencies at standstill of the system.

The first two mode shapes shown in figure 8. The first mode shape is characterized as a cylindrical mode. The second mode shape is called a conical mode. These motions are described with four degrees of freedom, two lateral and two rotational. The mode shapes look similar to the non rotating modes, but involve a circular motion instead of planar motion, the circular motion is called whirling. Due to gyroscopic effect the frequency line splits into two. There are three intersections between the excitation line and the forward(FW)and backward whirling(BW) curves, two at 390 rpm and one at 877 rpm, where the latter is a backward whirling. The first forward mode which crosses the excitation line at 390 rpm is critical speed. The intersection at backward whirl is not considered as critical speed because backward whirling modes not give any excitation.

C. Case 3 Offset Disc

Fig.

Disc with the same mass is placed at a distance from the centre as shown in the figure 9. Analysis is performed on the model between the speed range 0 rpm to 1000 rpm.
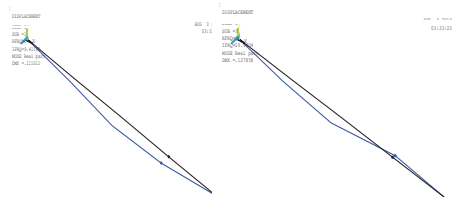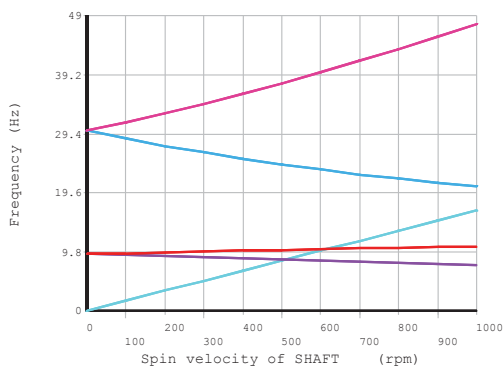
Fig.11 1

<div align="center">TABLE.4.</div>

Eigen frequency and critical speed	
Natural Frequencies (Hz)	*Critical speed(RPM)*
9.4119	509.80
29.940	616.24
63.205	-

From the Campbell diagram shown in figure10, frequency split is observed even for the first mode. That is when the disc placed off centre , which results in the wobbling for both the first and second mode shape. In the second mode the gyroscopic effect is more and as a result the frequency splits as forward and backward whirl. This is due to the fact that at zero speed, the modes of vibration are composed of a bending and a rotation one. When the system turns, these two modes still exist, but each of them is separated into two, one forward and one backward. From the equation of motion it is the coupling due to the diametric moment of inertia, that causes the splitting up of forward and backward whirl.

When the disc is placed offset natural frequencies of the system also changed. From the Campbell diagram shown in figure.8, the line of excitation intersect the frequency lines only in the first mode and only two critical speeds are observed one in forward whirl and other one in backward whirl.

D. Case4:Shaft with Two Rotor

1) Exploring Gyroscopic and Mass Effects

Considering two models, first model with the same mass at equal distance from the center and the in the second model, masses are replaced with two different masses.
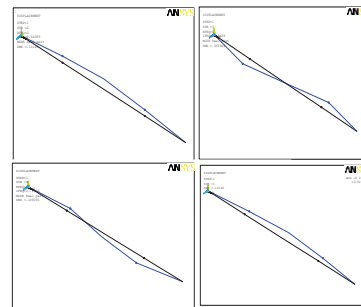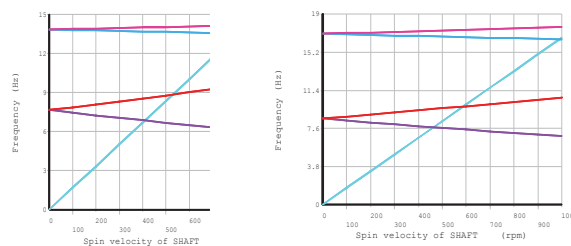

Fig.12 Two Rotor at equal distance

Fig.14 mode shapes

Figure 13 shows two sets of natural frequencies versus speed and from the natural frequencies, it can be see that the different mass model increases the first and second mode frequencies (mass is at a point of large whirling motion, mass moments of inertia changed). The reduced mass moment of inertia version (different mass) does not change the third mode (may be disk center of gravity has very little conical motion).The reduced mass moment of inertia increases the frequency of the second mode, and decreases the strength of the gyroscopic effect (disc center of gravity has substantial conical motion).

IV. RESULTS AND DISCUSSIONS

A. Non rotating Dynamics

When the machine is not spinning and the bearings have essentially no damping, in the modal test, we would find a set of natural frequencies/modes. At each frequency, the motion is planar. The ratio of bearing stiffness to shaft stiffness has a significant impact on the mode-shapes. For the soft and intermediate bearings, the shaft does not bend very much in the lower two modes. As the bearing stiffness increases (or as shaft stiffness decreases), the amount of shaft bending increases. In the first mode, the disk translates without rocking. In the second mode, it rocks without translation. This general characteristic repeats as the frequency increases. If we moved the disk off-centre, we would find that the motion is a mix of translation and rocking. This characteristic will give rise to some interesting behavior once the shaft starts rotating and is called gyroscopic effects

B. Rotating Dynamics

When shaft rotates the modes look like the non rotating modes, but involve circular motion rather than planar motion to see the effects of changing shaft speeds performed the analysis from

non spinning to a high spin speed and follow the two frequencies associated with the conical mode. The frequencies of the conical modes do change over the speed range. The backward mode drops in frequency, while the forward mode increases. The explanation for this behavior is a gyroscopic effect that occurs whenever the mode shape has an angular (conical/rocking) component. If consider forward whirl, shaft speed increases, the gyroscopic effects essentially act like an increasingly stiff spring on the central disk for the rocking motion. Increasing stiffness acts to increase the natural frequency. For backward whirl, the effect is reversed. Increasing rotor spin speed acts to reduce the effective stiffness, thus reducing the natural frequency .Also the gyroscopic terms are generally written as a skew-symmetric matrix added to the damping matrix, the net result is a stiffening or softening effect of the shaft. In the case of the cylindrical modes, very little effect of the gyroscopic terms was noted, since the centre disk was whirling without any conical motion. Without the conical motion, the gyroscopic effects do not appear. In conical type motion near the bearings a slight effect was noted.

The frequencies are affected by both the mass and diametric mass moment of inertia. The mass has the greatest effect at points of large circular motion (anti-nodes), while the mass moment of inertia has the greatest effect at points of large rocking motion (nodes).Changes in mass precisely at a node do not change the corresponding natural frequency, and changes in mass moment of inertia at points of no conical motion do not change the corresponding natural frequency.

V. CONCLUSIONS

The modes affected by the mass moment of inertia (the conical mode, for example), are strongly affected by changes in speed. Assuming the bearing characteristics do not change, the backward whirl mode will decrease in frequency with increasing shaft speed, while the forward mode frequency will increase. The extent to which this occurs is related to both the mode shape and the ratio of the polar mass moment of inertia to the diametric mass moment of inertia. Thus, a machine with a big disk/fan blade will probably show strong speed dependent effects in at least some modes. A fairly symmetric machine will probably have some modes that are relatively constant with shaft speed. A forward whirling increase the natural frequency and a backward whirling decreases the natural frequency. A forward whirling increases the stiffness of a system while backward whirling decreases the stiffness of a system. For a non rotating system ($\omega = 0$), the modes of vibration are composed of a bending and a rotation one. When the system turns, these two modes still exist, but each of them is separated into two, one forward and one backward.

Acknowledgment

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