OCR
Oxford Cambridge and RSA

Accredited

A Level Mathematics A
H240/02 Pure Mathematics and Statistics
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 2 hours

OCR supplied materials:
- Printed Answer Booklet

You must have:
- Printed Answer Booklet
- Scientific or graphical calculator

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INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r|<1$

Binomial series

$(a+b)^n = a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}b^2 + \ \ldots\ + {}^nC_r\, a^{n-r}b^r + \ \ldots\ + b^n \qquad (n \in \mathbb{N})$,

where ${}^nC_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ \ldots\ + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ \ldots \qquad \left(|x|<1,\ \ n \in \mathbb{R}\right)$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h\to 0}\dfrac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1 - \frac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \frac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x - \bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, Variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \le z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.575	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A: Pure Mathematics
Answer **all** the questions

1 Simplify fully.

(i) $\sqrt{a^3} \times \sqrt{16a}$ **[2]**

(ii) $(4b^6)^{\frac{5}{2}}$ **[2]**

2 A curve has equation $y = x^5 - 5x^4$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **[3]**

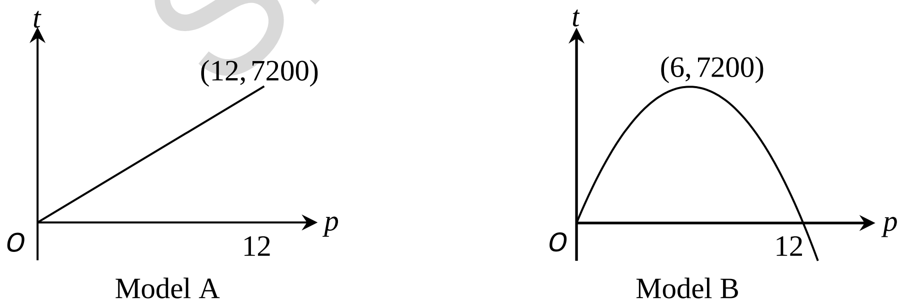
(ii) Verify that the curve has a stationary point when $x = 4$. **[2]**

(iii) Determine the nature of this stationary point. **[2]**

3 A publisher has to choose the price at which to sell a certain new book. The total profit, £t, that the publisher will make depends on the price, £p. He decides to use a model that includes the following assumptions.

- If the price is low, many copies will be sold, but the profit on each copy sold will be small, and the total profit will be small.
- If the price is high, the profit on each copy sold will be high, but few copies will be sold, and the total profit will be small.

The diagram shows the graphs of two possible models.



(i) Explain how model A is inconsistent with one of the assumptions given above. **[1]**

(ii) Given that the equation of the curve in model B is quadratic, show that this equation is of the form $t = k(12p - p^2)$, and find the value of the constant k. **[2]**

(iii) The publisher needs to make a total profit of at least £6400. Use the equation found in part **(ii)** to find the range of values within which model B suggests that the price of the book must lie. **[4]**

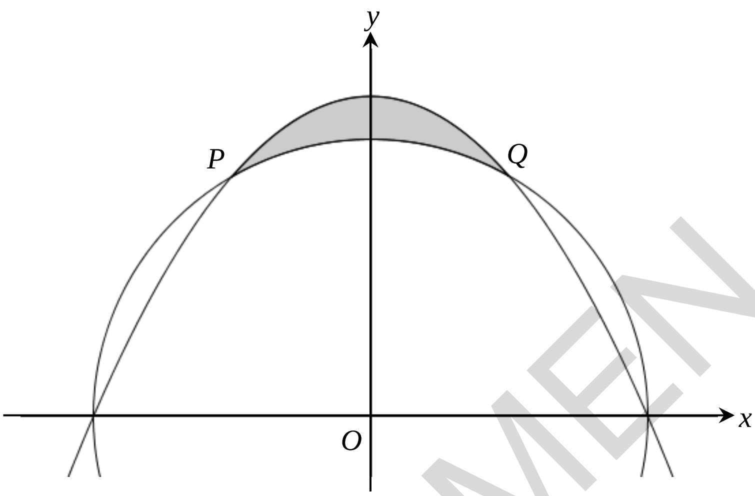
(iv) Comment briefly on how realistic model B may be in the cases $p = 0$ and $p = 12.1$. **[2]**

4 **(i)** Express $\frac{1}{(x-1)(x+2)}$ in partial fractions. **[2]**

(ii) **In this question you must show detailed reasoning.**

Hence find $\int_2^3 \frac{1}{(x-1)(x+2)}\,dx$. Give your answer in its simplest form. **[5]**

5



The circle with centre *O* and radius 2 meets the parabola $y = \frac{1}{\sqrt{3}}\left(4 - x^2\right)$ at points *P* and *Q*, as shown in the diagram.

(i) Verify that the coordinates of *Q* are $\left(1, \sqrt{3}\right)$. **[3]**

(ii) Find the **exact** area of the shaded region enclosed by the arc *PQ* of the circle and the parabola. **[8]**

6 Helga invests £4000 in a savings account. After *t* days, her investment is worth £*y*. The rate of increase of *y* is *ky*, where *k* is a constant.

(i) Write down a differential equation in terms of *t*, *y* and *k*. **[1]**

(ii) Solve your differential equation. Hence find the value of Helga's investment after *t* days. Give your answer in terms of *k*. **[4]**

It is given that $k = \frac{1}{365}\ln\left(1 + \frac{r}{100}\right)$ where *r*% is the rate of interest per annum. During the first year the rate of interest is 6% per annum.

(iii) Find the value of Helga's investment after 90 days. **[2]**

After one year (365 days), the rate of interest drops to 5% per annum.

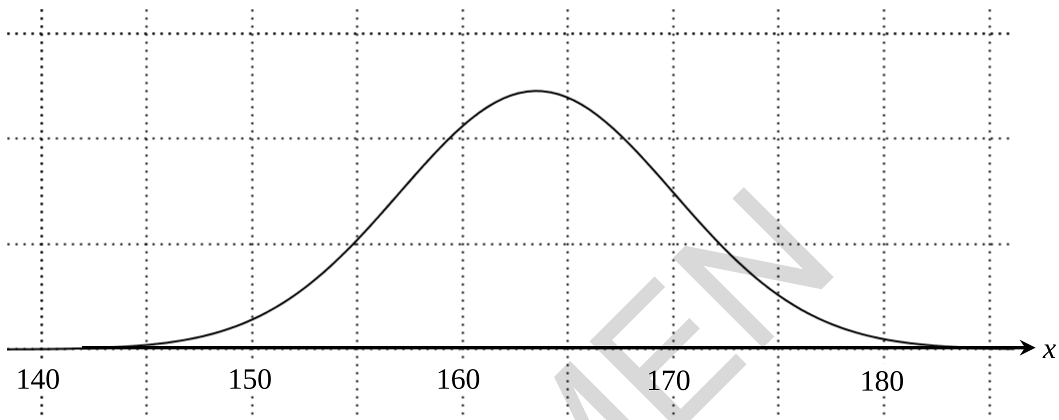
(iv) Find the total time that it will take for Helga's investment to double in value. **[5]**

Section B: Statistics

Answer **all** the questions

7 **(i)** The heights of English men aged 25 to 34 are normally distributed with mean 178 cm and standard deviation 8 cm. Three English men aged 25 to 34 are chosen at random. Find the probability that all three of them have a height less than 194 cm. **[3]**

(ii) The diagram shows the distribution of heights of Scottish women aged 25 to 34.



It is given that the distribution is approximately normal. Use the diagram in the Printed Answer Booklet to estimate the standard deviation of these heights, explaining your method. **[3]**

8 A market gardener records the masses of a random sample of 100 of this year's crop of plums. The table shows his results.

Mass, m grams	$m < 25$	$25 \le m < 35$	$35 \le m < 45$	$45 \le m < 55$	$55 \le m < 65$	$65 \le m < 75$	$m \ge 75$
Number of plums	0	3	29	36	30	2	0

(i) Explain why the normal distribution might be a reasonable model for this distribution. **[1]**

The market gardener models the distribution of masses by $N(47.5, 10^2)$.

(ii) Find the number of plums in the sample that this model would predict to have masses in the range

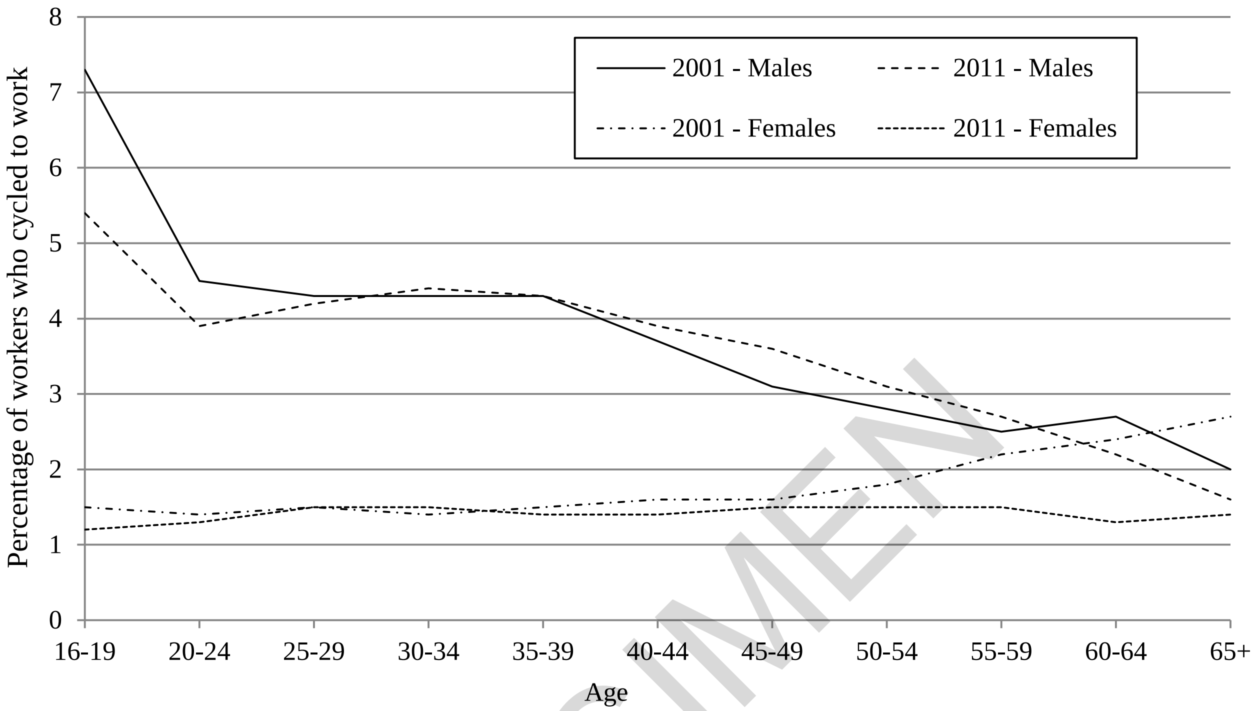
(a) $35 \le m < 45$, **[2]**

(b) $m < 25$. **[2]**

(iii) Use your answers to parts **(ii)(a)** and **(ii)(b)** to comment on the suitability of this model. **[1]**

(iv) The market gardener plans to use this model to predict the distribution of the masses of next year's crop of plums. Comment on this plan. **[1]**

9 The diagram below shows some “Cycle to work” data taken from the 2001 and 2011 UK censuses. The diagram shows the percentages, by age group, of male and female workers in England and Wales, excluding London, who cycled to work in 2001 and 2011.



The following questions refer to the workers represented by the graphs in the diagram.

(i) A researcher is going to take a sample of men and a sample of women and ask them whether or not they cycle to work. Why would it be more important to stratify the sample of men? **[1]**

(ii) A research project followed a randomly chosen large sample of the group of male workers who were aged 30-34 in 2001. Does the diagram suggest that the proportion of this group who cycled to work has increased or decreased from 2001 to 2011? Justify your answer. **[2]**

(iii) Write down one assumption that you have to make about these workers in order to draw this conclusion. **[1]**

10 In the past the time, in minutes, spent by customers in a certain library had mean 32.5 and standard deviation 8.2. Following a change of layout in the library, the mean time spent in the library by a random sample of 50 customers is found to be 34.5 minutes. Assuming that the standard deviation remains at 8.2, test at the 5% significance level whether the mean time spent by customers in the library has changed. **[7]**

11 Each of the 30 students in a class plays at least one of squash, hockey and tennis.

- 18 students play squash
- 19 students play hockey
- 17 students play tennis
- 8 students play squash and hockey
- 9 students play hockey and tennis
- 11 students play squash and tennis

(i) Find the number of students who play all three sports. **[3]**

A student is picked at random from the class.

(ii) Given that this student plays squash, find the probability that this student does not play hockey. **[1]**

Two different students are picked at random from the class, one after the other, without replacement.

(iii) Given that the first student plays squash, find the probability that the second student plays hockey. **[4]**

12 The table shows information for England and Wales, taken from the UK 2011 census.

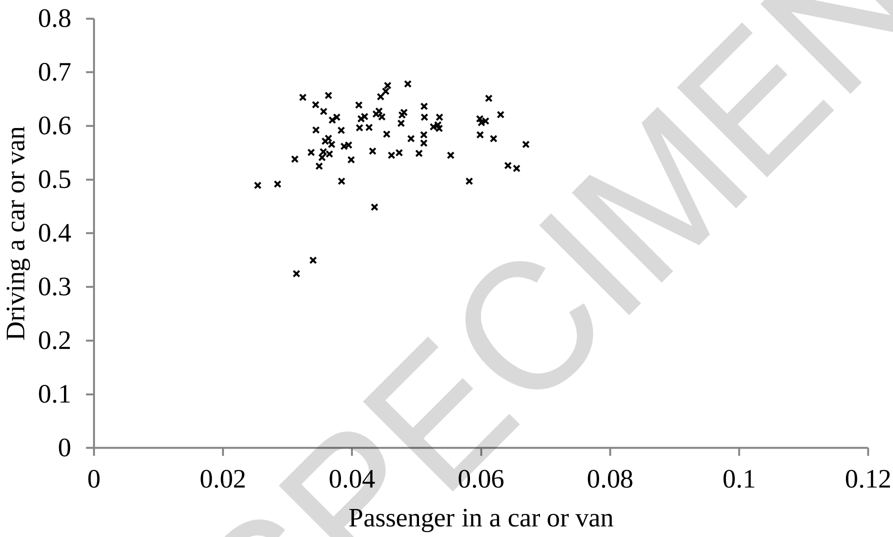
Total population	Number of children aged 5-17
56 075 912	8 473 617

A random sample of 10 000 people in another country was chosen in 2011, and the number, m, of children aged 5-17 was noted. It was found that there was evidence at the 2.5% level that the proportion of children aged 5-17 in the same year was higher than in the UK. Unfortunately, when the results were recorded the value of m was omitted. Use an appropriate normal distribution to find an estimate of the smallest possible value of m. **[5]**

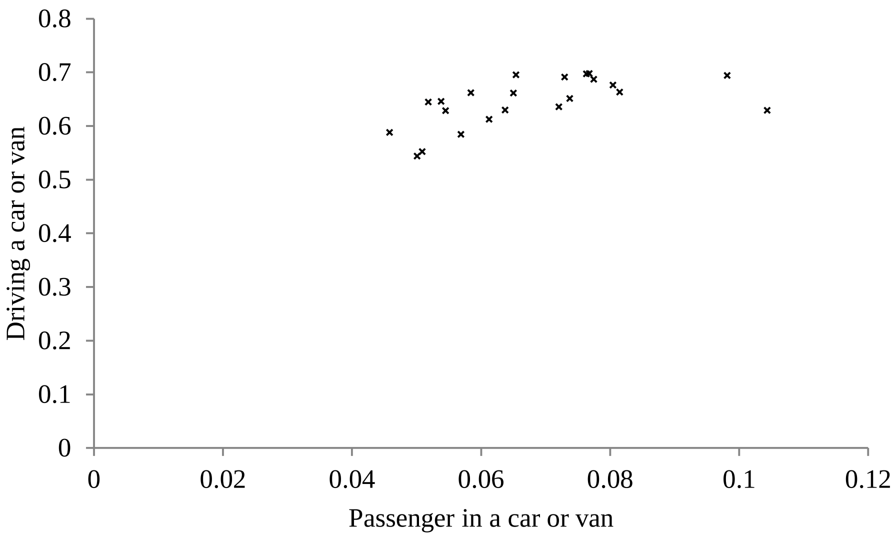
13 The table and the four scatter diagrams below show data taken from the 2011 UK census for four regions. On the scatter diagrams the names have been replaced by letters.
The table shows, for each region, the mean and standard deviation of the proportion of workers in each Local Authority who travel to work by *driving* a car or van and the proportion of workers in each Local Authority who travel to work as a *passenger* in a car or van.
Each scatter diagram shows, for each of the Local Authorities in a particular region, the proportion of workers who travel to work by *driving* a car or van and the proportion of workers who travel to work as a *passenger* in a car or van.

	Driving a car or van		Passenger in a car or van	
	Mean	Standard deviation	Mean	Standard deviation
London	0.257	0.133	0.017	0.008
South East	0.578	0.064	0.045	0.010
South West	0.580	0.084	0.049	0.007
Wales	0.644	0.045	0.068	0.015

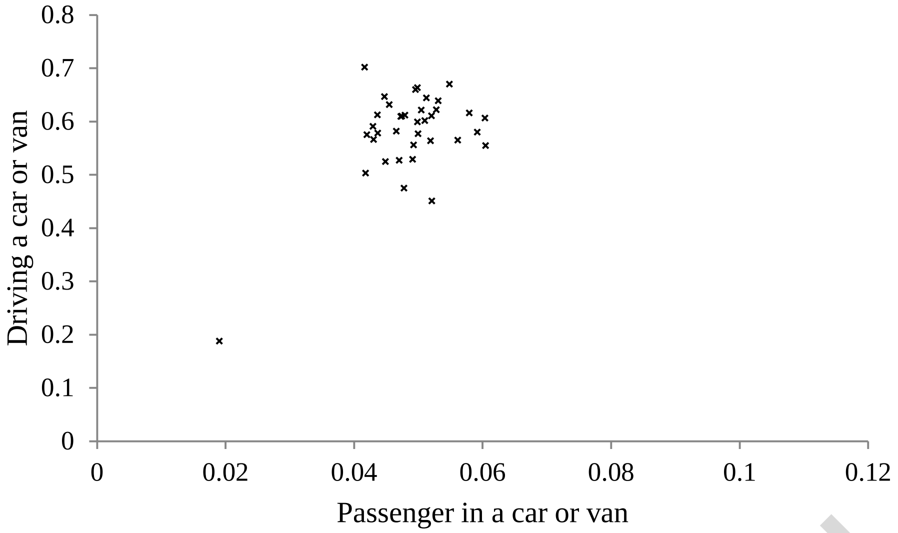
Region A



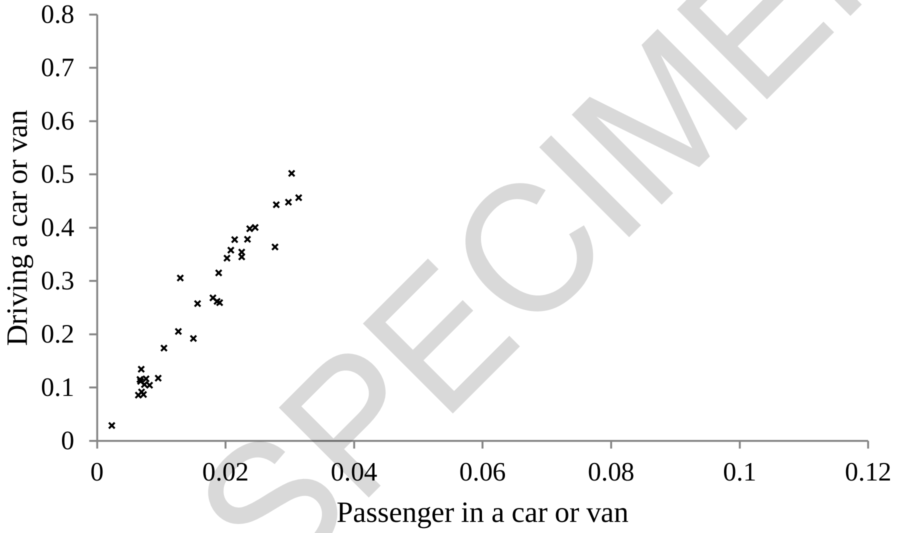
Region B



Region C



Region D



(i) Using the values given in the table, match each region to its corresponding scatter diagram, explaining your reasoning. **[3]**

(ii) Steven claims that the outlier in the scatter diagram for Region C consists of a group of small islands. Explain whether or not the data given above support his claim. **[1]**

(iii) One of the Local Authorities in Region B consists of a single large island. Explain whether or not you would expect this Local Authority to appear as an outlier in the scatter diagram for Region B. **[1]**

14 A random variable X has probability distribution given by

$$P(X = x) = \tfrac{1}{860}(1+x) \quad \text{for } x = 1, 2, 3, \ldots, 40.$$

(i) Find $P(X > 39)$. **[2]**

(ii) Given that x is even, determine $P(X < 10)$. **[6]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

SAMPLE MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

This document consists of 20 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(i)		$\sqrt{16a^4}$ or $4\sqrt{a^4}$ or $a\sqrt{a}\times 4\sqrt{a}$ $=4a^2$	M1 A1 [2]	1.1 1.1	Any correct first step	
1	(ii)		$32b^{15}$	B2 [2]	1.1 1.1	**B1** for 32 and **B1** for b^{15}	
2	(i)		$\frac{dy}{dx}=5x^4-20x^3$ oe $\frac{d^2y}{dx^2}=20x^3-60x^2$ oe	M1 A1 A1FT [3]	1.1a 1.1 1.1	For attempt at differentiation FT their $\frac{dy}{dx}$	Both indices decrease
2	(ii)		When $x=4$, $\frac{dy}{dx}=5x^4-20x^3=5\times4^4-20\times4^3$ = 0 hence there is a stationary point	M1 A1 [2]	1.1 2.1	Substitute into their $\frac{dy}{dx}$	
2	(iii)		When $x=4$, $\frac{d^2y}{dx^2}=20x^3-60x^2=20\times4^3-60\times4^2$ >0 hence the stationary point is a minimum	M1 E1FT [2]	1.1 2.2a	FT from their $\frac{d^2y}{dx^2}$ in part **(i)**	

Question			Answer	Marks	AO	Guidance	
3	**(i)**		Total profit (or *t*) is large when price (or *p*) is high	**B1** **[1]**	**3.5b**		
3	**(ii)**		Passes through (0, 0) and (12, 0) hence $t = kp(12 - p)$ $k = 200$	**B1** **B1** **[2]**	**3.1b** **3.3**	Or $t = 200p(12 - p)$ Or $t = 200(12p - p^2)$	
3	**(iii)**		$6400 = 200p(12 - p)$ oe $p^2 - 12x + 32 = 0$ $p = 4$, $p = 8$ $4 \le p \le 8$ Price must be between £4 and £8	**M1** **A1FT** **A1FT** **A1** **[4]**	**3.4** **1.1** **1.1** **3.4**	$6400 = (\text{their } k)\, p(12 - p)$ Any correct equation in form $ap^2 + bp + c = 0$ BC, but any method allowed Allow $4 < p < 8$	 FT **(ii)** FT **(ii)**
3	**(iv)**		E.g. $p = 0$ implies giving book for free. Unrealistic. oe E.g. When $p = 0$, $t = 0$; but *t* should be negative as would make a loss. Unrealistic. oe E.g. When $p = 12.1$, *t* is negative. Possibly realistic as could make a loss if *p* set too high. oe	**E1** **E1** **[2]**	**3.2b** **3.2b**	Valid comment about $p = 0$ Valid comment about $p = 12.1$	

Question			Answer	Marks	AO	Guidance	
4	(i)		$\frac{1}{(x-1)(x-2)}=\frac{A}{x-1}+\frac{B}{x+2}$ so $A(x+2)+B(x-1)=1$	M1	1.1	Attempt partial fractions with linear denominators, any method	
			so $A=\frac{1}{3}$ and $B=-\frac{1}{3}$ $\frac{\frac{1}{3}}{x-1}-\frac{\frac{1}{3}}{x+2}$ oe	A1	1.1		
				[2]			
	(ii)		**DR**	M1	1.2	Attempt integration using ln	Must be seen
			$\int_2^3 \frac{1}{(x-1)(x+2)}\,dx$ $=\left[\frac{1}{3}\ln(x-1)-\frac{1}{3}\ln(x+2)\right]_2^3$	A1FT	1.1	Correct integral in any equivalent form. FT their $A\ln(x-1)+B\ln(x+2)$	May have no limits at this stage
				M1	1.1a	Attempt to substitute 3 and 2 in their integral and subtract	Must be seen
			$=\frac{1}{3}(\ln 2-\ln 5-\ln 1+\ln 4)$	A1	1.1	All correct in any equivalent form	
			$=\frac{1}{3}\ln\frac{8}{5}$ or $\ln\sqrt[3]{\frac{8}{5}}$	A1	1.1	isw; must include one ln only	
				[5]			
5	(i)		$x^2+y^2=4$	B1	1.1	soi	
			When $x=1$ $1+y^2=4\Rightarrow y=\sqrt{3}$	E1	2.1	AG Check that Q lies on the circle	**OR** **B1** $x^2+\left(\sqrt{3}\right)^2=4\Rightarrow x=1$
			$y=\frac{1}{\sqrt{3}}(4-1)\Rightarrow y=\sqrt{3}$	E1	2.1	AG Check that Q lies on the parabola	**B1** $\sqrt{3}=\frac{1}{\sqrt{3}}\left(4-x^2\right)\Rightarrow x=1$
				[3]			

Question			Answer	Marks	AO	Guidance	
5	(ii)		$\frac{1}{\sqrt{3}}\int_{-1}^{1}\left(4-x^2\right)\mathrm{d}x$	M1	3.1a	Attempt correct integral and limits; may be implied by answer 4.23(39…)	**OR M1** $\frac{1}{\sqrt{3}}\int_{0}^{1}\left(4-x^2\right)\mathrm{d}x=$ 2.1169…
			$=\frac{22\sqrt{3}}{9}$	A1	1.1	BC	**A1** $=\frac{11\sqrt{3}}{9}$
			Let N be the point $(1, 0)$ Area $OQN=\frac{\sqrt{3}}{2}$ oe or 0.866 (3 s.f.)	B1	2.1		**OR** **B1** semi-circle: $y=\sqrt{4-x^2}$
			$QON=\tan^{-1}\sqrt{3}$	M1	3.1a	Or $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ or $\cos^{-1}\left(\frac{1}{2}\right)$ or $\frac{1}{3}\pi$ or 60°	**M1** attempt $\int_{-1}^{1}\sqrt{4-x^2}\mathrm{d}x$ by substitution, e.g. $x=2\sin u$
			$POQ=\frac{1}{3}\pi$ or 60°	A1	1.1	**M1A1** may be implied by seeing next line	**M1** Use trigonometric identity e.g. $\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi}4\cos^2u\,\mathrm{d}u=\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi}a\cos2u+b\,\mathrm{d}u$
			Area sector $POQ=\frac{1}{2}\times2^2\times\frac{1}{3}\pi$ oe $\left(=\frac{2}{3}\pi\right.$ oe or 2.09 (3 s.f.)$\left.\right)$	M1	1.1	FT their angle POQ	**A1** $\frac{2}{3}\pi+\sqrt{3}$
			Shaded area $=\frac{22\sqrt{3}}{9}-2\times\frac{\sqrt{3}}{2}-\frac{2}{3}\pi$ oe	M1	3.2a	Correct combination of their areas	**M1** Shaded area $=\frac{22\sqrt{3}}{9}-\frac{2}{3}\pi-\sqrt{3}$ oe
			$=\frac{13\sqrt{3}}{9}-\frac{2}{3}\pi$ oe	A1	1.1		**A1** $=\frac{13\sqrt{3}}{9}-\frac{2}{3}\pi$ oe
				[8]			

Question			Answer	Marks	AO	Guidance	
6	(i)		$\frac{\mathrm{d}y}{\mathrm{d}t} = ky$	B1 [1]	3.1b		
6	(ii)		$\frac{\mathrm{d}y}{y} = k\mathrm{d}t$	M1	1.1a	Attempt separation of variables	
			$[\ln y]_{4000}^{y} = k[t]_0^t$ or $\ln y = kt + c$	M1	1.1	Correct integrals and limits	
			$\ln \frac{y}{4000} = kt$ or $\ln 4000 = 0 + c$	A1	1.1	Correct substitution in correct integral	
			$y = 4000\mathrm{e}^{kt}$	A1 [4]	1.1		
6	(iii)		$4000\mathrm{e}^{\frac{90}{365}\ln 1.06}$	M1	1.1	FT their part (ii)	
			$= 4057.89$	A1 [2]	1.1	BC	
6	(iv)		After 1 year, increased by factor 1.06 Require further increase by factor $\frac{2}{1.06}$	M1	3.1b	May be implied	
				M1	1.1	Attempt to form equation with 1.05 and 1.06	
			$\mathrm{e}^{\frac{t}{365}\ln 1.05} = \frac{2}{1.06}$	A1	2.1	Correct equation	
			$\frac{t}{365}\ln 1.05 = \ln\frac{2}{1.06}$ $t = \frac{365}{\ln 1.05} \times \ln\frac{2}{1.06}$ $= 4750$	M1	1.1	Attempt to remove logs	**OR** BC
			Total number of days $= 5115$	A1 [5]	3.2a	isw	

Question			Answer	Marks	AO	Guidance	
7	(i)		$N(178, 8^2)$ and $X < 194$ oe $P(X < 194) = 0.977(249868...)$ $0.977249868...^3 = 0.933$ (3 s.f.)	M1 A1 A1 [3]	1.1 1.1 1.1	soi BC	
7	(ii)		E.g. $\lvert\text{inflection} - \text{mean}\rvert$ E.g. $\frac{1}{2}$ (97.5th percentile – mean) E.g. $\frac{1}{6}$ (99.7th percentile – 0.3th percentile) = 6 to 7 E.g. Point of inflection is 1 sd from mean E.g. 95% of values within (approx) 2 sds of mean E.g. Amost all within (approx) 3 sds of mean	M1 A1 E1 [3]	1.1a 1.1 2.4	E.g. $170 - 163$ E.g. $\frac{1}{2}(176 - 163)$ E.g. $\frac{1}{6}(183 - 145)$ Statement matching method used	Figures are illustrative only
8	(i)		Symmetrical, high in middle, tails off at ends	B1 [1]	2.4	Any two of these	Not just bell shaped
8	(ii)	(a)	$P(35 < m < 45) = 0.296$ Predicted no. $= 30$	M1 A1 [2]	3.4 1.1	Correct probability attempted Allow 29.6 or ‘29 or 30’	
8	(ii)	(b)	$P(m < 25) = 0.0122$ Predicted no. $= 1$	M1 A1 [2]	3.4 1.1	Correct probability attempted Allow 1.2 or ‘1 or 2’	
8	(iii)		29.6 close to 29 and 1.2 close to 0 Hence model (could be) suitable	B1 [1]	3.5a	Both needed	**OR B1** Model predicts some masses below 25 g, hence not suitable
8	(iv)		E.g. Weather may cause different distribution	B1 [1]	3.5b	Any sensible reason why next year may be different	

Question			Answer	Marks	AO	Guidance	
9	(i)		e.g. From the data given, the proportions of men who cycle to work show much more variability than women, with greater proportions of younger men cycling than older men.	**E1** **[1]**	**2.4**		
9	(ii)		The proportion decreased e.g. These workers were in the 40-44 group in 2011, which is a smaller proportion of the population than the 30-34 group in 2001.	**B1** **B1** **[2]**	**2.2a** **2.2b**		
9	(iii)		e.g. The age group is still approximately the same size in 2011 Very few (or no) males in this age group join the workforce between 2001 and 2011 Very few (or no) males in this age group leave the workforce between 2001 and 2011 The overall size of the workforce in this age group has not changed much The sample is representative of the whole population	**B1** **[1]**	**2.2b**	For any relevant assumption	

Question			Answer	Marks	AO	Guidance	
10			$H_0 : \mu = 32.5$	**B1**	**1.1**	Must be stated in terms of parameter values	
			$H_1 : \mu \neq 32.5$ where μ is mean time spent by all customers	**B1**	**2.5**	**B1B0** for one error, e.g. undefined μ or 1-tail	Use of 34.5 **B0B0**
			$X \sim N\left(32.5, \frac{8.2^2}{50}\right)$ and $X > 34.5$	**M1**	**3.3**	Stated or implied	**OR** **M1** $\frac{34.5-32.5}{8.2 \div \sqrt{50}}$ allow without square root
			$P(X > 34.5) = 0.0423$	**A1**	**3.4**	BC	**A1** $= 1.725$
			Comparison with 0.025	**A1**	**1.1**	Allow comparison with 0.05 if $H_1 : \mu > 32.5$	**A1** Comparison with 1.96 (allow comparison with 1.645 if $H_1 : \mu > 32.5$)
			Do not reject H_0	**M1**	**1.1**		
			Insufficient evidence that mean time in the library has changed	**A1FT**	**2.2b**	In context, not definite; FT their 0.0423, but not comparison with 0.05	FT their 1.725, but not comparison with 1.645
				[7]			

Question			Answer	Marks	AO	Guidance	
11	(i)		Attempt to represent information e.g. by Venn diagram with x in centre and 3 other correct values in terms of x	B1	3.3	Any equivalent method	**OR** **B1** $\frac{18}{30}+\frac{19}{30}+\frac{17}{30}-\left(\frac{8}{30}+\frac{9}{30}+\frac{11}{30}\right)\left(=\frac{26}{30}\right)$
			Attempt total (in terms of x) $=30$	M1	3.4		**M1** $1-"\frac{26}{30}"\left(=\frac{4}{30}\right)$
			$x=4$ so $n(\mathrm{S}\cap\mathrm{H}\cap\mathrm{T})=4$	E1	1.1	Or the number doing all three is 4. **E0** for just $x=4$	
				[3]			
11	(ii)		$\frac{5}{9}$ oe	B1FT	2.2a	FT their **(i)**	
				[1]			
11	(iii)		$\frac{5}{9}\times\frac{19}{29}$	B1	2.2a		
			$\frac{4}{9}\times\frac{18}{29}$	B1	2.2a		
			$\frac{5}{9}\times\frac{19}{29}+\frac{4}{9}\times\frac{18}{29}$	M1	2.2a	All correct	
			$=\frac{167}{261}$ oe or 0.640 (3 s.f.)	A1	1.1		
				[4]			

Question			Answer	Marks	AO	Guidance	
12			$p = 0.1511$ to 4 s.f.	**B1**	**3.1b**		**OR** **B1** $p = 0.1511$ to 4 s.f.
			$X \sim \text{Bin}(10000, 0.1511)$	**M1**	**3.3**	soi	**B1** $X \sim N(1511, 1283^2)$
			$np = 1511$ $np(1-p) = 1283$			Both; allow 3 s.f.	
			$1511 + 1.96 \times \sqrt{1283}$ (or $1511 + 2 \times \sqrt{1283}$)	**M1**	**3.4**	$\text{their}'np' + 2 \times \sqrt{\text{their}'np(1-p)'}$ or $\text{their}'np' + 1.96 \times \sqrt{\text{their}'np(1-p)'}$	**M1** P(X < *m*) = 0.975 Then use inverse normal to find…
			=1581 (or 1583)	**A1 FT**	**1.1**	FT *their* 3sf or better values	**A1 FT** 1581.203931… BC
			Minimum *m* is 1581	**A1**	**1.1**	Conclusion in context Allow 1580 to 1585	**A1** Minimum *m* is 1581
				[5]			

Question			Answer	Marks	AO	Guidance	
13	(i)		E.g. The only region with very low location on both variables is Region D which is therefore London.	**E1**	**2.2a**	Or any other valid reason to connect Region D with London	**OR E1** for one region correct with good reasoning
			E.g. The region with the lowest standard deviation is Region B, so this is Wales	**E1**	**2.2a**	Or any other valid reason to connect Region B with Wales	**OR E2** for two regions correct with good reasoning
			E.g. The only value where the other two differ much is sd of *driving*; the wider spread on Region C including the outlier suggests that this is the Southwest, so Region A is the South East.	**E1** **[3]**	**2.2b**	Careful argument involving mean and/or standard deviation	
13	(ii)		E.g. No the data only shows that this LA has low proportions of car use for travelling to work. E.g. No, many LAs in Region D (London) have similar proportions and they are not small islands.	**E1** **[1]**	**2.2b**	Or any other valid explanation of why the data given is insufficient to draw this conclusion	Identifying the LA as the Scilly Isles is not relevant; this requires information that is not in the supplied data.
13	(iii)		E.g. On a large island, methods of travel to work are unlikely to be different to any other LA; people will still be travelling to work on the roads, and provision of public transport will be similar to any other LA.	**E1** **[1]**	**2.2b**	Or any other valid explanation of how large islands are likely to have similar patterns of method of travel to other LAs	Candidates may, but need not, identify the LA as Anglesey, but this is not sufficient to award the mark

Question			Answer	Marks	AO	Guidance	
14	(i)		$P(X>39)=P(X=40) = \frac{1}{860}(1+40)$	M1	1.1	Attempt at evaluating $P(X=40)$	
			$=\frac{41}{860}$	A1	1.1		
				[2]			
14	(ii)		$P(X \text{ even})=\frac{1}{860}(20+(2+4+6+...+40))$ oe	M1	3.1a	Attempt Σ probabilities of all even values	Numerical sums may be evaluated BC throughout
			$=\frac{1}{860}\left(20+\frac{2+40}{2}\times 20\right)$	A1	1.1	Correct expression	
			$=\frac{22}{43}$	A1	1.1		
			$P(X=2,4,6,8)=\frac{1}{860}(4+2+4+6+8)$ $=\frac{12}{430}$ oe	M1	1.1	Attempt Σ probabilities for $X=2,4,6,8$	
			$\frac{P(X=2,4,6,8 \text{ and } X \text{ even})}{P(X \text{ even})}=\frac{P(X=2,4,6,8)}{P(X \text{ even})}$ $=\frac{12}{430}\div\frac{22}{43}=\frac{3}{55}$ oe or 0.0545 (3 s.f.)	A1	3.2a	$\frac{\text{their } P(X=2,4,6,8)}{\text{their } P(X \text{ even})}$	
				B1	2.1	For a clear solution allowing the line of reasoning to be followed, with each component of the conditional probability found clearly	
				[6]			

Assessment Objectives (AO) Grid

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1(i)	2				**2**
1(ii)	2				**2**
2(i)	3				**3**
2(ii)	1	1			**2**
2(iii)	1	1			**2**
3(i)				1	**1**
3(ii)			1	1	**2**
3(iii)	2			2	**4**
3(iv)			2		**2**
4(i)	2				**2**
4(ii)	5				**5**
5(i)	1	2			**3**
5(ii)	4	1	3		**8**
6(i)			1		**1**
6(ii)	4				**4**
6(iii)	2				**2**
6(iv)	2	1	2		**5**
7(i)	3				**3**
7(ii)	2	1			**3**
8(i)		1			**1**
8(ii)(a)	1			1	**2**
8(ii)(b)	1			1	**2**
8(iii)				1	**1**
8(iv)				1	**1**
9(i)		1			**1**
9(ii)		2			**2**
9(iii)		1			**1**
10	3	2		2	**7**
11(i)	1			2	**3**
11(ii)		1			**1**
11(iii)	1	3			**4**
12	2		1	2	**5**
13(i)		3			**3**
13(ii)		1			**1**
13(iii)		1			**1**
14(i)	2				**2**
14(ii)	3	1	2		**6**
Totals	**50**	**24**	**12**	**14**	**100**

PS = Problem Solving
M = Modelling

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